D-brane masses and the motivic Hodge conjecture

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We consider one complex structure parameter mirror families W of Calabi-Yau 3-folds with Picard-Fuchs equations of hypergeometric type. By mirror symmetry the even D-brane masses of the orginal Calabi-Yau M can be identified with four periods w.r.t. to an integral symplectic basis of $H_3(W, \mathbb{Z})$ at the point of maximal unipotent monodromy. It was discovered by Chad Schoen in 1986 that the singular fibre of the quintic at the conifold point gives rise to a Hecke eigen form of weight four f_4 on $\Gamma_0(25)$ whose Fourier coefficients a_p are determined by counting solutions in that fibre over the finite field $\mathbb{F}_{p^k}$. The D-brane masses at the conifold are given by the transition matrix T_{mc} between the integral symplectic basis and a Frobenius basis at the conifold. We predict and verify to very high precision that the entries of T_{mc} relevant for the D_2 and D_4 brane masses are given by the two periods (or L-values) of f_4. These values also determine the behaviour of the Weil-Petersson metric and its curvature at the conifold. Moreover we describe a notion of quasi periods and find that the two quasi period of f_4 appear in T_{mc}. We extend the analysis to the other hypergeometric one parameter 3-folds and comment on simpler applications to local Calabi-Yau 3-folds.