Quantum Speed Limits in Open Quantum Systems

Steve Campbell

ICTP School on Quantum Information Theory and Thermodynamics at the Nanoscale

Extending QSLs

So far we have focussed mostly on the so-called “minimal time approach”

what is the **shortest time** I can achieve some process

One can equivalently ask what is the **maximal speed** a given evolution can attain

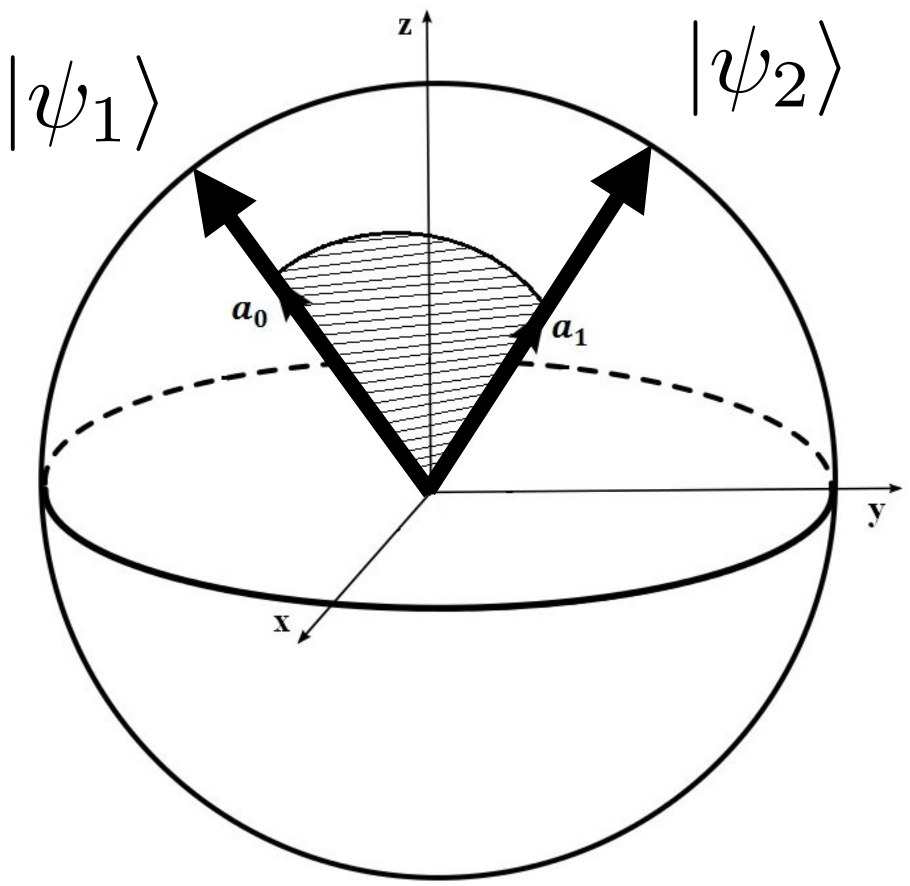
This is the basic starting point of applying a purely geometric approach to quantum speed limits

In general, we are interested in dynamics governed by a master equation

$$\dot{\rho}_t = L(\rho_t)$$

Defining a geometric quantum speed

We need a notion of distance in our state space



The shortest path connecting PURE states

$$\ell(\psi_1, \psi_2) = \arccos |\langle \psi_1 | \psi_2 \rangle|$$

$$v \equiv \dot{\ell}(\psi_0, \psi_t) \leq |\dot{\ell}(\psi_0, \psi_t)| \leq v_{\mathrm{QSL}}$$

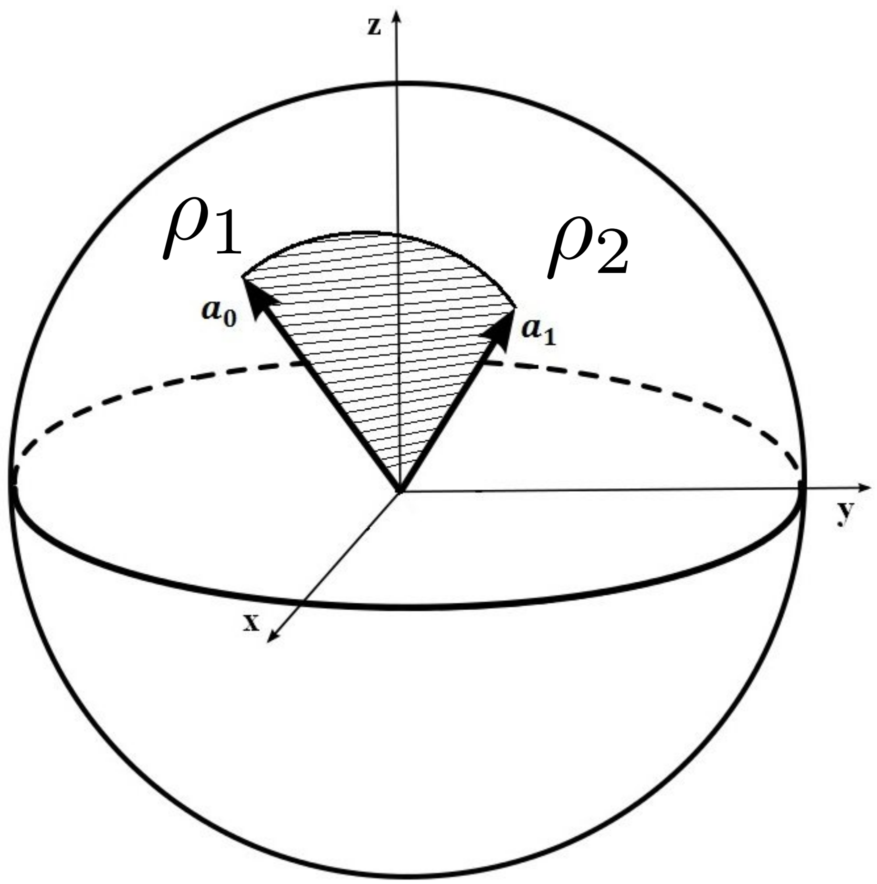
$$\tau_{\mathrm{QSL}} \equiv \frac{\tau}{\int_0^\tau \mathrm{d}t \; v_{\mathrm{QSL}}}$$

The speed and the QSL time characterise the dynamics and geometry of the dynamics

The interpretation of τ_{QSL} is still not entirely clear

Quantum speed for mixed states

The ‘correct’ statistical distance for mixed states is the Bures angle



$$\mathcal{L}(\rho_1, \rho_2) = \arccos \sqrt{F(\rho_1, \rho_2)}$$

$$F(\rho_1, \rho_2) = \left[\mathrm{tr}\sqrt{\sqrt{\rho_1}\rho_2\sqrt{\rho_1}}\right]^2$$

$$v \equiv \dot{\mathcal{L}}(\rho_0, \rho_t)$$

But it is, in general, a very difficult quantity to calculate

So we look for some bounds instead!

The ‘seminal’ year - 2013

PRL **110,** 050402 (2013) PHYSICAL REVIEW LETTERS week ending 1 FEBRUARY 2013

Quantum Speed Limit for Physical Processes

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(Received 29 August 2012; published 30 January 2013)

The evaluation of the minimal evolution time between two distinguishable states of a system is important for assessing the maximal speed of quantum computers and communication channels. Lower bounds for this minimal time have been proposed for unitary dynamics. Here we show that it is possible to extend this concept to nonunitary processes, using an attainable lower bound that is connected to the quantum Fisher information for time estimation. This result is used to delimit the minimal evolution time for typical noisy channels.

DOI: 10.1103/PhysRevLett.110.050402 PACS numbers: 03.65.Yz, 03.67.−a

$$\mathcal{L}(\rho_0, \rho_\tau) = \arccos \sqrt{F(\rho_0, \rho_\tau)} \leq \frac{1}{2} \int_0^\tau \mathrm{d}t \, \sqrt{\mathcal{F}_Q(t)}$$

The QSL is then given by the quantum Fisher information

$$v_{\mathrm{QSL}} = \frac{1}{2} \sqrt{\mathcal{F}_Q(t)}$$

The ‘seminal’ year - 2013

PRL **110,** 050403 (2013) PHYSICAL REVIEW LETTERS week ending 1 FEBRUARY 2013

Quantum Speed Limits in Open System Dynamics

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(Received 18 September 2012; published 30 January 2013)

Bounds to the speed of evolution of a quantum system are of fundamental interest in quantum metrology, quantum chemical dynamics, and quantum computation. We derive a time-energy uncertainty relation for open quantum systems undergoing a general, completely positive, and trace preserving evolution which provides a bound to the quantum speed limit. When the evolution is of the Lindblad form, the bound is analogous to the Mandelstam-Tamm relation which applies in the unitary case, with the role of the Hamiltonian being played by the adjoint of the generator of the dynamical semigroup. The utility of the new bound is exemplified in different scenarios, ranging from the estimation of the passage time to the determination of precision limits for quantum metrology in the presence of dephasing noise.

DOI: 10.1103/PhysRevLett.110.050403 PACS numbers: 03.65.Yz, 03.67.Lx

del Campo employed the relative purity as an easily computable metric to bound the speed

$$f(t) = \frac{\mathrm{tr}[\rho_0 \rho_t]}{\mathrm{tr}[\rho^2]} \qquad\qquad \tau_\theta \geq \frac{4\theta^2 \mathrm{tr}\rho_0^2}{\pi^2 \sqrt{\mathrm{tr}[(\mathcal{L}^\dagger \rho_0)^2]}}.$$

The ‘seminal’ year - 2013

PRL **111,** 010402 (2013) PHYSICAL REVIEW LETTERS week ending 5 JULY 2013

Quantum Speed Limit for Non-Markovian Dynamics

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(Received 26 February 2013; published 3 July 2013)

We derive a Margolus-Levitin-type bound on the minimal evolution time of an arbitrarily driven open quantum system. We express this quantum speed limit time in terms of the operator norm of the nonunitary generator of the dynamics. We apply these results to the damped Jaynes-Cummings model and demonstrate that the corresponding bound is tight. We further show that non-Markovian effects can speed up quantum evolution and therefore lead to a smaller quantum speed limit time.

DOI: 10.1103/PhysRevLett.111.010402 PACS numbers: 03.65.Yz

Without loss of generality Deffner+Lutz assume initially pure states and use the Bures angle

$$\mathcal{L}(\rho_0, \rho_\tau) = \arccos \sqrt{\langle \psi_0 | \rho_\tau | \psi_0 \rangle}$$

$$\frac{d}{dt}\mathcal{L}(\rho_0, \rho_\tau) \leq \frac{|\langle \psi_0 | \dot{\rho}_\tau | \psi_0 \rangle|}{2\sqrt{\langle \psi_0 | \rho_\tau | \psi_0 \rangle - \langle \psi_0 | \rho_\tau | \psi_0 \rangle^2}}$$

Deffner-Lutz M-L Type Bound

$$\frac{d}{dt}\mathcal{L}(\rho_0,\rho_\tau) \leq \frac{|\langle\psi_0|\dot{\rho}_\tau|\psi_0\rangle|}{2\sqrt{\langle\psi_0|\rho_\tau|\psi_0\rangle - \langle\psi_0|\rho_\tau|\psi_0\rangle^2}}$$

$$\implies 2\dot{\mathcal{L}}\cos\mathcal{L}\sin\mathcal{L} \leq |\langle\psi_0|\dot{\rho}_\tau|\psi_0\rangle|$$

This is the starting point for their deviations of both M-L + M-T type bounds

If we consider unitary dynamics again (just for illustration!)

$$\begin{aligned} 2\dot{\mathcal{L}}\cos\mathcal{L}\sin\mathcal{L} &\leq |\langle\psi_0|[H_t,\rho_t]|\psi_0\rangle| \\ &\leq |\mathrm{tr}\{H_t\rho_t\rho_0\}| + |\mathrm{tr}\{\rho_t H_t\rho_0\}| \end{aligned}$$

Triangle inequality

Now applying the von Neumann inequality we obtain the bound(s)

$$2\dot{\mathcal{L}}\cos\mathcal{L}\sin\mathcal{L} \leq 2\|H_t\rho_t\|_{\mathrm{op}} \quad (\leq 2\|H_t\rho_t\|_{\mathrm{tr}})$$

$$\implies \tau \geq \tau_{\mathrm{QSL}} = \frac{\hbar}{2E_\tau}\sin^2\left[\mathcal{L}(\rho_0,\rho_\tau)\right] \qquad E_\tau = \frac{1}{\tau}\int_0^\tau dt\langle H_t\rangle$$

(Margolus-Levitin type bound)

Deffner-Lutz M-L Type Bound

A virtually identical analysis can be followed if we instead consider an open system dynamics governed by the arbitrary master equation

$$\dot{\rho}_t = L_t(\rho_t)$$

And we arrive at the bound

$$\tau \geq \tau_{\mathrm{QSL}} = \max\left\{\frac{1}{\Lambda_\tau^{\mathrm{op}}}, \frac{1}{\Lambda_\tau^{\mathrm{op}}}\right\} \sin^2\left[\mathcal{L}(\rho_0, \rho_\tau)\right]$$

$$\Lambda_\tau^x = \frac{1}{\tau}\int_0^\tau dt \|L_t(\rho_t)\|_x$$

The operator norm provides the sharpest bound

Deffner-Lutz M-T Type Bound

Similarly a Mandalstam-Tamm-type bound can be derived

$$\begin{aligned} 2\dot{\mathcal{L}} \cos\mathcal{L} \sin\mathcal{L} &\leq \left|\langle\psi_0|\dot{\rho}_\tau|\psi_0\rangle\right| \\ &= \left|\mathrm{tr}\{L_t(\rho_t)\rho_0\}\right| \end{aligned}$$

Employing the Cauchy-Schwarz inequality

$$2\dot{\mathcal{L}} \cos\mathcal{L} \sin\mathcal{L} \leq \sqrt{\mathrm{tr}\left\{L_t(\rho_t) L_t(\rho_t)^\dagger\right\}} = \|L_t(\rho_t)\|_{\mathrm{hs}}$$

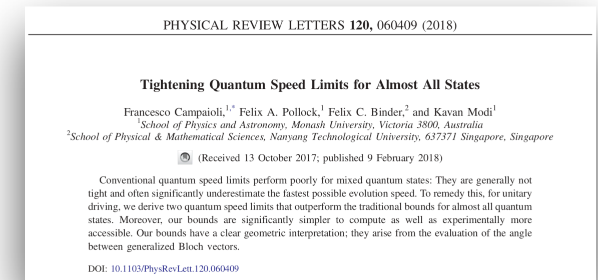
$$\Longrightarrow \quad \tau \geq \tau_{\mathrm{QSL}} = \frac{1}{\Lambda_\tau^{\mathrm{hs}}} \sin^2\left[\mathcal{L}(\rho_0, \rho_\tau)\right]$$

We arrive at the “unified” bound

$$\tau_{\mathrm{QSL}} = \max\left\{\frac{1}{\Lambda_\tau^{\mathrm{op}}}, \frac{1}{\Lambda_\tau^{\mathrm{tr}}}, \frac{1}{\Lambda_\tau^{\mathrm{hs}}}\right\} \sin^2\left[\mathcal{L}(\rho_0, \rho_\tau)\right]$$

Tightening quantum speed limits

If we once again restrict to only **unitary** dynamics we notice something about the Bures angle


PHYSICAL REVIEW LETTERS **120,** 060409 (2018)

Tightening Quantum Speed Limits for Almost All States

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(Received 13 October 2017; published 9 February 2018)

Conventional quantum speed limits perform poorly for mixed quantum states: They are generally not tight and often significantly underestimate the fastest possible evolution speed. To remedy this, for unitary driving, we derive two quantum speed limits that outperform the traditional bounds for almost all quantum states. Moreover, our bounds are significantly simpler to compute as well as experimentally more accessible. Our bounds have a clear geometric interpretation; they arise from the evaluation of the angle between generalized Bloch vectors.

DOI: 10.1103/PhysRevLett.120.060409

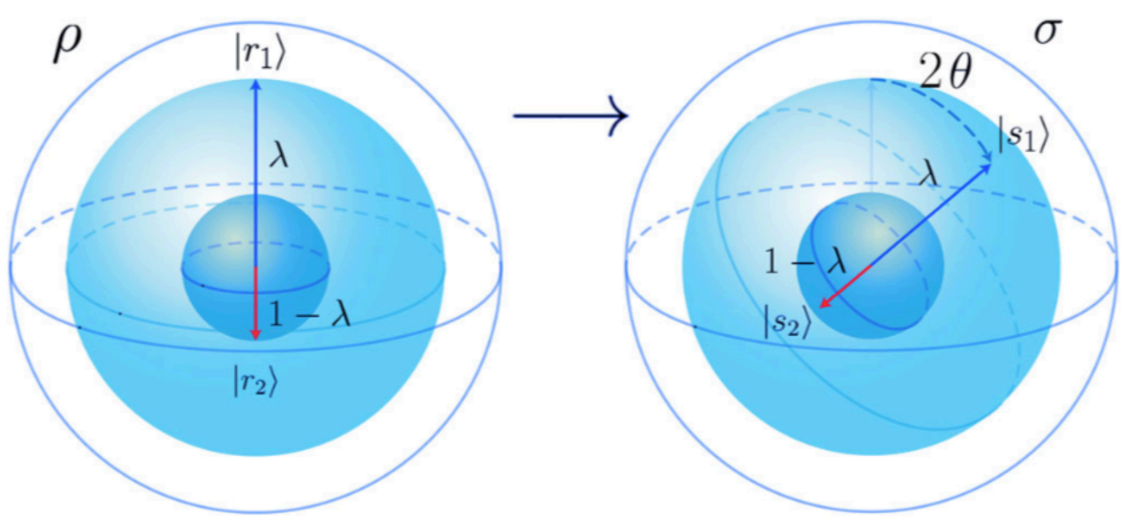

FIG. 1. Let ρ and σ be two mixed qubit states with the same spectrum, $\rho = \lambda|r_1\rangle\langle r_1| + (1-\lambda)|r_2\rangle\langle r_2|$, and $\sigma = \lambda|s_1\rangle\langle s_1| + (1-\lambda)|s_2\rangle\langle s_2|$, $\lambda \in (0,1)$, $\lambda \neq 1/2$, where $\{|r_1\rangle, |r_2\rangle\}$ and $\{|s_1\rangle, |s_2\rangle\}$ are two orthonormal bases. The problem of unitarily evolving ρ to σ can be mapped to evolving $|r_1\rangle$ to $|s_1\rangle$ (or equivalently, $|r_2\rangle$ to $|s_2\rangle$). Equation (1) is tight for pure states; thus, any Hamiltonian that takes $|r_1\rangle$ to $|s_1\rangle$ will also take ρ to σ in the same time. For any Hamiltonian with bounded standard deviation $\Delta E \leq \mathcal{E}$, this time is bounded from below by $\theta/\mathcal{E}$, where $\theta = d(|r_1\rangle, |s_1\rangle)$ is the distance between $|r_1\rangle$ and $|s_1\rangle$, i.e., half of the angle between the vectors associated with $|r_1\rangle$ and $|s_1\rangle$. However, Eq. (1) for the same constraint on the Hamiltonian suggests that $T_{\mathcal{L}} = \mathcal{L}(\rho,\sigma)/\mathcal{E}$, with $\mathcal{L}(\rho,\sigma) < \theta$ for every choice of $\lambda \neq 0$, 1 [see Eq. (9)], making the QSL unattainable for all mixed states.

$$\Phi(\rho,\sigma) = \arccos\sqrt{\frac{\mathrm{tr}[\rho\sigma]}{\mathrm{tr}[\rho^2]}}$$

$$\tau \geq \tau_{\mathrm{QSL}} = \frac{\Phi(\rho_0,\rho_\tau)}{Q_\tau}$$

$$Q_\tau = \frac{1}{\tau}\int_0^\tau dt\sqrt{\frac{\mathrm{tr}[\rho_t^2 H_t^2 - (\rho_t H_t)^2]}{\mathrm{tr}[\rho^2]}}$$

Reduces to the “standard” MT bound for pure states

An abundance of bounds

PHYSICAL REVIEW X **6,** 021031 (2016)

Generalized Geometric Quantum Speed Limits

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DOI: 10.1103/PhysRevLett.120.060409

New Journal of Physics
The open access journal at the forefront of physics
Deutsche Physikalische Gesellschaft DPG
IOP Institute of Physics
Published in partnership with: Deutsche Physikalische Gesellschaft and the Institute of Physics

PAPER

Geometric quantum speed limits: a case for Wigner phase space

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For **every metric** you can write down an associated quantity that can be viewed as a quantum speed limit.

Unravelling the meaning of these quantities is the focus of much current work

QSLs and non-Markovian dynamics

A classic example is the Jaynes-Cummings model

$$\dot{\rho}_t = L_t(\rho_t)$$
$$= -\frac{i}{\hbar}[H_q, \rho_t] - \frac{i}{\hbar}[\lambda_t \sigma_+ \sigma_-, \rho_t]\,\gamma_t \left(\sigma_- \rho_t \sigma_+ - \frac{1}{2}\sigma_+\sigma_-\rho_t - \frac{1}{2}\rho_t\sigma_+\sigma_-\right)$$

$$\rho_0 = |1\rangle\langle 1|$$

$$\tau_{\mathrm{QSL}} = \frac{(1 - P_\tau)\tau}{\int_0^\tau dt\ |\dot{P}t|}$$

Excited state population

$$\tau_{\mathrm{QSL}} = \frac{(1 - P_\tau)\tau}{2\mathcal{N} + (1 - P_\tau)}$$

BLP non-Markovianity measure

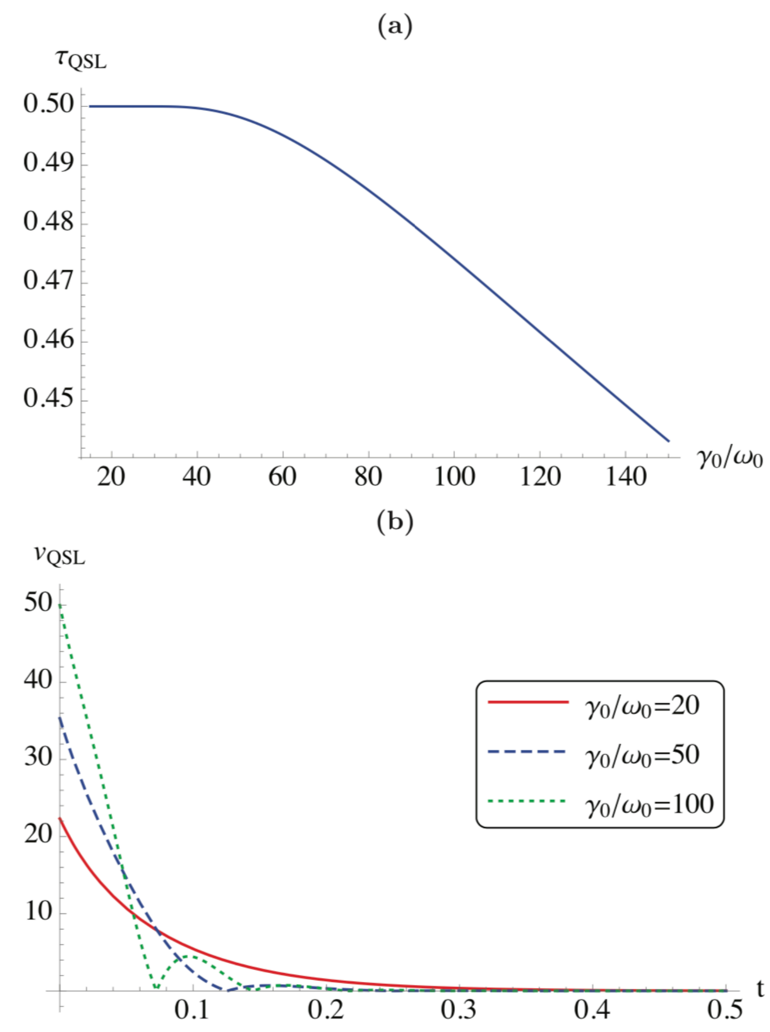

Figure 3. Quantum speed limit time τ_{QSL} (82) (upper panel, (a)) and quantum speed limit v_{QSL} (81) (lower panel, (b)) for the damped-Jaynes–Cummings model (91) and the initial state ρ_0 (96). Parameters are $\tau = 0.5$, $\hbar = 1$ and $\lambda = 50$.

QSLs and non-Markovian dynamics

A classic example is the Jaynes-Cummings model

$\rho_0 =$

$\tau_{QSL} = \frac{(\ldots}{\int \ldots}$

$\tau_{QSL} = \frac{(\ldots}{2\mathcal{N}}$

BLP non-Markovianity measure

IOP Publishing New J. Phys. 21 (2019) 123041 https://doi.org/10.1088/1367-2630/ab59fe

New Journal of Physics
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Deutsche Physikalische Gesellschaft DPG
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PAPER

There is no general connection between the quantum speed limit and non-Markovianity

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OPEN ACCESS

RECEIVED
30 July 2019

REVISED
23 October 2019

ACCEPTED FOR PUBLICATION
21 November 2019

PUBLISHED
17 December 2019

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Keywords: quantum speed limit, non-Markovian, open quantum systems, master equation

Abstract
The quantum speed limit (QSL) sets a bound on the minimum time required for a quantum system to evolve between two states. For open quantum systems this quantity depends on the dynamical map describing the time evolution in presence of the environment, on the evolution time τ, and on the initial state of the system. We consider a general single qubit open dynamics and show that there is no simple relationship between memory effects and the tightness of the QSL bound. We prove that only for specific classes of dynamical evolutions and initial states, there exists a link between non-Markovianity and the QSL. Our results shed light on the connection between information back-flow between system and environment and the speed of quantum evolution.

0.1 0.2 0.3 0.4 0.5 t

Figure 3. Quantum speed limit time τ_{QSL} (82) (upper panel, (a)) and quantum speed limit v_{QSL} (81) (lower panel, (b)) for the damped-Jaynes–Cummings model (91) and the initial state ρ_0 (96). Parameters are $\tau = 0.5$, $\hbar = 1$ and $\lambda = 50$.

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