Introduction to Mathematical Economics

Matteo Marsili, The Abdus Salam ICTP

Introduction: why is economics different from physics

1. *Individual Rationality* (Chapter 1 MasColell)

   - **Preferences**, **utility functions** and **choice structures**

2. Choice under uncertainty *(Chapter 6 MasColell)*

   - **Lotteries**, **expected utility** and **risk aversion**

   - **Bounded rationality and random utilities***

   - **Exercise: CARA vs CRRA gamblers***

3. *Social choice and voting* (Ch 21 Mas Colell, Ch 23 Easley)

   - **Social choice rules and Arrow's impossibility theorem**

   - **Beyond Arrow's theorem**

4. *Game theory: static games*

   - **Single stage games** (**Q&A**)

   - **Example: Cournot oligopoly**

   - **Extensive form games, backward induction and subgame perfection**

5. *Repeated games*

   - **With complete information and credible threats**

   - Incomplete information (**preliminaries on zero sum games** and **R. Aumann's lecture**)

6. **Evolutionary game theory** (by J. Maynard-Smith)

   - **Evolutionarily stable strategies**

   - **The replicator dynamics** (**errata**)

   - Imitation and learning

7. *General Equilibrium Theory*

- ■ **Competitive equilibria**

- ■ **Welfare theorems and the Edgeworth box**

- ■ *Statistical mechanics of general equilibrium theory*

8. **Financial markets** (**introduction. P. Millossovich**)

- ■ **Coping with risk, basic intuition**

- ■ **Many periods and the Black and Scholes equation**

- ■ **Fundamental theorem of Asset Pricing**

- ■ *The mechanics of the efficient market hypothesis and financial market (in)stability*

The aim of the course is to provide a (personal) conceptual map of how economists have tried to formalize economic behavior, from that of individuals to that of societies.

This corpus of ideas, that I call mathematical economics, provides a rather idealized picture of how individuals, societies, economies and financial markets behave. A central pillar of this approach is that individuals behave in a way to optimise their well being, i.e. that individuals are rational. So we shall discuss:

- individual rational decision making (choice theory)

- how this aggregates at the level of a society (social choice theory)

- rational behavior of interacting individuals (game theory)

- interactions mediated by markets (General Equilibrium Theory)

- market behavior in time under uncertainty (financial markets).

In reality, we know that individuals do not always take rational choices, that when they interact they do not behave as game theory would suggest and that economies and financial markets do not work as the theory predicts. Why did economics develop so much in spite of all its failures? Partly because it is not only a descriptive discipline - like physics - that aims at describing how a system works. There are also normative aspects, those that should inform policy. These need to be based on some explainable rationale. There is one way to be rational and many ways to be irrational, each of which is arbitrary. The effort to trace back economic behavior to rational choices is the least non-ambiguous manner to discipline our discussion on how economies should be. In addition, even if it fails in its predictions, mathematical economics still has its value because the way it fails indicates what needs to be understood.

Understanding the economic nature of private property or taxes, how elections work, can help us

discipline the discussion on these issues. Theorems cannot take responsibility for economic or political decisions, yet they may help create consensus on how to take complex decisions.

Finally, the collective behaviour of socio-economic systems is interesting in itself, for the variety of emergent behaviours that it displays.

Limited time only allows for a rather sketchy introduction to these topics, focusing on the main ideas. Yet I hope the course can serve as a guide for those interested to delve further into these subjects.

This page contains resources for the course, divided into lectures (and it may be updated during the course).

Italic links point to background and reasing material

Bold links point to video content

Link to the Exercises

Reference textbooks:

- **MasColell** Microeconomic Theory, A. Mas-Colell, M.D. Whinston and J.R. Green (OUP, '95)
- **Easley** Networks, Crowds and Markets, D. Easley, J. Kleinberg (CUP, 2010)
- **Gibbons** R. Gibbons, A primer in game theory (Prentice Hall, 1992)
- Introduction to mathematical finance, S. R. Pliska (Blackwell, 1997)
- The logic of asymmetric contests (JM Smith)