QCD Physics for Colliders

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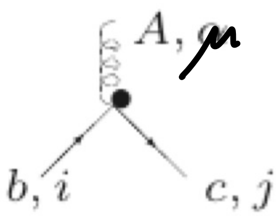
Lecture 3

Colour in QCD

Asymptotic freedom

First difference:

colour enters the quark-gluon interaction



colour indices (fund. rep.)

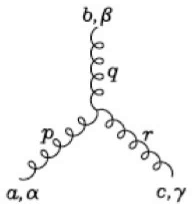
$$-ig\,\underbrace{(t^A)_{cb}}_{\text{Colour matrix}}\,\overbrace{(\gamma^\mu)}^{\text{Dirac matrix}}{}_{ji}$$

spinor indices

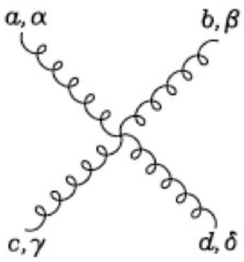
The colour changes in the interaction

Second difference

Gauge bosons self-coupling



$$= g f^{abc}\left[g^{\alpha\beta}(p-q)^\gamma + g^{\beta\gamma}(q-r)^\alpha + g^{\gamma\alpha}(r-p)^\beta\right]$$



$$\begin{aligned} = &-ig^2 f^{xac} f^{xbd}\left(g^{\alpha\beta}g^{\gamma\delta} - g^{\alpha\delta}g^{\beta\gamma}\right) \\ &-ig^2 f^{xad} f^{xbc}\left(g^{\alpha\beta}g^{\gamma\delta} - g^{\alpha\gamma}g^{\beta\delta}\right) \\ &-ig^2 f^{xab} f^{xcd}\left(g^{\alpha\gamma}g^{\beta\delta} - g^{\alpha\delta}g^{\beta\gamma}\right) \end{aligned}$$

Gluons interact with themselves →

→ direct consequence of non-abelianity

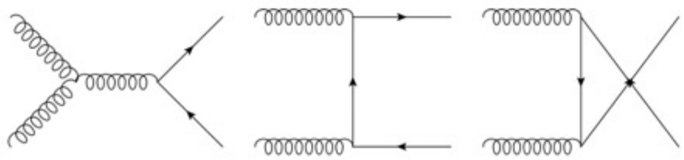
Third difference

Need for "ghosts" to cancel unphysical degrees of freedom that would otherwise propagate in covariant gauges

Example: in a tree level calc., we can either sum only over physical gluon polarisations

Ghosts: an example

$gg \to q\bar{q}$



In QED (i.e. replacing gluons with photons) we'd only have the second and third diagram, and we would sum over the photon polarisations using

$$\sum_{pol} \epsilon_i^\mu \epsilon_i^{*\nu} = -g_{\mu\nu}$$

In QCD this would give the wrong result

We must use instead

$$\sum_{phys\,pol} \epsilon_i^\mu \epsilon_i^{*\nu} = -g_{\mu\nu} + \frac{k_\mu \bar{k}_\nu + k_\nu \bar{k}_\mu}{k \cdot \bar{k}}$$

$\bar{k}$ is a light-like vector, we can use $(k_0,0,0,-k_0)$

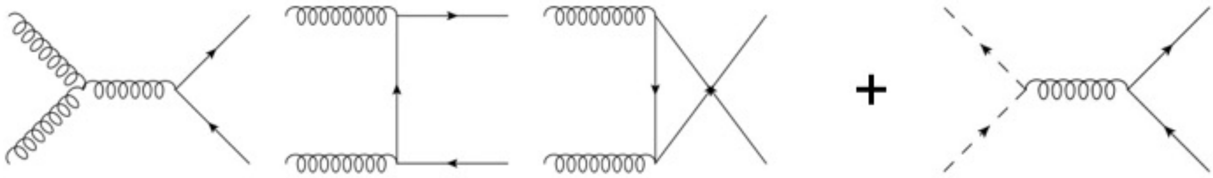
$$k^\mu = (k^0, \vec{k}) = (k^0, k^0)$$

$$\bar{k}^\mu = (k^0, -\vec{k}) = (k^0, -k^0)$$

or explicitly include the ghosts

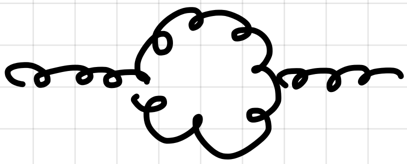
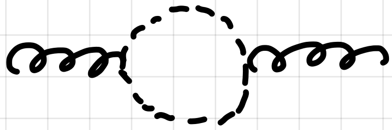
Ghosts: an example

An **alternative** approach is to include the ghosts in the calculation



Now we can safely use $\sum_{pol} \epsilon_i^{\mu} \epsilon_i^{*\nu} = -g_{\mu\nu}$

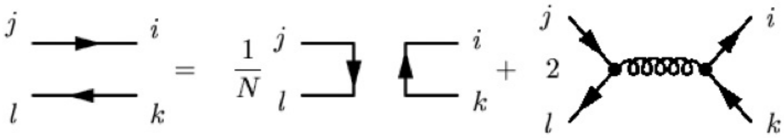
In a loop calculation, we have no choice: we MUST include ghost diagrams, e.g.

 AND 

Consequences of colour

First, consider

A fundamental colour relation



$$\delta_{ij}\delta_{lk} = \frac{1}{N}\delta_{ik}\delta_{lj} + 2\,t^A_{ik}t^A_{lj}$$

This is essentially the statement that

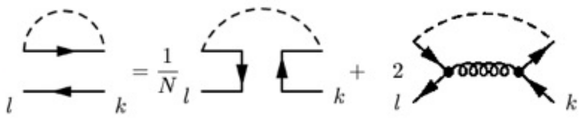
$$3 \otimes \bar{3} = 1 \oplus 8$$ (tensor product decomposition)

Now

C_F

Take i=j in

$$\delta_{ij}\delta_{lk} = \frac{1}{N}\delta_{ik}\delta_{lj} + 2\,t^A_{ik}t^A_{lj} \quad \longleftrightarrow$$



$$\Downarrow$$

$$N\delta_{lk} = \frac{1}{N}\delta_{lk} + 2\,t^A_{ik}t^A_{li}$$

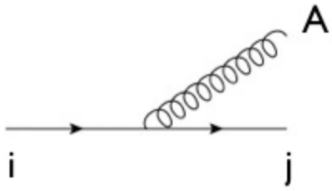
$$\Downarrow$$

$$(t^A t^A)_{lk} = \frac{1}{2}\left(N - \frac{1}{N}\right)\delta_{lk} = \frac{N^2-1}{2N}\delta_{lk} \equiv C_F\delta_{lk}$$

This defines C_F.
It is the Casimir of the fundamental representation of SU(N).
What is it, physically?

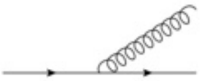
C_F

Gluon emission from a quark



$$\sim \quad t^A_{ji}$$

$$\text{Prob} \sim \sum_{jA} \left| \quad \right|^2 \sim \sum_{jA} t^A_{ij} t^A_{ji} = \sum_A (t^A t^A)_{ii} = C_F \delta_{ii}$$

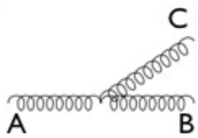


$C_F = (N^2-1)/(2N)$ is therefore the 'colour charge' of a quark, i.e. its probability of emitting a gluon (except for the strong coupling, of course)

C_A

Analogously, one can show that

$$\text{Prob} \sim \sum_{BC} \left| \quad \right|^2 \sim C_A \delta_{AA}$$

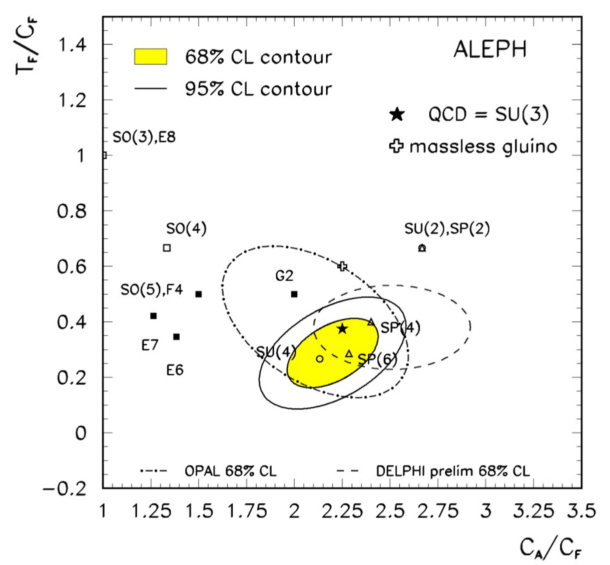


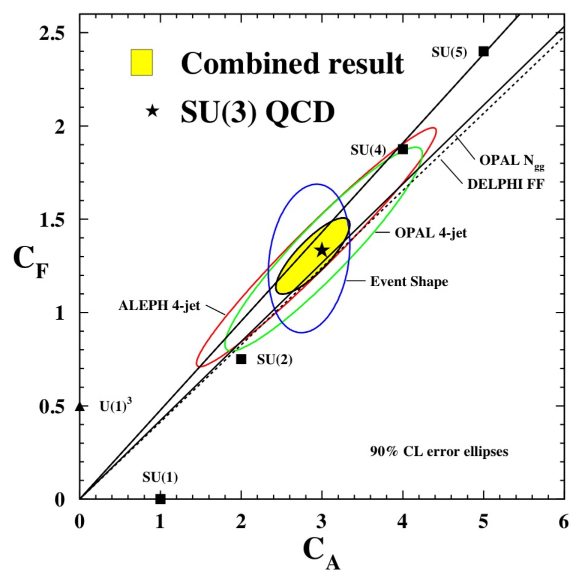
$C_A = N$ is the 'colour charge' of a gluon, i.e. its probability of emitting a gluon (except for the strong coupling, of course). It is also the Casimir of the adjoint representation.

N=3 is a largish number: gluons like to emit gluon!

The effects of colour factors can clearly be seen in observables, and experimentally tested. Many measurements have been performed at LEP, that clearly established SU(3) as the correct group for the theory of strong interactions.

For instance (see e.g. https://arxiv.org/pdf/hep-ex/9705016.pdf)



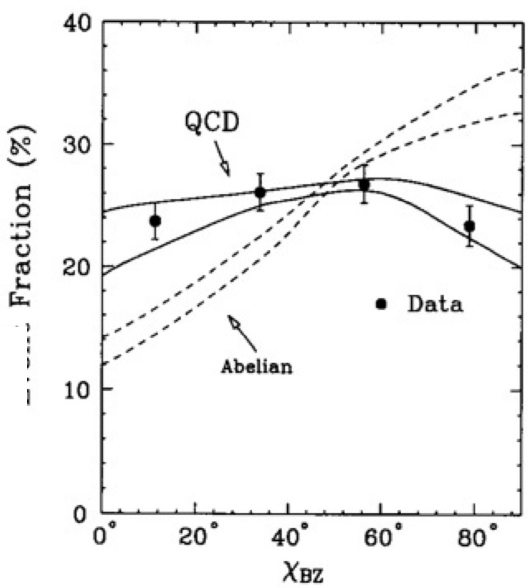


and, for gluon-gluon self-interactions,

Bengtsson-Zerwas angle

$$\chi_{BZ} = \angle[(\vec{p}_1 \times \vec{p}_2), (\vec{p}_3 \times \vec{p}_4)] = \frac{(\vec{p}_1 \times \vec{p}_2) \cdot (\vec{p}_3 \times \vec{p}_4)}{|(\vec{p}_1 \times \vec{p}_2)||(\vec{p}_3 \times \vec{p}_4)|}$$

(ordered moments in 4-jet events)



Macroscopic differences

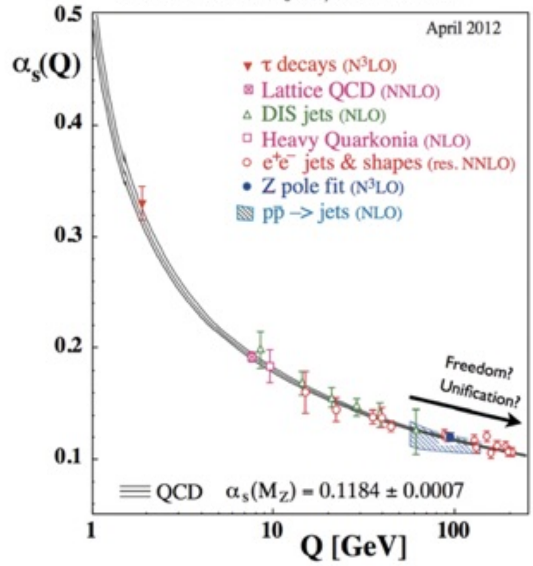
1. Confinement (probably -- no proof in QCD)

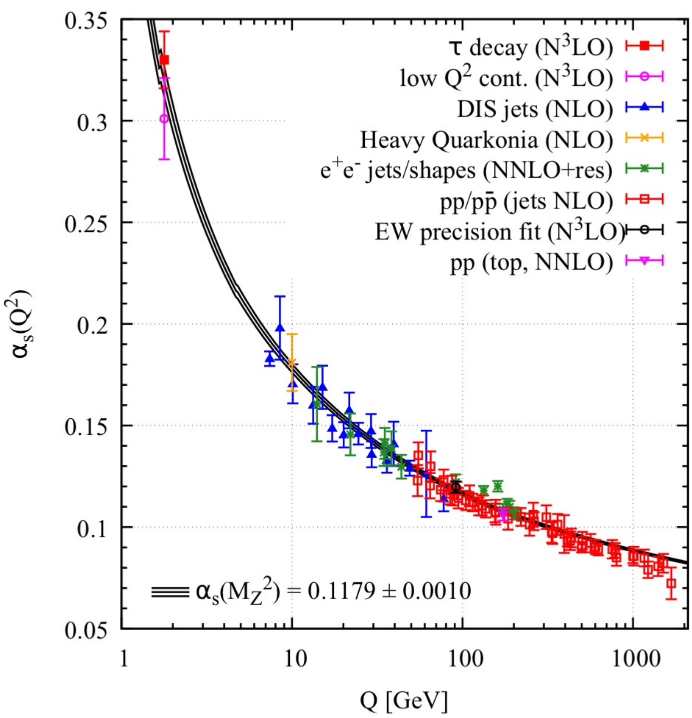
We never observe the fundamental degrees of freedom (quarks and gluons). They are always confined into hadrons.

2. Asymptotic Feedom

The running coupling of the theory, α_s, **decreases** at large energies

From S. Bethke, Nucl.Phys.Proc.Suppl. 234 (2013) 229
See also PDG Review on QCD. by Dissertori & Salam





From PDG 2019

$\alpha_s(M_Z) = 0.1179 \pm 0.0010$

Asymptotic freedom is what makes QCD a viable theory of strong interactions, as it can become strong at large distances ($\gtrsim 10^{-15}$ m) while allowing perturbative calculations at smaller distances ($\ll 10^{-15}$ m)

Asymptotic freedom is driven by the sign of the β function of the RGE for the strong coupling:

$$\mu_R \frac{dg}{d\mu_R} = \beta(g) < 0$$

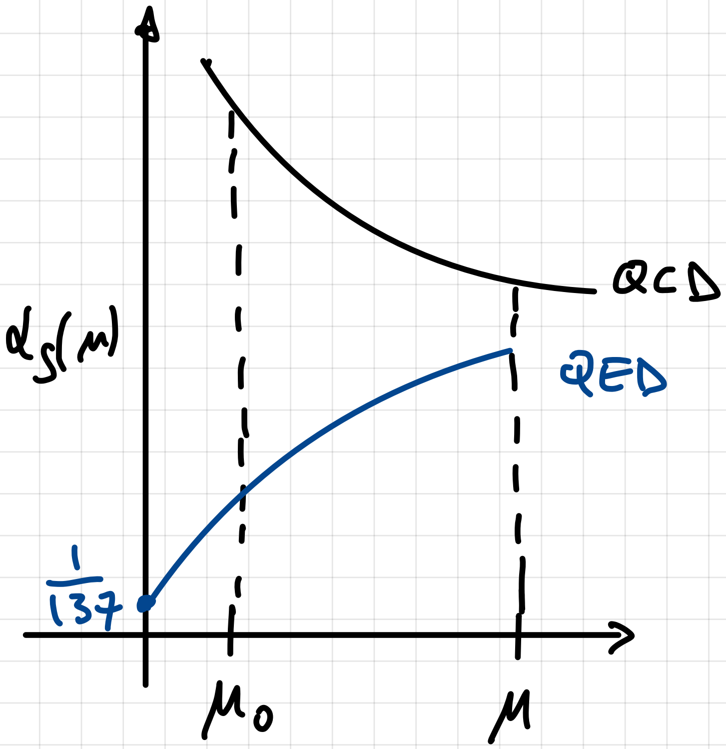
It is more common to write this for
$$\alpha_s \hat{=} \frac{g^2}{4\pi}$$

$$\mu_R^2 \frac{d\alpha_s}{d\mu_R^2} = \beta(\alpha_s) = -b_0 \alpha_s^2 - b_1 \alpha_s^3 - \ldots$$

and with $b_0 > 0$ (and good perturbative behaviour, i.e. $\alpha_s < 1$) we predict <u>perturbatively</u> asymptotic freedom, the solution of the RGE at the lowest order being

$$\alpha_s(\mu) = \frac{\alpha_s(\mu_0)}{1 + b_0 \alpha_s \log\left(\mu^2/\mu_0^2\right)}$$

and $\alpha_s(\mu) < \alpha_s(\mu_0)$ for $\mu > \mu_0$



NOT TO SCALE

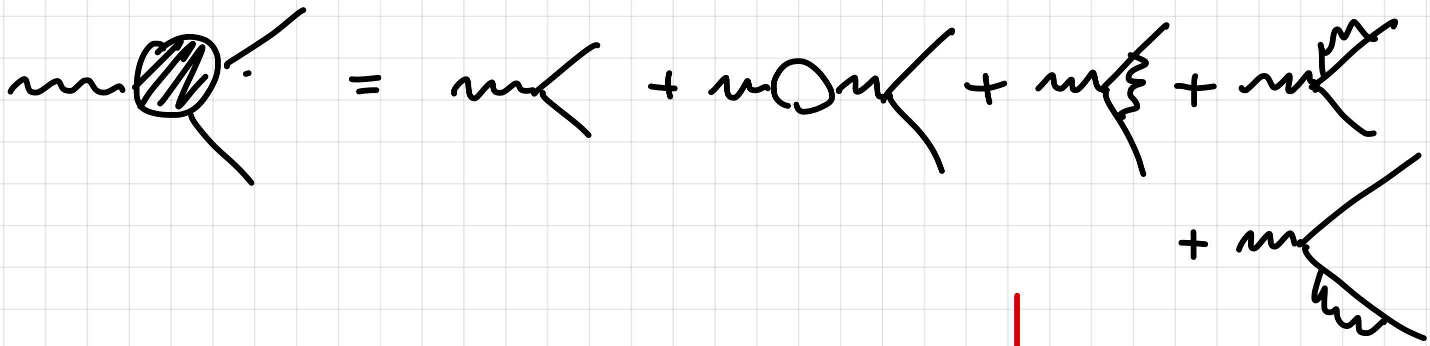
Small energy scale
→ large distance

large energy scale
→ small distance

This behaviour is opposite to the one of QED, that has positive β function and therefore increasing coupling with scale [i.e. decreasing coupling with distance → vacuum screening from virtual e^+e^- pairs]

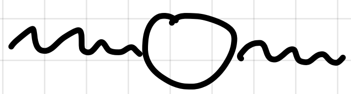
QED

Ren. coupling at one loop



Cancel due to a Ward identity in QED

$\Rightarrow$ in QED the β function at one loop is given only by

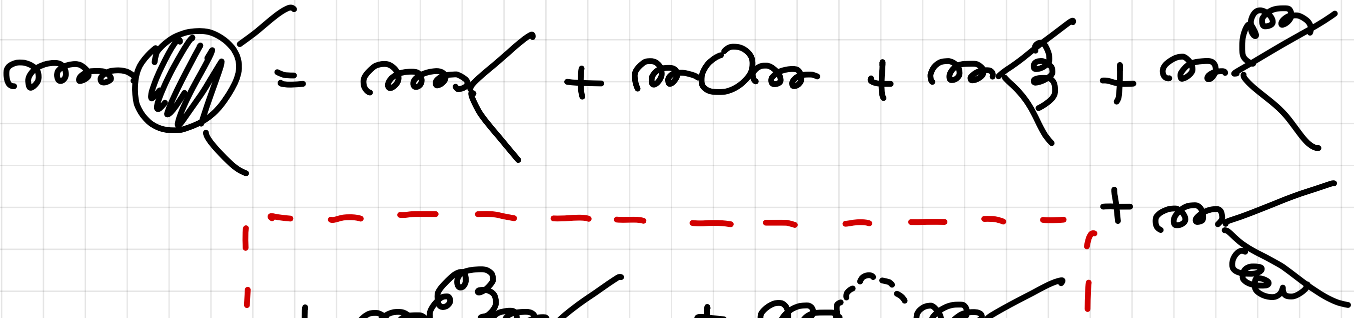


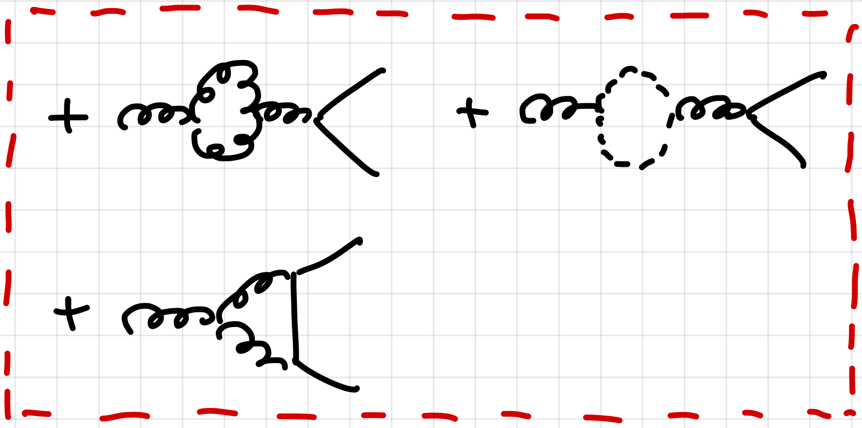
$$\Rightarrow \beta_{QED}(\alpha_{em}) = \frac{1}{3\pi}\alpha_{em}^2 + \dots > 0$$

(times number of particles species circulating in the loop)

QCD

More diagrams to consider





"non-abelian diagrams"

the result for the β_{QCD} function is (Gross, Wilczek, Politzer 1973)

(Nobel prize)

NB. Wilczek and Politzer PhD students at the time, like Peebles and Silk

$\to$ number of quark flavours

$$\beta_{QCD} = -\left(\frac{11 C_A - 4 n_f T_F}{12\pi}\right)\alpha_s^2 + \dots$$

< 0 for $2n_f < 33 \Rightarrow$ if less than 16 quark flavours

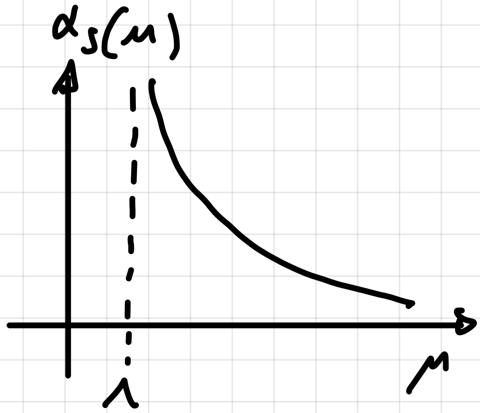
The solution of the RGE,

$$\alpha_s(\mu) = \frac{\alpha_s(\mu_0)}{1 + b_0 \alpha_s \log\left(\frac{\mu^2}{\mu_0^2}\right)}$$

can also be rewritten as

Dimensional transmutation

$$\alpha_s(\mu) = \frac{1}{b_0 \log \frac{\mu^2}{\Lambda^2}}$$



where Λ is defined as the scale at which the expression for $\alpha_s(\mu)$ diverges:

$$\alpha_s(\Lambda) \to \infty \iff 1 + b_0 \alpha_s \ln\left(\frac{\Lambda^2}{\mu_0^2}\right) = 0$$

$$\Rightarrow \Lambda = \mu_0 \, e^{-\frac{1}{2 b_0 \alpha_s(\mu_0)}}$$

Λ is where perturbation theory (which we used to calculate the β function) breaks down (the coupling used in the power expansion diverges, $\Rightarrow$ certainly > 1)

What is the value of the scale Λ?

$\rightarrow$ it's where strong interactions become strong

Estimate it using LEP data.

$\alpha_s(M_Z)$ can be measured from $\sigma(e^+e^- \to \text{hadrons})$ at $\sqrt{s} \simeq M_Z$

(more on this later)

$$\rightarrow \quad \alpha_s(M_Z \simeq 91\,\text{GeV}) \simeq 0.12$$

b_0 with 5 flavours ($m_{top} \gg M_Z$) is

$$b_0^{(n_f=5)} = \frac{33-10}{12\pi} \simeq 0.6$$

$$\Rightarrow \Lambda = \mu_0\, e^{-\frac{1}{2b_0\alpha_s(\mu_0)}} = 91\, e^{-\frac{1}{2\times 0.6\times 0.12}}$$

$$\simeq \underline{100\ \text{MeV}}$$

This is a <u>typical hadronic scale</u>, of the order of the pion mass ($m_\pi \simeq 139$ MeV) and of the proton radius (~ 200 MeV)

$\Rightarrow$ not surprisingly, pQCD breaks down at the scale of the size of hadrons

A very important point.

We said earlier that confinement is not proven in QCD

We are observing here that the coupling increases (and diverges) at small energy scales (= large distances)

Is this a proof of confinement!

NO!

The result for α_s is a perturbative one. It is only valid when $\alpha_s \ll 1$

Perturbation theory can indicate where it breaks down (more on this later), but it cannot make reliable predictions in such regions.

At best, we can take the α_s behaviour as an indication that confinement may be predicted by full QCD

One should note that Λ is <u>not</u> a physical observable. It depends on order of perturbation theory and on renormalisation scheme.

If we go beyond leading order in the β_{QCD} function we have

$$\mu^2 \frac{d\alpha_s}{d\mu^2} = \beta_{QCD}(\alpha_s) = -b_0 \alpha_s^2 - b_1 \alpha_s^3 + \cdots$$

$$\text{with } b_1 = \frac{17C_A^2 - 5C_A n_f - 3n_f C_F}{24\pi^2}$$

and a conventional solution for $\alpha_s(\mu)$ is (no exact closed form beyond LO)

$$\alpha_s(\mu) = \frac{1}{b_0 \log \frac{\mu^2}{\Lambda^2}} \left(1 - \frac{b_1}{b_0^2} \frac{\log\left(\log \frac{\mu^2}{\Lambda^2}\right)}{\log \frac{\mu^2}{\Lambda^2}}\right)$$

and note that this Λ, extracted from data, will be different from the LO one (but still ~ 100 – 200 MeV)