



# Experiments in ergodicity

Kamil Bonna

supervisors:

Ollie Hulme, Ole Peters, Alex Adamou, Yonatan Berman, Mark Kirstein

# Aim of the project

Improve the Copenhagen experiment.

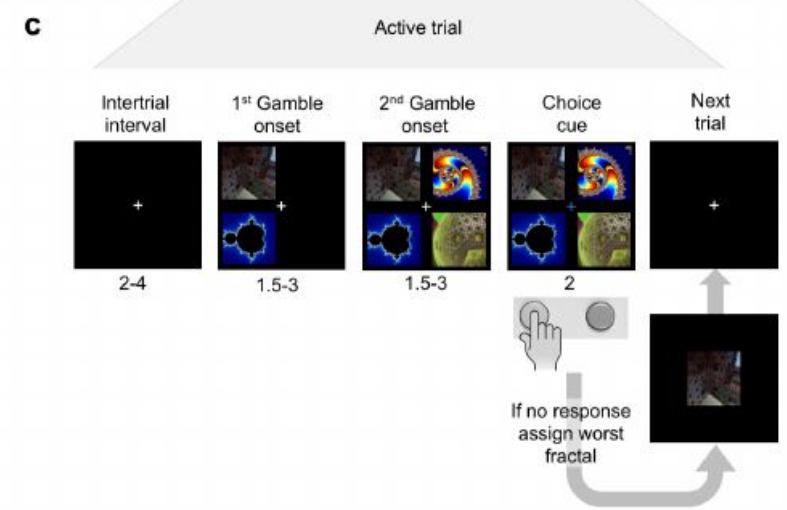
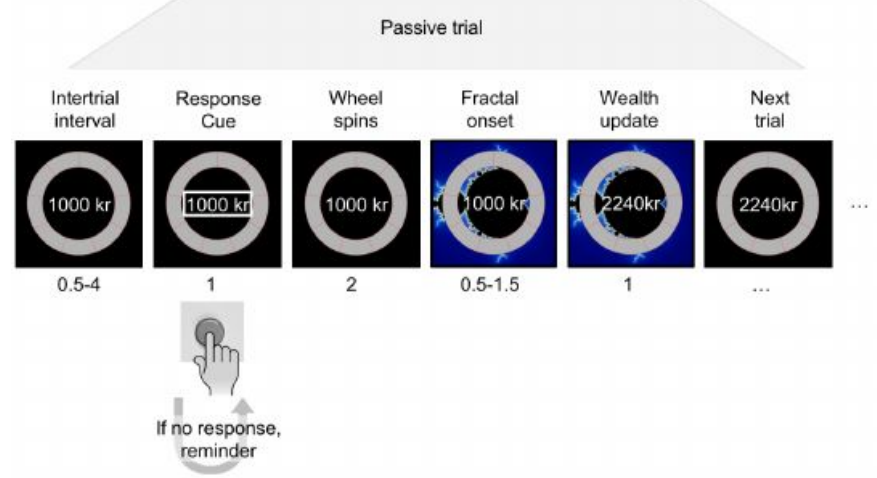
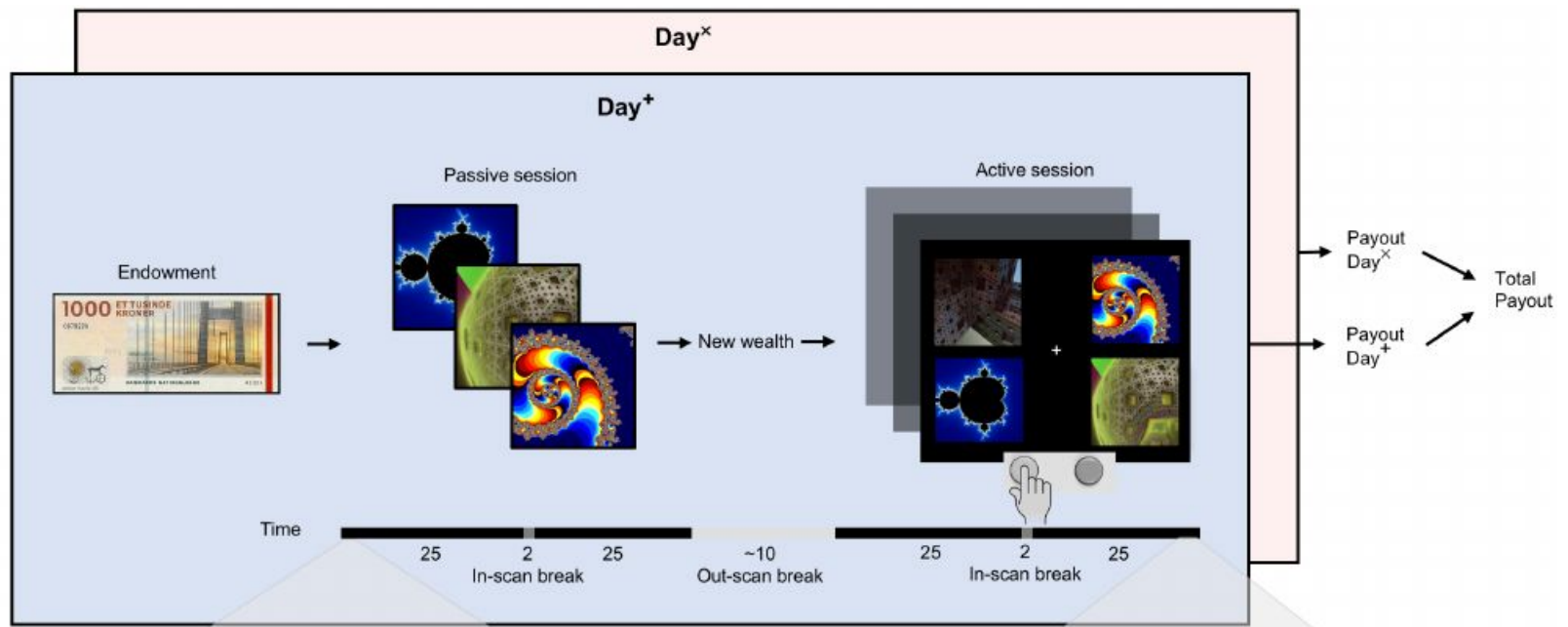
## Ergodicity-breaking reveals time optimal decision making in humans

David Meder<sup>1</sup>, Finn Rabe<sup>1,2</sup>, Tobias Morville<sup>1</sup>, Kristoffer H. Madsen<sup>1,3</sup>, Magnus T. Koudahl<sup>1,4</sup>, Ray J. Dolan<sup>6</sup> Hartwig R. Siebner<sup>1,7,8</sup>, Oliver J. Hulme<sup>1\*</sup>

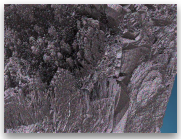


DANISH RESEARCH  
CENTRE FOR  
MAGNETIC RESONANCE

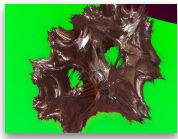
- validate ergodic theory of decision making
- address criticism, replicate findings



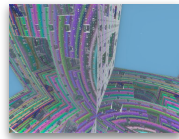
# Day<sup>+</sup> (additive)



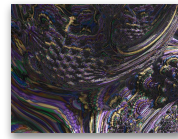
-428 kr



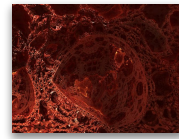
-321 kr



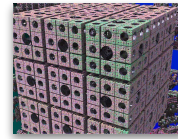
-214 kr



-107 kr



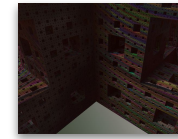
0 kr



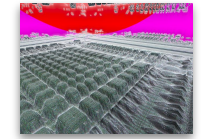
107 kr



214 kr

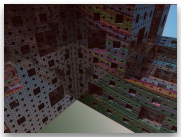


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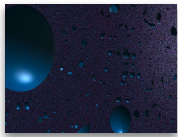


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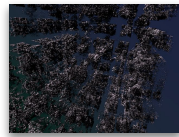
# Day<sup>x</sup> (multiplicative)



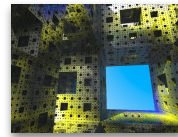
×0.45



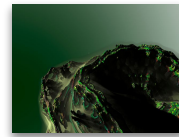
×0.55



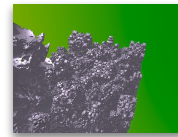
×0.67



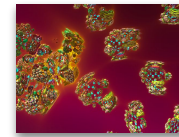
×0.82



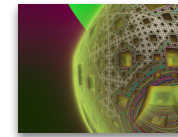
×1



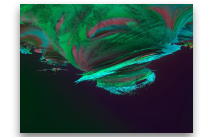
×1.22



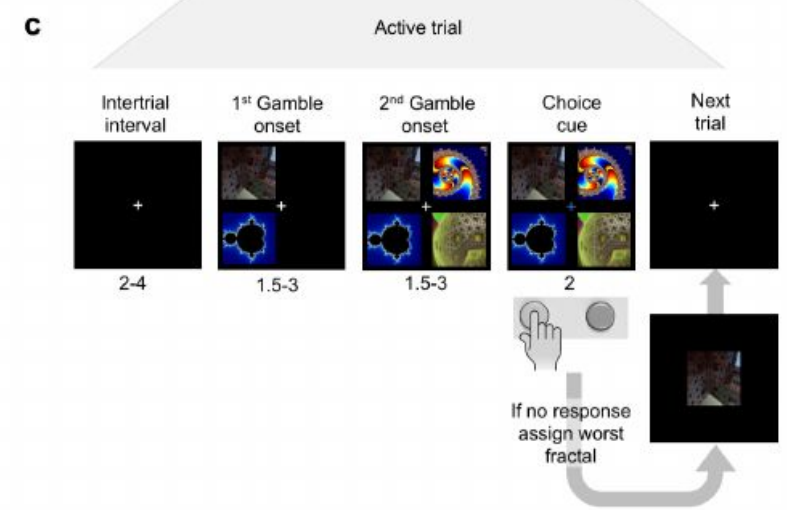
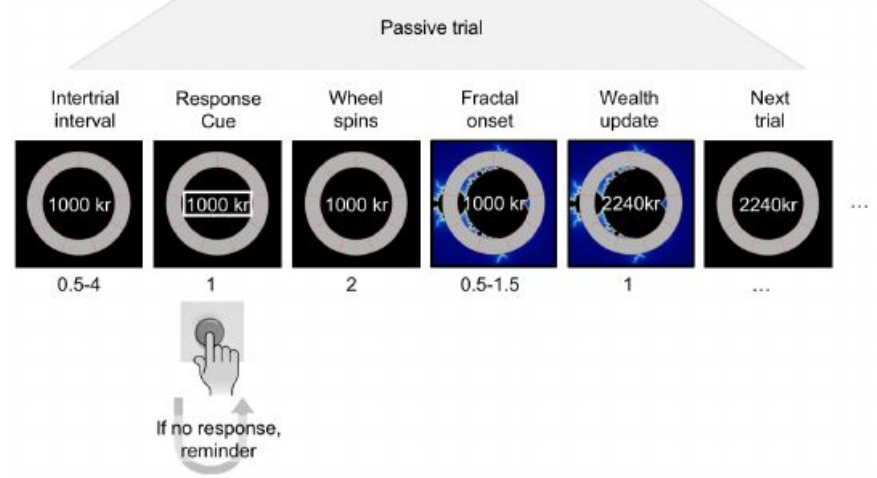
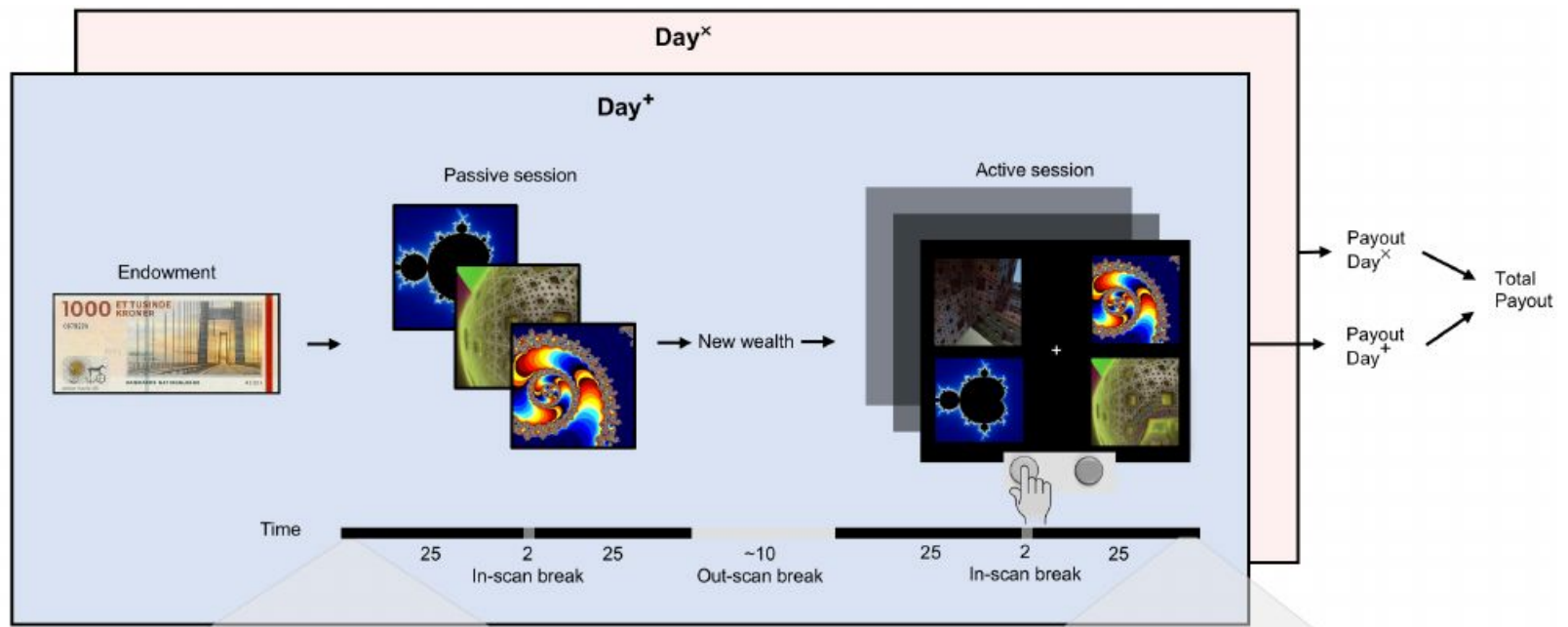
×1.49



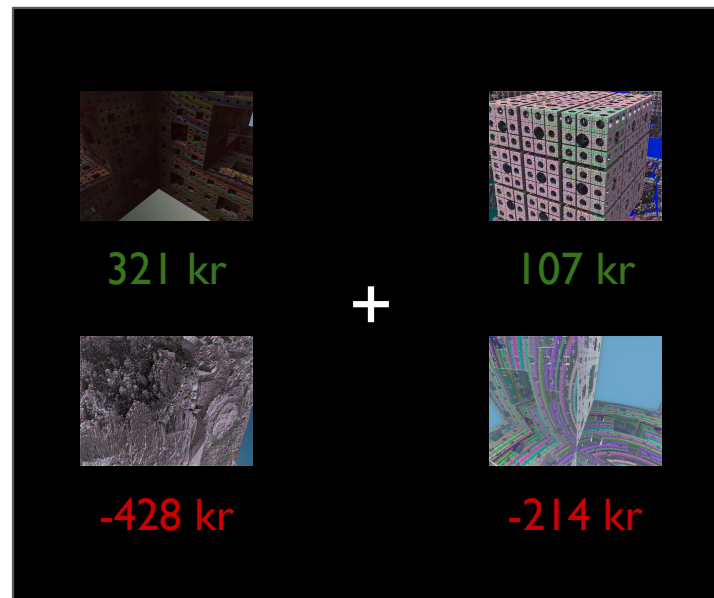
×1.83



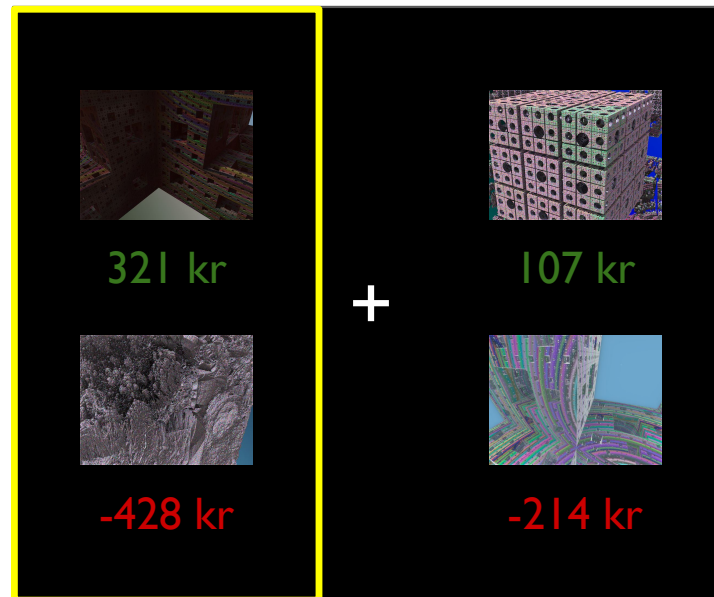
×2.23



# Example gamble

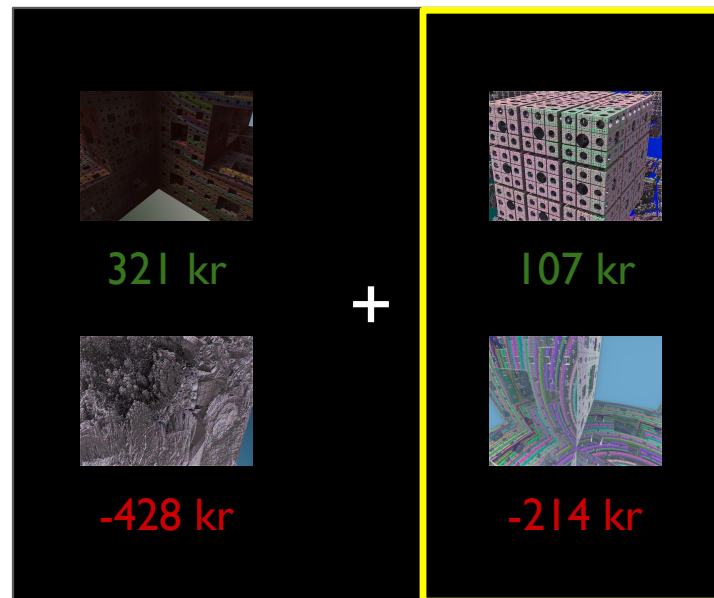


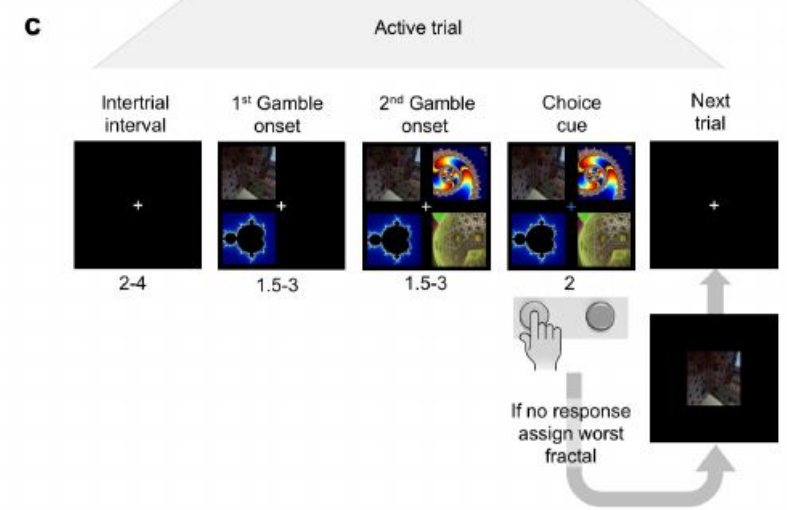
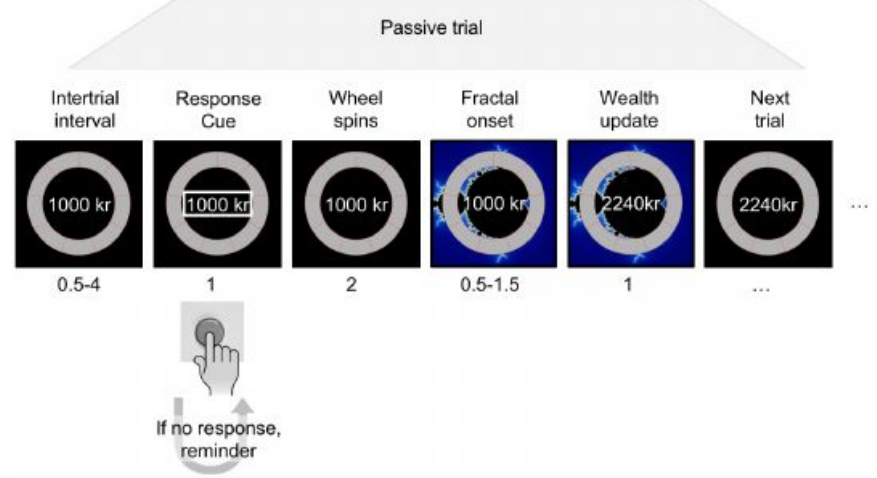
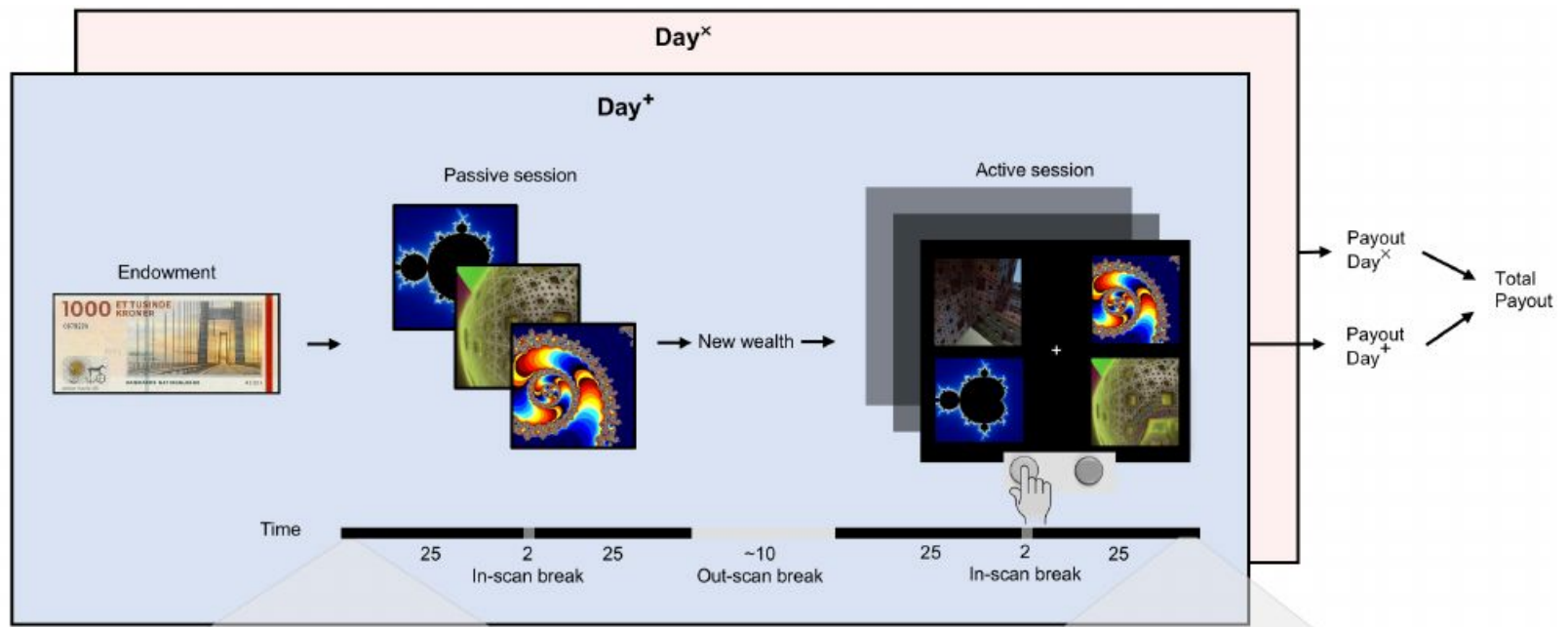
# Example gamble





# Example gamble





# Discriminating between models

risk attitude

Expected utility theory

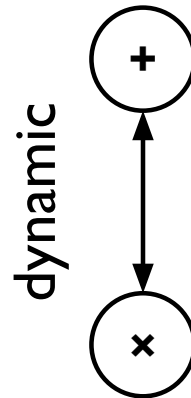
$$\eta = 0.6$$

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Ergodicity economics

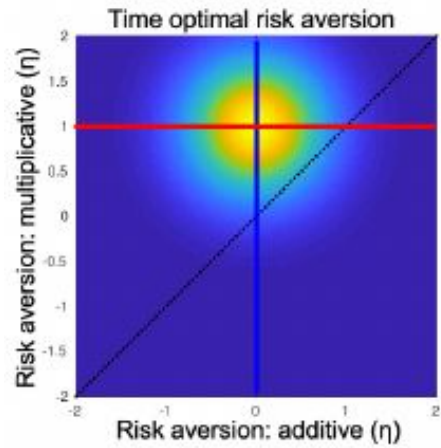
$$\eta = 0.0$$

$$\eta = 1.0$$

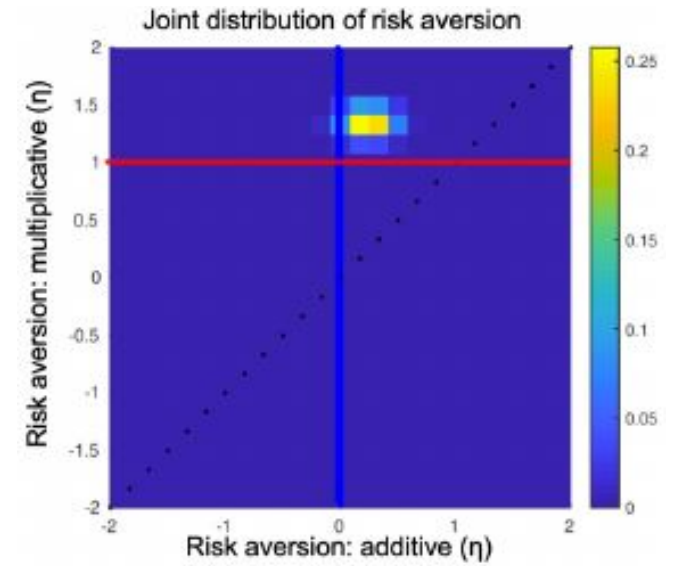
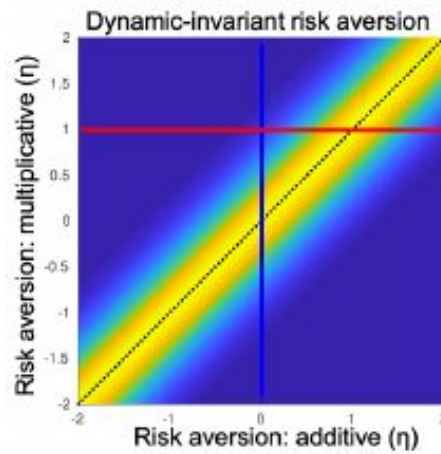


# Results

EE



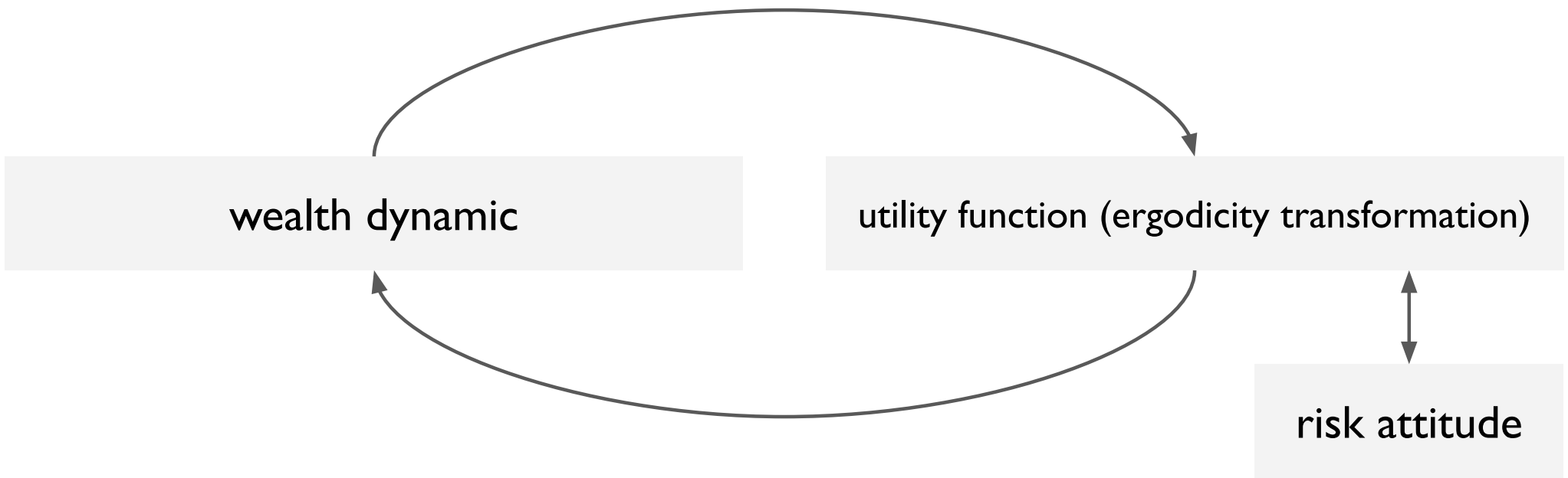
EUT

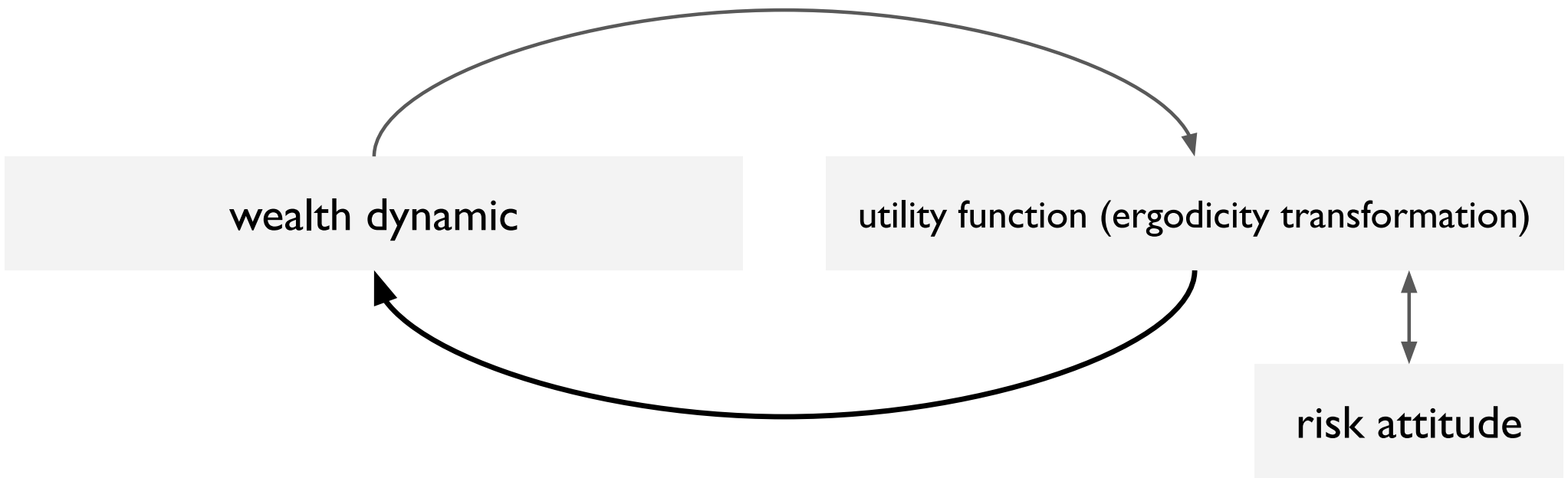


## Aim of the project

Improve the Copenhagen experiment.

- add more wealth dynamics
- show outcomes & realize all trials
- optimize a design





# Wealth dynamics

wealth dynamic

utility function (ergodicity transformation)

$$x(t) \longleftarrow u(x)$$

transformed wealth change at a constant rate

$$\frac{du(x(t))}{dt} = \gamma$$



# Wealth dynamics

wealth dynamic

utility function (ergodicity transformation)

$$x(t) \longleftarrow u(x)$$

transformed wealth change at a constant rate

$$\frac{u(x(t + \Delta t)) - u(x(t))}{\Delta t} = \gamma$$

# Wealth dynamics

wealth dynamic

utility function (ergodicity transformation)

$$x(t) \longleftarrow u(x)$$

transformed wealth change at a constant rate

$$x(t + \Delta t) = u^{-1}(u(x(t)) + \gamma \Delta t)$$

# Wealth dynamics

wealth dynamic

utility function (ergodicity transformation)

$$x(t) \longleftarrow u(x)$$

isoelastic utility

$$u(x) = \begin{cases} \frac{x^{1-\eta}-1}{1-\eta} & \eta \neq 1 \\ \ln x & \eta = 1 \end{cases}$$

$$x(t + \Delta t) = u^{-1}(u(x(t)) + \gamma \Delta t)$$

# Wealth dynamics

wealth dynamic

utility function (ergodicity transformation)

$$x(t)$$



$$u(x)$$

isoelastic utility

$$u(x) = \begin{cases} \frac{x^{1-\eta}-1}{1-\eta} & \eta \neq 1 \\ \ln x & \eta = 1 \end{cases}$$

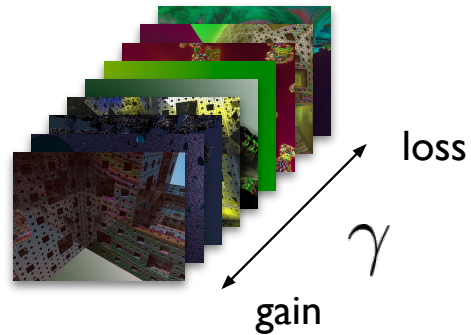
$$x(t+\Delta t) = \begin{cases} (x^{1-\eta} + (1-\eta)\gamma\Delta t)^{\frac{1}{1-\eta}}, & \eta \neq 1 \\ xe^{\gamma\Delta t}, & \eta = 1 \end{cases}$$

## Wealth dynamics

$$x(t+\Delta t) = \begin{cases} (x^{1-\eta} + (1-\eta)\gamma\Delta t)^{\frac{1}{1-\eta}}, & \eta \neq 1 \\ xe^{\gamma\Delta t}, & \eta = 1 \end{cases}$$

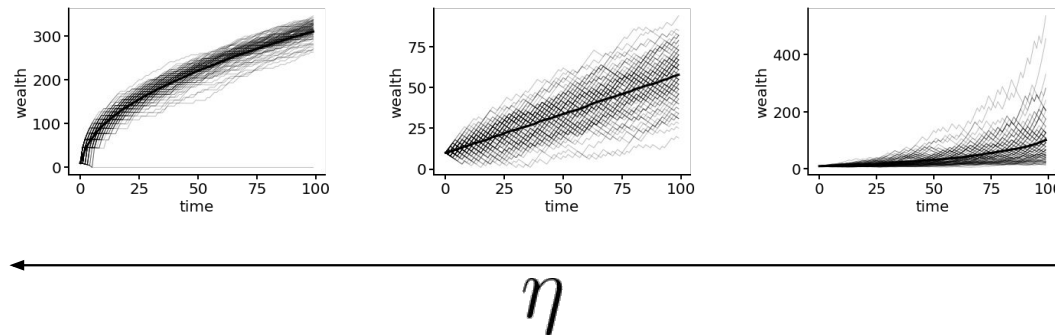
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# Wealth dynamics

$$x(t+\Delta t) = \begin{cases} (x^{1-\eta} + (1-\eta)\gamma\Delta t)^{\frac{1}{1-\eta}} & \eta \neq 1 \\ xe^{\gamma\Delta t}, & \eta = 1 \end{cases}$$



risk attitude  $\eta$

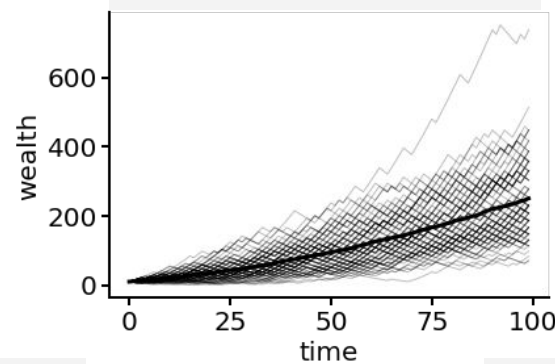
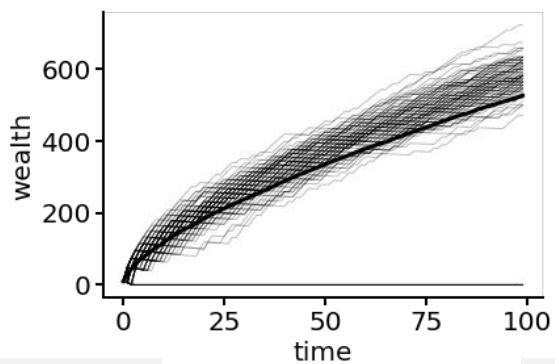
-1

-0.5

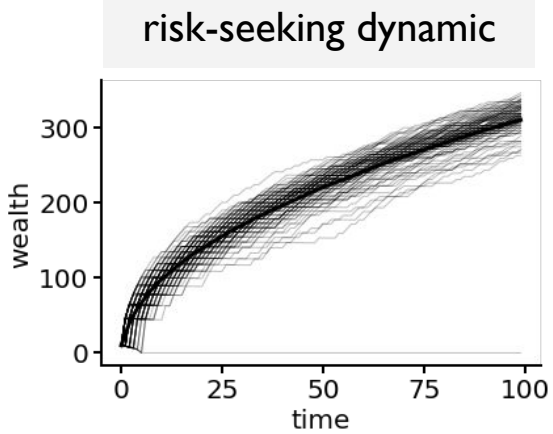
0

0.5

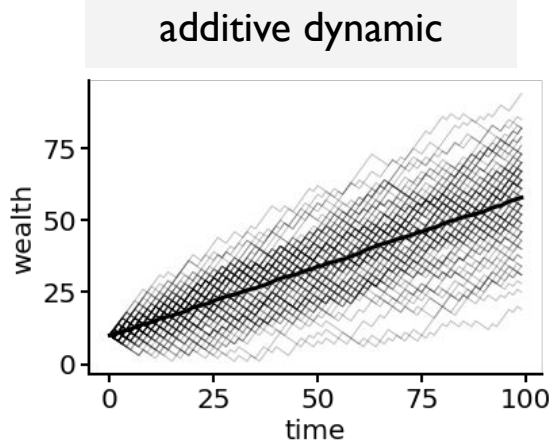
1



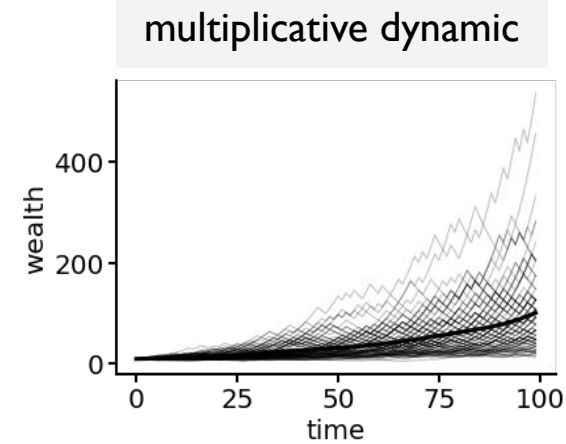
square-root dynamic



risk-seeking dynamic



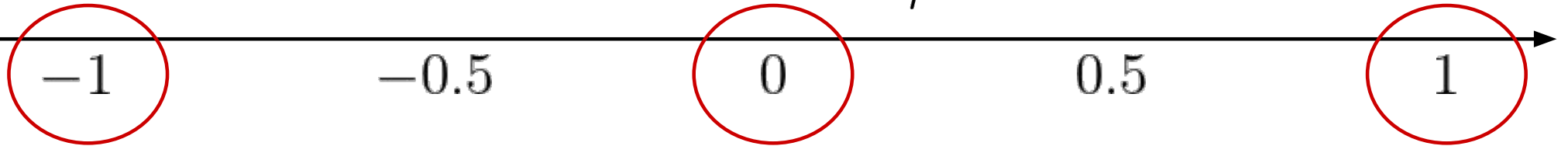
additive dynamic



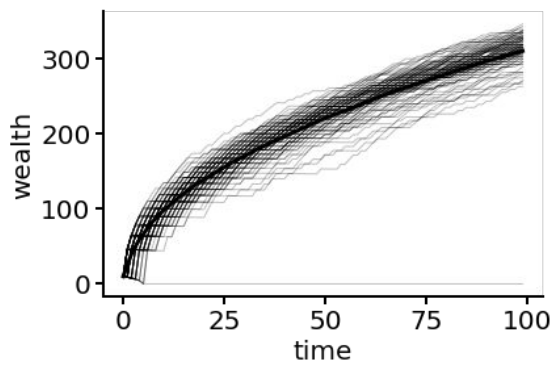
multiplicative dynamic



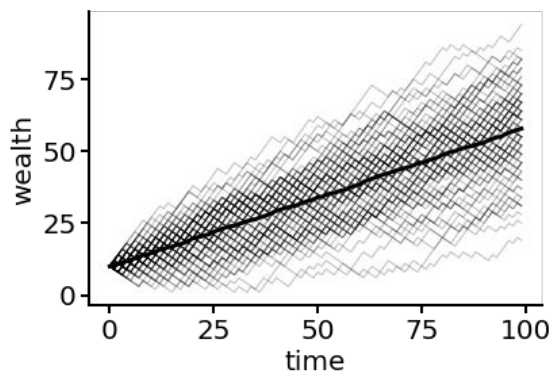
risk attitude  $\eta$



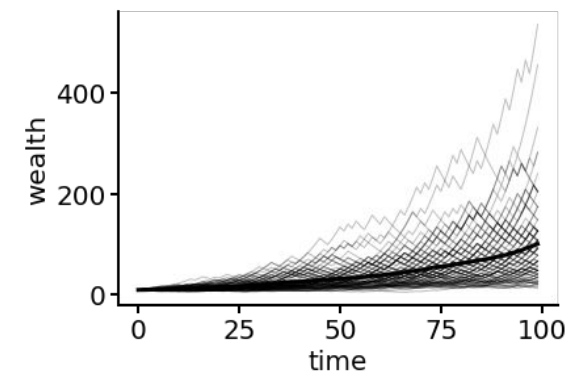
risk-seeking dynamic



additive dynamic



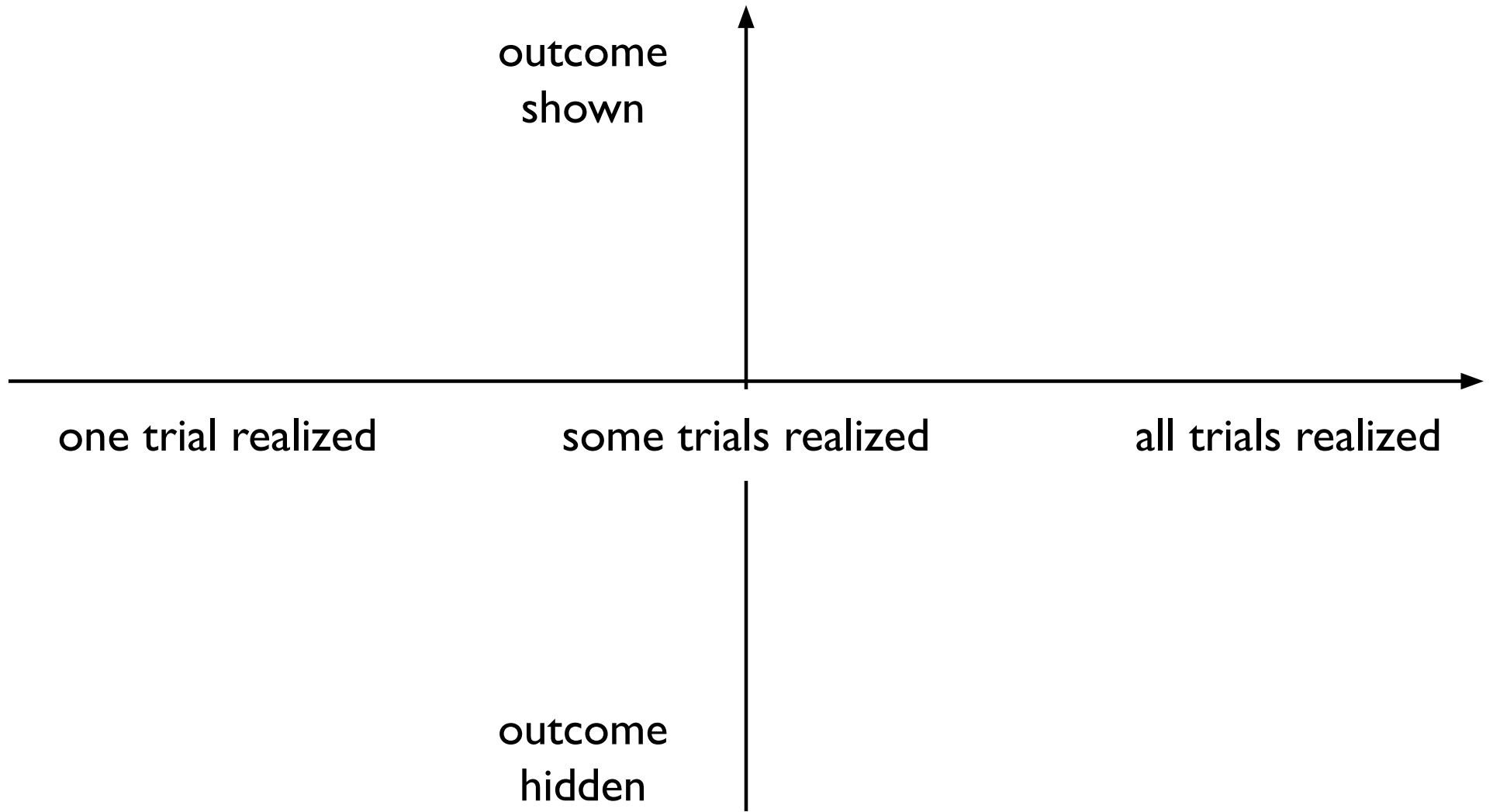
multiplicative dynamic

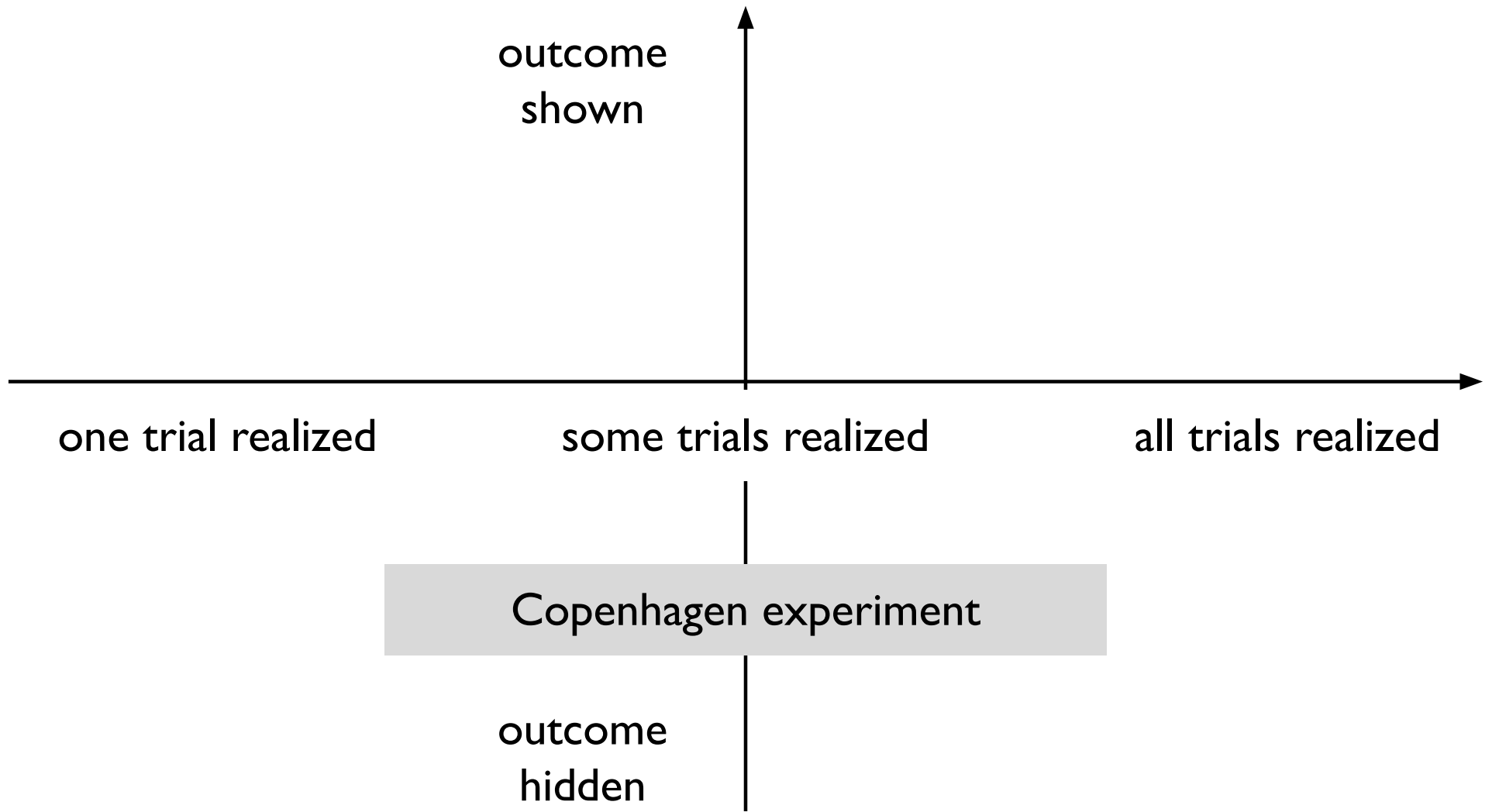


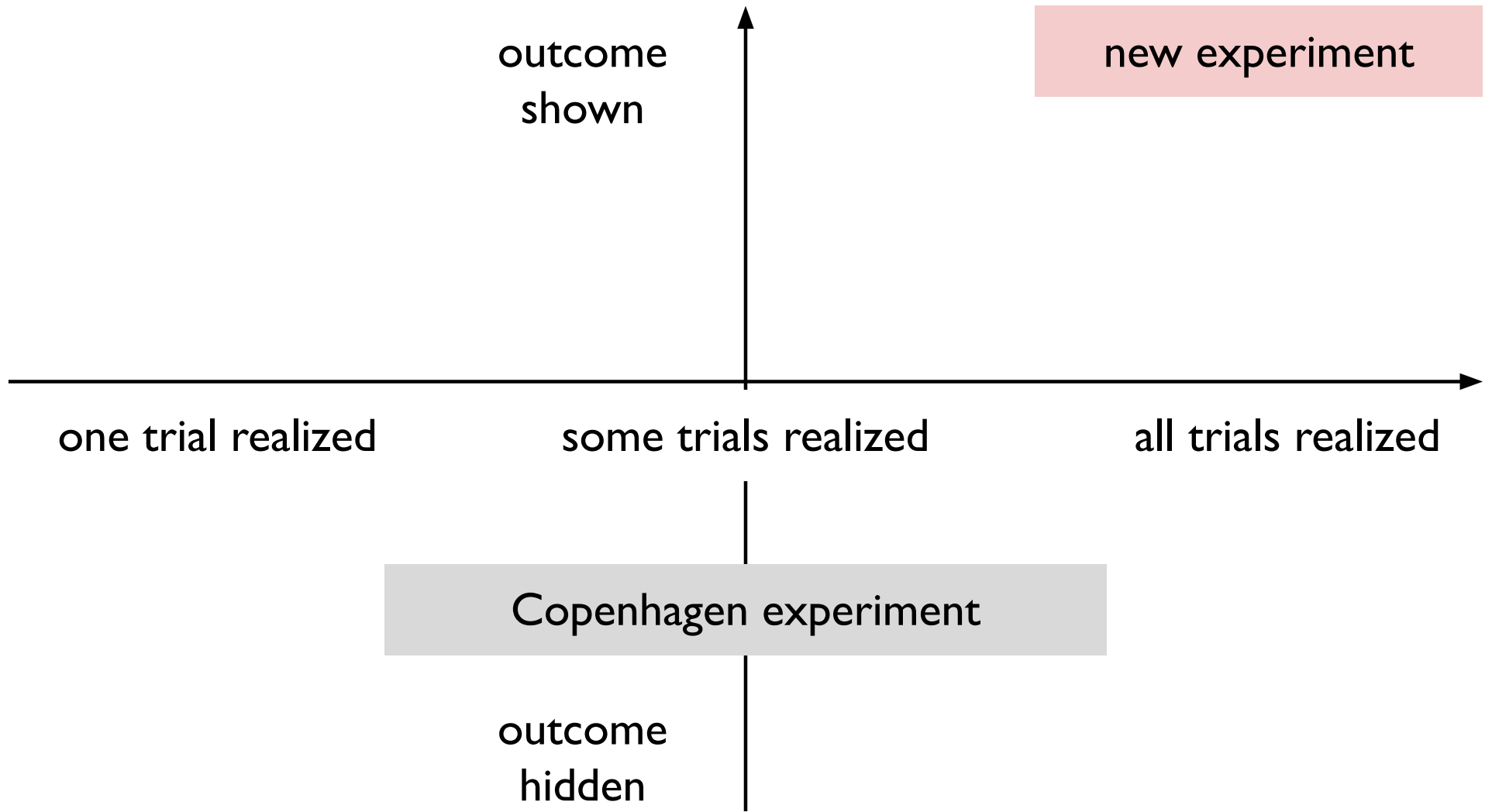
## Aim of the project

Improve the Copenhagen experiment.

- add more wealth dynamics
- show outcomes & realize all trials
- optimize a design







## Fully realized paradigm

### pros

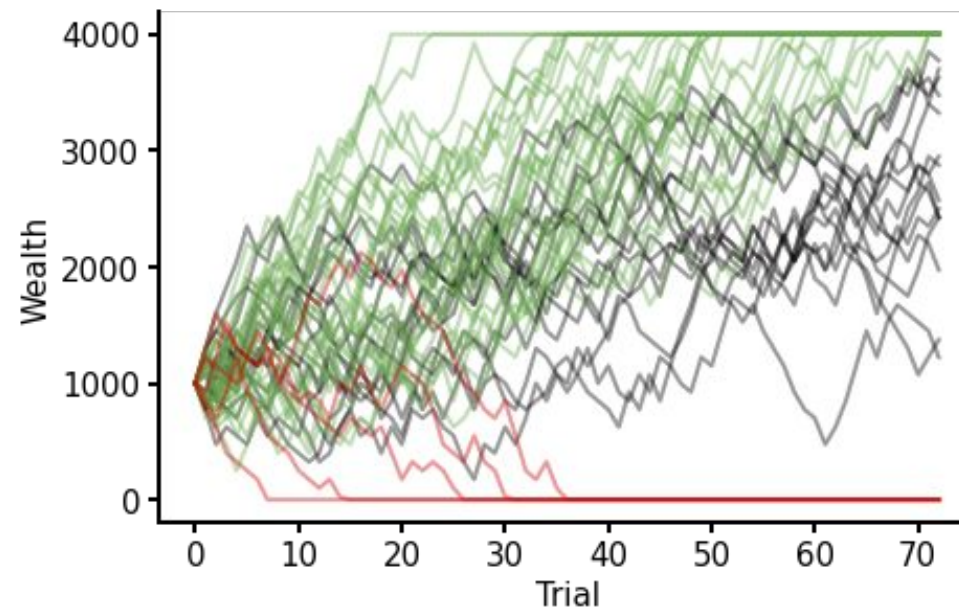
- realistic
- engaging
- temporal effects
- wealth dependency effects

### challenges

- difficult to control bankruptcy
- difficult to control payout
- potential confounds from emotions & probability matching

## Cost bounds

When subjects bankrupt or exceed max payout intended for single participant the experiment ends.



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## Disagreement

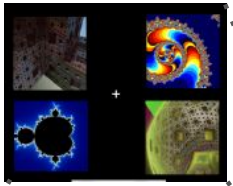
Good experiment would provide data allowing to discriminate between competing models.

## Disagreement

Simulated EE agent responses should differ from EUT agent responses.

# Disagreement

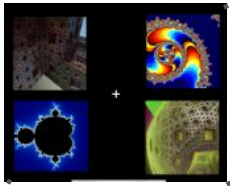
Simulated EE agent responses should differ from EUT agent responses.



- 1
- 2
- 3
- 4
- 5
- 6
- 7
- 8
- 9
- 10

# Disagreement

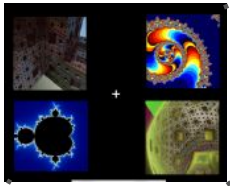
Simulated EE agent responses should differ from EUT agent responses.



	1	2	3	4	5	6	7	8	9	10
EUT	L	R	R	R	L	L	R	L	R	R
EE	L	L	R	R	L	R	R	L	L	R

# Disagreement

Simulated EE agent responses should differ from EUT agent responses.

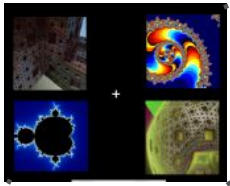


	1	2	3	4	5	6	7	8	9	10
EUT	L	R	R	R	L	L	R	L	R	R
EE	L	L	R	R	L	R	R	L	L	R

$$d = 0.3$$

# Disagreement

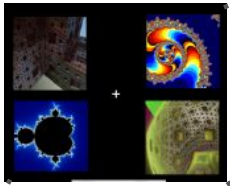
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	1	2	3	4	5	6	7	8	9	10
EUT	L	R	-	-	-	-	-	-	-	-
EE	L	L	R	-	-	-	-	-	-	-

# Disagreement

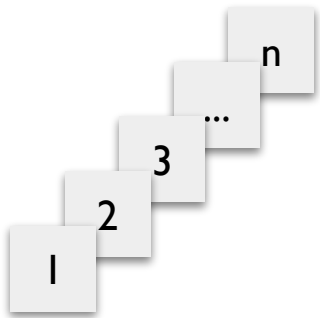
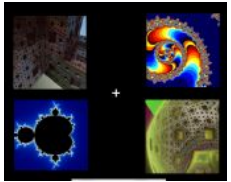
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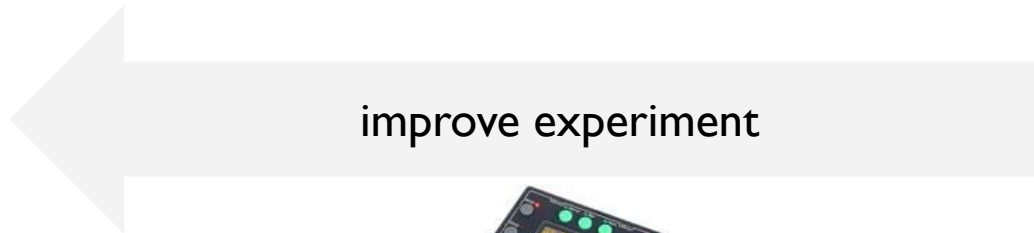
	1	2	3	4	5	6	7	8	9	10
EUT	L	R	-	-	-	-	-	-	-	-
EE	L	L	R	-	-	-	-	-	-	-

$$d = 0.1$$

# Optimization framework



experimental  
design

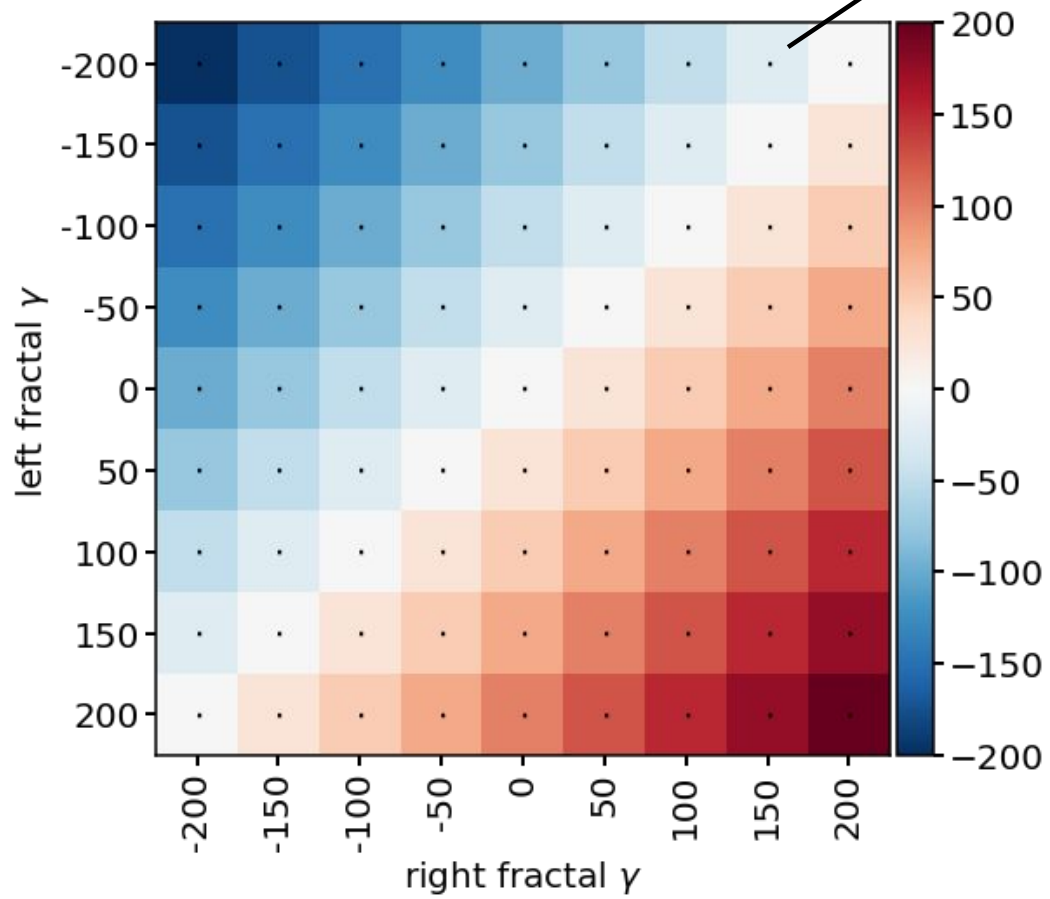


*d*

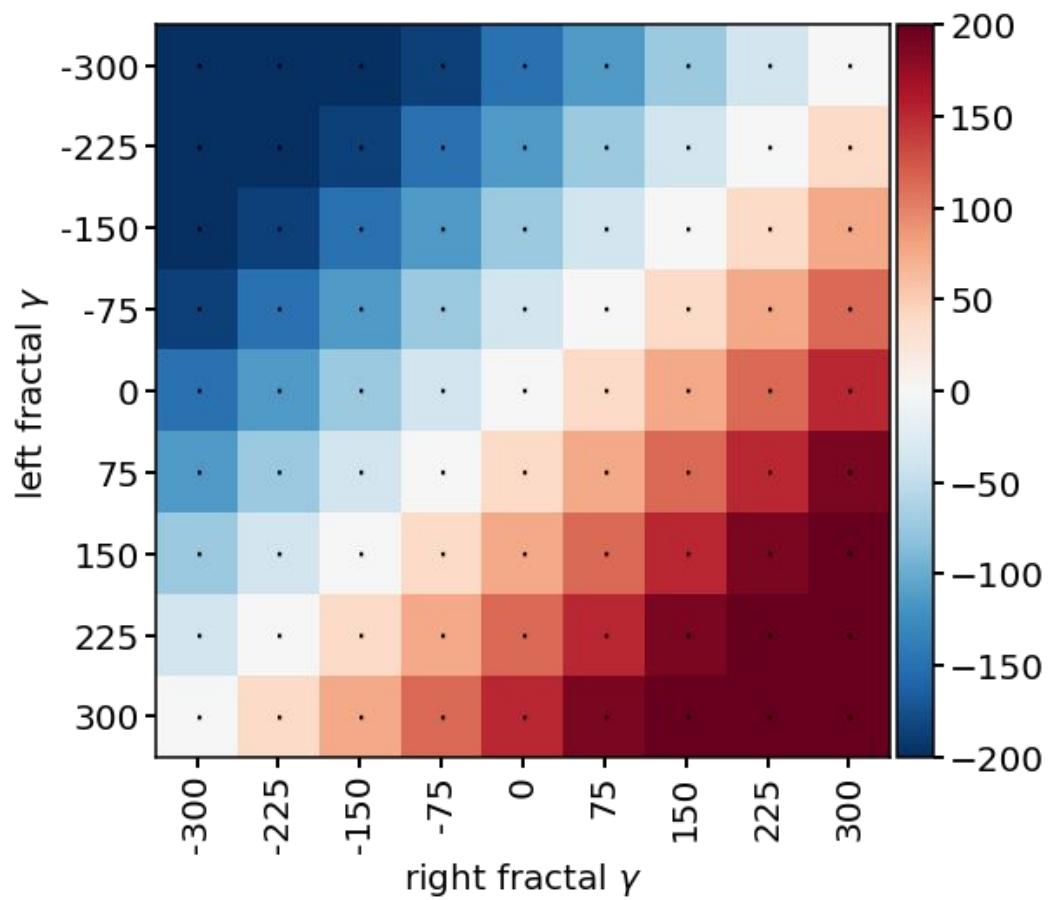




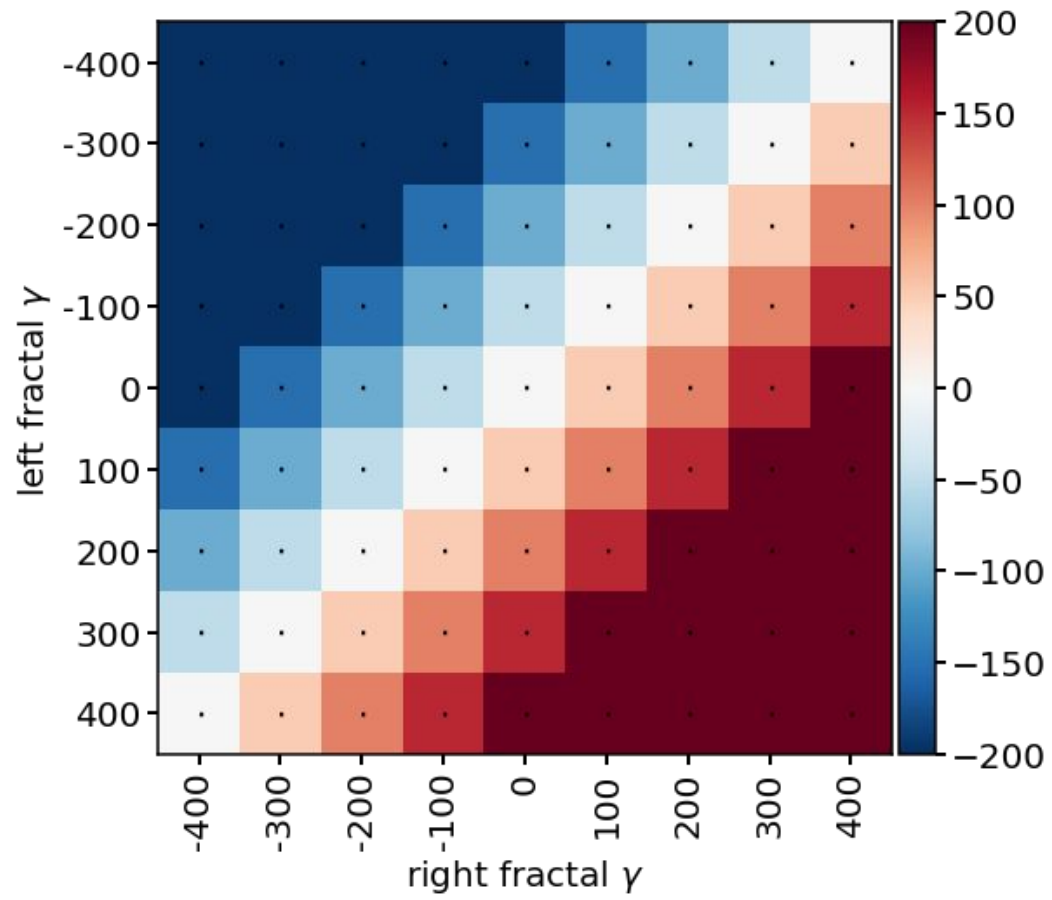
# Gamble space



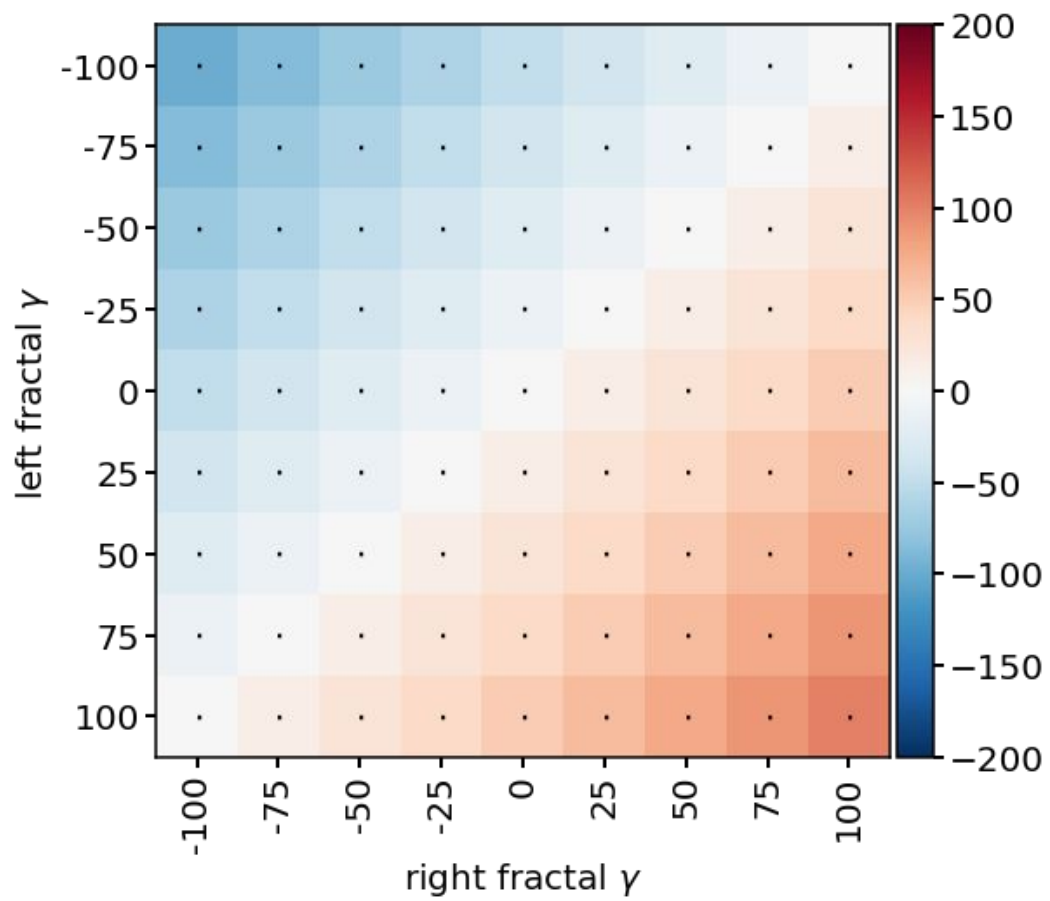
# Gamble space



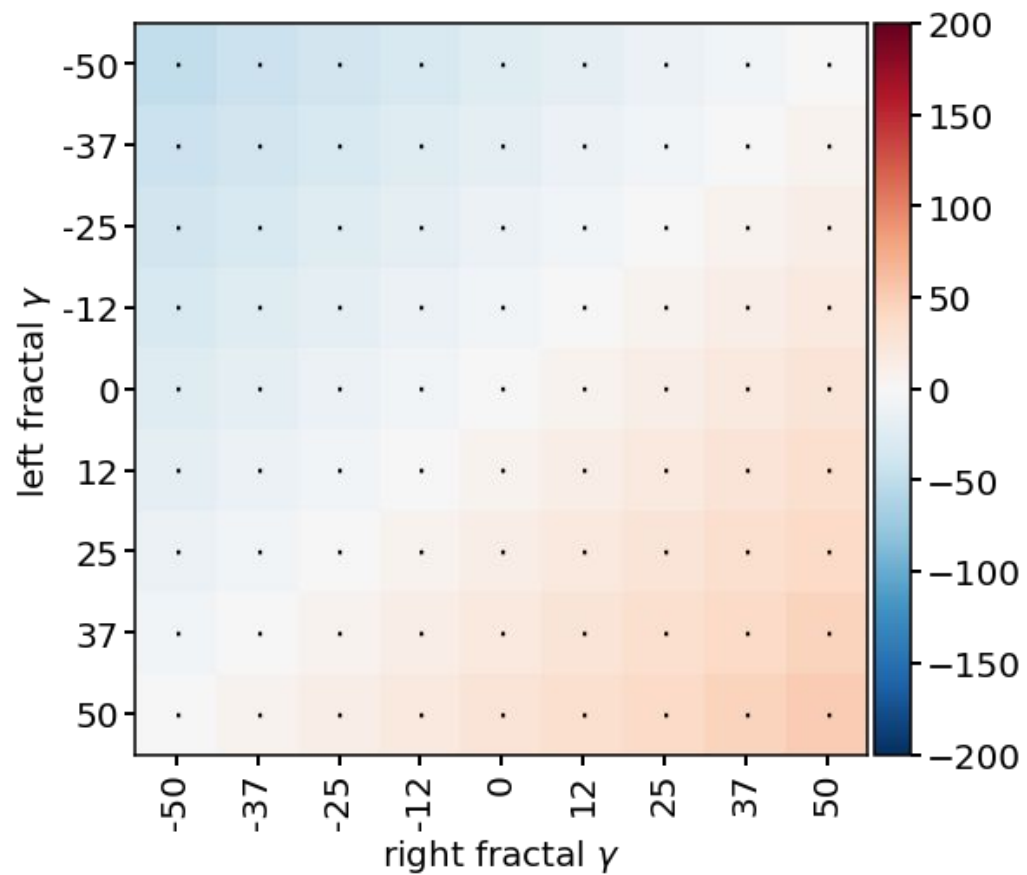
# Gamble space



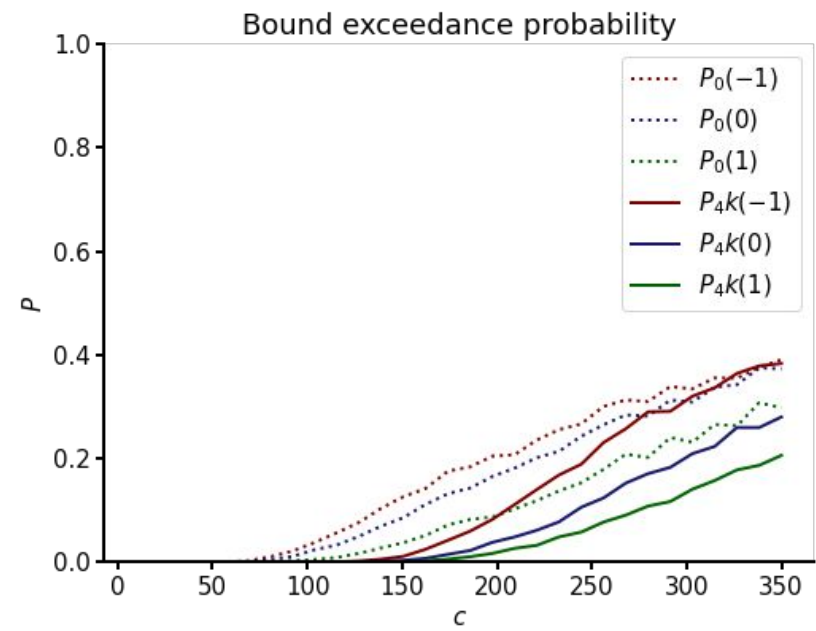
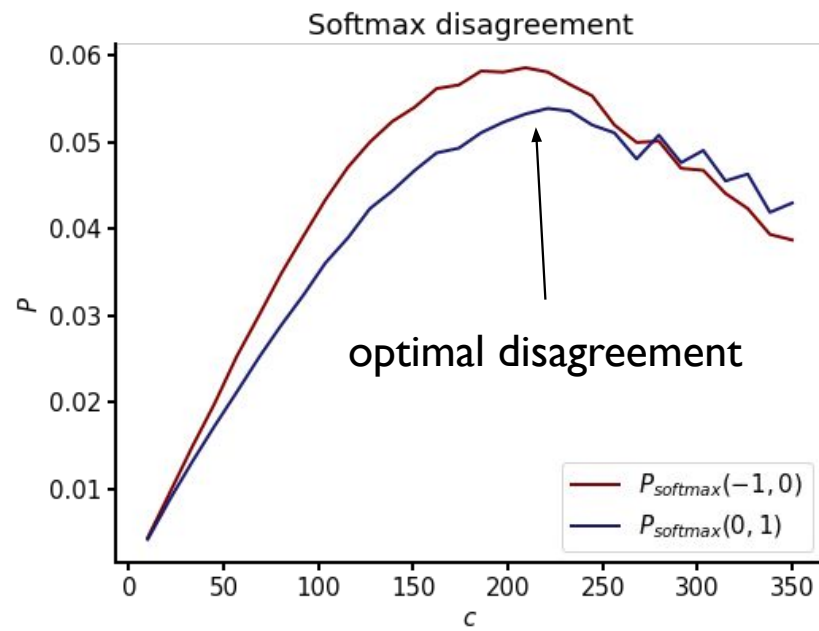
# Gamble space



# Gamble space



# Optimized growth rate



**The end**