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Trieste, 12 June 2020

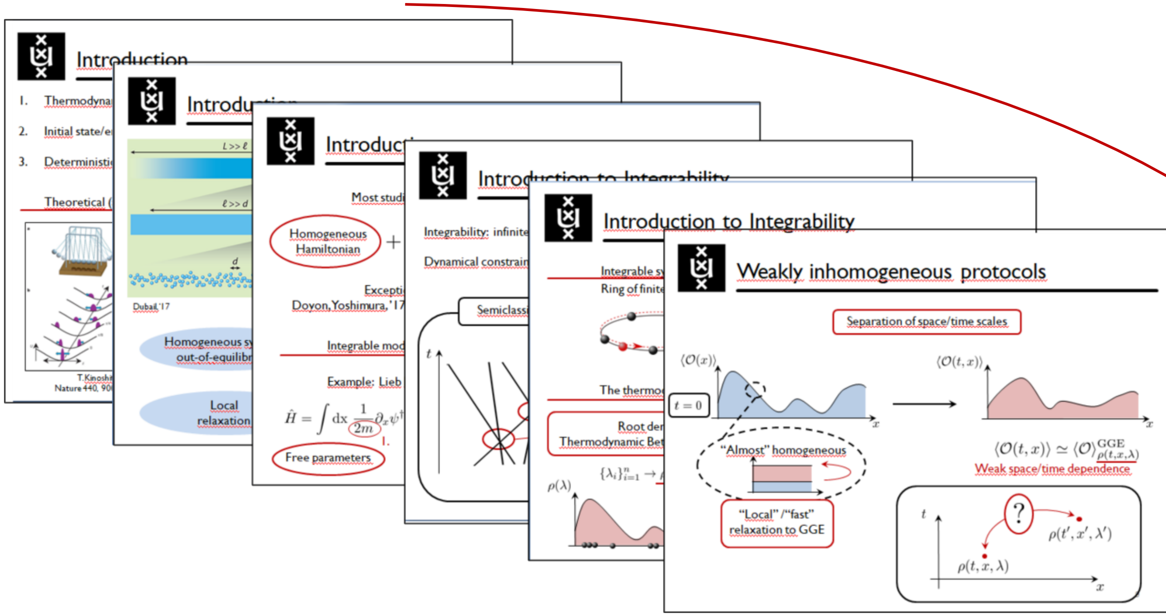
# Hydrodynamics of inhomogeneous locally integrable models

Based on:

AB, A. De Luca, PRL 122 (24), 240606 (2019)

AB, V. Alba, J.-S. Caux, PRL 123 (13) 130602 (2019)

# Introduction




Skip ≈ 20 min worth talk

Pros of a thematic workshop:
<u>no need of a general introduction</u>

From previous talks: traps (but not only)

Topics:

1 Generalized Hydrodynamics (GHD) can be extended to describe inhomogeneous smooth Hamiltonians

2 Inhomogeneities that look smooth sometimes are not: beyond GHD effects and bound-state recombination

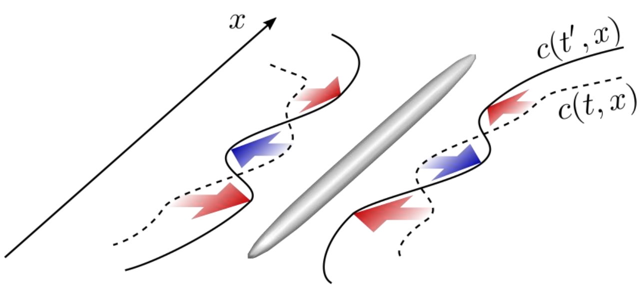
# Inhomogeneous interactions

A concrete example: the Lieb-Liniger model

Integrable for arbitrary (homogeneous) couplings

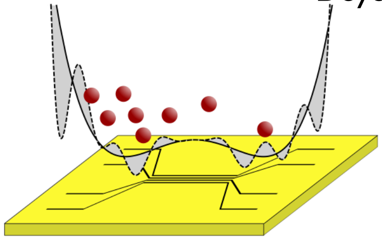
$$\hat{H} = \int \mathrm{d}x\, \partial_x\psi^\dagger\partial_x\psi + c\,\psi^\dagger\psi^\dagger\psi\psi - \mu\,\psi^\dagger\psi$$

Inhomogenous transverse trap




Inhomogenous longitudinal trap

Doyon, Yoshimura '17



Experimentally confirmed!

M. Schemmer, I. Bouchoule, B. Doyon, J. Dubail '19

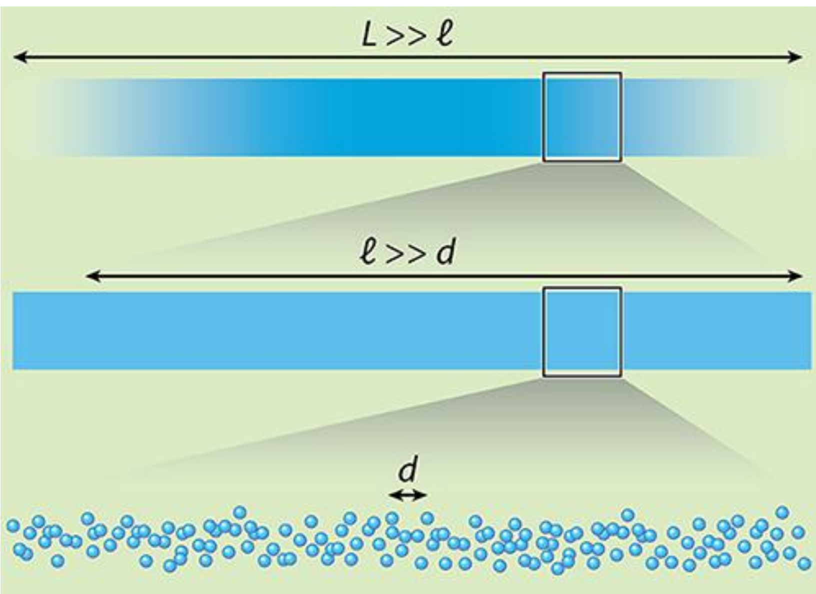
Solution with the Generalized Hydrodynamics (GHD)

# Inhomogeneous interactions

## GHD in a nutshell

Castro Alvaredo, Doyon, Yoshimura, '16

Bertini, Collura, De Nardis, Fagotti, '16




Dubail, '17

Smooth inhomogeneity

Local relaxation to the GGE of the local integrable model + Gluing together the smoothly varying GGEs

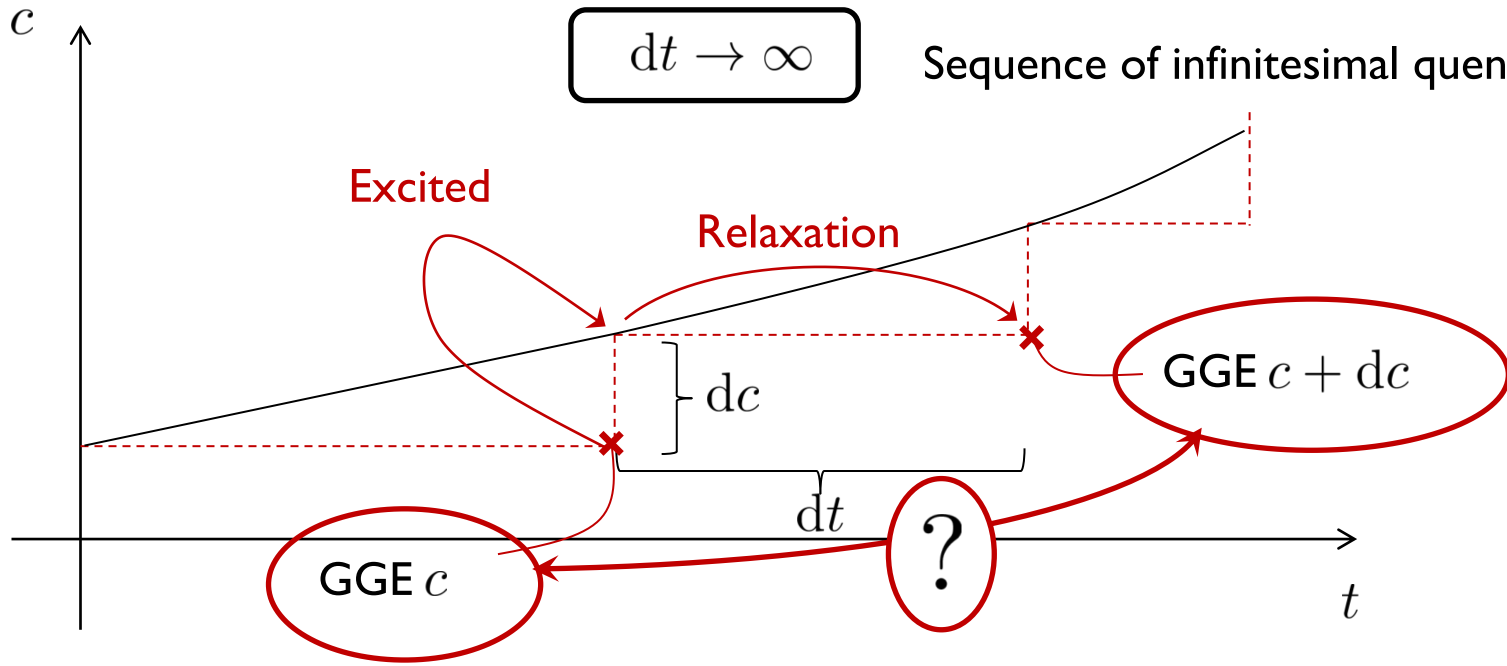
**GHD**

$$\partial_t \vartheta + v^{\rm eff}\partial_x \vartheta + F^{\rm eff}\partial_\lambda \vartheta = 0$$

# Inhomogeneous interactions

First step: slow homogeneous interaction changes

Sequence of infinitesimal quenches




Charges of the "post quench" model

$$\langle \mathbf{q}_j(c + \mathrm{d}c)\rangle_{\mathrm{GGE}\, c} = \langle \mathbf{q}_j(c + \mathrm{d}c)\rangle_{\mathrm{GGE}\, c+\mathrm{d}c}$$

Pre-quench state

Post-quench state

We need

$$\langle \partial_c \mathbf{q}_j(c)\rangle_{\mathrm{GGE}\, c}$$

Exactly computable

Hellmann-Feynman Theorem

# Inhomogeneous interactions

… + spatial inhomogeneities =…

$c \to \alpha$ "generic" coupling

**GHD with arbitrary inhomogeneities**

AB, V. Alba, J.-S. Caux, PRL 123 (13) 130602 (2019)

$$\partial_t \vartheta + v^{\text{eff}} \partial_x \vartheta + \frac{\partial_t \alpha f^{\text{dr}} + \partial_x \alpha \Lambda^{\text{dr}}}{(\partial_\lambda p)^{\text{dr}}} \partial_\lambda \vartheta = 0$$

"single particle" effect

"collective" effect

$$\Theta(\lambda) = -i \log S(\lambda)$$

$$f(\lambda) = -\partial_\alpha p(\lambda) + \int \frac{\mathrm{d}\mu}{2\pi} \partial_\alpha \Theta(\lambda - \mu)(\partial_\mu p)^{\text{dr}} \vartheta(\mu)$$

$$\Lambda(\lambda) = -\partial_\alpha \epsilon(\lambda) + \int \frac{\mathrm{d}\mu}{2\pi} \partial_\alpha \Theta(\lambda - \mu)(\partial_\mu \epsilon)^{\text{dr}} \vartheta(\mu)$$

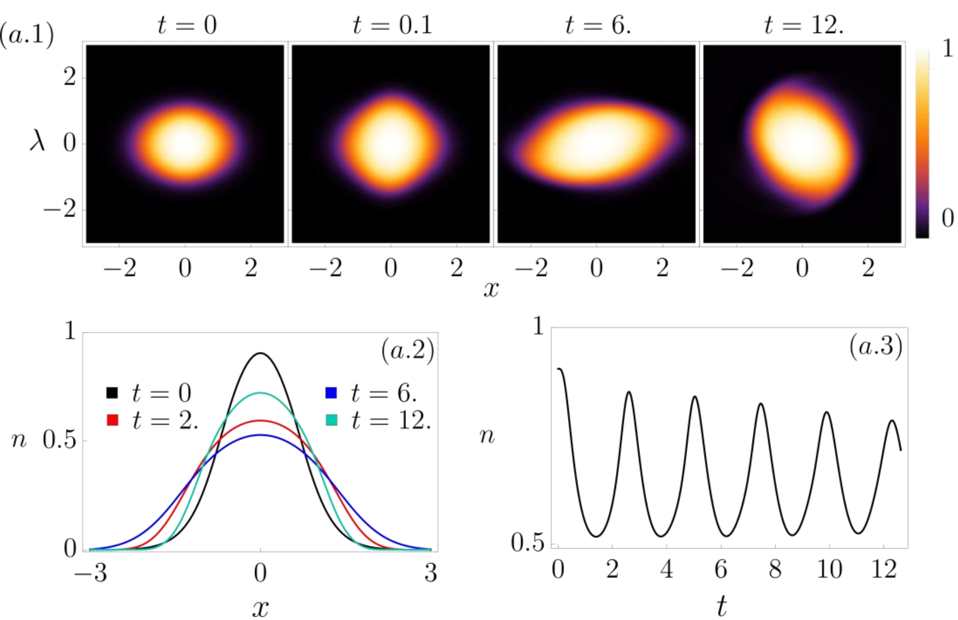
# Inhomogeneous interactions

## Applications

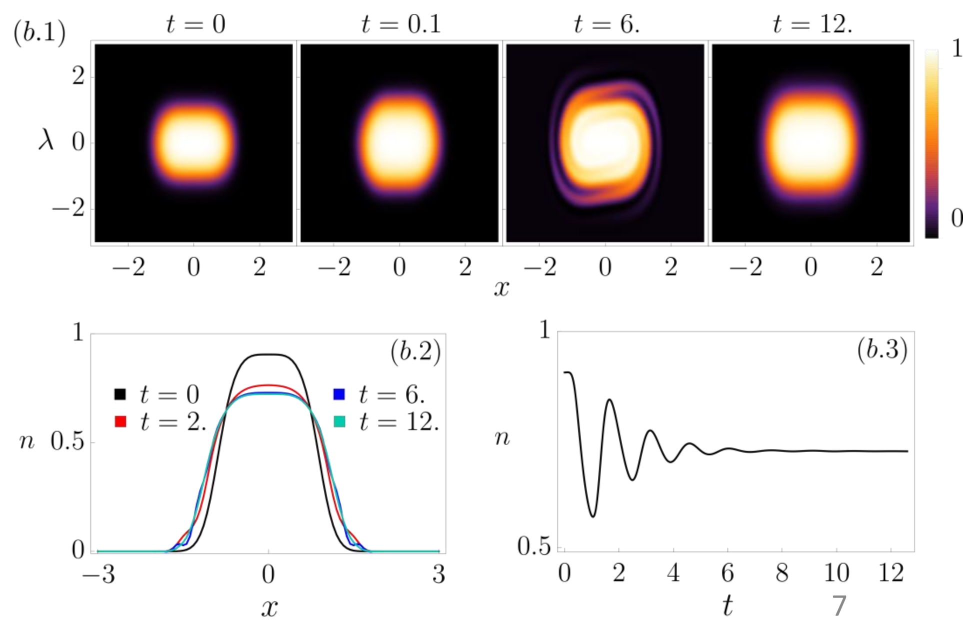
**Slow interaction changes in trapped Lieb-Liniger**

$$\hat{H} = \int \mathrm{dx}\ \partial_x \psi^\dagger \partial_x \psi + c(t/L)\, \psi^\dagger \psi^\dagger \psi \psi + V(x/L)\, \psi^\dagger \psi$$

### Harmonic trap




### Anharmonic trap






AB, V. Alba, J.-S. Caux, '19

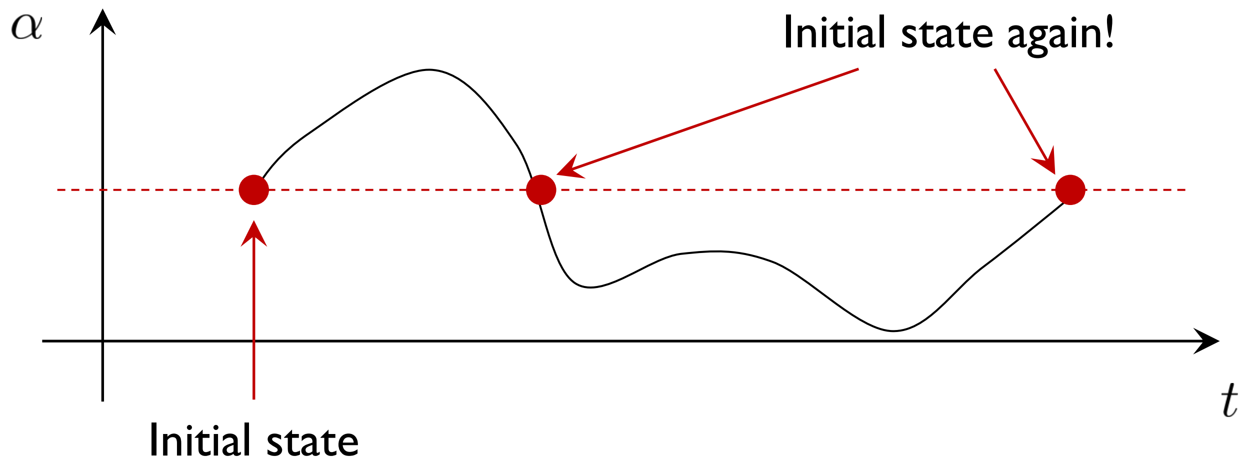
# GHD, reversibility and beyond

Homogeneous system, slow coupling changes

No explicit time dependence!

$$\partial_t \vartheta + \frac{\partial_t \alpha f^{\rm dr}}{(\partial_\lambda p)^{\rm dr}} \partial_\lambda \vartheta = 0 \quad \xrightarrow[t \to \alpha]{\text{Change variable}} \quad \partial_\alpha \vartheta + \frac{f^{\rm dr}}{(\partial_\lambda p)^{\rm dr}} \partial_\lambda \vartheta = 0$$

Reversibility under slow coupling changes



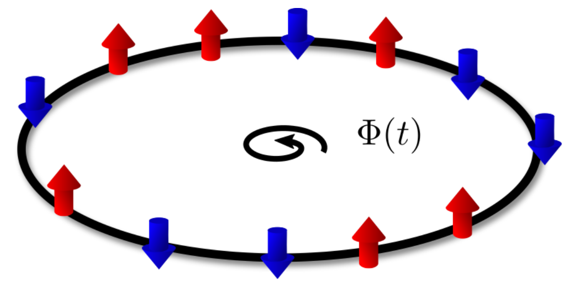

**… always?**

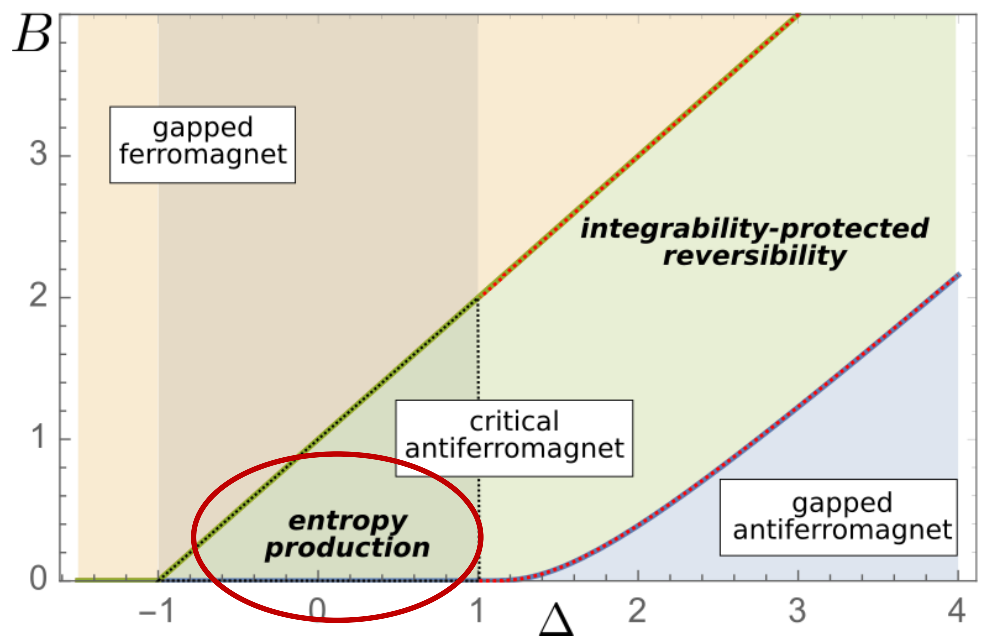
# GHD, reversibility and beyond

## Homogeneous magnetic flux in XXZ

AB, A. De Luca, PRL 122 (24), 240606 (2019)

$$\hat{H}(\Phi) = \sum_{j=1}^{N} \frac{1}{2}\left(e^{\imath\Phi}\hat{s}_j^+\hat{s}_{j+1}^- + \text{h.c.}\right) + \Delta\hat{s}_j^z\hat{s}_{j+1}^z - B\hat{s}_j^z$$




The XXZ chain is not "smooth" under flux changes for $|\Delta| < 1$

# GHD, reversibility and beyond

we can still write the GHD…

$$\partial_\Phi \vartheta_j + \frac{m_j^{\mathrm{dr}}(\lambda)}{(\partial_\lambda p_j(\lambda))^{\mathrm{dr}}} \partial_\lambda \vartheta_j = 0$$

| $\lvert\Delta\rvert \geq 1$ | $\lvert\Delta\rvert < 1$ |
| --- | --- |
| Infinitely many strings (bound states) | Number of strings $\Delta$-dependent |
| Brillouin zone $\lambda \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ | No Brillouin zone $\lambda \in \mathbb{R}$ |

Where does the entropy production come from?

Boundary conditions in the rapidity space

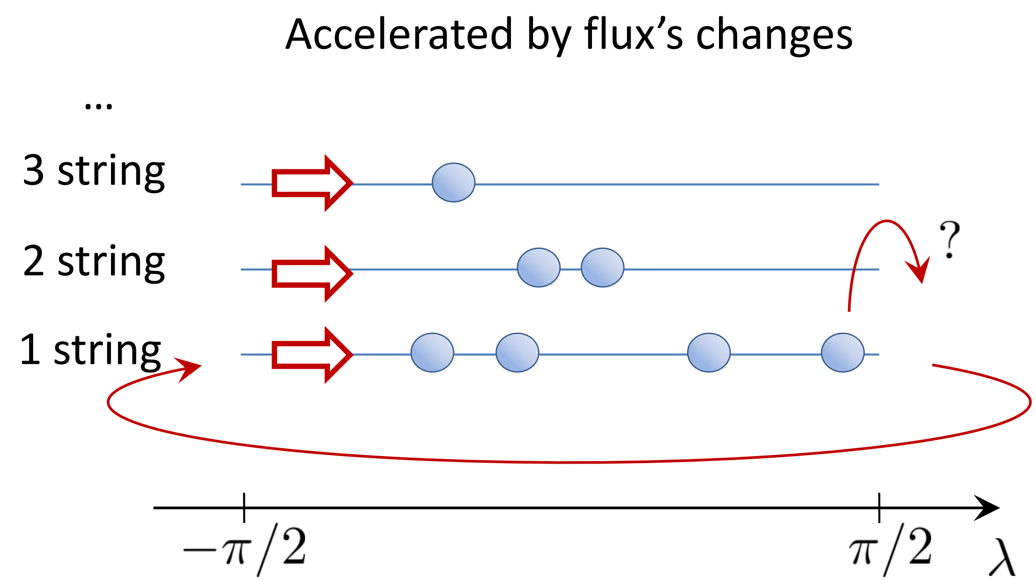
# GHD, reversibility and beyond

$|\Delta| \geq 1$

Infinitely many strings
(bound states)

Brillouin zone

$\lambda \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$



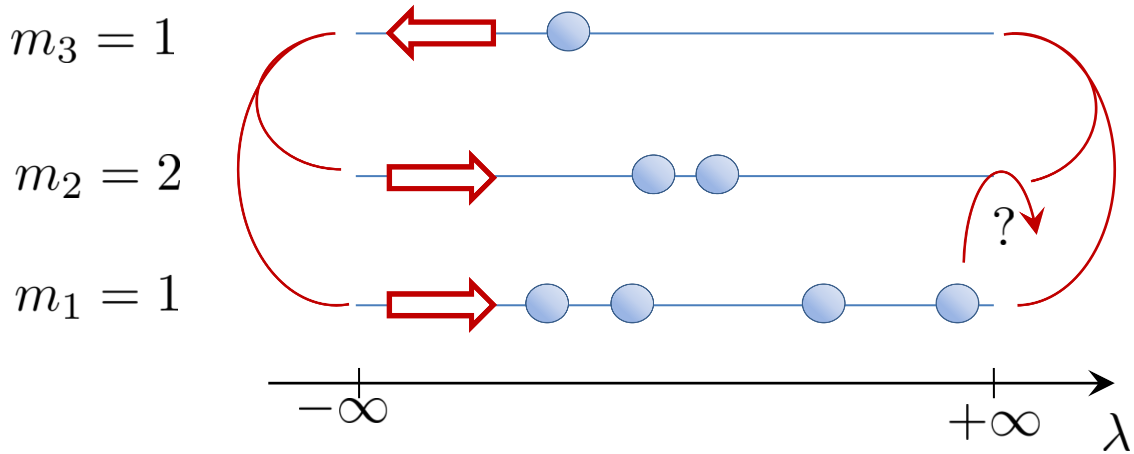

Time-reversible GHD equation + Time-reversible boundary conditions = Time-reversible dynamics

# GHD, reversibility and beyond

$\Delta = \frac{1}{2}$

3 strings
No Brillouin zone
$\lambda \in \mathbb{R}$

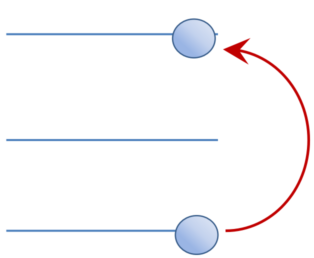
Accelerated by flux's changes



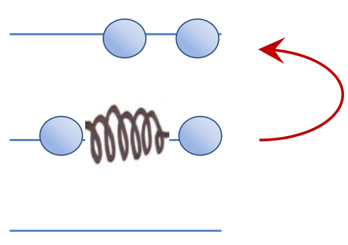

$$\lim_{\lambda\to\pm\infty} q_j(\lambda) = m_j \lim_{\lambda\to\pm\infty} q_1(\lambda)$$

Strings <u>indistinguishable</u> at the boundaries

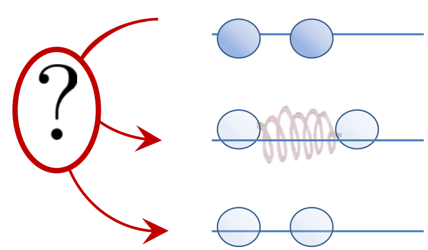
"Trivial" process



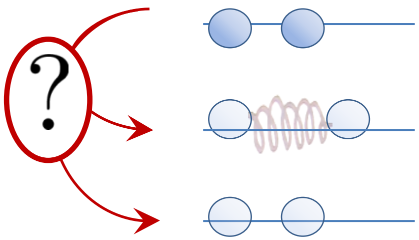
"Breaking" of a bound state



Possible "formation" of a bound state



# GHD, reversibility and beyond




GGEs = Charges conservation + Entropy maximization

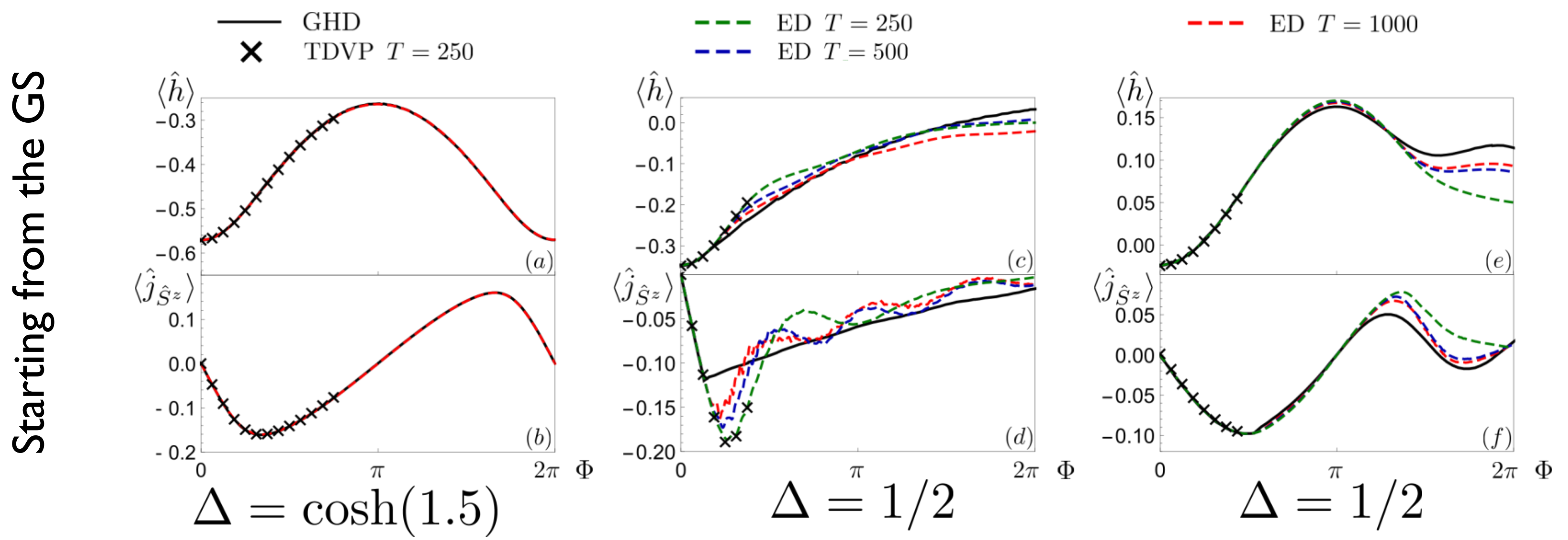
Entropy rate maximization fixes recombination rate

$$\mathcal{S} = \sum_j \int \frac{\mathrm{d}\lambda}{2\pi} (\partial_\lambda p_j)^{\mathrm{dr}} \eta(\vartheta_j(\lambda))$$

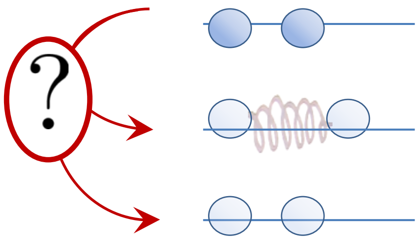
$$\eta(x) = -x \log x - (1-x)\log(1-x)$$

$\lambda = -\infty$ $\quad$ $\lambda = +\infty$

$$\partial_\Phi \mathcal{S} = \partial_\Phi \mathcal{S}^- + \partial_\Phi \mathcal{S}^+$$



# GHD, reversibility and beyond




GGEs $=$ Charges conservation $+$ Entropy maximization

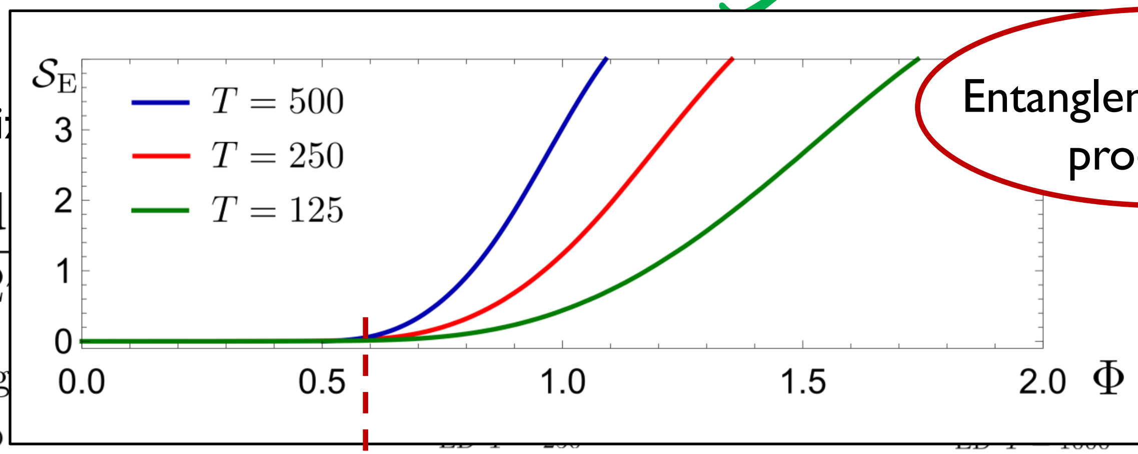
Entropy rate maximi

$\mathcal{S} = \sum_j \int \frac{\mathrm{d}}{2}$

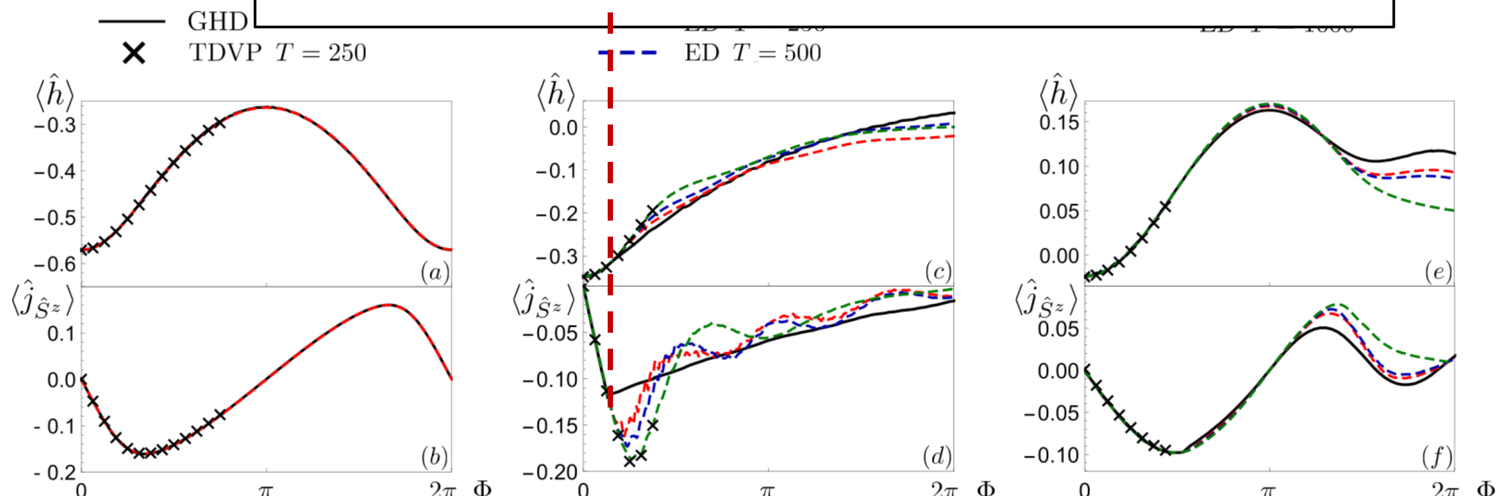
$\eta(x) = -x \log$

$= +\infty$

$\partial_\Phi \mathcal{S}^+$




Entanglement entropy production

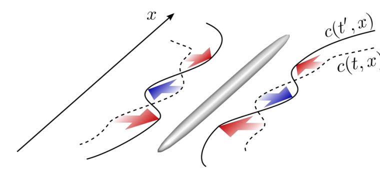



Starting from the GS

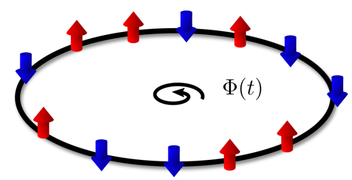
$\Delta = \cosh(1.5)$ $\Delta = 1/2$ $\Delta = 1/2$

# Conclusions and outlook

★ GHD can describe (locally) integrable systems with (smooth) inhomogeneous couplings




★ Sometimes smooth inhomogeneities are not smooth

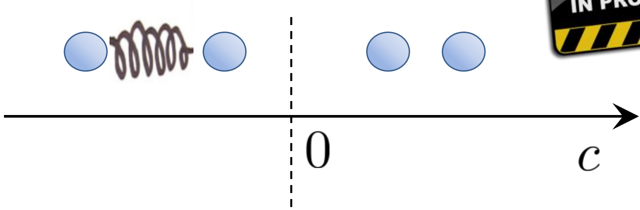



What's next?

General framework to handle smooth inhomogeneities of "non-smooth" integrable models ?

First step

Interaction changes from repulsive to attractive phase in Lieb-Liniger

# THANK YOU!

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