The Rogers-Ramanujan identities and the icosahedron

The two identities

$$\sum_{n=0}^{\infty} \frac{x^{n^2}}{(1-x)\cdots(1-x^n)} = \prod_{n\equiv\pm1 \pmod 5} \frac{1}{1-x^n}, \quad \sum_{n=0}^{\infty} \frac{x^{n(n+1)}}{(1-x)\cdots(1-x^n)} = \prod_{n\equiv\pm2 \pmod 5} \frac{1}{1-x^n},$$

discovered independently by Leonard Rogers and by Srinivasa Ramanujan more than a hundred years ago, are considered by many to be the most beautiful pair of formulas in all of mathematics. Their most striking aspect is the unexpected appearance of the number "5", which also appears in the theory of the two most complicated of the Platonic solids, the icosahedron and the dodecahedron. It turns out that these two appearances are intimately and intricately related through a wide variety of topics ranging from pure number theory and the theory of modular forms to combinatorics and continued fractions to conformal field theory and mirror symmetry, with guest appearances by various other gems of mathematics like Apéry's proof of the irrationality of $\zeta(2)$. In this series of talks, which will comprise three or four lectures depending on audience interest and also on whether I succeed in completing a planned application to the theory of the mirror quintic of Candelas *et al*, I want to give a survey of some of these topics that is intended to be accessible and of interest to mathematicians of all levels and persuasions, without any particular prerequisites.