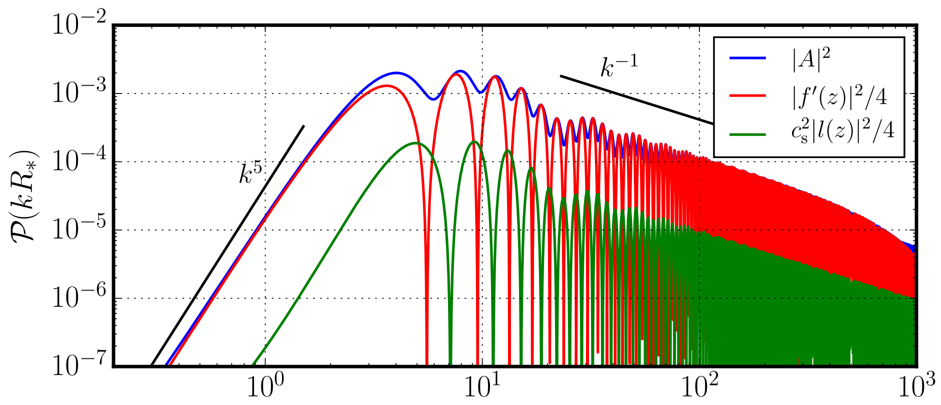
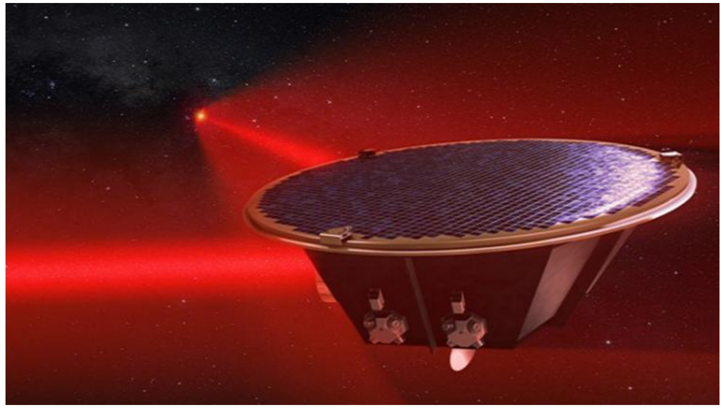




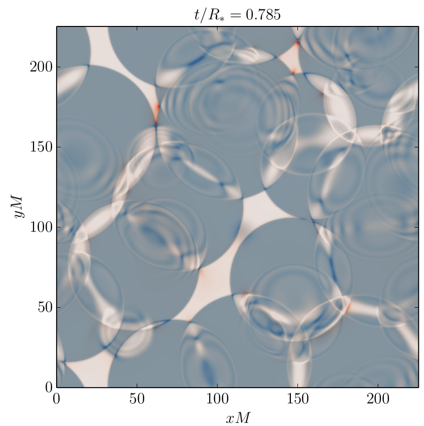
Gravitational waves from phase transitions

Mark Hindmarsh

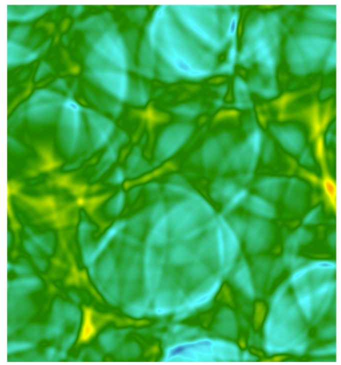
Helsinki Institute of Physics & Dept of Physics, University of Helsinki

and

Dept of Physics and Astronomy, University of Sussex



ICTP
15. lokakuuta 2019

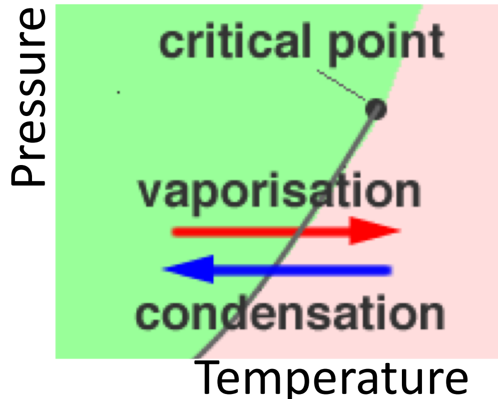


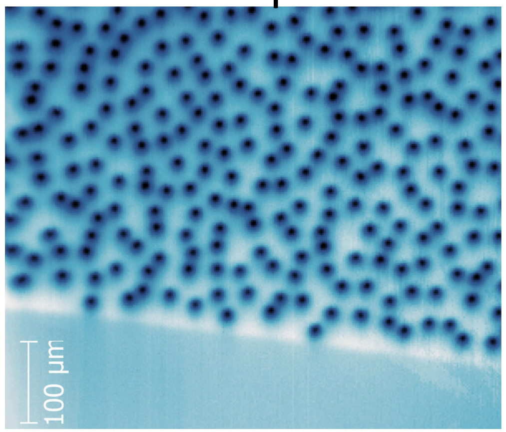
Phase transitions in the early Universe

- At very high temperatures and pressures, the state of matter in the Universe changes
  - T_c ~ 100 MeV (1 ms) QCD
  - T_c ~ 100 GeV (10 ps) Electroweak
  - T_c >> 100 GeV new symmetries?
- Departures from equilibrium and homogeneity (shear stress)
  - First order phase transition: relativistic condensation or `fizz'

    Steinhardt (1982)
  - Formation of topological defects

    Kibble (1976)

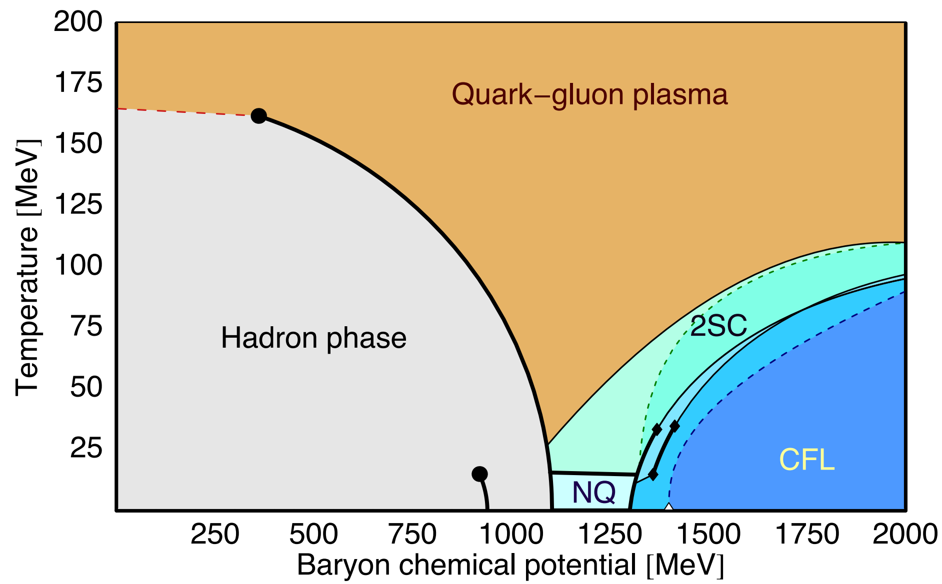


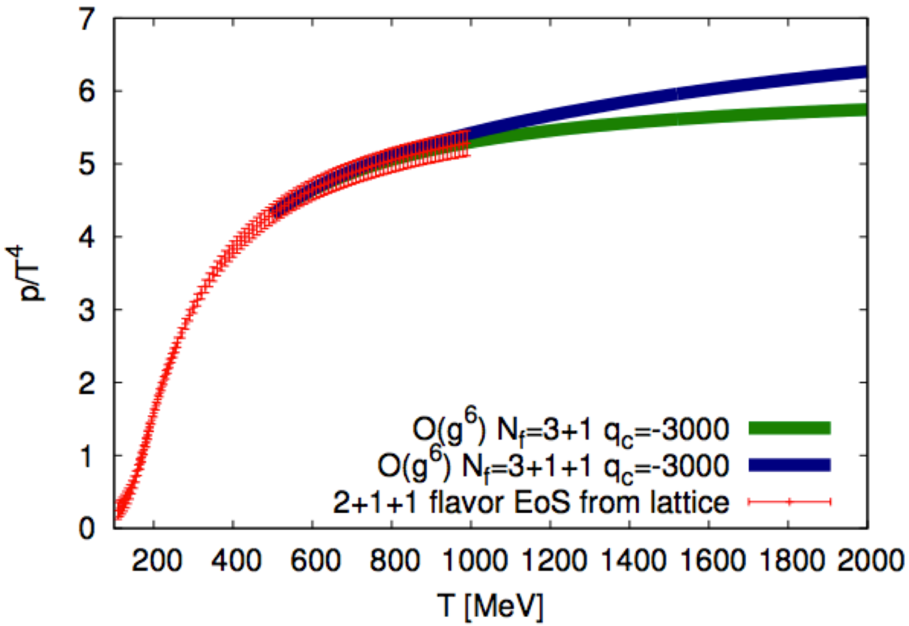


Abrikosov vortices

QCD phases

- QCD: rich phase diagram
- Universe: $n_B/n_\gamma \approx 6.1 \times 10^{-10}$
- Behaviour at low chemical potential well-established by lattice QCD   Borsanyi et al (2016)
- Transition from QGP to hadronic phase is a cross-over
- Departures from equilibrium very small: no GWs

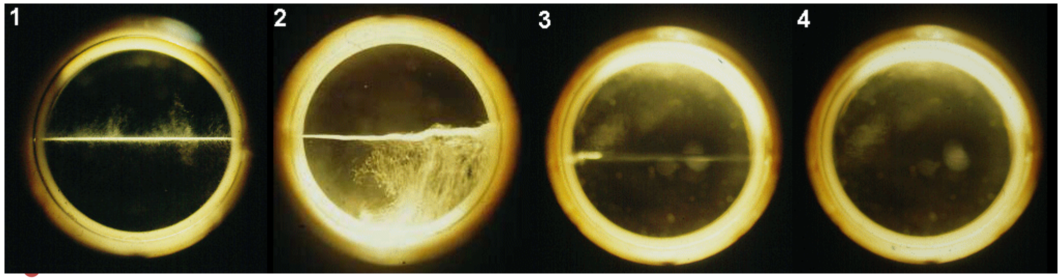


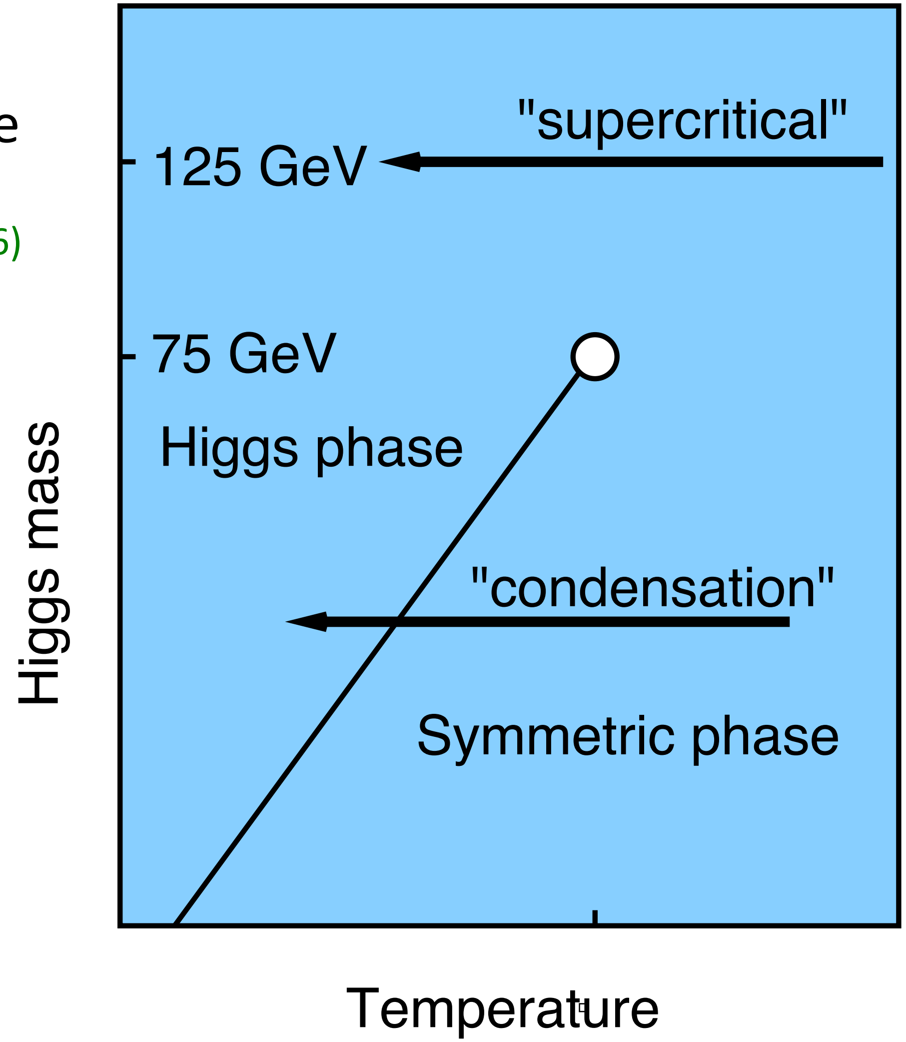


Borsanyi et al 2016

Electroweak transition

- SM is not weakly coupled at high T
- Non-perturbative techniques:
  - Dimensional reduction to 3D effective field theory + 3D lattice
    Kajantie, Laine, Rummukainen, Shaposhnikov (1995,6)
  - SU(2)-Higgs on 4D lattice
    Czikor, Fodor, Heitger (1998)
- SM transition at $m_h \approx 125$ GeV is a cross-over - a supercritical fluid

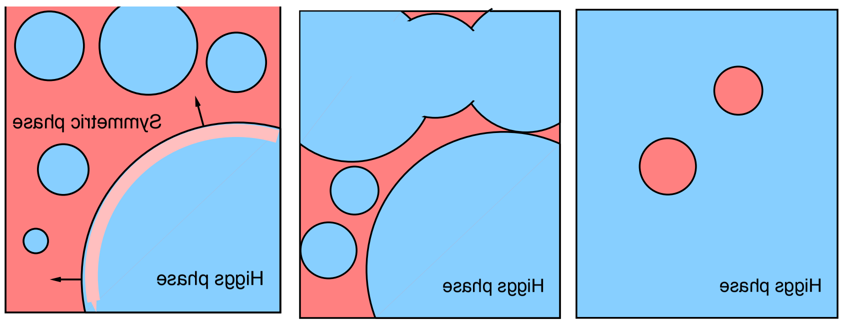




- Search for 1st order transition is a search for physics beyond SM

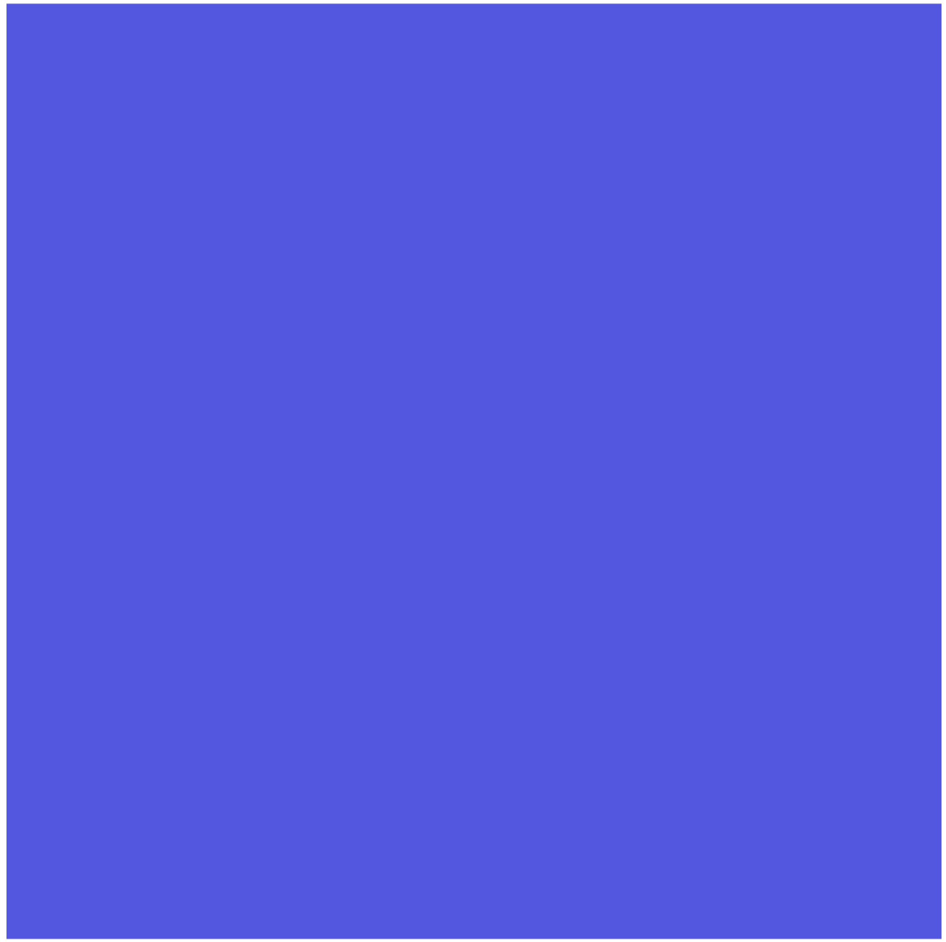
Little bangs in the Big Bang

- 1st order transition Langer 1969 proceeds by nucleation of bubbles of low-T phase
- Nucleation rate/volume $p(t)$ rapidly increases below T_c
- Expanding bubbles generate pressure waves in hot fluid
- Universal "fizz"



Steinhardt (1982); Hogan (1983,86); Gyulassy et al (1984); Witten (1984)

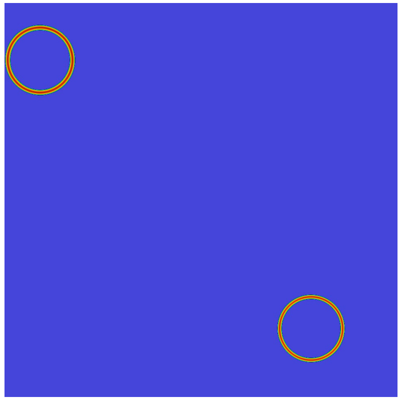
Fluid kinetic energy

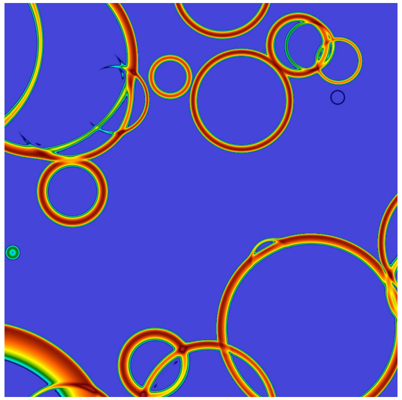


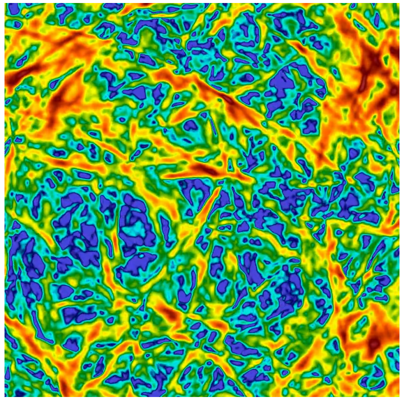
MH, Huber, Rummukainen, Weir (2013,5,7)
Cutting, MH, Weir (2018,9)

David Weir, Helsinki University

Phases of a phase transition







1 2 3 4

'exponential' nucleation

$$p(t) = p_n e^{\beta(t-t_n)}$$

$$\tau_{\rm co} = \beta^{-1}$$

Guth, Weinberg 1983; Enqvist et al 1992; Turner, Weinberg, Widrow 1992;

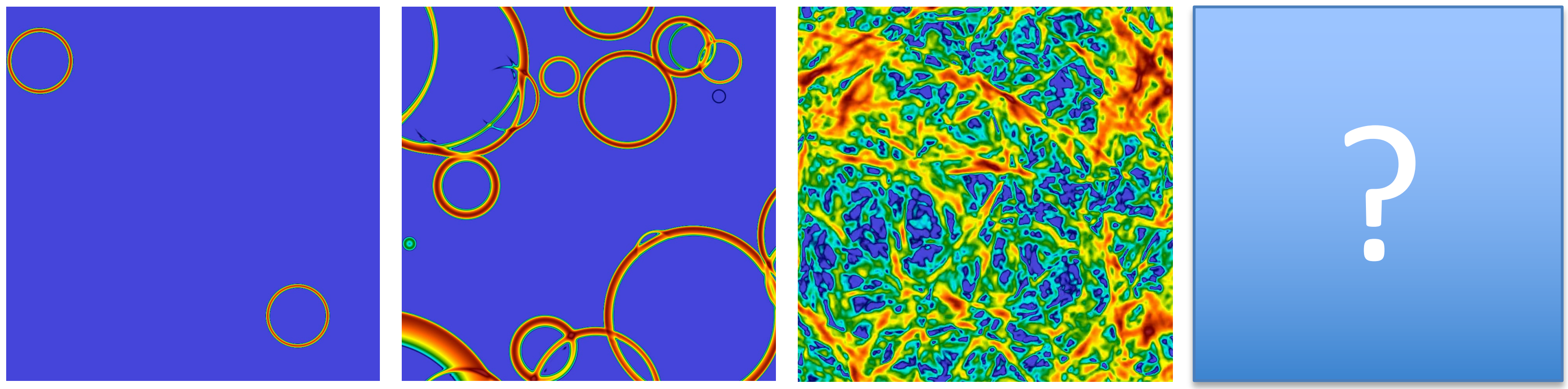
p – nucleation rate/volume
β – transition rate parameter

1. Nucleation and expansion
2. Collision
3. Acoustic
4. Non-linear (shocks, turbulence)

$$\tau_{\rm nl} \sim L_f/\bar{U}_f$$

$L_{\rm f}$ – fluid flow length scale
$U_{\rm f}$ – RMS fluid velocity

Phases of a phase transition



1 2 3 4

1. Nucleation and expansion
2. Collision
3. Acoustic
4. Non-linear (shocks, turbulence)

$$\tau_{\mathrm{tu}} \sim L_f / \bar{V}_\perp$$

L_f – fluid flow length scale
$\bar{V}_\perp$ – RMS rotational velocity

Not discussed here

- Generation of magnetic field?

  Baym, Bodeker, McLerran, 1996
  Vachaspati 2001
  Ferrer, Pogosian, Vachaspati 2019

- MHD turbulence

  Brandenburg et al 2017
  Roper Pol et al 2019

Dynamics of an early universe phase transition

- Ingredients:

Ignatius et al (1994), Kurki-Suonio, Laine (1996)

  - Higgs field $$-\ddot{\phi} + \nabla^2\phi - \frac{\partial V}{\partial \phi} = \eta W(\dot{\phi} + V^i\partial_i\phi)$$
    - η coupling to fluid (models energy transfer, friction)
  - Relativistic fluid

$$\dot{E} + \partial_i(EV^i) + P[\dot{W} + \partial_i(WV^i)] - \frac{\partial V}{\partial \phi}W(\dot{\phi} + V^i\partial_i\phi) = \eta W^2(\dot{\phi} + V^i\partial_i\phi)^2.$$

$$\dot{Z}_i + \partial_j(Z_iV^j) + \partial_i P + \frac{\partial V}{\partial \phi}\partial_i\phi = -\eta W(\dot{\phi} + V^j\partial_j\phi)\partial_i\phi.$$

    - *E* energy density, Z_i momentum density, V_i velocity, *W* γ-factor
  - Discretisation

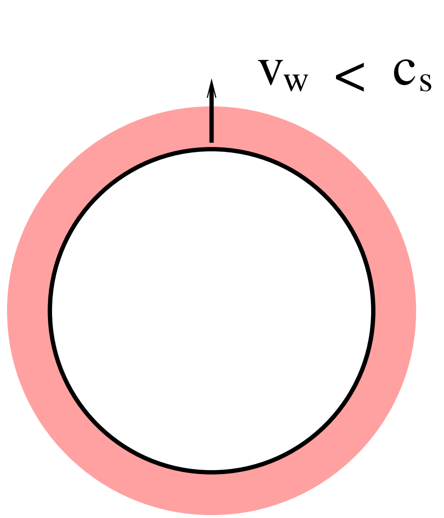
Wilson & Matthews (2003)
Different approach: Giblin, Mertens (2013)

  - Metric perturbation

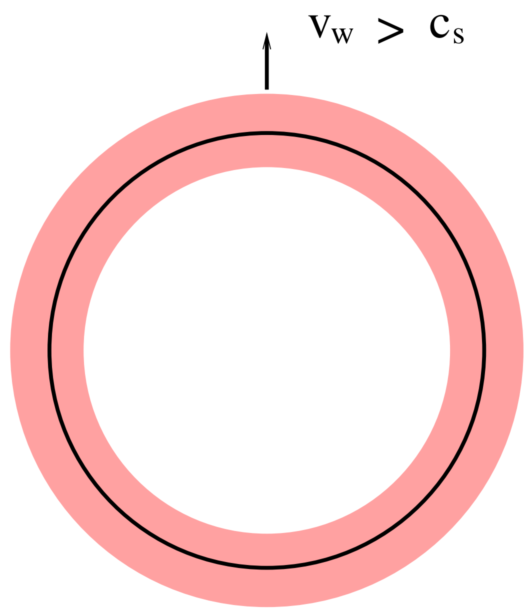
$$\ddot{h}_{ij} - \nabla^2 h_{ij} = 16\pi G T_{ij}^{\mathrm{TT}}$$

Garcia-Bellido, Figueroa, Sastre (2008)

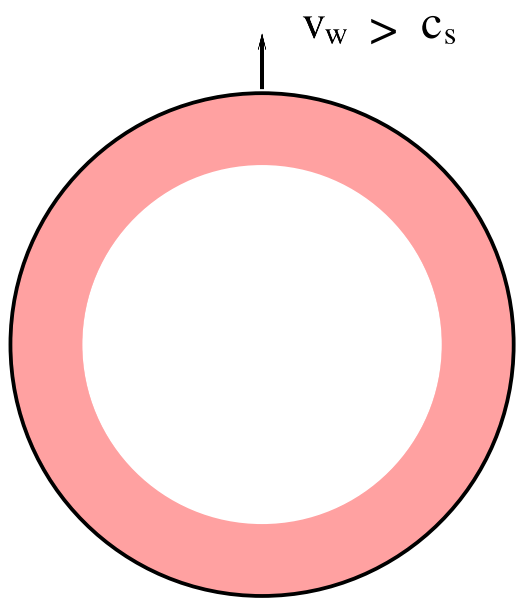
Relativistic combustion



Deflagration



Supersonic deflagration
("hybrid")



Detonation

Landau & Lifshitz
Steinhardt (1984)
Kurki-Suonio, Laine (1991)
Espinosa et al (2010)

- Scalar potential energy (free energy) to kinetic energy, heat energy
- Wall velocity v_w - pressure difference vs. scalar-fluid coupling $\eta(\phi)$
- Radial fluid velocity $v(r,t)$ and enthalpy distribution $w(r,t)$
- Similarity solution $v(r/t)$, $w(r/t)$
- Some cases … runaway ($v_w \to 1$) (near-vacuum transition)

Bodeker Moore 2010, 2017

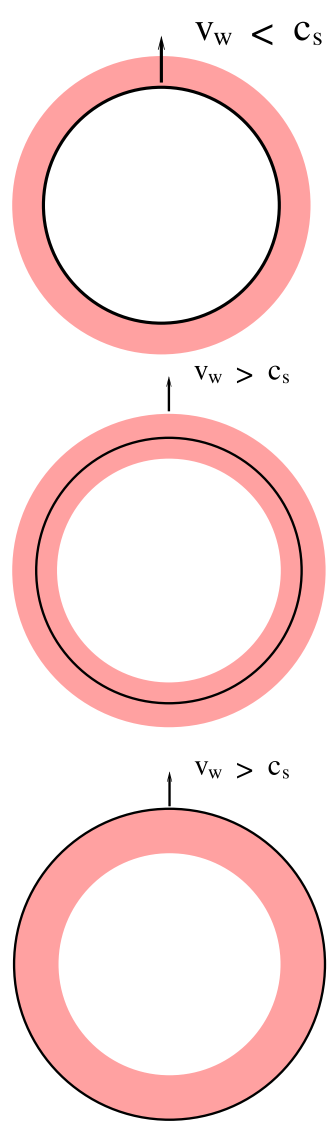
GWs from first order phase transitions

- Parameters of transition:
  - T_n = Temperature at nucleation
  - β = transition rate (= - d log p / dt)
  - v_w = Bubble wall speed
  - α = ("Potential energy")/("Heat energy")
- Derived parameters:
  - R_* = mean bubble separation $= (8\pi)^{1/3} v_w / \beta$
  - K = fluid kinetic energy fraction (depends on α, v_w)

Steinhardt '84
Espinosa et al 2010

- Aim: GW power spectrum

$$\frac{d\Omega_{\rm gw}}{d\ln f} = \frac{1}{\rho_{\rm tot}}\frac{d\rho_{\rm gw}}{d\ln f} = \frac{8\pi^2}{3H^2} f^3 S_h(f)$$



GWs from phase transitions

- Gravitational waves generated by shear stress fluctuations

$$\Omega_{\mathrm{GW}} \sim \frac{1}{G\rho} \left\langle \left| \dot{h}_{ij}(t) \right|^2 \right\rangle$$

$$\dot{h}_{ij} \sim G \int dt' \cos[k(t-t')] T_{ij}^{TT}(k,t')$$

- Shear stress ~ kinetic energy
- Kinetic energy from potential energy
  - $K(\alpha, v_w)$ = fluid kinetic energy fraction

$$T_{ij} \sim \rho U_i U_j$$

$$\dot{h} \sim \tau (G\rho) K$$

- Timescales τ_v and τ_c
  - τ_v duration of stresses from fluid velocity
  - τ_c coherence time of stress fluctuations

$$\Omega_{\mathrm{gw}} \sim \frac{\tau_v \tau_c}{G\rho} (G\rho)^2 K^2$$

$$\Omega_{\mathrm{gw}} \sim (H_n \tau_v)(H_n \tau_c) K^2$$

$$\Omega_{\mathrm{gw},0} \sim \Omega_{\mathrm{rad},0} (H_n \tau_v)(H_* \tau_c) K^2$$

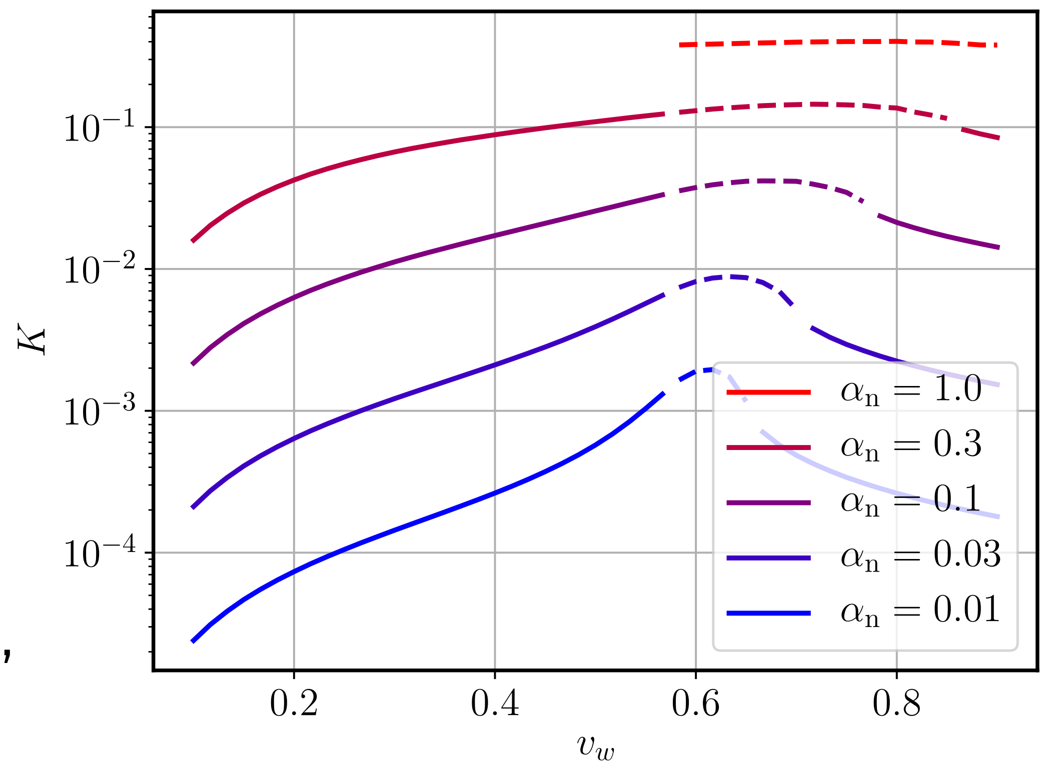
Estimating GW power

- Recall GW energy fraction:
  - τ_v duration of stresses
  - τ_c coherence time
- Numerical simulations:
  - $\tau_c \sim R_*$ (bubble separation)
- Analytical estimate:
  - $\tau_v = \min(H_n^{-1}, R_*/U_f)$
  - N.B. $K = (4/3)U_f^2$
  - U_f (weighted) RMS velocity
  - Pure acoustic ($H_n R_* >> U_f$)

$$\Omega_{\rm gw} \simeq (H_{\rm n} R_*) K^2 \tilde{\Omega}_{\rm gw},$$
$$\tilde{\Omega}_{\rm gw} \sim 10^{-2}$$

$$\Omega_{\rm gw} \sim (H_{\rm n}\tau_v)(H_{\rm n}\tau_c)K^2$$

K (kinetic energy fraction)
from self-similar hydro solution



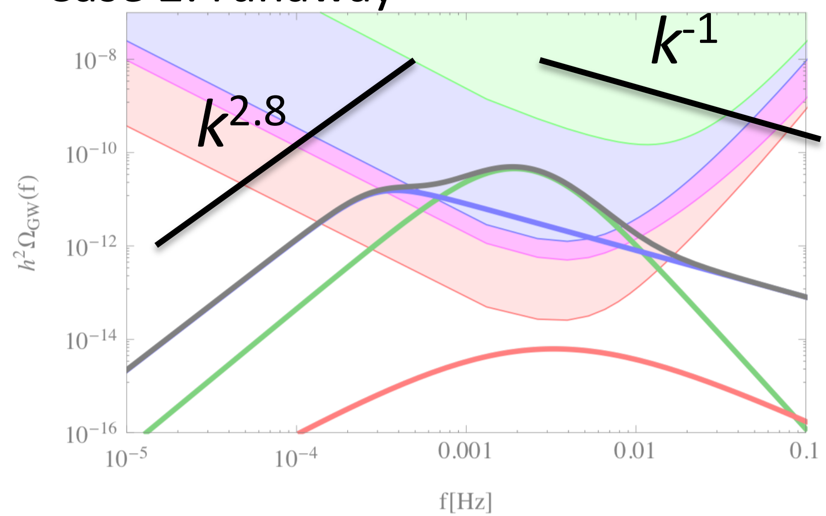
LISA CWG party line 2016

- Three contributions to total power:
  - Scalar field ϕ
  - Acoustic ac
  - Turbulent tu

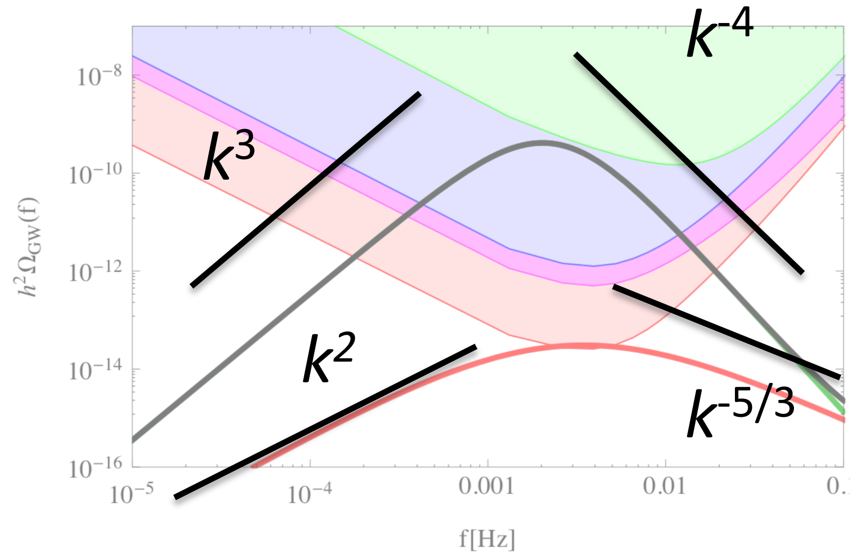
$$\Omega_{\rm gw} = \Omega_{\rm gw}^{\phi} + \Omega_{\rm gw}^{\rm ac} + \Omega_{\rm gw}^{\rm tu}$$

- Scalar field: bubble wall collisions
  - relevant only for runaway walls
  - "envelope approximation"
  - Kosowsky, Turner 1992
  - Huber, Konstandin 2008
- Acoustic production:
  - M.H. et al 2013, 2015, 2017, 2019
- Turbulent production:
  - Caprini, Durrer, Servant 2009

Case 2: runaway



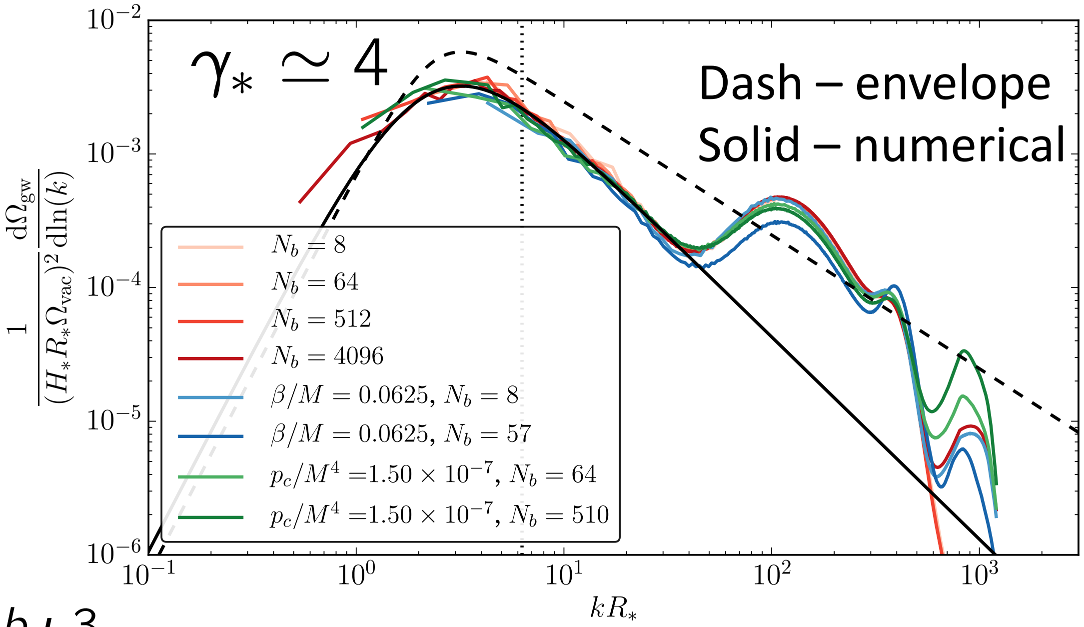
Case 1: constant v_w



Developments 1: scalar field

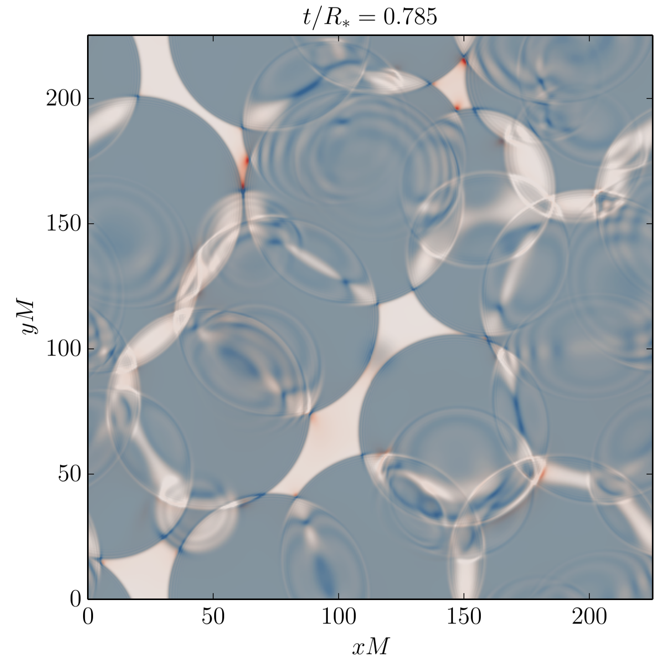
- Numerical simulations show differences from envelope approximation

Cutting, MH, Weir 2018



$$\frac{d\Omega_{\rm gw}^{\rm fit}}{d\ln k} = \Omega_{\rm p}^{\rm fit} \frac{(3+b)^c \tilde{k}^b k^3}{(b\tilde{k}^{(3+b)/c} + 3k^{(3+b)/c})^c}$$

$$\Omega_{\rm p}^{\rm fit} = (3.22 \pm 0.04) \times 10^{-3} (H_{\rm n} R_*)^2 \Omega_\phi^2,$$
$$\tilde{k} R_* = 3.20 \pm 0.04,$$
$$b = 1.51 \pm 0.04, \quad c = 2.18 \pm 0.15$$

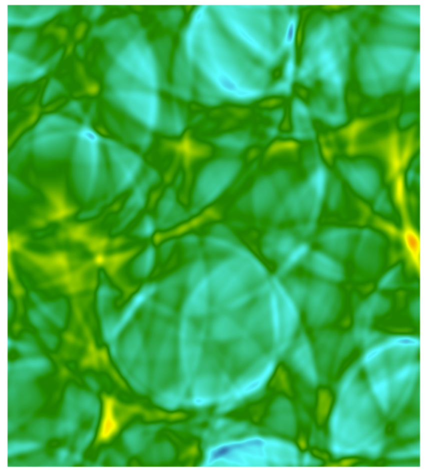


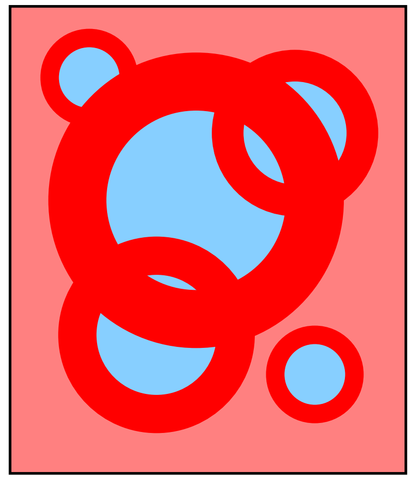
Developments 2: sound shell model

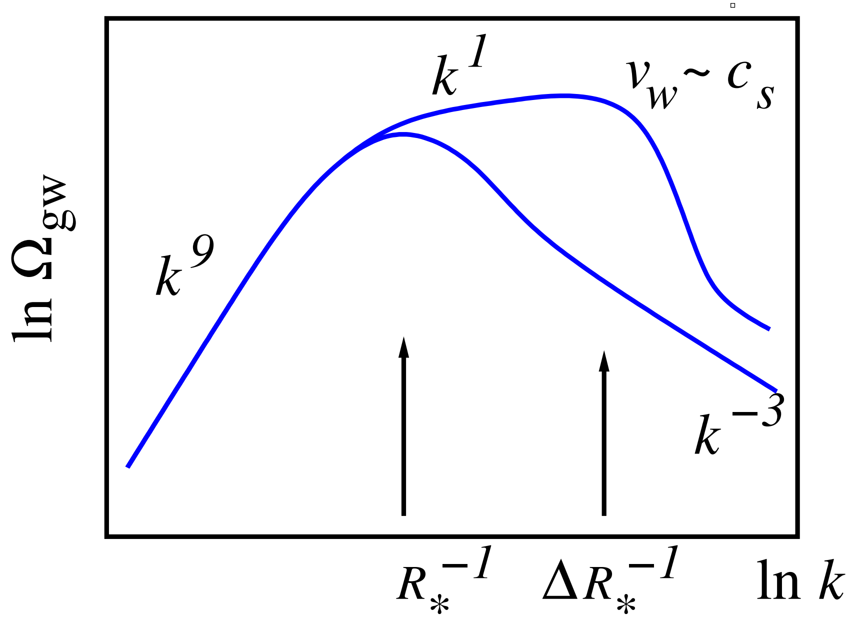
- Gaussian velocity field from weighted addition of sound shells $\mathbf{v}_{\mathbf{q}}(t_{\mathrm{i}})$

  MH 2017; MH, Hijazi 2019

- Two length scales:
  - Bubble spacing R_*
  - Shell width $R_* \frac{|v_w - c_s|}{v_w}$
- Double broken power law
  - $P_{gw} \sim k^9, k^1, k^{-3}$
- Amplitude:
  - Detonations: good (< 10%)
  - Deflagrations: overestimated

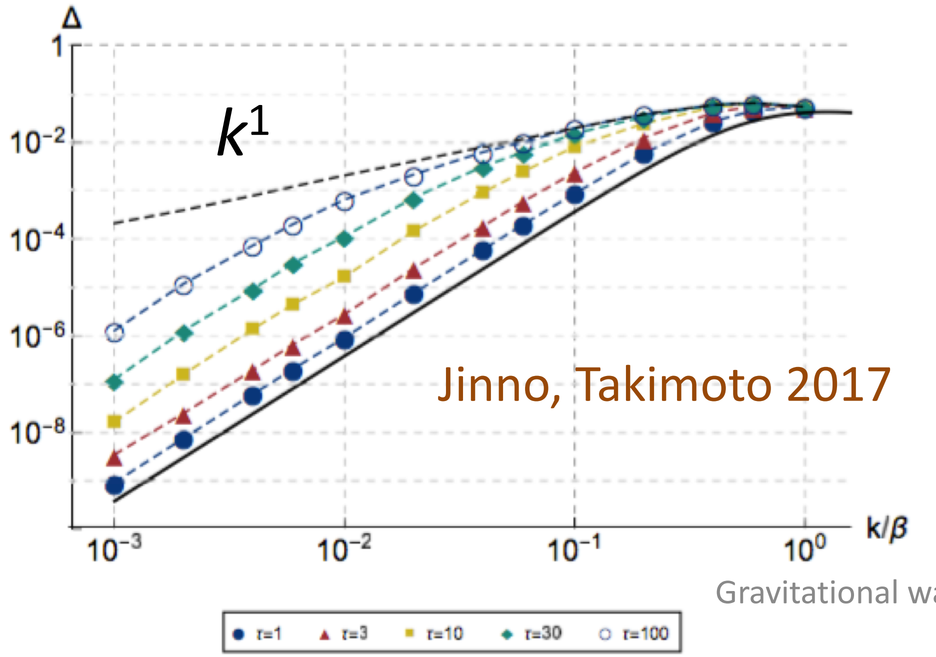




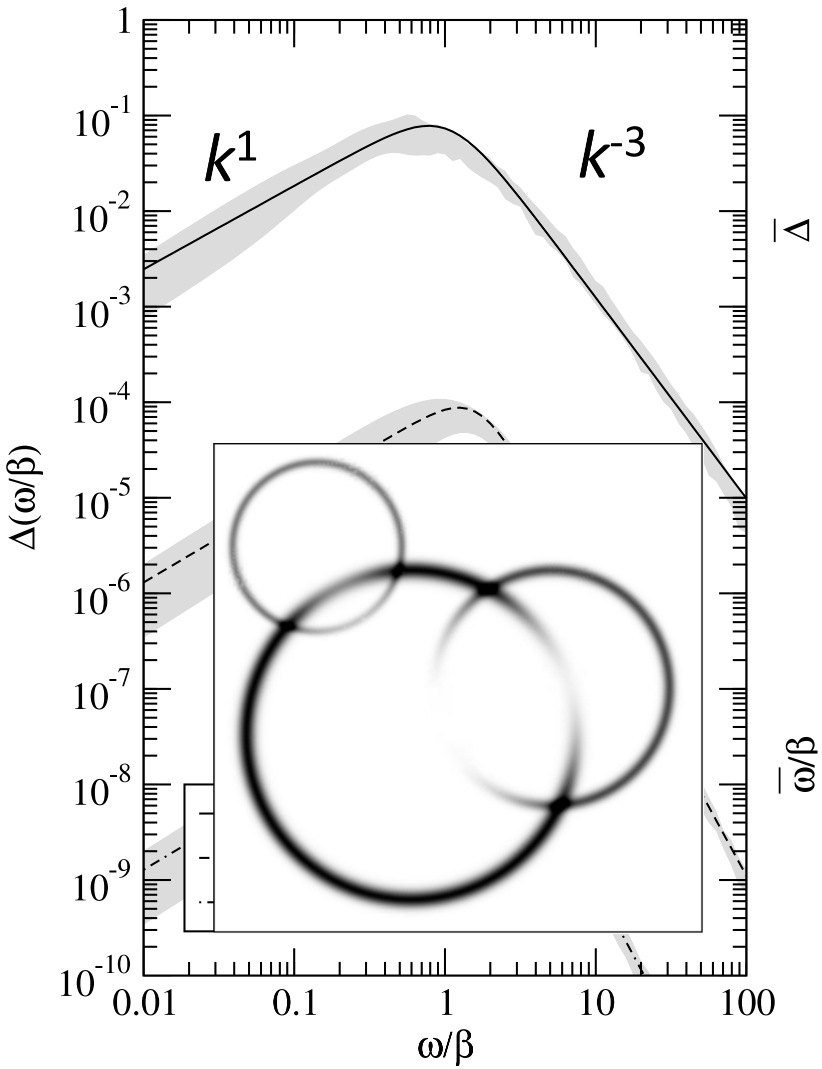


Developments 3: bulk flow models

- Based on envelope approximation
- Model overlapping energy shells in real space
- Disagrees with sound shell model at low *k*



Jinno, Takimoto 2017

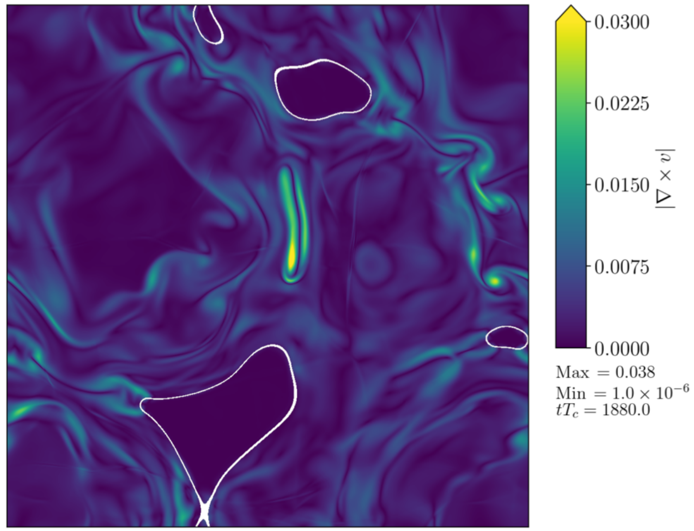


Konstandin 2017

Developments 4: non-linearities

- Longitudinal v
  - Kinetic energy suppression
- Transverse v
  - Vorticity production

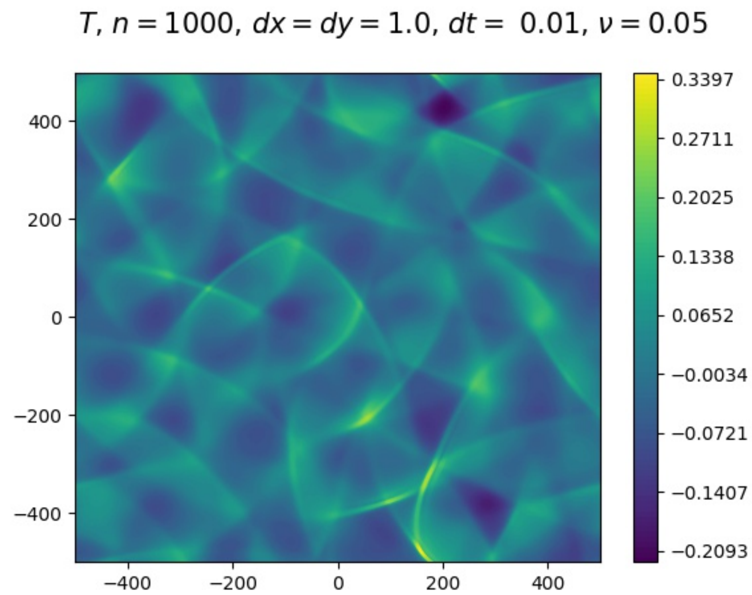
    Cutting, MH, Weir 2019



- Non-linearity timescale

  $$\tau_{\rm nl} \sim L_f / \bar{U}_f$$

  - Shock development
  - Further vorticity production
  - Decay of flows



J Dahl, U Helsinki

Developments 5: turbulence

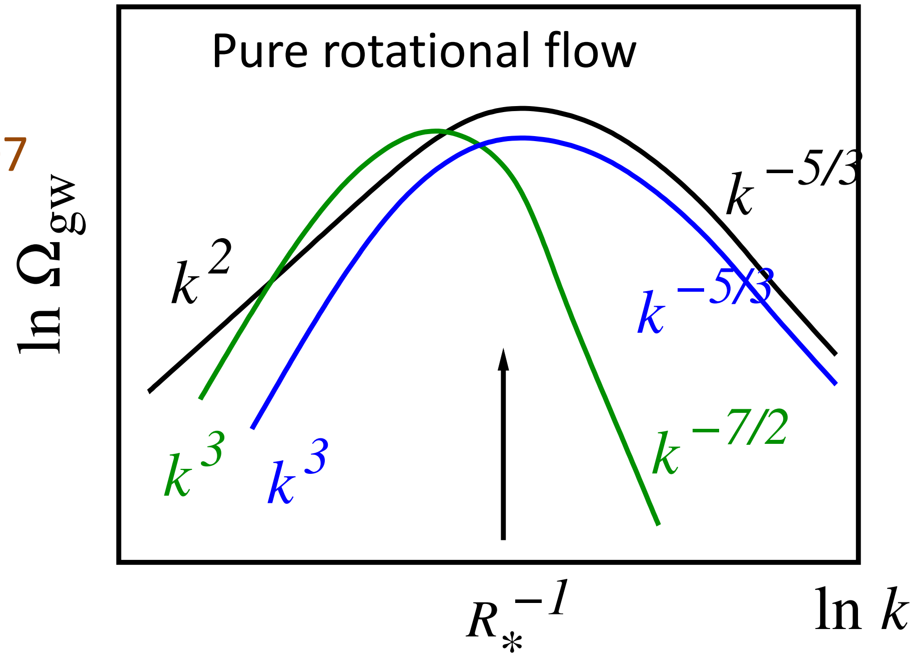
- Modelling

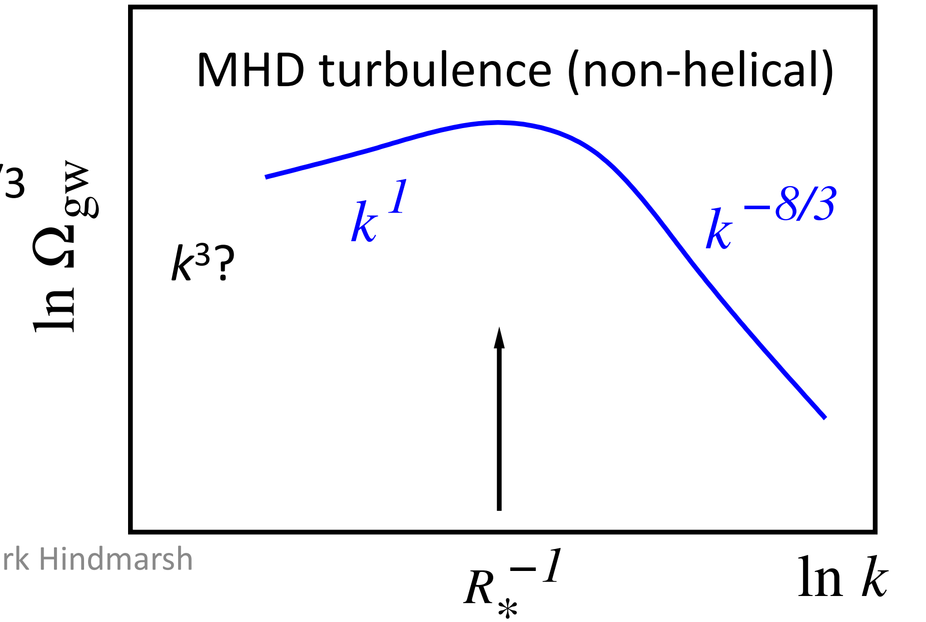
  Green: Gogoberidze, Kahniashvili, Kosowsky 2007
  Black: Caprini, Durrer, Servant 2008

  Blue: Niksa, Schlederer, Sigl 2018

  – Kraichnan sweeping model: velocity autocorrelation time
  $$\tau_k \sim 1/k\bar{v}_\perp$$
  – Pure rotational flow: high k GW power spectrum $k^{-5/3}$
  – Mixed acoustic-turbulent $k^{-8/3}$

- MHD simulation
  Roper pol et al 2019



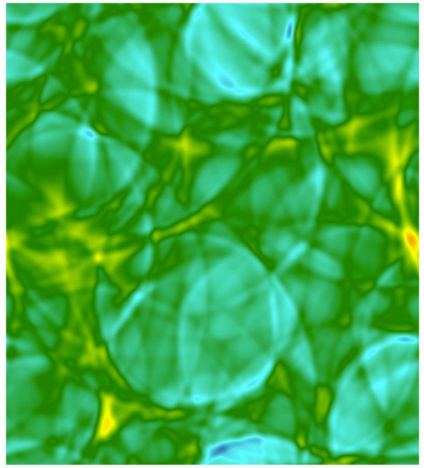


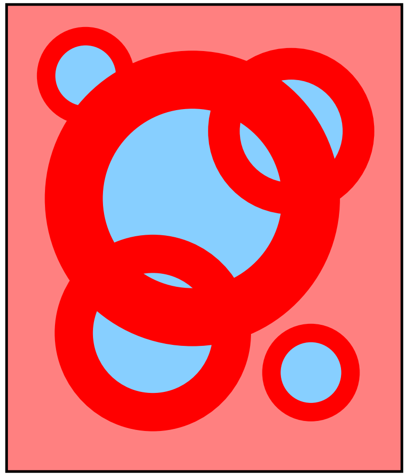
Sound shell model

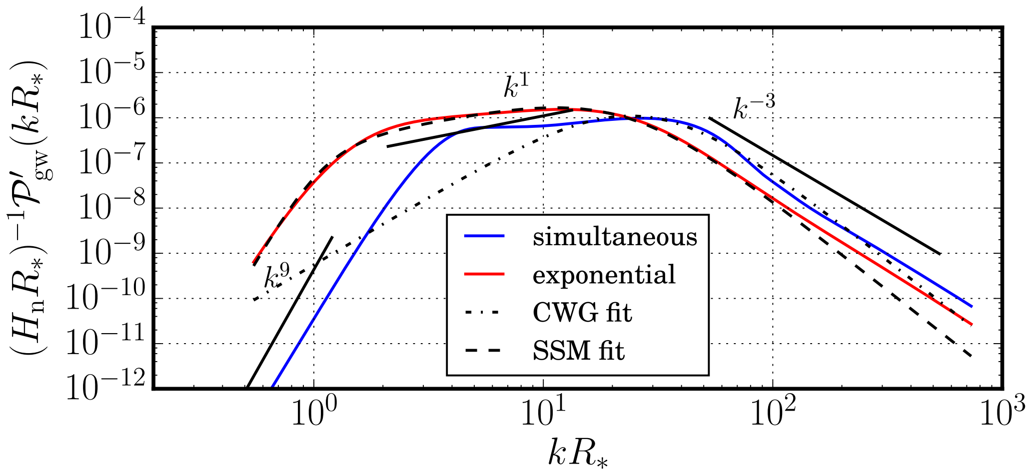
- Gaussian velocity field from weighted addition of sound shells $\mathbf{v}_{\mathbf{q}}(t_{\mathrm{i}})$

  MH 2017, MH, Hijazi (in prep 2019)

- Two length scales:
  - Bubble spacing R_*
  - Shell width $R_* \frac{|v_w - c_s|}{v_w}$
- Double broken power law
  - $P_{gw} \sim k^9, k^1, k^{-3}$
- Amplitude:
  - Bubble separation
  - (Kinetic energy)^2



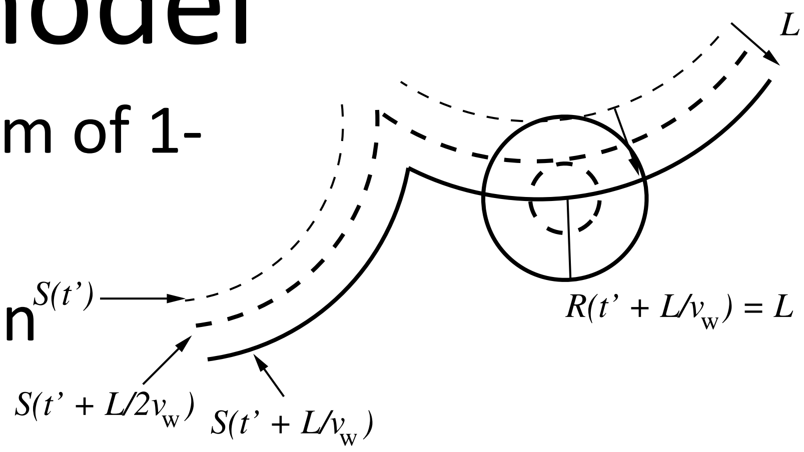




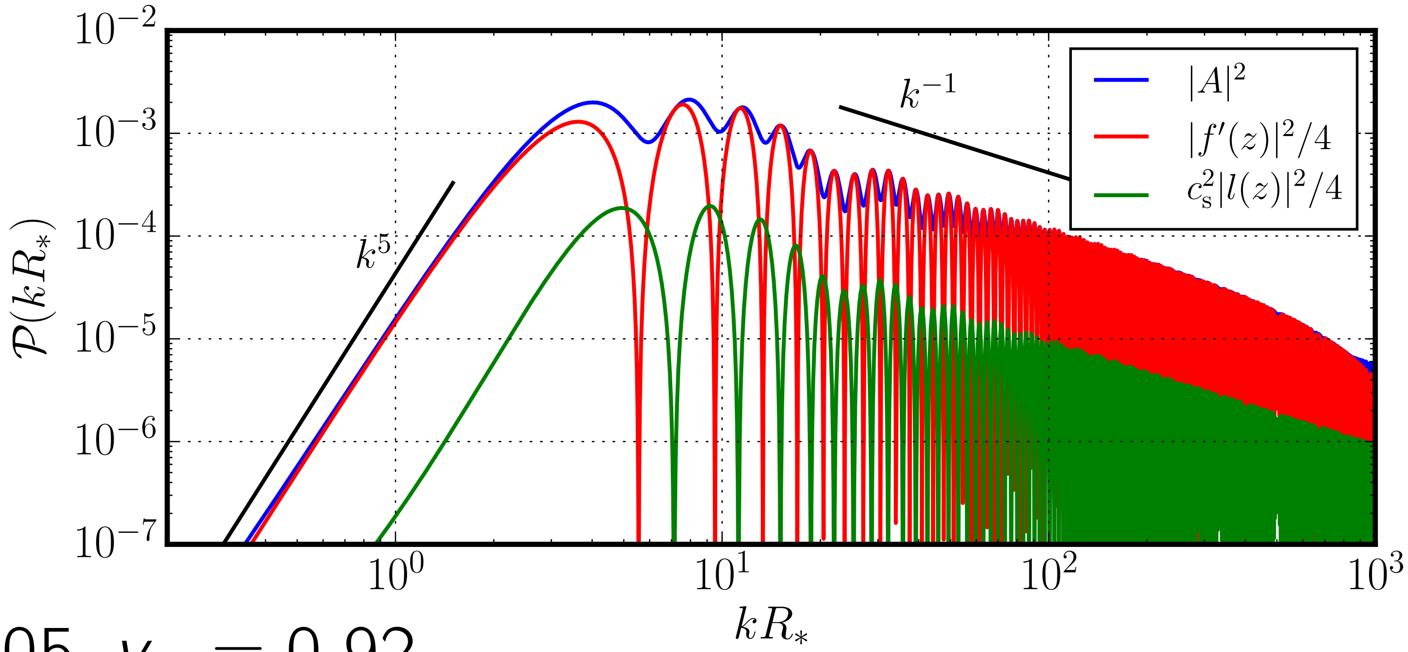
$\alpha_{\mathrm{n}} = 0.0046 \quad v_w = 0.56$

Sound shell model

- Velocity power spectrum is weighted sum of 1-bubble power spectra
- Weighting by bubble lifetime distribution



$$\nu(\beta T) = \begin{cases} \exp(-\beta T) & \text{exponential} \\ \frac{1}{2}(\beta T)^2 \exp(-\frac{1}{6}(\beta T)^3) & \text{simultaneous} \end{cases}$$

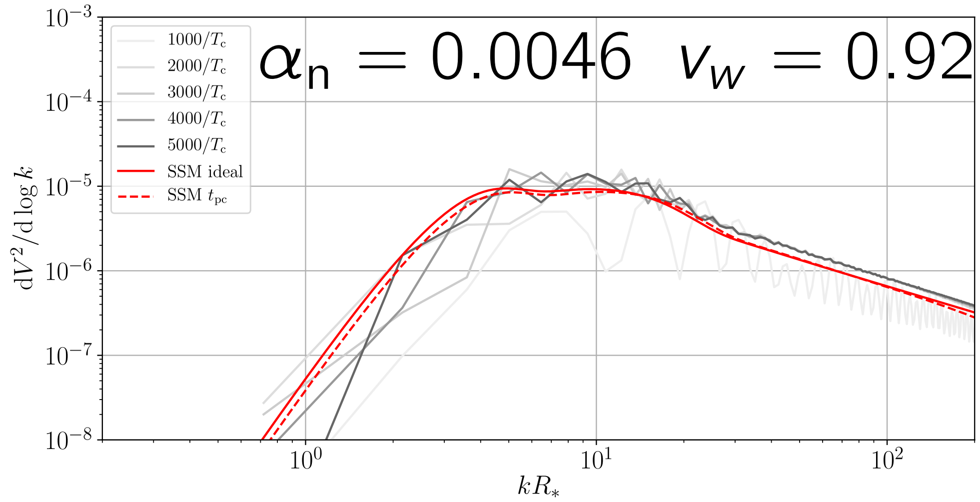


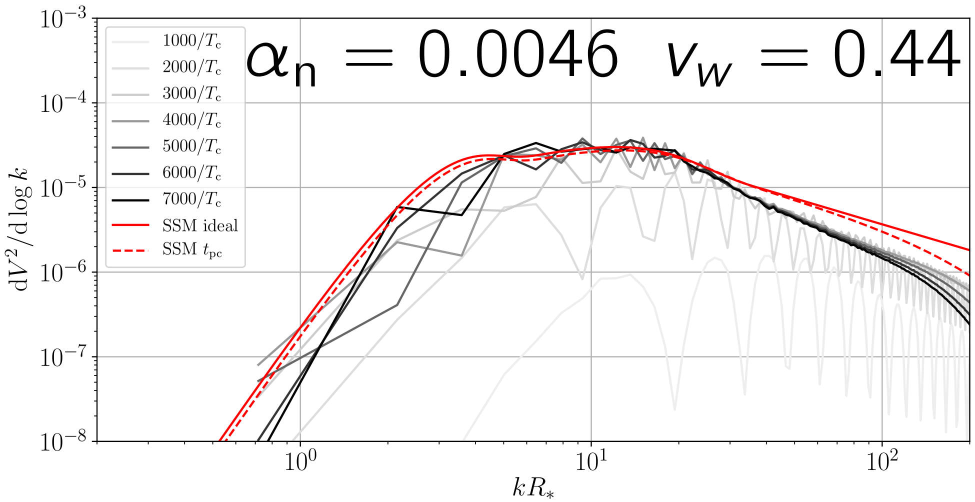
$\alpha_n = 0.05 \quad v_w = 0.92$

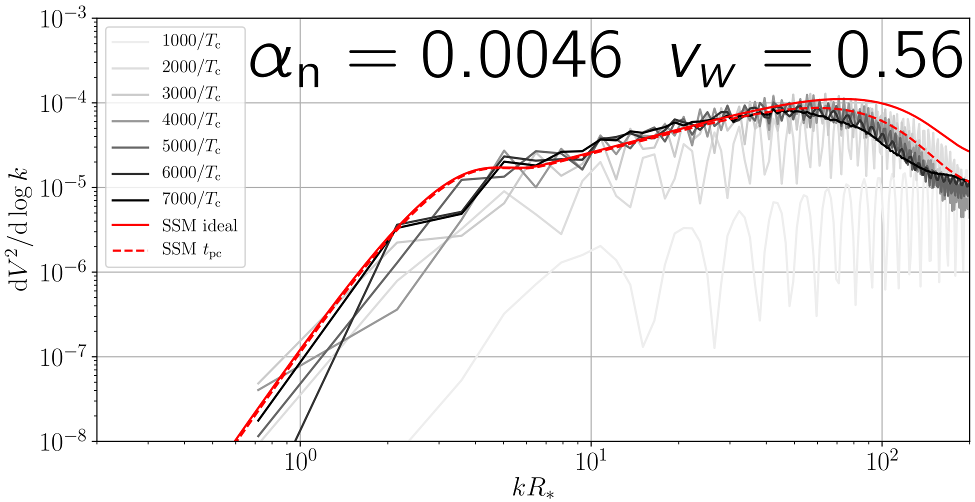
Sound shell model vs. simulations P_v

- Solid: self-similar sound shell
- Dash: evolving sound shell at peak collision time
- Simultaneous nucleation

MH et al in prep 2019



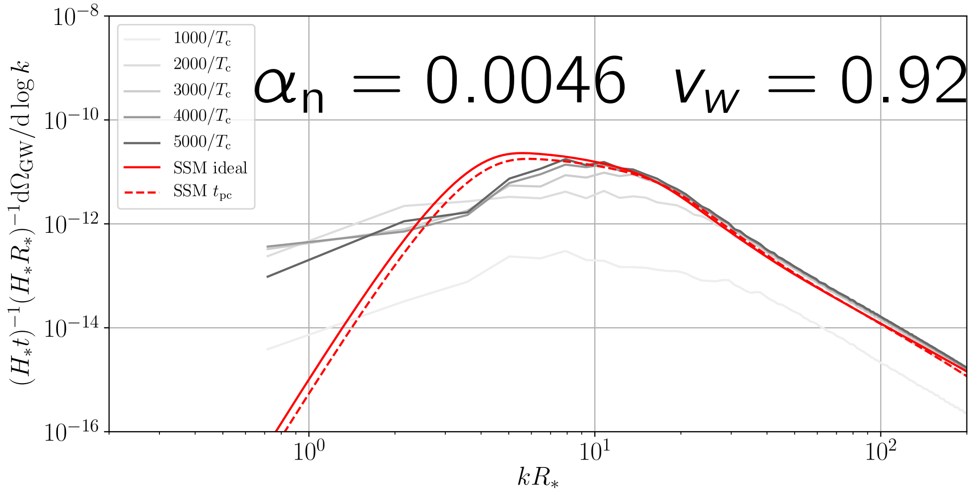


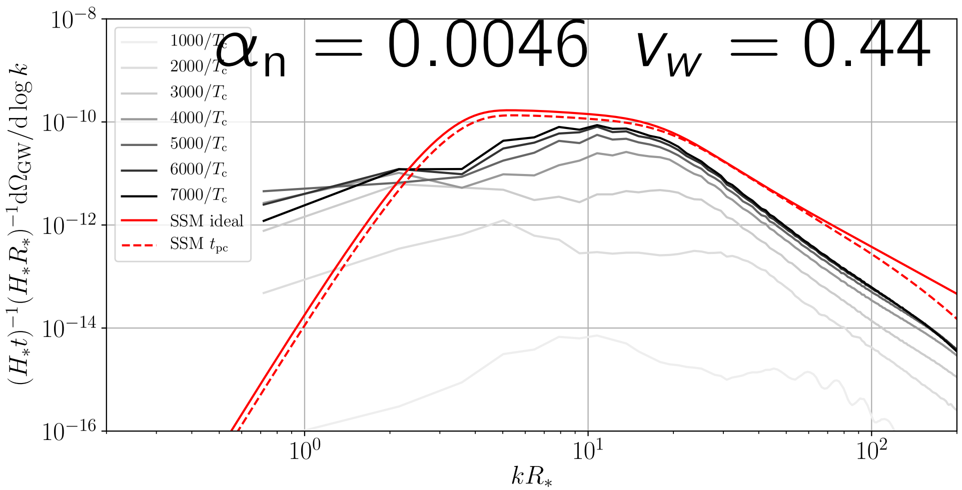


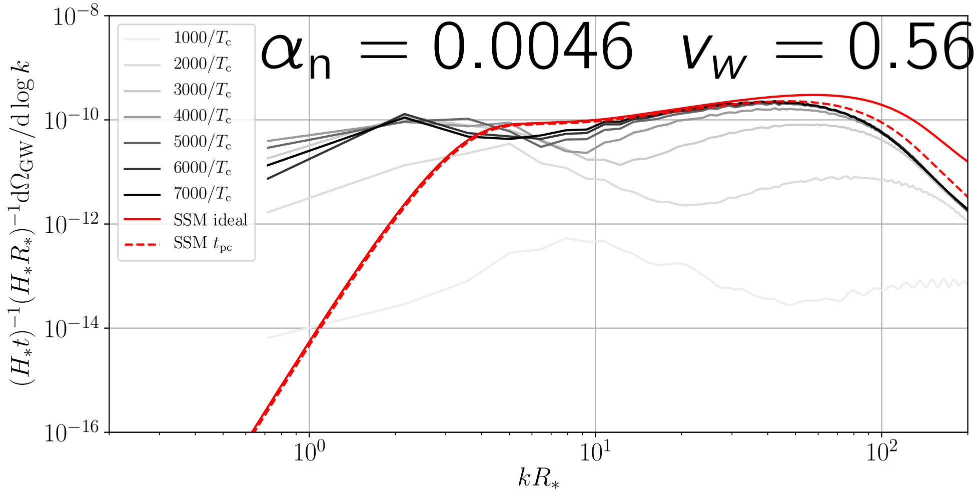
Sound shell model vs. simulations P_{gw}

- Solid: self-similar sound shell
- Dash: evolving sound shell at peak collision time
- Simultaneous nucleation

MH et al in prep 2019







GW PS in sound shell model

MH, Hijazi 2019

$$\Omega_{\rm gw}^{\rm ac}(f) = F_{\rm gw,0}(H_{\rm n}R_*)A_M\, M\left(\frac{f}{f_{p,0}}\right) \qquad A_M \simeq K^2\tilde{\Omega}_{\rm gw}$$

- Double broken power law

$$M(s) = s^9\left(\frac{r_{\rm p}^4+1}{r_{\rm p}^4+s^4}\right)^2\left(\frac{5}{5-m+ms^2}\right)^{5/2}, \qquad s = f/f_{p,0}$$

$$m = (9r_{\rm p}^4+1)/(r_{\rm p}^4+1)$$

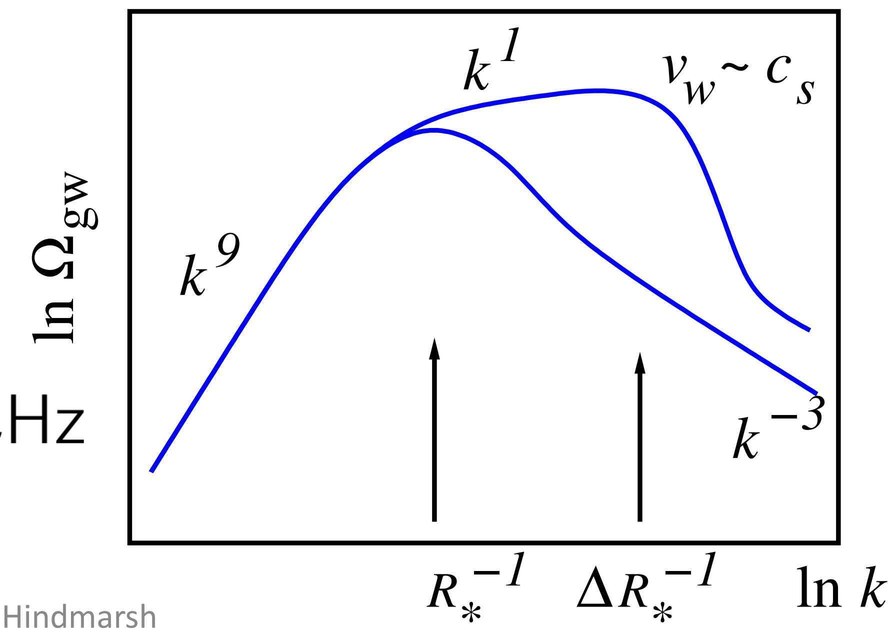
- Radiation energy density redshift

$$F_{\rm gw,0} \simeq 3.6\times10^{-5}$$

- Peak frequency now:

$$f_{p.0} \simeq 2.6\, z_{\rm p}(H_{\rm n}R_*)^{-1}(T_{\rm n}/100\,\mathrm{GeV})\,\mu\mathrm{Hz}$$

- Simulations: $z_{\rm p} = \mathrm{O}(10)$

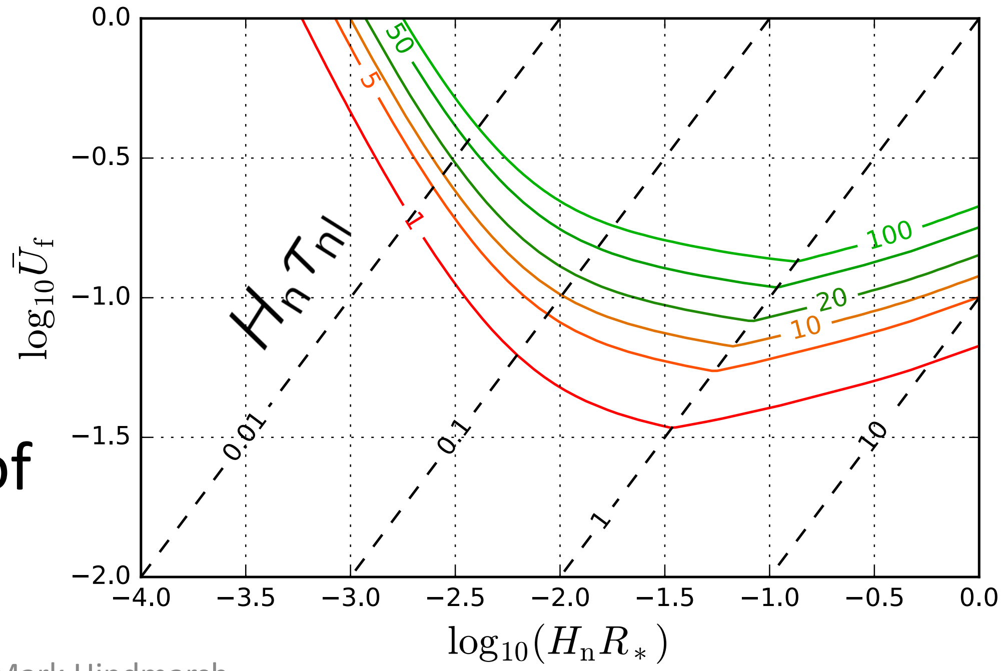


Flow lifetime uncertainty

- Non-linearities important after
  $$\tau_{\rm nl} \sim L_f / \bar{U}_f$$
  - Also effective flow lifetime

  - Estimate: multiply PS by
    $$\min(1, H_{\rm n}\tau_{\rm nl})$$
  - GW power parametric estimate
    $$\Omega_{\rm gw} \sim (H_{\rm n}R_*)^2 K^{3/2}$$
  - Estimate SNR for LISA in terms of $U_{\rm f}$, $H_{\rm n}R_*$

LISA SNR estimate (3 years observation)



Summary

- Good understanding of GWs from near-linear flows.
  - $\alpha \sim 0.1$, $v_w > 0.4$
- Dominant source is sound
- Total power estimate:

$$\Omega_{\mathrm{gw},0} \simeq F_{gw,0} \min\left(1, \frac{H_{\mathrm{n}} R_*}{\sqrt{K}}\right) (H_{\mathrm{n}} R_*) K^2 \tilde{\Omega}_{\mathrm{gw}}$$

Standard cosmology:

$$F_{\mathrm{gw},0} = 3.6 \times 10^{-5} \left(\frac{100}{g_{\mathrm{eff}}}\right)^{\frac{1}{3}}$$

Numerical simulations:

$$\tilde{\Omega}_{\mathrm{gw}} = \mathrm{O}(10^{-2})$$

- Naïve extrapolation: an upper bound on GWs from PTs: $\Omega_{\mathrm{gw},0} \lesssim 10^{-7}$

Connection to fundamental theory

- Scalar effective potential $V(\phi,T)$ → T_n, α, β, g_{eff} (equilibrium)
- Scalar-fluid coupling $\eta(\phi,T,v_w)$ → v_w (non-equilibrium)

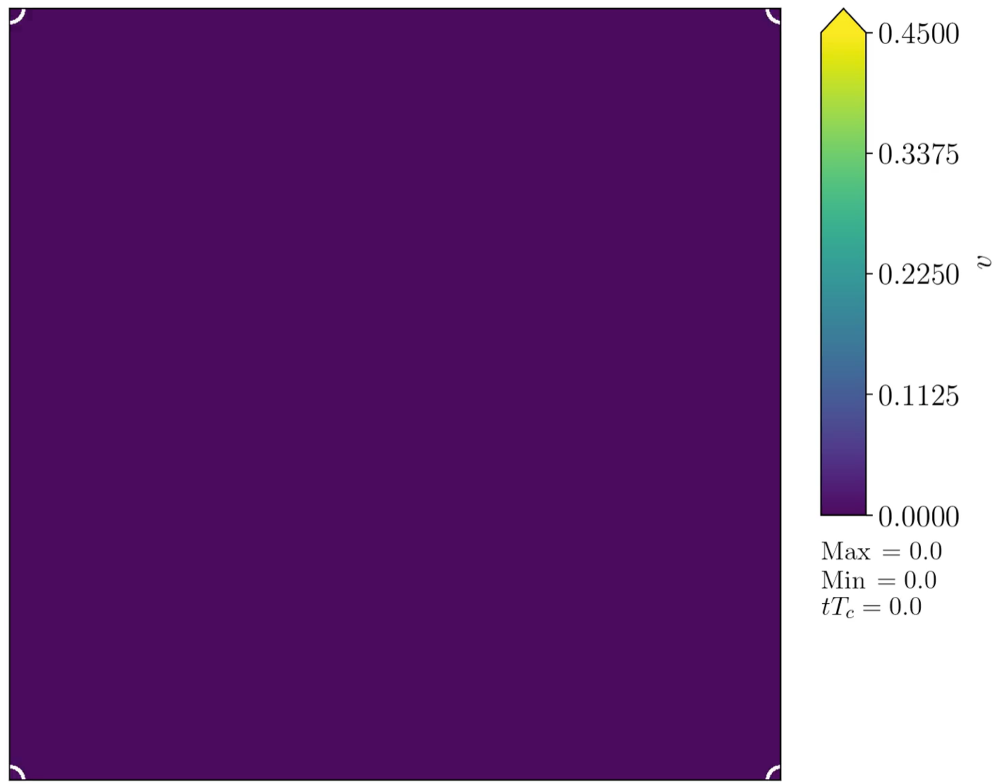
Phase transition parameters :

T_n = nucleation temperature
g_{eff} = effective d.o.f.
α = (scalar V)/(thermal energy)
v_w = bubble wall speed
β = transition rate

T_n, g_{eff} → H_n (Hubble rate at transition)
α, v_w → $K(v_w, \alpha)$ (kinetic energy fraction)
v_w, β → R_* (mean bubble separation)

Future challenges

- Stronger transitions lead to non-linear evolution, dynamics not understood
  - Longitudinal/compression modes
    - Kinetic energy suppression
    - Shocks, wave turbulence
  - Transverse/rotational modes
    - Vorticity generation
    - Turbulence
  - MHD turbulence
    - Turbulence less efficient at producing GWs?



Cutting, MH, Weir 2019

Roper pol et al 2019

Gravitational wave frequencies

- Shear stress at time t generate waves with minimum frequency $f \approx 1/t$ (Hubble rate)
- Redshifted to a frequency now: $f_0 = (a(t)/a(t_0))f$
- Redshifted Hubble rates:

Event	**Time/s**	**Temp/GeV**	**f_0/Hz**
QCD transition	10^{-3}	0.1	10^{-8}
EW transition	10^{-11}	100	10^{-5}
?	10^{-25}	10^{9}	100
End of inflation	$\geq 10^{-36}$	$\leq 10^{16}$	$\leq 10^{8}$

- Peak frequencies: $f_{p.0} \simeq 26(H_\mathrm{n} R_*)^{-1}(T_\mathrm{n}/100\,\mathrm{GeV})\,\mu\mathrm{Hz}$
- Typical range: $10^1 \lesssim (H_\mathrm{n} R_*)^{-1} \lesssim 10^5$

High frequency GWs from PTs?

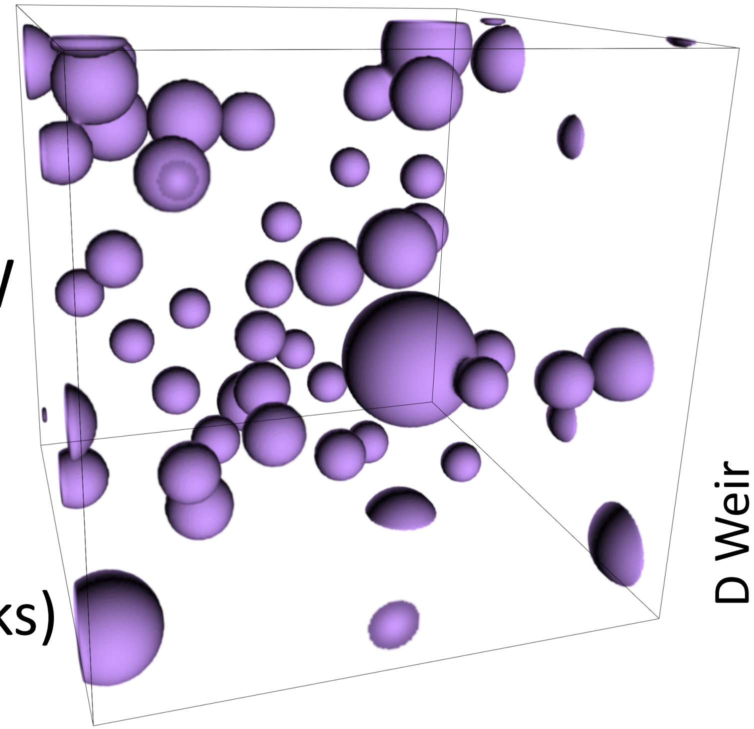
- Peak frequencies: $f_{p.0} \simeq 26(H_{\mathrm{n}}R_*)^{-1}(T_{\mathrm{n}}/100\,\mathrm{GeV})\,\mu\mathrm{Hz}$
- Typical range: $10^1 \lesssim (H_{\mathrm{n}}R_*)^{-1} \lesssim 10^5$
- Highest possible phase transition temperature?
  - Inflation (absence of GWs): $T_{\mathrm{n}} \lesssim 10^{15}\,\mathrm{GeV}$
- Corresponding frequencies: $10^8\,\mathrm{Hz} \lesssim f_{p,0} \lesssim 10^{12}\,\mathrm{Hz}$
- Higher frequency means smaller strain

$$h_c \simeq 0.4\sqrt{\Omega_{\mathrm{gw}}} \times 10^{-20}\,(f/100\,\mathrm{Hz})^{-1}$$

- No (known) astrophysical foregrounds

Conclusions

- GWs probe physics at very high energy
  - LISA will probe physics of Higgs transition from 2034
  - Measure/constrain phase transition parameters
- Towards accurate calculations of GW power spectrum from parameters
  - Some understanding of acoustic production, probably dominant
  - Non-linear evolution (turbulence, shocks) not well understood
- Applies to 1^{st} order PTs at all scales


D Weir

$$f_{p.0} \simeq 26(H_{\rm n}R_*)^{-1}(T_{\rm n}/100\,{\rm GeV})\,\mu{\rm Hz} \qquad h_c \lesssim 3\times 10^{-19}\,(10^2\,{\rm GeV}/T_{\rm n})$$