Interpolation inequality
Explicit universal estimate of finite Morse index solutions to polyharmonic equation

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Abstract

Let Ω be a domain of $\mathbb{R}^n$, $n \geq 2$, $u \in W^{r,p}_{loc}(\Omega)$ with $r,\ p \geq 2$. We establish interpolation inequality estimating $\int_\Omega |\nabla^q u|^p |\nabla^{r-q}(\psi)|^p$ where ψ is a "general" cut-off function and $1 \leq q \leq r-1$. When $p = 2$, we explore this inequality to provide an **explicit** universal estimate of finite Morse index solutions to

$$(-\Delta)^r u = f(x,u),\ \text{ in } \Omega.$$

Differently to [1, 2, 3, 4, 6, 7, 8, 10], We did not use a blow-up procedure which requires that Liouville-type theorem is available in the whole space and also $f(x,u)$ has an asymptotical behavior at infinity like $|u|^{q-1}u$. We propose here a direct proof under a large superlinear and subcritical growth conditions on f using a variant of the Pohozaev identity and a delicate boot strap argument from local L^p-$W^{2r,p}$ estimate. Particularly, we show that the universal constant (which does not depend on Ω) evolves as a polynomial function of the Morse index.
Also by virtue of our interpolation inequality, we extended and improved the integral estimate obtained in [6], to provide nonexistence results in the subcritical range of stable and stable at infinity weak solutions to the following p-polyharmonic equation

$$\Delta^r_p u = |u|^{q-1}u \text{ in } \mathbb{R}^n, \text{ where } r \geq 2,\ p \geq 2 \text{ and } n > rp.$$

Precisely we removed the exponential growth condition U_1 imposed on unbounded sable solutions in [6]. At last, we revised previous local L^p-$W^{2r,p}$ estimate stated in [5, 9].

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