LECTURE 3: NOISE MECHANISMS.

- Now that we have seen something about how to describe random systems we will turn to a quantitative discussion of some of the most important fundamental noise mechanisms: Shot Noise, Johnson Noise, and $1/f$ Noise.
- Once past these fundamental mechanisms, we will delve into practical noise issues that arise when designing physical systems (usually experiments).
- Such practical sources of noise include Amplifier noise, such as interference from unwanted signals etc.

* SHOT NOISE: A current, such as electrons in a wire or rain on a roof, is made up of the discrete arrival of many carriers.
- If their interactions can be ignored so that they arrive independently, this is a straightforward example of a Poisson process.

- For an electrical signal, the average current is $\langle I \rangle = qN/T$ for N electrons with charge q arriving over a time-interval T.
- If the electrons arrive far enough apart so that the duration during which they arrive is small compared to the time between the arrival of successive electrons, then the current can be approximated as a sum of delta functions.

$$I(t) = q\sum_{n=1}^{N} \delta(t - t_n) \longrightarrow (1)$$

where t_n is the arrival time for the n^{th} electron.
- The Fourier transform of this impulse train is:

$$I(f) = \lim_{T\to\infty} \int_{-T/2}^{+T/2} e^{i2\pi f t} q \sum_{n=1}^{N} \delta(t - t_n)\, dt = q\sum_{n=1}^{N} e^{i2\pi f t_n} \longrightarrow (2)$$

- Therefore, the power spectrum is:

$$S_I(f) = \langle I(f) I^*(f) \rangle$$
$$= \lim_{T \to \infty} \frac{q^2}{T} \left(\sum_{n=1}^{N} e^{i2\pi f t_n} \sum_{m=1}^{N} e^{-i2\pi f t_m} \right)$$
$$= \lim_{T \to \infty} \frac{q^2 N}{T} = q \langle I \rangle \longrightarrow (3)$$

(the cross-terms $m \neq n$ vanish in the expectation because their times are independent).

- We see that the power spectrum of carrier arrivals is white (flat) and that the magnitude is linearly proportional to the current.
- This is called Shot Noise or Schottky Noise.

- If the carriers do not arrive as delta functions then the broadening of the impulses will roll the spectrum off for high frequencies, so ~~that~~ the flat power spectrum is a good approximation up to the inverse of the characteristic times of the system.

- To find the fluctuations associated with shot noise, we can use Parseval's theorem to relate the average total energy in the spectrum to the average variance.
- If the bandwidth of the system is infinite this variance too will be infinite, because for ideal shot noise there is equal power at all frequencies.

- Any real measurement system will have a finite bandwidth, and this determines the amplitude of the noise.

- Multiplying the power spectrum by $2\Delta f$, where Δf is the bandwidth in hertz and the factor of 2 comes from including both positive and negative frequencies,

$$\langle I_{noise}^2 \rangle = 2q\langle I \rangle \Delta f \longrightarrow (4)$$

- Shot noise will be important only if the number of carriers is small enough for the rate of arrival to be discernible.

* JOHNSON NOISE: Johnson (or Nyquist) noise is the noise associated with the relaxation of thermal fluctuations in a resistor.
- Small voltage fluctuations are caused by the thermal motion of electrons, which then relax back through the resistance. The result is simple: $\langle V_{noise}^2 \rangle = 4kTR\Delta f \longrightarrow (5)$
where R is resistance, Δf is the bandwidth of the measuring system, T is the temperature, and k is Boltzmann's constant.

- Once again, this is white noise, but unlike shot noise it is independent of the current.
- The resistor is almost acting like a battery, driven by thermodynamic fluctuating.
- The voltage produced by these fluctuations is very real and very important: it sets the basic limit on the performance of many kinds of electronics.
- Unfortunately, it is not possible to take advantage of Johnson noise by rectifying the fluctuating voltage across a diode to use a resistor as a power source (Hint: What Temperature is the diode?)

- Johnson noise is an example of a <u>fluctuation-dissipation relationship</u> which we will cover soon - the size of a system's thermodynamic fluctuations is closely related to the rate at which the system relaxes to equilibrium from a perturbation.
- A system that is more strongly damped has smaller fluctuations, but it dissipates more energy.

* <u>1/f Noise and Switching Noise</u>: In a wide range of transport processes, from electrons in resistors, to cars on the highway, to notes in music, the power spectrum diverges at low frequencies inversely proportionally to frequency: $S(f) \propto f^{-1}$.
- Because such 1/f noise is scale invariant (the spectrum looks the same at all time scales) and is so ubiquitous, many people have been lured to search for profound general explanations for the many particular examples.
- While this has led to some bizarre ideas, there is a reasonable theory at least for the important case of electrical 1/f noise.
- We will look at a set of recent results that do illuminate upon more generic (but not universally applicable) cases a bit later.

- In a conductor there are usually many types of defects, such as lattice vacancies or dopant atoms.
- Typically, the defects can be in a few different inequivalent types of sites in the material, which have different energies.
- This means that there is a probability for a defect to be thermally excited into a higher-energy state, and then relax down to the lower-energy state.

- Because the different sites have different scattering cross-sections for the electron current, this results in a fluctuation in the conductivity of the material.
- A process that is thermally activated between two states, with a characteristic time τ to relax from the excited state, has a Lorentzian power spectrum of the form

$$S(f) = \frac{2\tau}{1+(2\pi f\tau)^2} \longrightarrow (6)$$

- If there is a distribution of activation times $p(\tau)$ instead of a single activation time in the material, and if the activated scatterers don't interact with one another, then the spectrum will be an integral over this:

$$S(f) = \int_0^\infty \frac{2\tau}{1+(2\pi f\tau)^2}\, p(\tau)\, d\tau \longrightarrow (7)$$

- If the probability of the defect having an energy equal to a barrier height E goes as $e^{-E/kT}$ (to be explained later), then the characteristic time τ to be excited over the barrier will be inversely proportional to probability $\tau = \tau_0 e^{E/kT} \longrightarrow (8)$
- This is called a <u>thermally activated process</u>.

- If the distribution of barrier heights is flat then $p(\tau) \propto 1/\tau$, and plugging this in eq^n 7 shows that $S(f) \propto 1/f$.

- This is the origin of $1/f$ noise: scatterers with a roughly flat distribution of activation energies.
- Cooling a sample to a low enough temperature can turn-off the higher-energy scatterers and reveal the individual Lorentzian

components in the spectrum.

- In this regime, the noise signal in time is made up of jumps between discrete values, called <u>switching noise</u>.
- This can be seen unexpectedly and intermittently at room temperature, for example if a device has a very bad wire-bond so that the current passes through a narrow constriction.

- Unlike Johnson noise, $1/f$ noise is proportional to the current in the material because it is a conductivity rather than a voltage fluctuation, and it increases as the cross-sectional area of the material is decreased because the relative influence of a single defect is greater.
- For this reason, $1/f$ noise is greater in carbon resistors, which have many small contacts between grains, than in metal-film resistors.
- Low-noise switches, for this reason, maintain large contact areas and wiping connections that slide against each other as the switch is closed, to make sure that the conduction is not constrained to small channels.

- The power spectrum of the noise from a resistor will be flat because of Johnson noise if there is no current flowing; as the current is increased the $1/f$ noise will appear, and the frequency below which it is larger than the Johnson noise will depend upon the applied current as well as the details of the material.
- $1/f$ noise is not an intrinsic property: the magnitude is a function of how a particular sample is prepared.

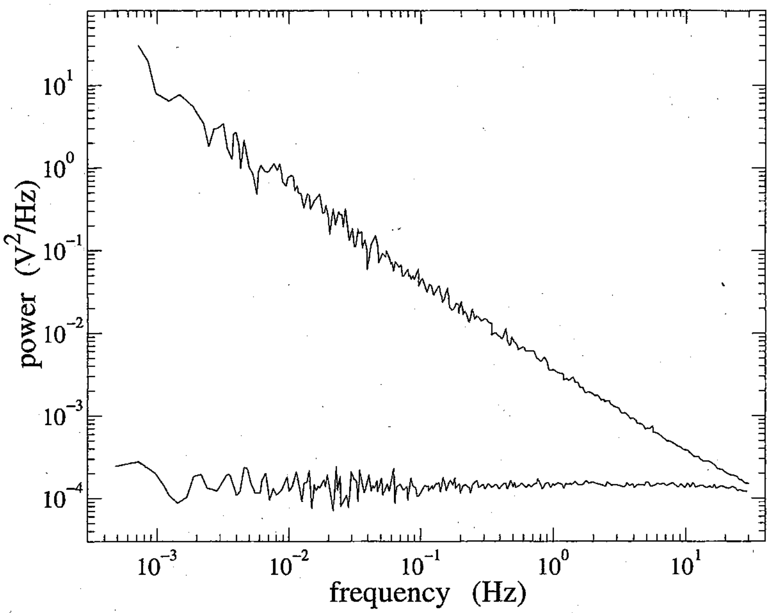


Fig 1: Noise in a 50Ω resistor with and without a current.

- Figure 1 shows the Johnson and 1/f noise for a carbon resistor.
- Because 1/f noise diverges at low frequencies, it sets a time limit below which measurements cannot be made; a common technique to avoid 1/f noise is to modulate the signal up to a higher frequency. Hopefully, we will have time to explore it later.

* AMPLIFIER NOISE: Any device that detects a signal must contend with one or more of the noise mechanisms we've discussed, in its workings.

- Johnson noise leads to the generation of voltage noise by an amplifier.
- Since the power spectral density is flat, the mean square noise magnitude will be proportional to the bandwidth, or the Root Mean Square (RMS) magnitude will increase as the square root of the bandwidth.
- The latter quantity is what is conventionally used to characterize an amplifier; for a low-noise device it can be on the order of $1\,nV/\sqrt{Hz}$.

- Likewise, shot noise is responsible for the generation of current noise at an amplifier's output; this is also flat and for a low-noise amplifier can be on the order of $1\ pA/\sqrt{Hz}$.

- Given the practical significance of detecting signals at (and beyond) these limits, it can be more relevant to relate the noise an amplifier introduces to the noise that is input to it.
- Signals and noise are usually compared on a logarithmic scale to cover a large dynamic range; the Signal-to-Noise Ratio (SNR), measured in decibels, is

$$SNR = 10 \log_{10}\left(\frac{\langle V_{signal}^2\rangle}{\langle V_{noise}^2\rangle}\right)$$

$$= 20 \log_{10}\left(\frac{\langle V_{signal}^2\rangle^{1/2}}{\langle V_{noise}^2\rangle^{1/2}}\right) \longrightarrow (9).$$

$$= 20 \log_{10}\left(\frac{V_{RMS\ Signal}}{V_{RMS\ Noise}}\right)$$

- It can be defined in terms of the mean square values of the signal and noise (equal to the variances if the signals have zero mean), or the RMS values by bringing out a factor of 2.

- One way to describe the performance of an amplifier is to ask how much more noise appears at its output than was present at its input, assuming that the input is responsible for Johnson noise due to its source impedance R_{source}. This ratio, measured in decibels is called the noise figure:

$$NF = 10 \log_{10}\left(\frac{\text{output noise power}}{\text{input noise power}}\right) = 10 \log_{10}\left(\frac{4kTR_{source}\Delta f + \langle V_{noise}^2\rangle}{4kTR_{source}\Delta f}\right) \longrightarrow (10)$$

$$= 10 \log_{10}\left(1 + \langle V_{noise}^2\rangle / (4kTR_{source}\Delta f)\right)$$

- V_{noise} is the noise added by the amplifier to the source; it is what would be measured if the input impedance was cooled to absolute zero.
- The noise figure is often plotted as noise contours as a function of input impedance and frequency, see fig. 2 below.

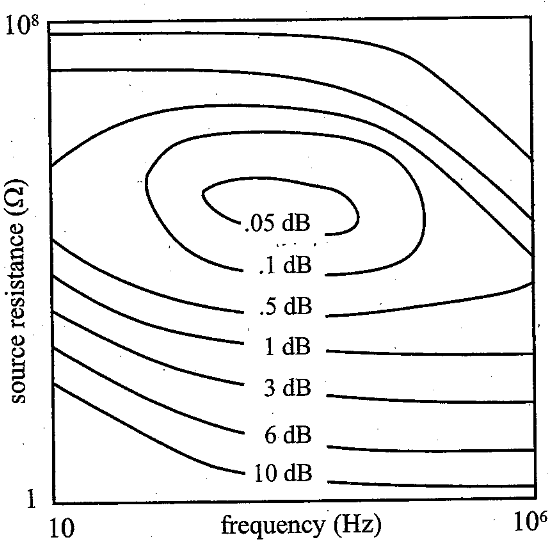


Fig 2: Noise contours for a low-noise amplifier.

- There is a "sweet spot" in the middle: it gets worse at low source impedances because the source thermal noise is small compared to the amplifier thermal noise; it gets worse at high source impedances and high frequencies because of capacitive coupling; and it degrades at low frequencies because of $1/f$ noise.

- Amplifier noise can also be measured by the noise temperature, defined to be the temperature T_{noise} to which the input impedance must be raised from its actual temperature T_{source} for its thermal noise to match the noise added by the amplifier:

$$NF = 10\log_{10}\left(1 + \frac{\langle V_{noise}^2\rangle}{4kT_{source}R\Delta f}\right)$$

$$= 10\log_{10}\left(1 + \frac{4kT_{noise}R}{4kT_{source}R\Delta f}\right) = 10\log_{10}\left(1 + \frac{T_{noise}}{T_{source}}\right) \longrightarrow (11)$$

- In a GaAs HEMT (High-Electron-Mobility Transistor) most of the electron scattering mechanisms have been eliminated and so the mean-free-path can be as large as the device.
- Since inelastic scattering is the origin of resistance and hence of the thermodynamic coupling of the conduction electrons to the material, this means that the noise temperature can be much lower than room temperature.
- In the best devices it gets down to just a few Kelvins.
- One of the places where this sensitivity is particularly important is for detecting the weak signals from space for satellite communications and radio astronomy.

** Grounding, Shielding, and Leads: An amplifier is only as good as it's leads.
- While this reasonable observation has led to unreasonable marketing of rather pathological cables to gullible audiophiles, it is true that small changes in wiring can have a very large impact on a system's performance (both good and bad).
- The goal is to make sure that as much of a signal of interest gets to it's destination, and as little as possible of everything else.
- The polite term for this area is electromagnetic compatibility, asking, say, how to ground your battleship so that it's electronics can withstand a nuclear electromagnetic pulse.

- Although the principles ~~of~~ for good wiring practice can appear to be closer to black magic than engineering design, they are really just an exercise in applying Maxwell's equations.

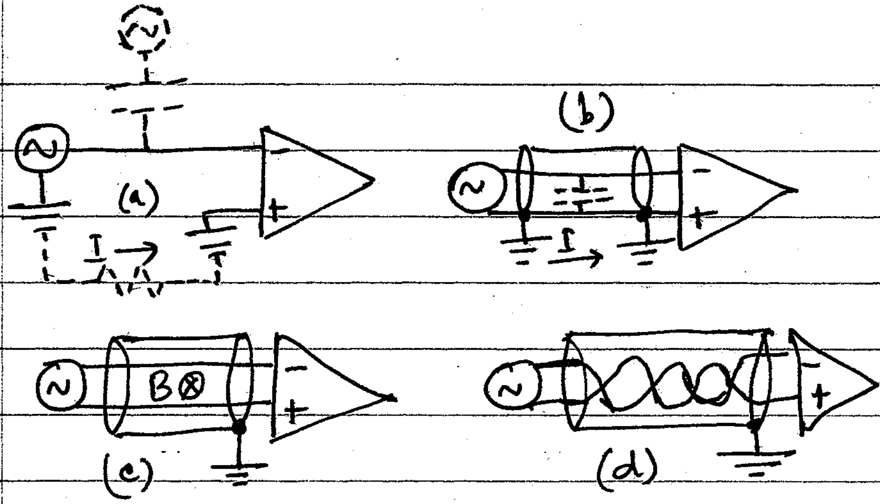


Fig 3: Grounding woes: (a) ground loop and capacitive pickup, (b) cross-talk and improper shield grounding, (c) magnetic pickup, and (d) shielded twisted pair.

- Consider the circuits shown in fig 3. In (a) a source is directly connected to a single-ended amplifier, introducing two serious problems.
- First, any other fluctuating voltages around the signal lead can capacitively couple into it, producing interference.
- Second, the source and amplifier are grounded in different locations.
- Current must flow through the pathway connecting the grounds, and so any resistance there will lead to a change in the relative potentials.
- Even worse, this difference will depend on the load, and on everything else using the ground.
- This is called a <u>ground loop</u>; thick conducting braid is a favorite tool for combating it by reducing the resistance between ground locations.
- Well designed systems ~~to~~ go further to maintain separate ground circuits for each function, with plenty of capacitance added to each as filters: one ground for digital logic with its high-frequency

noise, one for motors with their large current surges, a quiet one for sensors requiring little current but good voltage stability, and so forth.

- These join only at a single ground commonly called the <u>Mecca node</u>.

- Circuit (b) cures the capacitive pickup by surrounding the wire with a conducting shield, establishing an equipotential around it.
- Related tricks are building a conducting box around a sensitive circuit to provide electrostatic protection, and winding leads coming into and out of a circuit around a toroidal transformer core to provide inductive filtering of high-frequency noise.
- A cable shield comes at the cost of introducing a large capacitance from the source to the shield; for a typical coaxial cable this can be 10s of picofarads per foot, resulting in significant signal loss.
- That can be cured by using a unity-gain amplifier to drive the outer shield with the potential of the inner conductor.

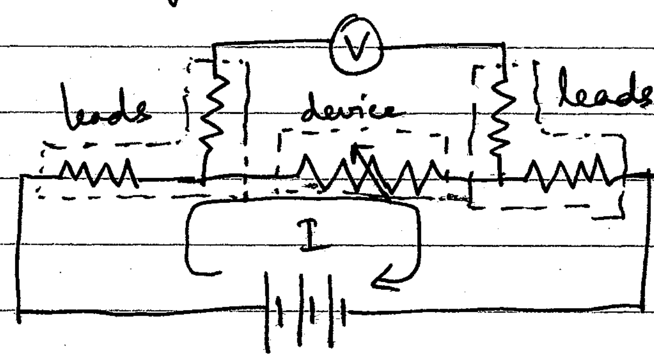
- As long as the amplifier is fast enough, the shield will track the signal, effectively removing the cable capacitance.
- Special followers are available for this purpose, because if the amplifier is not fast enough, or cannot source enough current, then the dynamics of the cable-shield system can swamp the signal of interest.
- Circuit (b) also grounds the shield at both ends.
- This is effective if a heavy shield is used, so that the resistance of the connection is very small, but otherwise it brings the ground loop even closer to the signal lead.

- In (c), both ends of the signal source are connected to a differential amplifier, and the cable shield is tied to one end.
- Not only is the shield not used as a continuous circuit, we don't want it to be available as one: its job is just to maintain the equipotential around the signal leads.
- And because the signals now arrive differentially, the amplifier can remove any common-mode interference that remains.
- That unfortunately does not help with another important noise source, time-varying magnetic flux linking the circuit, frequently coming from power lines.
- Even a high-permeability shield can't keep all the flux from threading between the conductors, and the induced potential appears as a voltage difference rather than a common mode shift.

- The straight conductors are replaced in (d) with <u>shielded twisted pairs</u>.
- The loops do two things: they reduce the cross-sectional area for flux pickup, and the direction of the induced current alternates between loops, approximately averaging it out.
- This is why shielded twisted pair, grounded at one end, is used most often for low-level signals.
- It ceases to be useful when signal wavelength becomes on order of the conductor spacing, but that cutoff can extend upto microwave frequencies.

- If the measurement apparatus need only respond to a signal, a high input-impedance amplifier can be used that does not load the source.

- But if the apparatus is also responsible for providing current to excite the measurement, there can be a substantial voltage drop across the connecting leads that will vary as the load changes.
- This problem is cured by making a <u>four-terminal measurement</u> shown in fig 4.



<u>fig 4</u>: A four-terminal measurement.

- Each lead on the device under test, here taken to be a variable resistor, has two connections.
- One goes to a voltage or current source that drives the current through the leads and the device.
- And the others are used to measure the voltage drop across the device.
- The resistance in the current loop does not matter, because the voltmeter draws essentially no current.
- This is why precision reference resistors have four terminals, even though they appear in two apparently identical pairs.

- When all these techniques fail, it's still possible to give up on electromagnetic shielding entirely and couple optically.
- For long runs, information can be sent in optical fibers, and many kinds of sensing are possible with all-optical devices.

- Even within an electronic circuit, Opto isolators pair an LED with a photodiode in a single package to provide a logical connection without an electrical one.
- These are used to prevent ground loops in audio equipment, and in medical instruments to prevent ground loops in people.