

## Outline

- Digital CMOS Design
- Arithmetic Operators
- Floating Point Arithmetic Operators
  - Square root



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February 2010

## Square root

An real number  $y$  using floating point representation

Find a real number  $x$  such as

$$(x + \varepsilon)^2 = y$$

- Calculation cannot be implemented in hardware
- Need iterative operation



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## Square root

- Direct method ➡ digit-by-digit
- Indirect method ➡ resolve a non-linear equation



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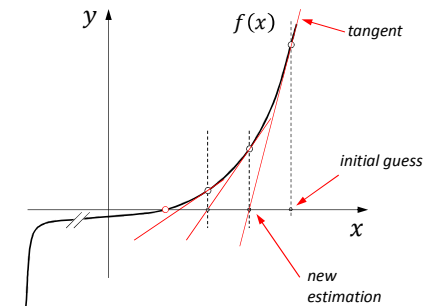
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## Square root - indirect

Resolving a non linear equation

$$f(x) = 0$$



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### Square root - indirect

Resolving a non linear equation  $f(x) = 0$

Taylor series in the neighborhood of  $x_0$

$$f(x) = f(x_0) + f'(x_0)(x - x_0) + \frac{1}{2}f''(x_0)(x - x_0)^2 + \dots$$

1<sup>st</sup> order :  $f(x) \approx f(x_0) + f'(x_0)(x - x_0)$



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### Square root - indirect

Resolving a non linear equation  $f(x) = 0$

Iterative resolution starting from an initial guess  $x_0$

$$f(x) \approx f(x_0) + f'(x_0)(x - x_0)$$

$$f(x) = 0$$

$$f(x_0) + f'(x_0)(x - x_0) = 0$$

$$x = \frac{-f(x_0)}{f'(x_0)} + x_0$$

Newton-Raphson method



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### Square root - indirect

Resolving  $x = \sqrt{y}$

Find a function  $f$  such as  $f(x) = 0$  for  $x = \sqrt{y}$

$$f(x) = x^2 - y$$

$$f(x) \approx f(x_0) + f'(x_0)(x - x_0)$$

$$f(x) \approx (x_0^2 - y) + 2x_0(x - x_0)$$

$$f(x) = 0 \quad x = x_0 + \frac{(y - x_0^2)}{2x_0}$$

$$x = \frac{1}{2} \left( \frac{y}{x_0} + x_0 \right)$$



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### Square root - indirect

Resolving  $x = \sqrt{y}$

$$\text{Each iteration } x_{i+1} = \frac{1}{2} \left( \frac{y}{x_i} + x_i \right)$$

~~$f(x) = x^2 - y$~~  division !!  
Hard to implement



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### Square root - indirect

Resolving  $x = \sqrt{y}$

Find a function  $f$  such as  $f(u) = 0$  for  $u = \frac{1}{\sqrt{y}}$

$$f(u) = u^{-2} - y$$

$$f(u) \approx f(u_0) + f'(u_0)(u - u_0)$$

$$f(u) \approx (u_0^{-2} - y) - 2u_0^{-3}(u - u_0)$$

$$f(u) = 0 \quad u = u_0 + \frac{(u_0^{-2} - y)}{2u_0^{-3}}$$

$$u = \frac{1}{2}u_0(3 - yu_0^2)$$



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### Square root - indirect

Resolving  $x = \sqrt{y}$

Each iteration  $u_{i+1} = \frac{1}{2}u_i(3 - yu_i^2)$

*multiply !!*

$$u = \frac{1}{\sqrt{y}}$$

$$x = \sqrt{y} = \frac{y}{\sqrt{y}} = u \cdot y$$



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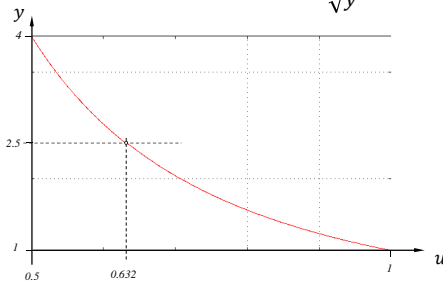
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### Square root - indirect

Resolving  $x = \sqrt{y}$

Initial guess

$$u = \frac{1}{\sqrt{y}} \quad u \in ]0.5, 1]$$



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