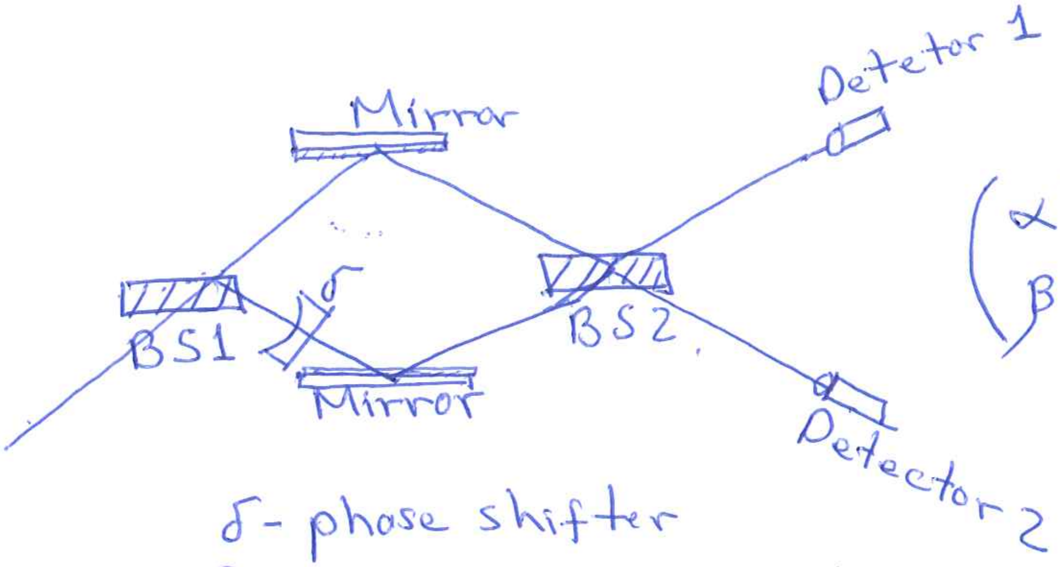
Interferometry.

Mach-Zehnder Interferometers



$\begin{pmatrix}\alpha \\ \beta\end{pmatrix}$ - Column Vector Output

δ - phase shifter
BS_1 - beam splitter 1
BS_2 - beam splitter 2

$\alpha, \beta \in \mathbb{C}$ → two complex numbers giving the probability amplitude for photons to be somewhere

α - probability amplitude to be up
β - " " " " down

$$|\alpha|^2 + |\beta|^2 = 1.$$

$|\alpha|^2$: probability to be up
$|\beta|^2$: probability to be down

States

$\begin{pmatrix}1\\0\end{pmatrix} \Rightarrow$ photon in the upper beam

$\begin{pmatrix}0\\1\end{pmatrix} \to$ " " the lower beam

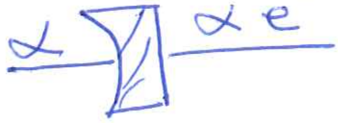
Superposition

$$\begin{pmatrix}\alpha\\\beta\end{pmatrix} = \alpha\begin{pmatrix}1\\0\end{pmatrix} + \beta\begin{pmatrix}0\\1\end{pmatrix}$$

state $\begin{pmatrix}\alpha\\\beta\end{pmatrix}$ is a superposition

Superposition with coefficient α for state in upper beam and coefficient β for state in the lower beam.

The shifter introduces a phase shift δ

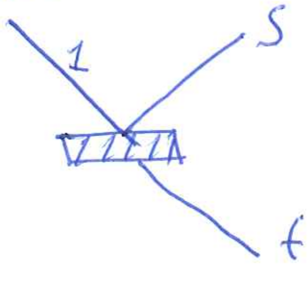


$\delta \in \mathbb{R}$

$|\alpha| = |\alpha e^{i\delta}|$

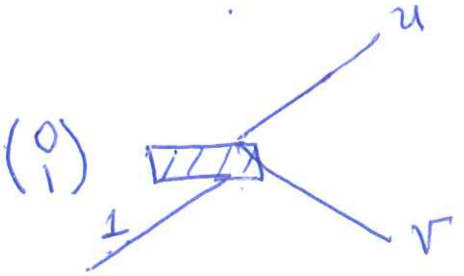
How does a beam splitter works?

$\begin{pmatrix}1\\0\end{pmatrix}$ photon



Beam splitter converts the $\begin{pmatrix}1\\0\end{pmatrix}$ photon in the $\begin{pmatrix}s\\t\end{pmatrix}$ photon

Probability conservation: $|s|^2 + |t|^2 = 1$



$\begin{pmatrix} 0 \\ 1 \end{pmatrix} \to \begin{pmatrix} u \\ v \end{pmatrix}$; $|u|^2 + |v|^2 = 1$

We need four number to characterize the beam splitter - because of linearity
What happens to $\begin{pmatrix} \alpha \\ \beta \end{pmatrix}$ state after BS?

$$\begin{pmatrix} \alpha \\ \beta \end{pmatrix} = \alpha \begin{pmatrix} 1 \\ 0 \end{pmatrix} + \beta \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$$\longrightarrow \alpha \begin{pmatrix} s \\ t \end{pmatrix} + \beta \begin{pmatrix} u \\ v \end{pmatrix} = \begin{pmatrix} \alpha s + \beta u \\ \alpha t + \beta v \end{pmatrix} = \begin{pmatrix} s & u \\ t & v \end{pmatrix} \begin{pmatrix} \alpha \\ \beta \end{pmatrix}$$

The effect of BS over any state $\begin{pmatrix} \alpha \\ \beta \end{pmatrix}$ is to multiply by the matrix $\underbrace{\begin{pmatrix} s & u \\ t & v \end{pmatrix}}_{BS}$

If we have balanced or 50/50% BS then we have:

$$|s|^2 = |t|^2 = |u|^2 = |v|^2 = \frac{1}{2}$$

Lets the matrix be:

$$BS = \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix} ?$$

It is satisfy if it acts on a normalized photon state it should give a normalized photon state $\Rightarrow$ probability conservation.

Any optical Engineer can built a BS satisfying this matrix if the probability conservation is kept. It does not contradicts any physical principle.

This is normalized $\left(\frac{1}{\sqrt{2}}\right)^2 + \left(\frac{1}{\sqrt{2}}\right)^2 = 1$

$$\begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix} \begin{pmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

However $1^2 + 1^2 = 2$

not normalized

state of photon

it cannot be a BS

So we propose another matrix

$$BS = \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{pmatrix} \begin{pmatrix} \alpha \\ \beta \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} \alpha + \beta \\ \alpha - \beta \end{pmatrix} \rightarrow$$

$$\rightarrow \underbrace{\frac{1}{2}|\alpha+\beta|^2 + \frac{1}{2}|\alpha-\beta|^2}_{\text{Normalization}} = \frac{1}{2}\left[(\alpha+\beta)(\alpha^*+\beta^*) + (\alpha-\beta)(\alpha^*-\beta^*)\right] =$$

The cross term vanish but $\underbrace{\alpha\alpha^* \ldots \beta\beta^*}_{\text{add.}}$

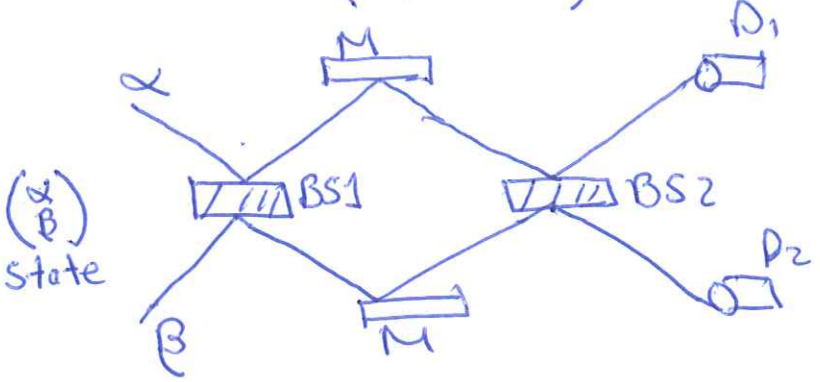
$$= |\alpha|^2 + |\beta|^2 = 1$$

Therefore this BS works perfectly.
It is not the unique solution but we can have two BS

$$BS_1 = \frac{1}{\sqrt{2}} \begin{pmatrix} -1 & 1 \\ 1 & 1 \end{pmatrix} \qquad BS_2 = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$$

Lets we use the two proposed BS in experiments

$$BS1 = \frac{1}{\sqrt{2}}\begin{pmatrix} -1 & 1 \\ 1 & 1 \end{pmatrix} \qquad BS2 = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$$



The two mirrors only multiply by minus (-1) and no any effect.

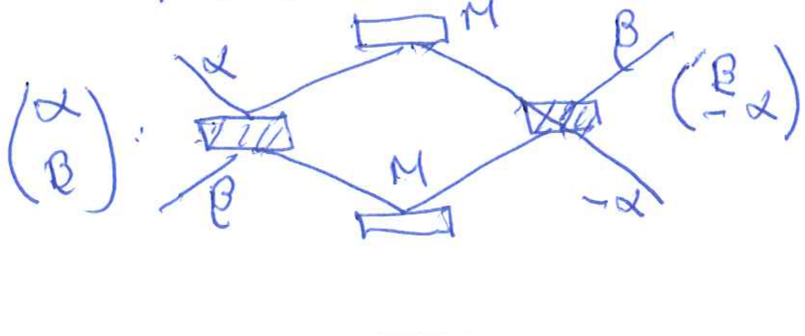
$$\text{output} = (BS2)_{\text{matrix}} \, (BS1)_{\text{matrix}} \begin{pmatrix}\alpha \\ \beta\end{pmatrix}$$

The output is a two components Vector with amplitude up and amplitude down

$$= \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \frac{1}{\sqrt{2}}\begin{pmatrix} -1 & 1 \\ 1 & 1 \end{pmatrix}\begin{pmatrix}\alpha \\ \beta\end{pmatrix} =$$

$$= \frac{1}{2}\begin{pmatrix} 0 & 2 \\ -2 & 0 \end{pmatrix}\begin{pmatrix}\alpha \\ \beta\end{pmatrix} = \begin{pmatrix}\beta \\ -\alpha\end{pmatrix}$$

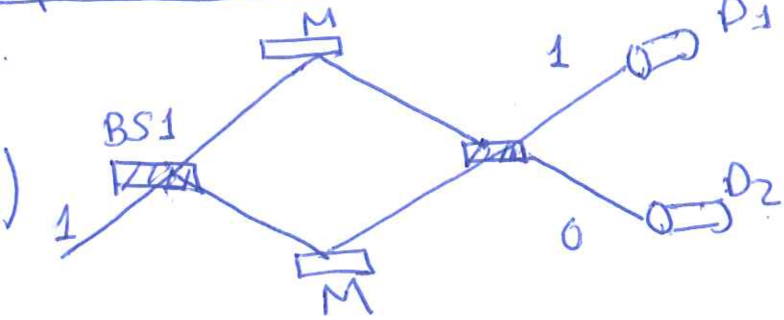
Then the rule is:



Lets do our first kind of experiment

Experiment 1

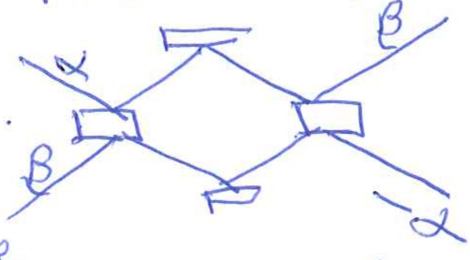
$\begin{pmatrix}0\\1\end{pmatrix}$



$\begin{pmatrix}1\\0\end{pmatrix}$

Aplying the rule:

$\begin{pmatrix}\alpha\\\beta\end{pmatrix}$



$\begin{pmatrix}\beta\\-\alpha\end{pmatrix}$

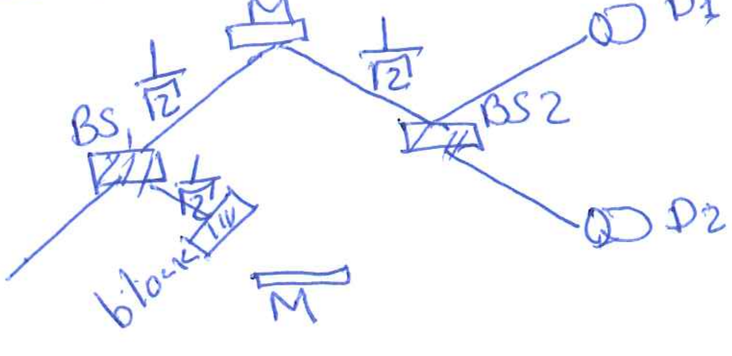
Interference

The photon from the botton reflected and the photon transmitted from the top add to give 0

the reflected from the top and transmitted from botton add coherently to give 1

Experiment 2

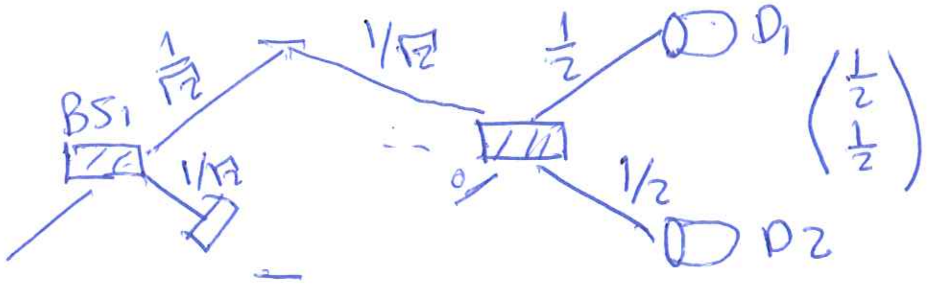
$\begin{pmatrix}0\\1\end{pmatrix}$



$$BS1\begin{pmatrix}0\\1\end{pmatrix}=\frac{1}{\sqrt{2}}\begin{pmatrix}-1 & 1\\1 & 1\end{pmatrix}\begin{pmatrix}0\\1\end{pmatrix}$$

$$=\begin{pmatrix}\frac{1}{\sqrt{2}}\\\frac{1}{\sqrt{2}}\end{pmatrix}$$

$$(BS_2)\begin{pmatrix}\frac{1}{\sqrt{2}}\\0\end{pmatrix}=\frac{1}{\sqrt{2}}\begin{pmatrix}1 & 1\\1 & -1\end{pmatrix}\begin{pmatrix}\frac{1}{\sqrt{2}}\\0\end{pmatrix}=\begin{pmatrix}\frac{1}{2}\\\frac{1}{2}\end{pmatrix}$$

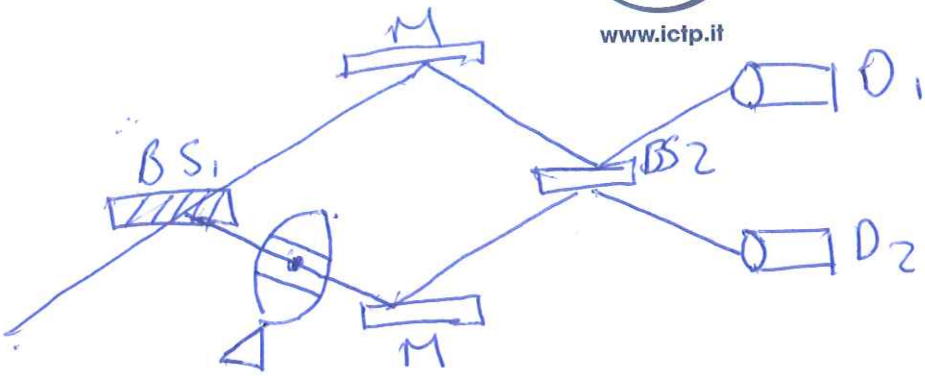


Results

Outcome (blocked down branch)	P
Photon at the block	1/2
Photon at D_1	1/4
Photon at D_2	1/4

Outcome (all open)	P
Photon D_1	1
Photon D_2	0

Elitzur-Vaidman bombs



Suppose the bomb is defective

Outcome	P
photon to D_1	1
photon to D_2	0
bomb explodes	0

Bomb is good

Outcome	P
bomb explodes	1/2
photon at D_1 (bomb ≠ exp	1/4
photon at D_2 (bomb ≠ exp (bomb does not exp but is good	1/4

$\frac{1}{2}$ probability bomb to explodes but if the bomb does not work, there is no way photon to reach D2, because if does not work all photons goes to D1

So, the fact that 1/4 of time photons reach D_2 implies that photons is at D_2 and bomb did not exploided but is good.