Title: Stable higher rank flag sheaves on surfaces

Abstract: We study moduli space of holomorphic triples f: E_{1}—> E_{2}, composed of (possibly rank > 1) torsion-free sheaves (E_{1}, E_{2}) and a holomorphic map between them, over a smooth complex projective surface S. The triples are equipped with Schmitt stability condition. We prove that when Schmitt stability parameter becomes sufficiently large, the moduli space of triples benefits from having a perfect relative and absolute obstruction theory in some cases (depending on rank of holomorphic torsion-free sheaf E_{1}). We further generalize our construction to higher-length flags of higher rank sheaves by gluing triple moduli spaces, and extend the earlier work, with Gholampur and Yau, where the obstruction theory of nested Hilbert schemes over the surface was studied. Here we extend the earlier results to the moduli space of flags E_{1}——>E_{2}—> …——> E_{n}, where the maps are injective (by stability). There is a connection, by wallcrossing, between the theory of such higher rank flags, and the theory of Higgs pairs on the surface, which provides the means to relate the flag invariants to the local DT invariants of threefold given by a line bundle over the surface, X :=Tot(L —> S).