GHD as the first order in a thermodynamic form factor expansion

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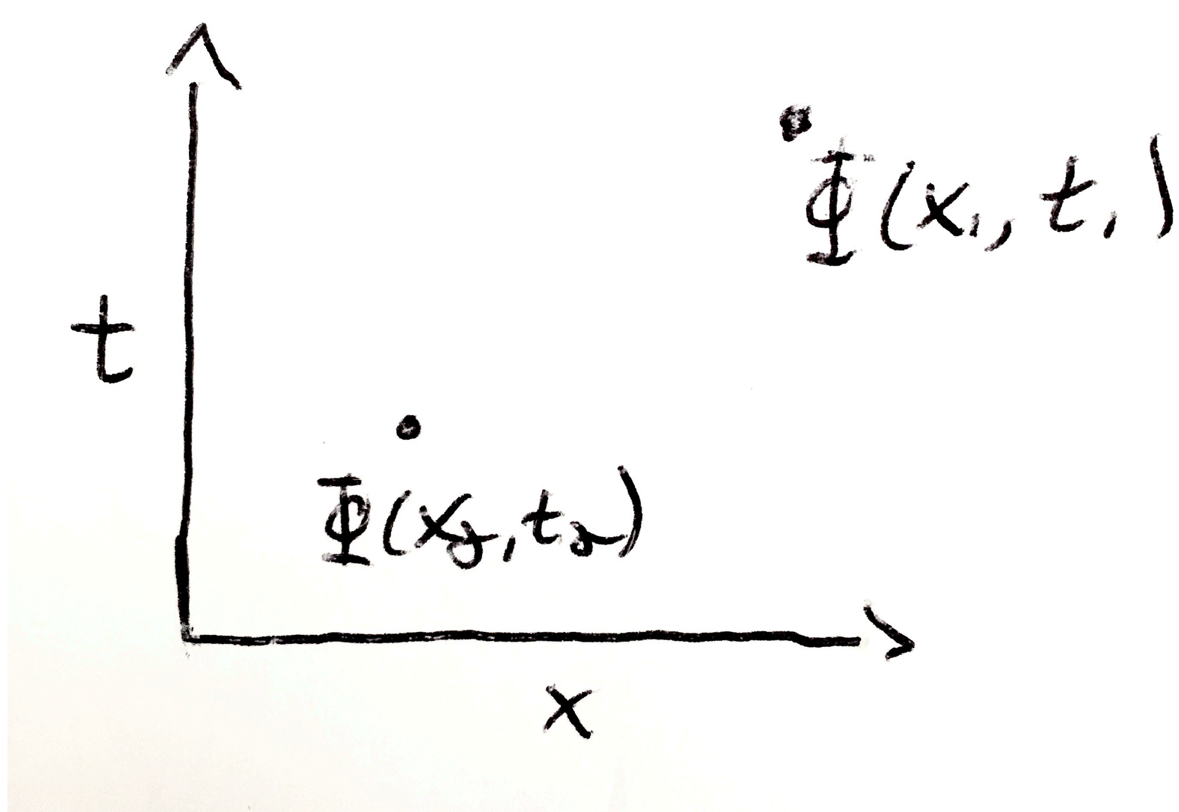
Based on A.C.C. and M. Panfil, *SciPost Phys. 8, 004 (2020)*
A.C.C., *arXiv:2001.03065*

(1+1)-d Integrable QFT

In a boring old vacuum



$$C^{\Phi_1\Phi_2}(x_1 - x_2, t_1 - t_2) = \langle 0 | \Phi_1(x_1, t_1)\Phi_2(x_2, t_2) | 0 \rangle$$

$$- \langle 0 | \Phi_1(x_1, t_1) | 0 \rangle \langle 0 | \Phi_2(x_2, t_2) | 0 \rangle$$

General strategy for IQFT correlators

Form factor expansion

$$C^{\Phi_1\Phi_2}(x_1 - x_2, t_1 - t_2) = \sum_{n \in \text{excited states}} \langle 0 | \Phi_1 | n \rangle \langle n | \Phi_2 | 0 \rangle \, e^{\mathrm{i} p_n \cdot (x_1 - x_2) - \mathrm{i} E_n \cdot (t_1 - t_2)}$$

Excited states in Relativistic IQFT

$$| n \rangle = | \theta_1, \theta_2, \ldots, \theta_n \rangle, \qquad E = m \cosh \theta, p = m \sinh \theta \,.$$

$$f^{\Phi_1}(\theta_1, \ldots, \theta_n) = \langle 0 | \Phi_1(0,0) | \theta_1, \ldots, \theta_n \rangle$$

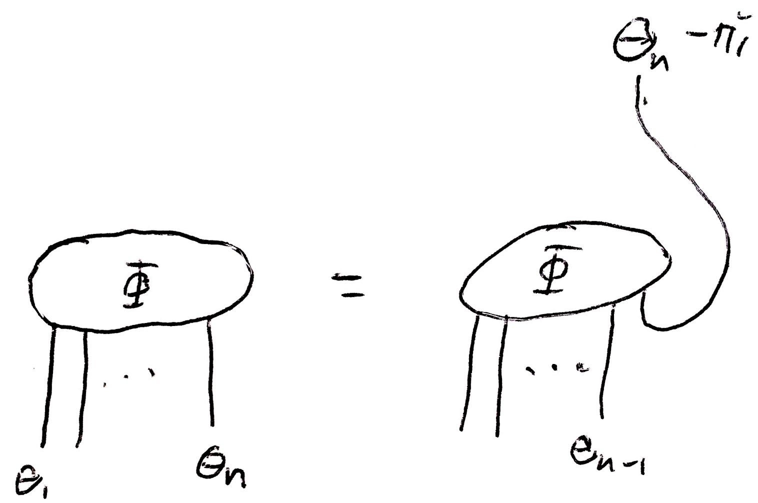
First few particle states are enough for long distance properties

$$C^{\Phi_1\Phi_2}(x_1 - x_2, t_1 - t_2) \approx \int d\theta \, f_1^{\Phi}(\theta) \left(f^{\Phi_2}(\theta) \right)^* \, e^{\mathrm{i} m \cosh \theta (x_1 - x_2) - \mathrm{i} m \sinh \theta (t_1 - t_2)}$$

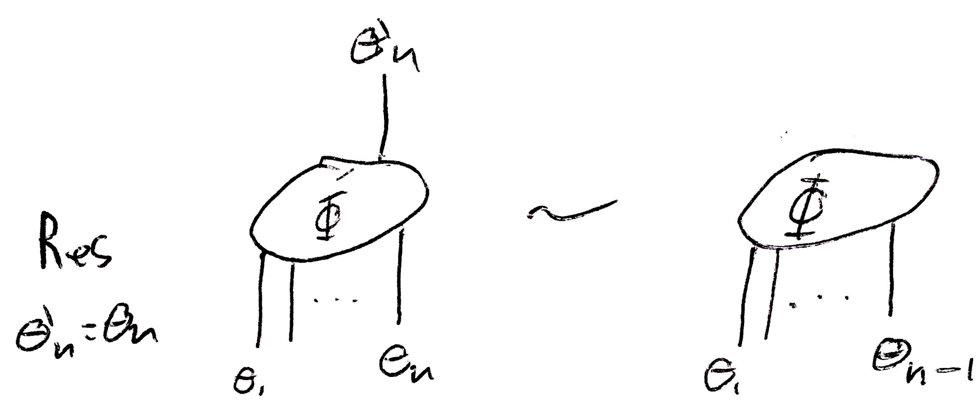
Leads generically to exponential behavior, $\sim e^{-mr}, \text{ or } \; e^{\mathrm{i} m r}$

A few of the useful form factor properties (axioms)

Crossing symmetry $\theta^{\mathrm{out}} \to \theta^{\mathrm{in}} + \pi \mathrm{i}$



Annihilation poles, $f(\theta_1, \ldots, \theta_{n-1}, \theta_n + \pi \mathrm{i}) \sim \dfrac{f(\theta_1, \ldots, \theta_{n-2})}{\theta_{n-1} - \theta_n}$



So, what is Generalized Hydrodynamics?

An effective theory which provides long distance correlation functions on highly excited and possibly inhomogeneous states

$$C_\rho^{\Phi_1\Phi_2}(x_1,x_2,t_1,t_2) = \frac{\langle\rho\,|\,\Phi_1(x_1,t_1)\Phi_2(x_2,t_2)\,|\,\rho\rangle}{\langle\rho\,|\,\rho\rangle}$$

Note the state can break translation invariance

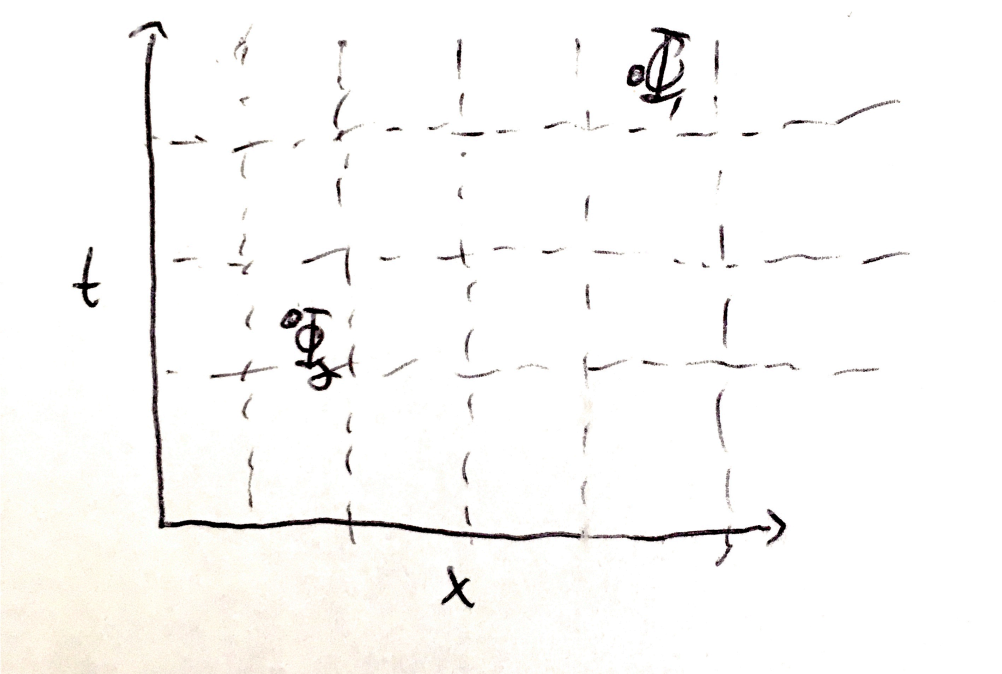
Particularly useful for slow space-time dependent states, for example:

$$\frac{\langle\rho\,|\,\Phi(x,0)\,|\,\rho\rangle}{\langle\rho\,|\,\rho\rangle} = \frac{1}{Z}\mathrm{Tr}\left(e^{-\int dx'\beta_i(x')q^i(x')}\Phi(x,0)\right)$$

$$Q =^i \int dx' q^i(x'), \quad [H, Q^i] = 0$$

How does GHD work?

Cut space-time into large (but also small) cells
Each cell is approximately a GGE



$$\frac{\langle \rho | \Phi(x,t) | \rho \rangle}{\langle \rho | \rho \rangle} \sim \frac{1}{Z_{x,t}} \mathrm{Tr} e^{-\beta_i^{x,t} Q^i} \Phi$$

Find effective evolution equation for $\beta_i^{x,t}$

Our goal:

We focus on two particularly simple predictions from GHD formalism, and we want to reproduce them Without using GHD assumptions, from a form factor expansion

Two point function on a *homogenous* excited state:

B. Doyon, SciPost Phys. 5, 054 (2018)

$$\lim_{\Delta x \sim \Delta t \to \infty} \Delta t \, \frac{\langle \rho \,|\, \Phi_1(x_1, t_1)\Phi_2(x_2, t_2) \,|\, \rho \rangle}{\langle \rho \,|\, \rho \rangle}$$

For causally connected operators

One point function on a *inhomogeneous* excited state:

$$\lim_{x_1 \sim t_1 \to \infty} \frac{\langle \rho[x,t] \,|\, \Phi(x_1, t_1) \,|\, \rho[x,t] \rangle}{\langle \rho[x,t] \,|\, \rho[x,t] \rangle}$$

What do *(homogeneous)* excited states look like?

Finite density of particles:

$$\lim_{N,L\to\infty} \frac{N}{L} = \bar{\rho} = \int d\theta\, \rho(\theta)$$

Thermodynamic "ground state" energy

$$E_\rho = mL \int d\theta\, \cosh(\theta)\rho(\theta)$$

Excitations over *thermodynamic ground state*

Pick a particular microscopic realization of $\rho(\theta)$

Insert additional particles

$$|\rho\rangle \to |\rho'; \theta_1, \theta_2, \ldots\rangle$$

Adding particles shifts the background particles,
Leads to "dressed"quantities.

We can also Remove particles now (add a hole)

Equivalent to $\theta_1 \to \theta_1 + \pi \mathrm{i}$

Low lying contributions to the two point function

One particle above thermodynamic ground state:

$$f_\rho^\Phi(\theta) \equiv \frac{\langle \rho \,|\, \Phi \,|\, \rho'; \theta \rangle}{\langle \rho \,|\, \rho \rangle}$$

Leads to qualitatively similar behavior as in vacuum (exponential)

One particle *removed* from thermodynamic ground state:

$$f_\rho^\Phi(\theta + \pi \mathrm{i})$$

Also leads to exponential behavior (supressed at low densities)

Both contributions vanish in the GHD regime

The relevant excitations are particle-hole pairs (no analog in vacuum)

$$f_\rho^\Phi(\theta_1, \theta_2 + \pi \mathrm{i})$$

A grotesque calculation of $f^{\Phi}_{\rho}(\theta_1, \theta_2 + \pi \mathrm{i})$

For long distance correlation function,
s.p. approximation: we just need the limit,

$$\lim_{\kappa \to 0} f^{\Phi}_{\rho}(\theta, \theta + \pi \mathrm{i} + \kappa)$$

Strategy: regularize at finite volume/number of background particles,
Find how all the background particles behave as $\kappa \to 0$

$$f^{\Phi}_{\rho}(\theta, \theta + \pi \mathrm{i} + \kappa) = \lim_{L,n \to \infty} \frac{f^{\Phi}(\{\theta'_i + \pi \mathrm{i}\}_n, \{\theta'_i + \Delta[\theta', \theta, \kappa]\}_n, \theta, \theta + \pi \mathrm{i} + \kappa)}{\rho_n(\{\theta'_i\}_n)}$$

We can calculate the shift on each background particle $\Delta[\theta'_i, \theta, \kappa] \sim \frac{\kappa}{L}$

We know how form factors behave around poles

Take thermodynamic limit

$$\lim_{\kappa\to 0} f_\rho^\Phi(\theta+\pi\mathrm{i},\theta+\kappa) = 2\pi\rho_s(\theta)V^\Phi(\theta) = \sum_{k=0}^{\infty}\frac{1}{k!}\int\prod_{j=1}^{k}\left(\frac{d\theta_j}{2\pi}\vartheta(\theta_j)\right)f_c^\Phi(\theta_1,\ldots,\theta_k,\theta)$$

Sum over standard (*connected)* form factors

Quickly convergent series,
similar to Leclair-Mussardo formula (if you know what that is)

$$C^{\Phi_1\Phi_2}(\xi,t) = t^{-1}\sum_{\theta} \in \theta_*(\xi)\frac{\rho_p(\theta)(1-\vartheta(\theta))}{|(v^{\mathrm{eff}})'(\theta)|}V^{\Phi_1}(\theta)V^{\Phi_2}(\theta) + \mathcal{O}(1/t^2),$$

Perfectly matches GHD correlation function,
Our formalism gives direct understanding higher corrections

One-point function, inhomogeneous background

$$\frac{\langle \rho \,|\, \Phi(x,t) \,|\, \rho \rangle}{\langle \rho \,|\, \rho \rangle} = \frac{1}{Z} \mathrm{Tr}\left(e^{-\int dx' \beta_i(x') q^i(x')} \Phi(x,t) \right)$$

Assume $\beta_i(x')$ **is slow enough that it can be split into large cells**

$$\int dx' \beta_i(x') q^i(x') = \sum_K \int_{K-l/2}^{K+l/2} dx' \beta_i^K q^i(x')$$

β_i^K **constant within each cell *K***

Extract an *arbitrary* overall $\bar{\beta}_i$**, redefine** $\tilde{\beta}_i^K = \beta_i^K - \bar{\beta}_i$

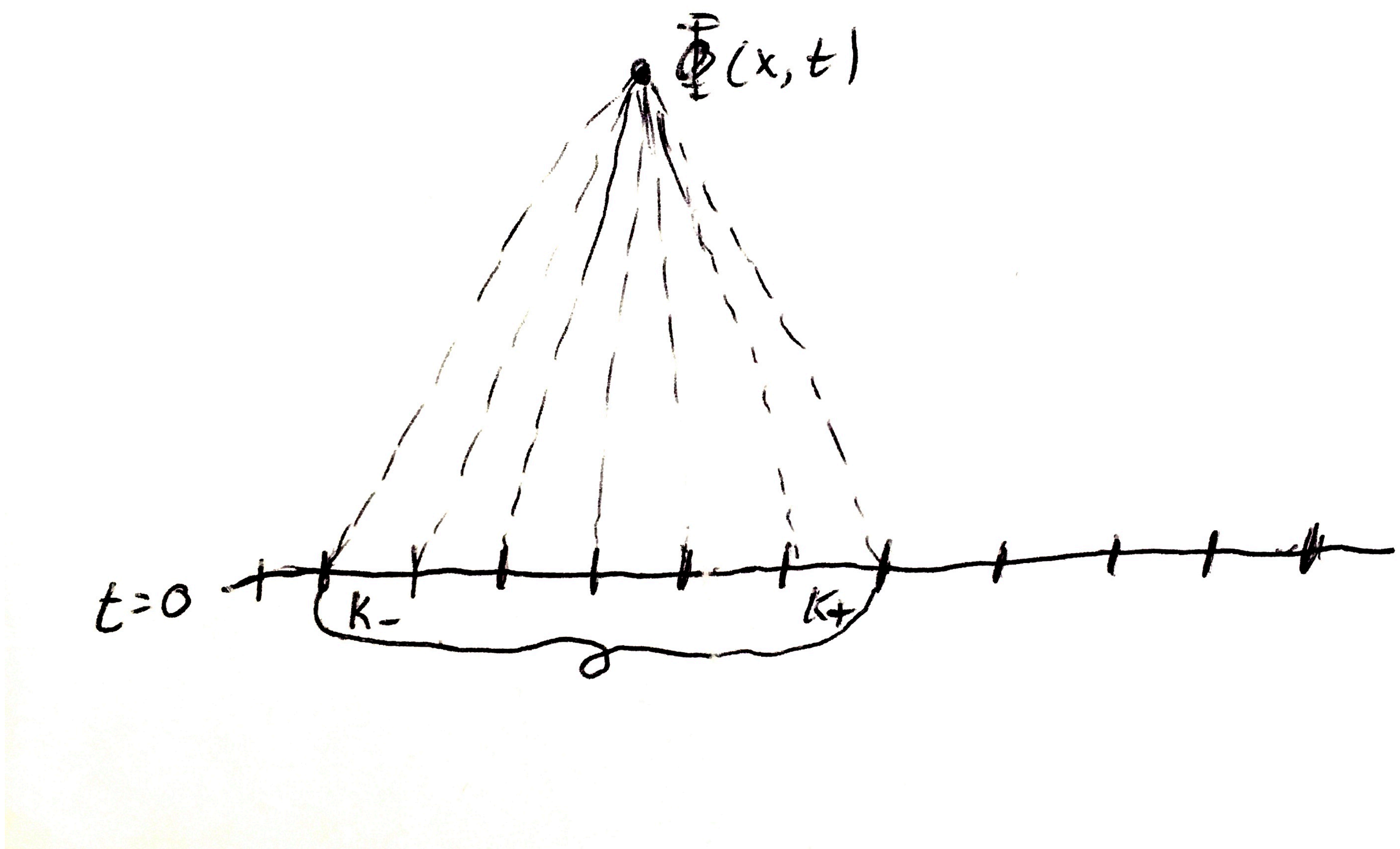
$$\frac{\langle \rho \,|\, \Phi(x,t) \,|\, \rho \rangle}{\langle \rho \,|\, \rho \rangle} \approx \frac{\mathrm{Tr}\left(e^{-\bar{\beta}_i Q^i} \left(\prod_K O_K \right) \Phi(x,t) \right)}{\mathrm{Tr}\left(e^{-\bar{\beta}_i Q^i} \prod_K O_K \right)}$$

At large ℓ

Now a ratio of many-point functions

Free to choose the most convenient $\bar{\beta}_i$

Many-point function



Not all of the many operators are Causally connected! (exponential decay)

Only correlations between $\Phi(x,t)$ **and each** O_K **are important!**

Sum of Homogeneous two-point functions

$$\frac{\langle \rho \,|\, \Phi(x,t) \,|\, \rho \rangle}{\langle \rho \,|\, \rho \rangle} = \frac{\langle \{\bar{\beta}_i\} \,|\, \Phi(0,0) \,|\, \{\bar{\beta}_i\} \rangle}{\langle \{\bar{\beta}_i\} \,|\, \{\bar{\beta}_i\} \rangle} + \sum_{K \in \text{light cone}} C^{\Phi O_K}_{\{\bar{\beta}_i\}}(x - x_K, t) + \mathcal{O}(1/\sqrt{t})$$

Each two-point correlator decays like *1/t,* but there are *~t terms*

We know how to compute the leading contributions to these correlators

The punchline: there is a unique optimal choice of $\{\bar{\beta}_i\}$
Which kills all of the *K* terms

Only the first term survives with $\{\bar{\beta}_i^{\text{Simplest}}\}$
Given by $\{\beta_i^{GHD}(x/t)\}$

Leading term is the GHD prediction, without using GHD!

Future wishlist

Two (and more) point function, inhomogeneous

Leading corrections/ compare with QGHD and Diffusive GHD

Classical and/or non-relativistic limits

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