Title: ``Topological String on genus one fibered Calabi-Yau 3-folds and string dualities"

Abstract:

We calculate the generating functions of BPS indices using their modular properties in Type II and M-theory compactifications on compact genus one fibered CY 3-folds with singular fibers and additional rational sections or just \$N\$-sections, in order to study string dualities in four and five imensions as well as rigid limits in which gravity decouples.

The generating functions are Jacobi-forms of $\Lambda = 1(N)$ with the complexified fiber volume as modular parameter.

The string coupling \$\lambda\$, or the \$\epsilon_\pm\$ parameters in the rigid limit, as well as the masses of charged hypermultiplets and non-Abelian gauge bosons are elliptic parameters. To understand this structure, we show that specific auto-equivalences act on the category of topological B-branes on these geometries and generate an action of \$\Gamma_1(N)\$ on the stringy K\"ahler moduli space. We argue that these actions can always be expressed in terms of the generic Seidel-Thomas twist with respect to the 6-brane together with shifts of the B-field and are thus monodromies.

This implies the elliptic transformation law that is satisfied by the generating functions. We use Higgs transitions in F-theory to extend the ansatz for the modular bootstrap to genus one fibrations with N-sections and boundary conditions fix the all genus generating functions for small base degrees completely. This allows us to study in depth a wide range of new, non-perturbative theories, which are Type II theory duals to the CHL Λ and Z_N orbifolds of the heterotic string on $K3 \times T_2$. In particular, we compare the BPS degeneracies in the large base limit to the perturbative heterotic one-loop amplitude with $R_+2F_+{2g-2}$ insertions for many new Type II geometries. In the rigid limit we can refine the ansatz and obtain the elliptic genus of superconformal theories in 5d.