TIME-REVERSAL-INVARIANT TOPOLOGICAL SUPERCONDUCTORS: PROPOSALS AND SIGNATURES

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Argentina

ICTP- 2019 -

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- William Herrera, Colombia
- Alfredo Levy Yeyati, Madrid
- Felix von Oppen, Berlin

CONICET

Ministerio de Ciencia, Tecnología e Innovación Productiva
Presidencia de la Nación

Alexander von Humboldt
Stiftung / Foundation

TRITOPS

BCS Hamiltonian- fermions with spin

$$H_{BCS} = \sum_k \Psi_k^\dagger H_{BdG}(k) \Psi_k$$

$$\left(\psi_{k,\uparrow} \; \psi_{k,\downarrow} \; \psi^\dagger_{-k,\downarrow} \; -\psi^\dagger_{-k,\uparrow} \right)^t$$

Particle-hole symmetry $\{H_{BdG}, \Xi\} = 0$

Time-reversal symmetry $[H_{BdG}, \Theta] = 0$

$$\Pi = \Theta\Xi \qquad \{\mathcal{H}_{BdG}, \Pi\} = 0,$$

$$\Theta^2 = 0, \pm 1, \quad \Xi^2 = \pm 1$$

$$\Xi^2 = 1$$

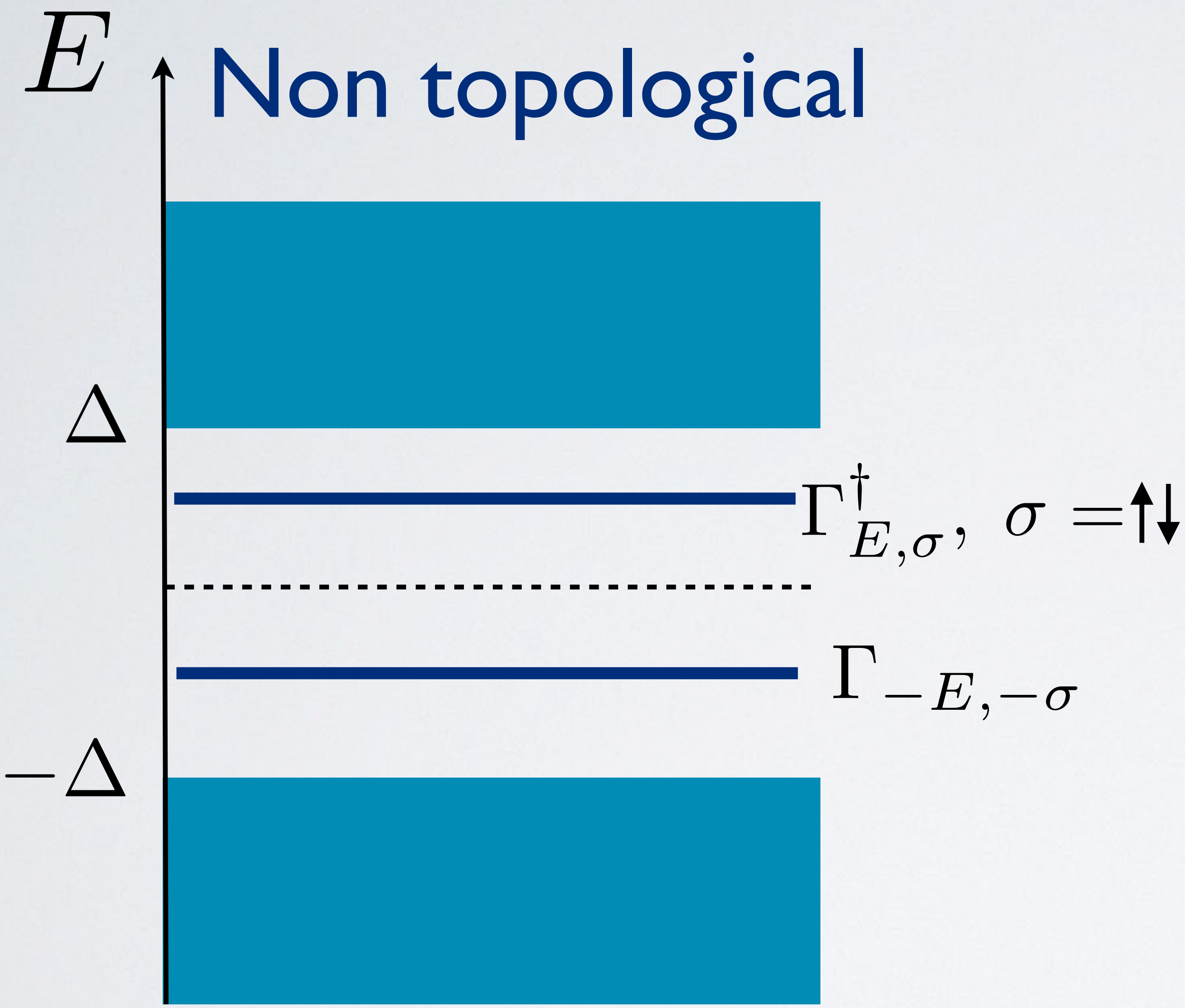
$$\Theta^2 = -1$$

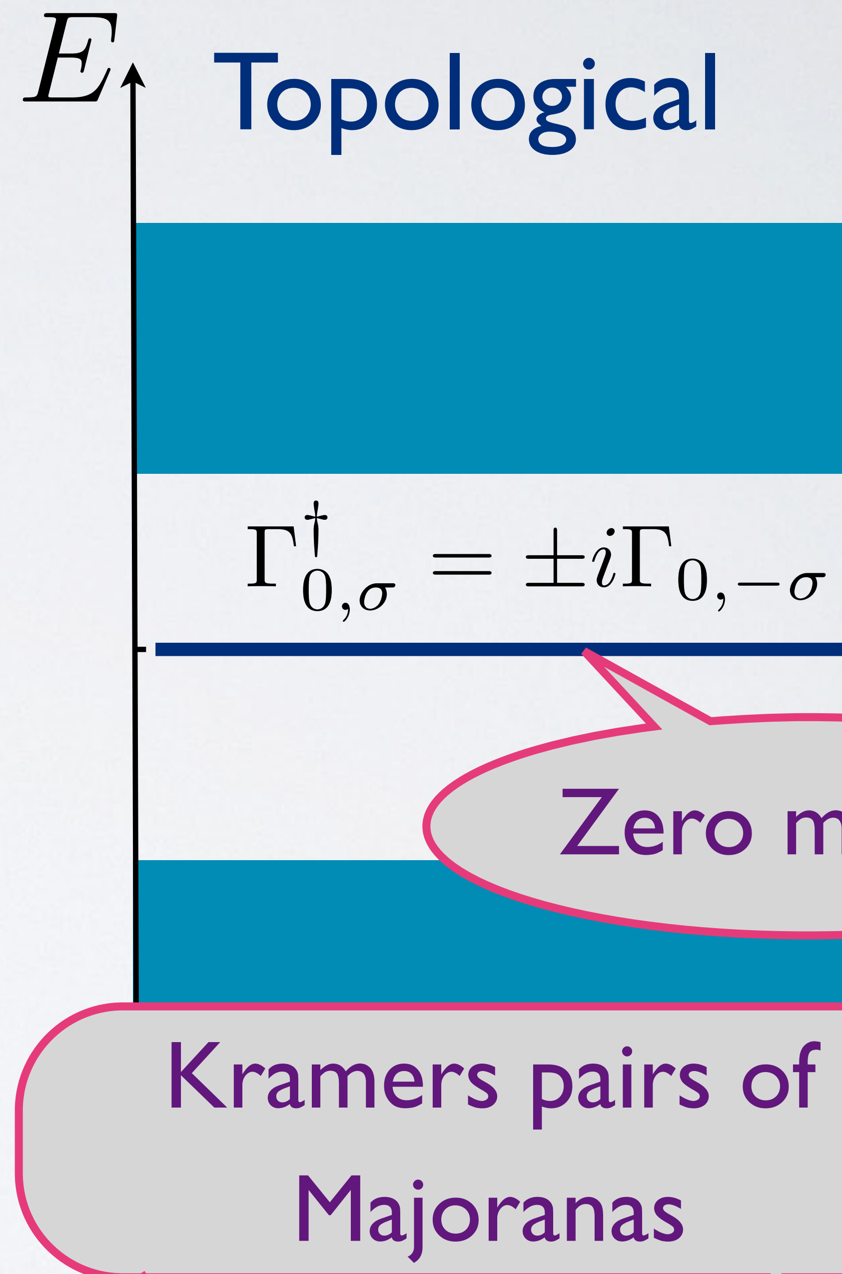
Class DIII

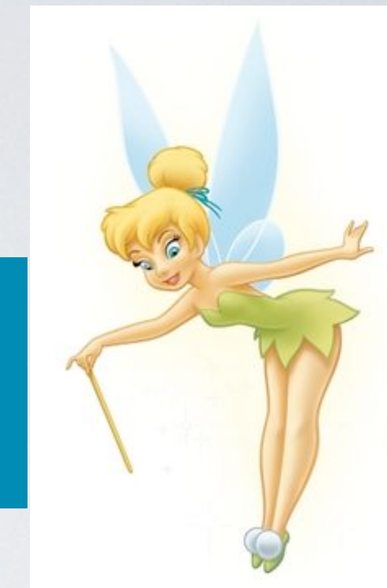
Altland, Zirnbauer, Phys, Rev. B 55,1145 (1997)

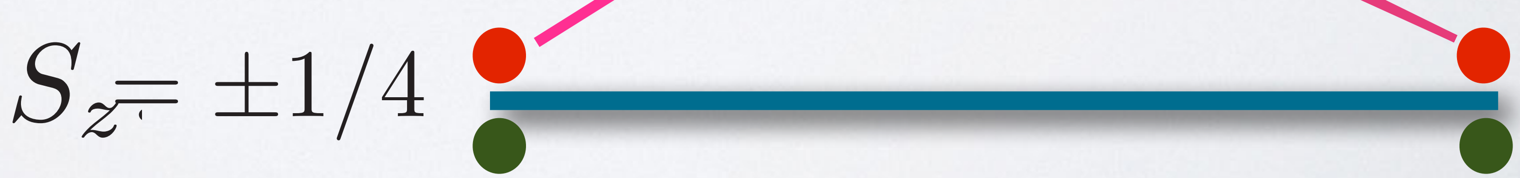
A. Schnyder, et al, Phys. Rev. B 78, 195125 (2008)

NORMAL SUPERCONDUCTIVITY vs TRITOPS (1D)









Zero modes have fractional spin!

HISTORICAL NOTE

MAJORANA IN ARGENTINA?

futuro

SÁBADO, 31 DE MAYO DE 2008

NOTA DE TAPA

La pista argentina

Bajo un manto de dudas subyace la historia, la parábola sobre la biografía de Ettore Majorana; quizás (quizás, quizás, eso al menos decía Fermi) uno de los grandes científicos de nuestra época (se anticipó al esbozo de la Teoría del Núcleo Atómico de Heisenberg que dio lugar al descubrimiento del neutrón) y que un buen día se esfumó por completo. Y bueno, hay malas o buenas lenguas que dicen que anduvo por aquí, allá por 1950.

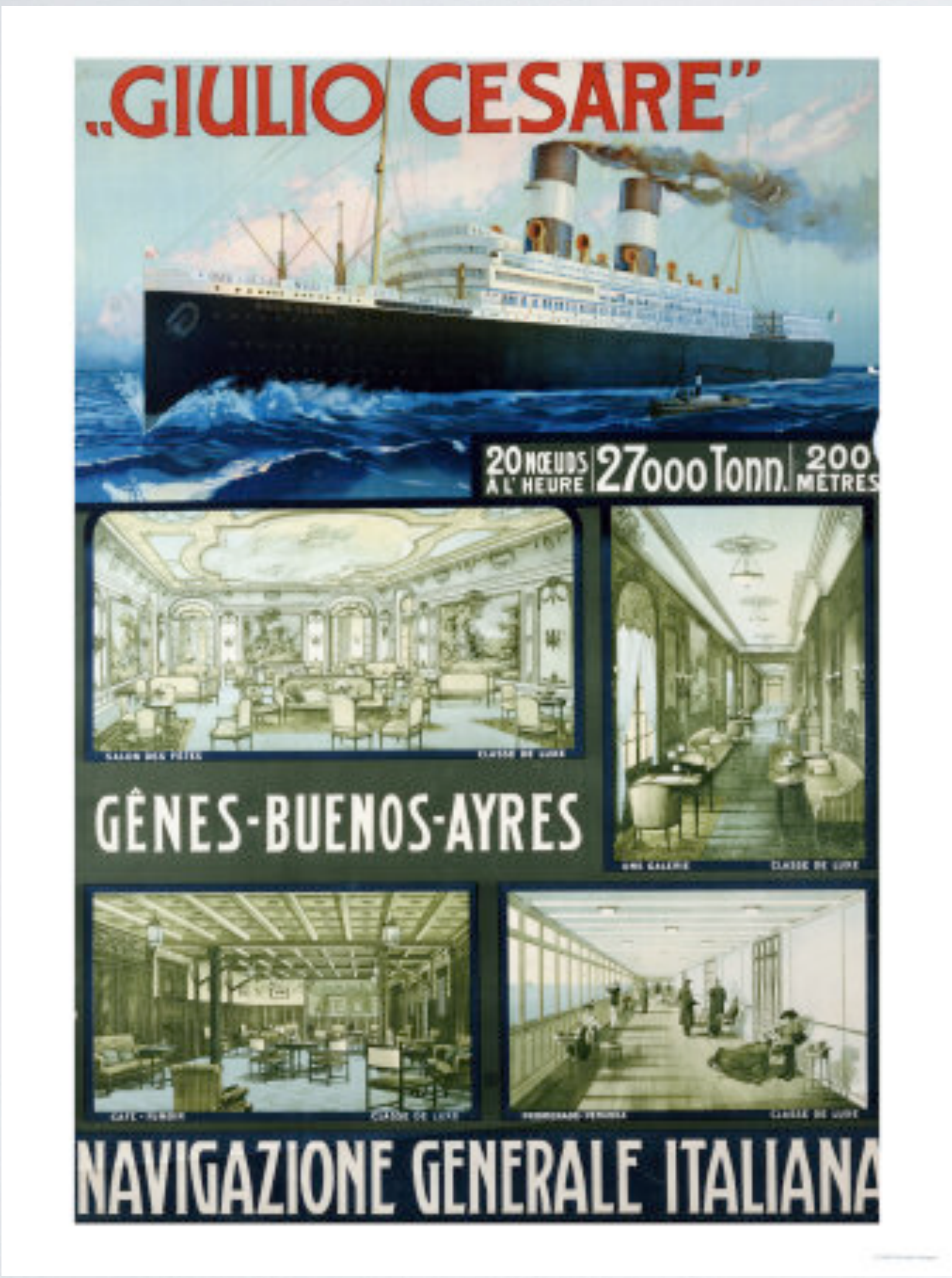
Por Matías Alinovi

Ettore Majorana siempre vuelve. En el suplemento Radar del 23 de marzo pasado, Juan Forn comentó la reedición de Tusquets de La desaparición de Majorana, libro de Leonardo Sciascia. Se refería también a la pista argentina sobre la desaparición del físico italiano, aunque de un modo lateral

FOTOGRAFIA FECHADA EL 3 DE NOVIEMBRE DE 1923, TOMADA DE SU LIBRETA UNIVERSITARIA.

MAJORANA'S ROUTE (1938) ?

NAPOLES-GENOVA-BUENOS AIRES?



EXAMPLES OF TRIPTOPS

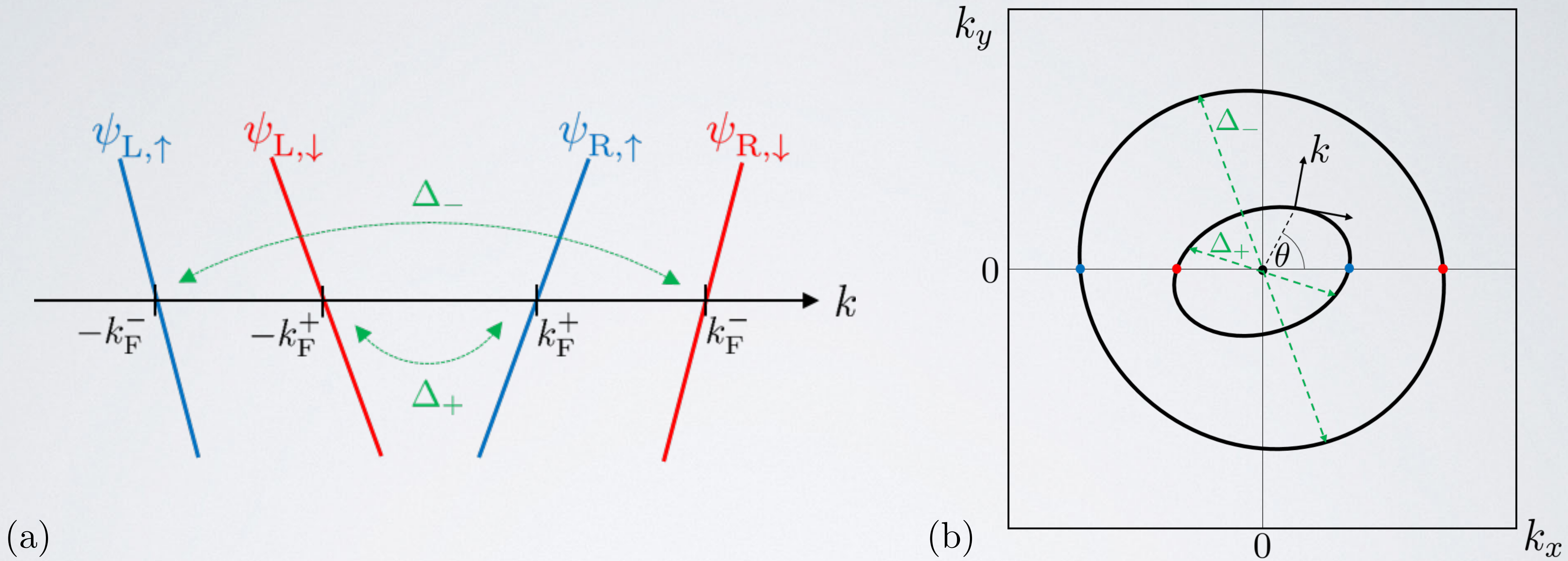
Time-reversal-invariant topological superconductivity

Arbel Haim[a], and Yuval Oreg[b]

[a] *Walter Burke Institute for Theoretical Physics and Institute for Quantum Information and Matter, California Institute of Technology, Pasadena, CA 91125, USA*
[b] *Department of Condensed Matter Physics, Weizmann Institute of Science, Rehovot 7610001, Israel*

Review Article, arXiv: 1809.06863

MINIMAL INGREDIENTS



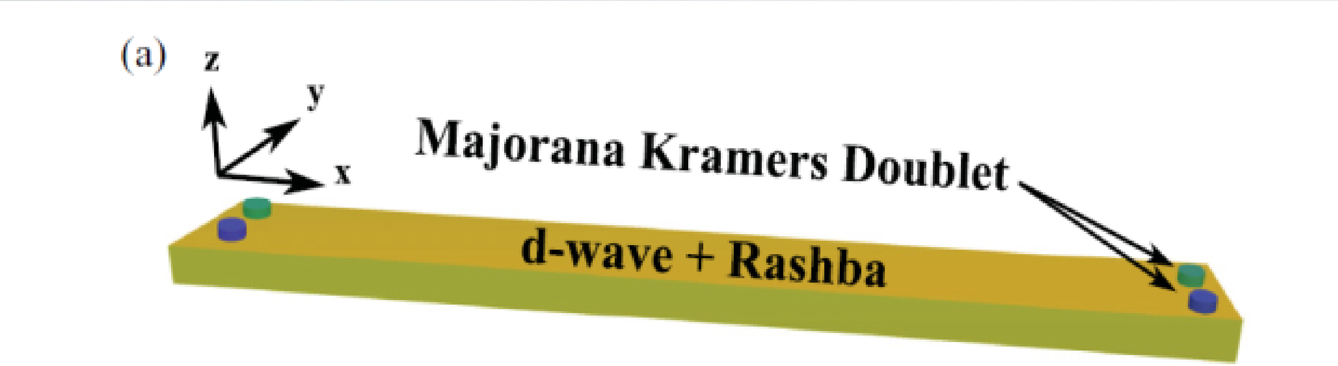
Topological invariant

$$\nu = \text{sgn}(\Delta_+)\text{sgn}(\Delta_-)$$

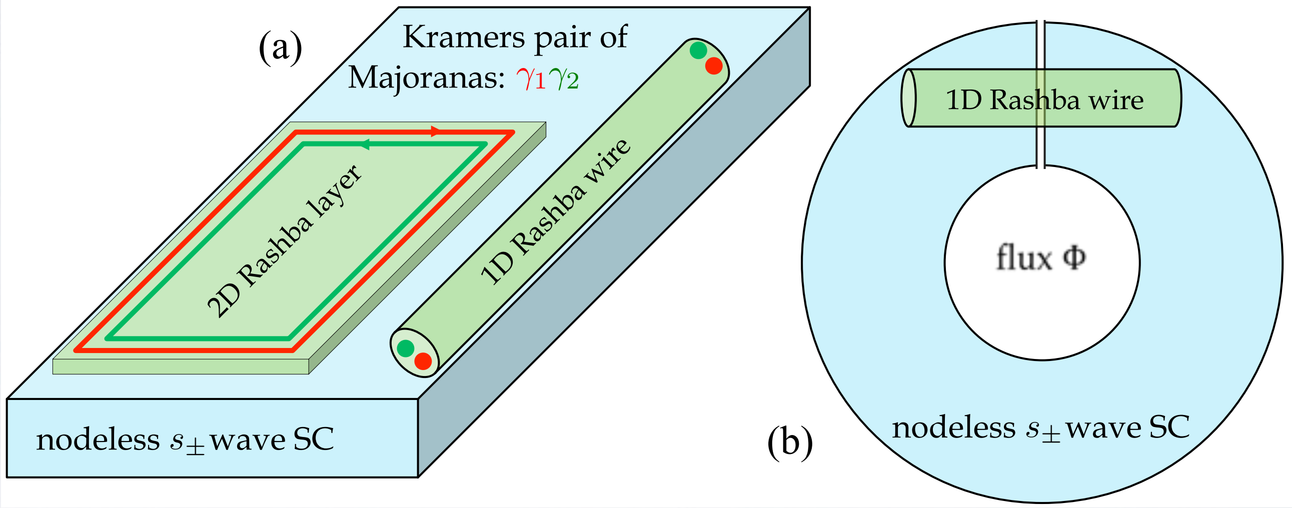
$\nu = 1$ Trivial

$\nu = -1$ Topological

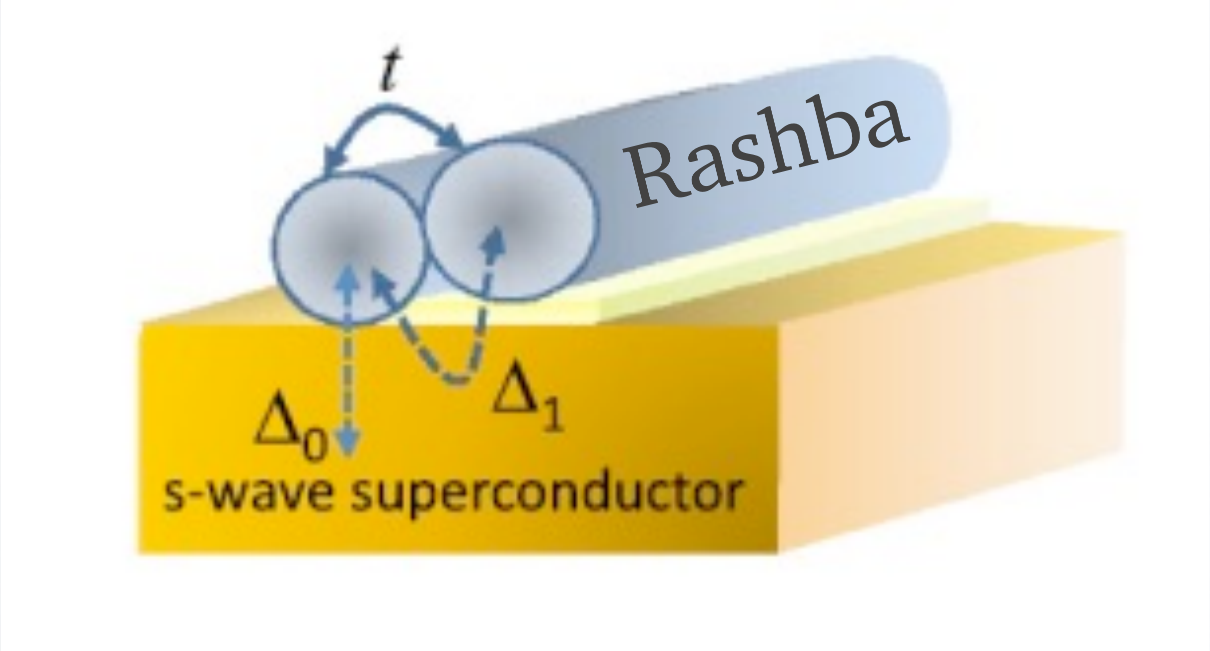
FROM PROXIMITY EFFECT



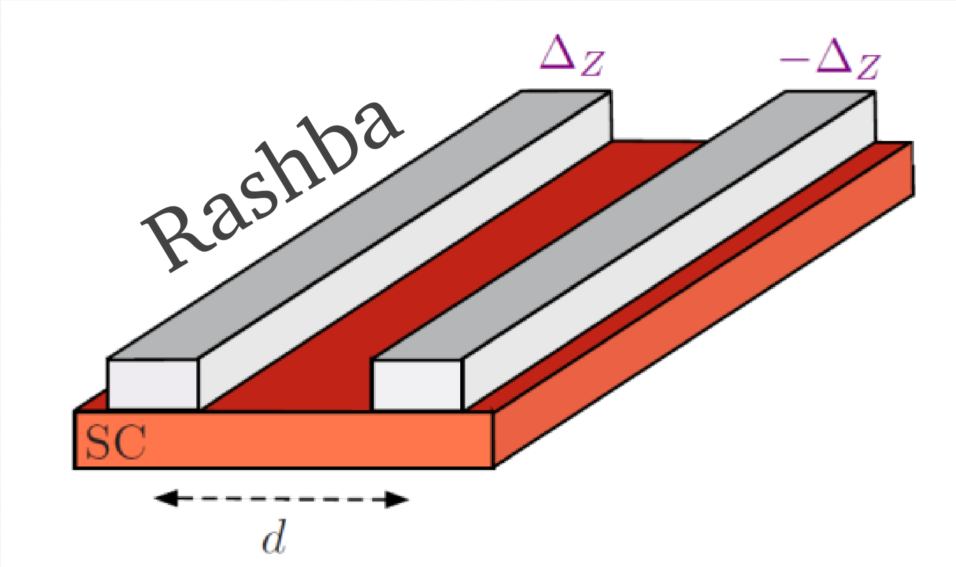
C.Wong, K.T.Law,
Phys, Rev. B 86, 184516 (2012)



F. Zhang, C. L. Kane, and E. J. Mele,
Phys, Rev. Lett. 111, 056402 (2013)

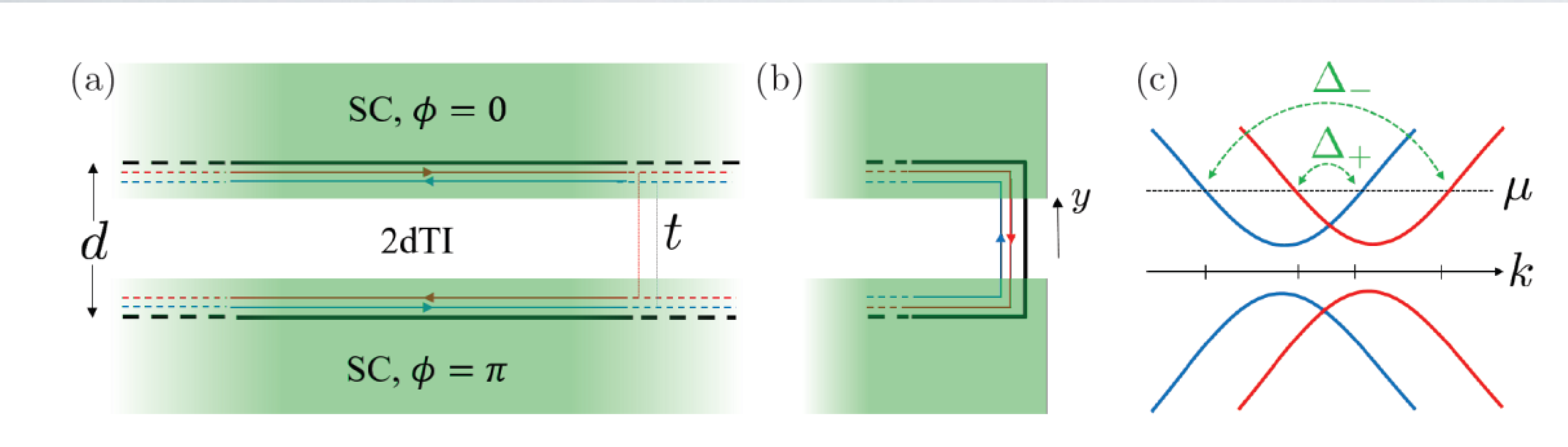


E. Gaidamauskas, J. Paaske, and K. Flensberg,
Phys, Rev. Lett 112, 126402 (2014)



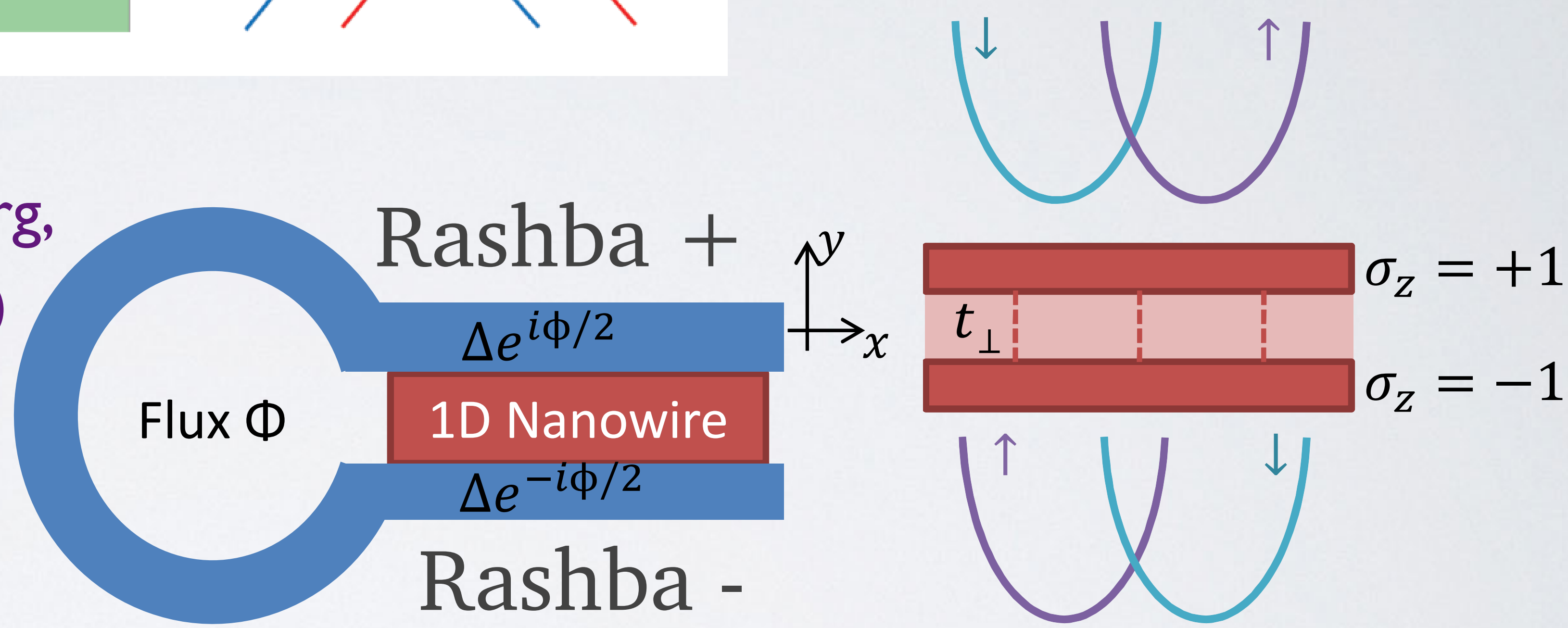
C.Reeg, C. Schrade, J. Klinovaja,
D. Loss, Phys, Rev. B (2017)

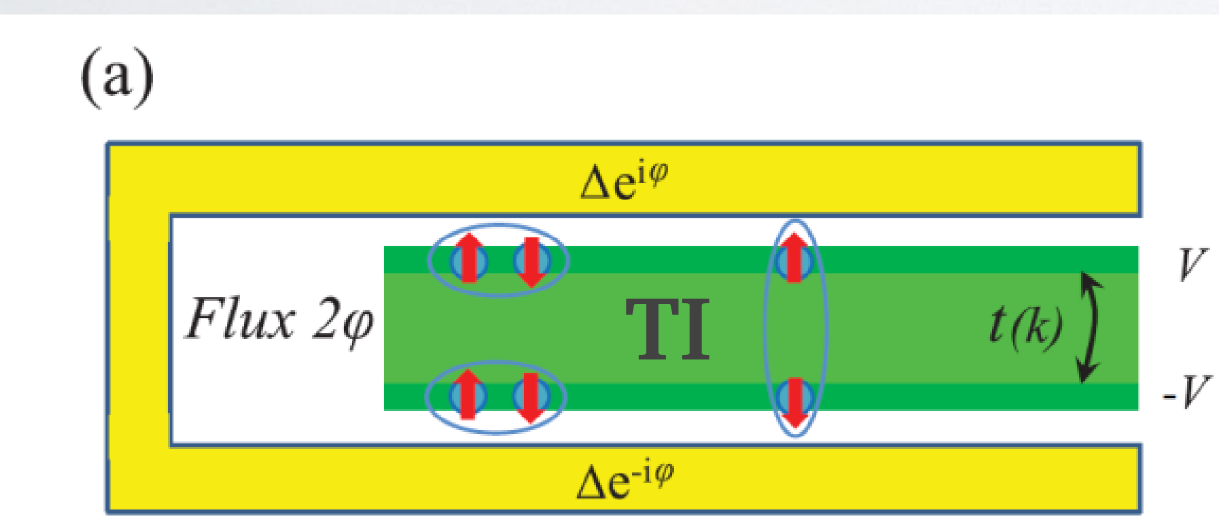
PROXIMITY+ PHASE TUNING



L. Fu, C. Kane, Phys. Rev. B 2009

A. Keselman, L. Fu, A. Stern and E. Berg, Phys. Rev. Lett. 111, 116402 (2013)



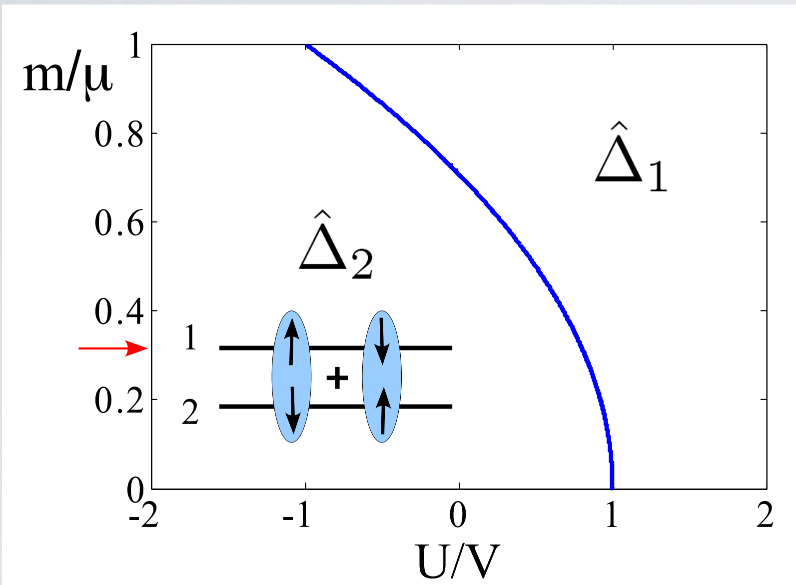


C-X Liu, B. Trauzettel , PRB 83, 229510 (2011)

F. Parhizgar, AM.Black-Schaffer, Sci. Rep. 7, 9817 (2017)

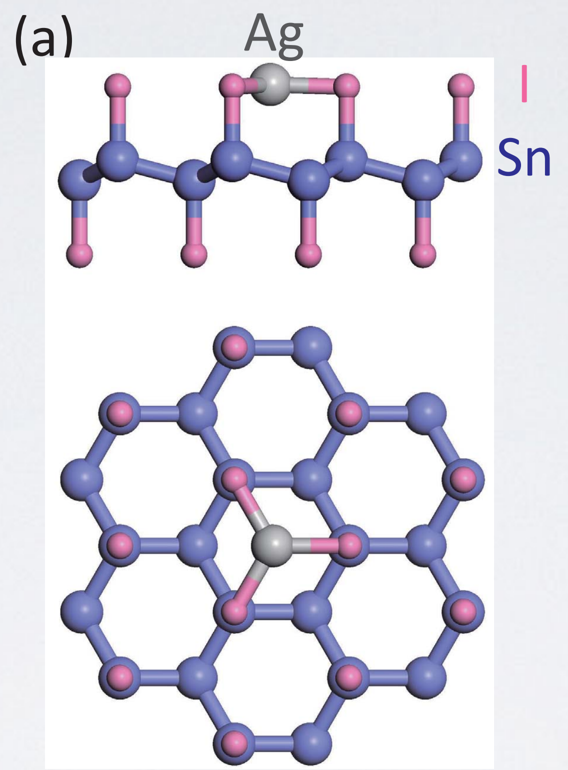
WITH MANY-BODY INTERACTIONS

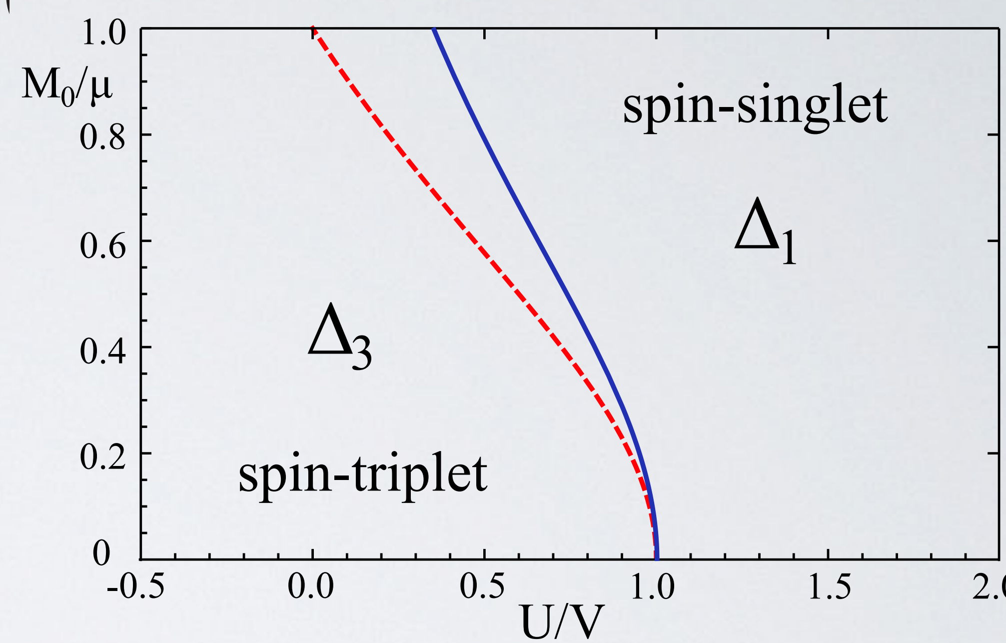
2D



$\mathbf{Cu_xBi_2Se_3}$

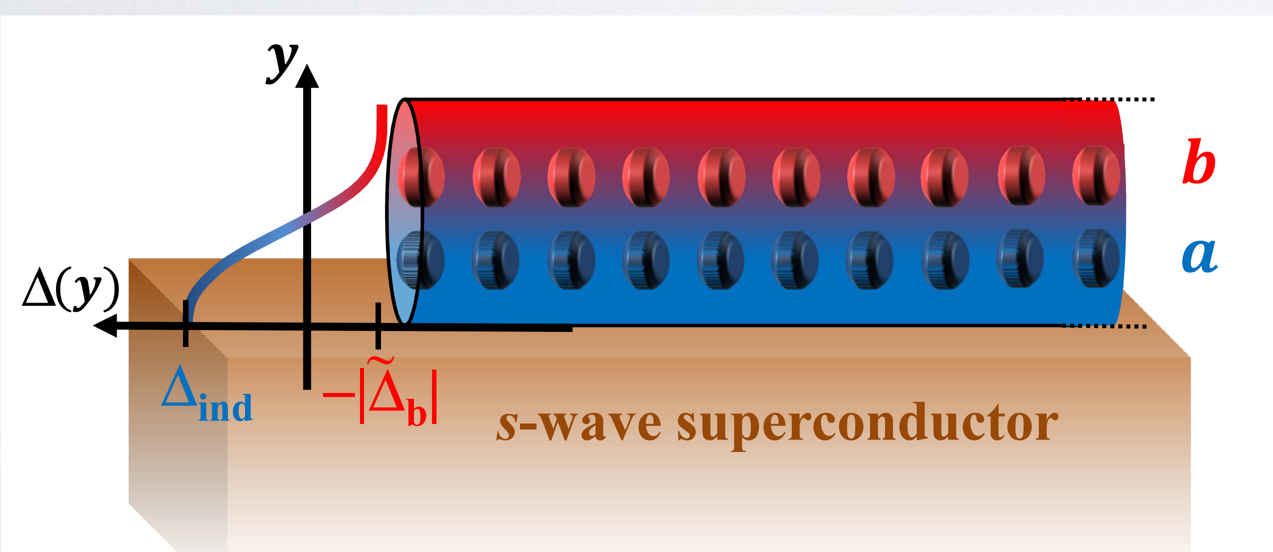
L. Fu and E. Berg, Phys. Rev. Lett. 105, 097001 (2010)





J. Wang, Y. Xu, S-C Zhang, Phys. Rev. B 90, 054503 (2014)

S.Nakosai, Y. Tanaka, N.Nagaosa, PRL 108, 147003 (2012)



1D

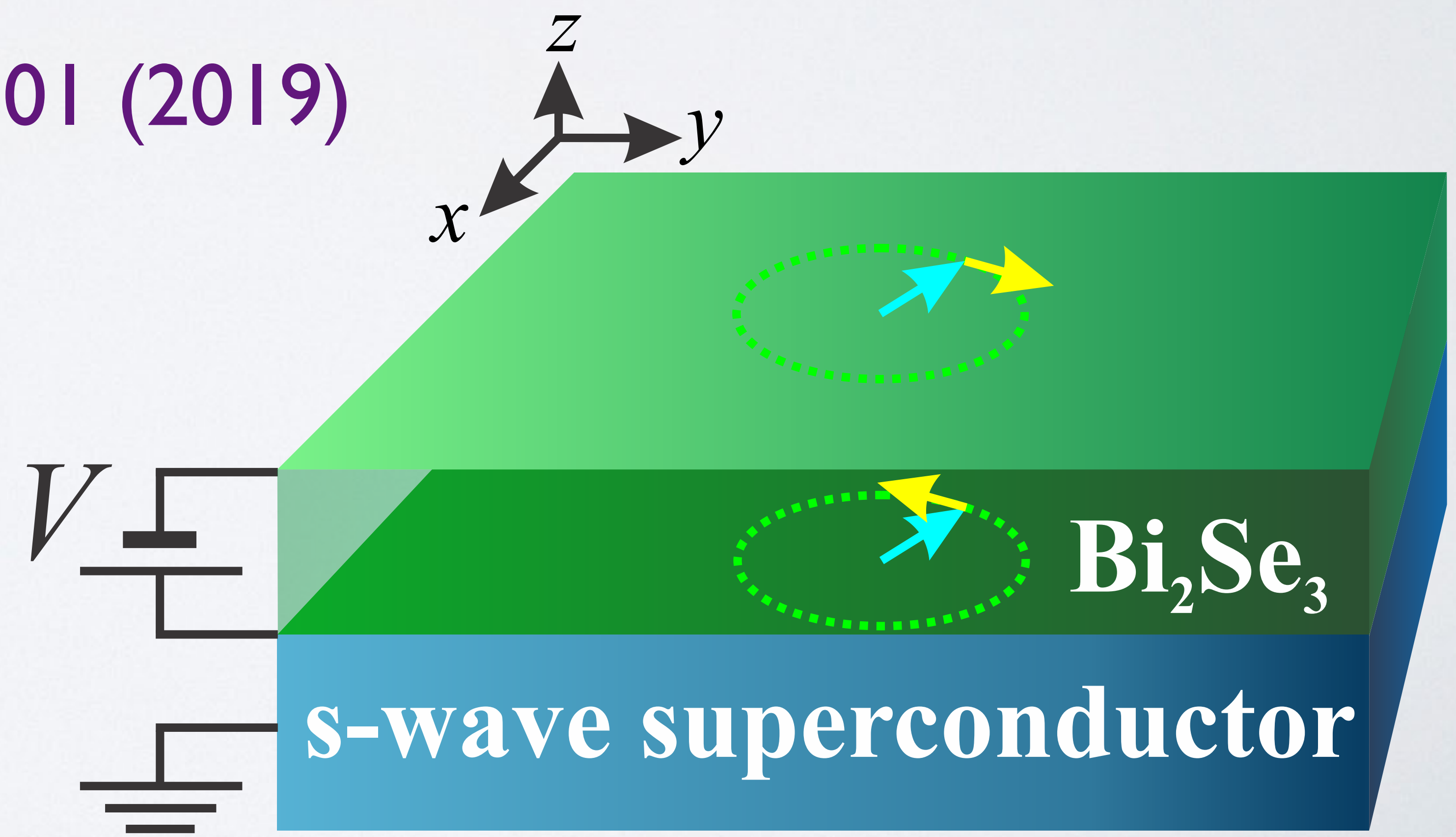
A. Haim, A. Keselman, Y. Oreg, Phys. Rev. B. 89, 220504 (2014)

TRIPTOPS 2D WITHOUT PHASE-TUNING

Proximity induced time-reversal topological superconductivity in Bi_2Se_3 films without phase tuning

Oscar E. Casas,[1,2] Liliana Arrachea,[3] William J. Herrera,[1] and Alfredo Levy Yeyati[2]

Phys. Rev. B (RC) 99, 161301 (2019)
arXiv:1812.00931



3D TOPOLOGICAL INSULATORS

ARTICLES
nature physics
PUBLISHED ONLINE: 10 MAY 2009 | DOI: 10.1038/NPHYS1270

Topological insulators in Bi_2Se_3, Bi_2Te_3 and Sb_2Te_3 with a single Dirac cone on the surface

Haijun Zhang[1], Chao-Xing Liu[2], Xiao-Liang Qi[3], Xi Dai[1], Zhong Fang[1] and Shou-Cheng Zhang[3]*

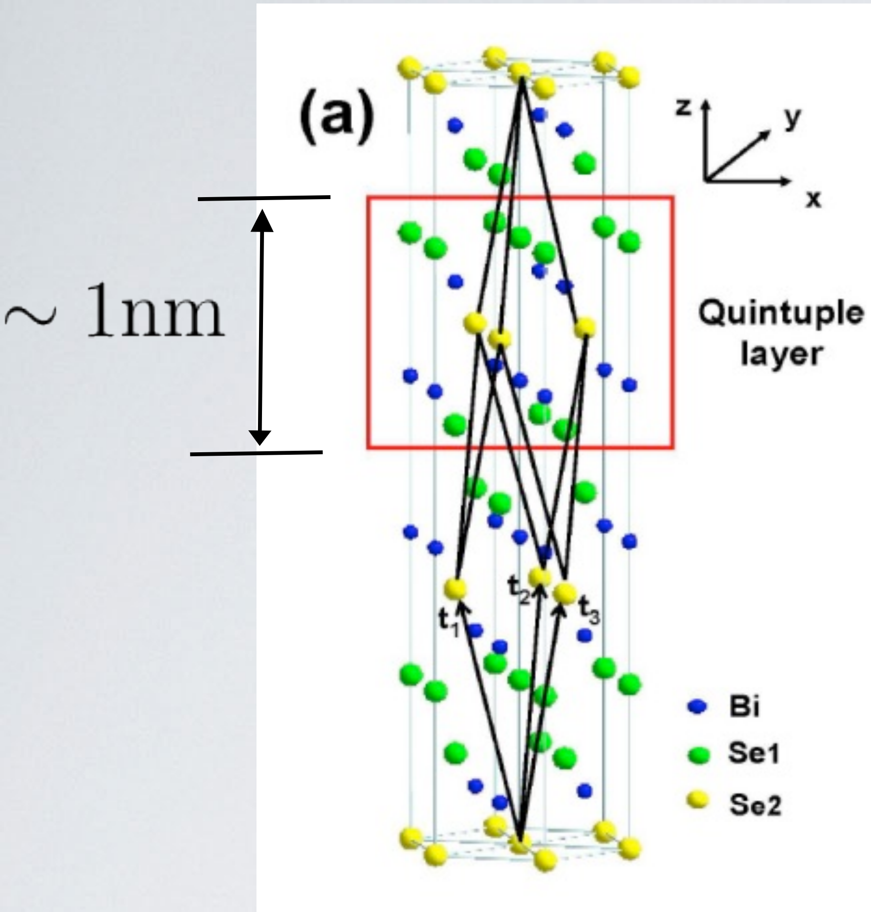
Nat. Phys. 5,438 (2009)

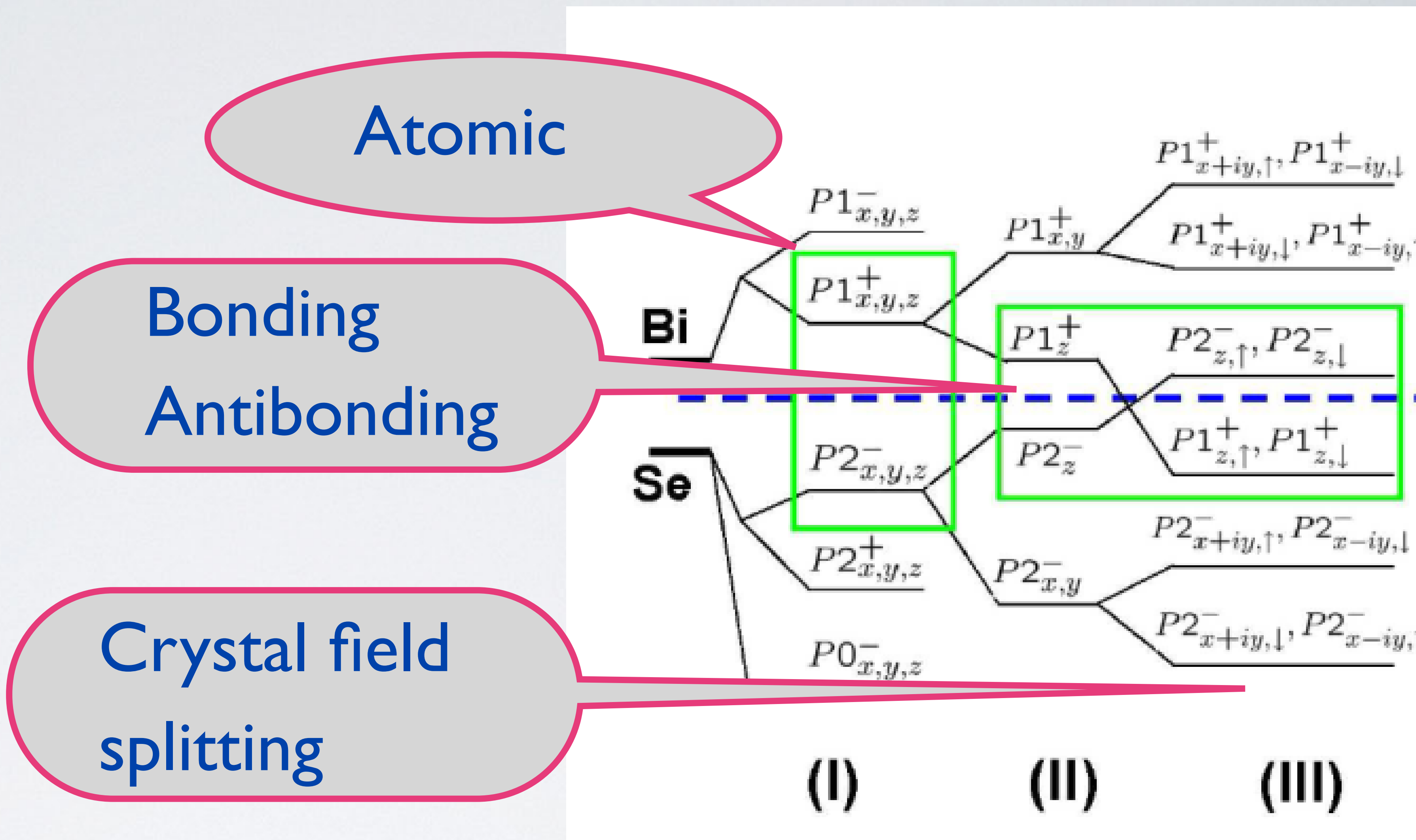
Model Hamiltonian for Topological Insulators

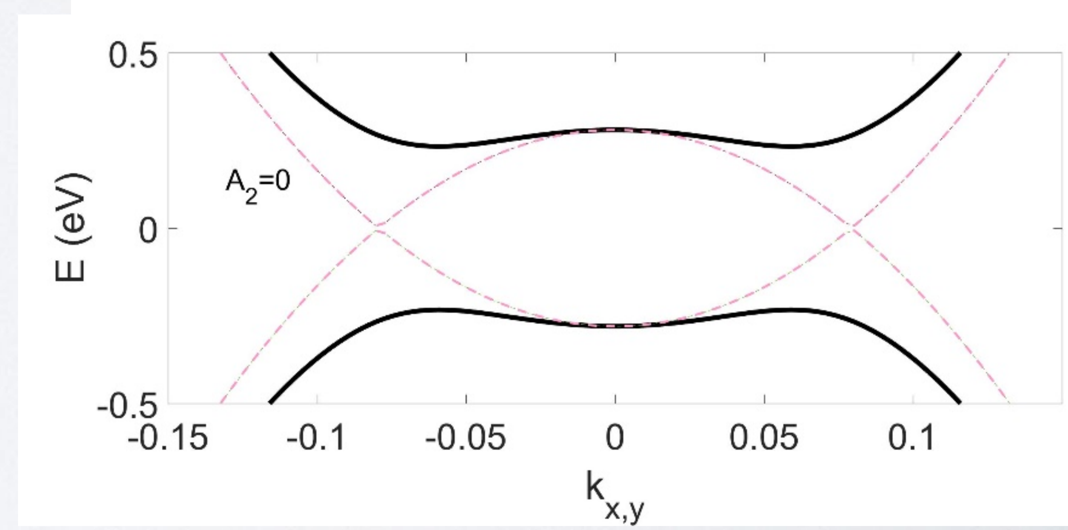
Chao-Xing Liu[1], Xiao-Liang Qi[2], HaiJun Zhang[3], Xi Dai[3], Zhong Fang[3] and Shou-Cheng Zhang[2]

Phys. Rev. B 82, 045122 (2010)

Bi_2Se_3







Basis $\{|P1^+_z,\uparrow\rangle, |P1^+_z,\downarrow\rangle, |P2^-_z,\uparrow\rangle, |P2^-_z,\downarrow\rangle\}$

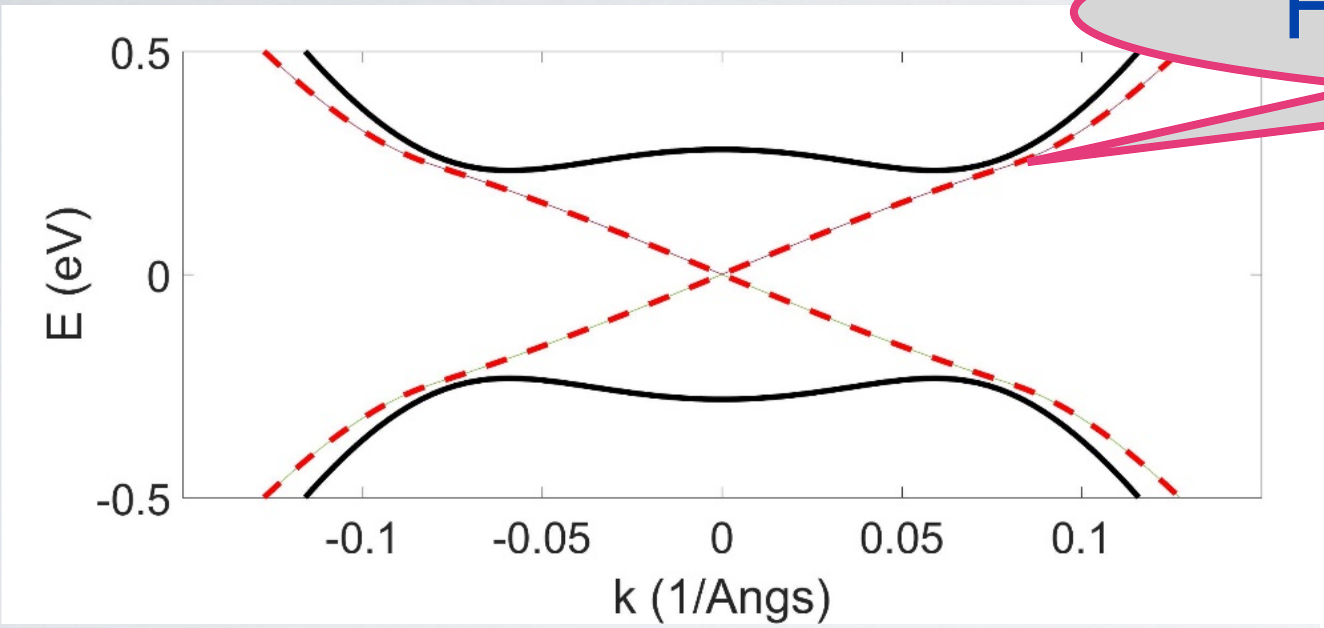
$$H(\mathbf{k}) = \begin{pmatrix} -\mathcal{M}(\mathbf{k}) & 0 & A_1 k_z & A_2 k_- \\ 0 & -\mathcal{M}(\mathbf{k}) & A_2 k_+ & -A_1 k_z \\ A_1 k_z & A_2 k_- & \mathcal{M}(\mathbf{k}) & 0 \\ A_2 k_+ & -A_1 k_z & 0 & \mathcal{M}(\mathbf{k}) \end{pmatrix}$$

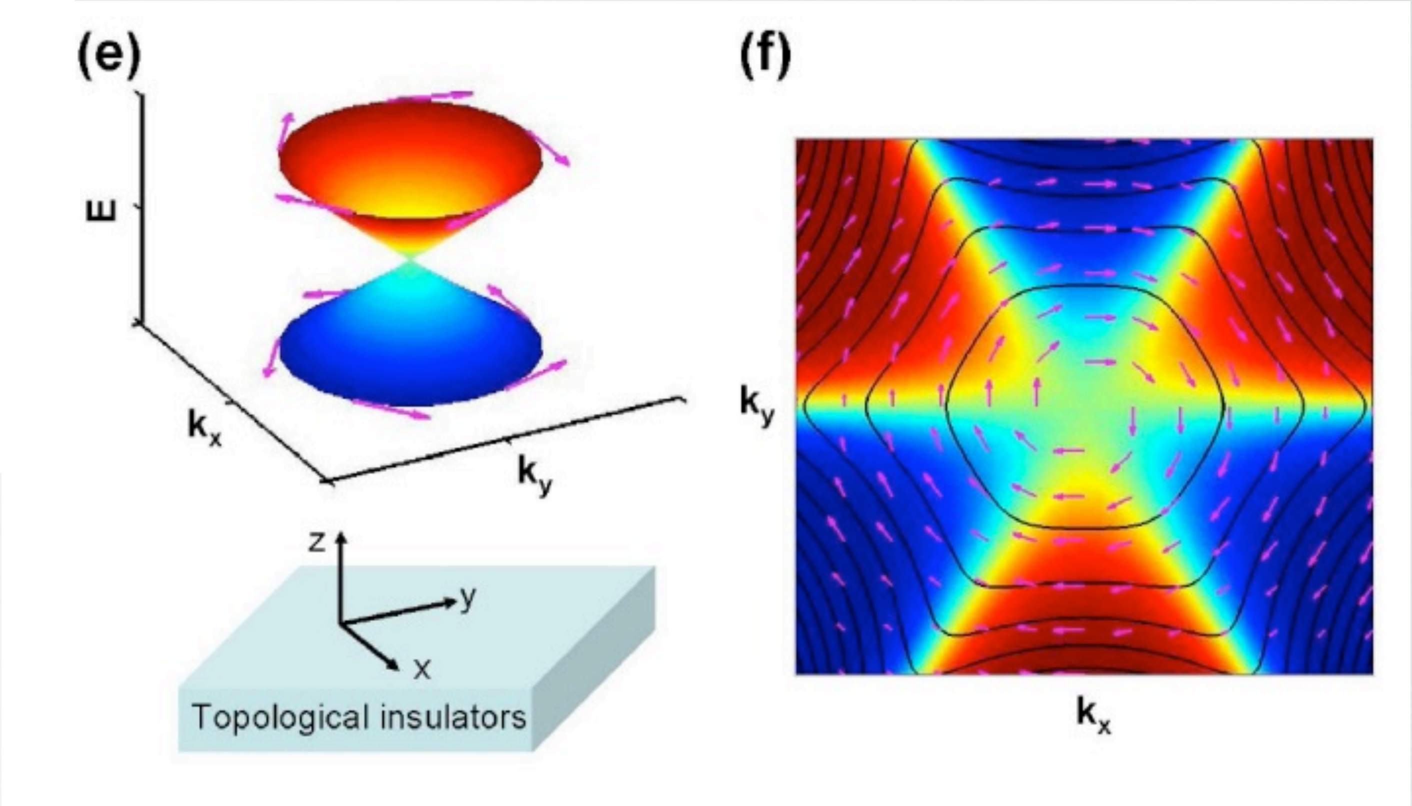
$$k_\pm = k_x \pm i k_y$$

$$\mathcal{M}(\mathbf{k}) = M - B_1 k_z^2 - B_2 k_\perp^2$$

SURFACE STATES Bi_2Se_3

Helicity-degenerate $\hat{h} = (\hat{\sigma} \times \mathbf{k})_z / k$





Well defined helicity

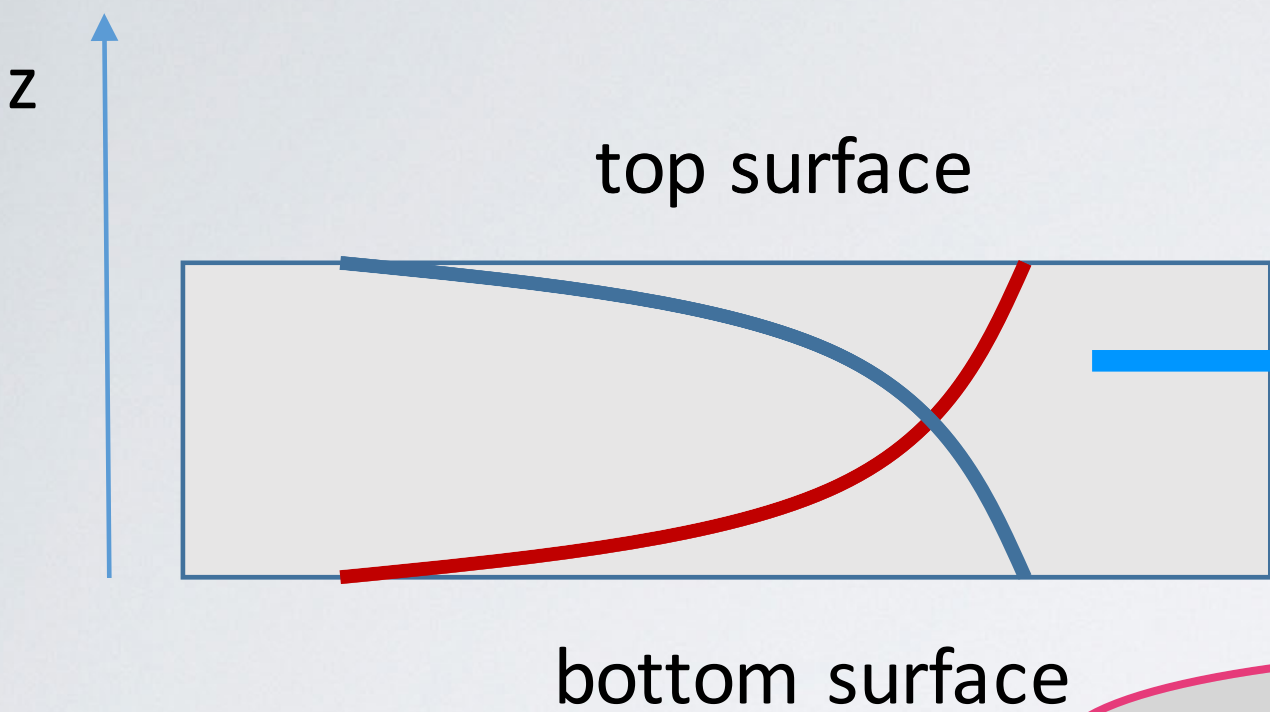
Parity structure:

$$\Psi_{top}(k=0) \sim \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \end{pmatrix} \begin{matrix} +\uparrow \\ +\downarrow \\ -\uparrow \\ -\downarrow \end{matrix} \qquad \Psi_{bot}(k=0) \sim \begin{pmatrix} 1 \\ 1 \\ -1 \\ -1 \end{pmatrix} \begin{matrix} +\uparrow \\ +\downarrow \\ -\uparrow \\ -\downarrow \end{matrix}$$

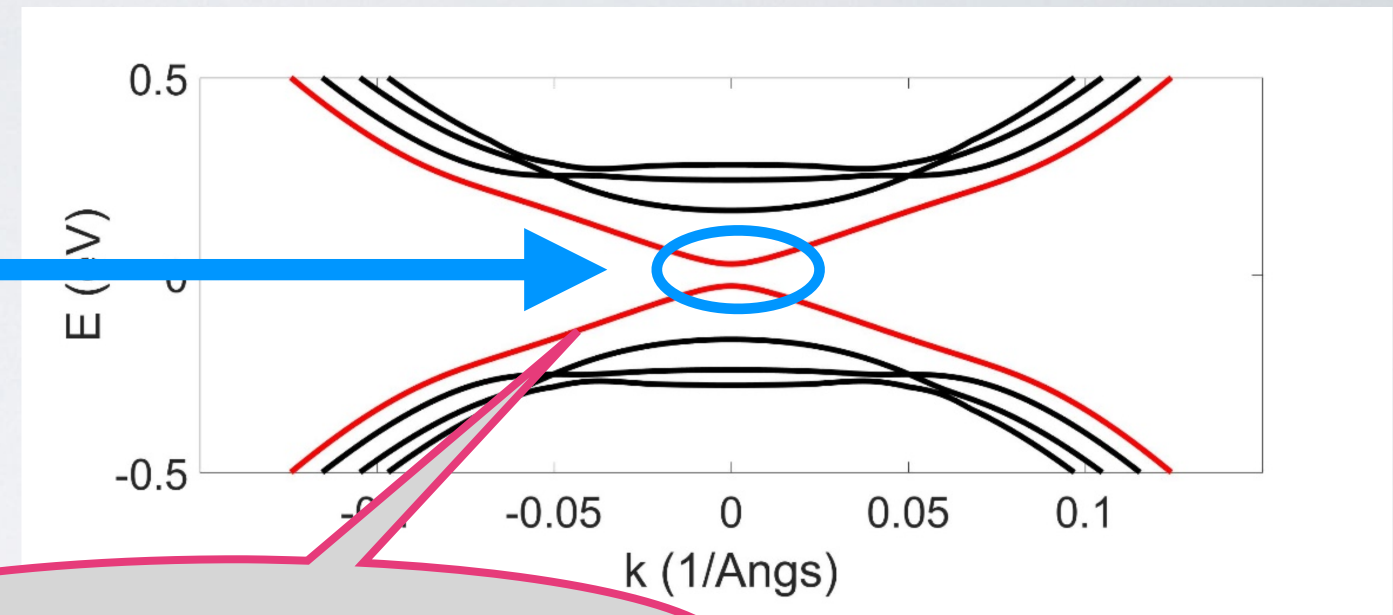
Even

Odd

Bi_2Se_3 THIN FILMS



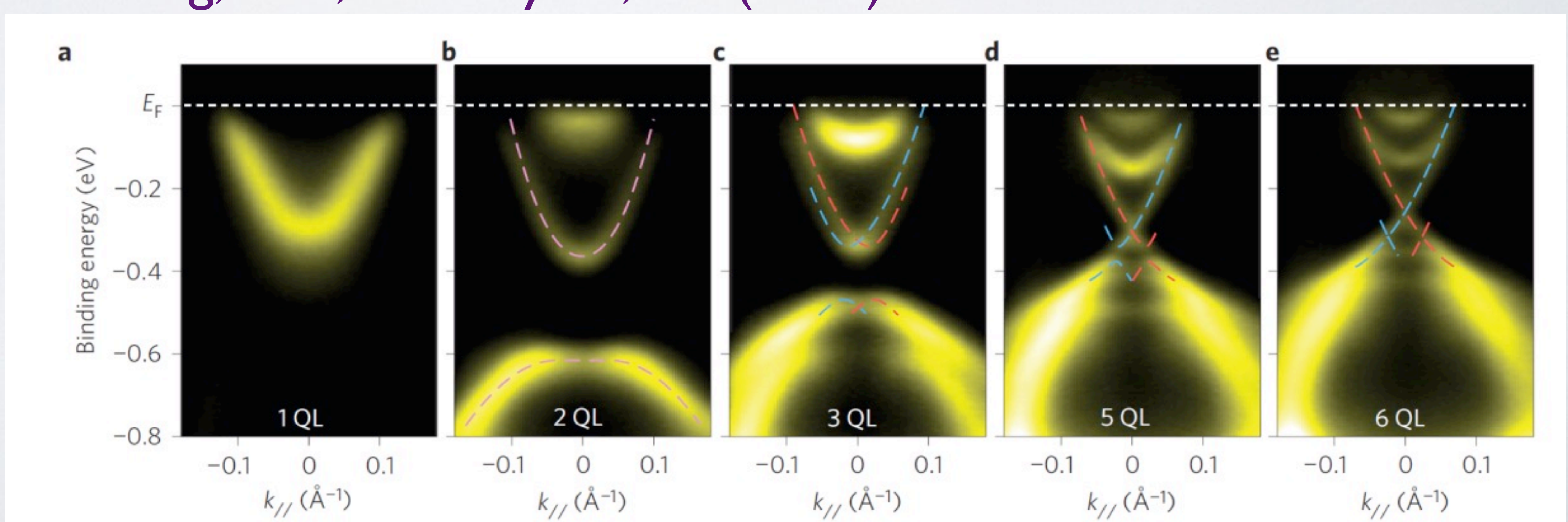
A gap appears



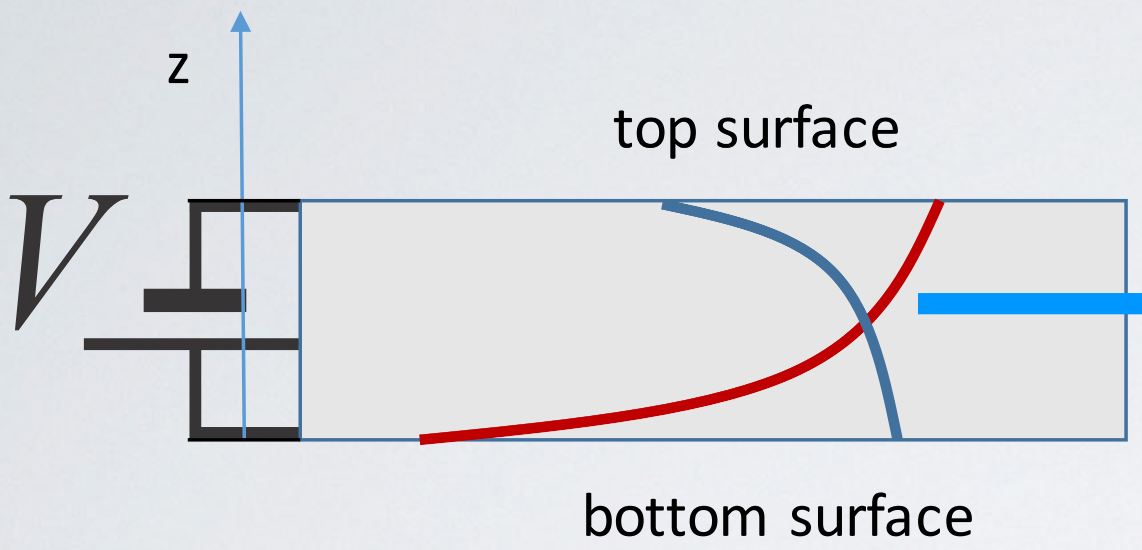
Helicity-degenerate

Y Zhang, et al, Nat. Phys. 6,584 (2010)

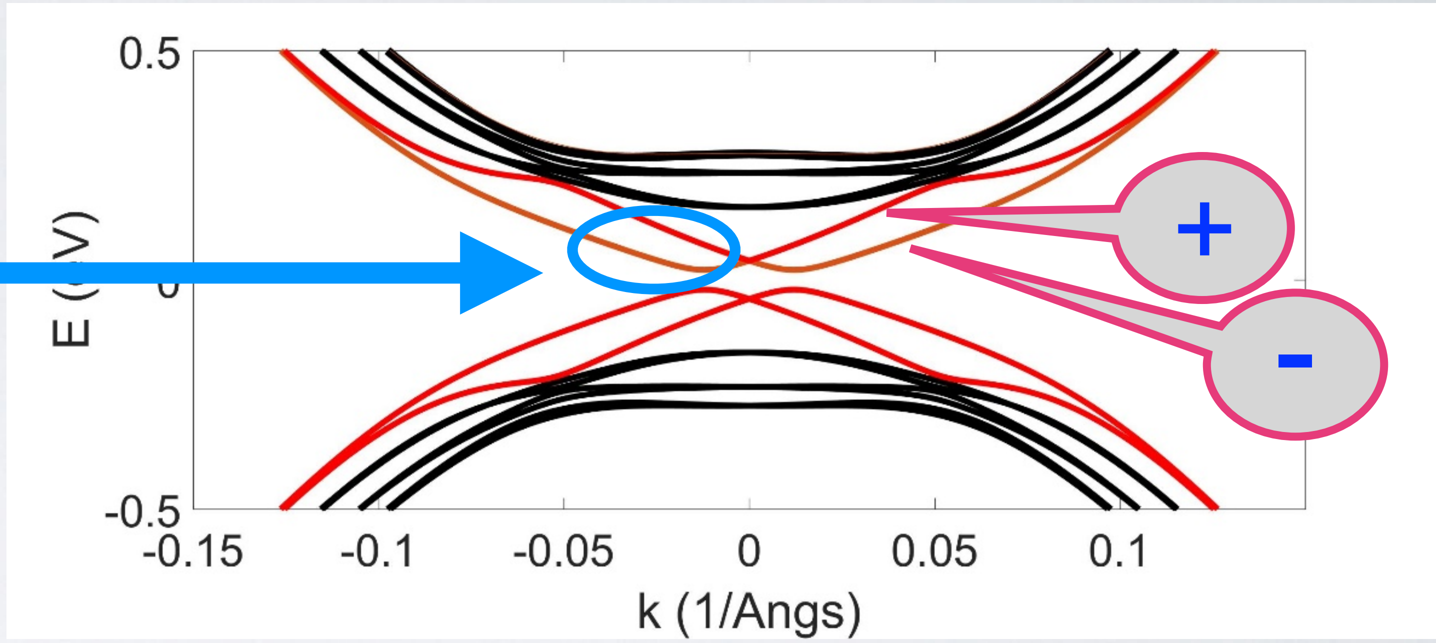
ARPES Spectra



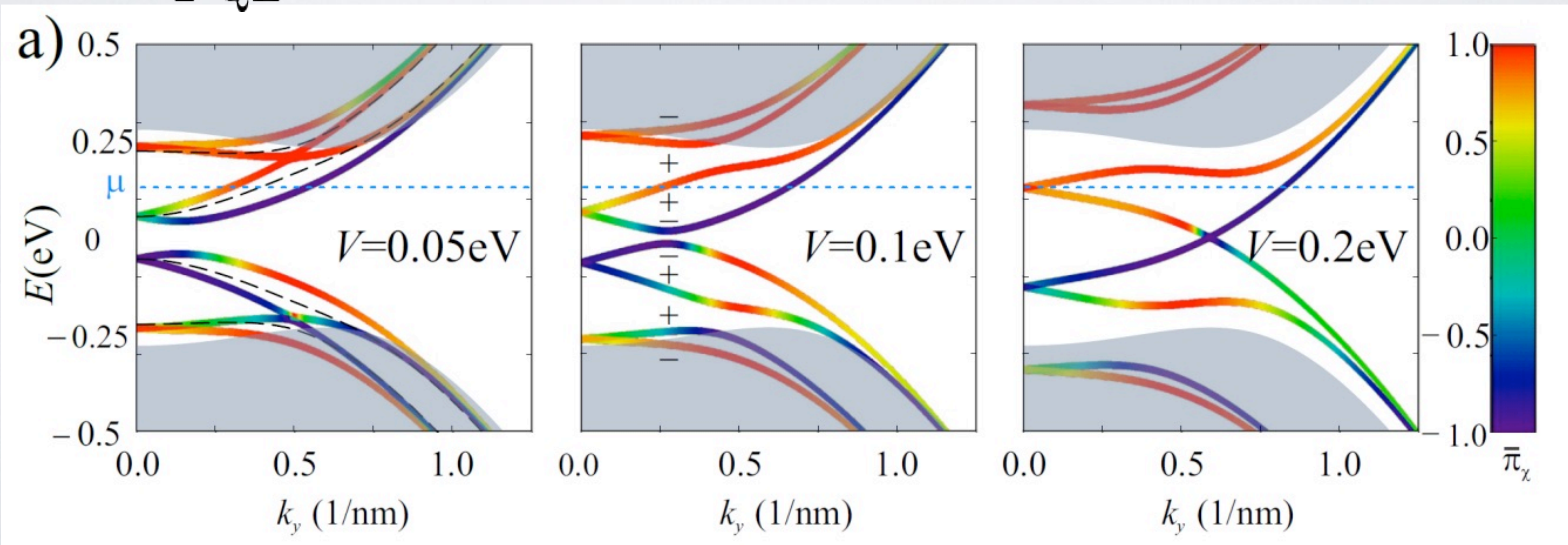
Bi_2Se_3 THIN FILMS+ELECTRIC FIELD



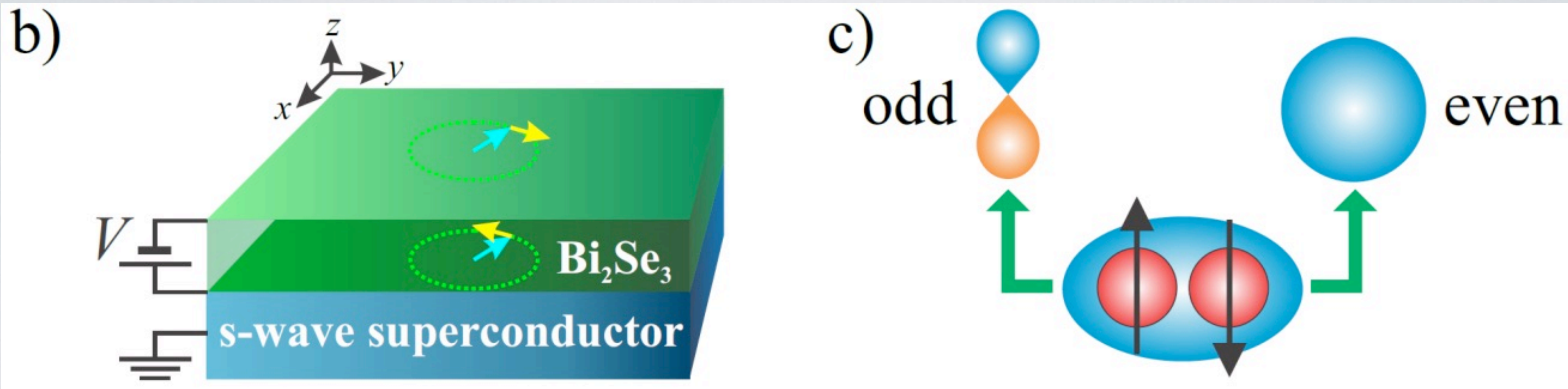
Degeneracy is broken

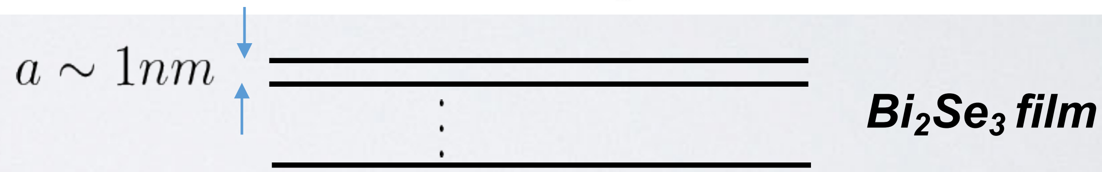


2 QL



PROXIMITY EFFECT IN THIN FILMS





Discretized model

$$\psi_{k_\parallel,i} = (c_{k_\parallel,i+\uparrow}, c_{k_\parallel,i+\downarrow}, c_{k_\parallel,i-\uparrow}, c_{k_\parallel,i-\downarrow})^T \qquad \hat{H}^{TB}(k_\parallel) = \sum_{k_\parallel,ij} \psi^\dagger_{k_\parallel,i} \hat{\mathcal{H}}^e(k_\parallel)_{ij} \psi_{k_\parallel,j}$$

$$\hat{\mathcal{H}}^{BdG}(k_\parallel)_{ij} = \begin{pmatrix} \hat{\mathcal{H}}^e(k_\parallel)_{ij} - \mu \hat{I}\delta_{ij} & \hat{\Delta}_{ij} \\ \hat{\Delta}^\dagger_{ij} & \mu \hat{I}\delta_{ij} - \hat{\mathcal{H}}^h(k_\parallel)_{ij} \end{pmatrix} \qquad i,j = 1, \ldots, N_z$$

$$\hat{\Delta}_{ij} = \delta_{i1}\delta_{j1}\hat{\Delta}_1 \qquad \hat{\Delta}_1 = \hat{\Delta}_{intra} + \hat{\Delta}_{inter}$$

$\hat{\Delta}_{intra} \rightarrow \Delta_+ \, , \Delta_-$

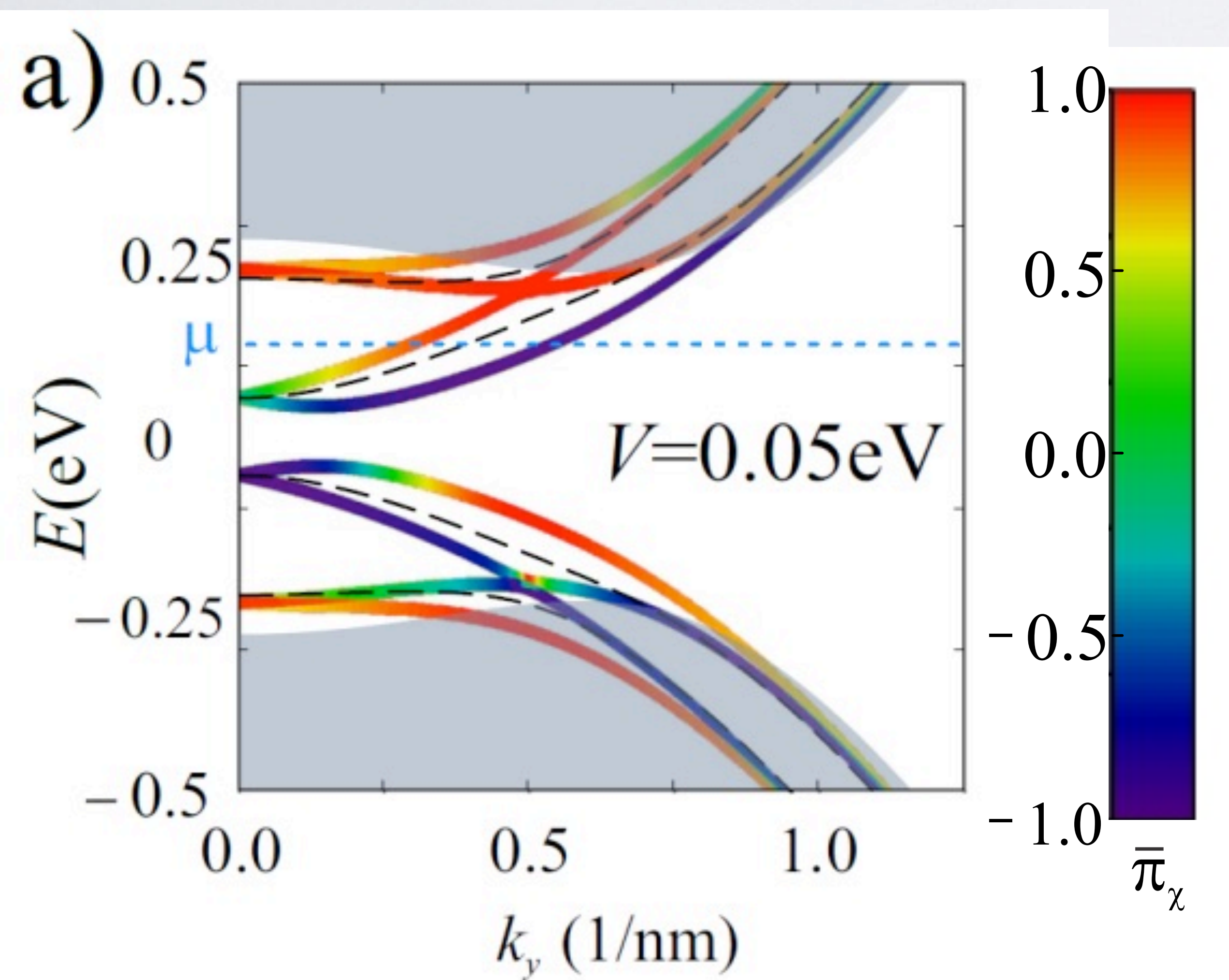
$\hat{\Delta}_{inter} \rightarrow \Lambda$

TOPOLOGICAL INVARIANT

Weak coupling $\Delta_{+,-}, \Lambda \ll \mu$

$$N = \prod_n \text{sgn}\left(\left\langle \psi_n(k_{F,n}) | \mathcal{T}\hat{\Delta}^\dagger | \psi_n(k_{F,n}) \right\rangle\right) \quad \begin{cases} +1 \text{ triv} \\ -1 \text{ topo} \end{cases}$$

$$\mathcal{T} = \tau_0 \otimes i\sigma_y K$$



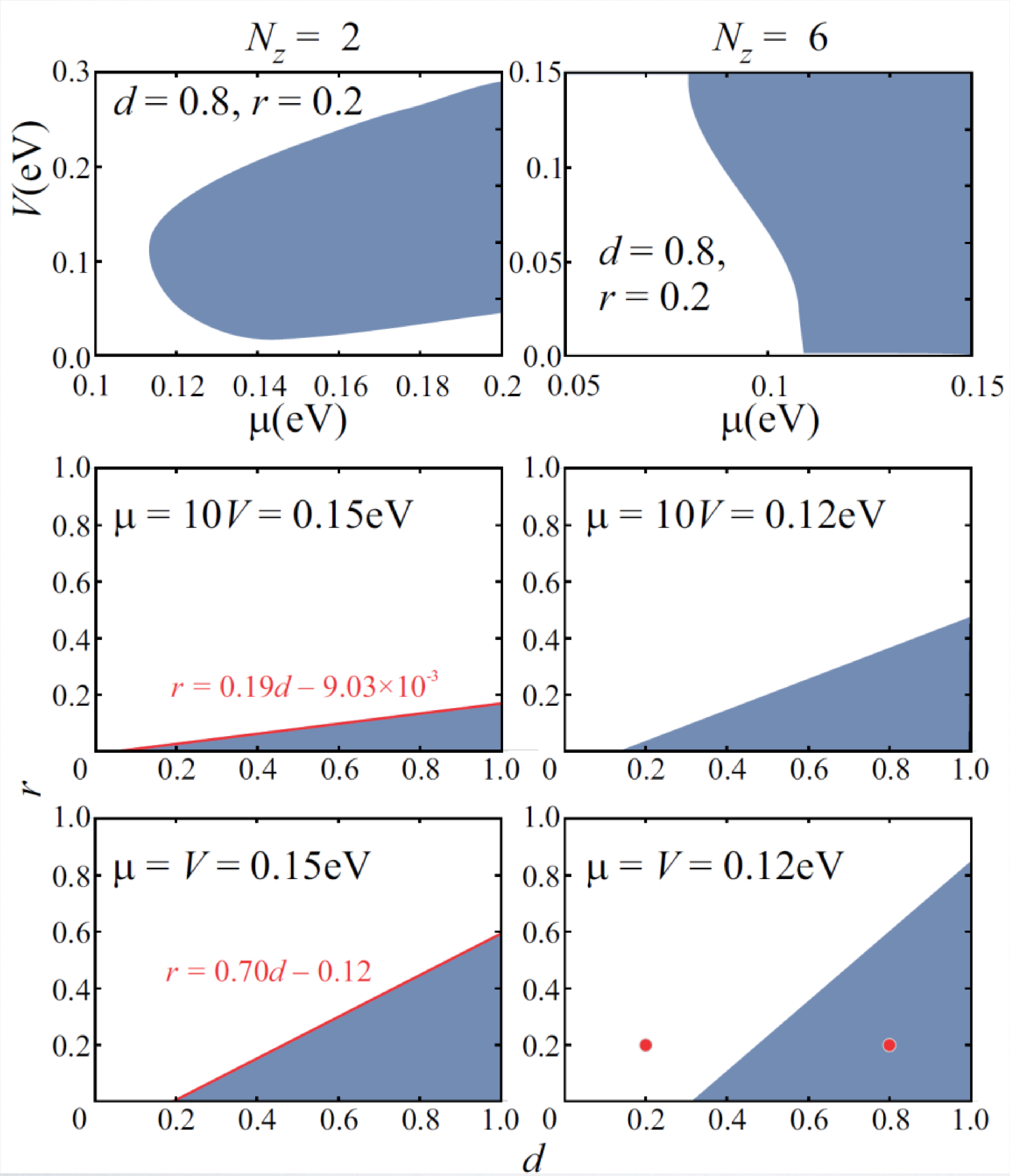
Analytical result $N_z = 2$

$$\left\langle \psi_\chi | \mathcal{T}\hat{\Delta}^\dagger | \psi_\chi \right\rangle \propto (1 - \beta_\chi \Lambda)$$

$$\beta_\chi = 2\pi_\chi / (\Delta_+ + \Delta_- \pi_\chi)$$

$$\pi_\chi \sim \frac{\langle 1, \text{parity} = + | \psi_\chi \rangle}{\langle 1, \text{parity} = - | \psi_\chi \rangle}$$

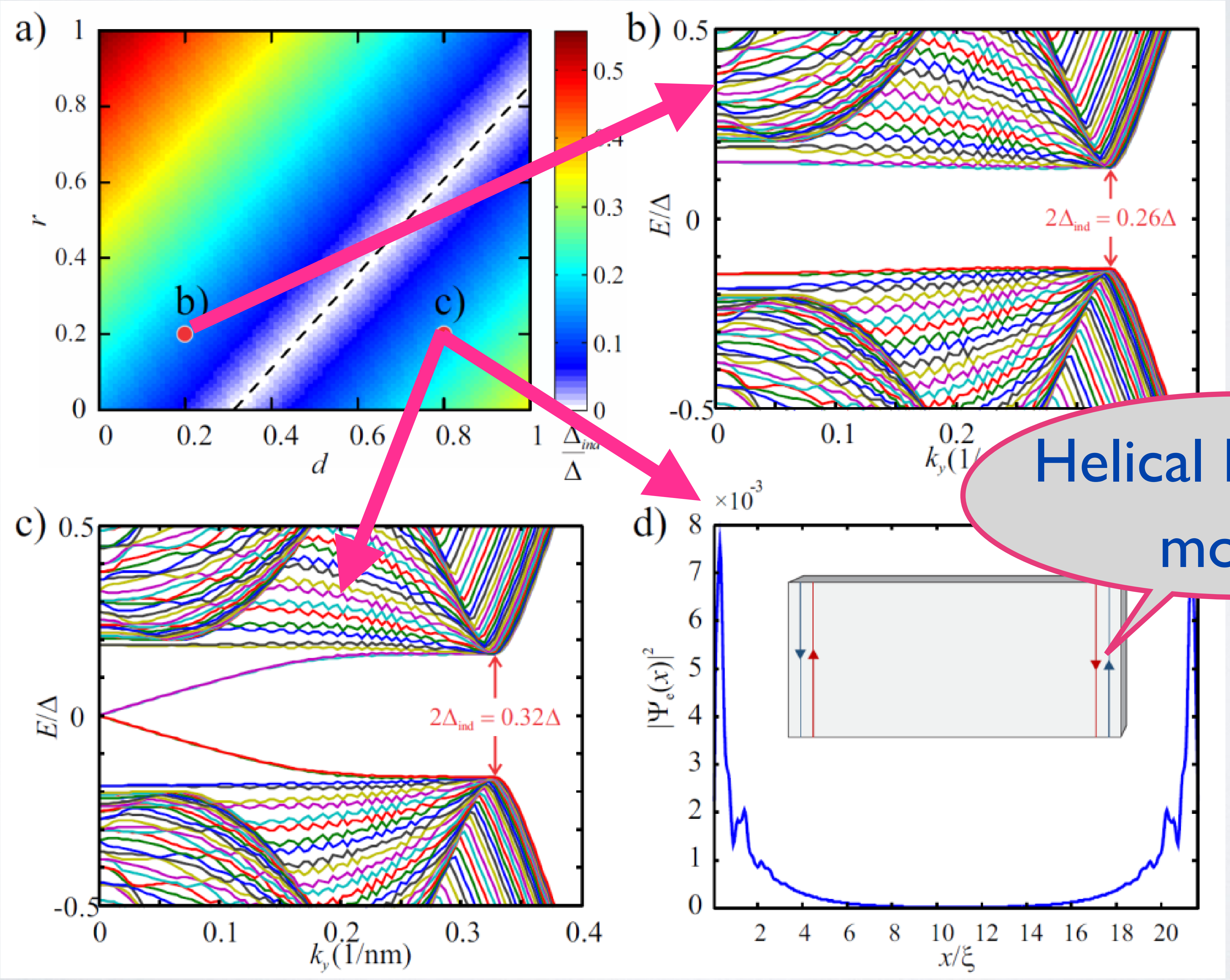
PHASE DIAGRAM



$$r = \Delta_+/\Delta_-$$

$$d = \Lambda/\Delta_-$$

INDUCED GAP N=6



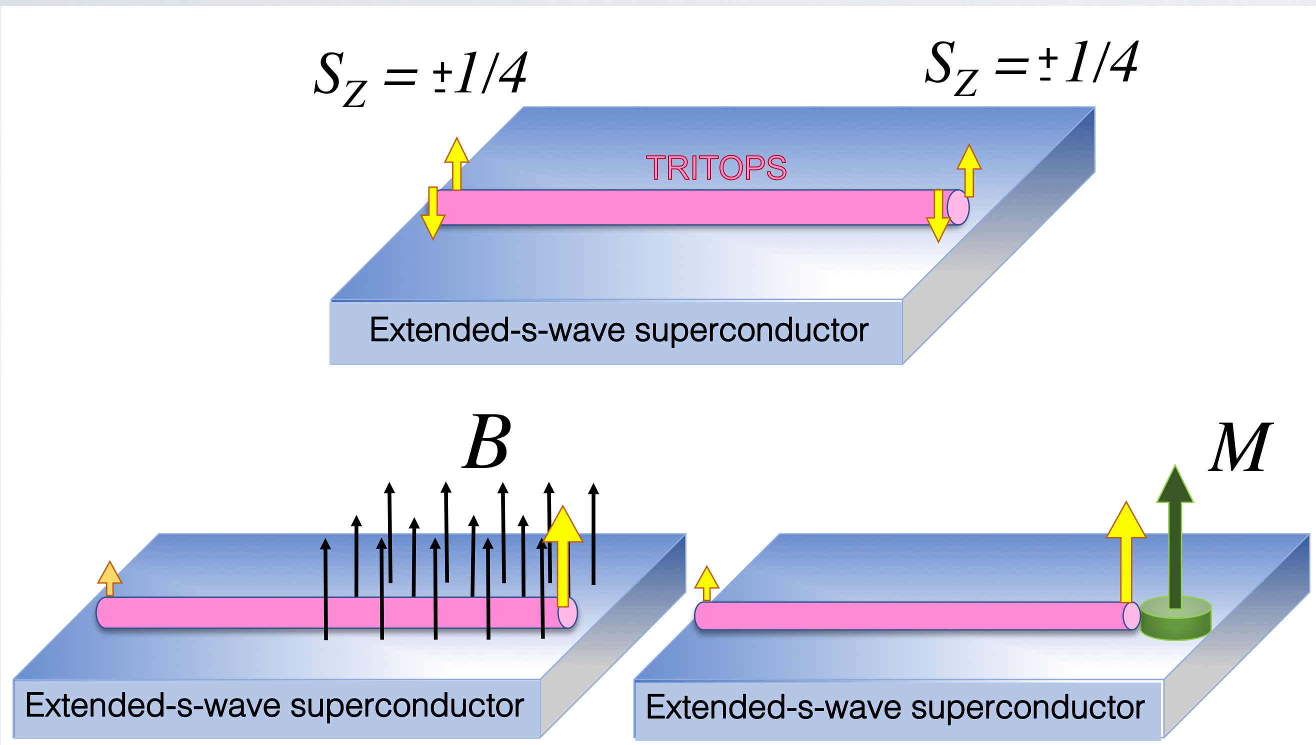
TRIPTOPS 1D
FRACTIONAL SPIN PROJECTION

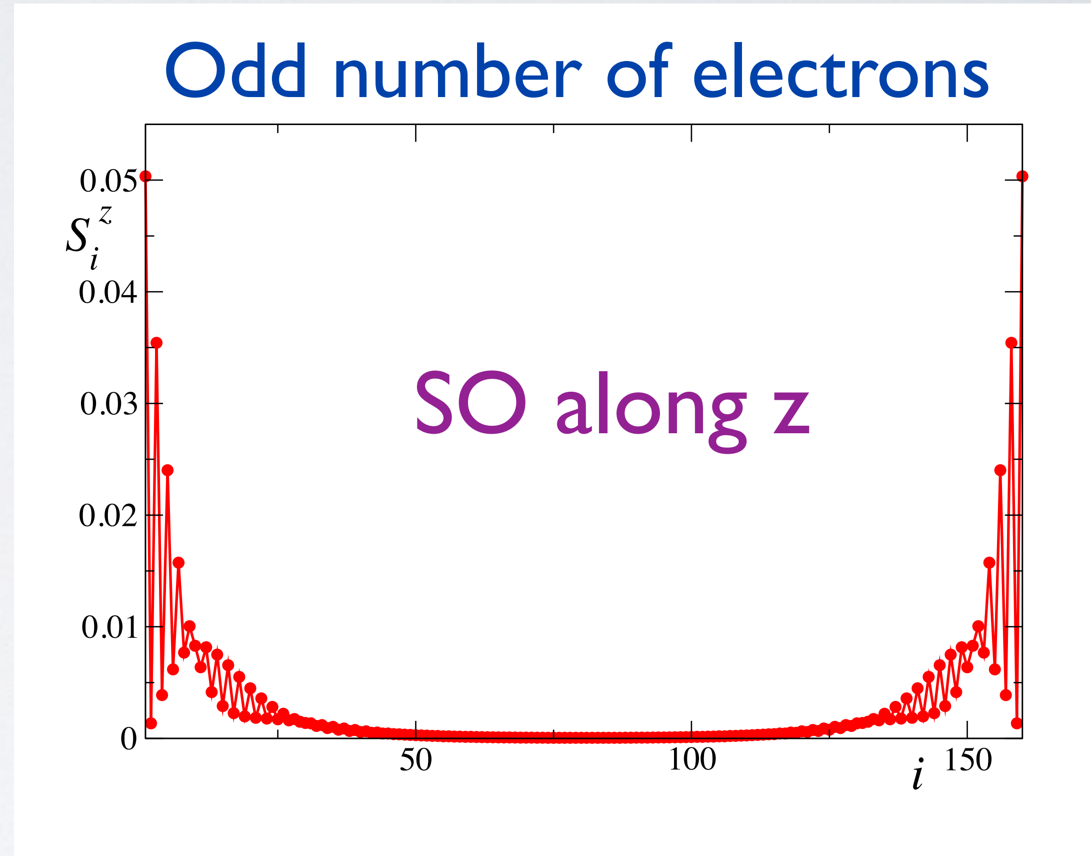
A. Keselman, L. Fu, A. Stern and E. Berg,
B. Phys. Rev. Lett. 111, 116402 (2013)

Entangled end states with fractionalized spin projection in a time-reversal-invariant topological superconducting wire

Armando A. Aligia[1] and Liliana Arrachea[2]

Phys. Rev. B 98, 174507 (2018) arXiv:1806.06104





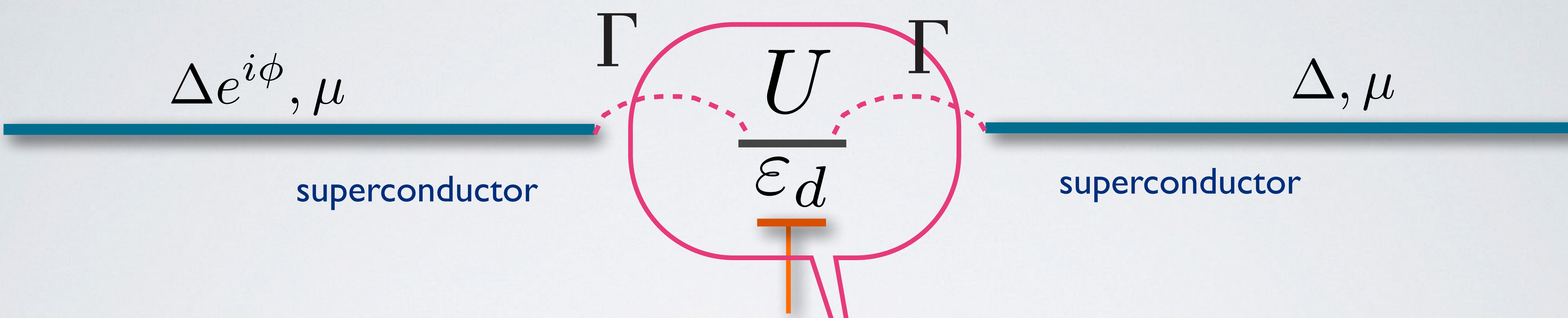
$\Delta_Z \gg E$ **Zeeman splitting** $\Delta_Z \ll E$

$$E_\uparrow = \frac{2E^2}{\Delta_Z},$$
$$E_\downarrow = \frac{2E^2}{\Delta_Z} + \frac{\Delta_Z}{2}.$$

$$E_\uparrow = E\left[1 + \frac{1}{2}\left(\frac{\Delta_Z}{4E}\right)^2\right] - \frac{\Delta_Z}{4},$$
$$E_\downarrow = E\left[1 + \frac{1}{2}\left(\frac{\Delta_Z}{4E}\right)^2\right] + \frac{\Delta_Z}{4}.$$

TRIPTOPS 1D SIGNATURES IN JOSEPHSON JUNCTIONS WITH EMBEDDED Q-DOTS

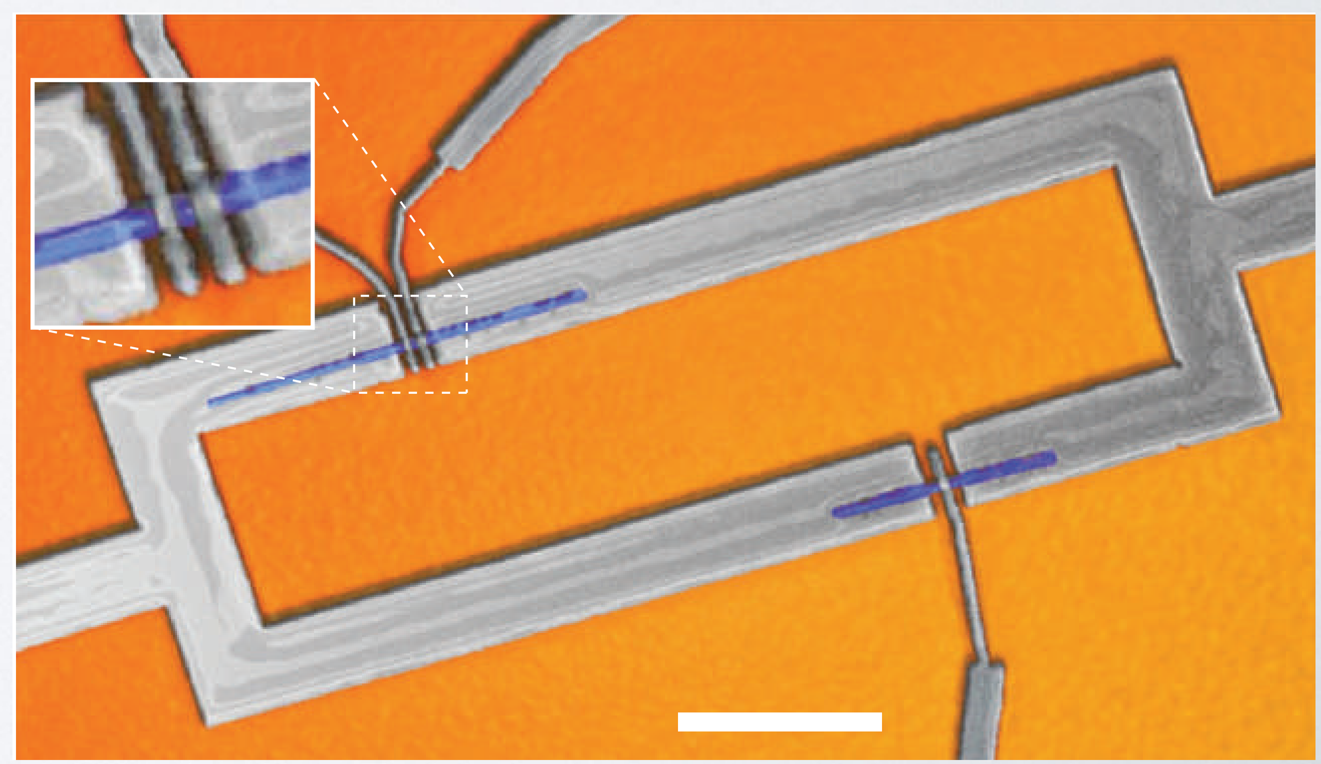
S-D-S JUNCTIONS



nature nanotechnology
REVIEW ARTICLE
PUBLISHED ONLINE: 19 SEPTEMBER 2010 | DOI: 10.1038/NNANO.2010.173

Hybrid superconductor–quantum dot devices

Silvano De Franceschi[1]*, Leo Kouwenhoven[2], Christian Schönenberger[3] and Wolfgang Wernsdorfer[4]



JOSEPHSON CURRENT

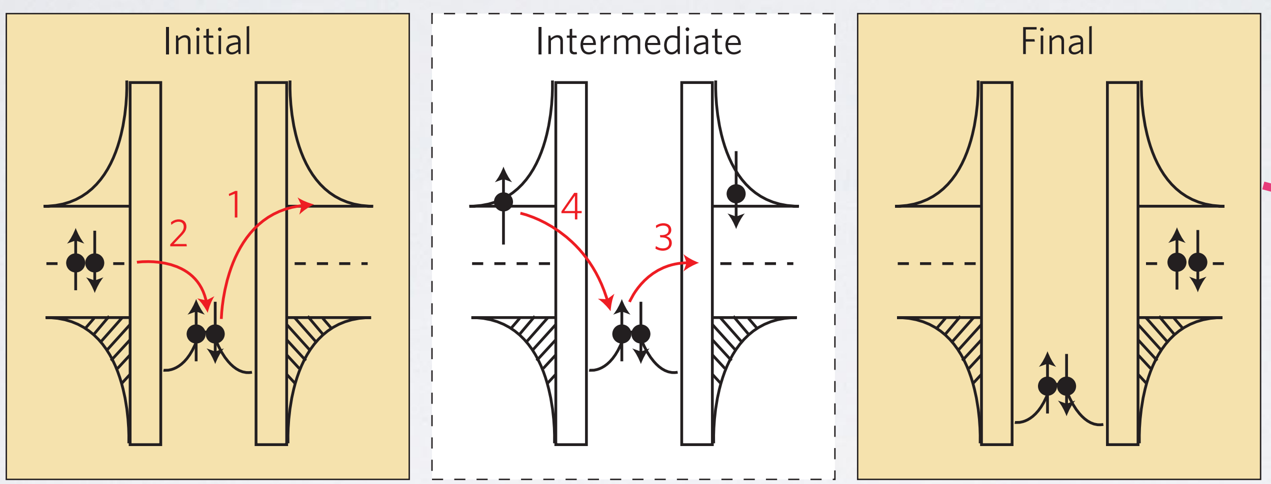
$$k_B T_K \gg \Delta$$

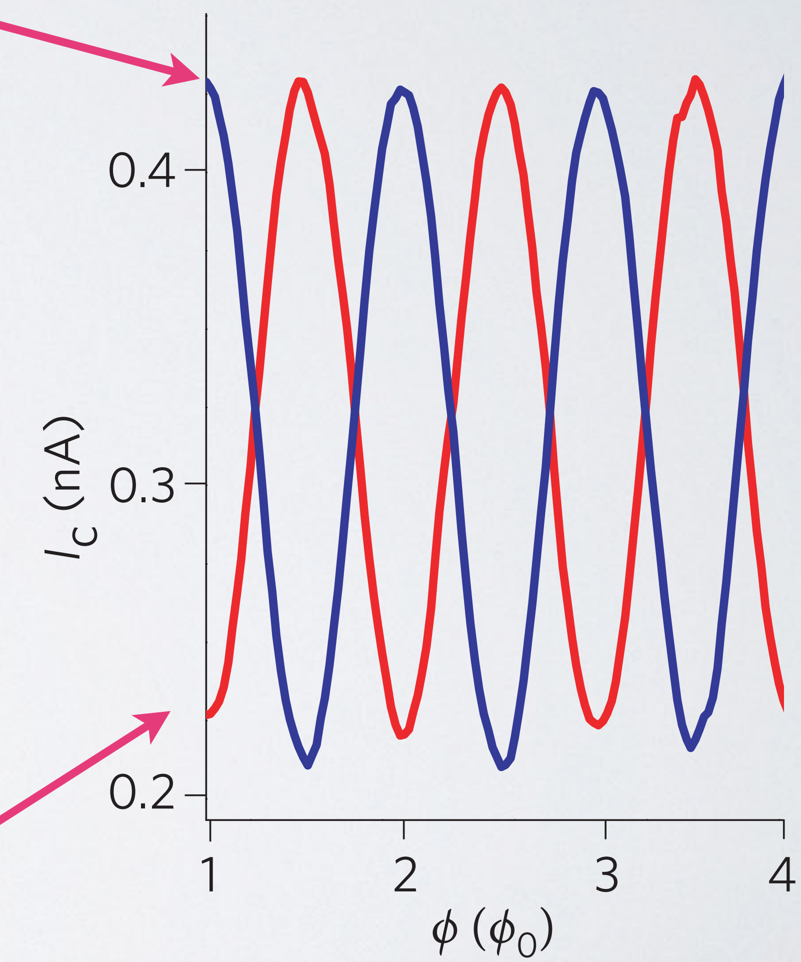
Kondo $k_B T_K = \sqrt{\Gamma U/2}\exp(-\pi U/8\Gamma)$

Singlet

a

N = even, S = 0 $\longrightarrow I_s = I_c \sin(\Delta\varphi)$



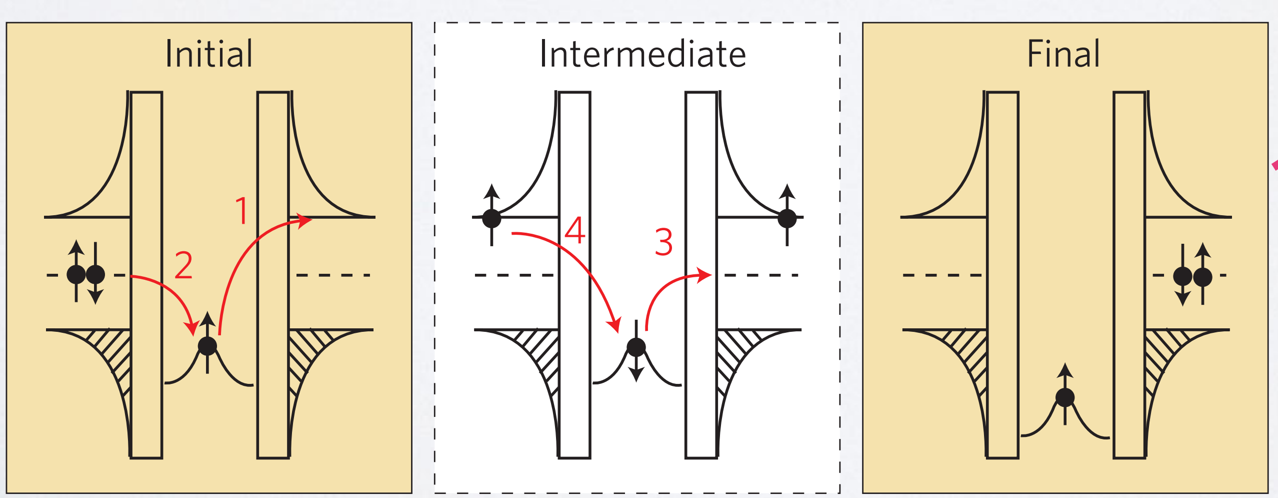


$$k_B T_K \ll \Delta$$

S=1/2

π **junction**

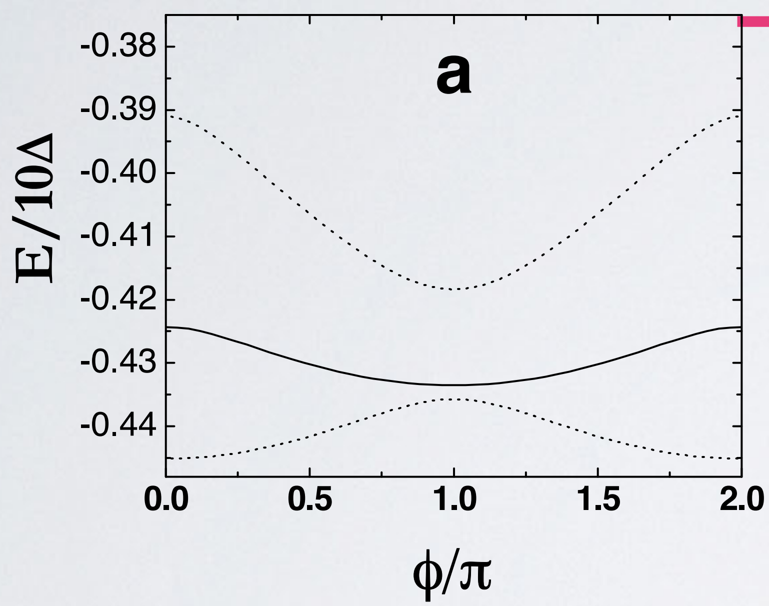
N = odd, S = 1/2 $\longrightarrow I_s = I_c \sin(\Delta\varphi + \pi)$



0-PI TRANSITION S-D-S JUNCTIONS

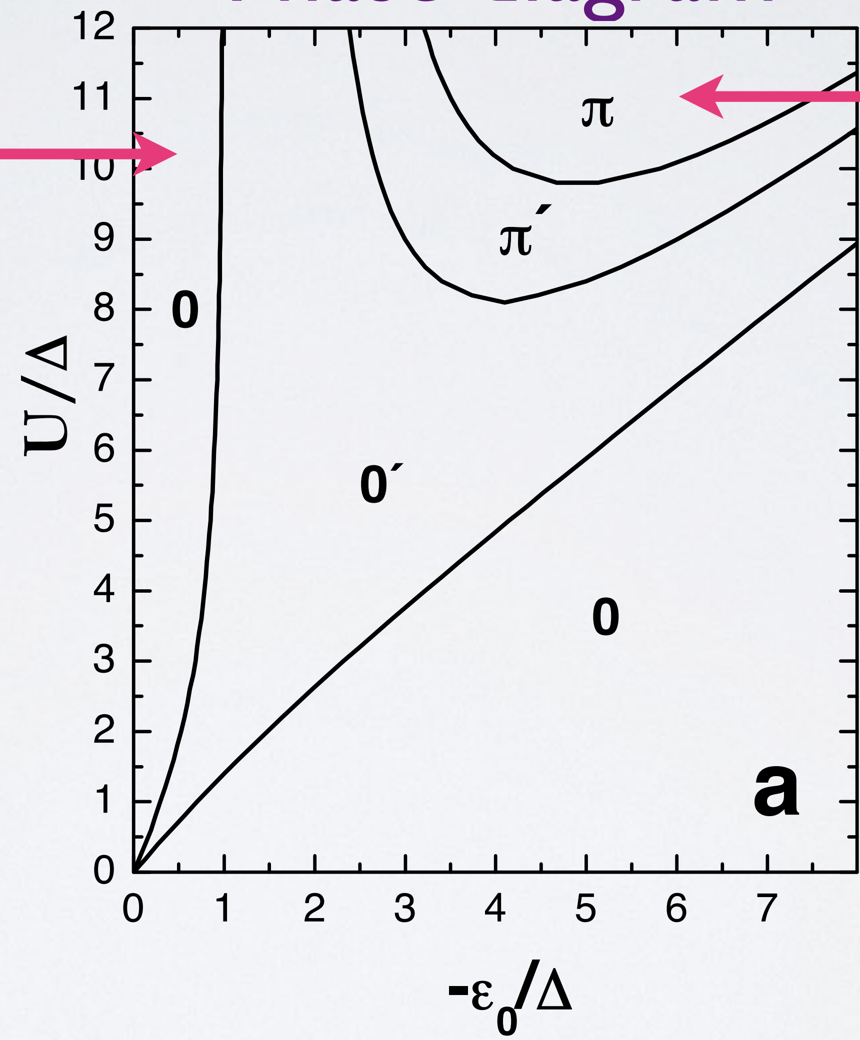
E. Vecino, A. Martín-Rodero, and A. Levy Yeyati, Phys. Rev. B 68, 035105 (2003)

$\Delta < k_B T_K$

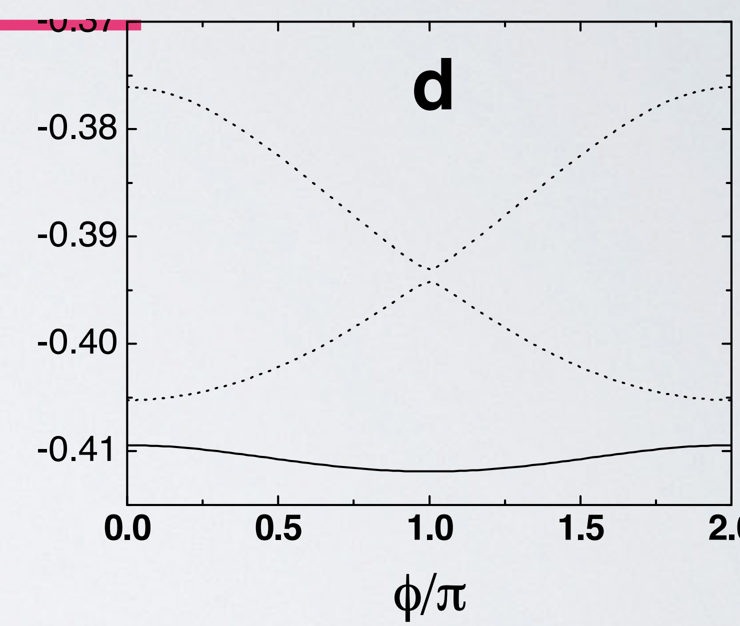


Singlet GS
(Kondo)

Phase diagram



$\Delta > k_B T_K$



Doublet GS
(~isollated S=1/2)

E. Vecino, A. Martín-Rodero, and A. Levy Yeyati, Phys. Rev. B 68, 035105 (2003) Perturbation theory
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M.-S. Choi, M. Lee, K. Kang, and W. Belzig, Phys, Rev. B 70, 020502 (R) (2004) NRG
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Fractional spin and Josephson effect in time-reversal-invariant topological superconductors

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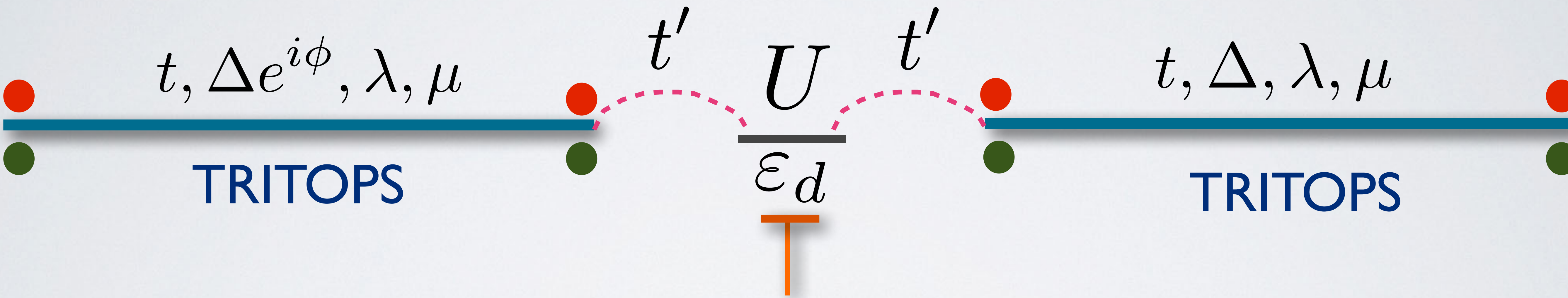
Phys. Rev. Lett. 119, 046801 (2017)



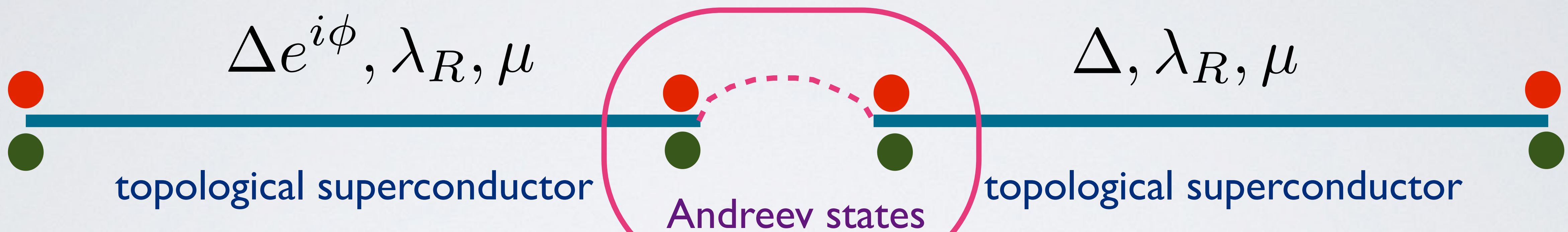
$$H = \sum_{\alpha=L,R} (H_\alpha + H_{c,\alpha}) + H_d$$

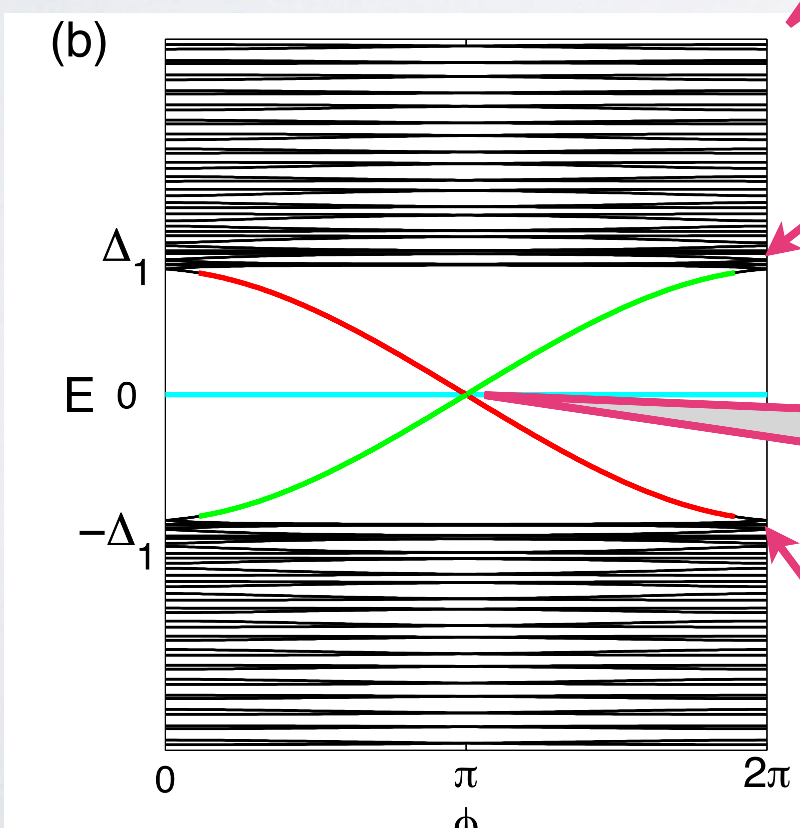
$$H_\alpha = \sum_{\sigma,j=1}^{N} \Big(-t c^\dagger_{\alpha,j+1,\sigma} c_{\alpha,j,\sigma} + i\lambda s_\sigma c^\dagger_{\alpha,j+1,\sigma} c_{\alpha,j,\sigma} - \mu\, n_{\alpha,j,\sigma} + \Delta e^{i\varphi_\alpha} s_\sigma c^\dagger_{\alpha,j+1,\sigma} c^\dagger_{\alpha,j\bar{\sigma}} + H.c.\Big), \quad (1)$$

$$H_{c,\alpha} = -t' \sum_\sigma \left(c^\dagger_{\alpha,1,\sigma} d_\sigma + H.c.\right)$$

$$H_d = \varepsilon_d \sum_{\sigma=\uparrow,\downarrow} n_{d,\sigma} + U n_{d\uparrow} n_{d\downarrow}.$$

JOSEPHSON JUNCTION





Odd parity

$$|\mu| < 2\lambda_{\mathrm{R}}$$

4-fold symmetry protected crossing: 4π periodicity

Even parity

TRITOPS-QD-TRITOPS. U=0

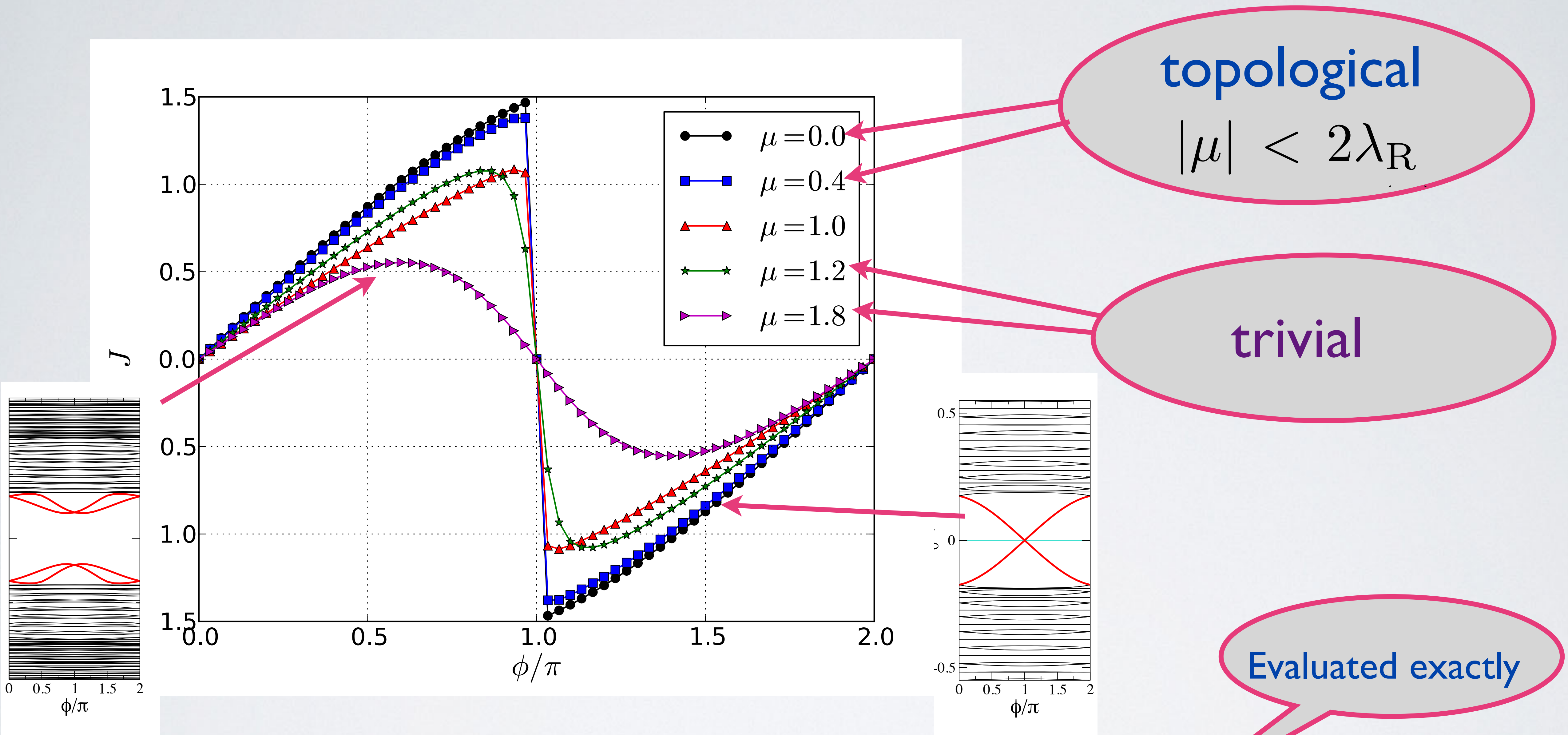
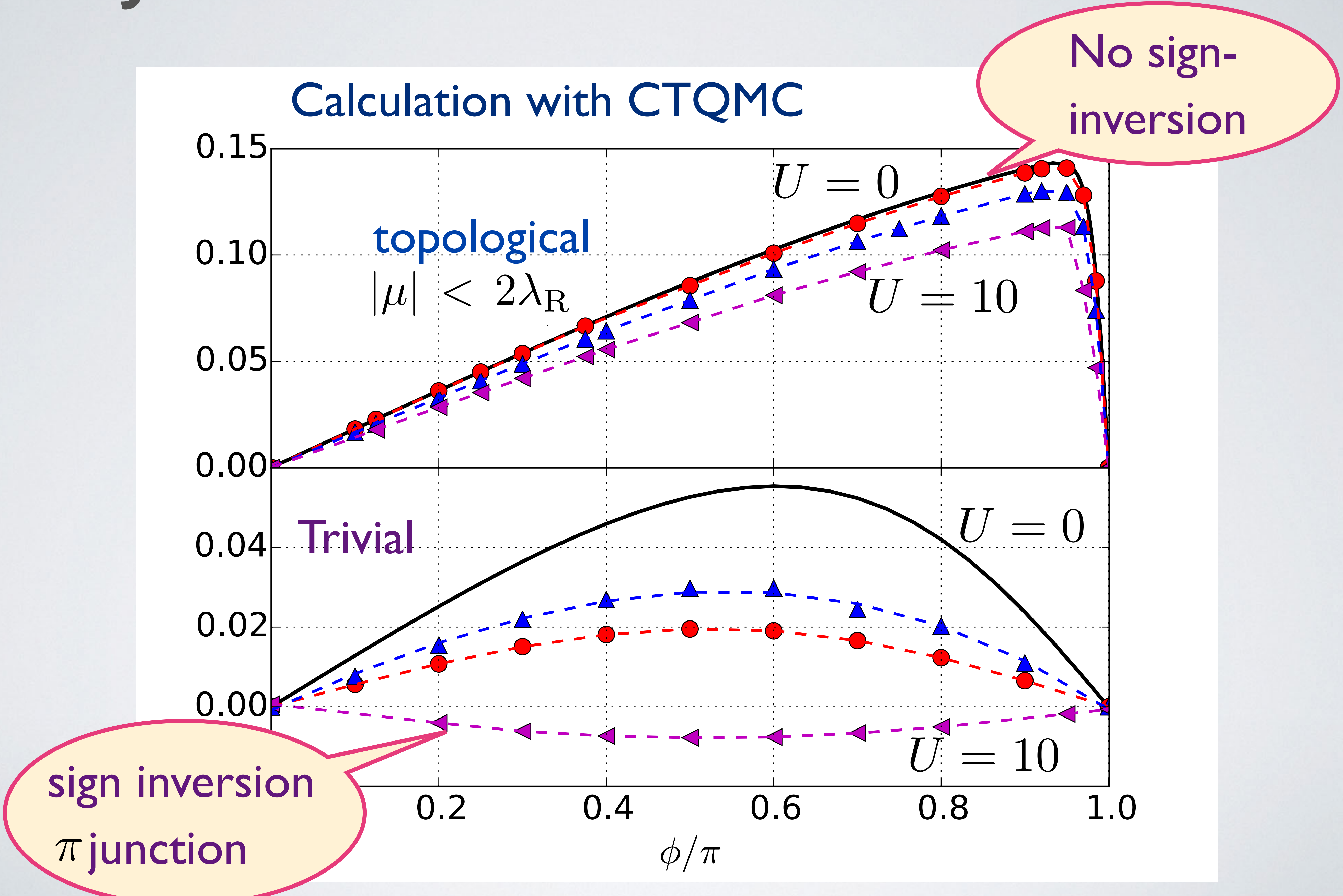


FIG. 2. (Color online) Josephson current for the quantum dot with $U = 0$, $t' = t$, $\varepsilon = 0$, $\lambda = t/2$. The length of the superconducting wires is $N = 100$ sites. The inverse of the temperature is $\beta = 400$. Energies are expressed in units of $t = 1$.

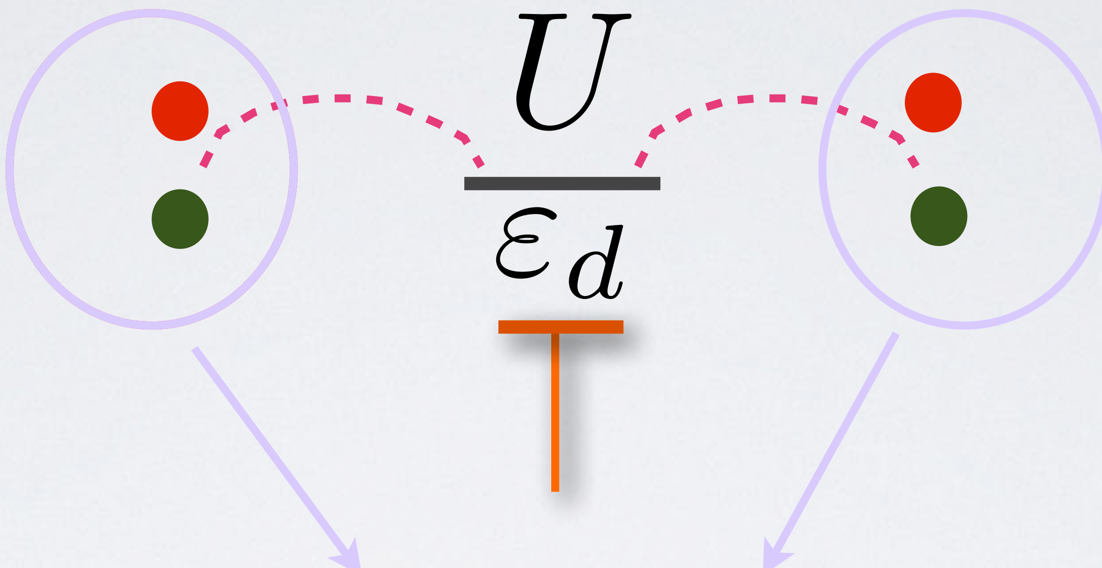
$$J = -2t' \sum_{\sigma} \mathrm{Im}\left[\langle c^{\dagger}_{\alpha,1,\sigma} d_{\sigma}\rangle\right]$$

$$= \frac{2t'^2}{\beta} \sum_{\sigma} \sum_{n} \mathrm{Im}\left[g^{(12)}_{1\alpha,\sigma}(i\omega_n) G^{(21)}_{d,\sigma}(i\omega_n)\right]$$

JOSEPHSON CURRENT U#0



EFFECTIVE HAMILTONIAN I



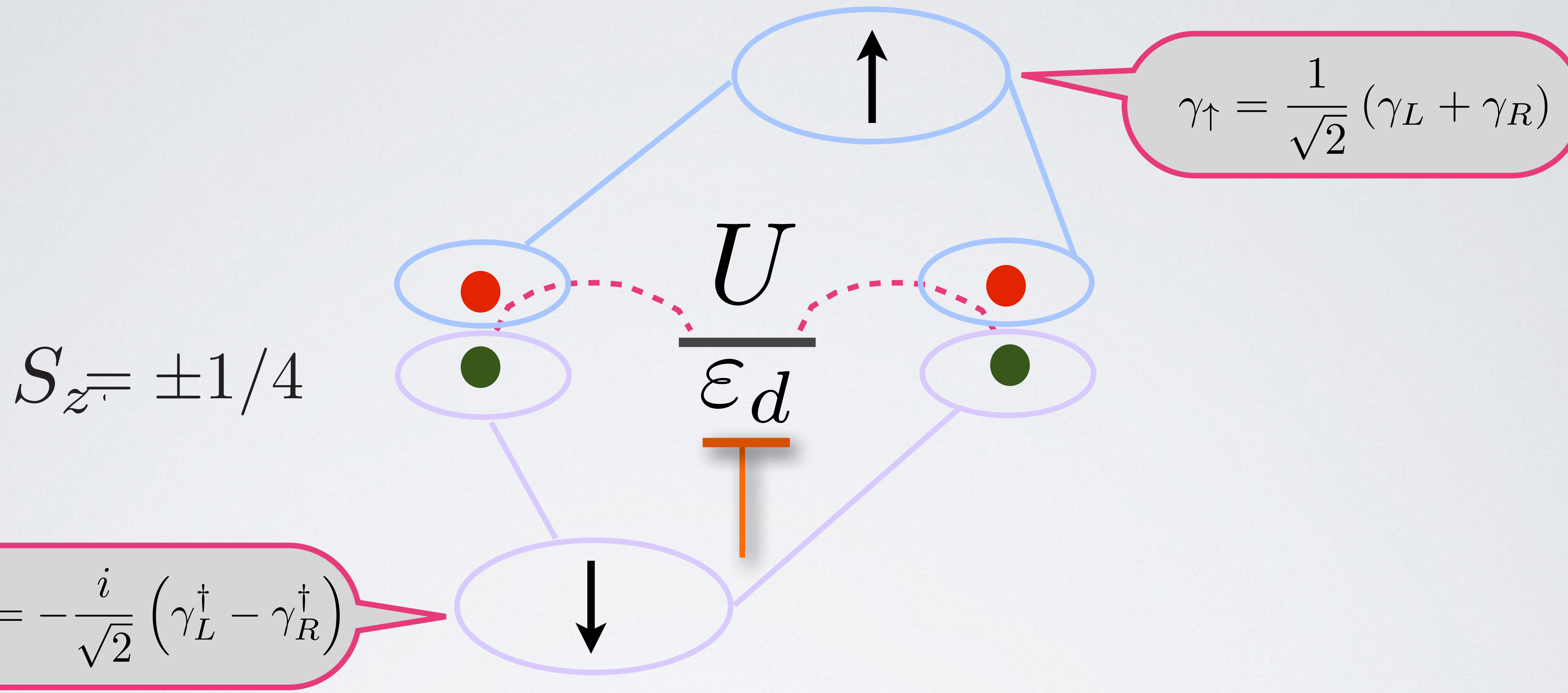
$$H_{\rm eff} = t_0 e^{i\phi/4} \sum_\sigma \left(\gamma^\dagger_{L,\sigma} d_\sigma + d^\dagger_\sigma \gamma_{R,\sigma}\right) + H.c. + H_d.$$

$$\gamma^\dagger_L = \gamma^\dagger_{L,\uparrow} = i\gamma_{L,\downarrow}$$

$$\gamma^\dagger_R = \gamma^\dagger_{R,\uparrow} = -i\gamma_{R,\downarrow}$$

Zero-energy states
(Bogoliubov q-particles)

EFFECTIVE HAMILTONIAN II



$$H_{\text{eff}} = \sum_\sigma \left(t_c \gamma_\sigma^\dagger d_\sigma - t_s \gamma_\sigma d_{\overline{\sigma}}\right) + H.c + H_d$$

$$H_{low} = J\{S_d^z \left[(n_L + n_R - 1) + i \sin\frac{\phi}{2}\left(\gamma_L^\dagger \gamma_R - \gamma_R^\dagger \gamma_L\right)\right]$$
$$+ i \cos\frac{\phi}{2}\left(S_d^- \gamma_L^\dagger \gamma_R^\dagger - S_d^+ \gamma_R \gamma_L\right)\}$$

GS is always singlet!!

H_{eff} vs MONTE CARLO

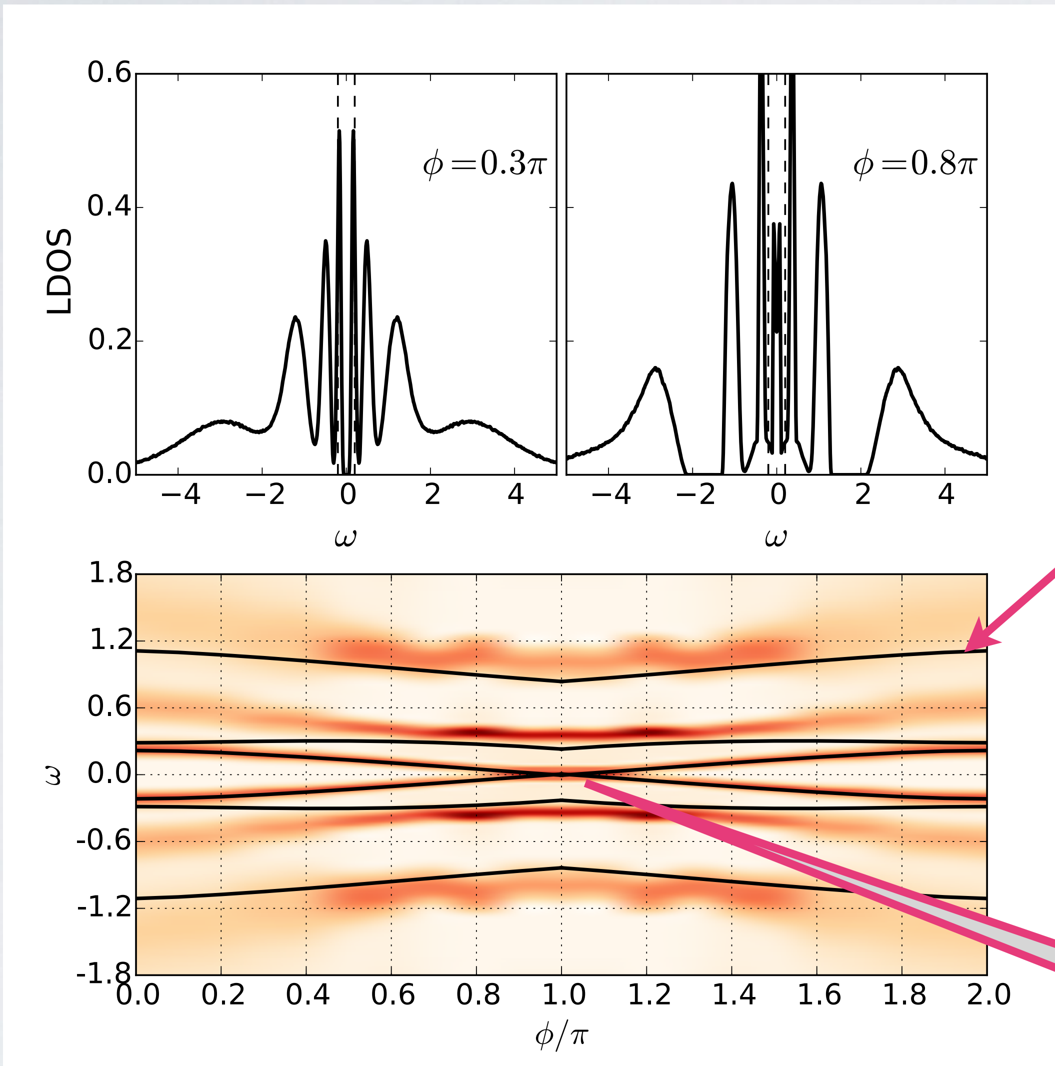


FIG. 5. (Color online) Upper panel: Local density of states at the quantum dot in the topological phase with $t' = t$, $\varepsilon = U/2$, $\lambda = t/2$, $\Delta = t/5$ and $\mu = 0$. On the left sub-panel $\phi = 0.3\pi$ and on the right $\phi = 0.8\pi$, as indicated. Lower panel: evolution of the spectrum as a function of ϕ with the same values of the parameters as above. The dark lines correspond to the prediction of H_{eff} with $t_0 = 0.3$, $U = 1.2$ with an additional on-site energy $\varepsilon_0 = 0.04$ at the non-interacting site, in order to simulate the coupling to the continuum at $\phi = 0$.

Effective Hamiltonian

$$\rho_\sigma(\omega) = -2\text{Im}[G^R_{d,\sigma}(\omega)]$$

Symmetry protected crossing

4-fold level degeneracy at $\phi = \pi$

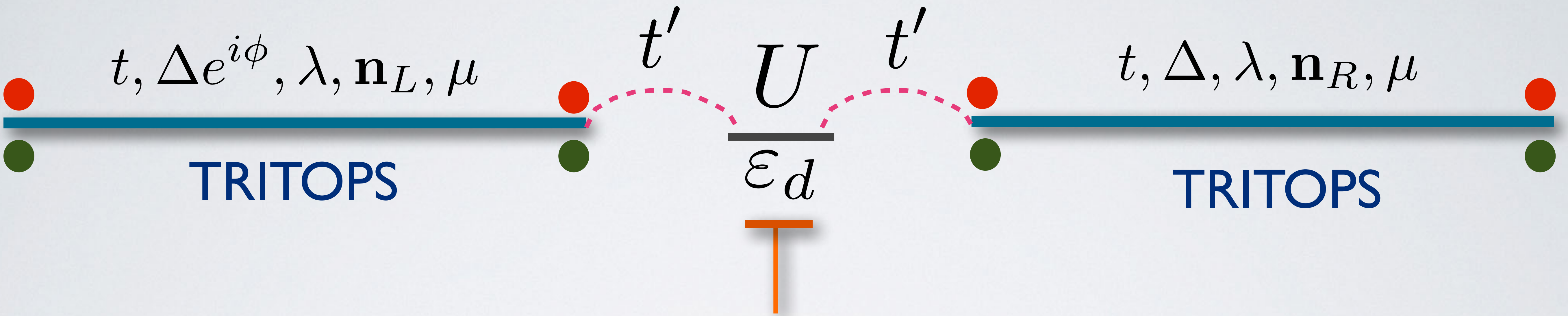
TRIPTOPS 1D
SIGNATURES OF ORIENTATION OF FRACTIONAL SPIN IN JOSEPHSON EFFECT

Catalogue of Andreev spectra and Josephson effects in structures with time-reversal-invariant topological superconductor wires

Liliana Arrachea,[1] Alberto Camjayi,[2] Armando A. Aligia,[3] and Leonel Gruñeiro[1]

Physical Review B 99, 085431 (2019)

TRITOPS-D-TRITOPS



$$H = \sum_{\alpha=L,R} (H_\alpha + H_{c,\alpha}) + H_d$$

$$H_\alpha = \sum_{i,j} \left[\psi^\dagger_{\alpha,i}\, h^\alpha_{ij}\, \psi_{\alpha,j} + \psi^\dagger_{\alpha,i}\, \Delta^\alpha_{ij}\, \psi^\dagger_{\alpha,j}\right] + \text{H.c.},$$

$$h^\alpha_{ij} = -\left(t_\alpha + i\lambda_\alpha \mathbf{n}_\alpha \cdot \boldsymbol{\sigma}\right) \delta_{j,i+1} - \mu_\alpha \delta_{i,j}$$

$$\Delta^\alpha_{ij} = \left(\tilde{\Delta}_\alpha\, \delta_{j,i+1} + \Delta_\alpha\, \delta_{i,j}\right)\, i\sigma_y$$

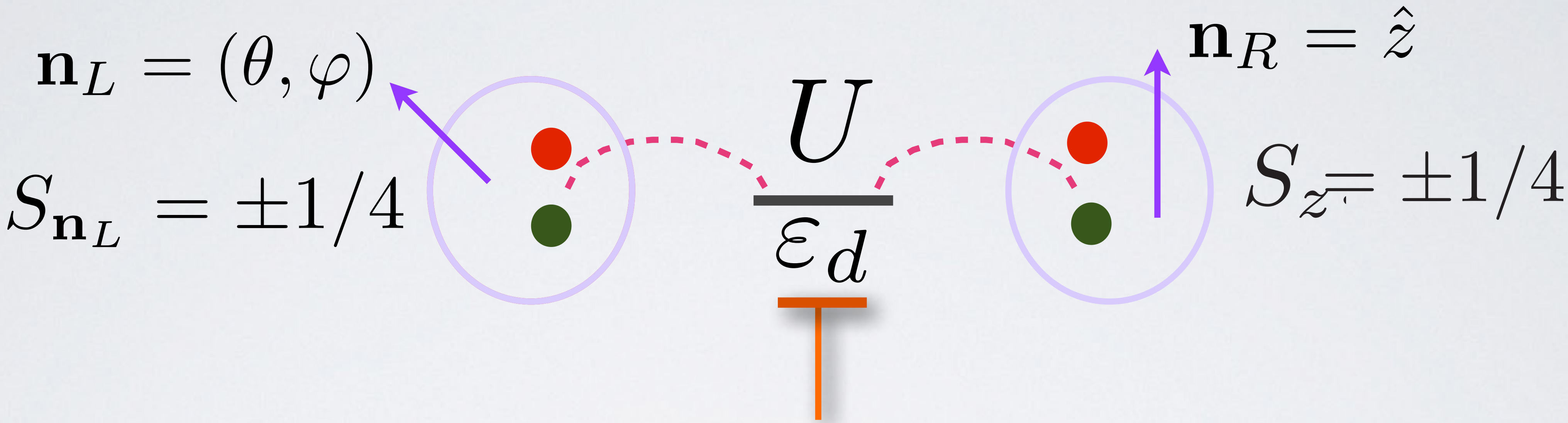
$$\psi^\dagger_{\alpha,j} = \left(c^\dagger_{\alpha,j,\uparrow}, c^\dagger_{\alpha,j,\uparrow}\right)$$

$$H_d = \varepsilon_d \sum_{\sigma=\uparrow,\downarrow} n_{d,\sigma} + U n_{d\uparrow} n_{d\downarrow}.$$

$$H_{c,\alpha} = -t' \sum_\sigma \left(c^\dagger_{\alpha,1,\sigma} d_\sigma + H.c.\right)$$

EFFECTIVE HAMILTONIAN II

$$\gamma^{\dagger}_{\alpha,+} = i\,\mathrm{sgn}\left(\lambda_\alpha \tilde{\Delta}_\alpha\right)\gamma_{\alpha,-}, \qquad \tilde{\gamma}^{\dagger}_{\alpha,+} = -i\,\mathrm{sgn}\left(\lambda_\alpha \tilde{\Delta}_\alpha\right)\tilde{\gamma}_{\alpha,-}.$$



$$H^{\mathrm{eff}}_{\mathrm{J,dot}} = H_L + t_\phi d^{\dagger}_{\uparrow}\gamma - i t_\phi d^{\dagger}_{\downarrow}\gamma^{\dagger} + \mathrm{H.c.} + H_d$$

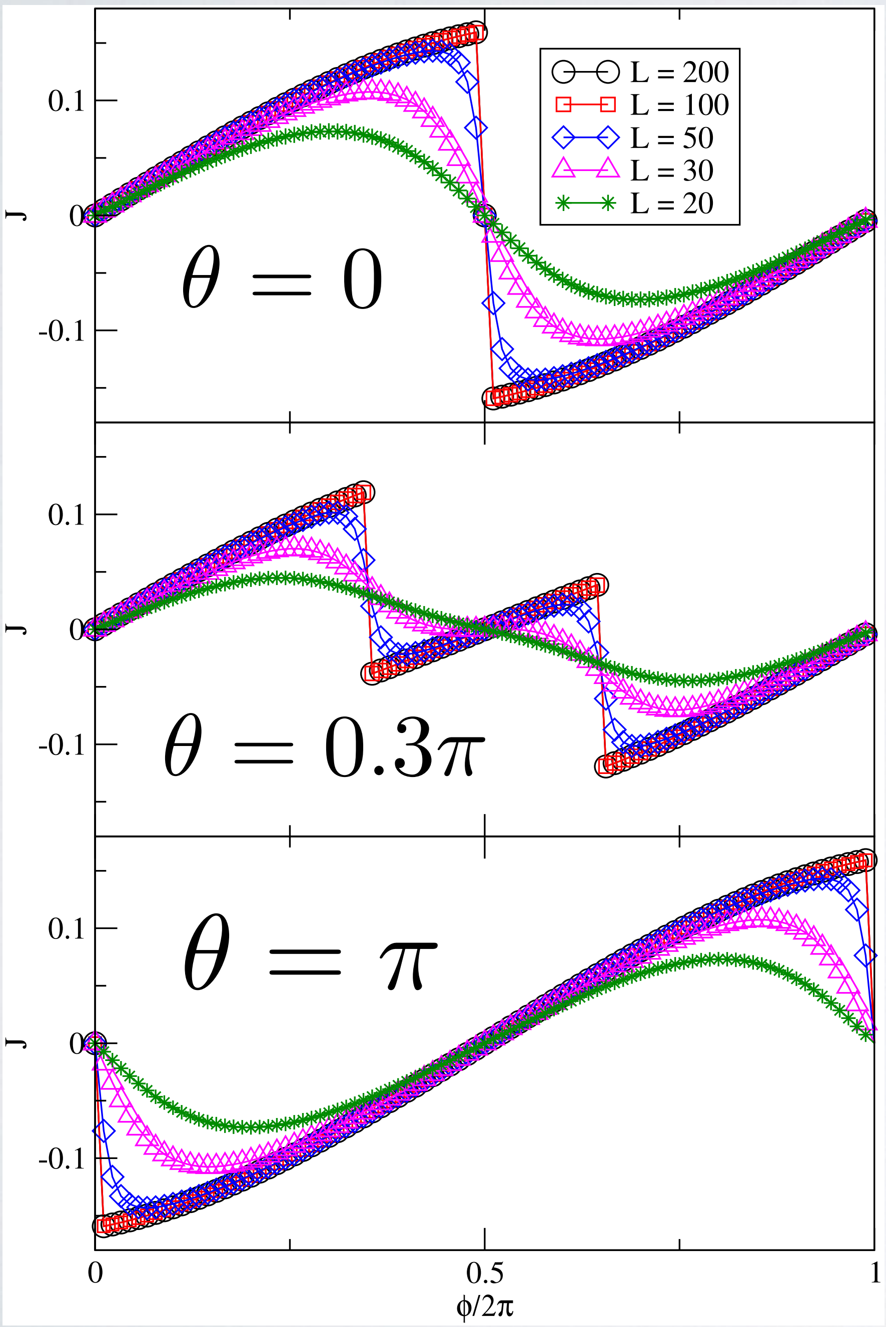
$$H_L = \sum_{s=\uparrow,\downarrow}\left(t_s \tilde{\gamma}^{\dagger} d_s + \delta_s \tilde{\gamma} d_s\right)$$

$$t_\uparrow = t_\phi \cos\frac{\theta}{2}, \quad t_\downarrow = t_\phi e^{i\varphi}\sin\frac{\theta}{2}, \quad t_\phi = t_J e^{i\phi/4}, \qquad \tilde{\gamma}_{L,\uparrow} = \cos\frac{\theta}{2}\tilde{\gamma} - i e^{i\varphi}\sin\frac{\theta}{2}\tilde{\gamma}^{\dagger}, \qquad \gamma_{R,\uparrow} = \gamma,$$

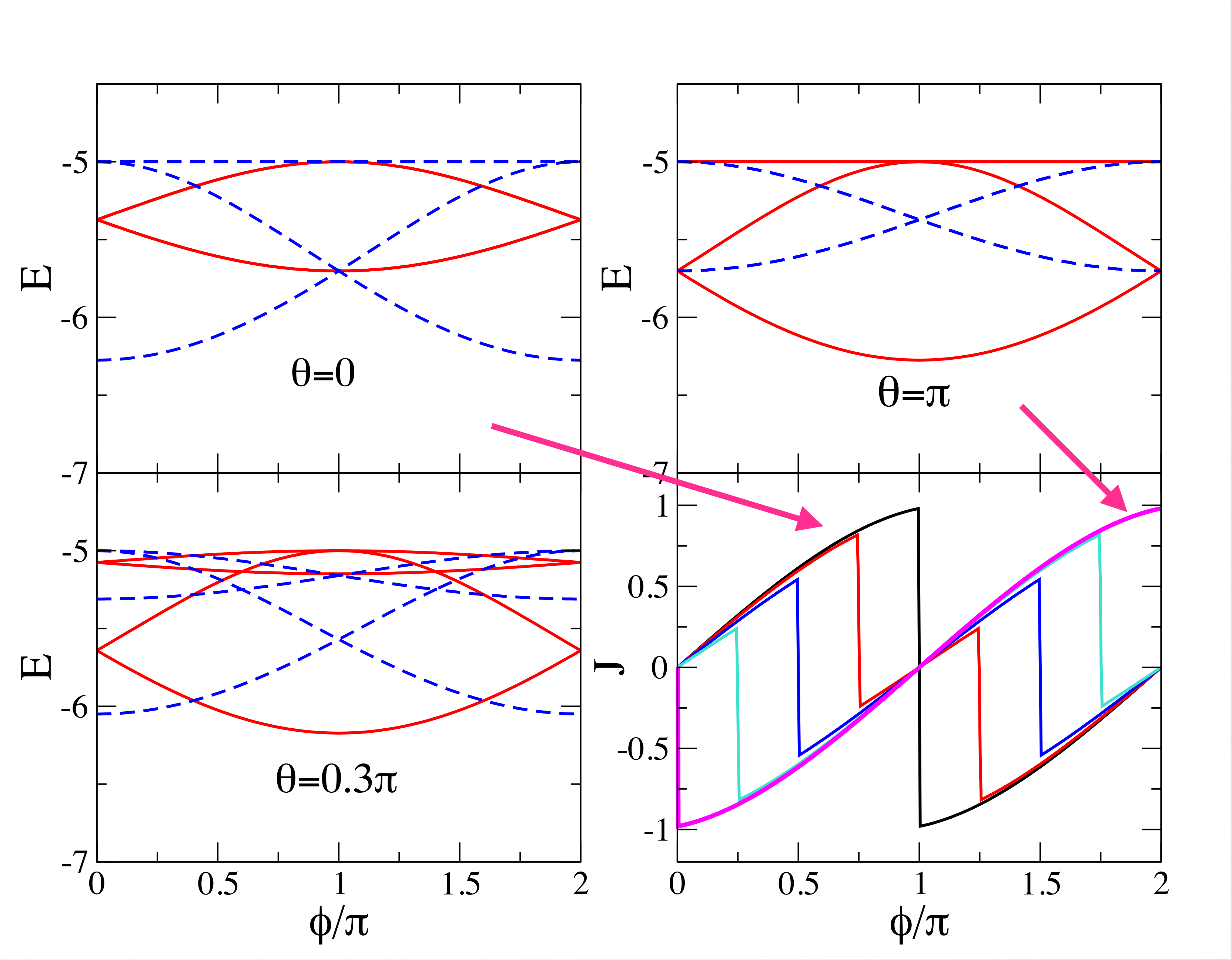
$$\delta_\uparrow = i t_\phi e^{-i\varphi}\sin\frac{\theta}{2}, \quad \delta_\downarrow = -i t_\phi \cos\frac{\theta}{2}. \qquad \tilde{\gamma}_{L,\downarrow} = e^{-i\varphi}\sin\frac{\theta}{2}\tilde{\gamma} + i\cos\frac{\theta}{2}\tilde{\gamma}^{\dagger}, \qquad \gamma^{\dagger}_{R,\downarrow} = i\gamma,$$

TRITOPS-QD-TRITOPS.

Exact, U=0

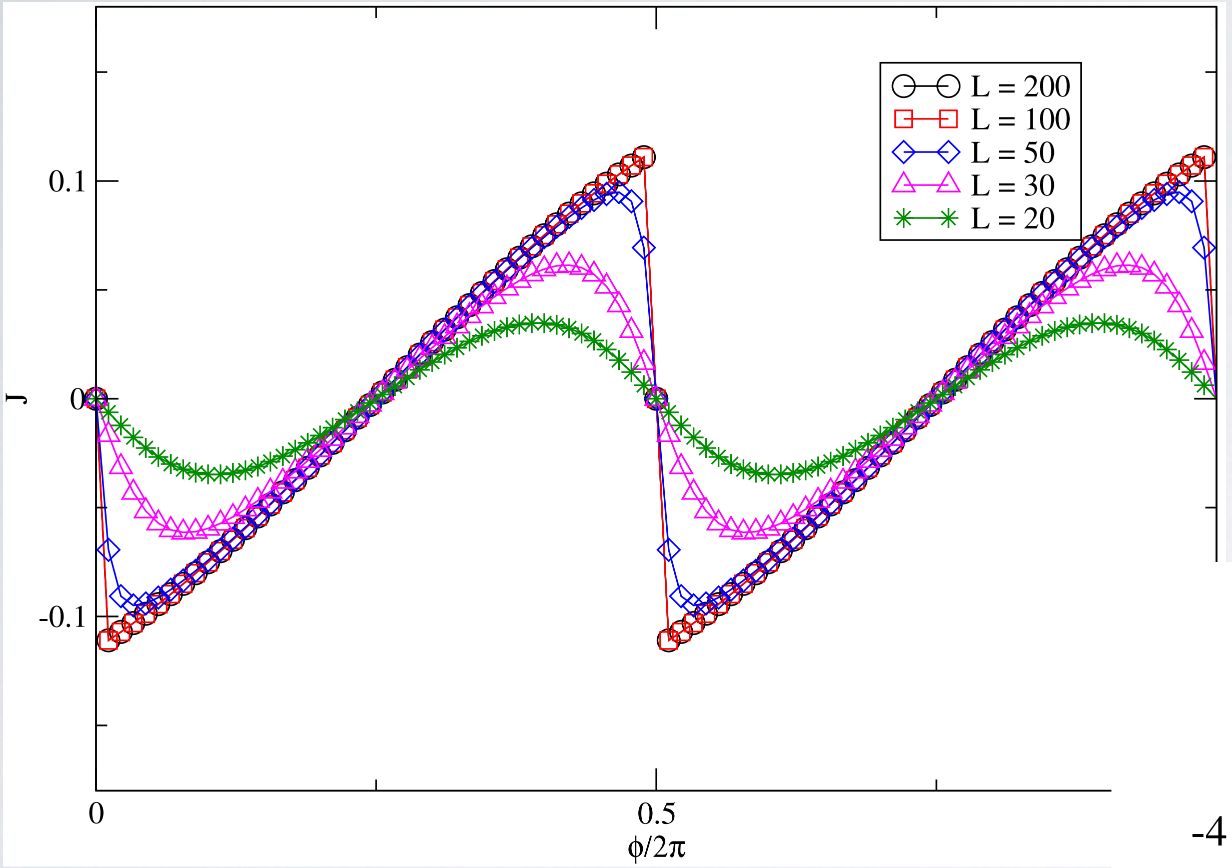


Effective Hamiltonian $U \neq 0$



No signatures of $0-\pi$ transition induced by U

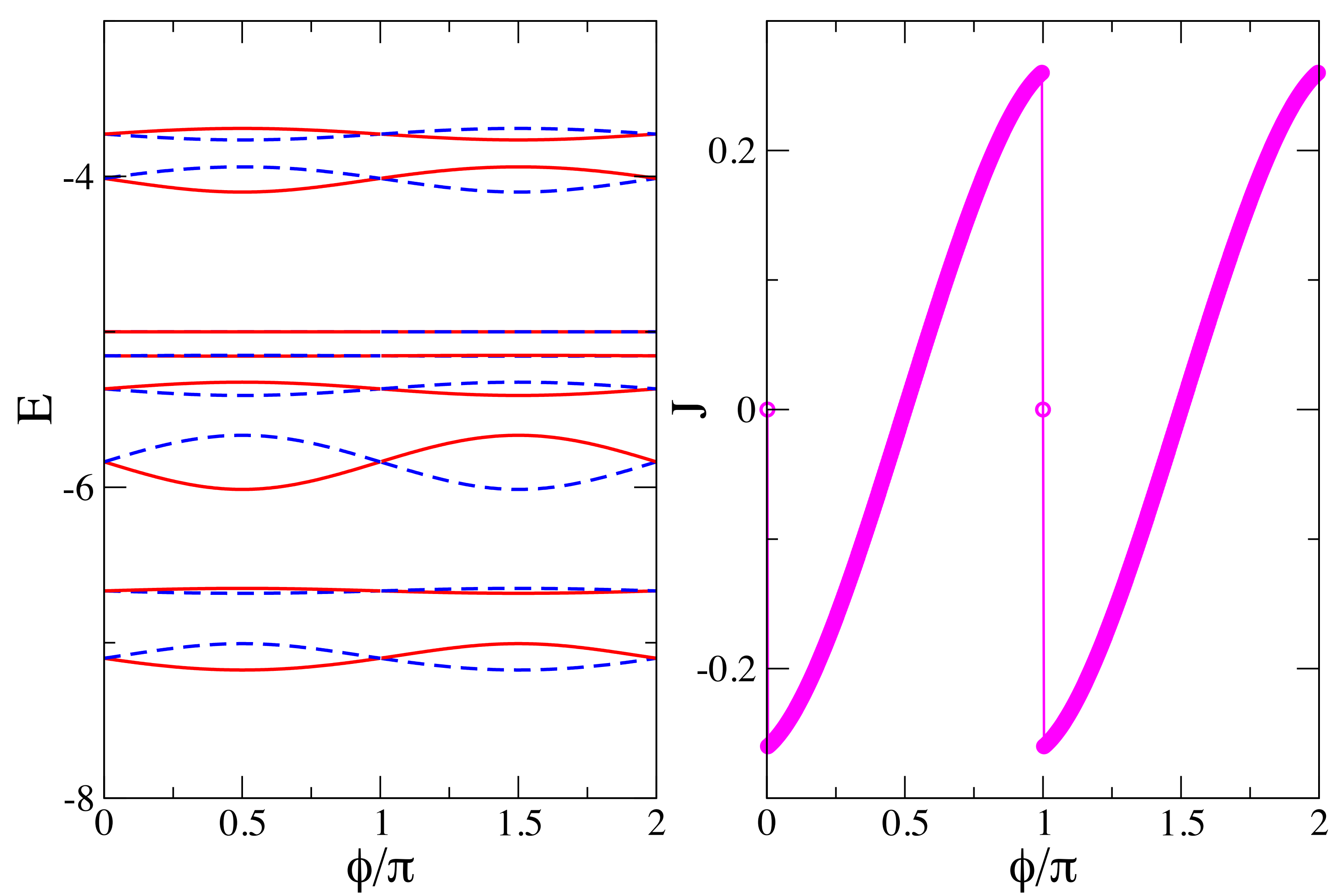
TRITOPS-QD-S.



Exact, U=0

Effective Hamiltonian

$U \neq 0$



OUTLOOK

- TRITOPS phase induced in thin films of BiSe by proximity to s-wave superconductors.
- Zero-energy states with Sz=1/4 at the ends combine at the junction to form 1/2-spin that screen the localized spin of the quantum dot: No transition to pi-junction!
- Signatures in Josephson junctions TRITOPS-TRITOPS and TRITOPS-TRS. Main features well described by low-energy effective Hamiltonians.
- To do: experimental setups to realize the TRIPTOPS phase.

THANK YOU!

Xul Solar, Argentina, 1937-1963