Adiabatic theorem for unbounded Hamiltonians,

with applications to superconducting circuits

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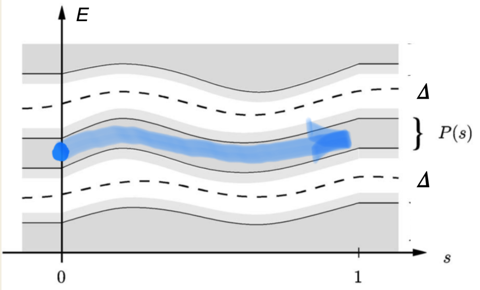
Adiabatic theorem

If a subspace $P(t)\mathscr{H}$ of eigenstates of $H(t)$ is separated by a gap $\Delta(t)$ from the rest of the spectrum, then

$$\|(1 - P(t))\psi(t)\| = O(1/t) \tag{1}$$

where $\psi(t)$ is the solution of $\dot{\psi} = -iH(t)\psi$, $\psi(0) = P(0)\psi(0)$.



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$$\|(1 - P(t))\psi(t)\| = O(1/t) \tag{2}$$

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Big-O notation means $\exists\theta$:

$$\|(1 - P(t))\psi(t)\| \leq \theta/t \tag{3}$$

This talk is about the adiabatic timescale $\theta = \theta(H', \Delta \dots)$

Our results

We present an explicit expression for θ^{new}, improving on the existing result θ^{JRS} [S. Jansen, M.-B. Ruskai, and R. Seiler, (2007)] :

- for an unbounded $\|H'\| = \infty$, $\theta^{\text{JRS}} = \infty$ while $\theta^{\text{new}} < \infty$
- for an n-qubit subspace $P(t)\mathscr{H}$, $\theta^{\text{JRS}} \sim 2^n$ while $\theta^{\text{new}} \sim 1$
- First practical application of both bounds to a circuit model of a flux qubit

$$\omega_q \theta^{\text{JRS}} = \frac{11}{\sqrt{2}} \frac{\omega_q}{\omega_{\text{pl}}^{s=1} \delta} \tag{4}$$

Unbounded H' (e.g. Harmonic oscillator)

For $\theta < \infty$, an assumption is needed.

Assumption of $\|R'(z=i)H\| < \infty$ [J. E. Avron and A. Elgart, (1999)]:, where the resolvent is:

$$R(z=i) = \frac{1}{i-H} = (i-H)^{-1} \tag{5}$$

No explicit $\theta(\|R'(z=i)H\|, \ldots)$ is presented.

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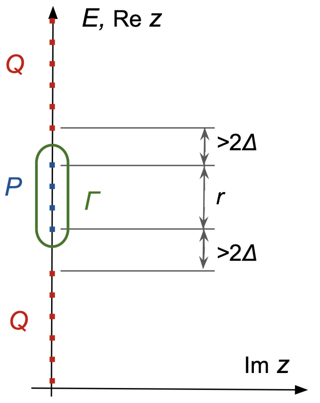
Our assumption: $H'^2 \leq \sum_{k=0}^{k_{max}} c_k H^{2k}$ *(easier to work with)*
Explicit $\theta(c_k, \dots)$ is presented.

n-qubit low-energy subspace (e.g. D-wave)



A replacement

$$\frac{\sqrt{d}}{\Delta} \to \min\left(\frac{\sqrt{d}}{\Delta}, \frac{2r + 2\pi\Delta}{2\pi\Delta^2}\right)$$

can be made in θ^{JRS}, where $d = 2^n$ for n-qubit subspace $P\mathscr{H}$.

Application to superconducting qubits

$$H_{\text{CSFQ,sin}} = E_C \hat{n}^2 + E_J b \cos \hat{\phi} - E_\alpha \sin \frac{1}{2} \hat{\phi} \sin \frac{1}{2} f \quad \phi \in [-2\pi, 2\pi].$$

- The $\hat{n}$ and $\hat{\phi}$ are canonically conjugate operators.
- The E_J, E_C and E_α are fabrication parameters.
- The $b(t)$ and $f(t)$ are time dependent controls.

We follow the experimental procedure that aims at implementing:

$$H_{\text{q}} = \omega_{\text{q}}((1 - s + \delta)X + sZ), \quad s \in [0, 1] \tag{6}$$

Note that there's always nonzero tunneling under the barrier $\delta\omega_{\text{q}}$.
The gap to the non-qubit states $\Delta \sim \omega_{\text{pl}} \sim \sqrt{E_J E_C b}$ (plasma frequency).

Application to superconducting qubits

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We find:

$$\omega_q\theta^{\text{JRS}} = \frac{11}{\sqrt{2}}\frac{\omega_q}{\omega_{\text{pl}}^{s=1}\delta}, \quad \omega_q\theta^{\text{new}} = O\left(\frac{\omega_q}{\omega_{\text{pl}}^{s=1}\delta\ln\frac{\omega_{\text{pl}}^{s=1}}{\delta\omega_{\text{q}}}}\right) \tag{7}$$

where

- $\omega_{\text{q}}\delta$ – tunneling under the barrier at the end of the anneal
- $\omega_{\text{pl}} \sim \sqrt{E_J E_C b}$ – gap to the non-qubit states, plasma frequency
- ω_{q} – the qubit frequency