بسم الله الرحمن الرحیم

Institute for Research in Fundamental Sciences

Department of Physics

A pipeline for searching Stochastic Gravitational Wave by Pulsar Timing Residuals

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The main goal of my talk:

1) How can detect GWB?
2) How can determine the type of GWs?
3) What is the robust sequences of computational tools to detect and to determine the type of GWB?

For more details see:

1) I. Eghdami and SMSM, APJ, 864:162 (18 pp), 2018
2) M. Mozafarilagha and SMSM, Scientific Reports, 9 (1), 2019
3) A Vafaei Sadr, M Farhang, SMSM, B Bassett, M Kunz, MNRAS, 478.1 (2018): 1132-1140.
4) A Vafaei Sadr, SMSM, M Farhang, C Ringeval, F R Bouchet, MNRAS, 475.1 (2017): 1010-1022.
5) Ferreira, Paulo, Andreia Dionísio, and SMSM, Physica A: Statistical Mechanics and its Applications 486: 730-750, 2017.

Outline

1) Self-similar and self-affine Processes

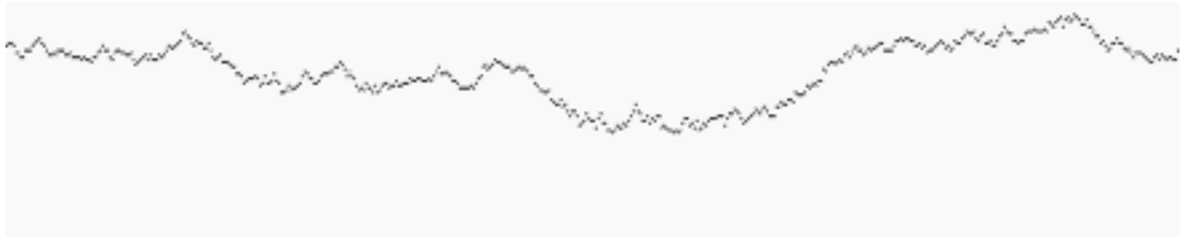
2) Pulsars Timing Residuals and GWB

3) Our Pipeline for GWB detection

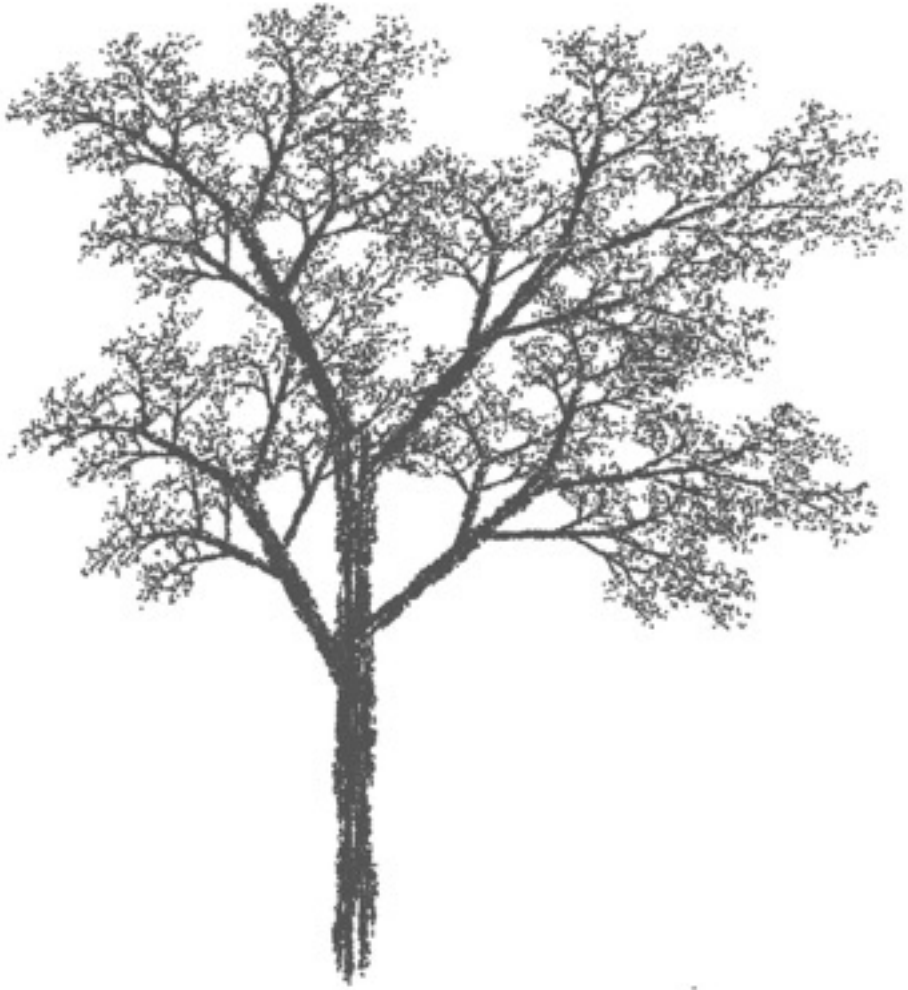
4) Research in progress

Part 1
Self-similar Processes

Self-similar process



Scale dependency (Multifractal) ???
Self-similarity and Self-affinity
Complexity



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Self-similar
Regular time series

- Suppose a time series as:

$$y:\{y(i)\} \quad i=1,\ldots,N$$

$$i \rightarrow a \times i$$

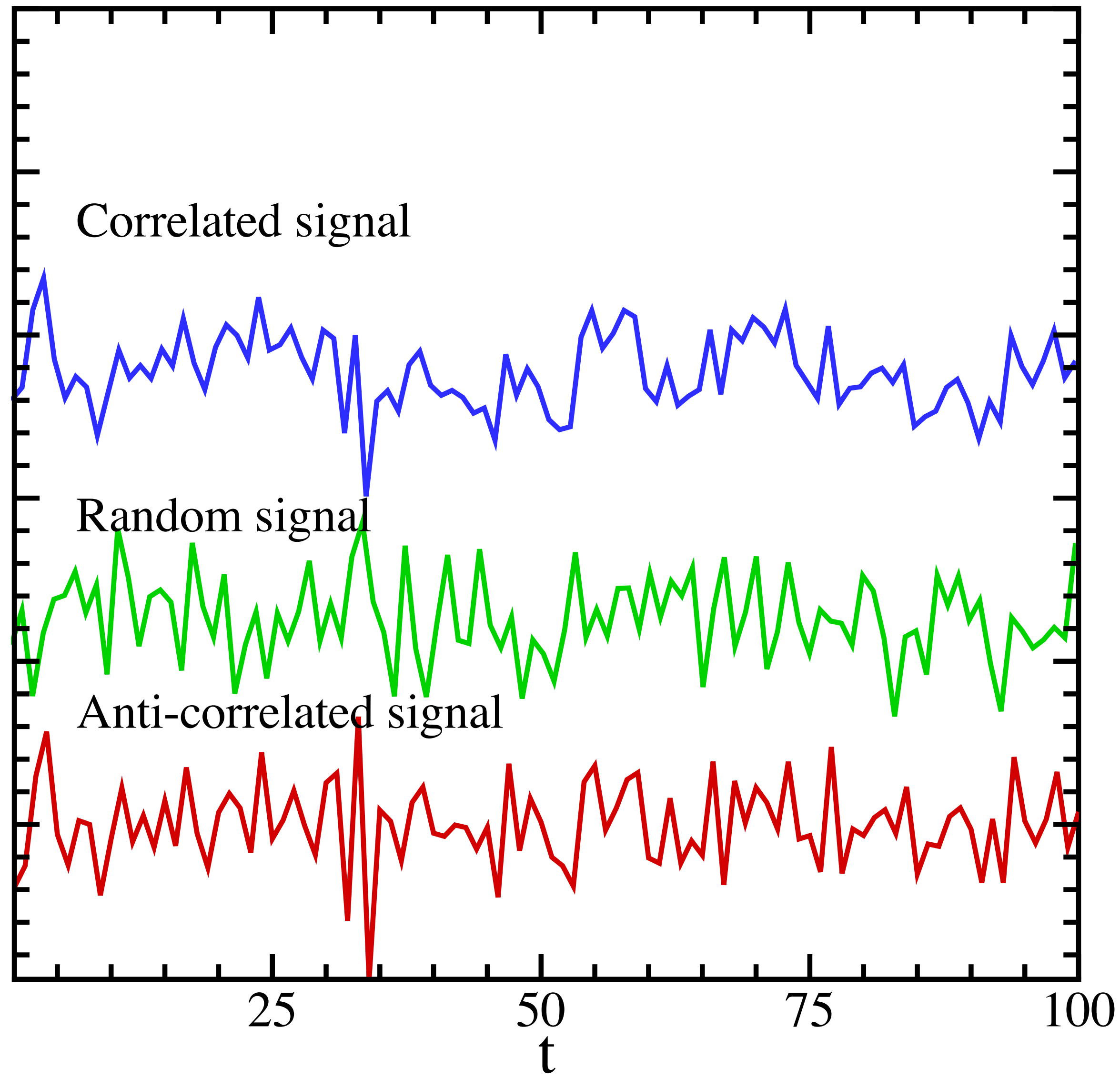
$$y(a \times i) = a^{H} y(i)$$

So-called Hurst exponent

Classification of time series based on Hurst exponent

0<H<1

- Correlated: H>0.5
- Uncorrelated: H=0.5
- Anti-correlated : H<0.5



Scaling exponents

- Multifractal scaling exponent $\tau(q) = qh(q) - 1$
- Generalized multifractal dimension $D(q) = \dfrac{\tau(q)}{q-1}$
- Autocorrelation exponent $\begin{cases} C(s) \sim s^{-\gamma} \\ C(i,j) \sim i^{-\gamma} + j^{-\gamma} - |i-j|^{-\gamma} \end{cases}$
- Power spectrum scaling exponent $S(\omega) \sim \omega^{-\beta}$
- Holder exponent $\alpha = \tau'(q)$, $\alpha = h(q) + qh'(q)$
- Singularity spectrum $f(\alpha) = q\left[\alpha - h(q)\right] + 1$

Exponent	1D-fGn	1D-fBm	2D-Cascade	2D-fBm
γ	$2-2H$	$-2H$	$1-2H$	$-1-2H$
β	$2H-1$	$2H+1$	$2H$	$2H+2$

Hosseinabadi, S., et al., Physical Review E 85.3 (2012): 031113.

Novelties

1) (Irregular)-MF-DFA, MF-DMA and MF-DXA combined by either SVD or AD methods to characterize (1+1)D PRTs.
2) We have developed a systematic approach for Noise modeling
3) Quadrupolar signature of spatial cross-correlation
4) Searching the footprint of GWs not only for amplitude but also for determining the type of GWs

MF-DXA algorithm for joint analysis

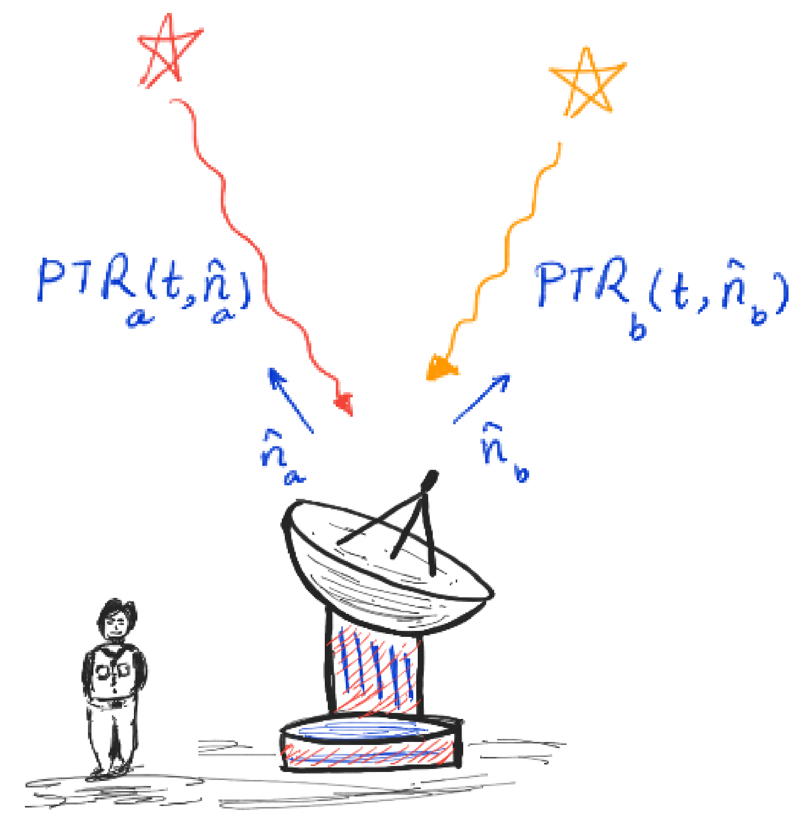
(1) We consider two typical PTR series, named PTR_a and PTR_b, located at $\hat{n}_a$ and $\hat{n}_b$ with respect to the line of sight, respectively, as the input data sets to study their mutual multifractal property:

$$\mathrm{PTR}_a(i), \quad \mathrm{PTR}_b(i), \qquad i = 1, \ldots, N. \tag{1}$$

(2) To magnify the hidden self-similarity property, we make profile series according to

$$X_\Diamond(j) = \sum_{i=1}^{j} [\mathrm{PTR}_\Diamond(i) - \langle \mathrm{PTR}_\Diamond \rangle], \quad j = 1, \ldots, N. \tag{2}$$

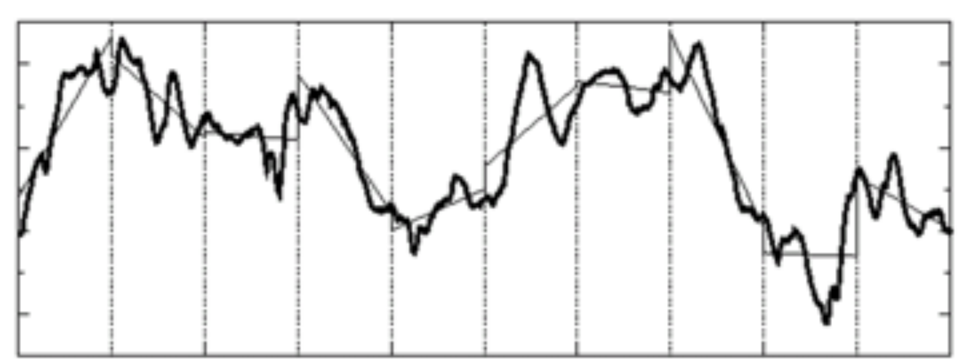
Here the subscript $\Diamond$ can be replaced by "a" or "b."



$$\mathcal{E}_\times(s, \nu) = \frac{1}{s} \sum_{i=1}^{s} [X_a(i + (\nu - 1)s) - \tilde{X}_a^{(\nu)}(i)] \times [X_b(i + (\nu - 1)s) - \tilde{X}_b^{(\nu)}(i)], \tag{3}$$

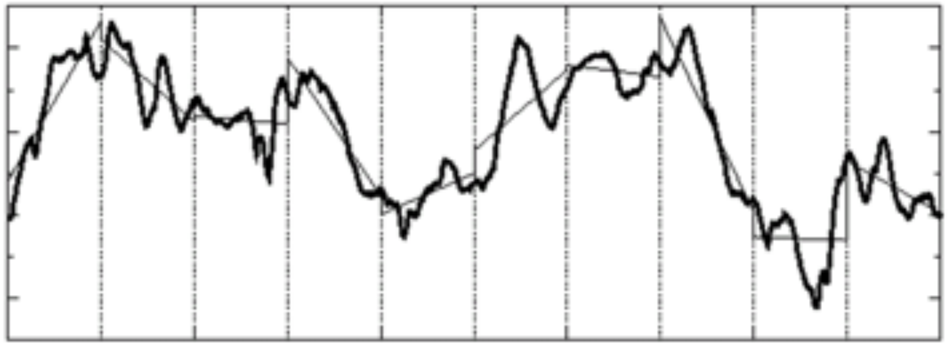
for segments $\nu = 1, \ldots, N_s$. For the opposite end, we have

$$\mathcal{E}_\times(s, \nu) = \frac{1}{s} \sum_{i=1}^{s} [X_a(i + N - (\nu - N_s)s) - \tilde{X}_a^{(\nu)}(i)] \times [X_b(i + N - (\nu - N_s)s) - \tilde{X}_b^{(\nu)}(i)], \tag{4}$$



Algorithm for MF-DXA

(3b) For each moving window with size s, we calculate the moving average function,



$$\widetilde{X_{\Diamond}(j)} = \frac{1}{s} \sum_{k=-s_1}^{s_2} X_{\Diamond}(j-k), \tag{5}$$

where $s_1 = \lfloor (s-1)\theta \rfloor$ and $s_2 = \lceil (s-1)(1-\theta) \rceil$. The sym-

$$\varepsilon_{X_{\Diamond}}(i) = X_{\Diamond}(i) - \widetilde{X_{\Diamond}(i)},$$

Algorithm for MF-DXA

$$\mathcal{E}_{\times}(s, \nu) = \frac{1}{s}\sum_{i=1}^{s} \varepsilon_{X_a}(i + (\nu - 1)s) \times \varepsilon_{X_b}(i + (\nu - 1)s).$$

$$\mathcal{F}_{\times}(q, s) = \left(\frac{1}{2N_s}\sum_{\nu=1}^{2N_s} |\mathcal{E}_{\times}(s, \nu)|^{q/2}\right)^{1/q}.$$

For $q = 0$, we have

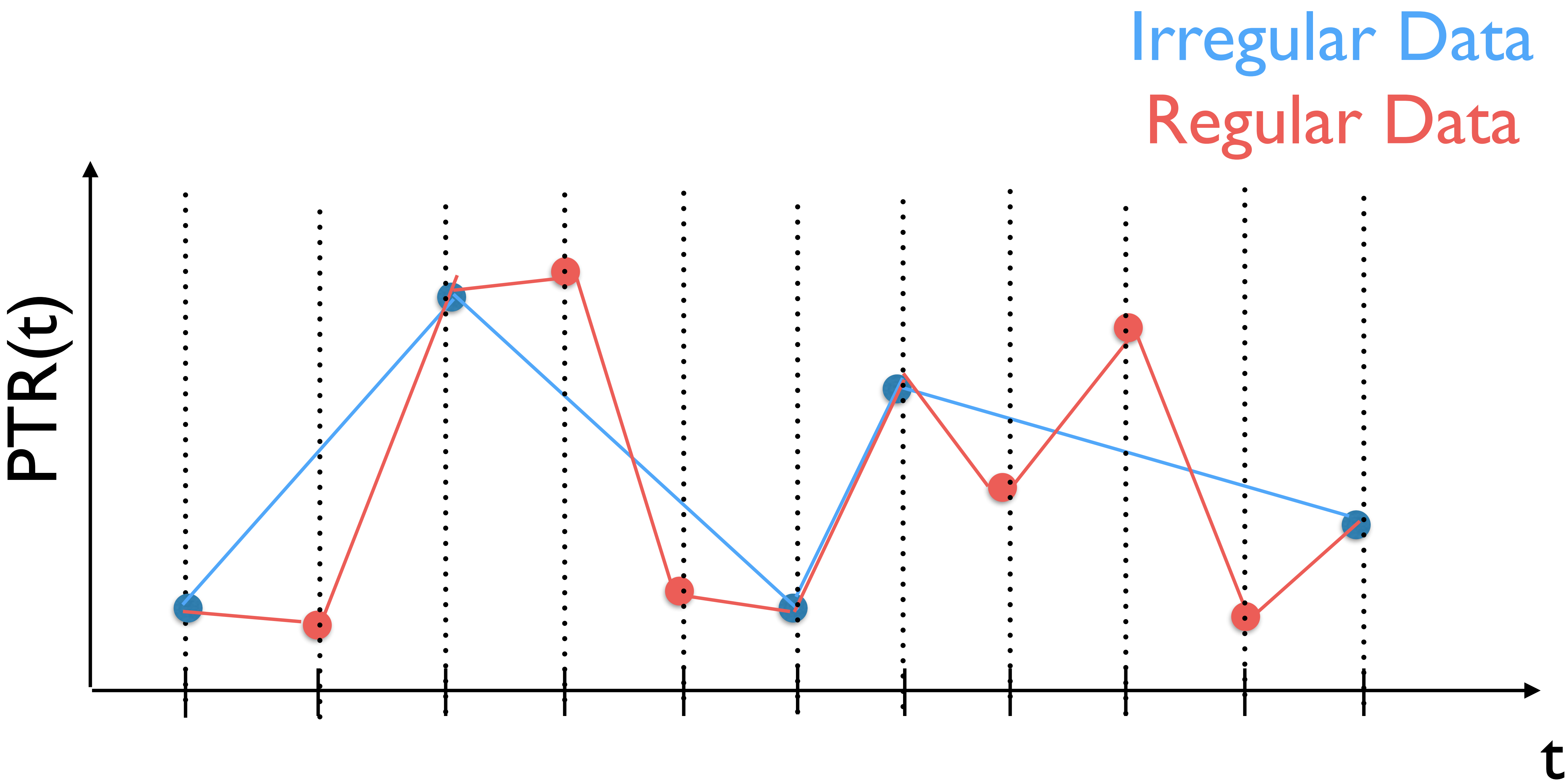
$$\mathcal{F}_{\times}(0, s) = \exp\left(\frac{1}{4N_s}\sum_{\nu=1}^{2N_s} \ln|\mathcal{E}_{\times}(s, \nu)|\right).$$

$$\mathcal{F}_{\times}(q, s) \sim s^{h_{\times}(q)}$$

$$\mathcal{F}_q(s) = \mathcal{G}_{h(q)} s^{h(q)};$$

for $q = 2$, $\mathcal{G}$ is

$$\mathcal{G} = \frac{\sigma^2}{2H + 1} - \frac{4\sigma^2}{2H + 2} + 3\sigma^2\left(\frac{2}{H + 1} - \frac{1}{2H + 1}\right)$$
$$- \frac{3\sigma^2}{H + 1}\left(1 - \frac{1}{(H + 1)(2H + 1)}\right)$$



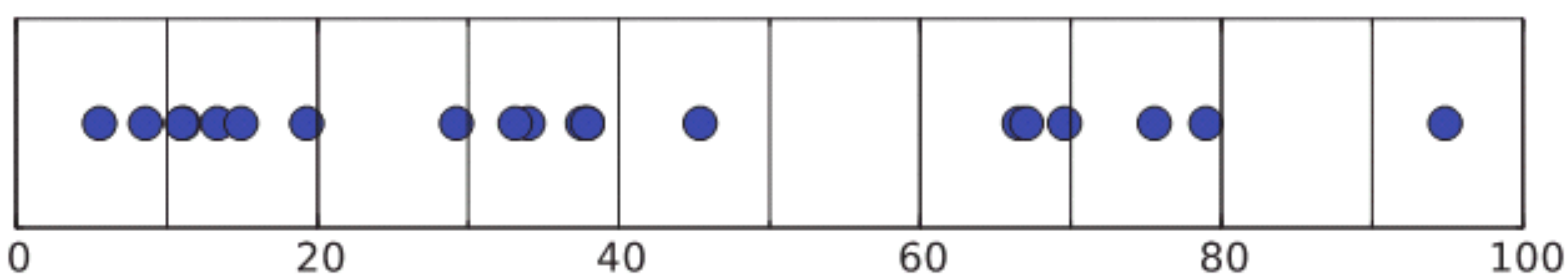
$$\mathrm{PTR}_{\mathrm{reg}}(t) = \int dt' \mathrm{PTR}_{\mathrm{irre}}(t') \mathcal{W}(t - t'),$$

Regularization

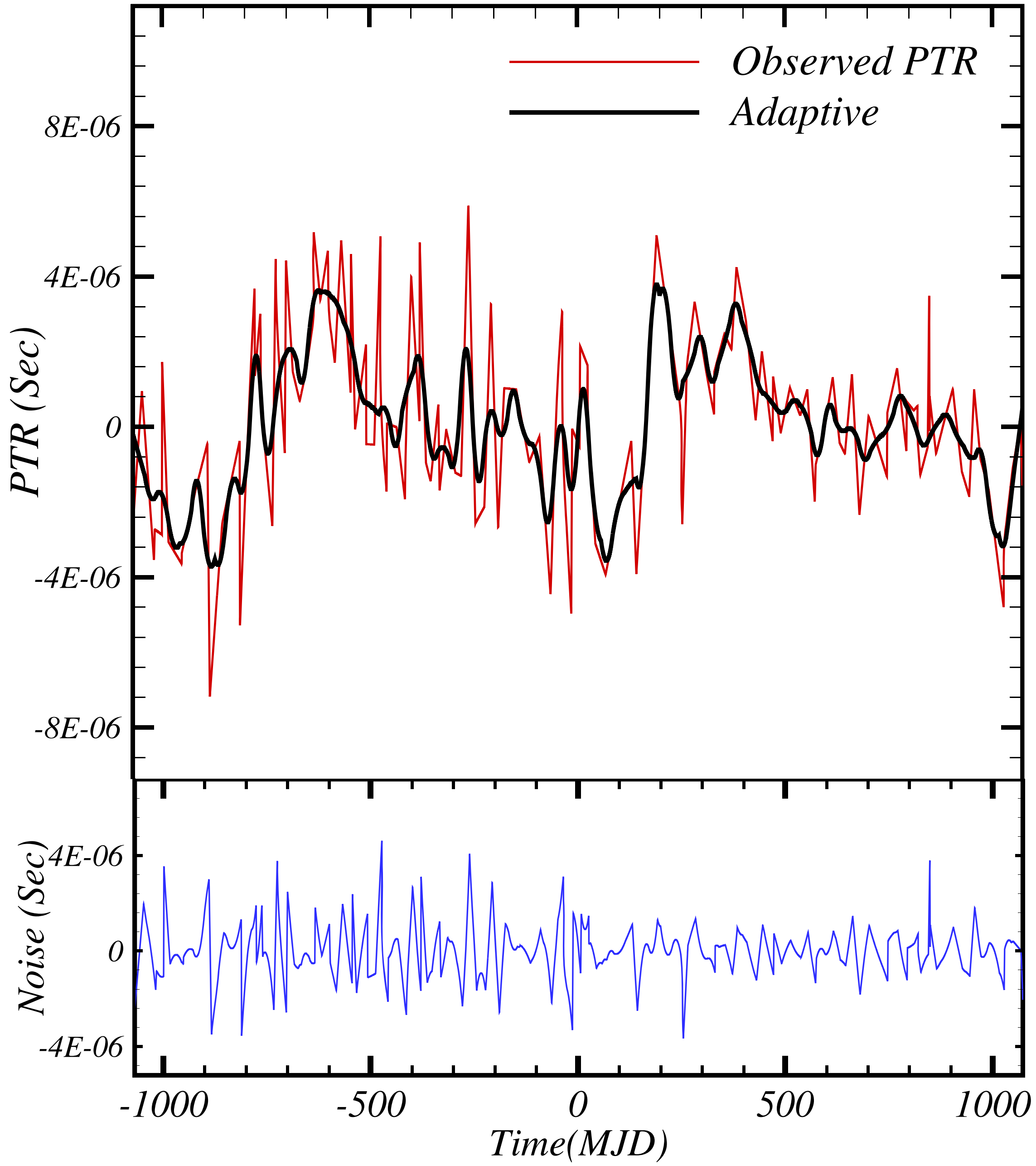
$$\mathrm{PTR}_{\mathrm{reg}}(t) = \int dt' \mathrm{PTR}_{\mathrm{irre}}(t') \mathcal{W}(t - t'),$$

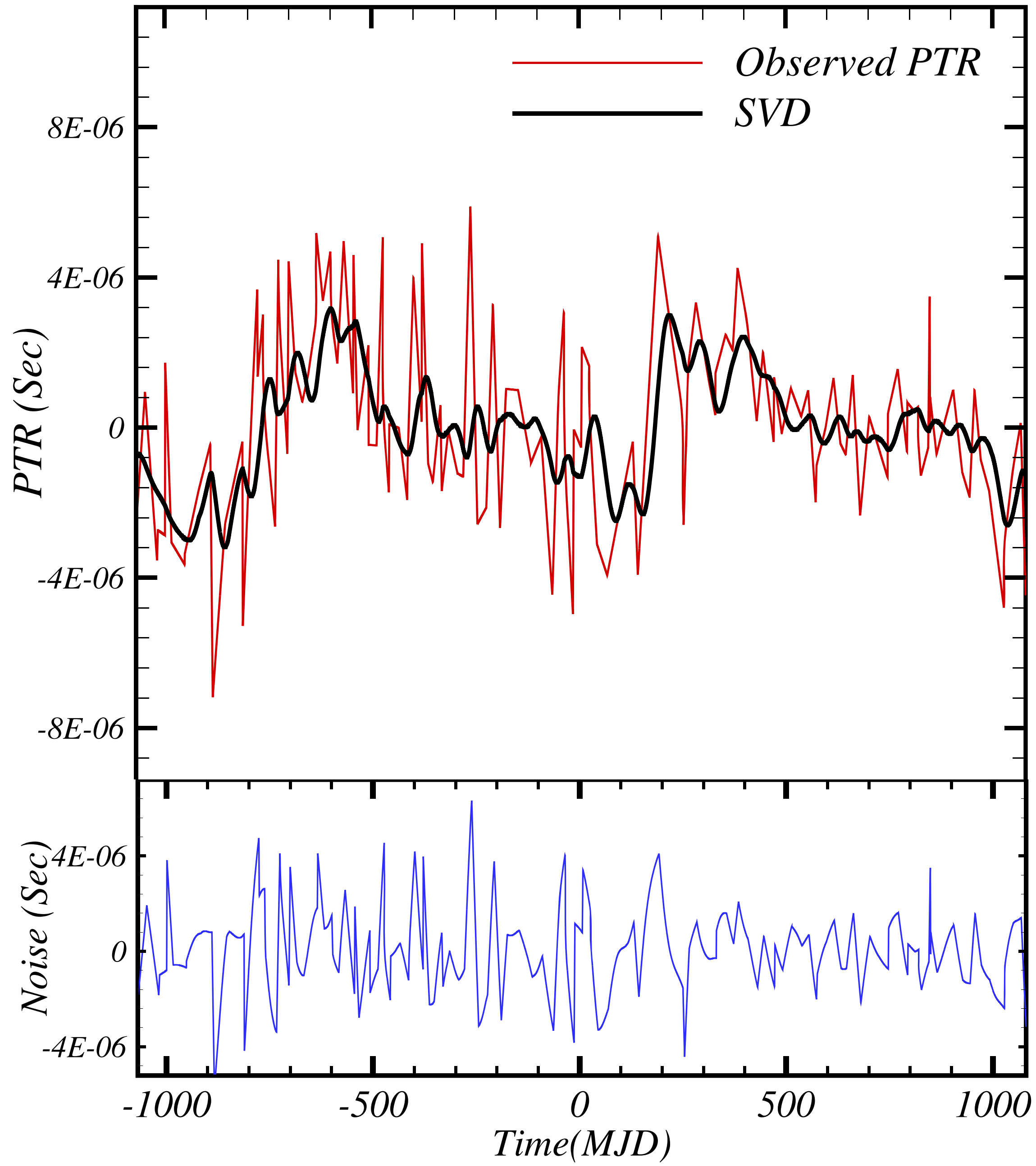
$$\mathcal{E}^2(s, \nu) = \frac{1}{s'_\nu(s)} \sum_{i=1}^{s'_\nu(s)} [X(i + (\nu - 1)s') - \tilde{X}_\nu(i)]^2.$$

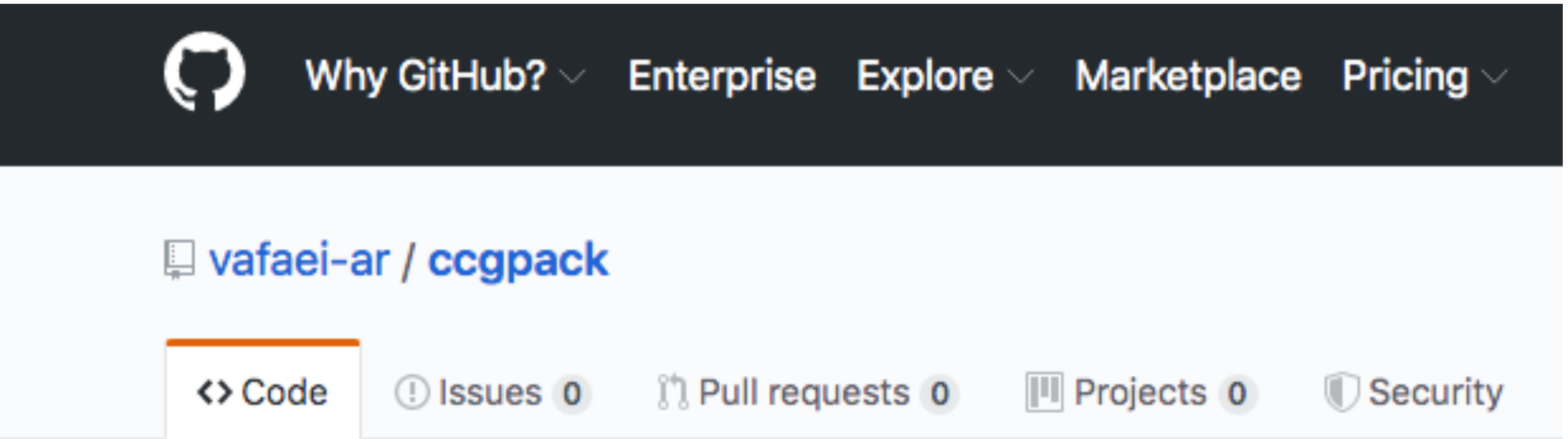
$$\mathcal{F}_\times(q, s) = \left(\frac{1}{2N_s} \sum_{\nu=1}^{2N_s} |\mathcal{E}_\times(s, \nu)|^{q/2} \right)^{1/q} \Rightarrow \mathcal{F}_q(s) = \left(\frac{\sum_{\nu=1}^{2N_s} \frac{[\mathcal{E}^2(s,\nu)]^{q/2}}{\sigma^2_{\mathcal{E}}(s,\nu,q)}}{\sum_{\nu=1}^{2N_s} \frac{1}{\sigma^2_{\mathcal{E}}(s,\nu,q)}} \right)^{1/q}$$



Trend and Noise models







CCGPack

Computational Cosmology Group
Of Shahid Beheshti University

User manual for MF-DFA code written by Sadegh Movahed

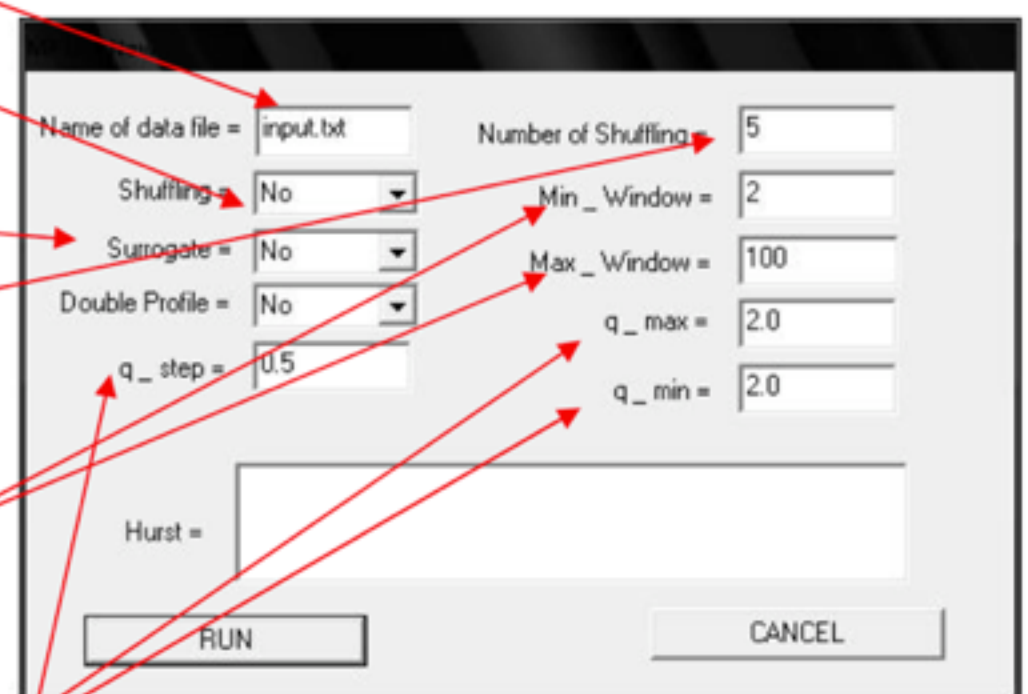
1: You should write the name of your data file in it

2: To shuffled data set you should select YES here.

3: If you want to surrogate your data, select YES for this option

4: This value shows the number of shuffling data set.

5: Here you should determine the maximum and minimum no. of windows, i.e. if you select "10" for maximum and "2" for minimum, your data set is divided to 2 up to 10 non-overlapping windows.



6: If you want to calculate just H=h(q=2) you should determine q=2., namely, q_max=q_min=2. To find the generalized Hurst exponent i.e. h(q) versus q(moment exponent), must q_min and q_max to be different. Just in this case you can find the singularity spectrum for data set.

7: Here the step of moment exponent is determined.

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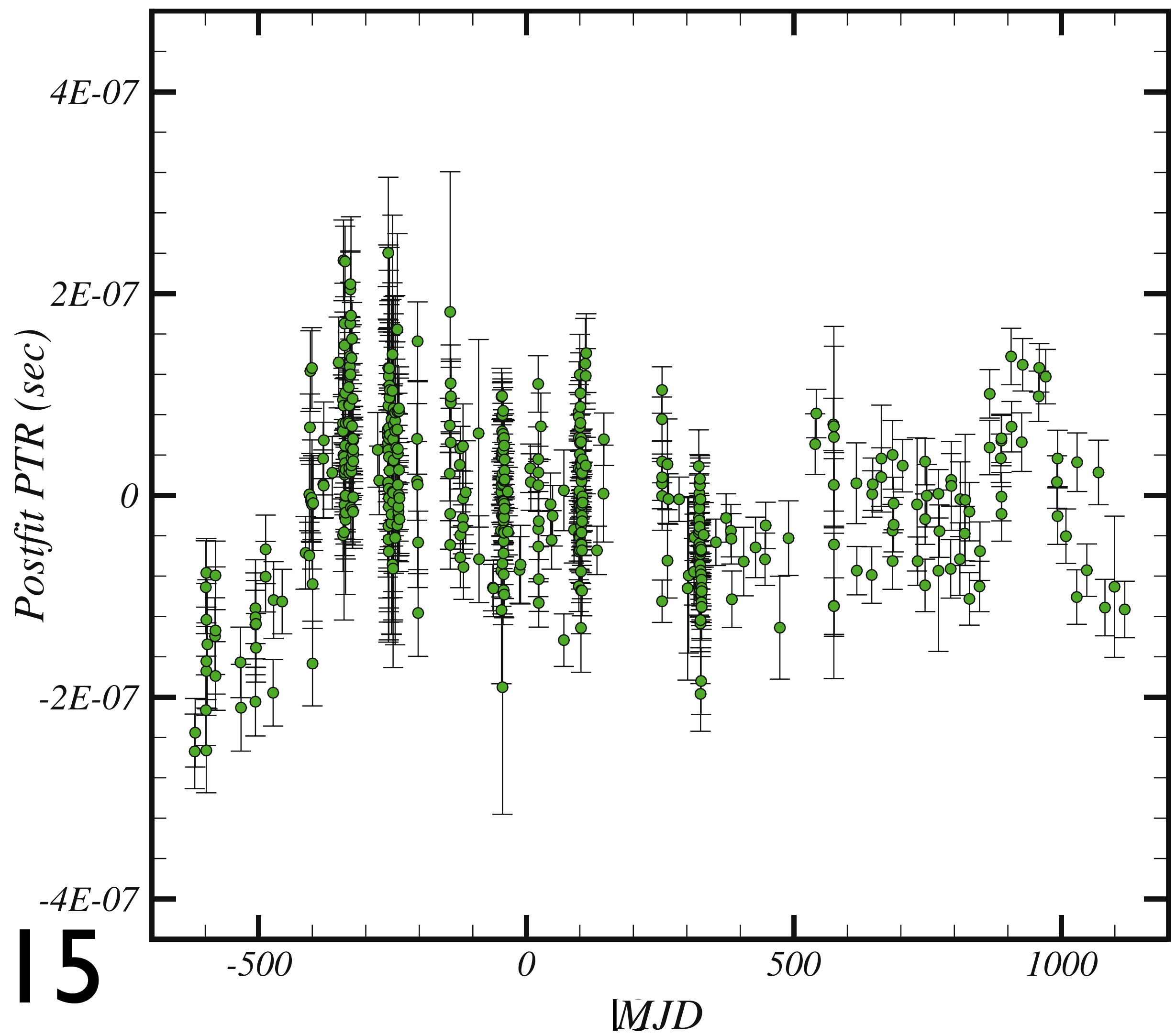
Part 2
PTRs and GWB

Pulsar's Timing residual

Residual is difference between measured pulse's time of arrival and expected time of arrival:

Residual=Observed ToA-Computed ToA or vise versa

$$\Delta t = \Delta_c + \Delta_A + \Delta_{E_\odot} + \Delta_{R_\odot} + \Delta_{S_\odot} - D/f^{٢} + \Delta_{VP} + \Delta_B$$

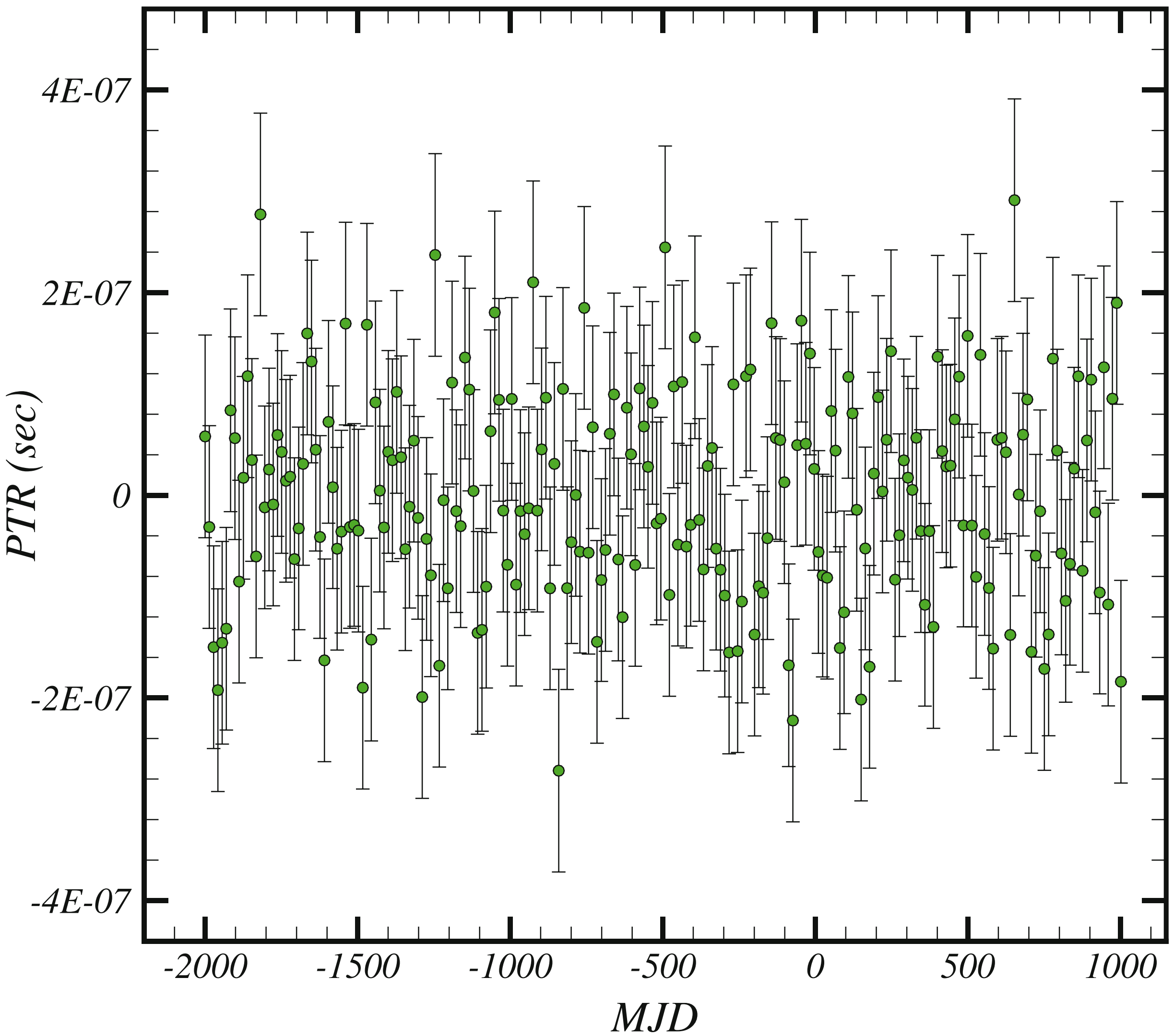


PSR J0437-4715

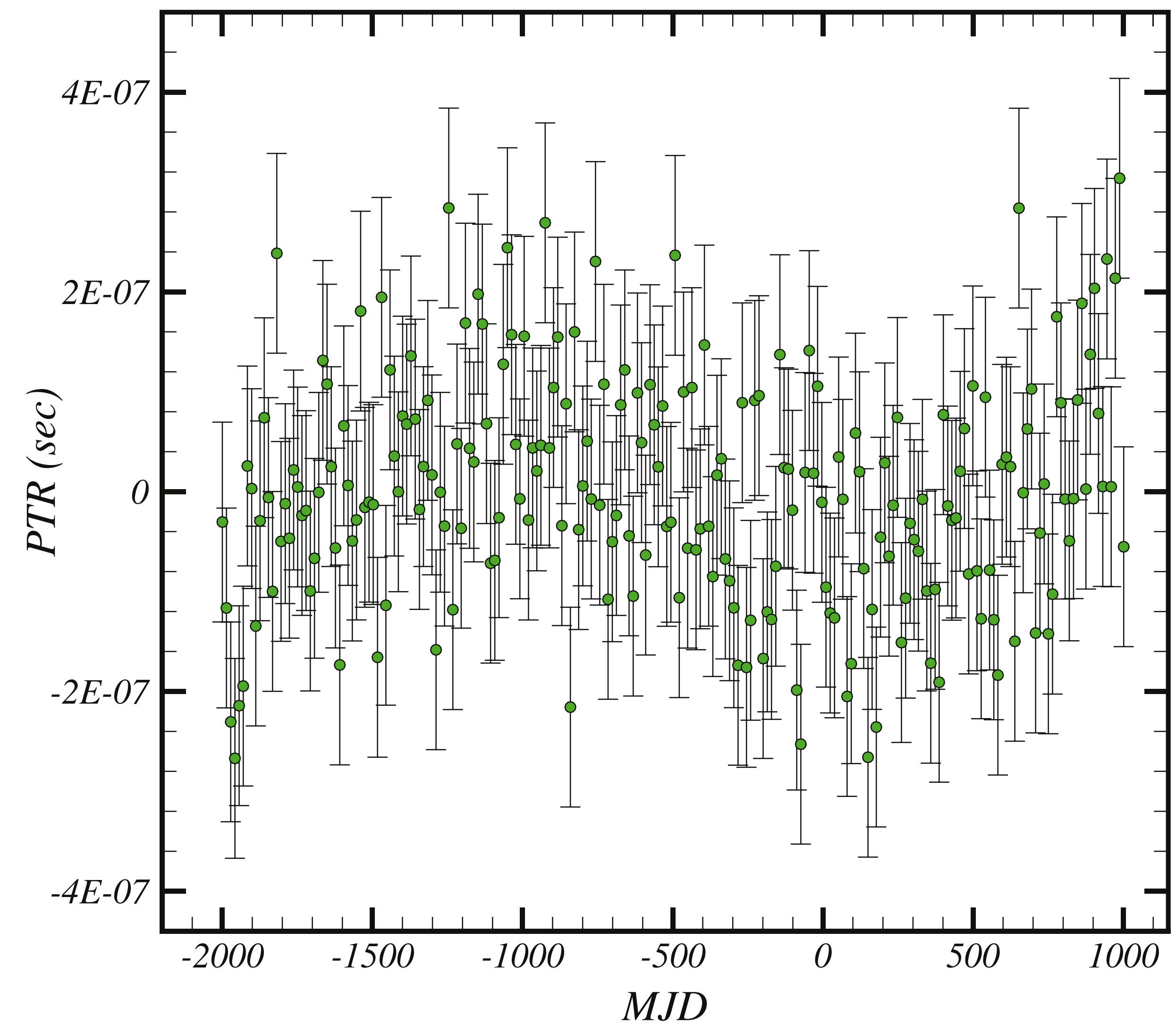
Synthetic Datasets

$$\mathcal{A}_{\mathrm{yr}} = 10^{-15}$$
$$\zeta = -2/3$$

Pure PTR



PTR+GW



TEMPO2 software package

Generalized form of Hellings & Downs (1983)

$$\mathcal{C}_\times(\tau,\Theta_{ab}) = \langle PTR_a(t,\hat{n}_a) PTR_b(t+\tau,\hat{n}_b)\rangle_t \qquad \Theta_{ab} = \arccos|\hat{n}_a.\hat{n}_b|$$

$$PTR_a(t) = PTR_a^{\text{pure}}(t) + \mathcal{B}_a R_{\text{GWB}}(t)$$

$$PTR_b(t) = PTR_b^{\text{pure}}(t) + \mathcal{B}_b R_{\text{GWB}}(t)$$

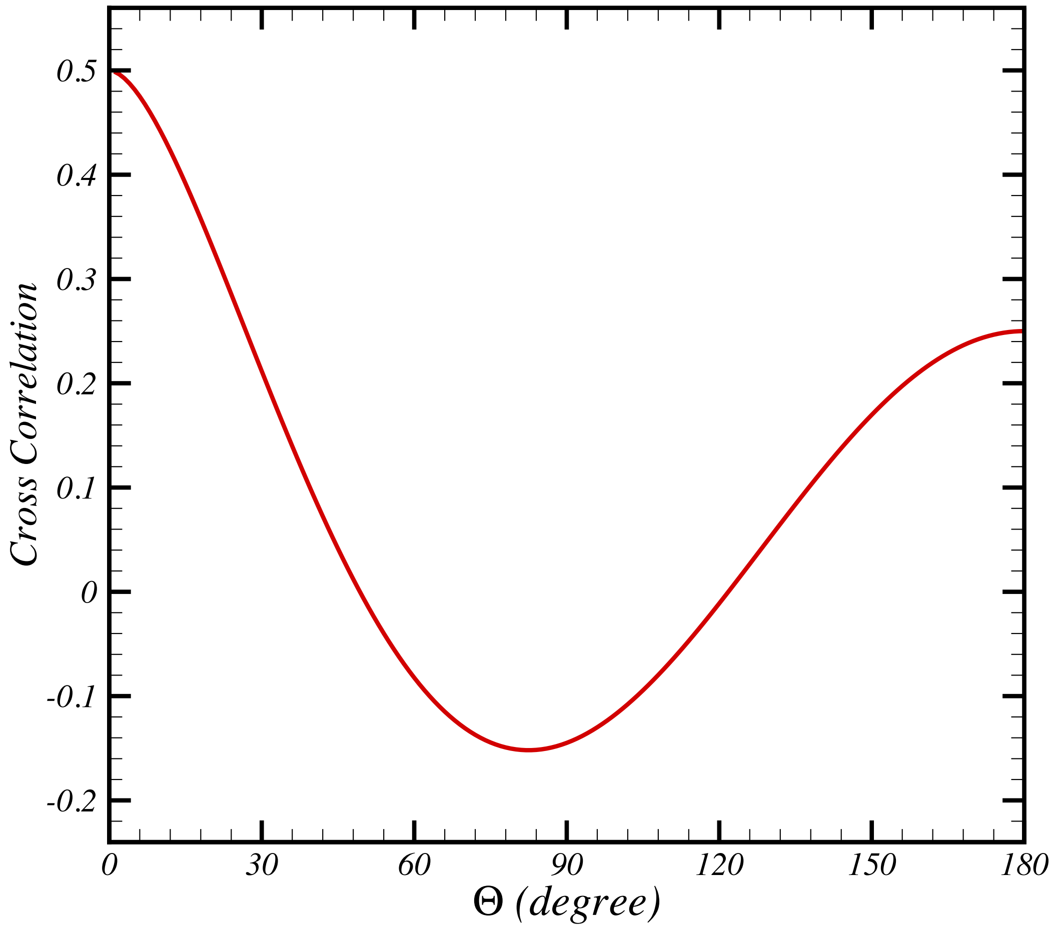
$$\mathcal{B}_i \equiv -\frac{1}{2}\cos(2\phi_i)(1-\cos(\theta_i))$$

$$\overline{\mathcal{C}}_\times(\tau,\Theta) = \langle\, \mathcal{C}_\times(\tau,\Theta_{ab})\rangle_{\text{pairs}} \sim \overline{\Gamma}(\Theta) \times \tau^{\gamma_\times}$$

$$\overline{\Gamma}(\Theta) = \frac{3}{2}\psi\ln(\psi) - \frac{\psi}{4} + \frac{1}{2},$$

$$\psi \equiv [1-\cos(\Theta)]/2$$

$$\gamma_\times = 2 - 2H_\times = 2 - 2h_\times(q=2)$$



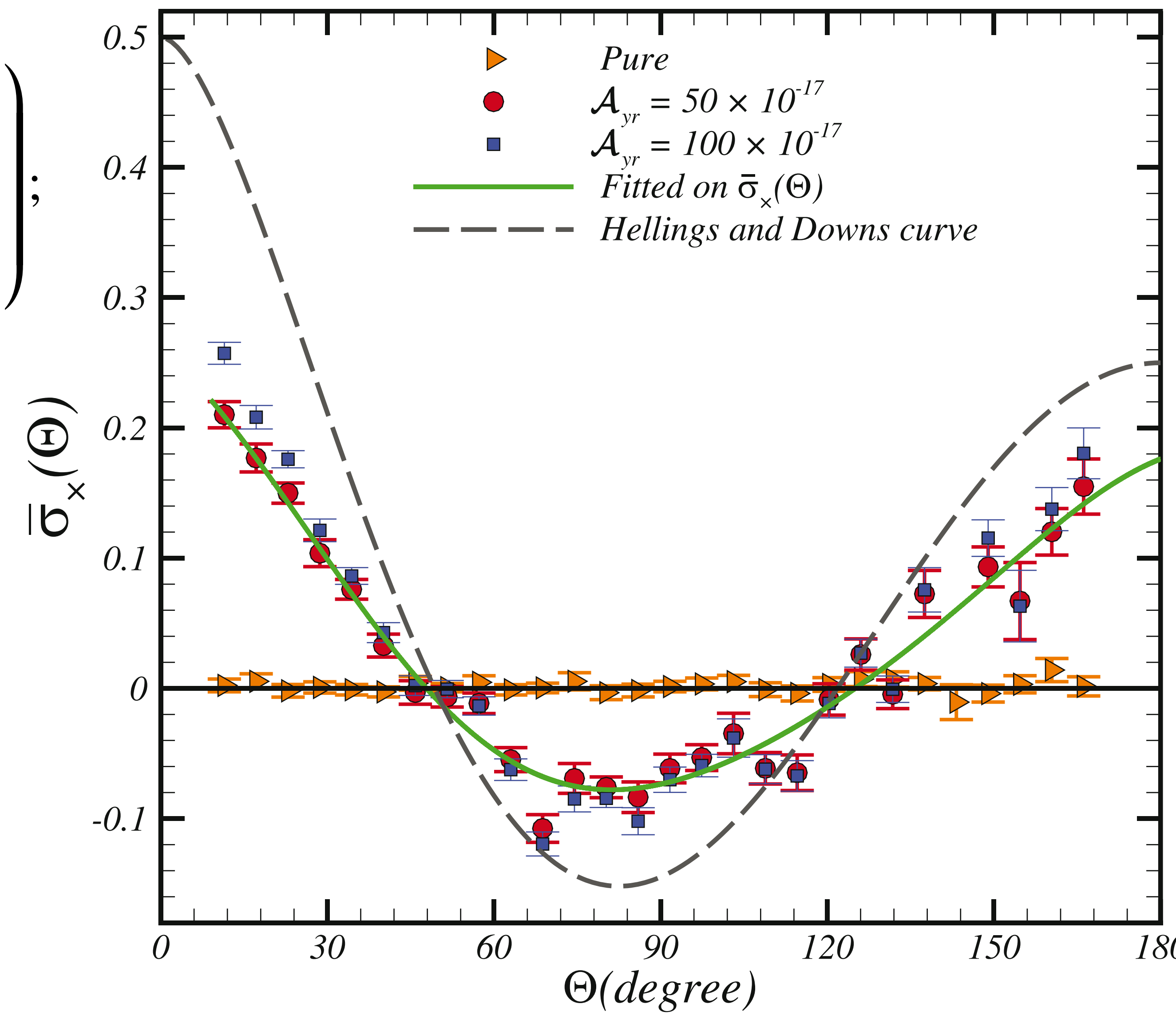
arXiv:1410.8256

New measure

$$\sigma_\times(\Theta_{ab}) \equiv \sum_s \left(\frac{\sum_{\nu=1}^{2N_s} \mathcal{E}_\times(s,\,\nu)}{\sqrt{\left[\sum_{\nu=1}^{2N_s} \mathcal{E}_a(s,\,\nu)\right]\left[\sum_{\nu=1}^{2N_s} \mathcal{E}_b(s,\,\nu)\right]}} \right);$$

$$\Theta_{ab} = \arccos|\hat{n}_a.\hat{n}_b|$$

$$\bar{\sigma}_\times(\Theta) = \langle \sigma_\times(\Theta_{ab}) \rangle_{ab}$$



4 Strategies for GWB detection

1) $$\Delta h_1(\mathcal{A}_{yr},\zeta) \equiv \sum_{q=q_{min}}^{q=q_{max}} |\mathcal{G}_{h(q)}(\mathcal{A}_{yr},\zeta) - \mathcal{G}_{h(q)}(\mathcal{A}_{yr}=0)|$$

$$\left(\frac{\mathcal{A}_{yr}}{10^{-17}}\right) = a\Delta h_1^2 + b\Delta h_1 + c$$

2) $$\Delta h_2(\mathcal{A}_{yr},\zeta) \equiv \sum_{q=q_{min}}^{q_{max}} |h(q;\mathcal{A}_{yr},\zeta) - h_{\rm shuf}(q;\mathcal{A}_{yr},\zeta)|$$

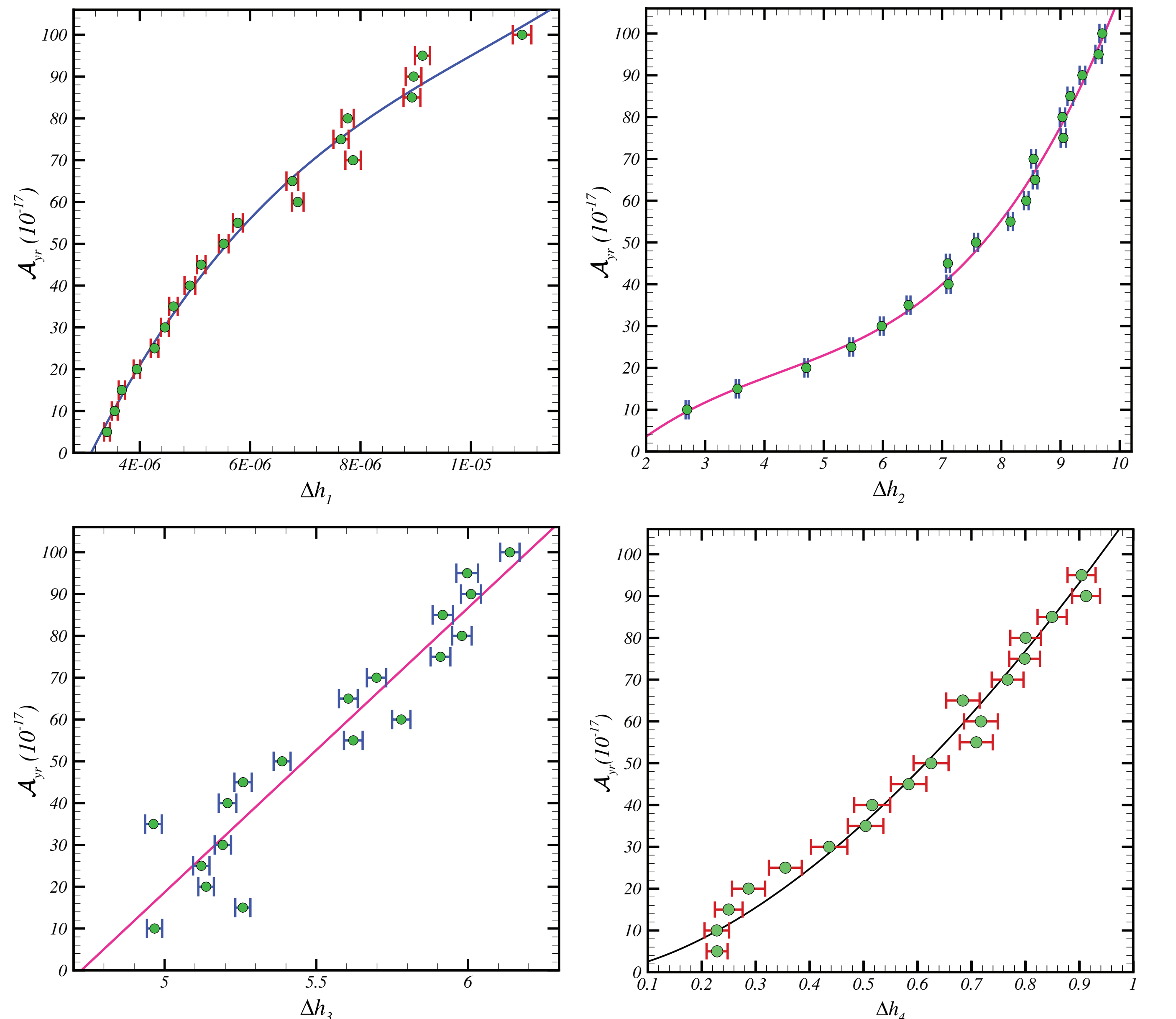
$$\left(\frac{\mathcal{A}_{yr}}{10^{-17}}\right) = a\Delta h_2^3 + b\Delta h_2^2 + c\Delta h_2$$

3) $$\Delta h_3(\mathcal{A}_{yr},\zeta) \equiv \sum_{q=q_{min}}^{q_{max}} |h(q;\mathcal{A}_{yr},\zeta) - h_{\rm sur}(q,\mathcal{A}_{yr},\zeta)$$

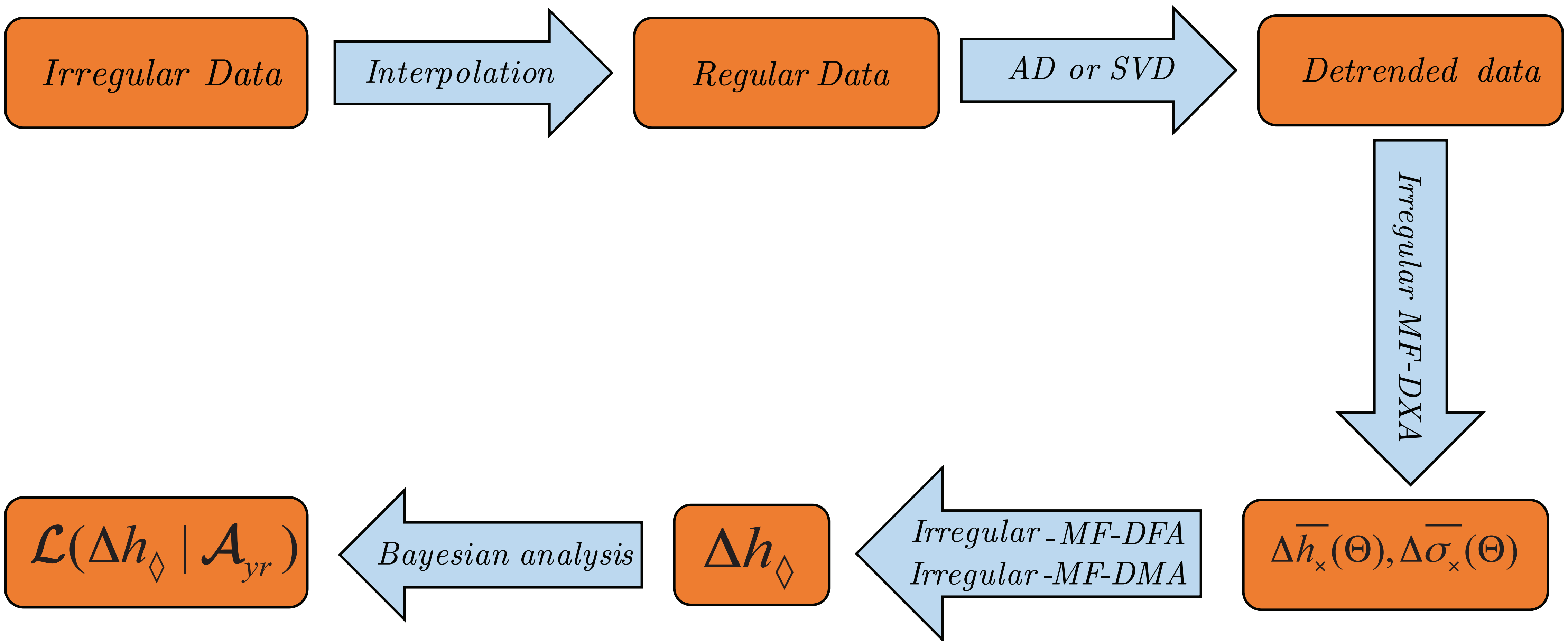
$$\left(\frac{\mathcal{A}_{yr}}{10^{-17}}\right) = a\Delta h_3 + b$$

4) $$\Delta h_4(\mathcal{A}_{yr},\zeta) \equiv |\Delta\alpha(\mathcal{A}_{yr},\zeta) - \Delta\alpha(\mathcal{A}_{yr}=0)|$$

$$\left(\frac{\mathcal{A}_{yr}}{10^{-17}}\right) = a\Delta h_4^b + c$$



Pipeline for GWB detection



Eghdami, I., et al., APJ, 864:162 (18 pp), 2018

Observed PTR, Parkes Pulsar Timing Array (PPTA)

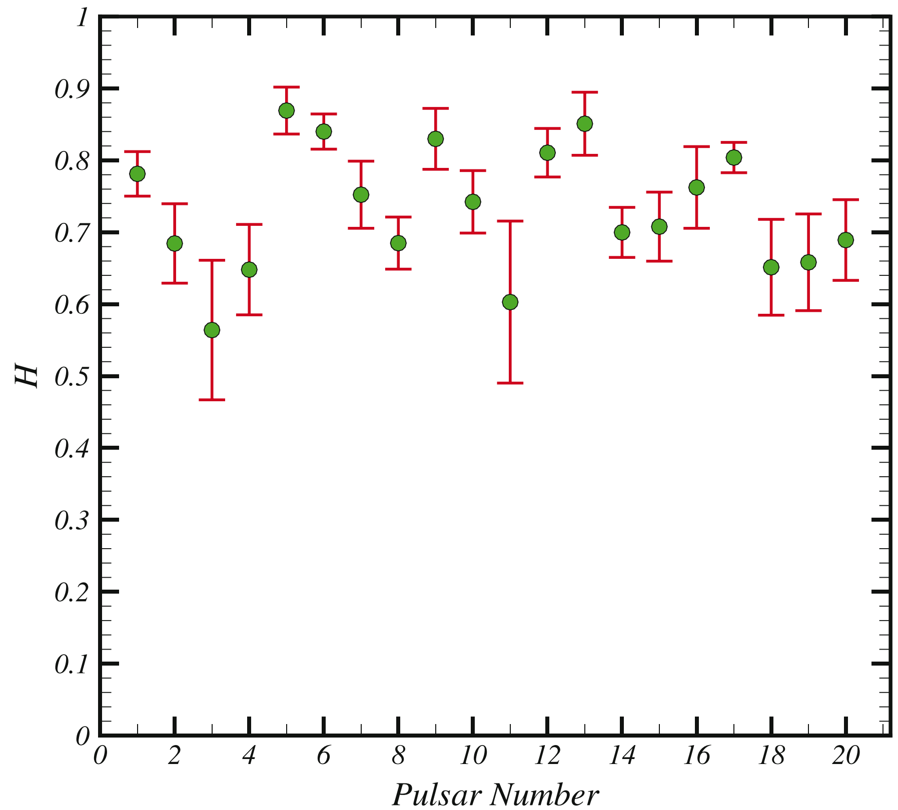
Table 1

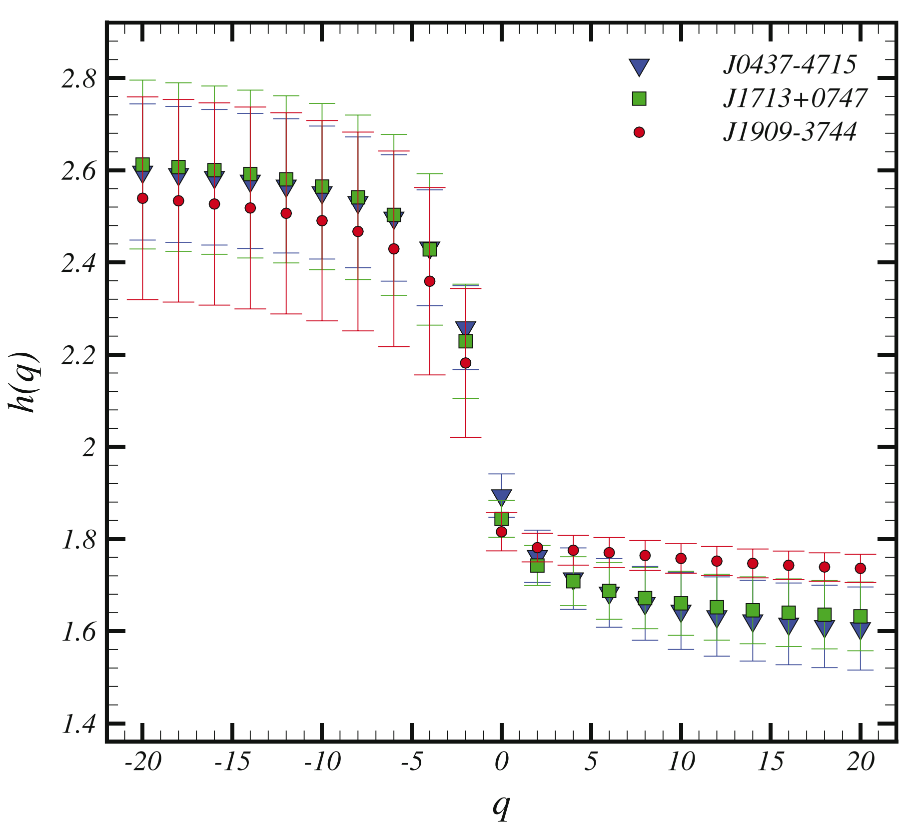
Hurst Exponent, H, Width of Singularity Spectrum, $\Delta\alpha$, Scaling Exponent of Temporal Autocorrelation, γ, rms, Total Time Span (TTS) of Post-fit Timing Residuals, and Upper Limit on Dimensionless Amplitude of GWB of 20 MSPs Observed in PPTA Project

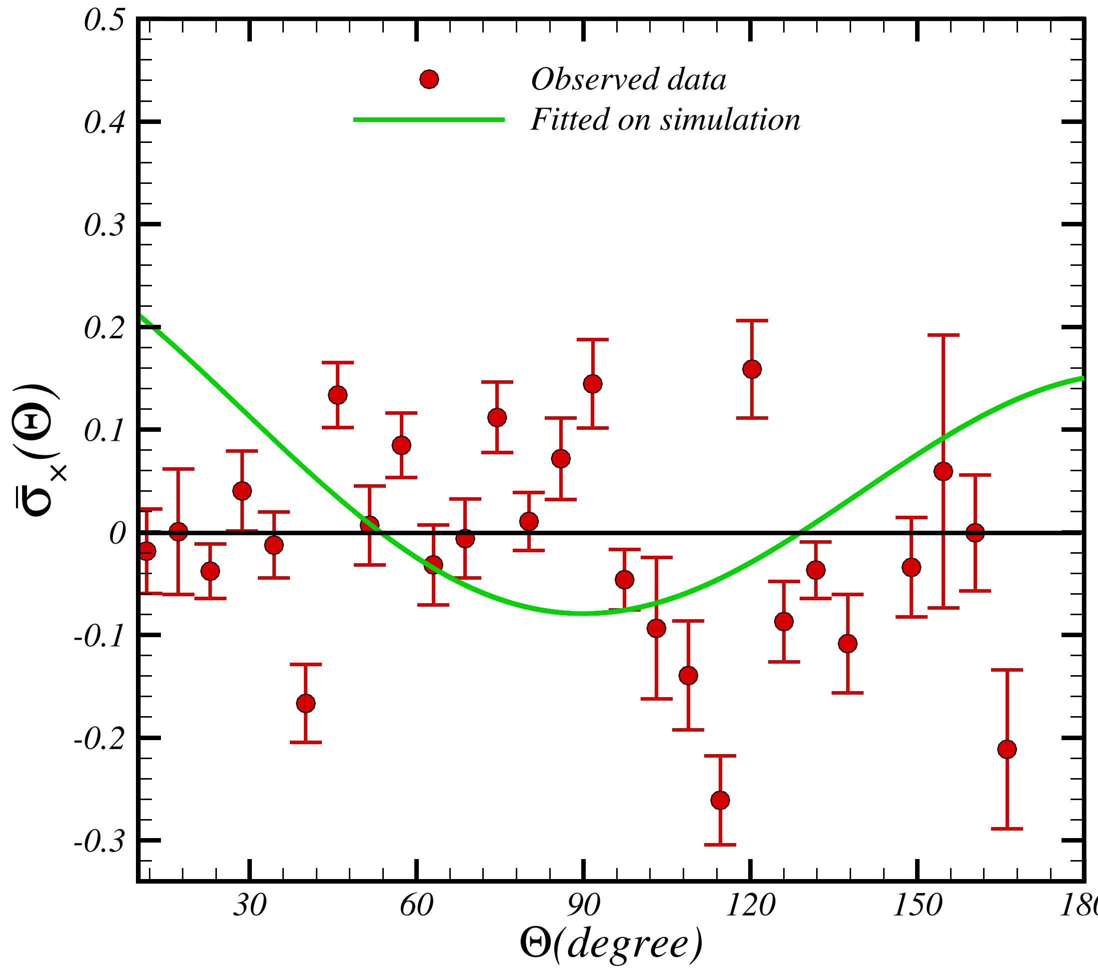
PSR Number	PSR Name	H	$\Delta\alpha$	γ	rms (μs)	TTS (yr)	$\mathcal{A}_{\rm yr}^{\rm up}$(95%)
1	J0437–4715	0.78 ± 0.03	0.89 ± 0.06	−1.56 ± 0.06	0.08	4.76	5.0×10^{-15}
2	J0613–0200	0.68 ± 0.06	1.22 ± 0.04	−1.37 ± 0.11	1.07	5.99	7.0×10^{-15}
3	J0711–6830	0.56 ± 0.10	1.40 ± 0.08	−1.13 ± 0.19	0.89	5.99	6.0×10^{-15}
4	J1022+1001	0.65 ± 0.06	1.04 ± 0.04	−1.30 ± 0.13	1.72	5.88	8.5×10^{-15}
5	J1024–0719	0.87 ± 0.03	1.60 ± 0.03	−1.74 ± 0.07	1.13	5.99	⋯
6	J1045–4509	0.84 ± 0.02	1.29 ± 0.04	−1.68 ± 0.05	2.77	5.94	⋯
7	J1600–3053	0.75 ± 0.05	1.34 ± 0.04	−1.50 ± 0.09	0.68	5.93	⋯
8	J1603–7202	0.68 ± 0.04	1.29 ± 0.05	−1.37 ± 0.07	2.14	5.99	2.5×10^{-15}
9	J1643–1224	0.83 ± 0.04	0.89 ± 0.02	−1.66 ± 0.08	1.64	5.87	⋯
10	J1713+0747	0.74 ± 0.04	1.20 ± 0.05	−1.48 ± 0.09	0.31	5.71	2.0×10^{-15}
11	J1730–2304	0.60 ± 0.11	1.79 ± 0.04	−1.21 ± 0.23	1.47	5.93	⋯
12	J1732–5049	0.81 ± 0.03	1.56 ± 0.03	−1.62 ± 0.07	2.22	5.08	2.0×10^{-15}
13	J1744–1134	0.85 ± 0.04	1.52 ± 0.03	−1.70 ± 0.09	0.32	5.87	⋯
14	J1824–2452A	0.70 ± 0.03	1.26 ± 0.05	−1.40 ± 0.07	2.44	5.75	10.0×10^{-15}
15	J1857+0943	0.71 ± 0.05	1.45 ± 0.02	−1.42 ± 0.10	0.84	5.93	⋯
16	J1909–3744	0.76 ± 0.06	1.32 ± 0.06	−1.52 ± 0.11	0.13	5.75	6.0×10^{-15}
17	J1939+2134	0.80 ± 0.02	1.25 ± 0.02	−1.61 ± 0.04	0.68	5.88	⋯
18	J2124–3358	0.65 ± 0.07	1.23 ± 0.04	−1.30 ± 0.13	1.90	5.99	6.0×10^{-15}
19	J2129–5721	0.66 ± 0.07	1.54 ± 0.04	−1.32 ± 0.13	0.80	5.86	7.0×10^{-15}
20	J2145–0750	0.69 ± 0.06	1.29 ± 0.05	−1.38 ± 0.11	0.78	5.99	⋯

Eghdami, I., et al., APJ, 864:162 (18 pp), 2018

Observed PTR, Parkes Pulsar Timing Array (PPTA)



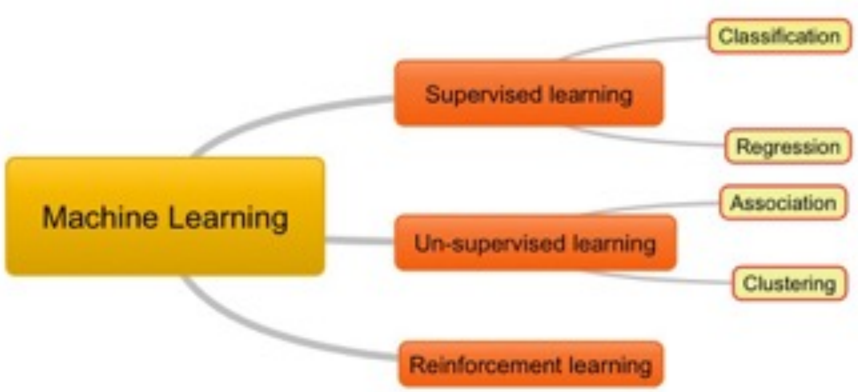


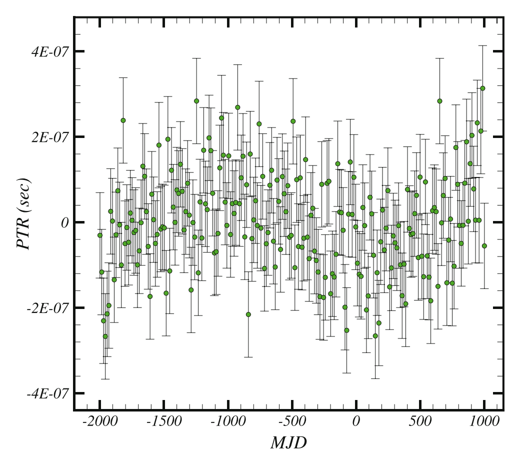


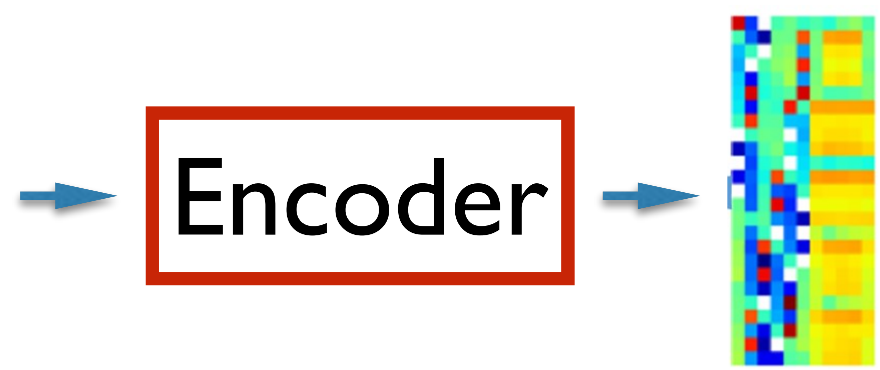
Eghdami, I., et al., APJ, 864:162 (18 pp), 2018

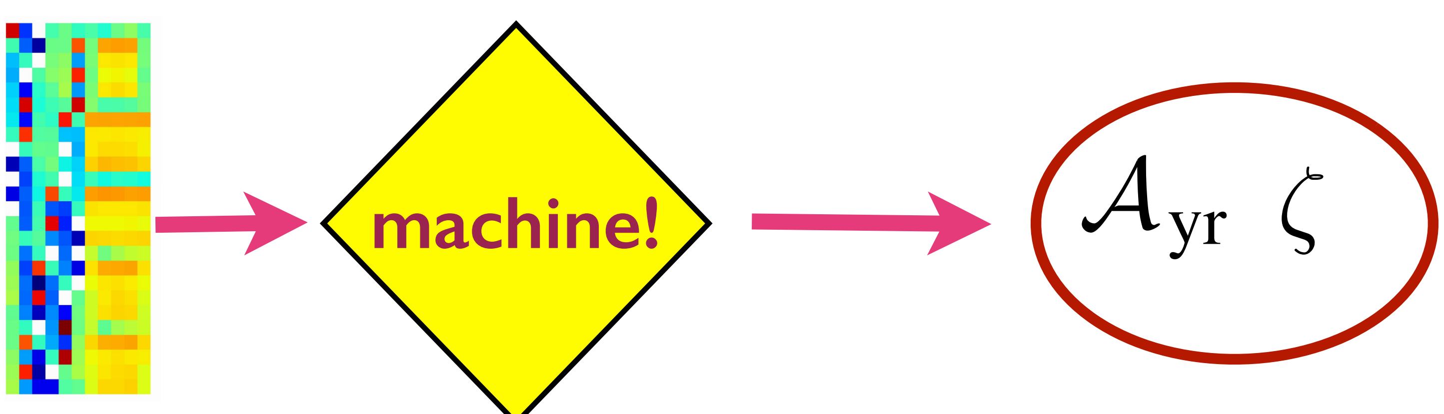
What the next?

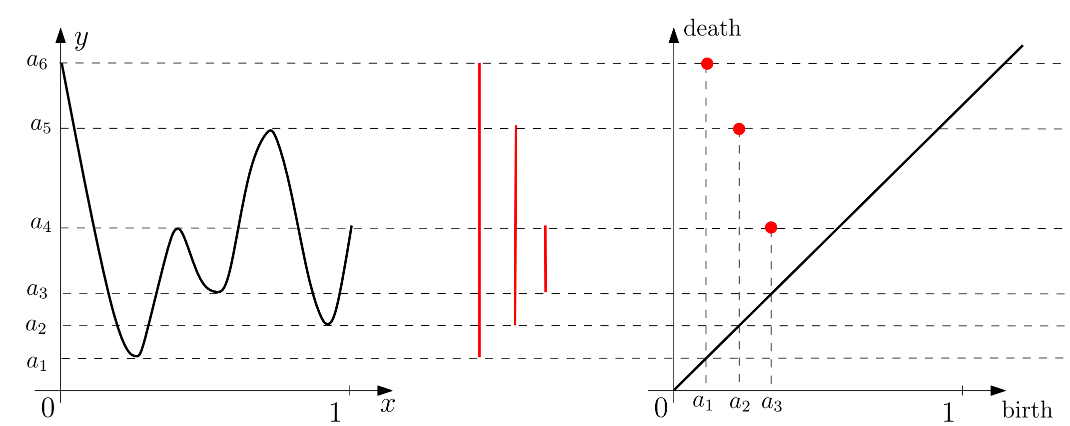
1) Based on Machine Learning, we look for optimum pipeline for GW detection
2) Applying Topological and geometrical measures on PTRs for further analysis
3) TDA based analysis (Persistence Homology)











Frédéric Chazal and Bertrand Michel , arXiv:1710.04019

Take-home message

1- Data and various observations provide opportunity to evaluate our models to explain our cosmos

2- Pipelines may help compared to doing single tools.

3- Machines may help as well

Vafaei Sadr, A., et al., MNRAS, 478.1 (2018): 1132-1140;
Eghdami, I., et al., APJ, 864:162 (18 pp), 2018

Summary

1) Self-similar and self-affine Processes

2) Pulsars Timing Residuals and GWB

3) Our Pipeline for detection GWB

Thank you

از توجه شما سپاسگزارم

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