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The Average-Case Complexity of Counting Cliques in Erdos-Renyi Hypergraphs

The complexity of clique problems on Erdos-Renyi random graphs has become a central topic in average-case complexity. Algorithmic phase transitions in these problems have been shown to have broad connections ranging from mixing of Markov chains to information-computation gaps in high-dimensional statistics. We consider the problem of counting k-cliques in s-uniform Erdos-Renyi hypergraphs $G(n,c,s)$ with edge density c, and show that its fine-grained average-case complexity can be based on its worst-case complexity. We prove the following:

1. Dense Erdos-Renyi graphs and hypergraphs: Counting k-cliques on $G(n,c,s)$ with k and c constant matches its worst-case complexity up to a polylog(n) factor. Assuming ETH, it takes $\Omega(n^k)$ time to count k-cliques in $G(n,c,s)$ if k and c are constant.

2. Sparse Erdos-Renyi graphs and hypergraphs: When $c=\Theta(n^{-\alpha})$, our reduction yields different average-case phase diagrams depicting a tradeoff between runtime and k for each fixed α. Assuming the best known worst-case algorithms are optimal, in the graph case of $s=2$, we establish that the exponent in n of the optimal running time for k-clique counting in $G(n,c,s)$ is $\omega k/3 - C\alpha \binom{k}{2} + O_{k,\alpha}(1)$, where $\omega/9 \le C \le 1$ and ω is the matrix multiplication constant. In the hypergraph case of $s \ge 3$, we show a lower bound at the exponent of $k - \alpha \binom{k}{s} + O_{k,\alpha}(1)$, which surprisingly is tight exactly for the set of c above the Erdos-Renyi k-clique percolation threshold.

Our reduction yields the first known average-case hardness result on Erdos-Renyi hypergraphs based on worst-case hardness conjectures. We also analyze algorithms for counting k-cliques in $G(n,c,s)$ to prove our upper bounds in the sparse case $c=\Theta(n^{-\alpha})$.