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Phase retrieval in high dimensions: Statistical and computational phase transitions

We consider the phase retrieval problem of reconstructing a n-dimensional real or complex signal X from m (possibly noisy) observations of the modulus (taken element-wise) of F X, for a large class of correlated real and complex random sensing matrices F, in a high-dimensional setting. First, we derive sharp asymptotics for the lowest possible estimation error achievable statistically and we unveil the existence of sharp phase transitions for the weak- and full-recovery thresholds as a function of the singular values of the matrix F. This is achieved by providing a rigorous proof of a result first obtained by the replica method from statistical mechanics. Secondly, we analyze the performance of the best-known polynomial time algorithm for this problem — approximate message-passing— establishing the existence of a statistical-to-algorithmic gap depending, again, on the spectral properties of. Our work provides an extensive classification of the statistical and algorithmic thresholds in high-dimensional phase retrieval for a broad class of random matrices.