Magnet Design

Joint ICTP-IAEA Workshop on Accelerator Technologies, Basic Instruments and Analytical Techniques

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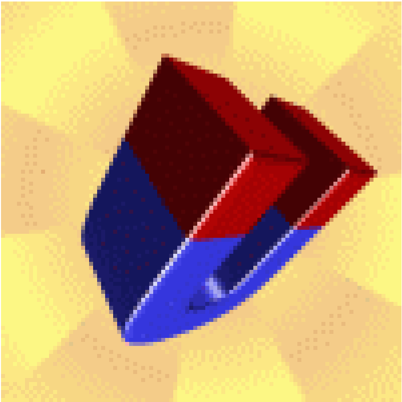
MAGNETS

1. Introduction
   Magnets in everyday life, in history,
   Understanding magnetism, Glossary,
   Units
2. General Principles of magnets
   Type, number of poles, Field shapes
   Pole shape, Fringe fields, Saturation,
   Shims, Field quality, Magneto-motive
   force
3. Magneto-motive force
   Dipole and Quadrupole
4. Magnetization of iron
   Hysteresis, permeability, materials
6. Magnet design
   Computer programs
   Steps in designing a magnet
   Design a magnet (example - tutorial)

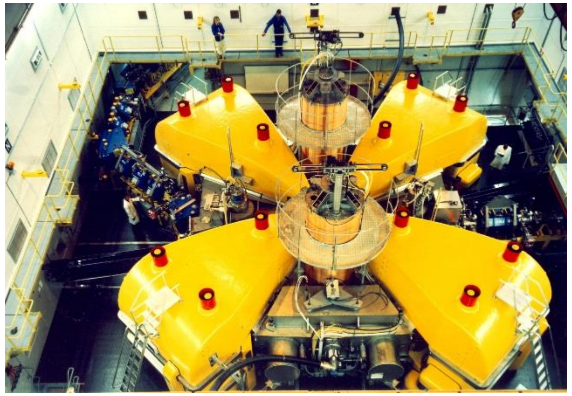


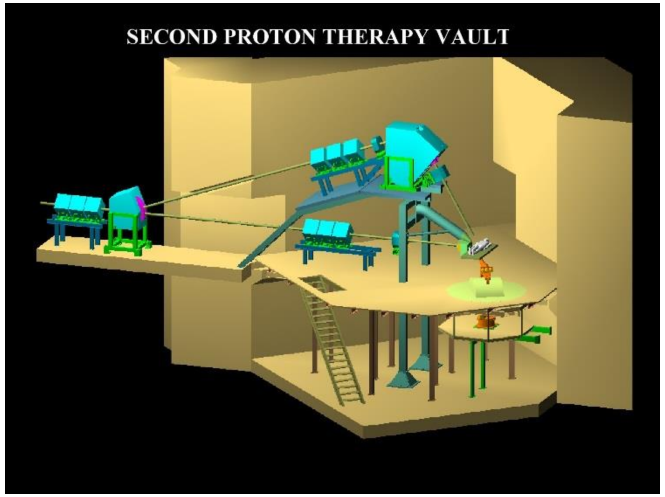
Magnets in Everyday Life

Rubber mat magnets

Magnets in Everyday Life

- Accelerators – mainly electromagnets.
  - •Dipoles for bending a charged particle beam
  - •Quadrupoles for focusing a beam
  - •Sextupoles, octupoles, etc for higher order beam corrections
  - •Fast deflecting magnets for beam injection and extraction
- Permanent magnets in vacuum pumps, gauges and sweeping devices, but nowadays also as beam optical devices
- Particle detectors





How strong are magnets?

Typical Values

Here is a list of how strong some magnetic fields can be:

Smallest value in a magnetically shielded room	10^{-14} Tesla
Interstellar space	10^{-10} Tesla
Earth's magnetic field	0.00005 Tesla = 0.5 Gauss
Small bar magnet	0.01 Tesla
Within a sunspot	0.15 Tesla
Small NIB magnet	0.2 Tesla
Big electromagnet	2 Tesla
Surface of neutron star	100,000,000 Tesla
Magstar	100,000,000,000 Tesla

What is a Tesla? It is a unit of magnetic flux density. It is also equivalent to these other units:

1 weber per square meter
10,000 Gauss (10 kilogauss)
10,000 magnetic field lines per square centimeter
65,000 magnetic field lines per square inch.

1Gauss is about 6.5 magnetic field lines per square inch.

If you place the tip of your index finger to the tip of your thumb, enclosing approximately 1 square inch, four magnetic field lines would pass through that hole due to the earth's magnetic field!

Some Units and Conversion Numbers in Electromagnetism

- Charged Particle properties
- Particle energy : 1eV = (1.6×10^{-19} C)(1V) = 1.6×10^{-19} J
- Current ***i*** in ampere (A), current density ***j*** in (A/m^2)
- Number of conductor turns in a coil is N
- Magnetic Field Strength **H** : 1 Oe = ($10^3/4\pi$) A/m = 79.58 A/m (mmf)
- Magnetic Flux ϕ : 1 Wb = 1 Vs
- Magnetic Flux Density **B** : 10^4 G = 1 Wb/m^2 = 1 Vs/m^2 = 1 T
- Permeability of any material = $\mu = \mu_0 \mu_r$ (unit = Vs/Am = H/m)
- Permeability of vacuum = $\mu = \mu_0 \mu_r = (4\pi \times 10^{-7}) \times \mathbf{1} = 4\pi \times 10^{-7}$ H/m

Magnetic Flux Density in relation to its magneto-motive force (mmf) :

$\mathbf{B} = \mu \mathbf{H}$

TYPES OF MAGNETISM

A. DIAMAGNETISM

- due to the modification of the electron orbit magnetic moment by an external field (a pure orbit effect)
- Present in all materials, independent of temperature
- Shows no hysteresis
- Very weak

B. PARAMAGNETISM

- atoms present a permanent magnetic moment, e.g. odd number of electrons
  (mostly an electron spin effect)
- Incomplete inner electronic shells (transition and rare earth elements)
- Can be orders of magnitude bigger than diamagnetism
- No hysteresis effect

C. FERROMAGNETISM

- Larger inter-atomic distances
- Electron spin effect – line up from atom to atom – polarization
- “conduction electrons” from the 4s-shell free to wander between atoms

TYPES OF MAGNETS

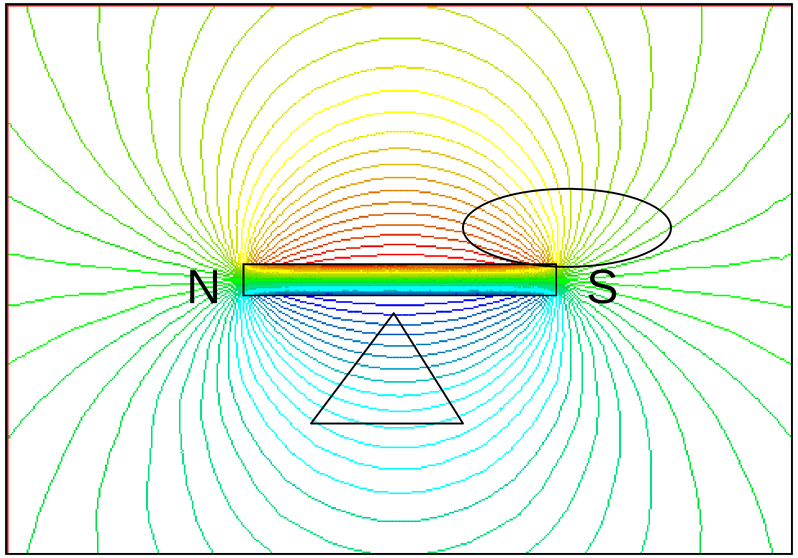
A. Permanent Magnets (magneto-motive force from intrinsic material properties)

B. Electro-magnets (magneto-motive force generated from applied electric current)

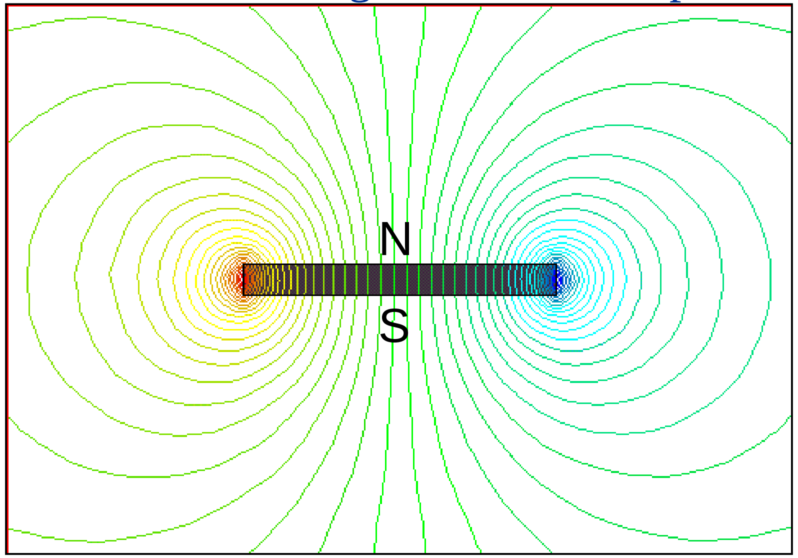
    DC-current

    AC-current (pulsed, eddy-currents, laminations)

    Super-conducting electro-magnets and materials



Permanent Magnets : Field Shape



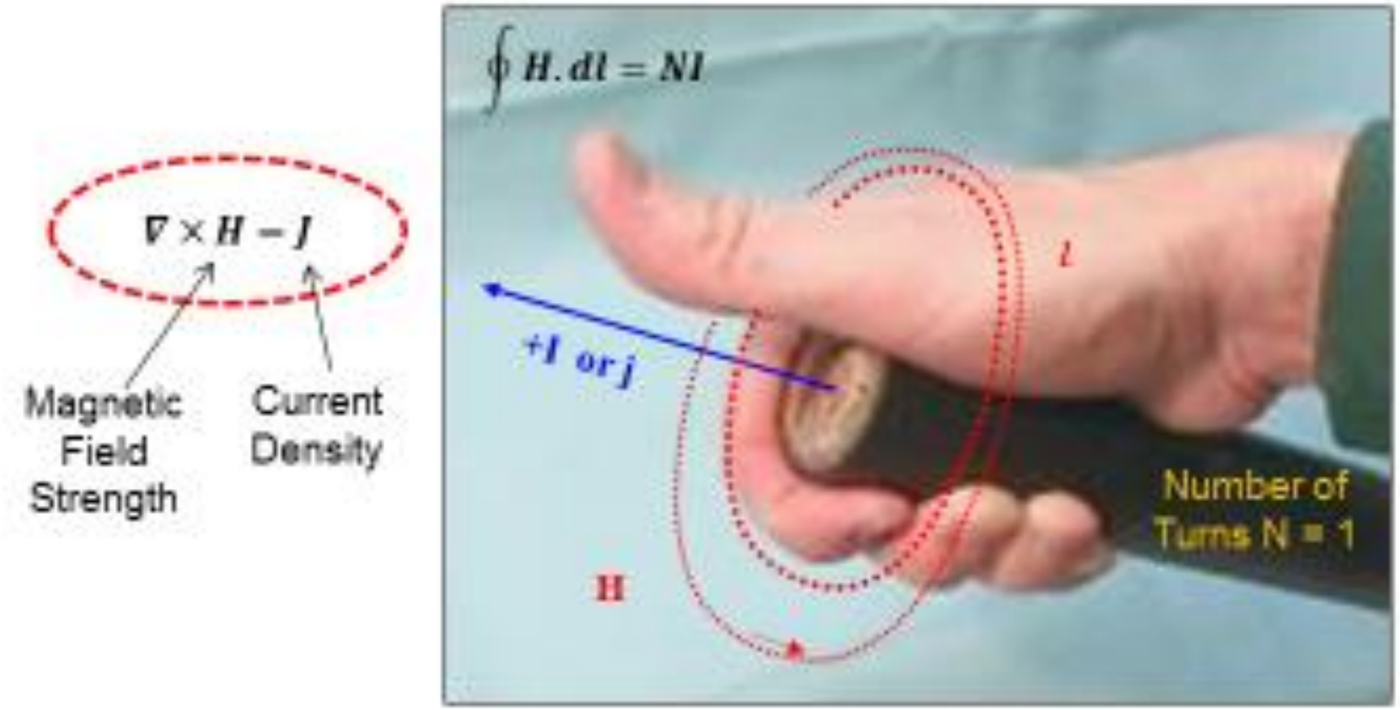
VISUAL PERCEPTION - FIELD LINES

GENERAL RULES FOR USING LINES TO VISUALIZE MAGNET FIELDS

1. Any line (all lines) must close on itself or end according to a specified boundary condition.
2. Lines may NOT cross or touch.
3. Lines usually cross an air/iron interface perpendicularly.
4. The higher the field density, the denser the line representation.

$\nabla . B = 0$

SIMPLE METHOD TO FIND THE SOURCE AND DIRECTION OF MAGNETIC FIELD LINES



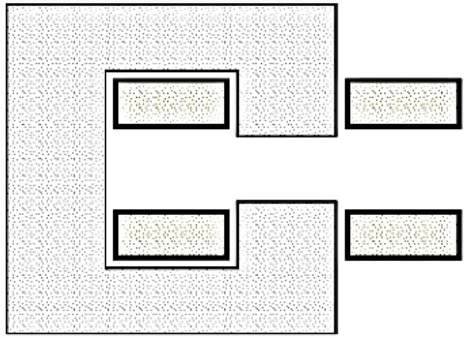
Electro-Magnets : n = 2, 4, 6, etc. poles

Different Dipole geometries

C Magnet

Advantages:
- Easy asses
- Simple design

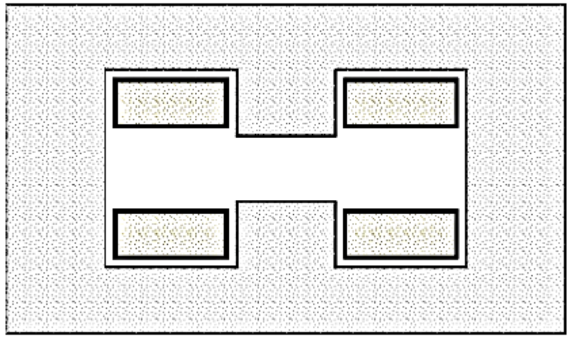
Disadvantages:
- Pole shims needed
- Field asymmetric
- Less rigid



H Magnet

- Advantages
- Symmetric
- Rigid

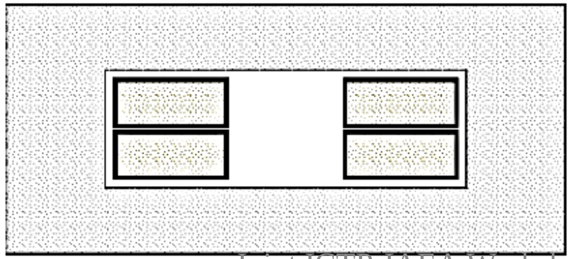
Disadvantage:
- Need shims
- Difficult to access

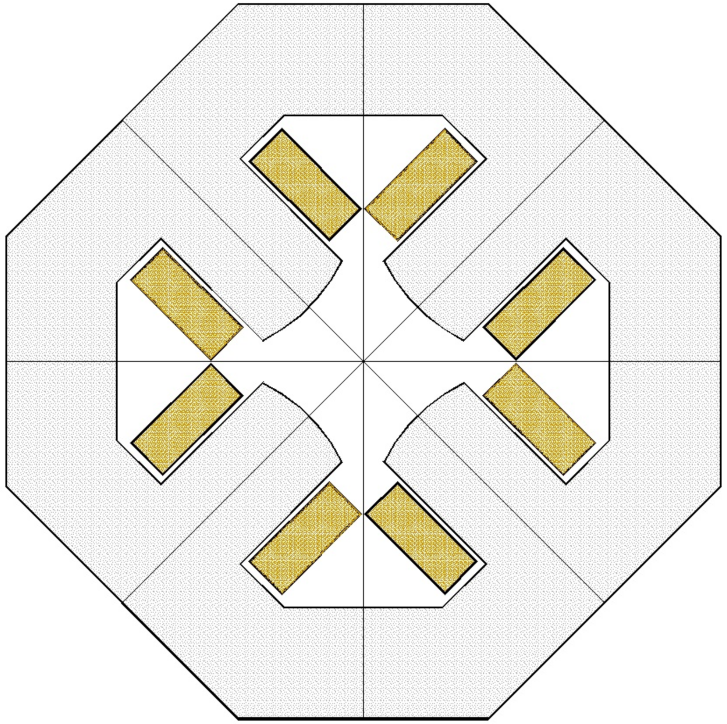


Window frame Magnet

Advantages:
- No shims
- Symmetric
- Compact
- Rigid

Disadvantages:
- Access problems
- Insulation thickness





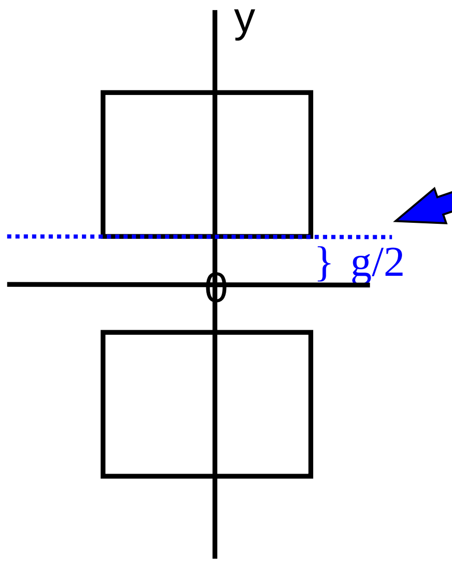
Dipole for bending/steering a beam

Quadrupole for focussing/defocussing a beam

Higher orders for creating magnetic bottles, beam profile shaping and corrections to inadequate fields from other magnets

Combinations, active and passive components

Electro-Magnets : Pole Shape



For normal fields:

Dipole:

$Y = \pm g/2$;

(g is pole gap).

Quadrupole:

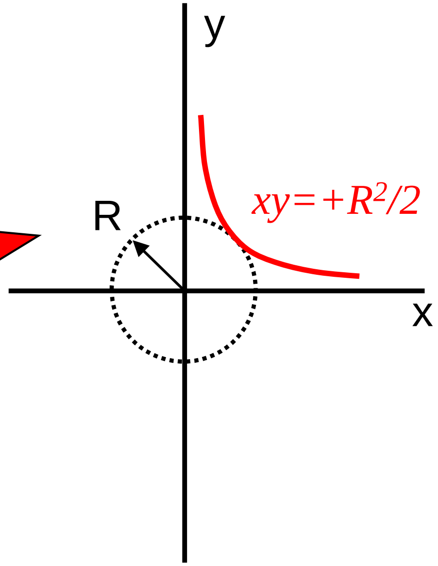
$xy = \pm R^2/2$;

(R is radius of pole opening).

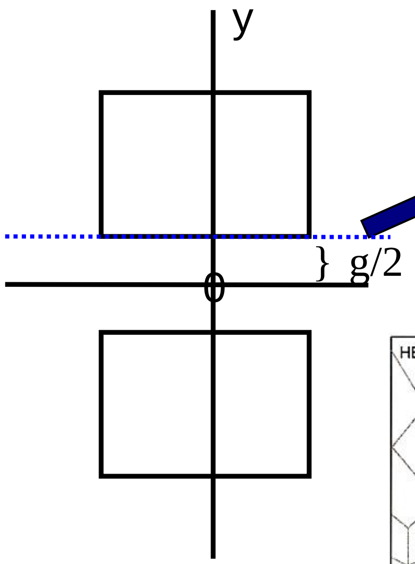
Sextupole:

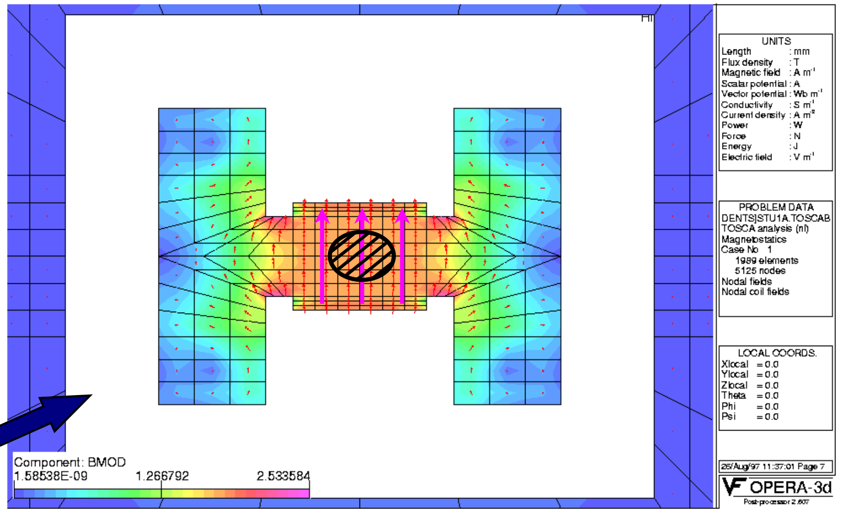
$3x^2y - y^3 = \pm R^3$;

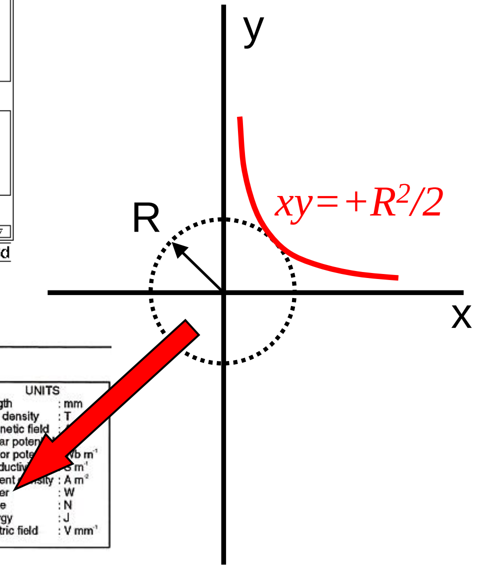
Equations of ideal pole shape

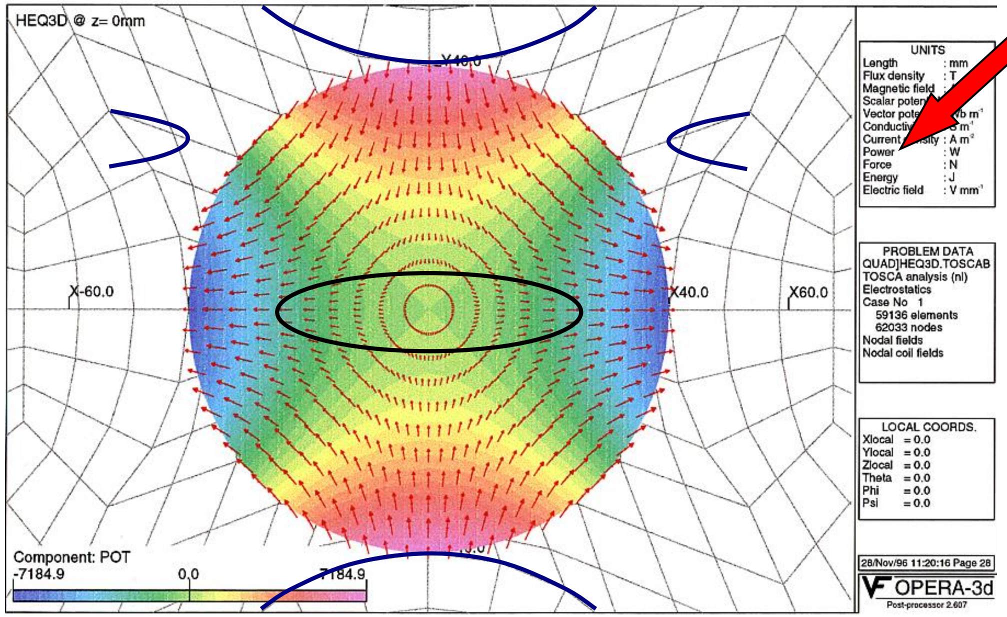


Electro-Magnets : Field Shape

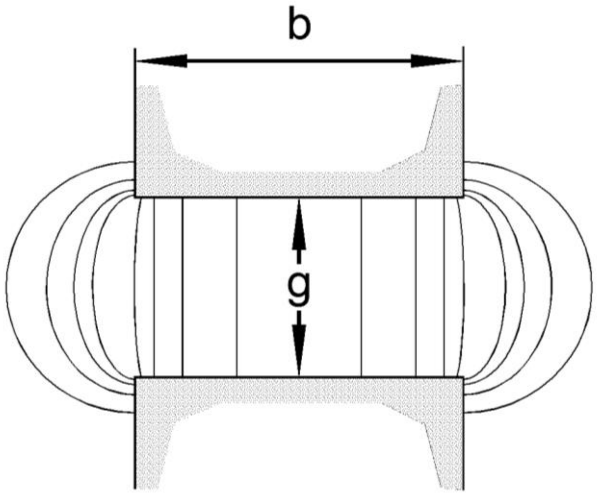




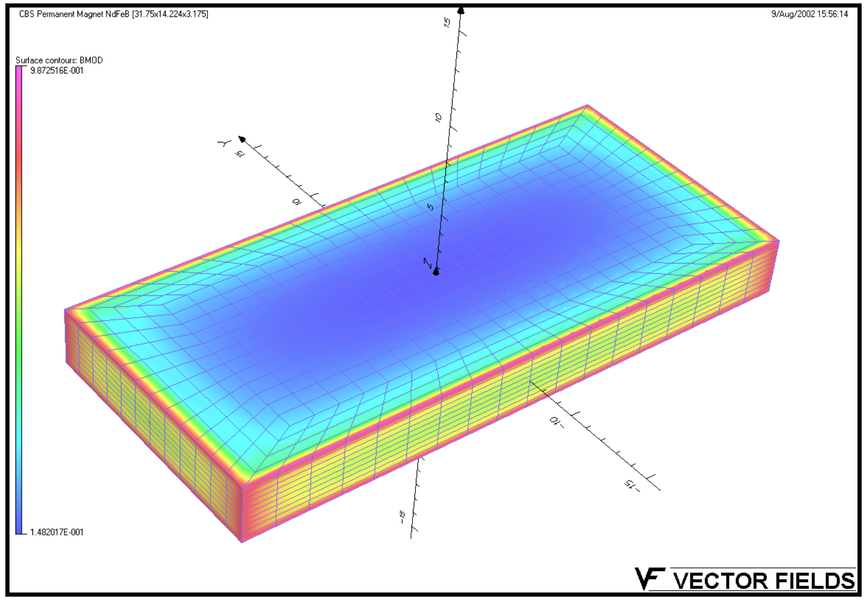




Electro-Magnets : Fringe Fields & Field Saturation

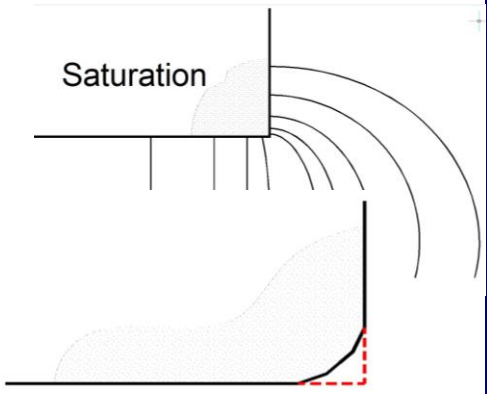


Magnetic field distribution and magnet ends



Square ends:

- Display non linear effects due to saturation
- Influence the radial distribution in the fringe field



Chamfered ends:

- Magnetic length better define
- Prevent saturation

Control of the longitudinal field at magnet ends

Control of the longitudinal field at magnet ends

MAGNETO-MOTIVE FORCE : DIPOLE MAGNET

Ampere's Law $\oint \boldsymbol{H}.\boldsymbol{dl} = \boldsymbol{NI}$ (ampere-turns)

$NI = \oint \boldsymbol{H}.\boldsymbol{dl} \quad = (H_{air}.g_{air} + H_{iron}.l_{iron})$;

$\boldsymbol{H}=\boldsymbol{B}/\mu$

$NI = \oint \boldsymbol{B}/\mu.\boldsymbol{dl} = B_{air}.g_{air}/\mu_0 + B_{iron}.l_{iron}/\mu_{iron}$

neglect 2nd term with μ_{iron} about 5000 larger then` μ_0

$NI = B_{air}.g/\mu_0$

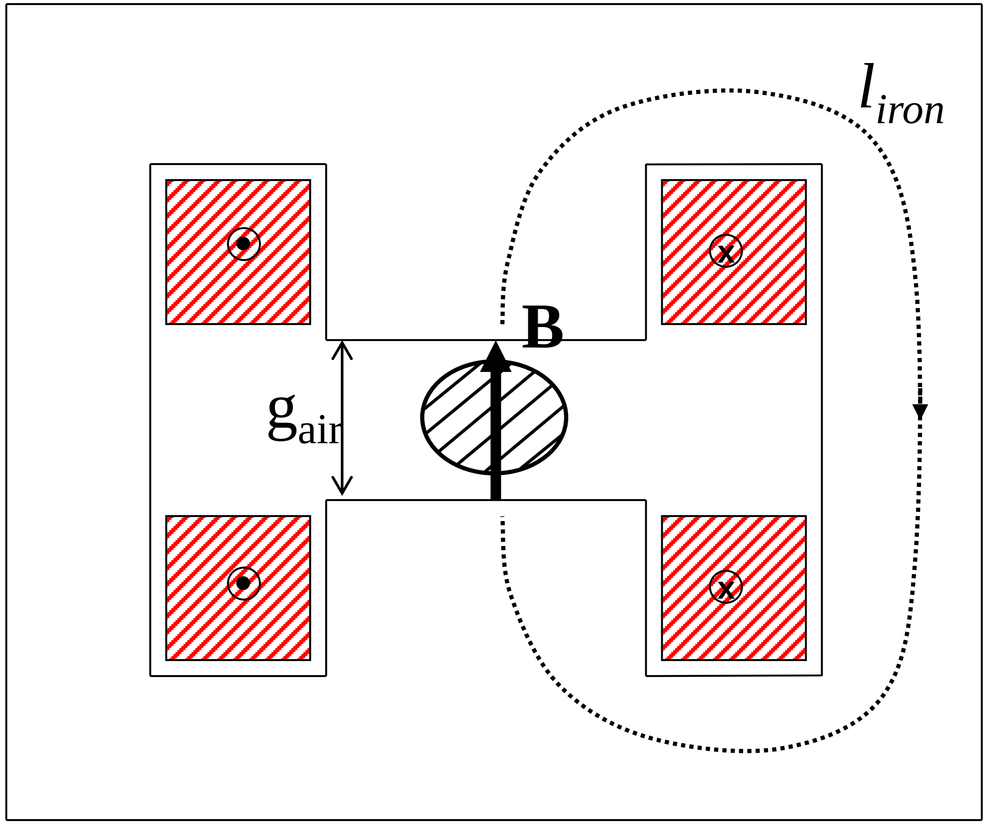
$\mu_0 = 4\pi \times 10^{-7}$ *(webers/amperemeter)*

Electrical power $P = I^2 R_0 \propto g^2$

$R_0 = \rho\ L/A,$

with ρ = resistivity of conductor material

L = length of the conductor and A the crossectional area of the coil



Saturation effect : keep field in yoke < 1.5 T by providing enough area of steel.

MAGNETO-MOTIVE FORCE : QUADRUPOLE MAGNET

Quadrupole with hyperbolic pole faces and with aperture a, such that the field at radius r from the axis is $\boldsymbol{B(r)} = K.r$

Ampere's Law

$\int \boldsymbol{H}.d\boldsymbol{l} = N\boldsymbol{I}$ (ampere-turns)

$NI = \int \boldsymbol{H}.d\boldsymbol{l} \quad = (\frac{B_{air}}{\mu_0} g_{\text{air}} + \frac{B_{iron}}{\mu_0 \mu_r} l_{\text{iron}})$;

$NI = {}_0\int^{a} B(r)/\mu_0 .dr$ + (iron path)

+ (path perpendicular to field)

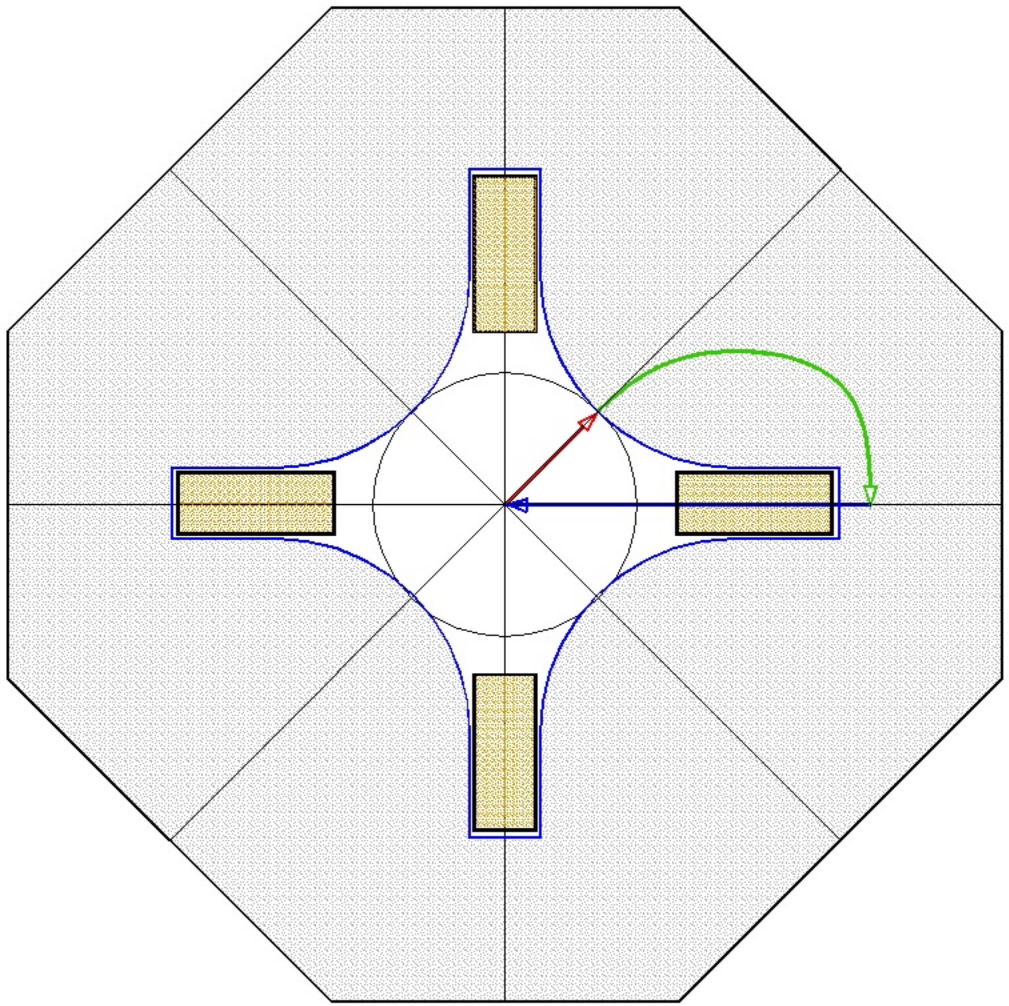
On the first path (red) B(r) = K.r/μ_0. The second integral (green) is very small for $\mu_r >> 1$. The third integral (blue) vanishes since B is perpendicular to the direction of integration, ds. So we get in good NI

$NI = (1/\mu_0)\ {}_0\int^{a} K\boldsymbol{r}.d\boldsymbol{r}$

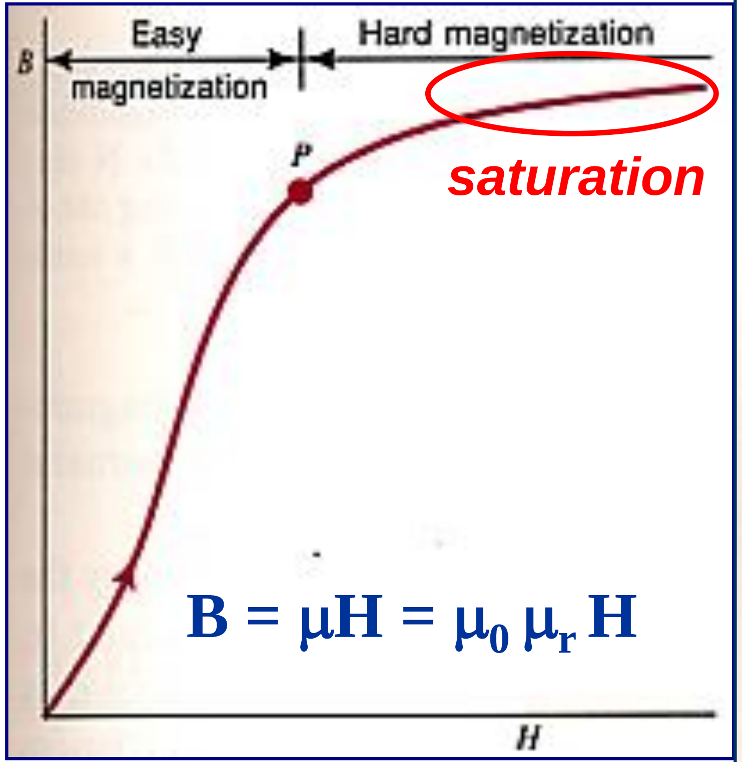
$NI = (1/\mu_0)\ Ka^2/2,$ but $Ka = B_{poletip}$

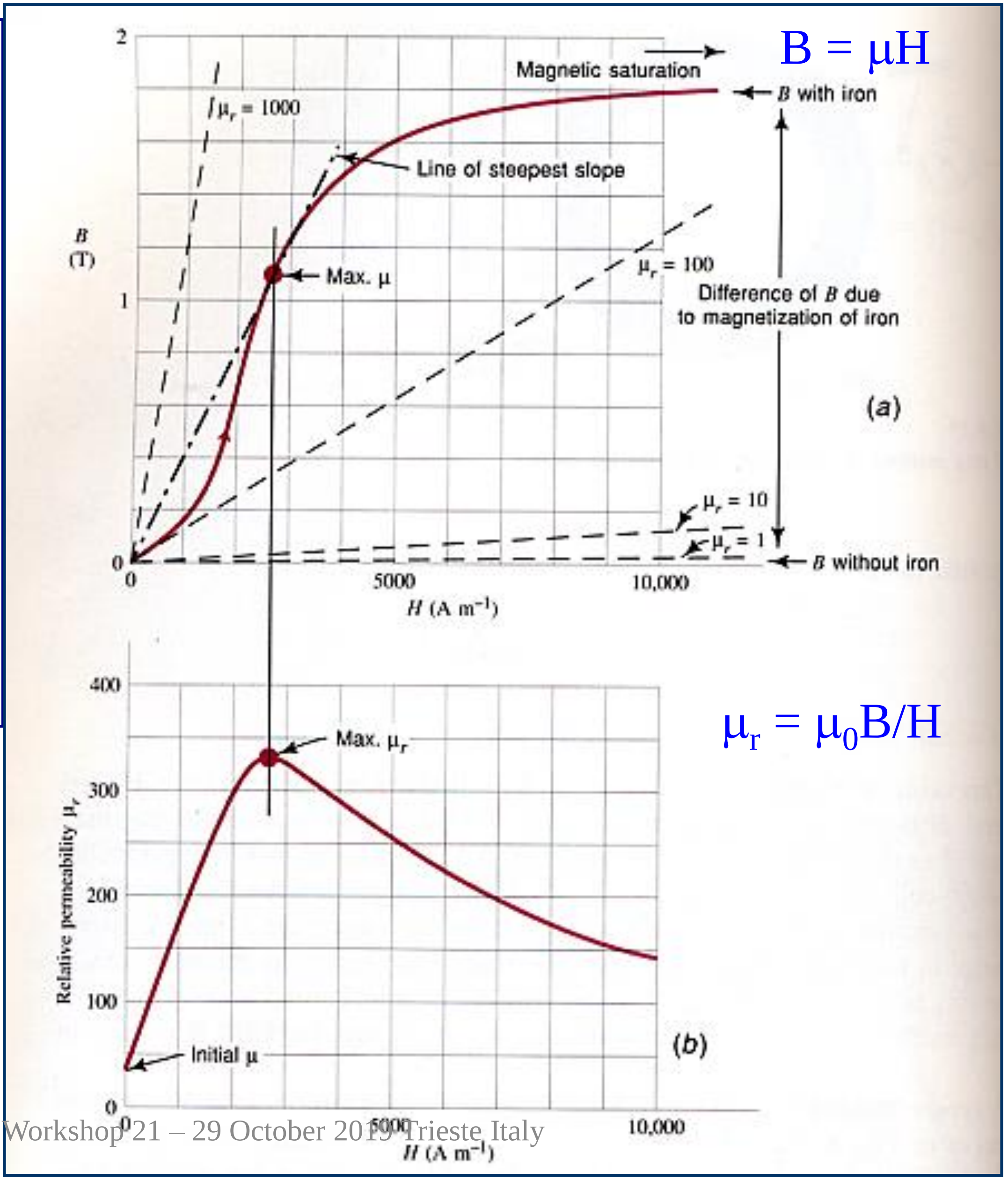
$NI = (B_{poletip}\ .a)/(2\mu_0)$

Power $\propto (I)^2 \propto a^4$



MAGNETIZATION CURVE and PERMEABILITY





Magnetic Materials: relative permeability

Relative permeabilities	**$\mu = \mu_0 \mu_r = (4\pi x 10^{-7}) \mu_r$**	
Substance	**Group type**	**Relative permeability, μ_r**
Bismuth	Diamagnetic	0.99983
Silver	Diamagnetic	0.99998
Lead	Diamagnetic	0.999983
Copper	Diamagnetic	0.999991
Water	Diamagnetic	0.999991
Vacuum	Nonmagnetic	1^{+}
Air	Paramagnetic	1.0000004
Aluminium	Paramagnetic	1.00002
Palladium	Paramagnetic	1.0008
2-81 Permalloy powder (2 Mo, 81 Ni) ‡	Ferromagnetic	130
Cobalt	Ferromagnetic	250
Nickel	Ferromagnetic	600
Ferroxcube 3 (Mn-Zn-ferrite powder)	Ferromagnetic	1,500
Mild steel (0.2 C)	Ferromagnetic	2,000
Iron (0.2 impurity)	Ferromagnetic	5,000
Silicon iron (4 Si)	Ferromagnetic	7,000
78 Permalloy (78.5 Ni)	Ferromagnetic	100,00
Mumetal (75 Ni, 5 Cu, 2 Cr)	Ferromagnetic	100,000
Purified iron (0.05 impurity)	Ferromagnetic	200,000
Superalloy (5 Mo, 79 Ni) ‡	Ferromagnetic	1,000,000

HYSTERESIS

Permanent magnets defined by curve in 2nd quadrant

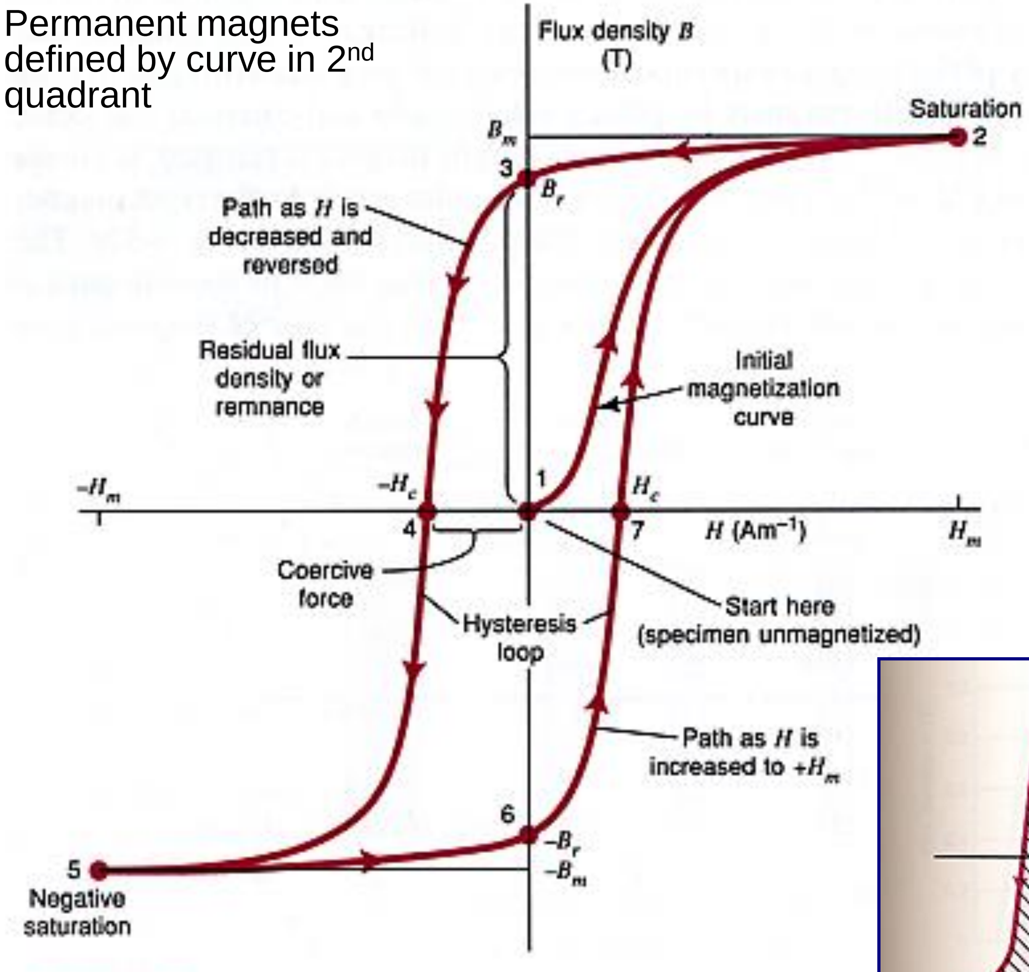
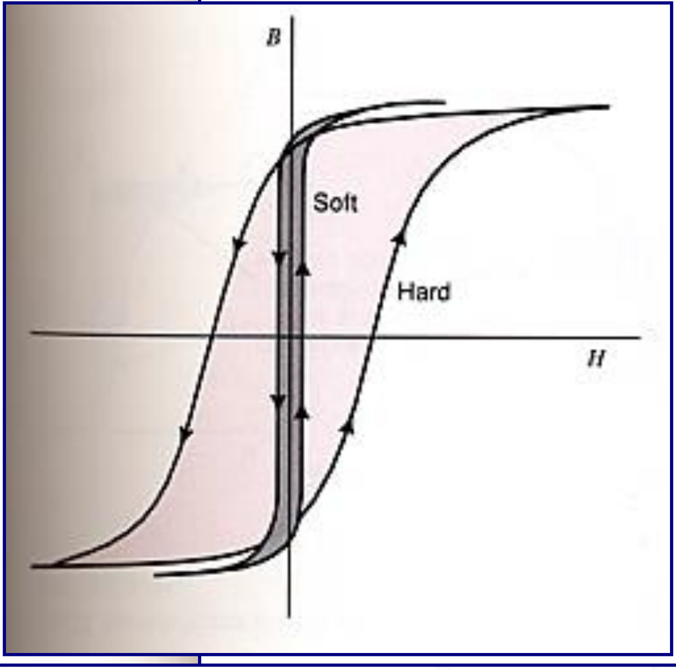


FIGURE 8-33
Hysteresis loop showing path of **B** as **H** is changed.



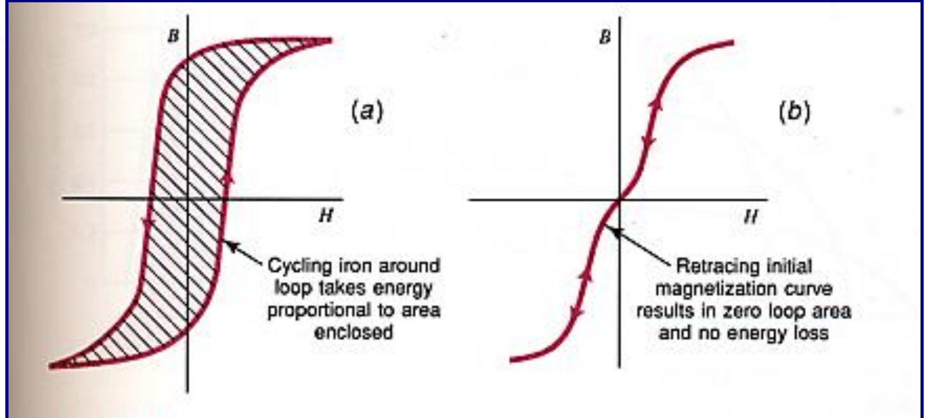


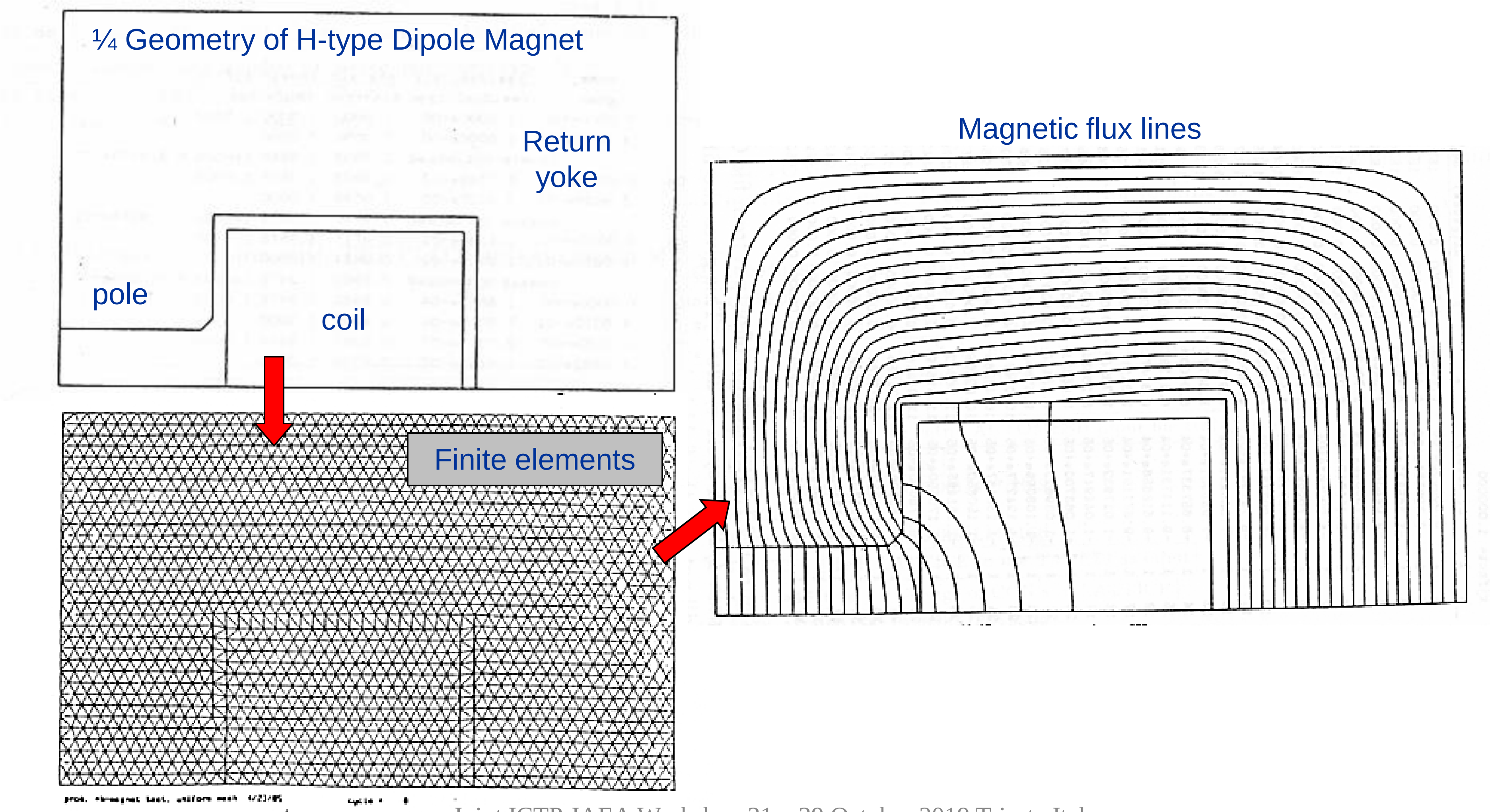
FIGURE 8-37
Energy lost in magnetization cycle is proportional to area enclosed by hysteresis loop.

SOME USEFUL GUIDES FOR DESIGN OF CONVENTIONAL MAGNETS

A. Magnet steel begins to saturate around 1.5 T
B. Coils with current density < 1 A/mm^2 may not need cooling
C. Max. current density for normal water cooled conductor is < 10 A/mm^2
D. Water flow should be turbulent (v > 1.5-2 m/s)
E. Know the price of Power consumption
F. Cost of putting magnet into service (measurement, installation, cables, power supply) is the same as the capital cost of the magnet

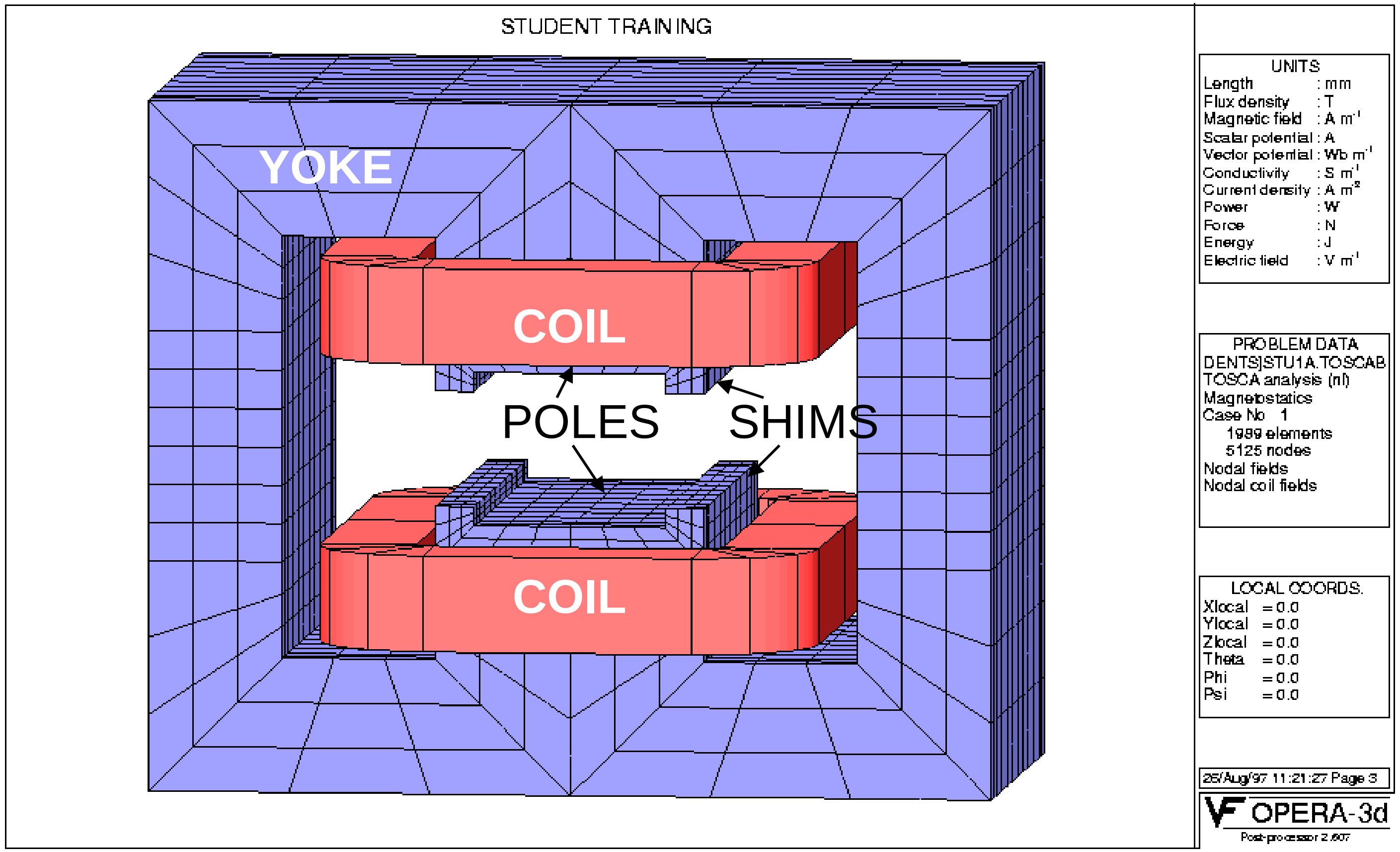
COMPUTER PROGRAMS for MAGNET DESIGN

2d : POISSON / SUPERFISH & OPERA-2d



COMPUTER PROGRAMS for MAGNET DESIGN

3d : OPERA-3d (Pre- and Post-Processor, TOSCA, ELEKTRA, SOPRANO, Geometric Modeller, SCALA, CONCERTO, TEMPO)



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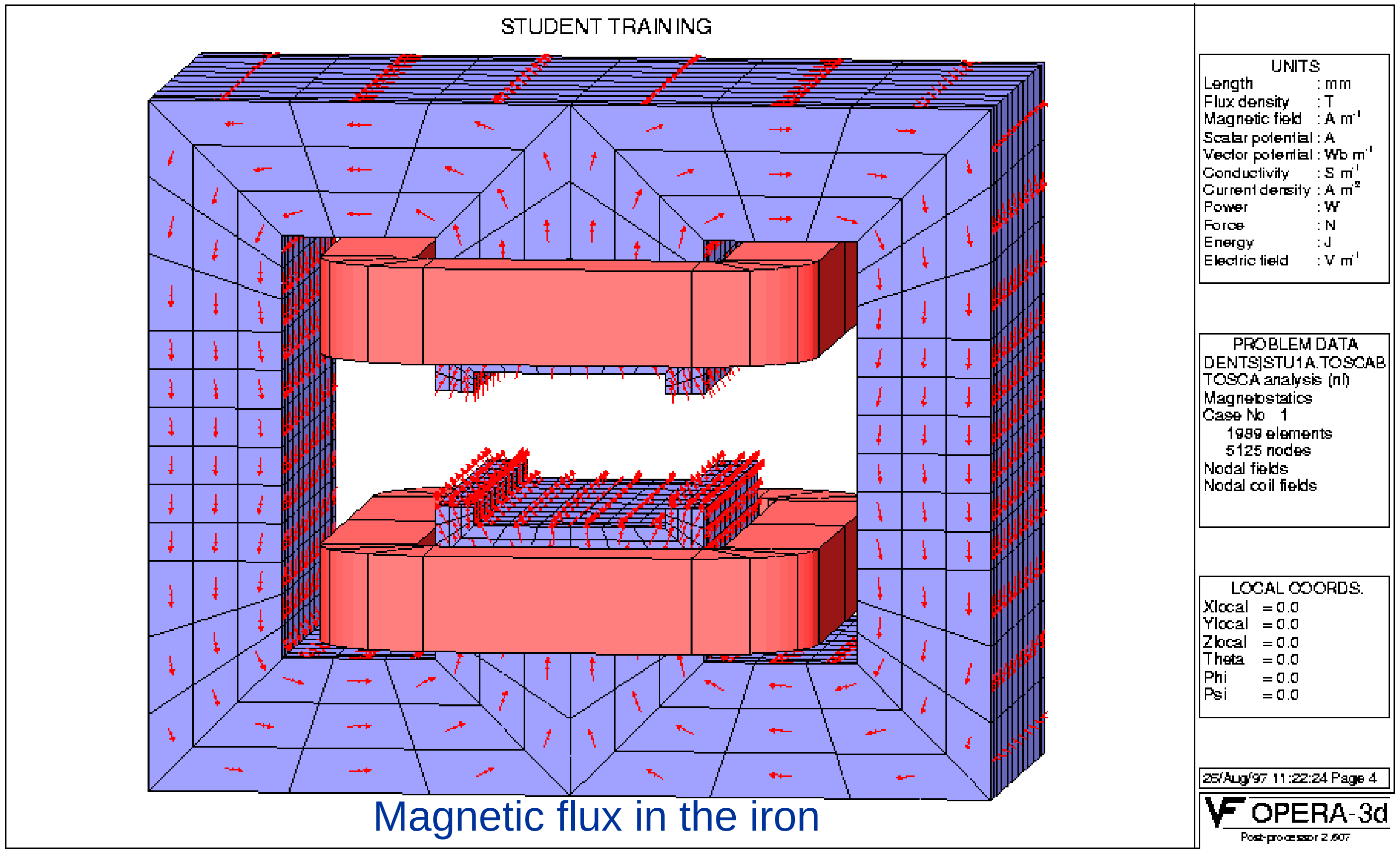


Magnetic flux in the iron

DESIGNING A BENDING MAGNET

Assignment: Design a 90-degree bending magnet for beam analysis with the duoplasmatron ion source and injection into an accelerator.

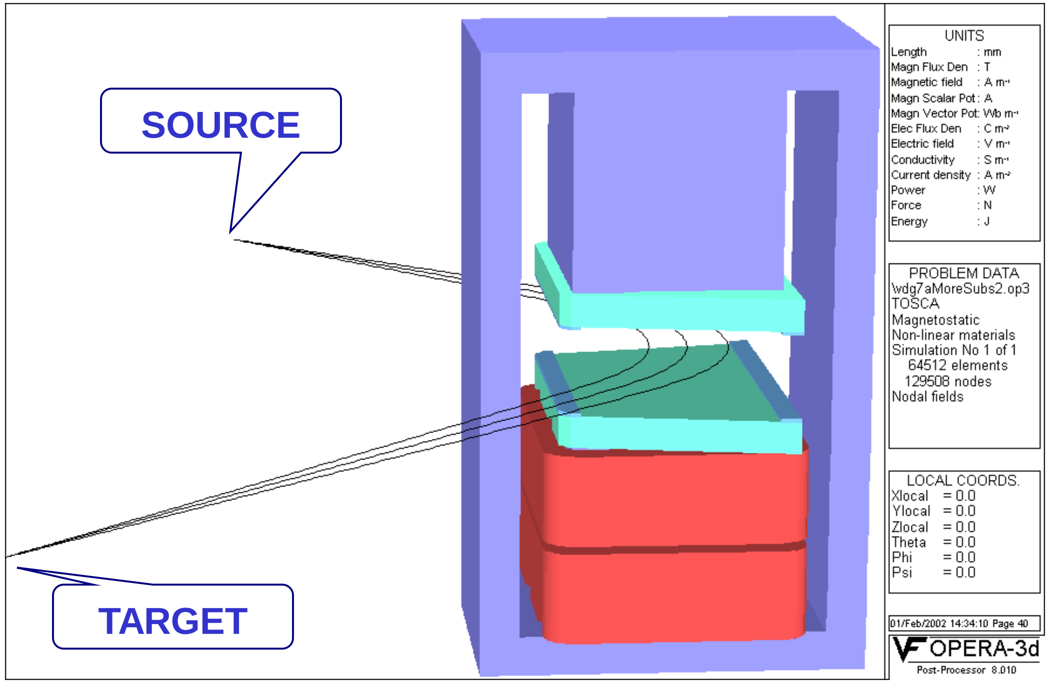
The magnet must adhere to the following requirements:

- Bending angle = 90 degrees, Radius of curvature = 220 mm
- Pole gap = 70 mm, Beam width in pole gap ≈ 40 mm
- Maximum energy of protons injected into the accelerator = 20 keV
- Maximum current by power supply is 6 A

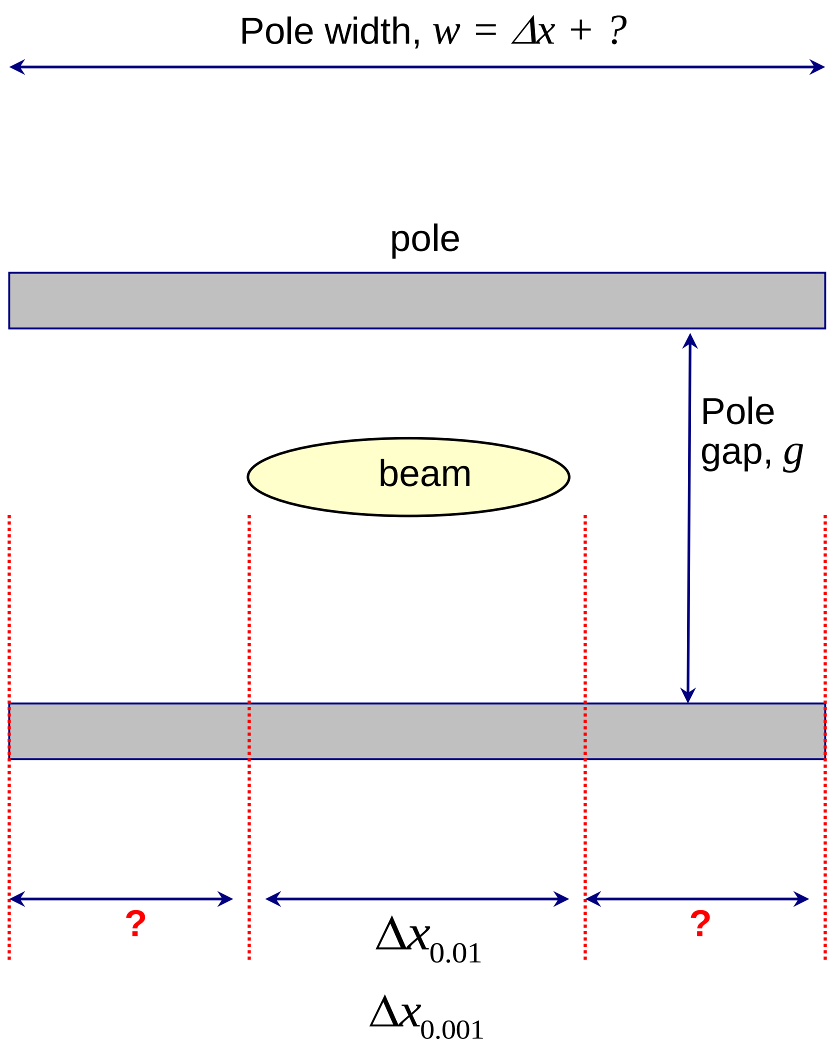
Therefore : Calculate the main parameters of the magnet that will transfer the beam from the source to the target.

- rigidity and magnetic flux density
- maximum B field
- pole width (homogenity of the field)
- thickness of iron yoke
- the mmf
- the number of coil turns
- voltage and power at a max. 6 A

Then: Measure and calculate the excitation curve, effective length and field homogenity



MAGNET DESIGN : POLE WIDTH



For a magnet with a pole width *w* and gap *g* the width $\Delta x_{0.01}$ over which the field varies less than 1%, is more of less given by:

$$\Delta x_{0.01} = w - g$$

And for a variation of less than 0.1% it becomes

$$\Delta x_{0.01} = w - 2g$$

If the horizontal beam diameter is about 40 mm then the minimum pole gap width can be computed within 0.1% variation in the magnetic flux density region across the beam width, using the above relation, is

$$w = 2g + \Delta x_{0.001} = 2 \times 70\ mm + 40\ mm = 180\ mm$$

And for a maximum variation of 1% the pole width is

$$w = g + \Delta x_{0.01} = 70\ mm + 40\ mm = 110\ mm$$

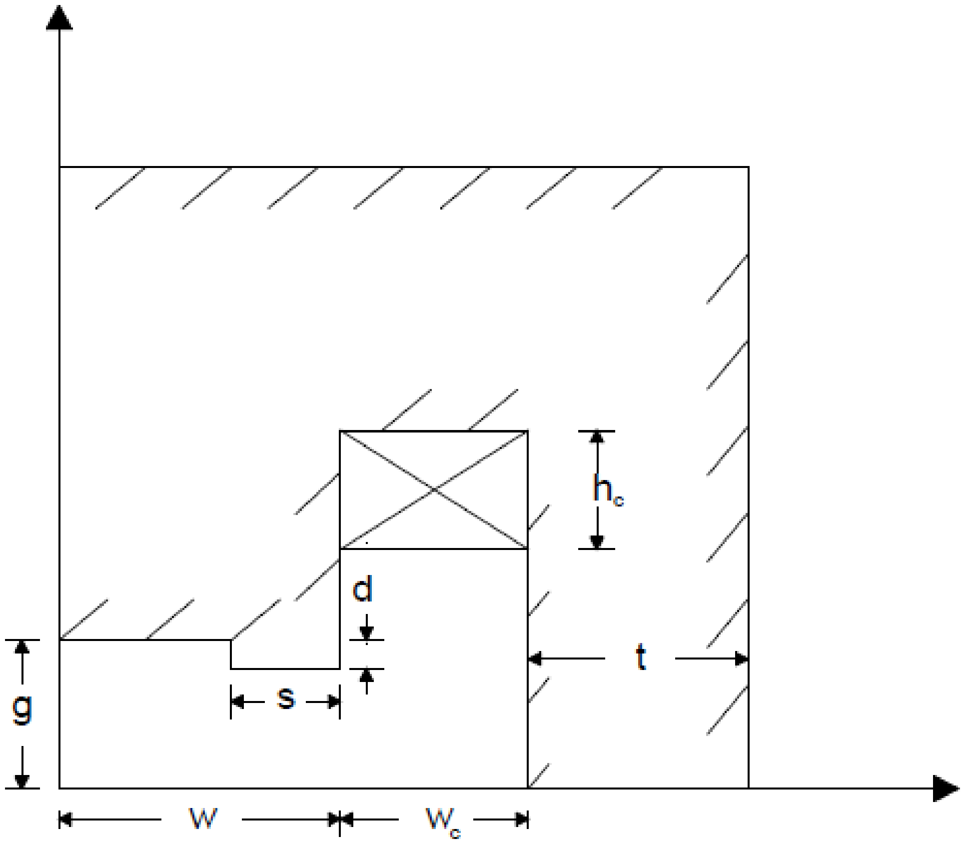
Shims for dipole magnet to improve the uniformity of magnetic field

Shim area = s x d

Shim Area ≅ $.021g^2$

With 0.2≤ $s/g \leq 0.6$

g = pole gap
w = width of pole
s = width of shim
d = height of shim



MAGNET DESIGN : MAGNETIC RIGIDITY, FLUX DENSITY, YOKE THICKNESS

For a particle with charge q, mass m and speed v moving in a uniform, time-independent magnetic field, B, on a circular orbit with radius of curvature, ρ, at a right angle to the uniform magnetic field, the Lorentz force equal to the centrifugal force:

$$qvB = \frac{mv^2}{\rho}$$

$$\rho = \frac{mv}{qB}$$

The momentum, mv, can, for a given charge q, be expressed by the product $B\rho$. The product $B\rho$ is called the magnetic rigidity of the particle and is a direct measure of the particle's momentum. The expression relating the total energy E and the momentum p of a particle is:

$$E^2 = p^2c^2 + m^2c^2$$

Using the energy relations $E = E_0 + E_k$ and $E = mc^2$

with c = speed of light,

E_0 = the rest energy of the particle

E_k = the kinetic energy of the particle

The rigidity (in SI-units) $B\rho = \frac{mv}{q} = \frac{\sqrt{E^2{}_k + 2E_0E_k}}{qc}$

and with an absolute charge state Q and energy in eV, it becomes

$$B\rho = \frac{\sqrt{E^2{}_k + 2E_0E_k}}{Qc}$$

The magnetic rigidity of a 20 keV proton (maximum injection energy), is

$$B\rho = 0.0204Tm$$

With the radius of the magnet known (the radius was fixed by the double focusing distance) the maximum flux density for the magnet is:

$$B = \frac{0.0204Tm}{0.22m} = 0.0927T$$

If we assume that saturation will only be reached when the magnetic flux density in the iron is about 1.2 T, and that the flux that passes through the iron is the same as that which passes through the pole gap, the following calculation can be used to determine the minimum thickness of the iron yoke pieces.

Magnetic flux through air (pole gap) = Magnetic flux through iron

$$A_g \times B_g \cong y \times A_i \times B_i$$

where,

A_g = cross sectional area of the pole gap,

A_i = cross sectional area of the iron yoke,

B_g = magnetic flux density in the pole gap,

B_i = magnetic flux density in the iron yoke,

y = number of yoke pieces for closing of the magnetic flux loop, which is determined by the magnet shape (i.e. y=1 for a C-magnet and y=2 for an H-magnet)

MAGNET DESIGN : YOKE THICKNESS, MAGNETO-MOTIVE FORCE

The path-length, L, of a 90^0 circular bend is given by:

$$L = \alpha\rho = \frac{2\pi\rho}{4} = \frac{2\pi \times 220mm}{4} \approx 346mm$$

Assuming a H-shape magnet, the minimum thickness of the yoke pieces can thus be calculated as follows:

$$A_g \times B_g \cong y \times A_i \times B_i = 2 \times (length \times thickness)_i \times B_i$$

$$Thickness_i = \frac{A_g \times B_g}{2 \times B_i \times length_i}$$

$$= \frac{346\ mm \times 110\ mm \times 0.0927\ T}{346\ mm \times 2 \times 1.2\ T}$$

$$= 4.3\ mm$$

The yoke thickness was chosen, a practical 20 mm.

To create the magnetic field in the magnet a certain magneto-motive force (mmf) per centimeter length (or field strength) that will give the maximum flux density through the magnet has to be applied. The required mmf is generated by the coil windings in the magnet through which a current is sent. Following Ampere's law (the integral of the magnetic field along a closed path equals the enclosed total current):

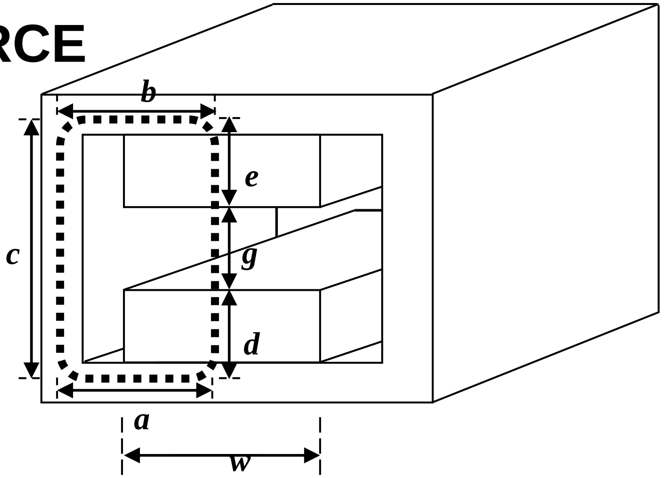
$$\oint \vec{H} \cdot d\vec{l} = NI$$

where,

H = magnetic intensity or the field strength in A/m,

N = number of turns in the windings,

I = current in A through the windings.



It is assumed that the magnetic flux density in the iron is constant and the flux density between the poles is constant and that the direction of the field is parallel to the arrows of path a-b-c-d-e-g (as shown in the figure). The mmf is

$$H_g \times g \quad + \quad H_l \times l \quad = \quad NI$$

where,

H_g = magnetic intensity in the air between the poles,

H_i = magnetic intensity in the iron,

l = $a + b + c + d + e$,

g = pole-gap.

With the path $l = n \times g$ it becomes $g(H_g + nH_l) \cong NI$

The relationship between the magnetic flux density B and magnetic intensity H *is*

$$B = \mu_0 \mu_r H$$

Where μ_r = relative permeability of the material,

And μ_0 = permeability of free space.

MAGNET DESIGN : MAGNETO-MOTIVE FORCE, MAXIMUM CURRENT, NUMBER OF COIL TURNS, CURRENT DENSITY, LENGTH OF COIL, COIL RESISTANCE, VOLTAGE, POWER CONSUMPTION

The mmf now becomes: $g\left(\frac{B_g}{\mu_0\mu_{air}}+\frac{nB_i}{\mu_0\mu_{iron}}\right)=NI$

The second term in the relation can be neglected for flux densities below 1.2 T. Hence the following approximation for mmf:

$$NI \approx \frac{gB_g}{\mu_0\mu_{air}}$$

To obtain a magnetic flux density of 0.0927 T in the air gap (μ_{air} = 1), the mmf is then :

$$mmf = NI = \frac{0.07\ m \times 0.0927\ T}{4\pi \times 10^{-7}\ Tm/A} = 5614\ ampere.turns$$

The maximum tolerable current density for air-cooled coils is between 2 and 3 A/mm^2, depending on the coil geometry and we decided not to exceed 2 A/mm^2. The available copper wire with 2 mm diameter can thus have a maximum current of 6.28 A. With the maximum current known the minimum number of windings for the required magnetic field can be calculated as:

$$N = \frac{mmf}{I} = \frac{5164\ ampere.turns}{6.28\ A} \approx 822\ turns$$

A safety factor of 25% is added to the number of windings, hence the minimum total number of about 1027 windings for the coils. Round up to say 1040 and split the turns equally between the upper and lower coils with 520 turns each.

At 6.28 A the magnetic field B will now be 0.11725 T.

Require only 0.0927 T and therefore 4.965 A is adequate (1.58 A/mm^2).

A rough estimate of the length of one turn in the coils is 1 m. Total length of coil required is about 1040 m.

The resistance of the 1040 m copper wire with thickness of 2 mm is:

$$R = \frac{L_{coil}}{A_{wire}}\rho = \frac{1040\ m \times 0.01754\ \Omega\, mm^2/m}{3.14\ mm^2} \approx 5.81\,\Omega$$

where,

- L_{coil} = length of the coils in m,
- ρ = resistivity of copper (= 0.01754 Ω mm^2/m at 25^o C),
- A_{wire} = cross-sectional area of the coil.

The voltage required for the maximum current of 6 A is:

$$V = I \times R = 6\ A \times 5.81\,\Omega = 34.86\ V$$

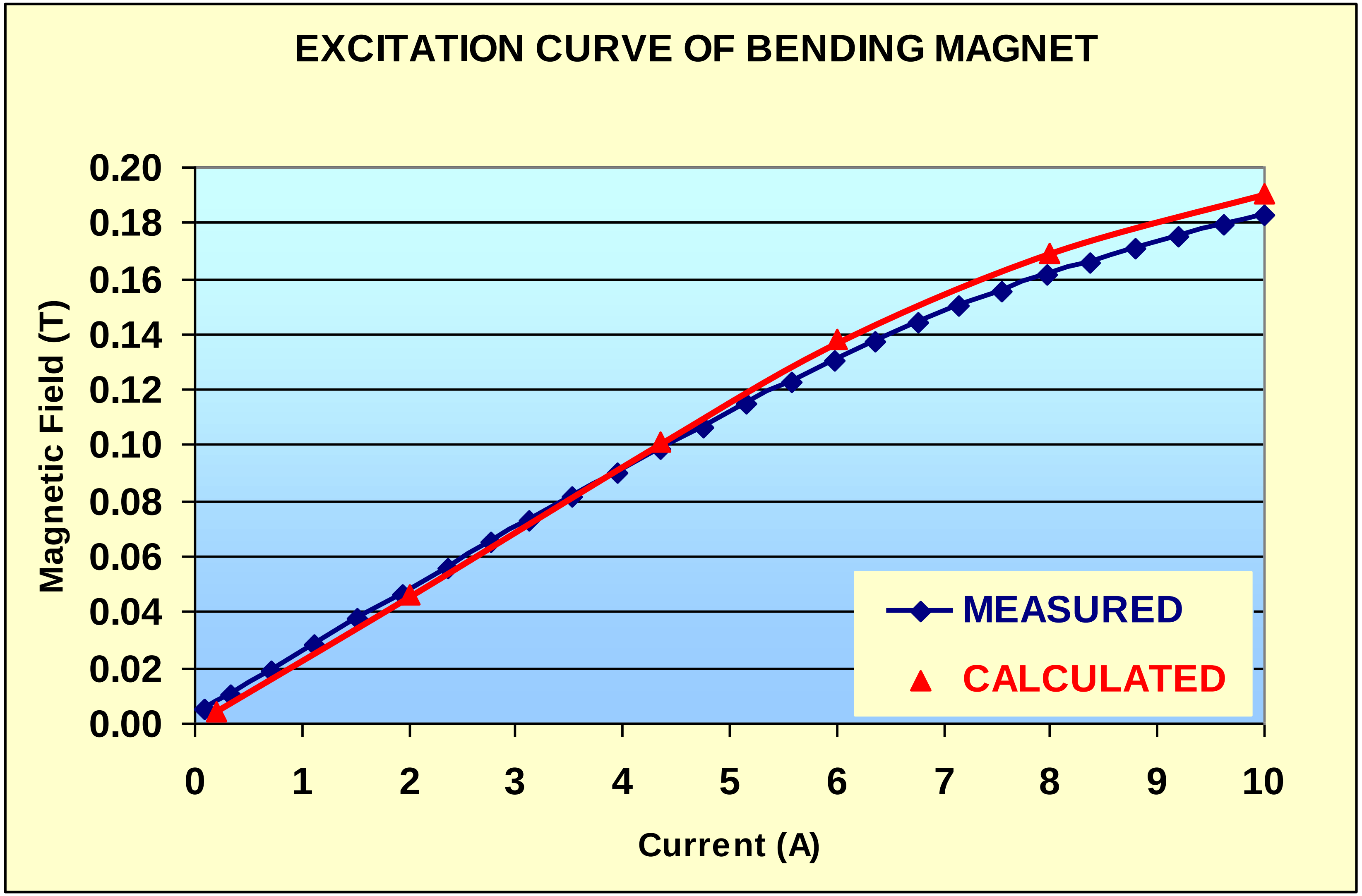
And the power consumption of the magnet is:

$$P = I \times V = 6\ A \times 34.86\ V = 209\ W$$

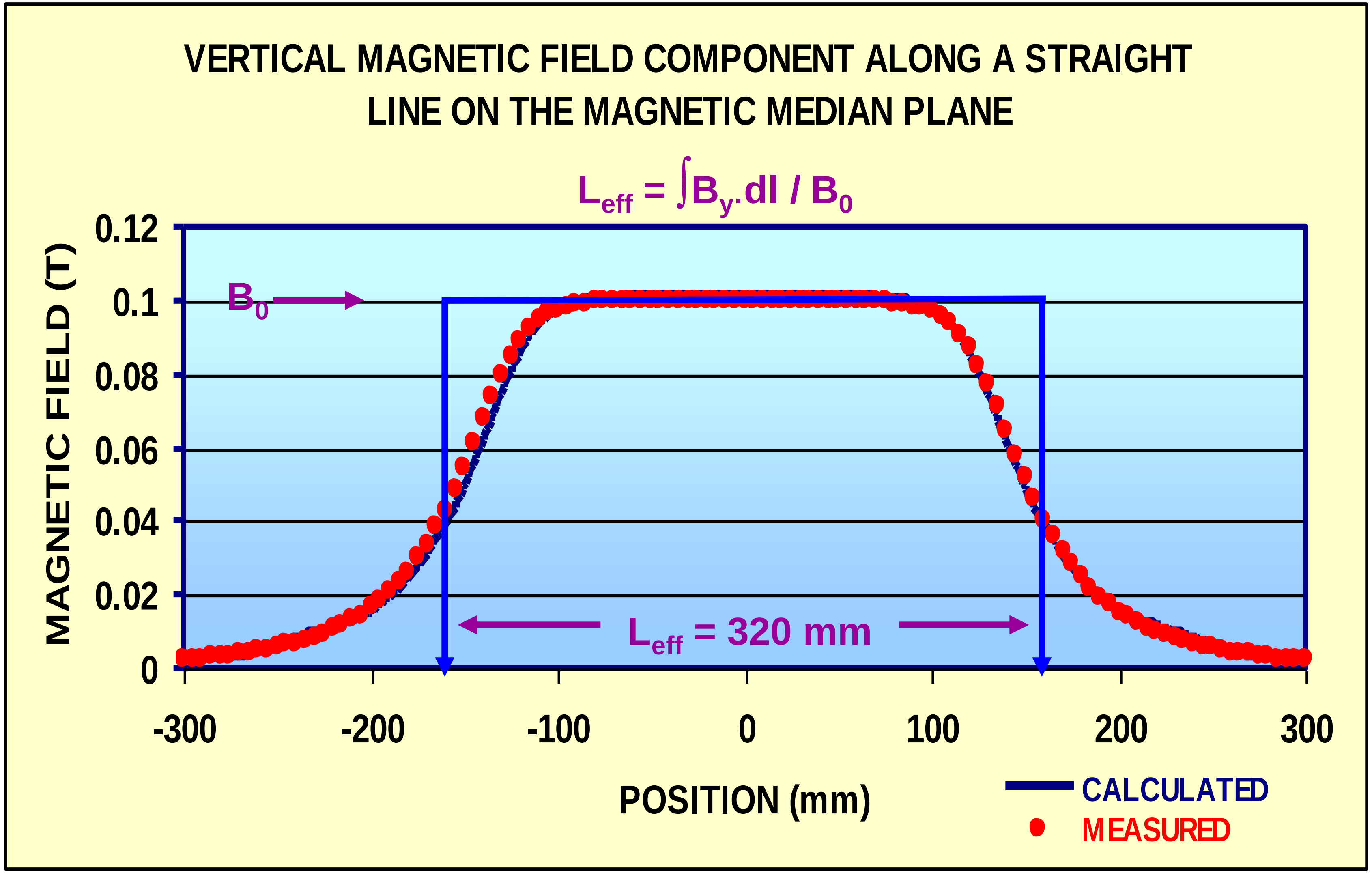
Must still determine the following parameters through calculation and measurement on the manufactured magnet :

- Analyzing power of the magnet (resolution)
- Edge angles
- Excitation curve
- Effective length
- Field homogenity

MAGNET DESIGN : EXCITATION CURVE



MAGNET DESIGN : EFFECTIVE LENGTH



Area under BLUE = Area under RED

MAGNET DESIGN : EFFECTIVE LENGTH

<u>The empirical result (for small gaps):</u>

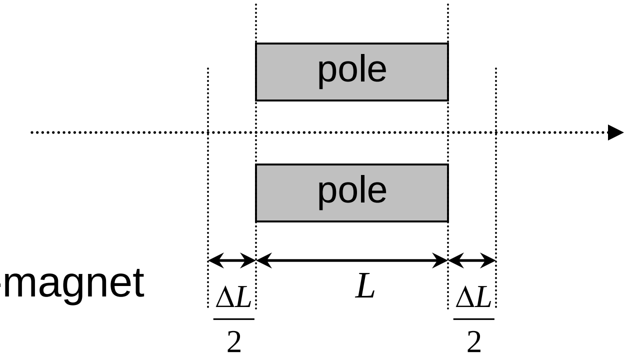
The effective length (L_{eff}) of the magnet can be taken as the region in the magnet median plane through which the magnetic field is almost constant and is approximated by the following expression:

$$L_{eff} = \Delta L + L$$

With a magnet pole gap (g) = 70 mm

$$\frac{\Delta L}{2} = 1.1g = 77mm$$

$$\frac{\Delta L}{2} = 0.63g = 44.1mm$$



where $\Delta L/2$ is the contribution of one end to the effective length for an H-magnet

With a magnet length (L) = 256 mm

$$L_{eff} = \begin{cases} 410mm \\ 344mm \end{cases}$$

$$k = \begin{cases} 0.63 \\ 1.1 \end{cases}$$

where k is a parameter that varies from 0.63 g to 1.1 g

<u>Effective length results:</u>

- Numerical field analysis : 320 mm
- Measured : 324 mm
- Empirical : 344 – 410 mm

Calculate the number ampere turns required for a quadrupole magnet with aperture of 100mm and maximum field gradient of 12 T/m

$NI = Ka^2 / 2\mu_0 = 12 \times (.05)^2 / (2 \times 4 \times \pi \times 10^{-7}) = 11940$ ampere turns

With
N = the number of turns on a pole
I = Current in the coils for desired gradient
K = Field gradient

For a quadrupole power supply that can deliver a maximum current of 100A the required number of turns on each coil is:

Number or turns = $NI/I_{max} = 11940/100 = 119.4$ turns (make it 120 turns)

Calculate the magnetic field at the pole tip of a quadrupole magnet with aperture of 100 mm and current of 100 A The number of turns per pole is 120

$B = N \times I \times 2 \times \mu_0 / r = 120 \times 100 \times 2 \times 4\pi \times 10^{-7} / 0.05 = 0.6032$ T

The gradient $K = B_{pole} / r = 0.6032/.05 = 12.064$ T/m

If the number of turns on the quadrupole is not known one needs a tesla meter to measure the pole field. For calculations with the program TRANSPORT the gradient of a quadrupole as function of current must be known. Normally there is a calibration table of current against the measured magnetic field for each quadrupole magnet available.

Specify the quality of the magnetic field of a quadrupole magnet

The quality of a multipole magnet is normally specified by setting a limit on the amplitude of the higher order multipoles contributing to the quadrupole field. The radial and tangential component of a infinite long multipole magnet is given by:

$$B_r = \sum_{n=1}^{\infty} n a_n r^{n-1} \sin(n\theta + \theta_n)$$

$$B_\theta = \sum_{n=1}^{\infty} n a_n r^{n-1} \cos(n\theta + \theta_n)$$

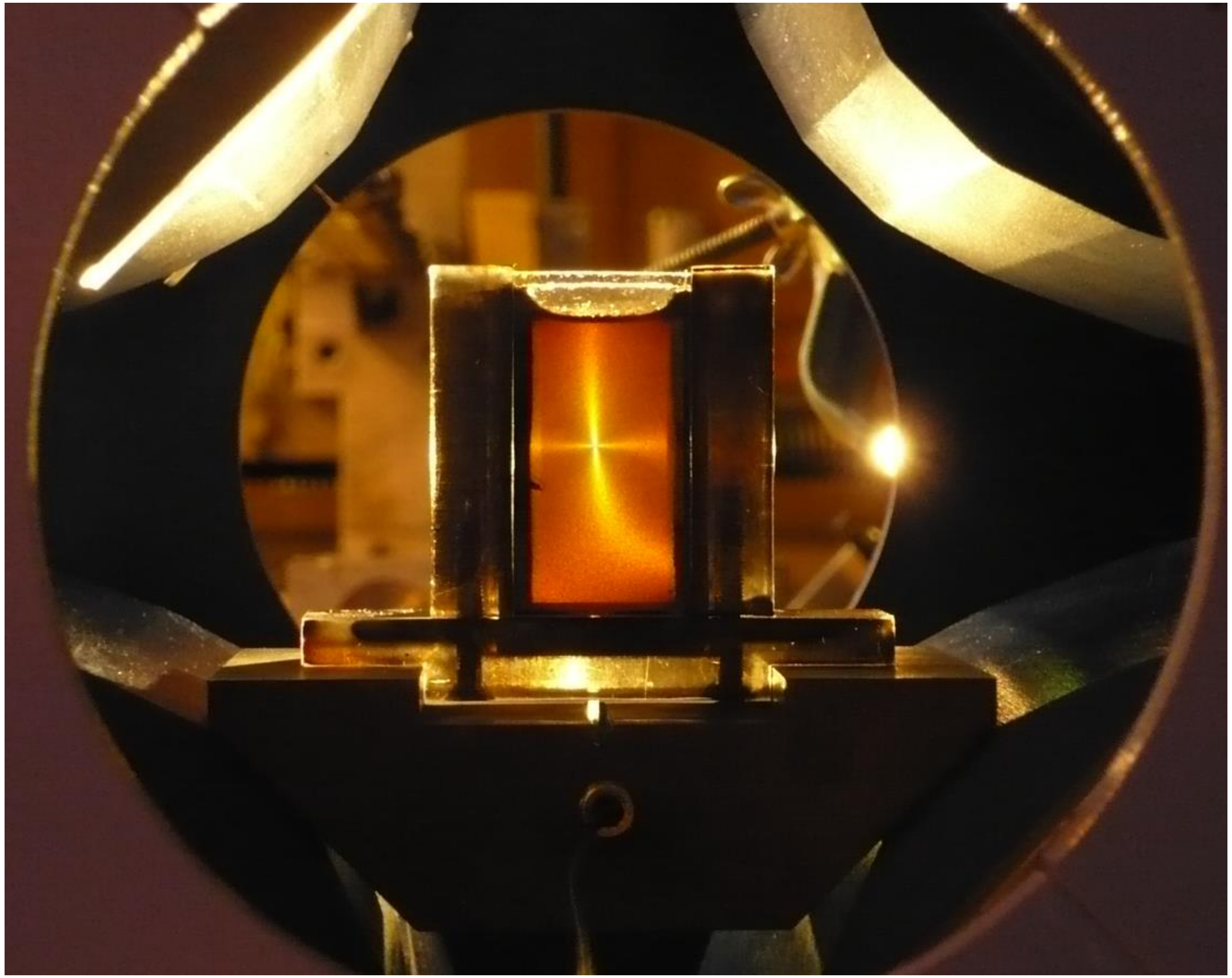
For a quadrupole magnet n = 2 with a_2 the amplitude of the quadrupole component. iThemba LABS specify that at 75% of the quadrupole radius, the amplitude of higher order multipoles must not exceed the following criteria

$$\frac{a_n}{a_2} < 0.5\% \; for \; n \geq 3$$

$$\frac{\sum_{n=3}^{\infty} a_n}{a_2} \leq 1.5\%$$

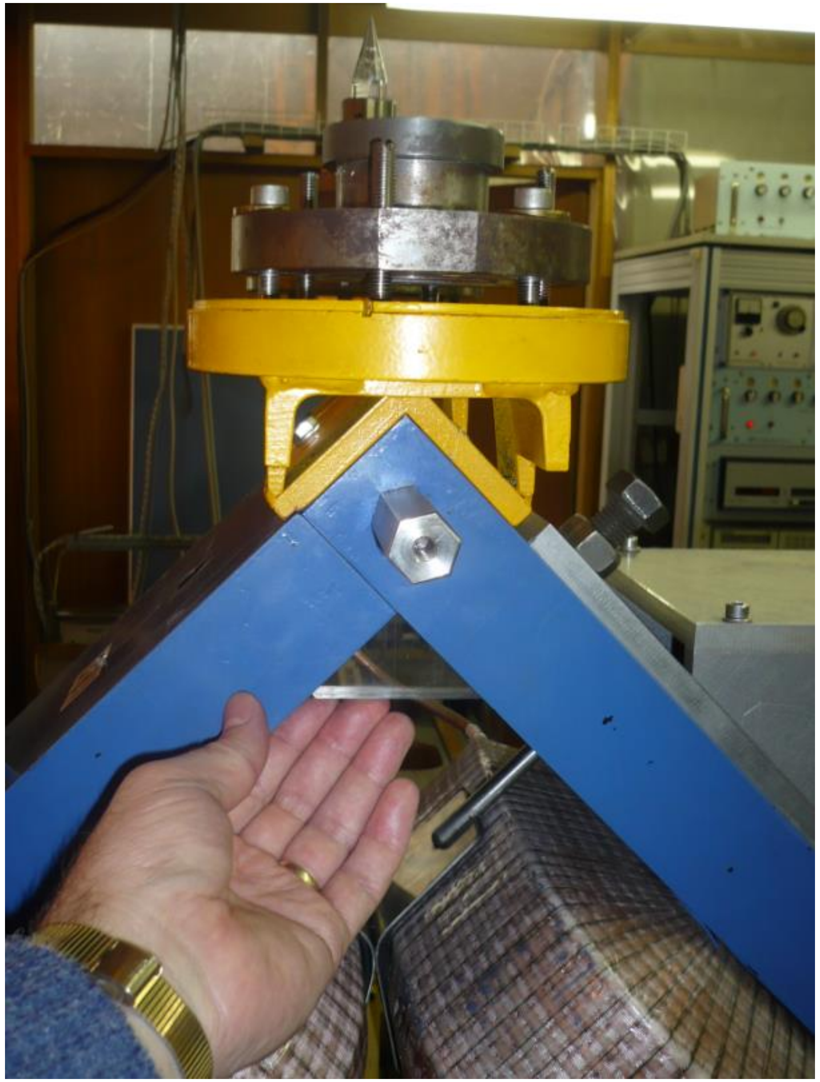
Measurement on Quadrupole magnet

Determine the magnetic centre of the Quadrupole magnet with a suspension of magnetite in glycerol and two light sources



Measurement on Quadrupole magnet

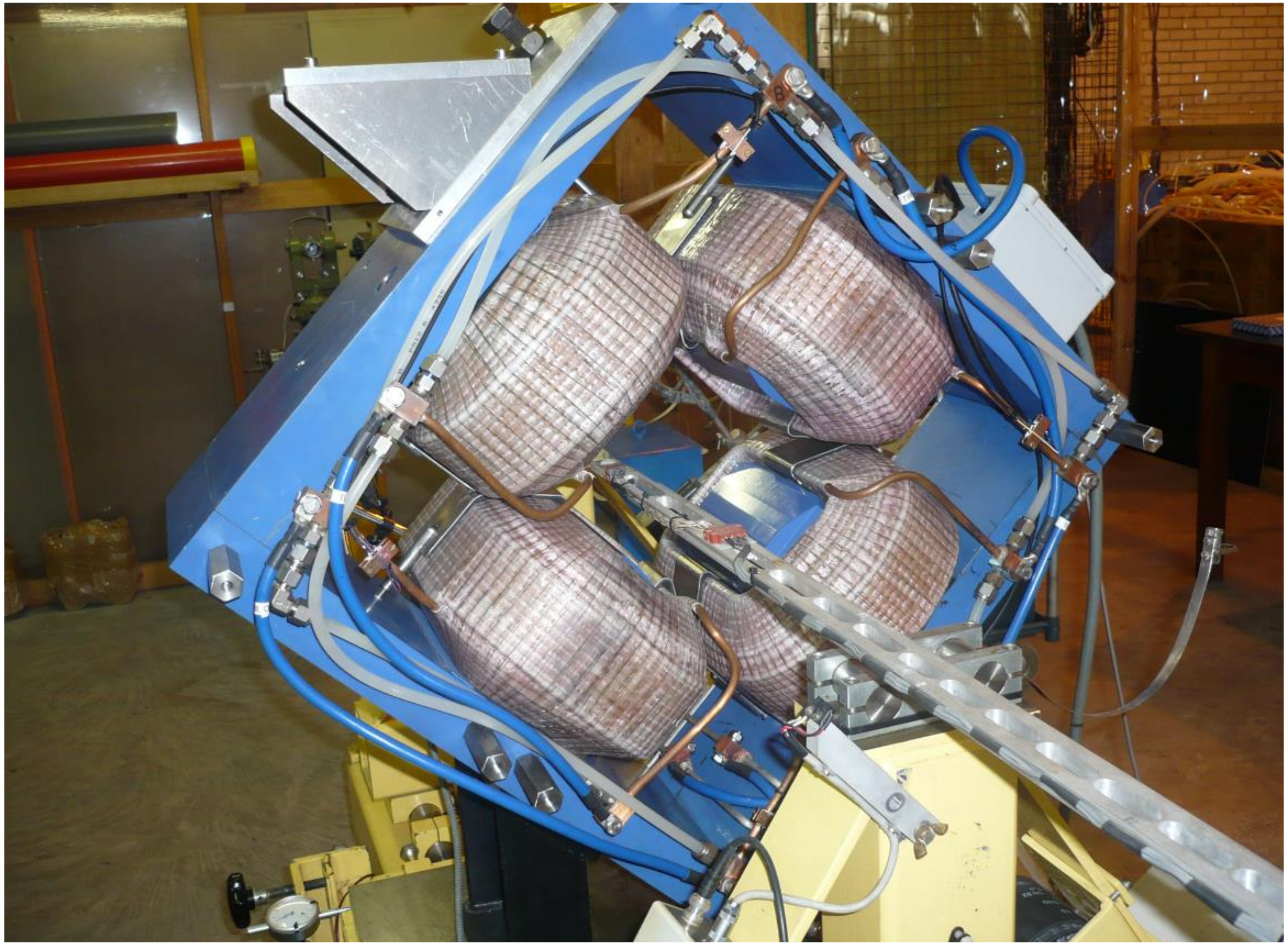
Transfer of the magnetic centre to top of magnet for future alignment of quadrupole magnet in a beam line



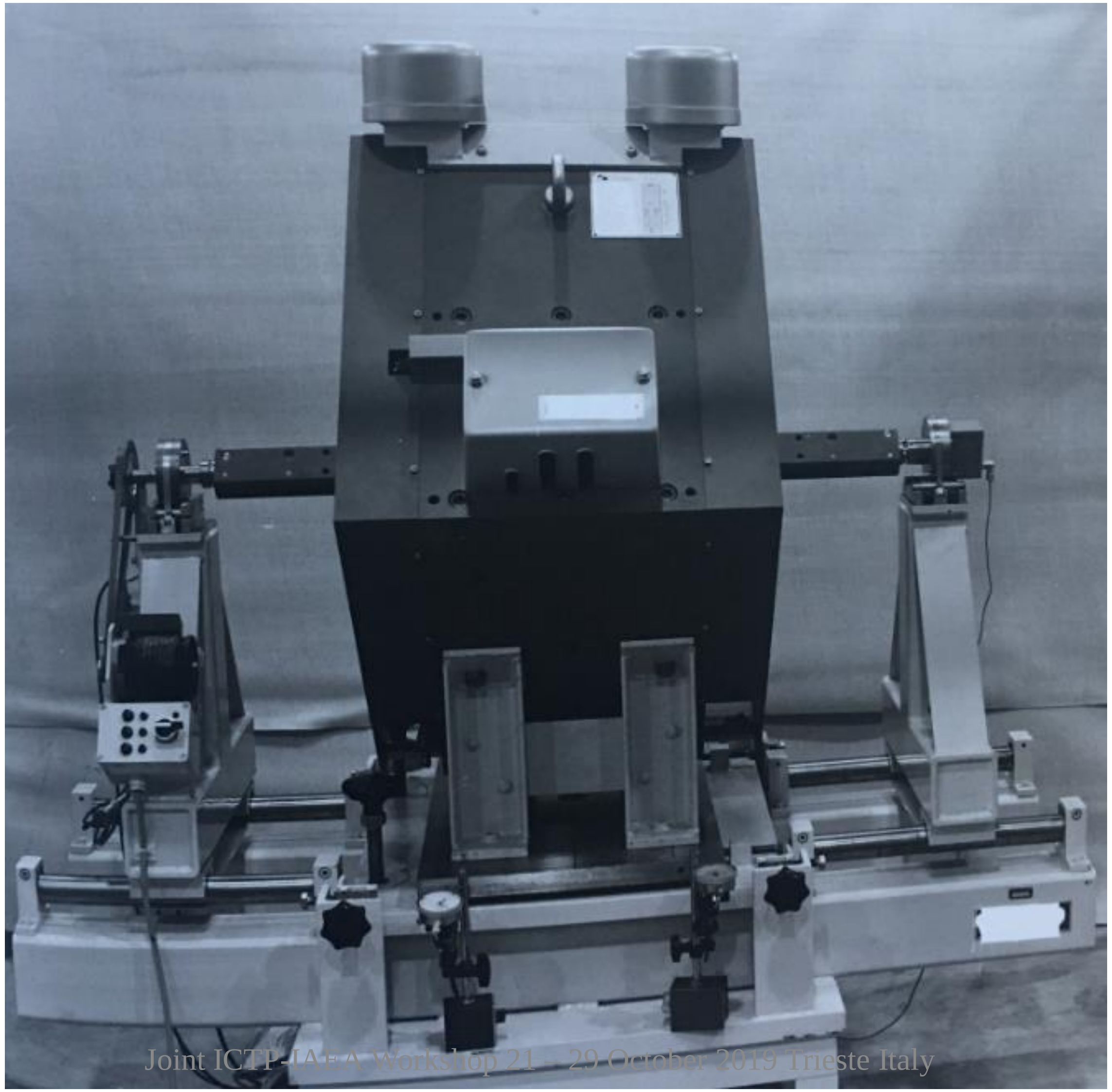
Measurement on Quadrupole magnet

- Measure the relation between the magnetic field and current

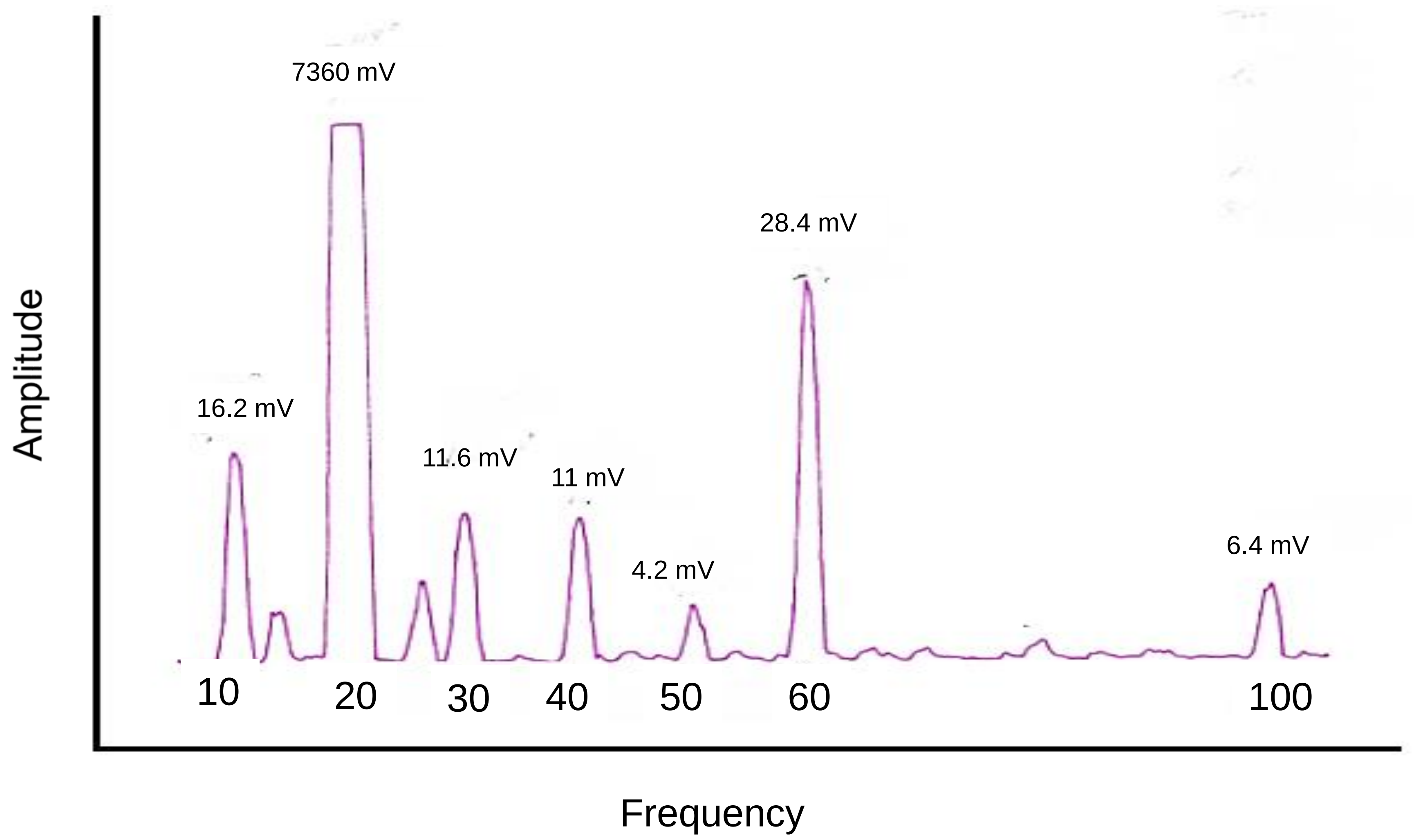
- Measure the effective length of the magnet



Measuring the multipole components of quadrupole magnet



Measured multipole components of Quadrupole magnet



FIELD MEASURING METHODS
(used at iThemba LABS)

1. **Fluxmeter (based on induction law)**

   - rotating coil in fixed field

   - fixed coil in dynamic field

   - accurate with field value and direction

   - harmonic analysis (multipole analysis)

2. **Hall Effect Method**

   - simple and fast

   - requires temperature stabilization

   - requires frequent recalibration of probes

   - well suited for fields of all gradients

3. **Nuclear Magnetic Resonance (NMR)**

   - classical method for measuring absolute value of field

   - high precision

   - restricted field gradients

Nuclear Magnetic Resonance (NMR) tesla meter

The nuclear magnetic resonance (NMR) phenomenon was first described experimentally by both Bloch and Purcell in 1946, for which they were both awarded the Nobel Prize for Physics in 1952

Advantages of NMR

- Can make absolute measurements
- resolution of 1 mG (0.1 μT)
- Use for calibration of other magnetic field measuring devices

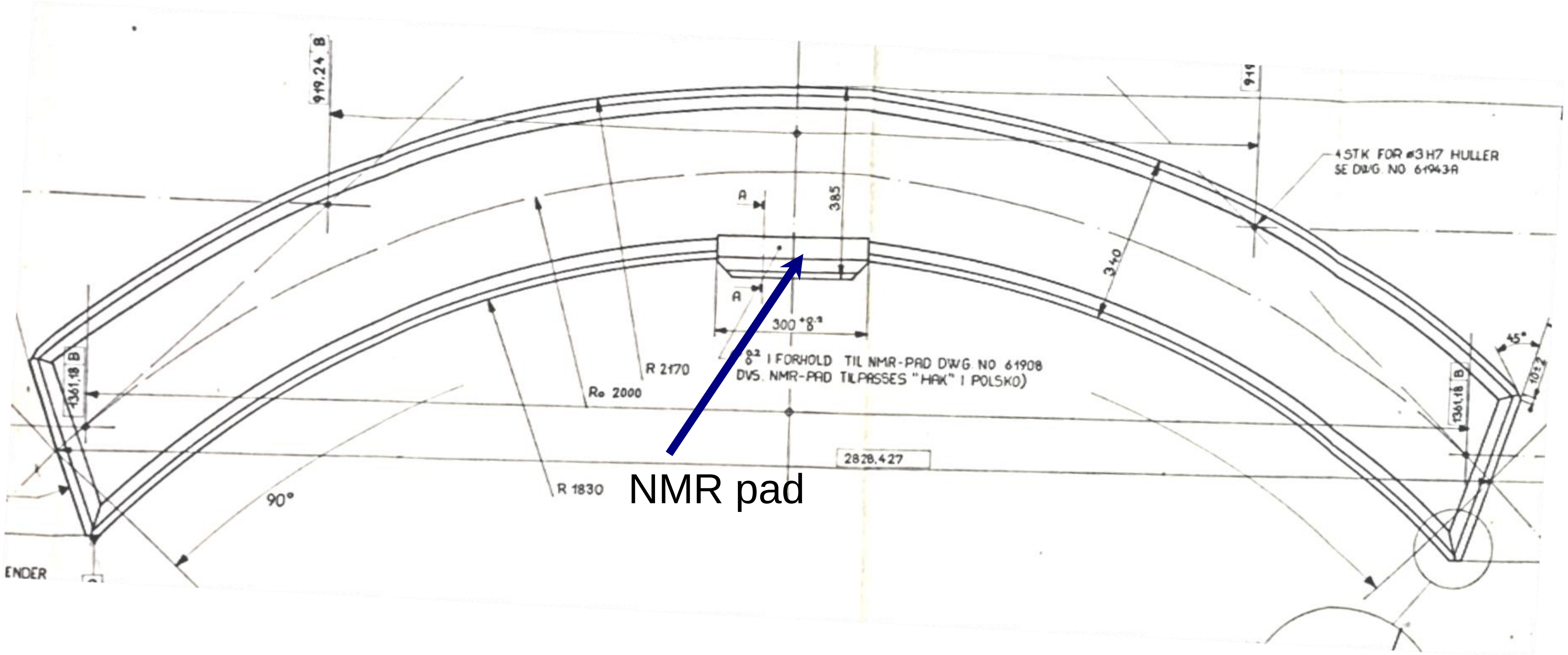
Disadvantages

- Probe is relatively large
- Cannot measure magnetic field with a large gradient
- Need a number of different probes to measure magnetic fields from low to high field values
- Expensive

- The NMR20 gaussmeter (NMR teslameter) have the characteristic of measuring weak fields **from 140 G (14 mT) up to 13 T.**

- Possibility to measure fields from **14 mT** to 13 Tesla with only 6 probes. And from 14mT to 2.1T with only 3 probes.

Magnet that will be used for energy measurements of the beam make provision for mounting a NMR Tesla meter



Magnetic field measurement

Only discuss Nuclear Resonance Meter and the Hall probe

Hall probe

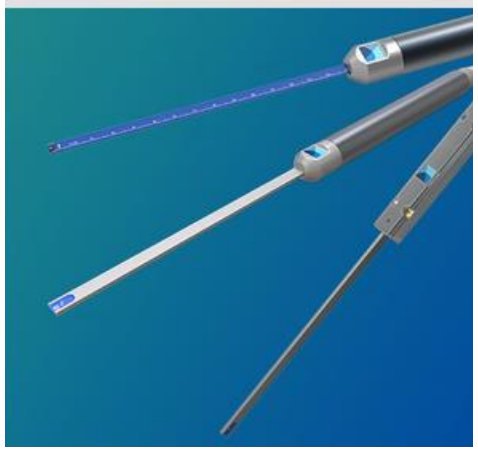
Advantages of hall probe

- Small active area can be used to map magnetic field and make point measurements
- Can be used to measure fast varying magnetic fields
- Can measure field less than 1 gauss
- One can measure over a large range with only one probe
- Relatively cheap compared to an NMR meter
- Three axis measurement possible

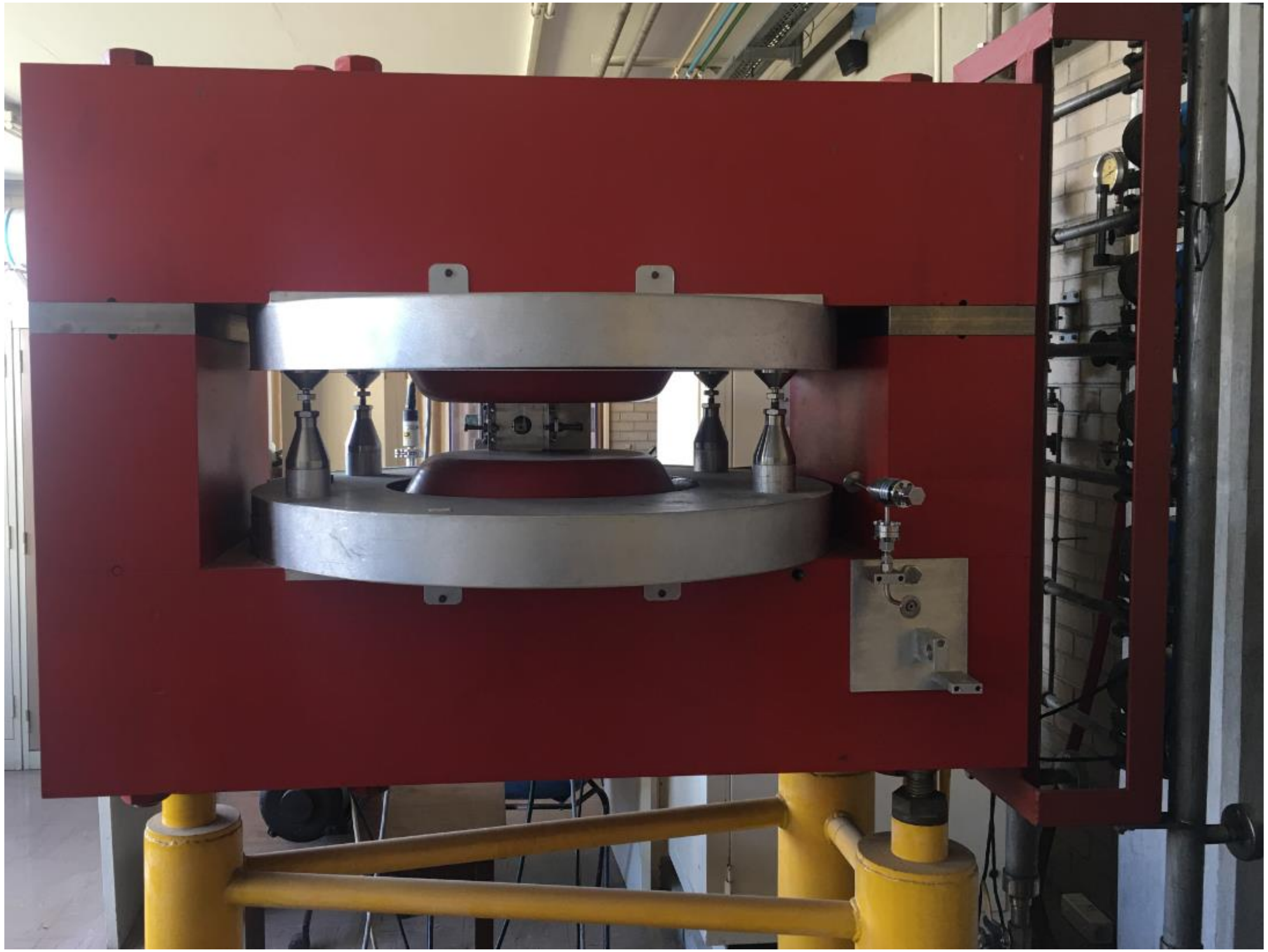
Disadvantages of hall probe

- Have to be calibrated against an NMR meter
- Have to be calibrated on a regular basis
- Sensitive to temperature changes

Hand-held Gauss meter measures magnetic fields up to 2 T down to fine resolution (0.1G).



Hall probe active area of 0.1 mm^2



MAGNET DESIGN : BY YOURSELF

Your accelerator lab requires the design of a H-type electro-magnet which can bend a 220 MeV proton beam by $3x10^{-3}$ radians. The beam pipe that must fit in the pole gap has an outer diameter of 104 mm. The length of the magnet in the direction of the beam, must not exceed 300 mm (use 200 mm). The maximum horizontal and vertical space available for the magnet is 1m in both directions. The beam diameter inside the pole gap will be about 40 mm and we require a magnetic field homogenity of about 0.8% over the width of the beam. Assume you have a good quality magnet steel available for the manufacturing of the magnet and 2 mm copper wire for the coil. The current from the power supply may not exceed 6 A. Assume the average length per turn is 0.924 m.

Guidelines to assist you :

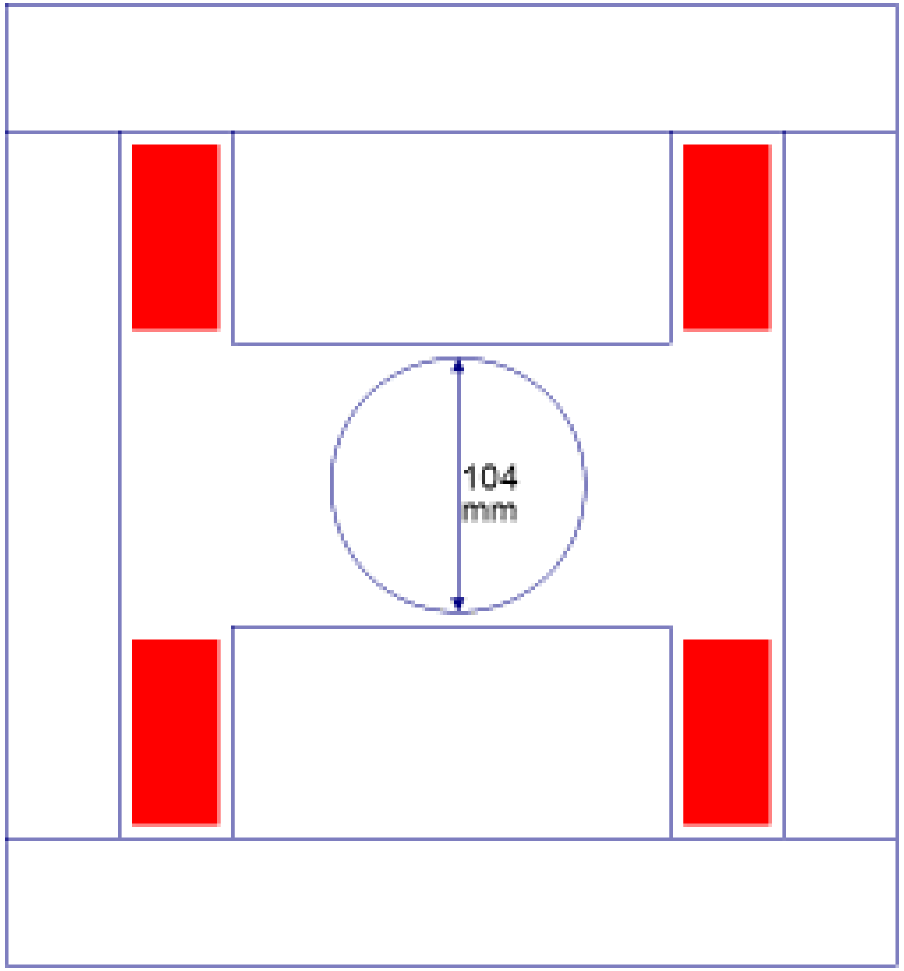
1. Make a simple sketch of the magnet you intend to design (yoke, pole and coils)
2. Calculate the rigidity
3. Calculate the pole gap
4. Determine the effective length of the magnet (and then use a value of 250 mm for further calculations)
5. Calculate the bending radius
6. Calculate the Magnetic flux density B
7. Calculate the width of the pole
8. Calculate the cross sectional surface of the yoke
9. Decide on acceptable values to use for the yoke dimensions
10. Calculate the required mmf
11. Add 25% extra mmf for safety margin

- Decide on a final practical number of turns to be used for the coil
- Calculate current and current density in the coil
- Calculate the total length of conductor
- Calculate resistance of the coil
- Calculate voltage required from power supply
- Calculate power consumption of the coils
- Mass of the magnet (assume the iron volume as $1.2328x10^4$ cm^3)
- Tabulate the magnet specification parameters

Determine the pole polarity for a deflection of the beam to the right from its original direction and also the current direction in the coils.

Thank you

1. Sketch



2. Rigidity

Particle = proton

Proton rest mass energy $\boldsymbol{E_0}$ = 938.25 MeV

Proton kinetic energy $\boldsymbol{T}$ = 220 MeV

Proton charge $\boldsymbol{q}$ = 1.6 x 10^{-19} C

Charge state **Q** = +1

Velocity of light $\boldsymbol{c}$ = 2.9979 x 10^{8} m/s

$$BR = \frac{1}{Qc}\sqrt{T^2 + 2E_0T}$$

$$= \frac{1}{1\times(2.9979\times10^8)}\sqrt{(220\times10^6)^2 + 2(938.25\times10^6)(220\times10^6)}$$

$$= 2.265 \quad T.m$$

3,4,5,6. Pole Gap, Magnet effective length, Radius of curvature, Magnetic flux density

Pole gap

The beam pipe must obviously fit into the pole gap. With the outer diameter of the beam pipe at 104 mm, select a pole gap value of 106 mm

Magnet Length

Physical length = 200 mm (given)

Effective length = physical length + (extra)

(use 250 mm)

Radius of curvature

$$R = path\ length(effective\ length) \div angle(radians)$$

$$R = S / \theta = 0.250\ m / 3\times10^{-3} rad = 83.333\ m$$

Magnetic flux density

$$B = BR / R = 2.265\ Tm / 83.333\ m = 0.02718\ T$$

$$S = \frac{2.265\times\theta}{B}$$

7. Pole width

With the pole gap = 106 mm and the beam width about 40 mm, the pole width for a field homogenity of 1% is

$$w \approx gap + \Delta x_{0.01} = 106 + 40 = 146\ mm$$

And for a field homogenity of 0.1% it is

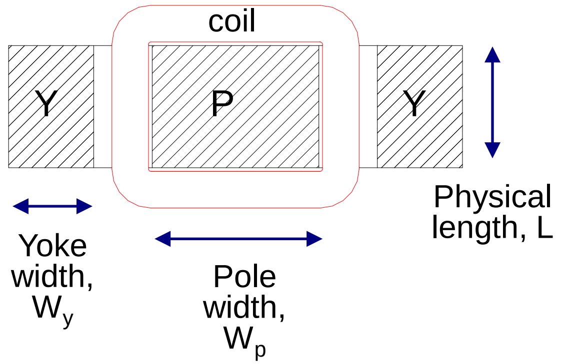
$$w \approx 2 \times gap + \Delta x_{0.001} = 212 + 40 = 252\ mm$$

In order to obtain about 0.8% field homogenity, linear interpolation gives a total width of about 170 mm, which is the same as if a beam width of 64 mm with 1% homogenity was assumed.

$$w \approx gap + \Delta x_{<0.01} = 106 + 64 = 170\ mm$$

8,9. Cross sectional surface of the yoke

In our magnet, with a magnetic flux density much less than 1.5 T (iron saturation point), we can assume that all the magnetic flux through the pole surface, P, will return through the yoke surfaces, Y.



Magnetic flux through the air gap = magnetic flux through the iron

$$area_{air} \times flux\ density_{air} = area_{iron} \times flux\ density_{iron}$$
$$area_{A} \times flux\ density_{air} = area_{2B} \times flux\ density_{iron}$$
$$W_p \times L \times B_{air} = 2 \times W_y \times L \times B_{iron}$$

The minimum yoke width, which will ensure that the flux through the return yokes, is at saturation of 1.5 T, is

$$W_y = \frac{170mm \times 200mm \times 0.02718T}{2 \times 1.5T \times 200mm} = 1.54\ mm$$

A yoke width of 1.54 mm is impractical and therefore select any dimension > 1.54 mm that is readily available in the commercial market.

Select (say) 30 mm. It will provide a stable, rigid construction to support the coil weight and definitely have no problem with magnetic flux saturation in the yokes.

10,11. Magneto-motive force

Use Ampere's law $NI = \oint H.dl$

where,

H = magnetic intensity in A/m,

N = number of coil turns

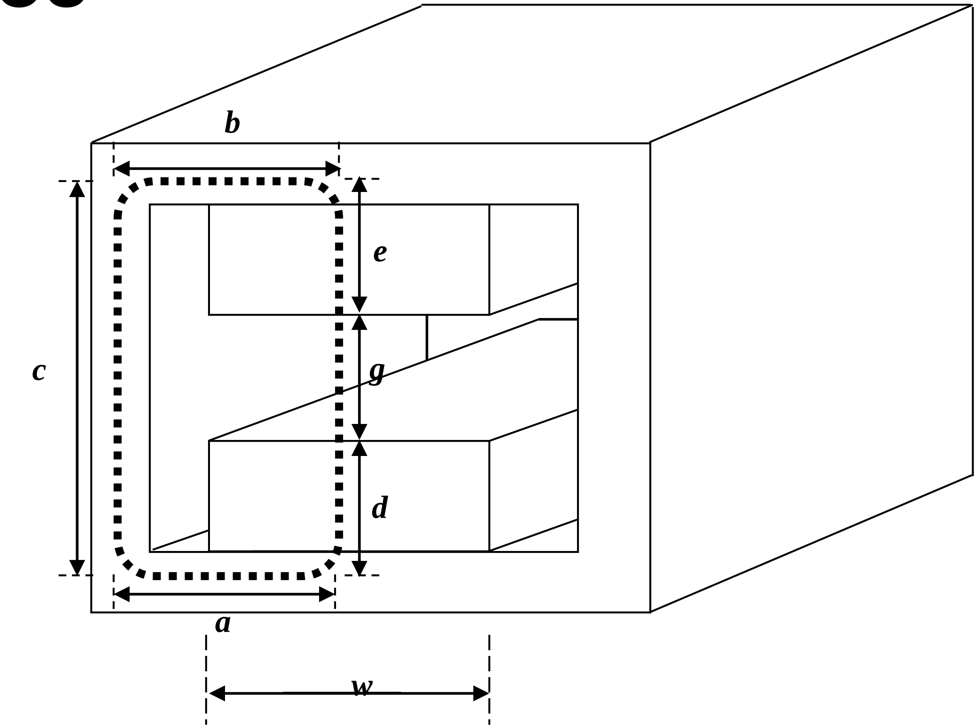
I = current in the coil

$H_{air} \times g + H_{iron} \times l = NI, \qquad l = a + b + c + d + e$

With $l \approx n \times g$ in our magnet, it becomes :

$$NI \approx g\left(H_g + nH_l\right)$$

$$NI \approx g\left(\frac{B_g}{\mu_0 \mu_{air}} + \frac{nB_{iron}}{\mu_0 \mu_{iron}}\right)$$



In order to make sure that we can ignore the second term, we can use the iron yoke width calculation to estimate the magnetic flux density in the iron.

$$W_{pole} \times L_{pole} \times B_{air} = 2 \times W_{yoke} \times L_{yoke} \times B_{iron}$$

$$B_{iron} = \frac{W_{pole} \times L_{pole} \times B_{air}}{2 \times L_{yoke} \times W_{yoke}} = \frac{0.170m \times 0.2m \times 0.02718T}{2 \times 0.2m \times 0.03m} = 0.07701\,T < 1.5T$$

In ferromagnetic materials is μ_{iron} > 1000 for flux densities < 1.5 T. Therefore the second term is ignored.

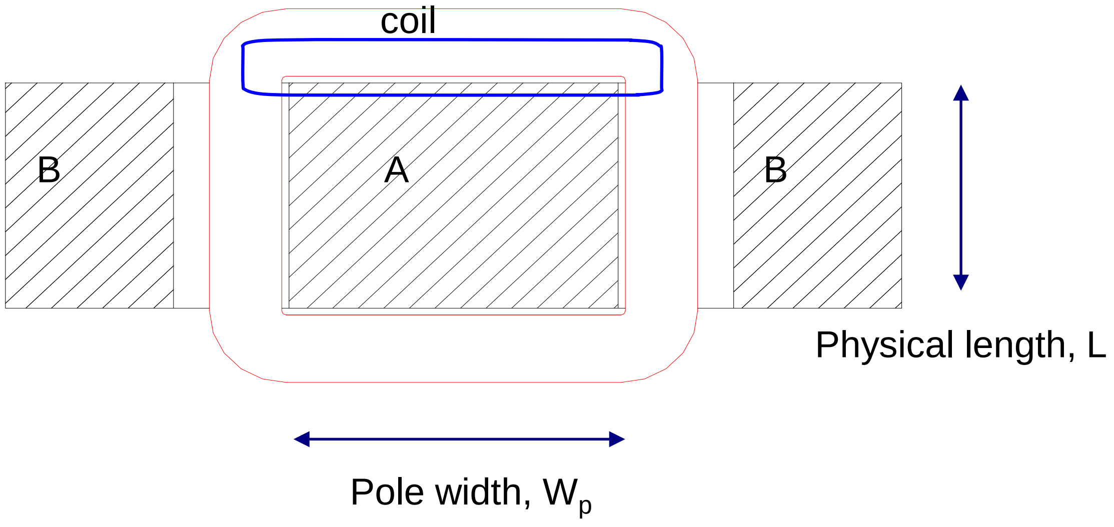
$$NI \approx g\left(\frac{B_g}{\mu_0 \mu_{air}}\right) = \frac{0.106m \times 0.02718T}{4\pi \times 10^{-7}} = 2293\,ampere.turns$$

Add 25% for extra bending power:

Total mmf = 2866 ampere. turns

With the total mmf divided between a coil around each pole tip, the mmf for each coil is 1433 ampere.turns

12,13,14. Number of turns, Currents, Total length of coil



The final choice of number of turns depends on the power supply. With 652 turns x 4.4 A will provide the 2866 ampere.turns, which is the required mmf.

The current density in a 2 mm diameter wire will then be $4.4/(\pi r^2) = 1.4$ A/mm^2, which is a comfortable number for air-cooled coils. [If the current density is > 2 to 3 A/mm^2, a conductor with another type of cooling has to be considered.]

The total length of coil can be calculated by taking the average length per turn x number of turns – AFTER DECIDING ON THE WIDTH AND HEIGHT TO BE USED FOR THE COIL.

Length per one coil = 0.924 (given for our calculation purposes) x 326 = 301 m

Total length of wire required for 2 coils = 602 m

15,16,17. Coil Resistance, Power supply current, Voltage and Power

The resistance of each coil is $R = \rho \frac{l}{A} = 0.01754 \times \frac{301m}{\pi r^2} = 1.6805\,\Omega$

with l = length of conductor in meter

ρ = resistivity of copper at 25° C

= 0.01754 Ω.mm^2/m

A = cross sectional conducting area of a single conductor

r = radius of wire conductor

Current from power supply: $I = \frac{mmf}{N} = \frac{2866}{652} = 4.4\,A$

Voltage required for one coil: $V = I \times R = 4.4 \times 1.6805 = 7.39\,V$

With the two coils in series, the total voltage required: $V_{tot} = 2 \times 7.39 \approx 14.8\,V$

NOTE: Care should be taken to provide extra voltage for the connection cables as well as the effect of temperature rise in the conductors.

Power: $P = V \times I = 14.8 \times 4.4 = 65.12\,W$

18. Mass of the magnet

Volume of iron = volume of pole + volume of yoke	$= 1.2328 \times 10^4$ cm^3 (given)
Mass of magnet (iron only) = volume x density of Fe	$= 1.2328 \times 10^4$ cm$^3 \times 7.85$ g/cm^3
	= 97 kg
Total length of wire	$= 2 \times 3.05 \times 10^4$ cm $= 6.1 \times 10^4$ cm
Volume of copper wire = cross sectional area x length	$= \pi \times 10^{-2}$ cm$^2 \times 6.02 \times 10^4$ cm
	= 1891 cm^3
Mass of conductor = Volume x density of copper	= 1891 cm$^3 \times 8.96$ g/cm^3
	$\approx$ 17 kg
Total mass of magnet = mass iron + mass copper	= 97 + 17 kg
	= 114 kg

19. Table of parameters

Pole gap	= 106 mm
Mass of magnet	= 114 kg
Mass of conductor	= 17 kg
Mass of iron	= 97 kg
Maximum Voltage	= 17 V
Maximum Current	= 5 A
Maximum Power	= 85 W
Maximum magnetic flux density	= 0.0431 T
Physical length	= 200 mm (given)
Effective length	= 0.25 m (empirical calculation)
Bending of 220 MeV protons	= 3.0×10^{-3} rad