Variational problems and spectral curves for the hermitian matrix model with external source

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Abstract.

A spectral curve for a matrix model is, in very loose terms, an equation with unknown being the Cauchy (a.k.a. Stieltjes) transform of the limiting spectral density. Sometimes also called master loop equation or string equation, it commonly appears as an algebraic equation, hence the name "curve" as it determines an algebraic curve. A classical situation is given by the celebrated semicircle law, whose Cauchy transform satisfies a very simple algebraic equation of degree 2. In this context, it also turns out that this limiting spectral density is the minimizer of a weighted log energy on the real line.

In this talk we plan to discuss spectral curves for matrix models and how they can be used in the construction of variational problems that describe the limiting spectral density for the model. Our key novel technique is to translate the determination of the solutions to the variational problem into the problem of geometrically describing trajectories of a canonical quadratic differential that lives on the curve.

We will devote particular attention to the hermitian matrix model with external source. Using techniques from integrable systems, we will discuss how one can associate a spectral curve for this model at the level of finite random matrix size. By letting the size of the random matrix grows large, we then show that any limiting eigenvalue distribution is, in fact, the solution of a spectral curve. By then applying the aforementioned techniques on quadratic differentials, we are thus able to obtain a variational problem characterizing such limiting eigenvalue distributions. Also as a consequence of our results, we are able to describe all possible critical local behaviors that can arise in this external source model.

This is a joint work with Andrei Martínez-Finkelshtein (Baylor University/Universidad de Almería)