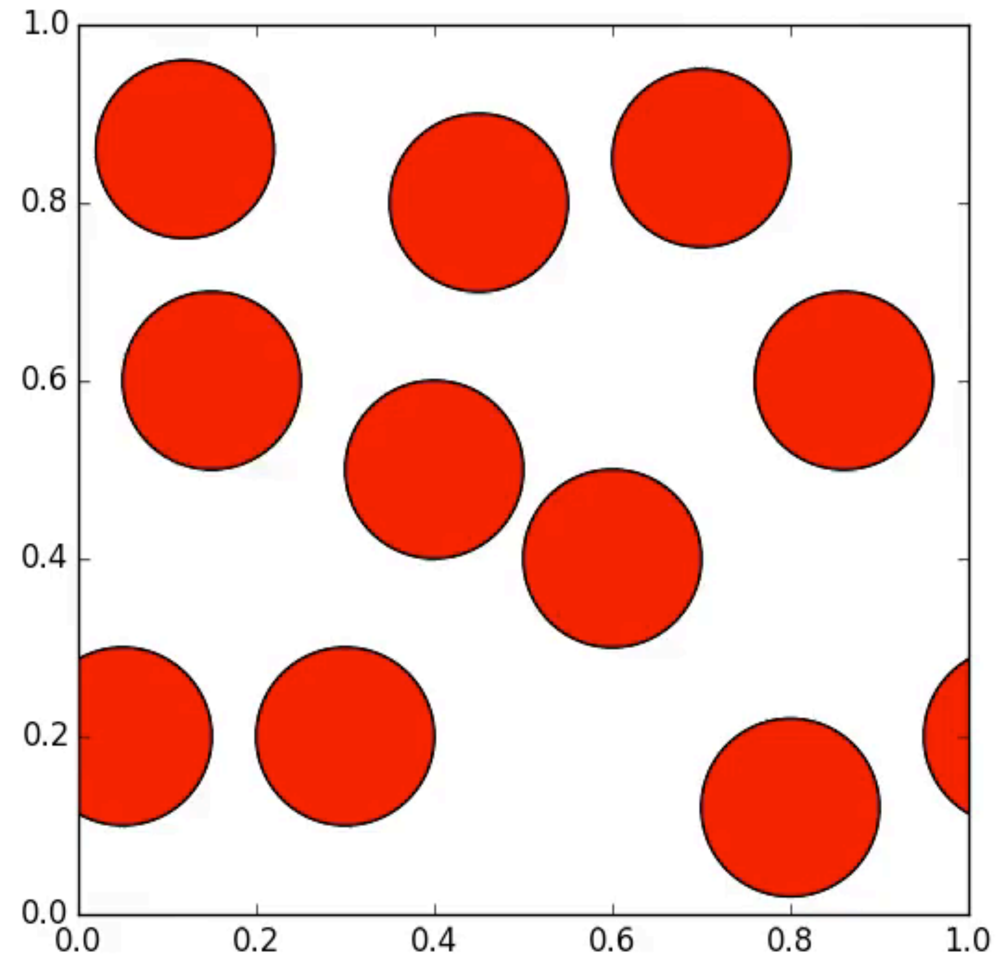
Evolution of the distribution of rapidities under atom losses in the 1D Bose gas

Jérôme Dubail
LPCT, CNRS and Université de Lorraine, Nancy

Based on **arXiv:2006.03583** with:

Isabelle Bouchoule (Institut d'Optique, Palaiseau)
Benjamin Doyon (King's College, London)

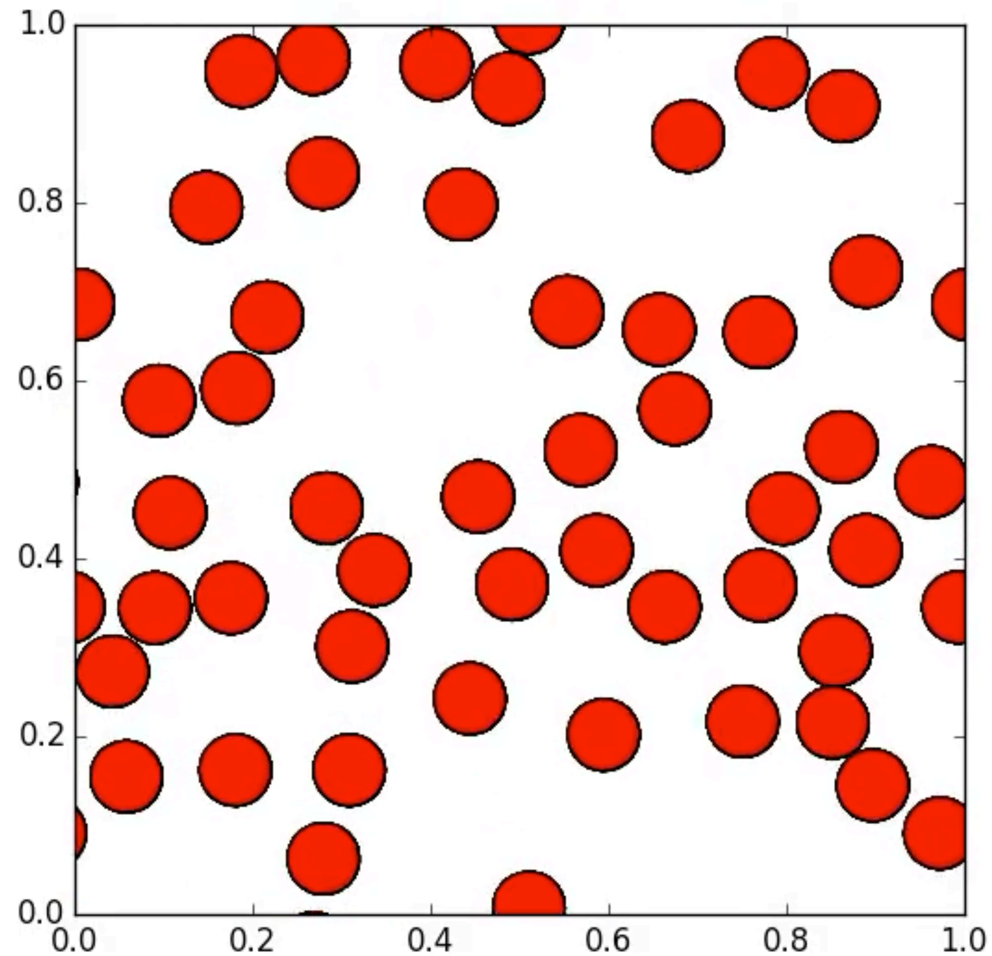
Slow losses in a chaotic/ergodic gas (2D hard spheres)



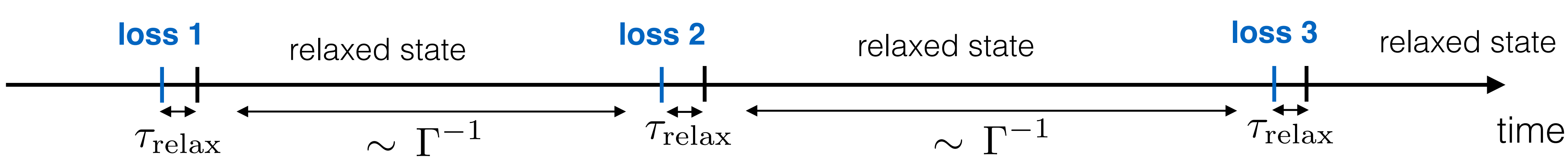
after short relaxation time τ_{relax} , the macrostate in the box is entirely characterized by $n,\ u,\ e$
(particle density, mean velocity, mean energy per particle)

Slow losses in a chaotic/ergodic gas (2D hard spheres)

imagine that some of the particles are lost, at a rate Γ



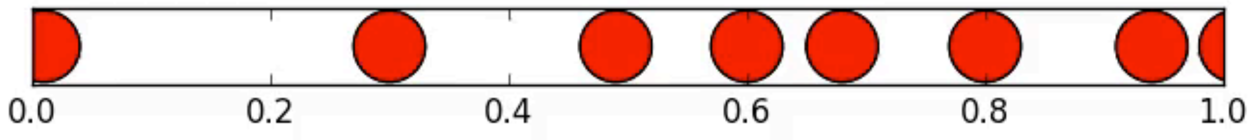
as long as $\Gamma \ll 1/\tau_{\text{relax}}$, the system always has time to relax.



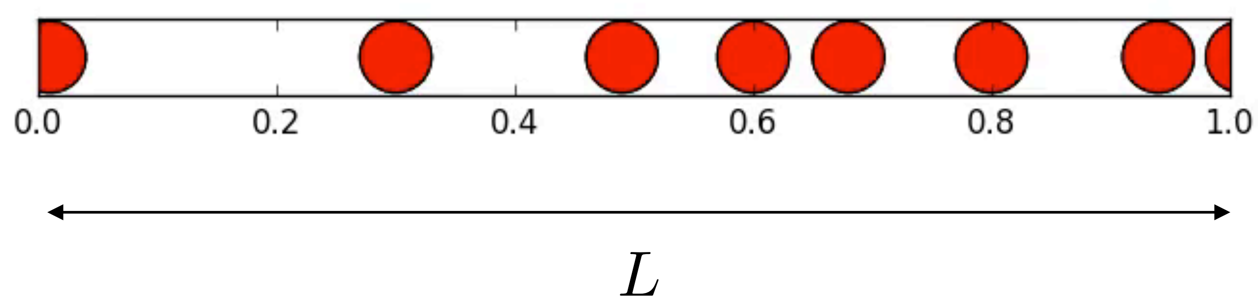
So the macrostate in the box is still characterized by $n,\ u,\ e$, but:

$$\frac{dn}{dt} = -\Gamma n \qquad \frac{du}{dt} = -\Gamma u \qquad \frac{de}{dt} = -\Gamma e$$

Slow losses in a integrable gas (1D hard spheres or hard rods)



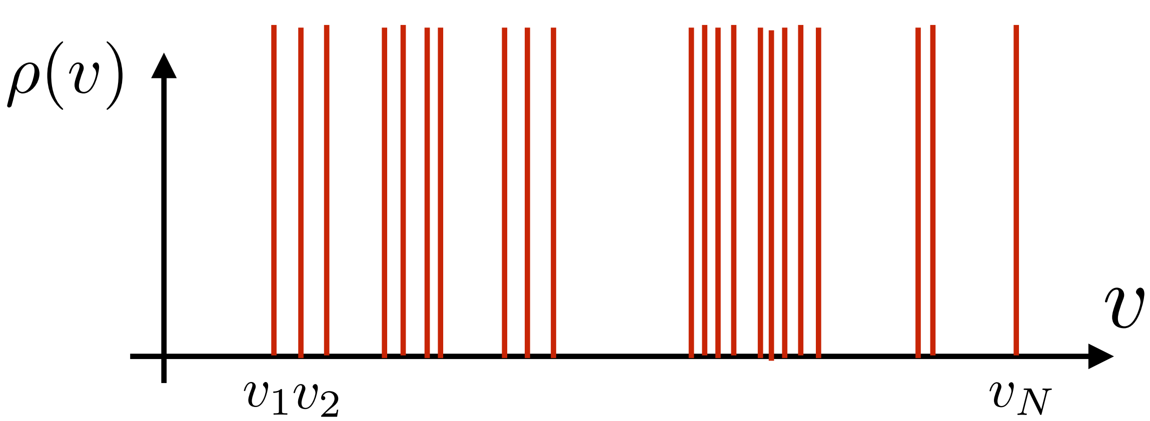
Slow losses in a integrable gas (1D hard spheres or hard rods)

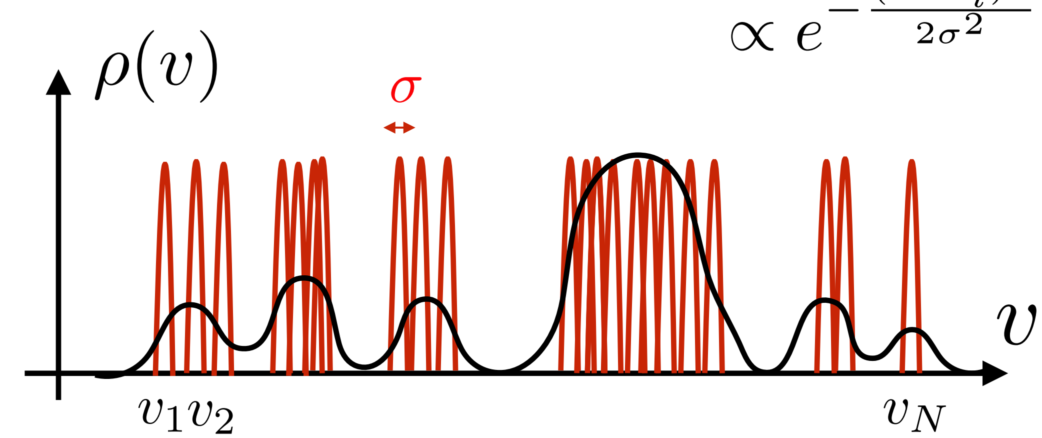


to characterize the macrostate in the box, one needs the entire distribution of velocities:

$$\rho(v) \;=\; \frac{1}{L}\sum_{i=1}^{N}\delta(v-v_i)$$

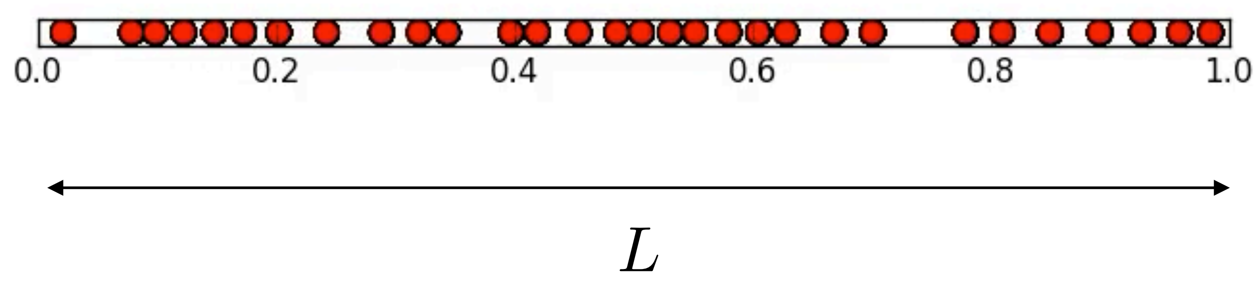
$$\delta(v-v_i) \rightarrow \delta_\sigma(v-v_i)$$
$$\propto e^{-\frac{(v-v_i)^2}{2\sigma^2}}$$





Slow losses in a integrable gas (1D hard spheres or hard rods)

imagine that some of the particles are lost, at a rate $\Gamma \ll 1/\tau_{\mathrm{relax}}$

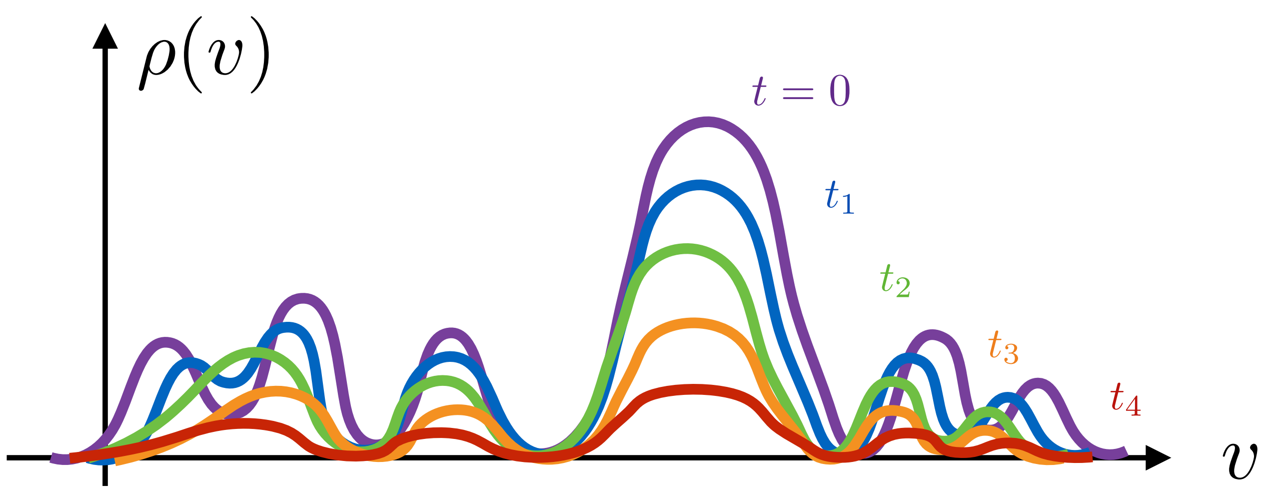


The distribution of velocities evolves according to $\frac{d}{dt}\rho(v) = -\Gamma\rho(v)$ in this toy model.

More generally, in this talk we will have:

$$\boxed{\frac{d}{dt}\rho(v) = -\Gamma F[\rho](v)}$$

and the problem will be to determine the functional $F[\rho]$.



Losses in the 1D Bose gas

The 1D Bose gas is described by the Hamiltonian $H = \int_0^L \psi^\dagger \left(-\frac{1}{2}\partial_x^2 + \frac{g}{2}\psi^\dagger \psi \right) \psi \, dx$

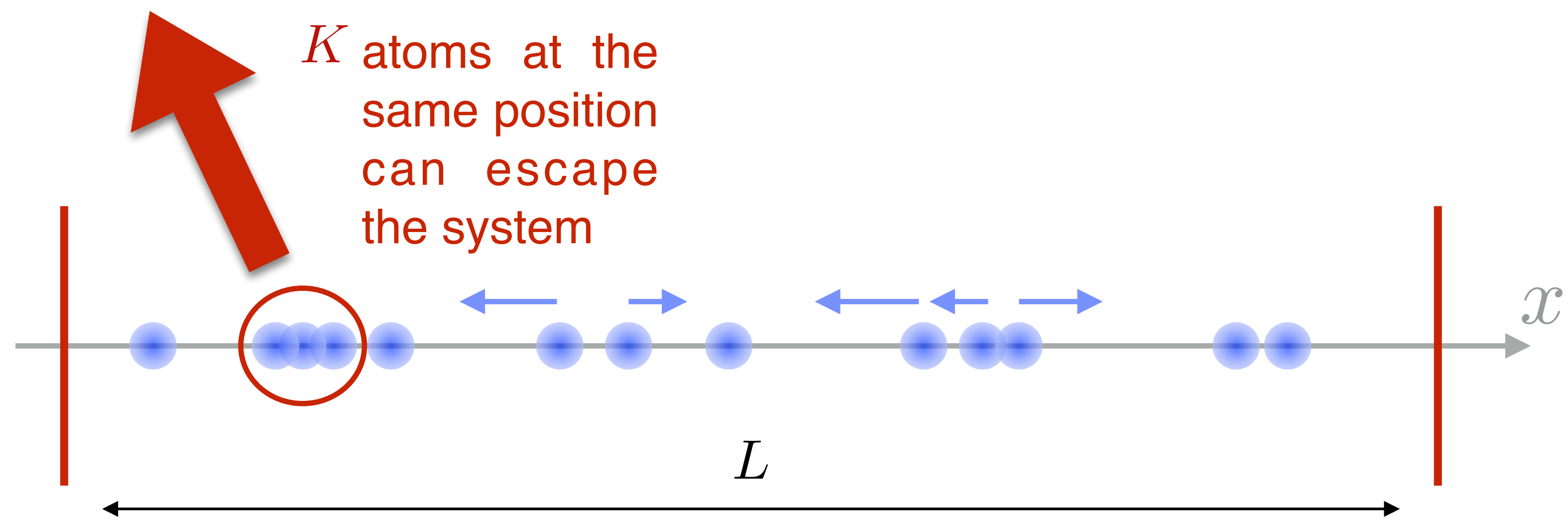
where $[\psi(x), \psi^\dagger(y)] = \delta(x-y)$

kinetic term

contact repulsion

$(\hbar = m = 1)$

K-body losses: when K atoms are at the same position, they can escape the system



This is described by the Lindblad equation for the density matrix:

$$\frac{d\hat{\rho}}{dt} = -i[H, \hat{\rho}] + G \int \left(\psi^K \hat{\rho} \psi^{\dagger K} - \frac{1}{2}\{\psi^{\dagger K}\psi^K, \hat{\rho}\} \right) dx$$

unitary evolution

K-body loss rate
$[G] = [\mathrm{length}^{K-1}/\mathrm{time}]$

removes K atoms

enforces $\mathrm{tr}\ \hat{\rho} = 1$

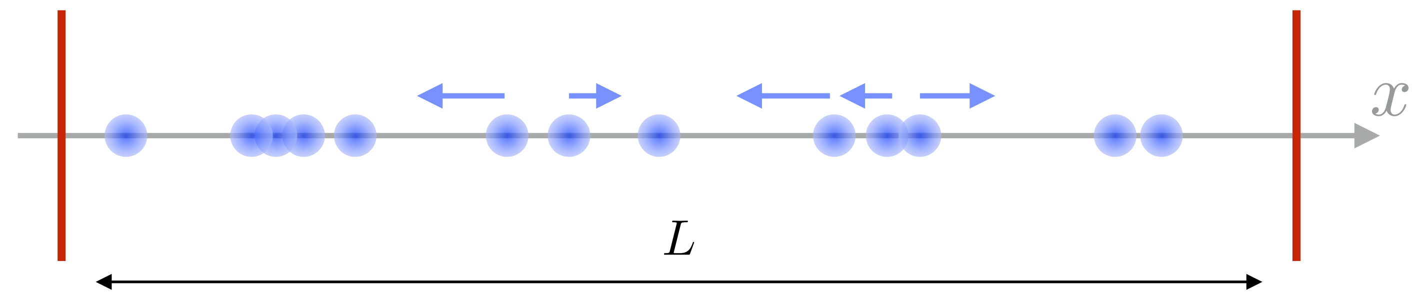
typical loss rate: $\Gamma = G n^{K-1}$

with atom density $n = \langle \psi^\dagger \psi \rangle$

Rapidities in the 1D Bose gas

The eigenstates of the Hamiltonian are superpositions of products of plane waves labeled by the **rapidities**

$$|\{v_1, v_2, v_3, \dots, v_N\}\rangle$$

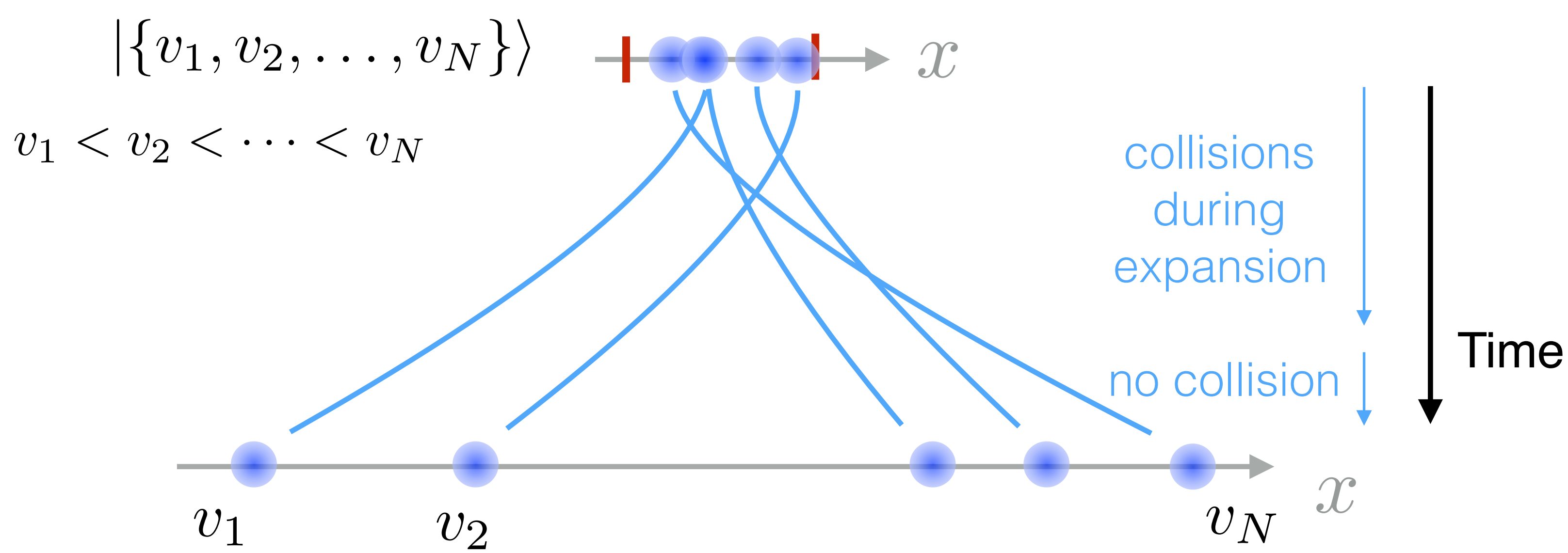


i.e., for each eigenstate the wave function is of the form [Lieb, Liniger, Phys. Rev. 130 (1963)]

$$\langle 0|\, \psi(x_1) \dots \psi(x_N) \,|\{v_1, \dots, v_N\}\rangle = \sum_{\mathrm{perm.}\, \sigma} e^{i\varphi_\sigma(\{v_j\})} e^{iv_{\sigma(1)}x_1 + iv_{\sigma(2)}x_2 + \dots + iv_{\sigma(N)}x_N}$$

where $e^{i\varphi_\sigma(\{v_j\})}$ is a known phase. These eigenstates are called **'Bethe states'**.

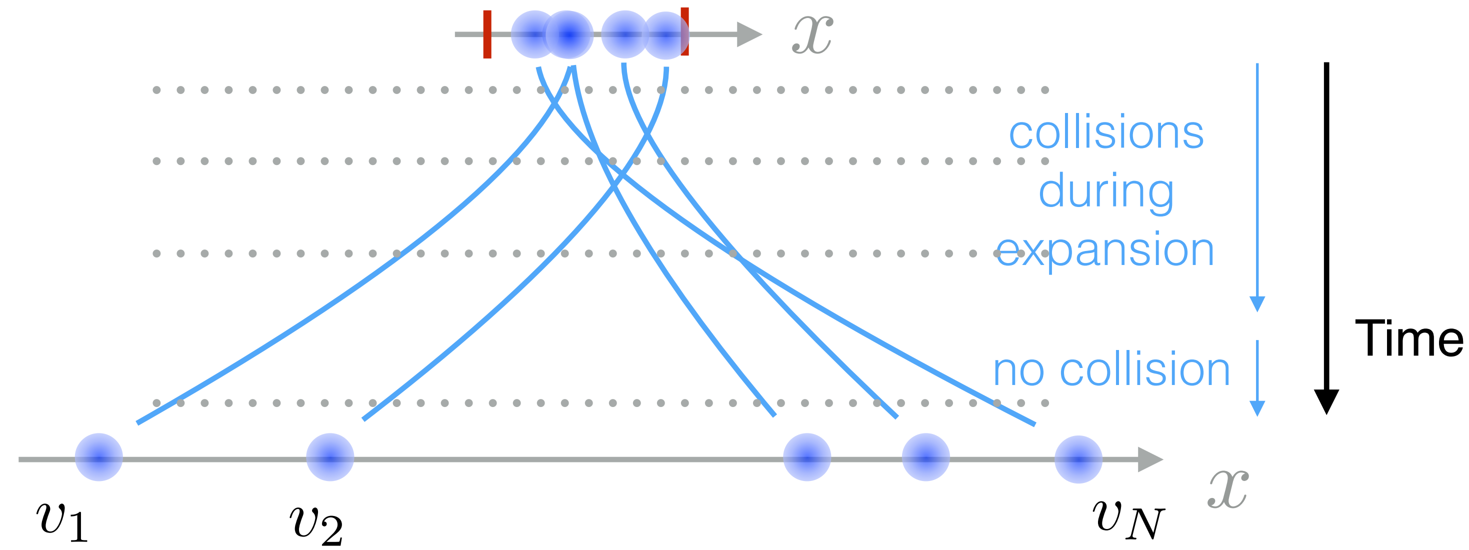
The rapidities can be thought of the **asymptotic velocities, when one lets the system expand in 1D**:



The rapidity distribution is: $\boxed{\rho(v) = \frac{1}{L} \sum_{j=1}^{N} \delta(v - v_j)}$

Rapidities in the 1D Bose gas

The rapidities (i.e. asymptotic velocities) can be measured by letting the gas expand in 1D [Rigol-Muramatsu, PRL 94, 2005; Minguzzi-Gangardt, PRL 94, 2005; Jukic-Pezer-Gasenzer-Buljan, PRA 78, 2008; Bolech-Heidrich-Meisner-Langer-McCulloch-Orso-Rigol, PRL 109, 2012; Bolech-Heidrich-Meisner-Langer-McCulloch-Orso-Rigol, J.o. Physics: Conference Series 414 2013, Campbell-Gangardt-Kheruntsyan, PRL 114, 2015; Caux-Doyon-JD-Konik-Yoshimura, SciPost 6, 2019, …]



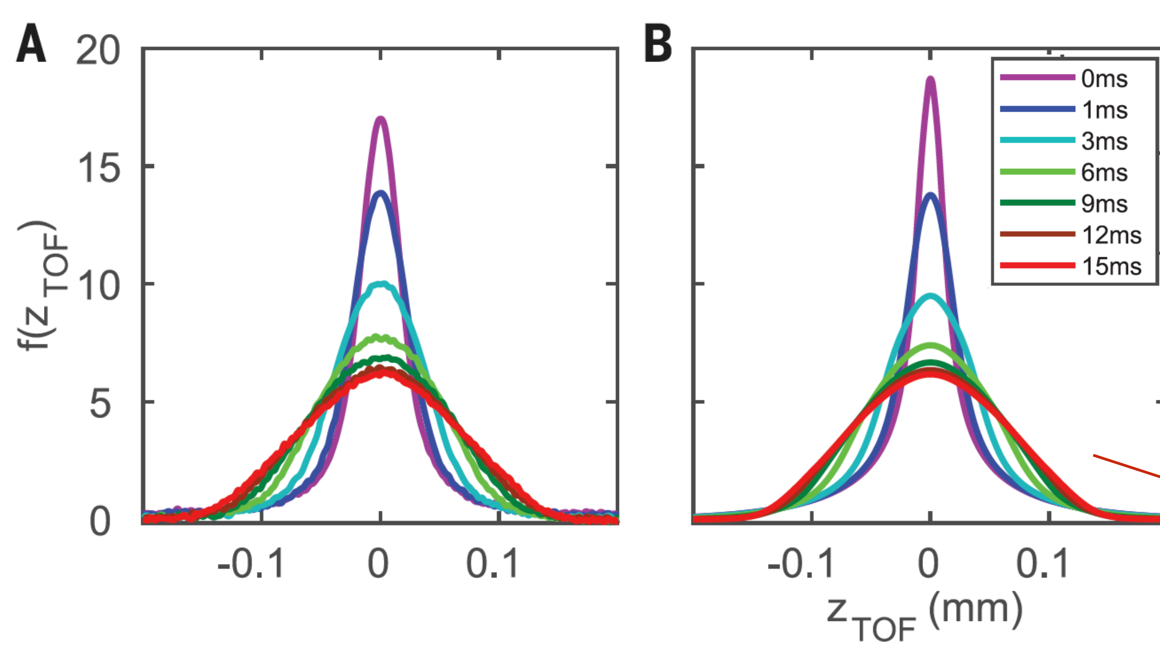
Science

QUANTUM GASES

Observation of dynamical fermionization

Joshua M. Wilson, Neel Malvania, Yuan Le, Yicheng Zhang, Marcos Rigol, David S. Weiss*

from bosonic to fermionic after its axial confinement is removed. The asymptotic momentum distribution after expansion in one dimension is the distribution of rapidities, which are the conserved quantities associated with many-body integrable systems. Our measurements agree well with T-G gas theory. We



experimental measurement of rapidity distribution

$$\rho(v) = \frac{1}{L}\sum_{i=1}^{N}\delta(v - v_i)$$

From Lindblad to the rapidity distribution

The charges Q that would be conserved under unitary evolution (i.e. $[Q, H] = 0$) are no longer conserved under Lindblad evolution. Assuming losses are slow compared to relaxation, i.e.

$$\boxed{\Gamma \ll 1/\tau_{\mathrm{relax}}}$$

after each loss event the density matrix quickly relaxes towards a density matrix such that $[\hat{\rho}, H] = [\hat{\rho}, Q] = 0$ then the expectation value $\langle Q \rangle = \mathrm{tr}[\hat{\rho} Q]$ evolves as follows:

$$\frac{1}{L} \frac{d \langle Q \rangle}{dt} = G \left\langle [\psi^{\dagger K}, Q] \psi^K \right\rangle$$

In particular, we can apply this to the rapidity distribution itself (up to some smoothening)

$$Q_\sigma(v) \left| \{v_1, \dots, v_N\} \right\rangle = \sum_{j=1}^{N} \delta_\sigma(v - v_j) \left| \{v_1, \dots, v_N\} \right\rangle$$

where δ_σ is a smoothed Dirac delta function (e.g. a Gaussian of width σ). This gives the evolution of the rapidity distribution:

$$\boxed{\frac{d}{dt} \rho(v) = -\Gamma \, F[\rho](v) \qquad \text{with} \qquad F[\rho](v) = \lim_{\sigma \to 0^+} n^{1-K} \left\langle \underline{[Q_\sigma(v), \psi^{\dagger K}] \psi^K} \right\rangle}$$

Big challenge: evaluate the functional F

expectation value w.r.t macrostate parameterized by $\rho(v)$

local operator (exp. decaying tails)

The functional $F[\rho]$: results

1. Ideal Bose gas regime: when $e \gg gn, mg^2/\hbar^2$, the atoms behave like non-interacting bosons.

energy per atom

repulsion energy

scattering energy

In that regime, the rapidities are nothing but the (non-interacting) atoms' velocities. Then:

$$\boxed{F[\rho](v) \;=\; K\,K!\,\rho(v)}$$

K atoms lost in each loss event

comes from $\dfrac{\langle \psi^{\dagger K} \psi^K \rangle}{\langle \psi^\dagger \psi \rangle^K} = K!$

(Wick's theorem for bosons)

2. Hard-core regime: when $e \ll mg^2/\hbar^2$, an exact calculation is possible by mapping to free fermions:

1-body loss:

$$\boxed{F[\rho](v) = \rho(v) + \frac{2n}{\pi} \int \frac{\rho(v) - \rho(w)}{(v-w)^2} dw - 2\pi \left(\rho(v)^2 - \left(\frac{1}{\pi} \int \frac{\rho(w) dw}{v - w} \right)^2 \right)}$$

(and $F[\rho] = 0$ for $K \geq 2$ because two or more atoms cannot be at the same position).

This formula shows that:
— in general, the effect of losses is **non-linear in the rapidity distribution**
— it is **non-local in rapidity space**, i.e. $F[\rho](v)$ is a function of $\rho(w)$ for all w

The functional $F[\rho]$: results

3. General case: Markov chain summation over Bethe states [J.-S. Caux and P. Calabrese, Physical Review A74,031605 (2006), J.-S. Caux, P. Calabrese, and N. A. Slavnov, Journalof Statistical Mechanics: Theory and Experiment2007,P01008 (2007), J.-S. Caux, J. Math. Phys. 50, 095214 (2009), J.-S. Caux and R. M. Konik, Physical review letters109,175301 (2012), V. Alba, arXiv preprint arXiv:1507.06994 (2015), …]

$$\boxed{F[\rho](v) = \sum_{|\{v_i\}\rangle} \sum_{|\{w_j\}\rangle} p(\{v_i\})p(\{w_j\}|\{v_i\}) \left(n\, g_K(\{v_i\}) \times \left[\sum_{i=1}^{N} \delta_\sigma(v - v_i) - \sum_{j=1}^{N-K} \delta_\sigma(w - w_i) \right] \right)}$$

sum over pre-loss states

sum over post-loss states

The conditional probability of a post-loss state given the pre-loss state $|\{v_i\}\rangle$ is:

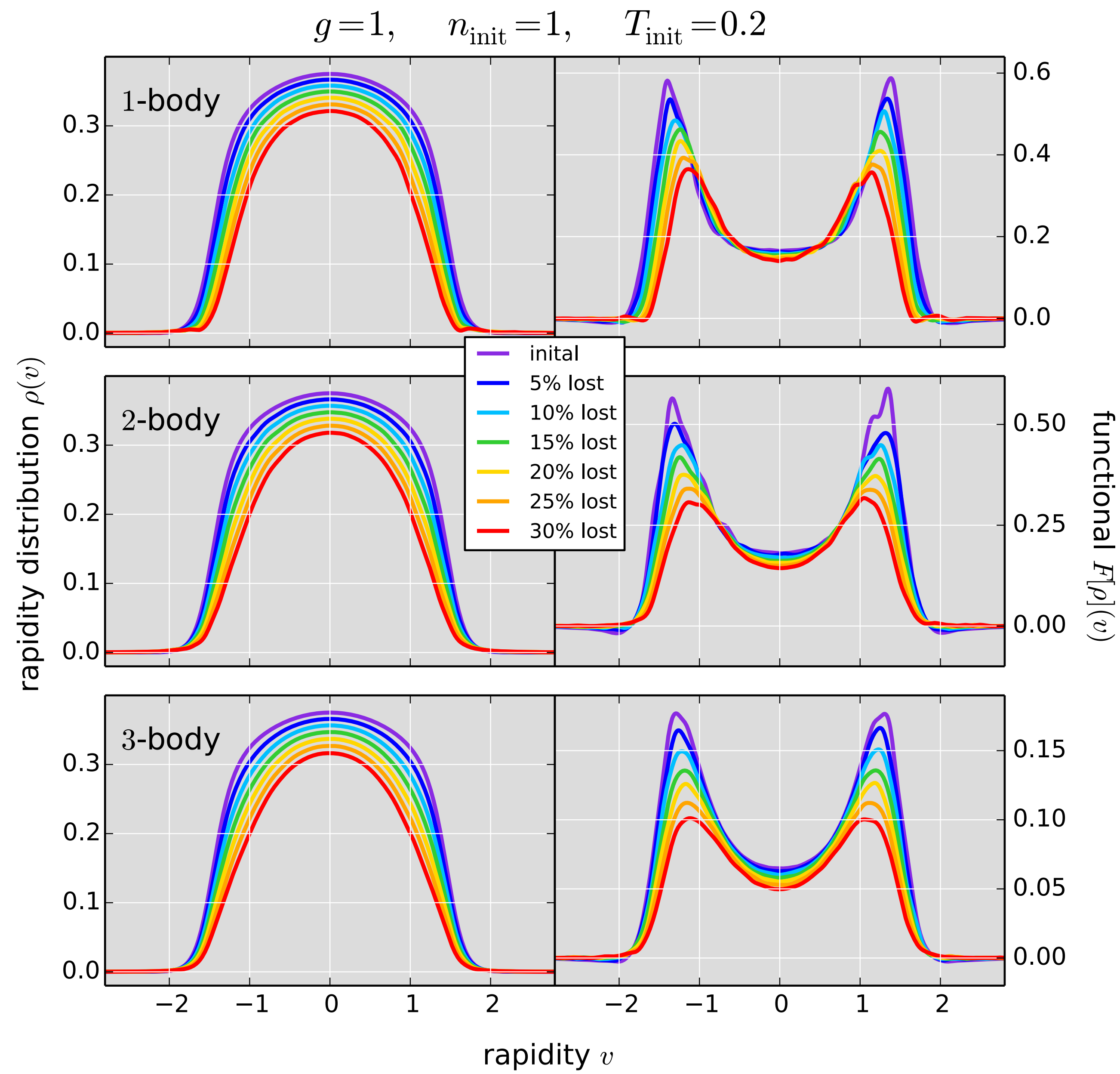
$$p(\{w_j\}|\{v_i\}) = \frac{\left|\langle\{w_j\}|\,\psi^K\,|\{v_j\}\rangle\right|^2}{\langle\{v_i\}|\,\psi^{\dagger K}\psi^K\,|\{v_i\}\rangle}$$

[Piroli-Calabrese, J. Phys. A 48, 2015]

and $p(\{v_i\})$ is taken as the weight in the GGE parameterized by the rapidity distribution $\rho(v)$.

Here $g_K(\{v_i\}) = \dfrac{\langle\{v_i\}|\,\psi^{\dagger K}\psi^K\,|\{v_i\}\rangle}{n^K}$.

[Pozsgay, J. Stat. Mech. P11017 2011]



It works, but it's computationally heavy…

Summary

1. It is important (experimentally relevant) to understand the effect of atom losses on the rapidity distribution in the 1D Bose gas:

$$d\rho(v)/dt = -\Gamma F[\rho](v)$$

Evaluating the functional $F[\rho]$ is **challenging**.

2. In arXiv:2006.03583, we have computed $F[\rho]$ in the ideal Bose gas regime (trivial result) and hard-core regime (non-trivial result), and implemented a numerical method based on sampling of Bethe states, which works for arbitrary parameters but is computationally heavy.

3. Many open questions remain: what about the quasi-condensate regime ([Rauer-Grisins-Mazets-Schweigler-Rohringer—Langen-Schmiedmayer, PRL 116, 2016; Grisins-Rauer-Langen-Schmiedmayer-Mazets, PRA 93, 2016; Bouchoule-Schemmer-Henkel, SciPost 5, 2018; Johnson, Szigeti, Schemmer, Bouchoule, PRA 96, 2017; Schemmer-Bouchoule, PRL121, 2018])?
Can one find good approximations to evaluate $F[\rho]$ more efficiently? [Caux-Doyon-JD-Konik-Yoshimura, SciPost 6, 2019; Mallayya-Rigol-de Roeck, PRX 9, 2019; Friedman-Gopalakrishnan-Vasseur, PRB 101, 2020; Bastianello-De Nardis-De Luca, arXiv:2003.01702; Durnin-Bhaseen-Doyon, arXiv:2004.11030; Lopez-Piqueres-Ware-Gopalakrishnan-Vasseur, arXiv:2005.13546]
Analytical progress needed on form factors and their summation…

Thank you!

Calculating $F[\rho]$: remarks

A. Using typicality of eigenstates ('Generalized ETH'), one might replace the double sum by a single sum over post-loss states. For a given 'representative eigenstate' $|\{v_i\}\rangle$:

$$F[\rho](v) \propto \underbrace{\sum_{|\{w_j\}\rangle}}_{\text{sum over post-loss states}} \left|\langle\{w_j\}|\,\psi^K\,|\{v_i\}\rangle\right|^2 \left[\sum_{i=1}^{N} \delta_\sigma(v-v_i) - \sum_{j=1}^{N-K} \delta_\sigma(w-w_j)\right]$$

However, numerically, this does not work well: we find that, to evaluate $F[\rho]$ numerically, it is better to sum also over pre-loss states. But, analytically, this would be the way to go.

B. Analytical progress on summation of form factors crucially needed!

C. What about the quasi-condensate regime, $\underbrace{e}_{\text{energy per atom}} \simeq \underbrace{gn}_{\text{repulsion energy}} \gg \underbrace{mg^2/\hbar^2}_{\text{scattering energy}}$? How to calculate $F[\rho]$ there? Connection to theoretical and experimental works in that regime? [Rauer-Grisins-Mazets-Schweigler-Rohringer—Langen-Schmiedmayer, PRL 116, 2016; Grisins-Rauer-Langen-Schmiedmayer-Mazets, PRA 93, 2016; Bouchoule-Schemmer-Henkel, SciPost 5, 2018; Johnson, Szigeti, Schemmer, Bouchoule, PRA 96, 2017; Schemmer-Bouchoule, PRL121, 2018]

D. In recent related works, various approximations have been proposed to tackle effects of integrability breaking. Is it possible to use one of these to describe atom losses? [Caux-Doyon-JD-Konik-Yoshimura, SciPost 6, 2019; Mallayya-Rigol-de Roeck, PRX 9, 2019; Friedman-Gopalakrishnan-Vasseur, PRB 101, 2020; Bastianello-De Nardis-De Luca, arXiv:2003.01702; Durnin-Bhaseen-Doyon, arXiv:2004.11030; Lopez-Piqueres-Ware-Gopalakrishnan-Vasseur, arXiv:2005.13546]

This is not obvious. In particular, for losses, the sum over particle-hole excitations does not truncate.

E. In the hard-core limit, the equation $d\rho(v)/dt = -\Gamma F[\rho](v)$ turns out to be exactly solvable. A special role is played by the (non-hermitian) charges:

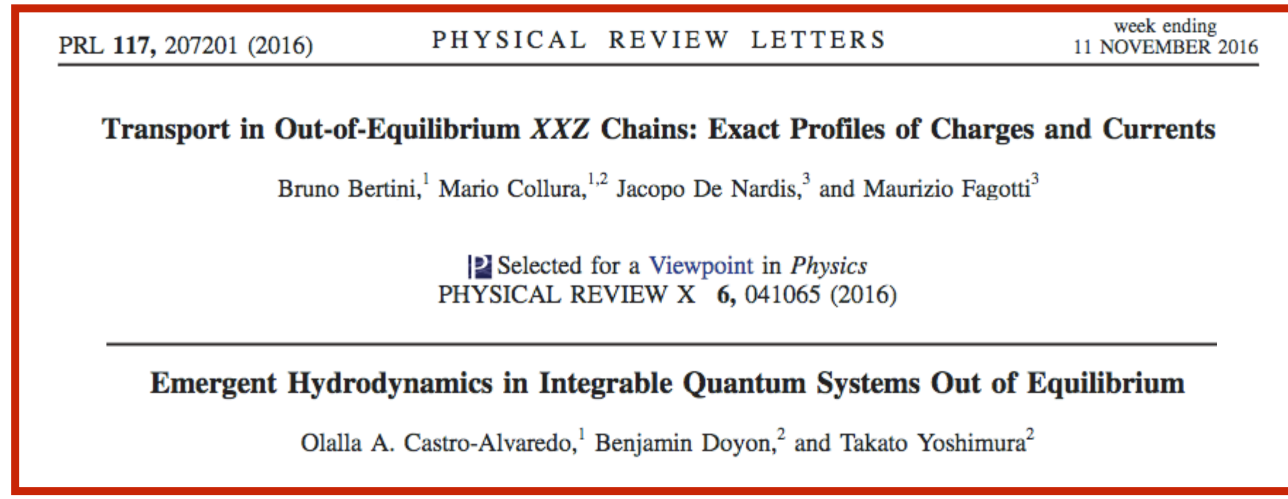
$$Q(z;\{v_i\}) = \sum_{i=1}^{N} \frac{1}{z-v_i}, \quad z \in \mathbb{C}.$$

Why is that so?

Inhomogeneous profiles: GHD with losses

For comparison with experimental data, it is also important to consider inhomogeneous settings. Thanks to the 2016 breakthrough of GHD, nowadays this is straightforward:

$$\partial_t \rho + \partial_x [v^{\mathrm{eff}} \rho] - (\partial_x V)\partial_v \rho = -G n^{K-1} F[\rho]$$


PRL 117, 207201 (2016) PHYSICAL REVIEW LETTERS week ending 11 NOVEMBER 2016

Transport in Out-of-Equilibrium *XXZ* Chains: Exact Profiles of Charges and Currents

Bruno Bertini,[1] Mario Collura,[1,2] Jacopo De Nardis,[3] and Maurizio Fagotti[3]

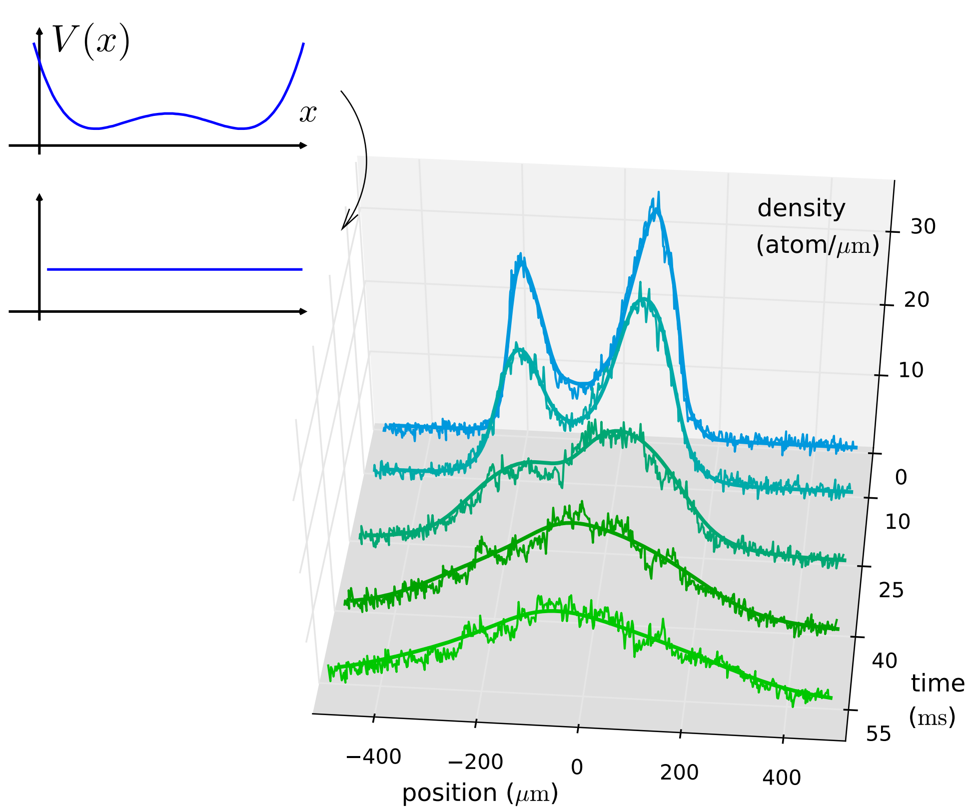
Selected for a Viewpoint in *Physics*

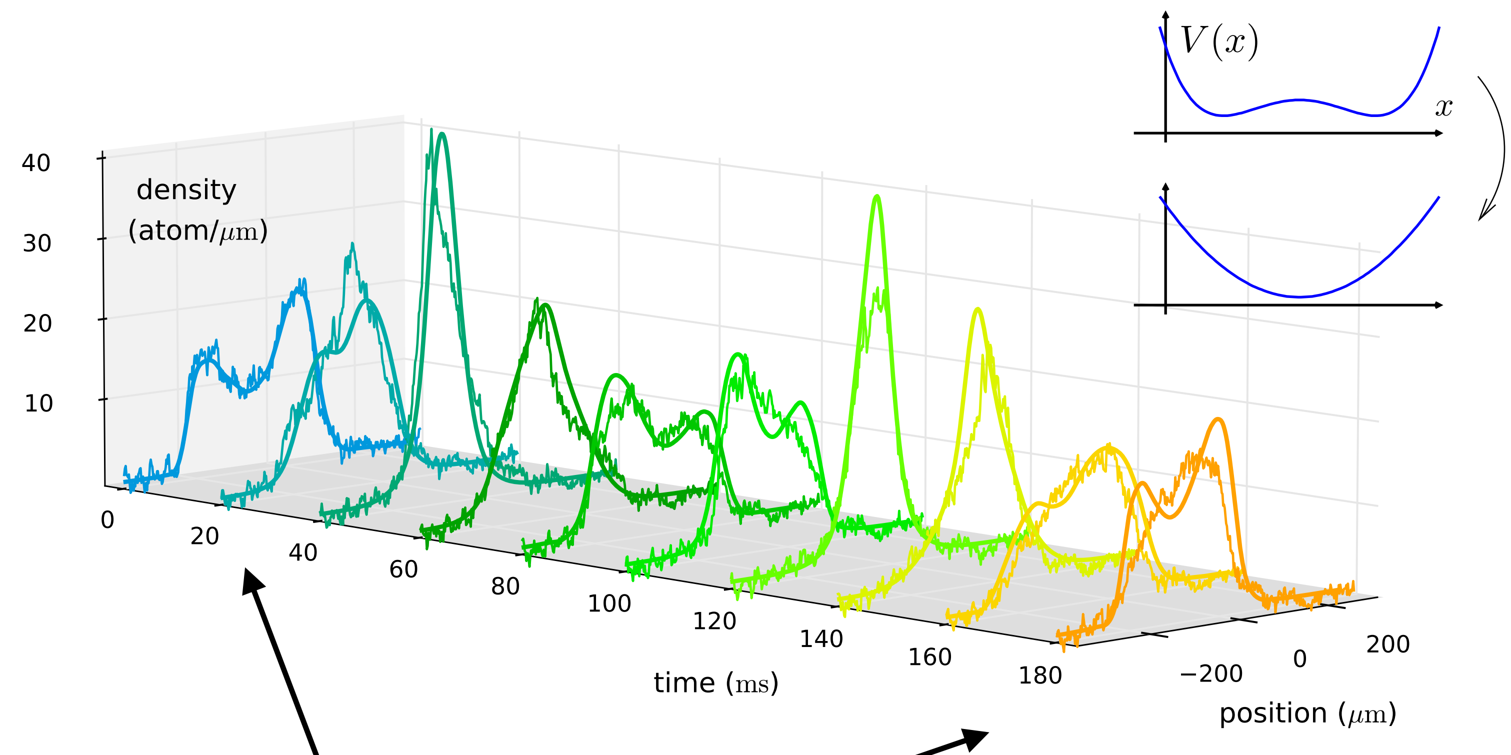
PHYSICAL REVIEW X 6, 041065 (2016)

Emergent Hydrodynamics in Integrable Quantum Systems Out of Equilibrium

Olalla A. Castro-Alvaredo,[1] Benjamin Doyon,[2] and Takato Yoshimura[2]

(Also [Caux-Doyon-JD-Konik-Yoshimura, SciPost 6, 2019; Friedman-Gopalakrishnan-Vasseur, PRB 101, 2020; Bastianello-De Nardis-De Luca, arXiv:2003.01702; Durnin-Bhaseen-Doyon, arXiv:2004.11030; Lopez-Piqueres-Ware-Gopalakrishnan-Vasseur, arXiv:2005.13546] for similar equations for other problems -not losses-)



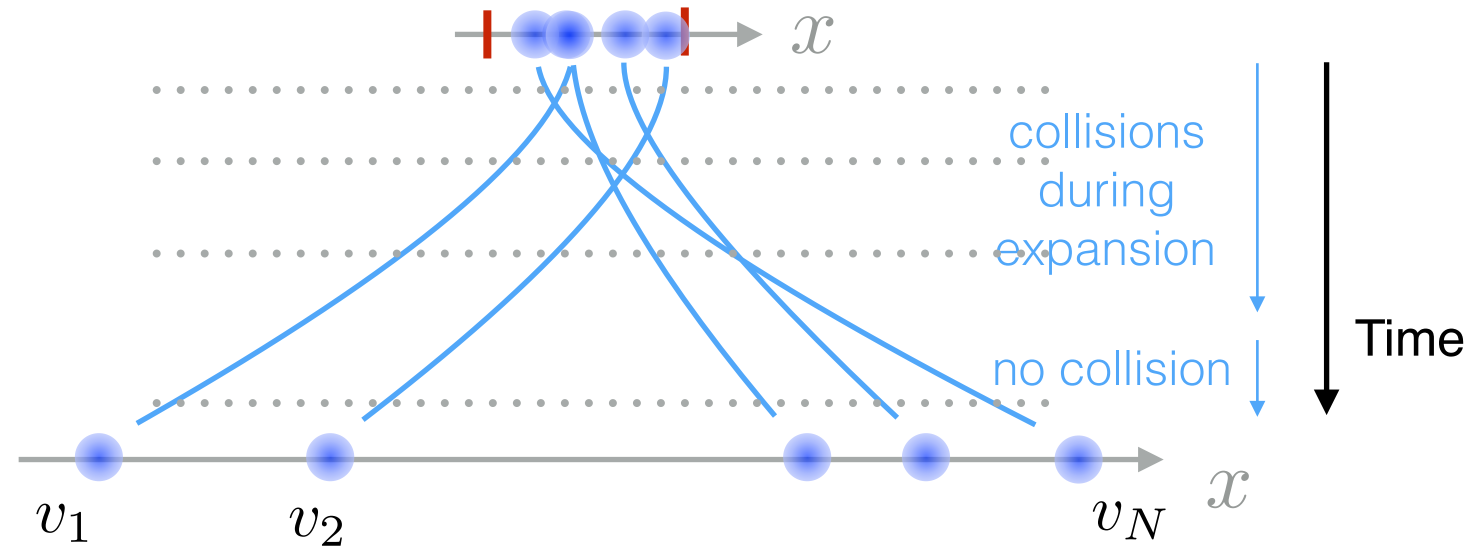


number of atoms N has dropped by 15%, compared to t=0. Losses cannot be neglected

another good reason to calculate $F[\rho]$!

Rapidities in the 1D Bose gas

The rapidities (i.e. asymptotic velocities) can be measured by letting the gas expand in 1D [Rigol-Muramatsu, PRL 94, 2005; Minguzzi-Gangardt, PRL 94, 2005; Jukic-Pezer-Gasenzer-Buljan, PRA 78, 2008; Bolech-Heidrich-Meisner-Langer-McCulloch-Orso-Rigol, PRL 109, 2012; Bolech-Heidrich-Meisner-Langer-McCulloch-Orso-Rigol, J.o. Physics: Conference Series 414 2013, Campbell-Gangardt-Kheruntsyan, PRL 114, 2015; Caux-Doyon-JD-Konik-Yoshimura, SciPost 6, 2019, …]

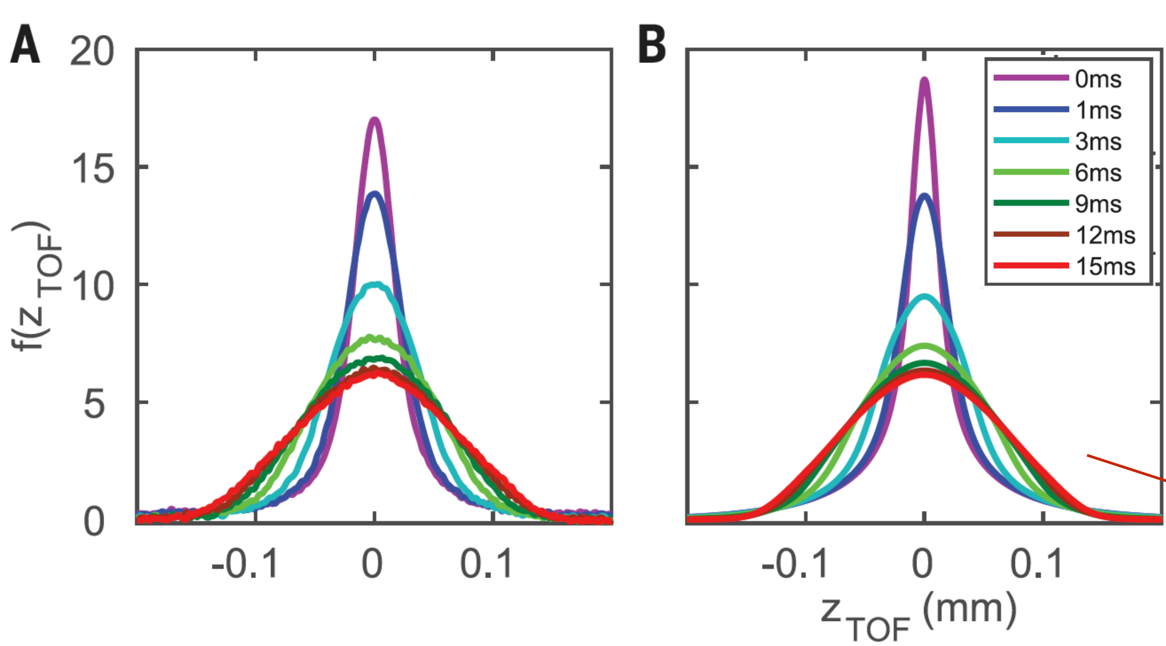


Science
QUANTUM GASES

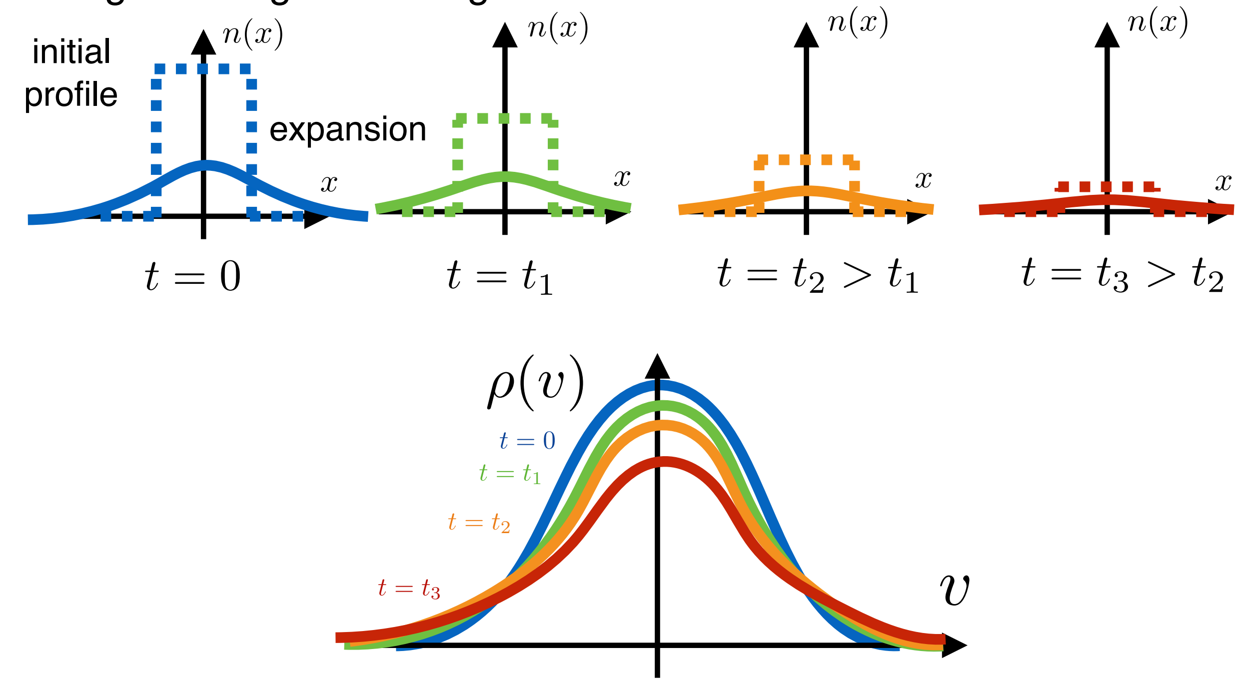
Observation of dynamical fermionization

Joshua M. Wilson, Neel Malvania, Yuan Le, Yicheng Zhang, Marcos Rigol, David S. Weiss*

from bosonic to fermionic after its axial confinement is removed. The asymptotic momentum distribution after expansion in one dimension is the distribution of rapidities, which are the conserved quantities associated with many-body integrable systems. Our measurements agree well with T-G gas theory. We



To measure the effect of losses on the rapidity distribution, one imagine doing something like this:



experimental measurement of rapidity distribution

$$\rho(v) = \frac{1}{L}\sum_{i=1}^{N}\delta(v - v_i)$$