Rule 54 reversible cellular automaton

An exactly solvable microscopic model of interacting dynamics

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KK, M. Medenjak, T. Prosen, M. Vanicat, Commun. Math. Phys. **371**, 651–688 (2019)
KK, M. Vanicat, J. P. Garrahan, T. Prosen, arXiv:1912.09742 (2019)
KK, T. Prosen, arXiv:2004.01671 (2020)

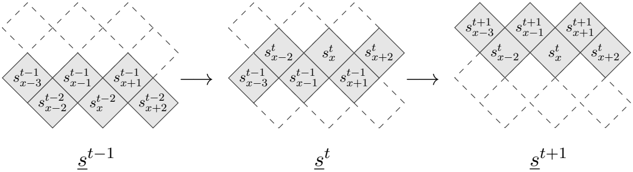
Motivation

- A model in which "everything" can be done exactly and very explicitly.
- A classical cellular automaton, introduced in 1993

  A. Bobenko, M. Bordemann, C. Gunn, U. Pinkall, Commun. Math. Phys. **158**, 127–134 (1993)
- Became popular in recent years:
  - Classical:
    T. Prosen, C. Mejía-Monasterio, J. Phys. A: Math. Theor. **49**, 185003 (2016)
    T. Prosen, B. Buča, J. Phys. A: Math. Theor. **50**, 395002 (2017)
    A. Inoue, S. Takesue, J. Phys. A: Math. Theor. **51**, 425001 (2018)
    B. Buča, J. P. Garrahan, T. Prosen, M. Vanicat, Phys. Rev. E **100**, 020103 (2019)
    KK, M. Medenjak, T. Prosen, M. Vanicat, Commun. Math. Phys. **371**, 651–688 (2019)
    KK, M. Vanicat, J. P. Garrahan, T. Prosen, arXiv:1912.09742 (2019)
    KK, T. Prosen, arXiv:2004.01671 (2020)
  - Quantum:
    S. Gopalakrishnan, Phys. Rev. B **98**, 060302 (2018)
    S. Gopalakrishnan, D. A. Huse, V. Khemani, R. Vasseur, Phys. Rev. B **98**, 220303 (2018)
    A. J. Friedman, S. Gopalakrishnan, R. Vasseur, Phys. Rev. Lett. **123**, 170603 (2019)
    V. Alba, J. Dubail, M. Medenjak, Phys. Rev. Lett. **122**, 250603 (2019)
    V. Alba, arXiv:2006.02788 (2020)

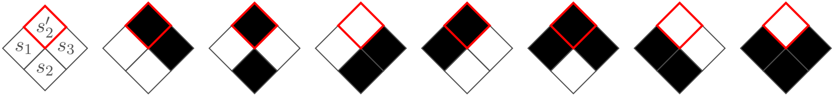
Definition of dynamics

1-dim lattice of binary variables, with staggered time evolution:

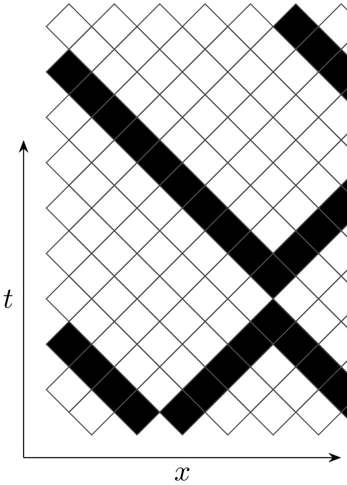


Local time evolution maps:

$$s'_2 = \chi(s_1, s_2, s_3) = s_1 + s_2 + s_3 + s_1 s_3 \pmod 2$$



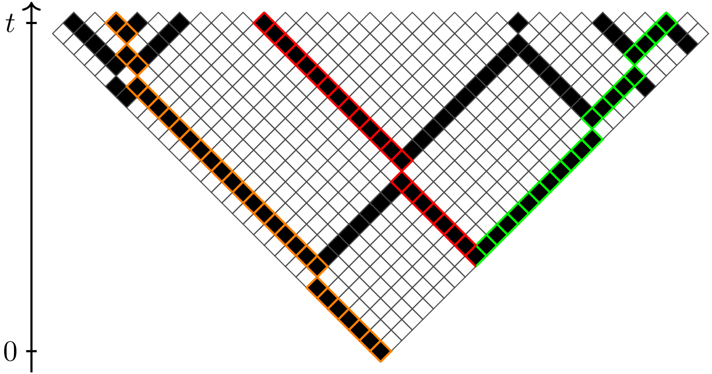
Solitons move with fixed velocities ± 1 and obtain a delay while scattering.



MPS for time evolution of local observables

KK, M. Medenjak, T. Prosen, M. Vanicat, Commun. Math. Phys. **371**, 651–688 (2019)

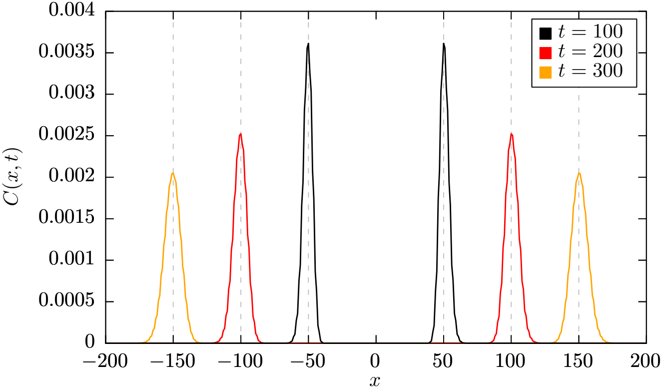
Time evolution of local observables is mapped onto the problem of counting solitons in a section of the lattice of length $2t+1$.



The computational complexity of the procedure grows as t^2.

Example: Spatio-temporal density-density correlation function

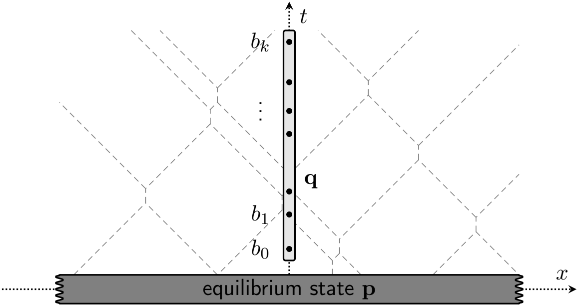
$$C(x,t) = 2^{-t-1} \sum_{m=0}^{\frac{t-|x|-2}{2}} 4^m \left(2\binom{t-2m-3}{m} - \binom{t-2m-2}{m} \right)$$



Multi-time correlation functions at the same position

KK, M. Vanicat, J. P. Garrahan, T. Prosen, arXiv:1912.09742 (2019)

Can we efficiently encode the probabilities of finding a given configuration in time?



Generically the complexity grows exponentially.
Our case: $\mathbf{q}$ is an MPS with bond dimension 3 (the same as $\mathbf{p}$).

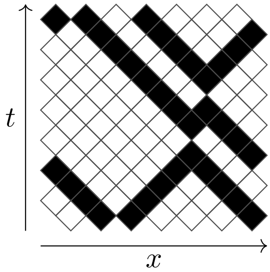
Time-space duality

KK, T. Prosen, arXiv:2004.01671 (2020)

What happens when the roles of space and time are reversed?

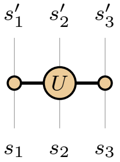
Motivation: *dual unitary* circuits

B. Bertini, P. Kos, T. Prosen, Phys. Rev. Lett. 123, 210601 (2019)



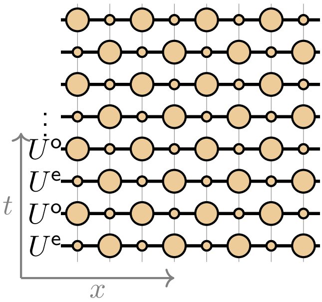
Solitons speed up while scattering (instead of slowing down)

Circuit representation of dynamics

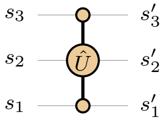


$$U^{s'_1 s'_2 s'_3}_{s_1 s_2 s_3} = \delta_{s'_1, s_1} \delta_{s'_2, \chi(s_1,s_2,s_3)} \delta_{s'_3, s_3}$$

Time evolution is then:

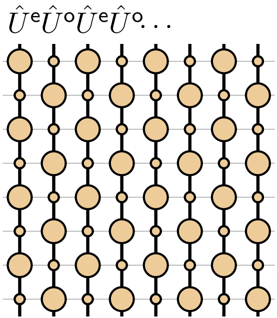


We define *dual gates* $\hat{U}$ as



$$\hat{U}^{s'_1 s'_2 s'_3}_{s_1 s_2 s_3} = \delta_{s'_1, s_1} \delta_{s'_3, s_3} U^{s_2 s_1 s'_2}_{s_2 s_3 s'_2}$$

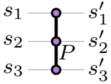
and obtain the rotated picture:



Problem: $\hat{U}$ non-deterministic

$$\hat{U} = \begin{bmatrix} 1 & & & & & & & \\ & 0 & & 1 & & & & \\ & & 0 & & & & & \\ & 1 & & 1 & & & & \\ & & & & 0 & & 1 & \\ & & & & & 1 & & \\ & & & & 1 & & 1 & \\ & & & & & & & 0 \end{bmatrix} = \hat{U} P = P \hat{U}$$

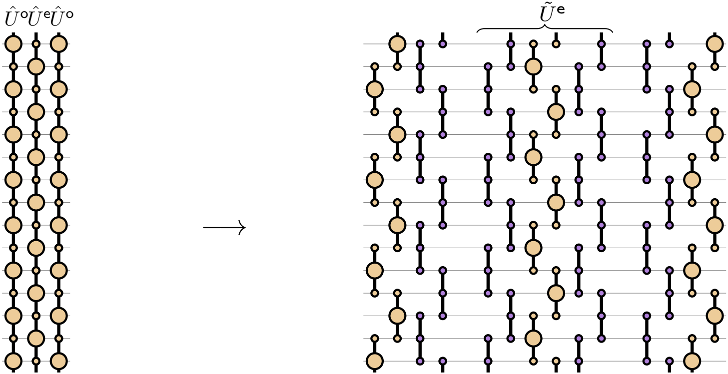
P projector on the *allowed subspace*



$$P^{s_1' s_2' s_3'}_{s_1 s_2 s_3} = \delta_{s_1', s_1} \delta_{s_2', s_2} \delta_{s_3', s_3} (1 - \delta_{s_1, s_3} \delta_{s_2, 1})$$

"Time-configurations" $(0, 1, 0)$ and $(1, 1, 1)$ are forbidden!

Evolution in space is local and deterministic on the *reduced* subspace of allowed configurations.

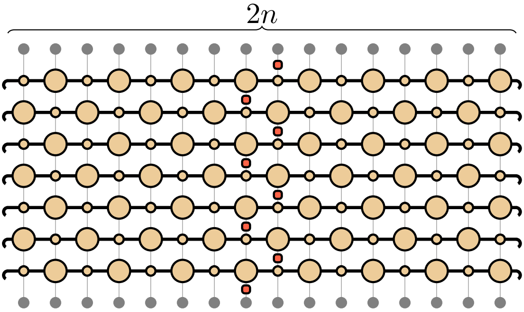


$\tilde{U}^{\mathsf{e}}$ and $\tilde{U}^{\mathsf{o}}$ can be expressed in terms of *deterministic* gates with support 7.

Revisiting multi-time correlation functions

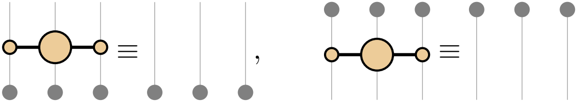
- Multi-time correlation function of $2m$ observables in maximum entropy state:

$$C^{(2n)}_{a_1,a_2,a_3,\ldots,a_{2m}}(\mathbf{p}_\infty) = 2^{-2n}$$

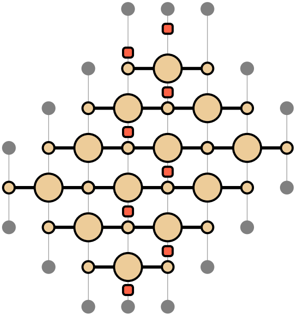
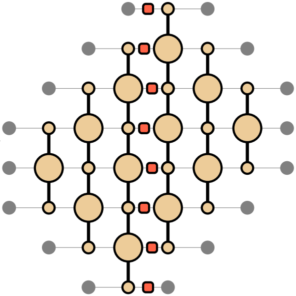


- Gray dots: one-site vectors $\omega = \begin{bmatrix} 1 & 1 \end{bmatrix}$

- U is deterministic,

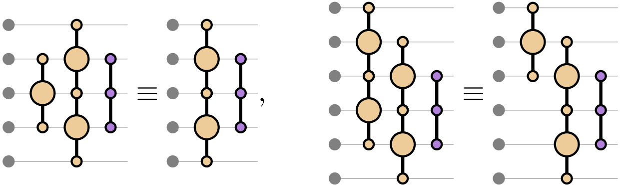


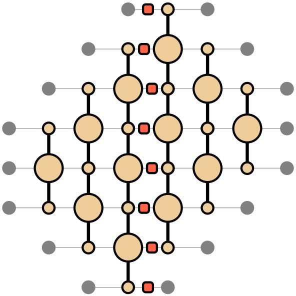
- Light-cone structure

2^{-2m}  $\equiv 2^{-2m}$ 

- This is *general*

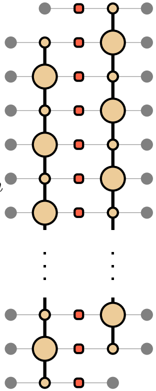
- RCA54 dual operator $\hat{U}$ is not deterministic, but it has some nontrivial structure:



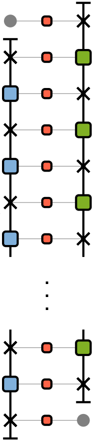


Layer after layer of dual gates can therefore be removed:

$$C_{a_1,a_2,a_3,\ldots,a_{2m}}(\mathbf{p}_\infty) = 2^{-2m}$$



$$= 2^{-2m}$$



This is equivalent to the MPS for correlation functions corresponding to the maximum entropy state. It can be generalized to a class of equilibrum states.

Summary and outlook

- A more algebraic formulation that does not explicitly depend on the quasiparticle interpretation of dynamics
- Open questions:
  - How far can we push this approach?
  - Can this be done for a class of models?

    Other cellular automata?

    J. W. P. Wilkinson, KK, T. Prosen, J. P. Garrahan, arXiv:2006.06556 (2020)

    Stochastic generalizations?

    Quantum generalizations?

    A. J. Friedman, S. Gopalakrishnan, R. Vasseur, Phys. Rev. Lett. 123, 170603 (2019)