A Minicourse on Non-equilibrium Statistical Physics

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A. Master Equation Approach (1.5 lectures)

(i) Statement of the Equation and its Consequences

(ii) Examples

  (a) Two-state system
  (b) Random walks on small graphs—continuum and discrete time
  (c) Random walks in one dimension
    - The Poisson process
    - Nearest-neighbor hopping—Laplace and by Fourier transform solutions
    - The birth-death process

B. Irreversible Aggregation (1.5 lectures)

(i) The Smoluchowski Equation and its Underlying Assumptions

(ii) Constant-Kernel Aggregation

  (a) Recursive solution
  (b) Generating function solution
  (c) Asymptotic properties

(iii) Scaling Solutions

  (a) Preliminary: the diffusion equation
  (b) Aggregation with homogeneous kernels

(iv) Product Kernel Solution

  (a) Generating function solution
  (b) The gelation transition

C. Irreversible Adsorption (1 lectures)

(i) Warmup: Monomer Adsorption

(ii) Dimer Adsorption in One Dimension

  (a) Basic dynamical variables
  (b) Solution for the time-dependent coverage

(iii) Extension to k-mer Adsorption in One Dimension

(iv) Higher Dimensions

D. Dynamics of the Ising Model (2 lectures)

(i) Statement of the Problem

(ii) Master Equation and the Spin-Flip Rates of the Ising Model

(iii) Time Dependence of the Magnetization on the Complete Graph

(iv) Glauber Solution of the One-Dimensional Model
   (a) Average magnetization
   (b) Two-spin correlation function and its geometric interpretation
   (c) Continuum solution

(v) The Voter Model
   (a) Discrete solution in one dimension
   (b) Continuum solution in general spatial dimensions

(vi) Order-Parameter Conserving (Kawasaki) dynamics

E. Coarsening (1.5 lectures)

(i) Time-Dependent Ginzburg-Landau and Cahn-Hilliard equations
   (a) Basic Assumptions
   (b) Analogy with a Reaction-Diffusion Process
   (c) Mechanical Analog

(ii) Case studies
   (a) Planar interface
   (b) Droplet shrinking
   (c) Droplet evolution for conservative dynamics

(iii) Lifshitz-Slyozov-Wagner coarsening

F. Dynamics of Complex Networks (1.5 lectures)

(i) Kinetic Approach for the Erdős-Rényi Random Graph
   (a) The Degree Distribution as a Poisson Process
   (b) The Cluster-Size Distribution from Product-Kernel Aggregation

(ii) Random Recursive Trees
   (a) Formulation of the master equation
   (b) The degree distribution

(iii) Preferential Attachment Networks
   (a) Sublinear attachment rate
   (b) Superlinear attachment rate
   (c) Strictly linear attachment rate
   (d) Redirection and the correspondence with shifted linear attachment rate