Hamiltonian truncation methods
for the study of
continuous quantum field dynamics

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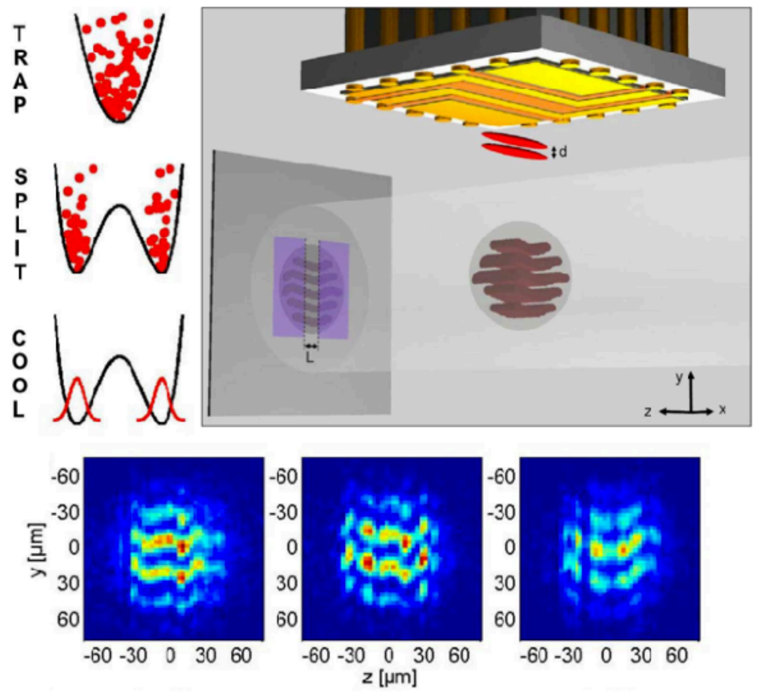
Freie Universität Berlin

Outline

Introduction

- motivation
- quantum many-body dynamics
- why one spatial dimension?



An analog quantum field simulator

- how cold-atom experiments can help us solve the mysteries of QFT

Classical "simulation" of a quantum simulator

- a numerical RG method for QFT

Effects of topological excitations in and out of equilibrium

- solitons and non-locality

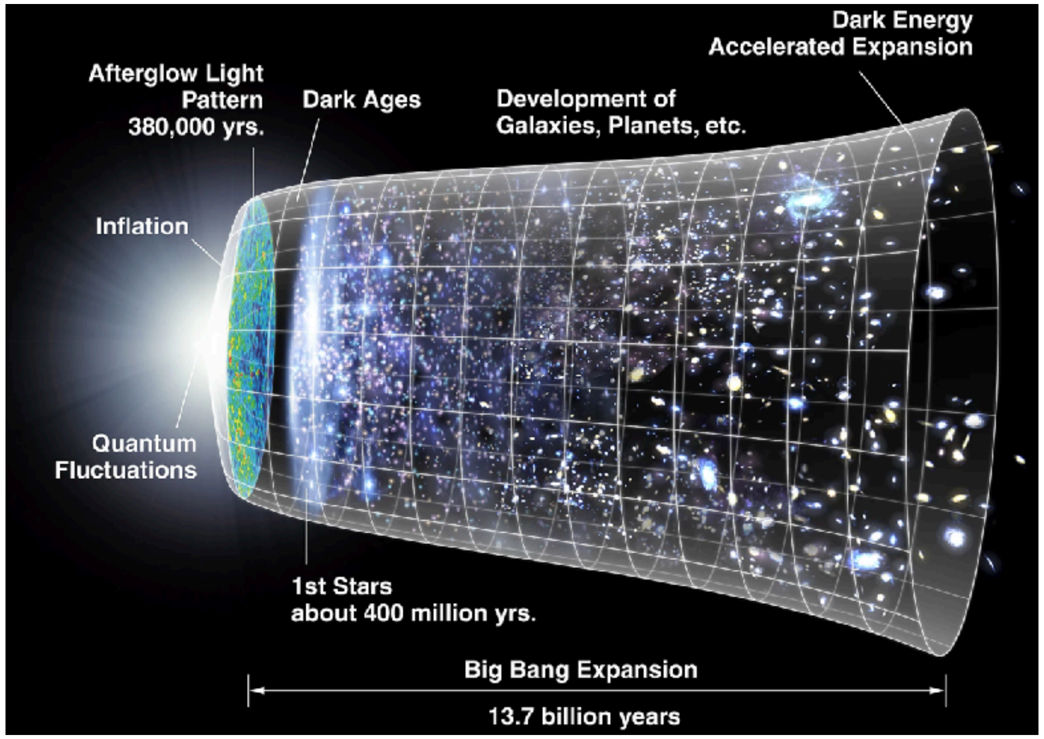
Quantum equilibration and recurrences

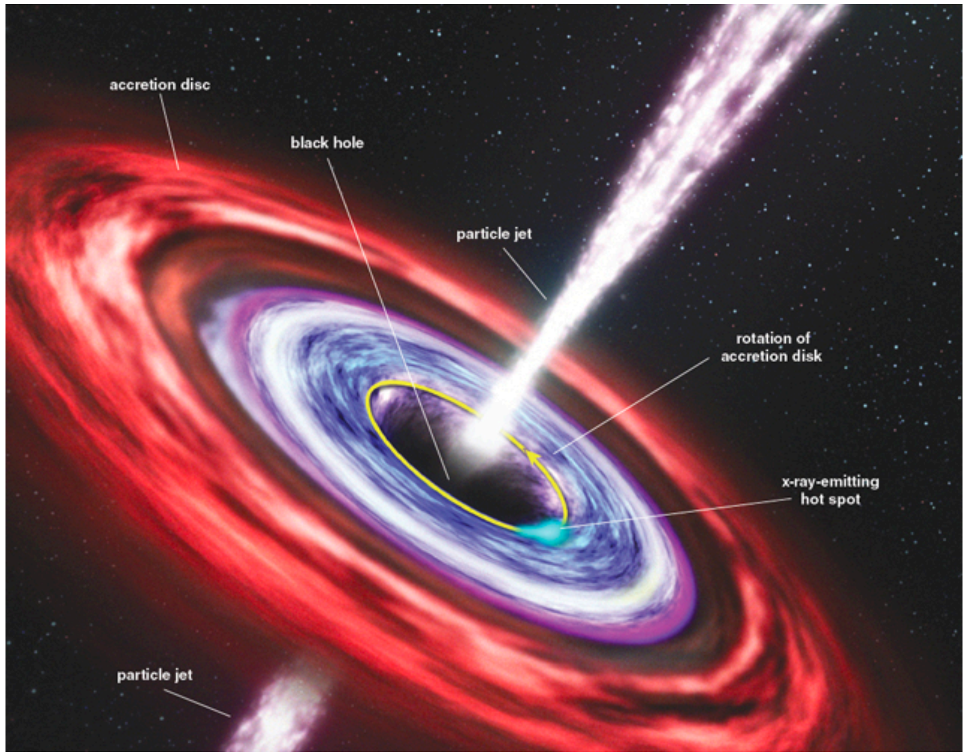
- a quantum central limit theorem

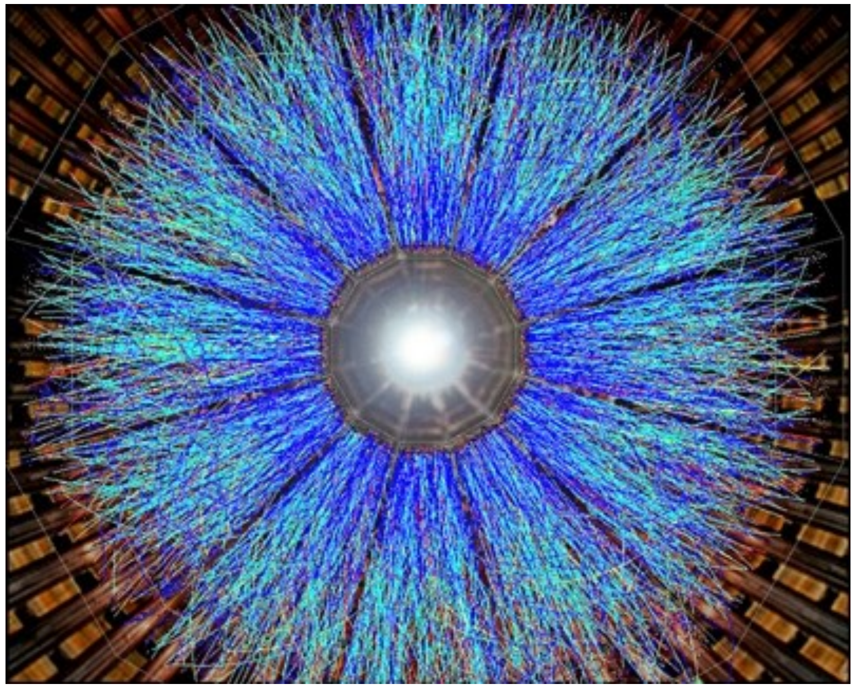
Quantum Chaos

- level spacing & eigenvector statistics

Motivation

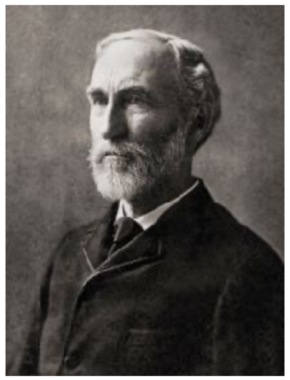

Afterglow Light Pattern 380,000 yrs.
Dark Ages
Development of Galaxies, Planets, etc.
Dark Energy Accelerated Expansion
Inflation
Quantum Fluctuations
1st Stars about 400 million yrs.
Big Bang Expansion
13.7 billion years

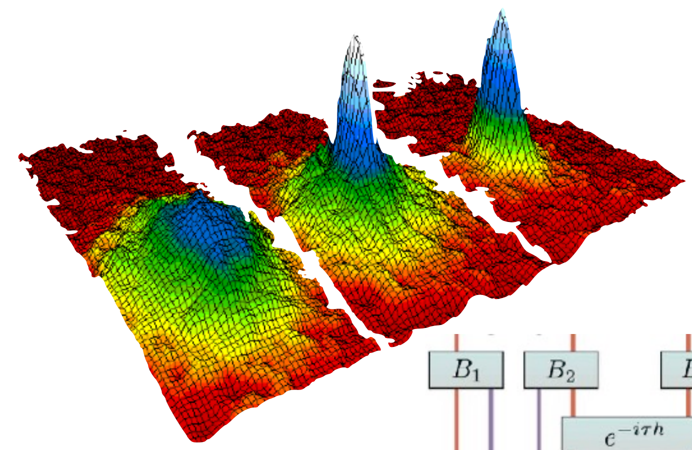

accretion disc
black hole
particle jet
rotation of accretion disk
x-ray-emitting hot spot
particle jet

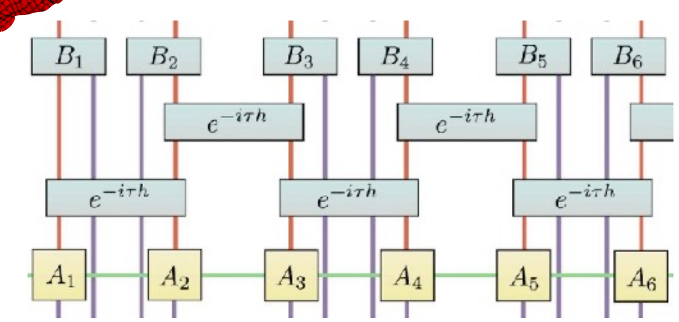


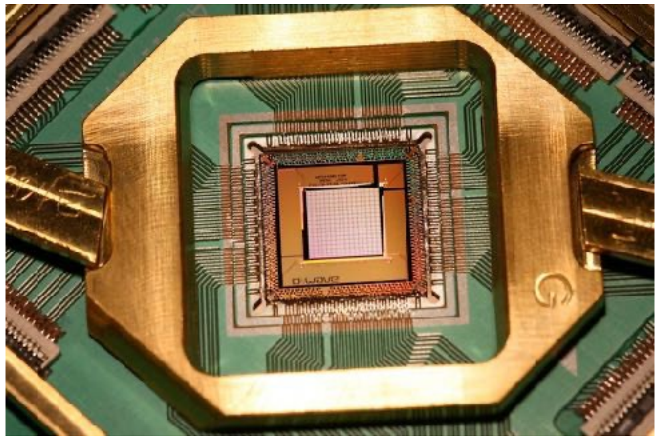
Motivation

- Quantum equilibration is a *fundamental* and *long-standing* question of statistical mechanics
- Reach the *ultimate limits of classical thermodynamics* expectations and unveil *novel quantum effects* at macroscopic level
- Recent progress in experimental (*ultra-cold atoms*) and numerical (*tDMRG, MPS*) techniques for study of quantum many-body dynamics
- Applications to quantum technologies: quantum thermal engines, quantum information processing & computing



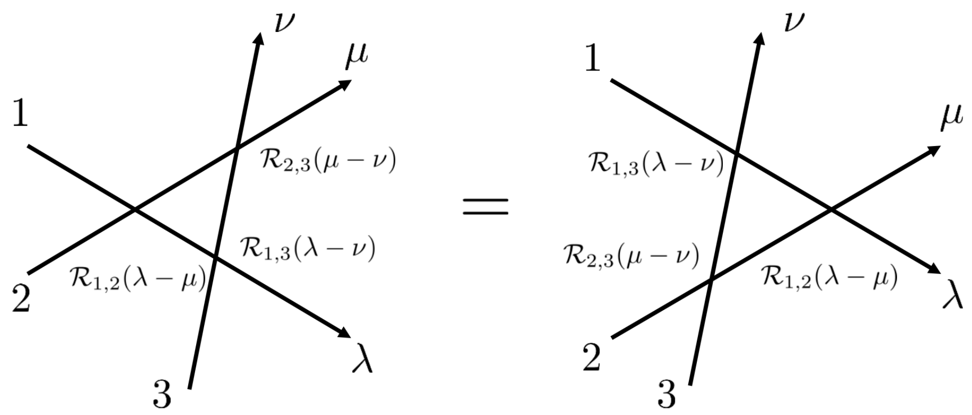




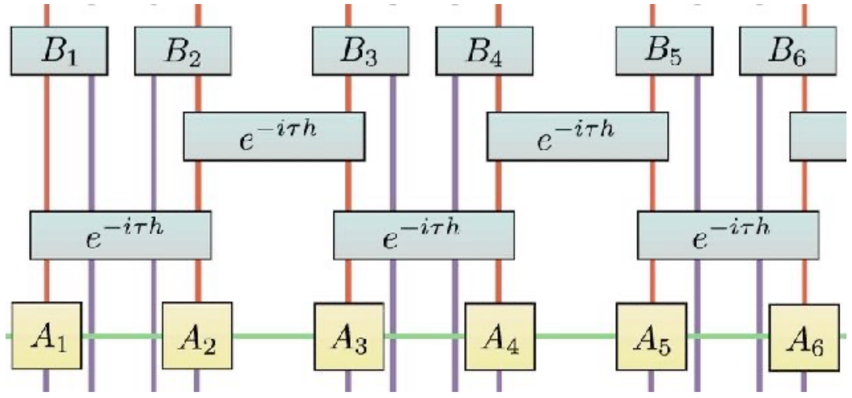


Why focus on one spatial dimension?

- Exact analytical tools
  - Integrability
  - Dualities



- Efficient numerical tools



An analog quantum field simulator

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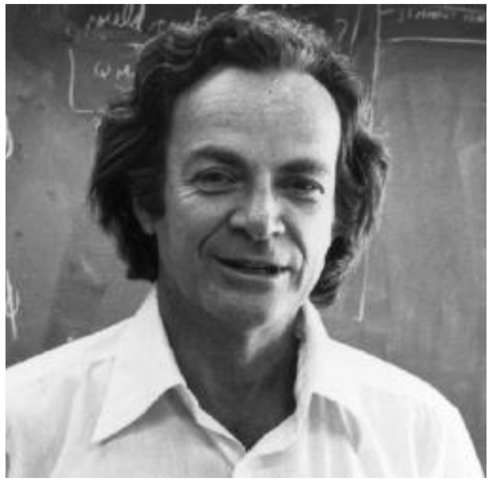
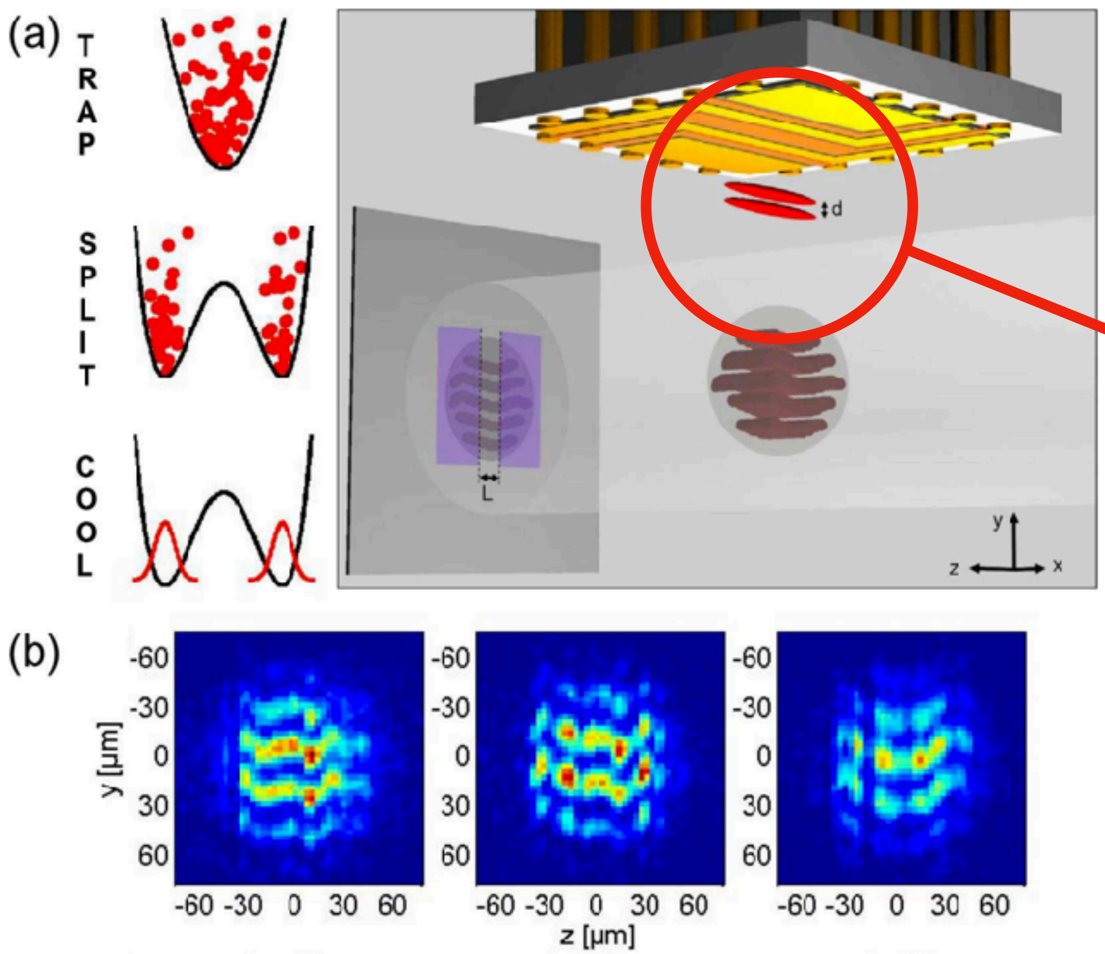
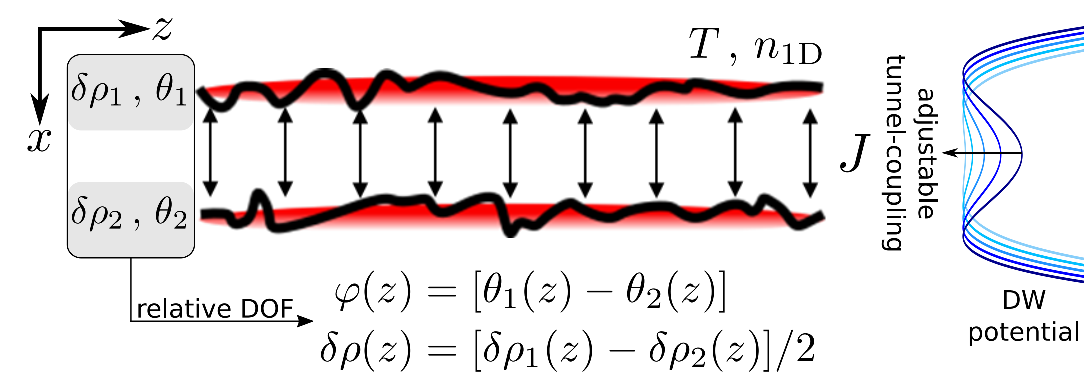


Quantum simulator

A quantum system of many particles is described by a Hilbert space whose dimension is exponentially large in the number of particles. Therefore, the obvious approach to simulate such a system requires exponential time on a classical computer. However, it is conceivable that a quantum system of many particles could be simulated by a quantum computer using a number of quantum bits similar to the number of particles in the original system.

An analog quantum field simulator





sine-Gordon model

$$H_{SGM} = \int \left(\frac{1}{2}(\partial_t \phi)^2 + \frac{1}{2}(\partial_x \phi)^2 - \frac{m^2}{\beta^2} \cos \beta\phi \right) dx$$

Schweigler et al., Nature (2017)

The quantum sine-Gordon model

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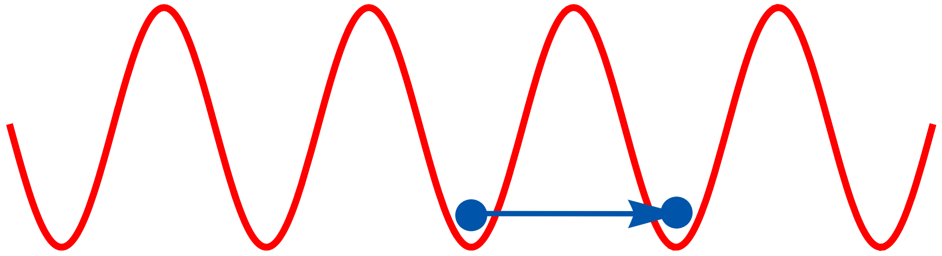
- Non-trivial field topology



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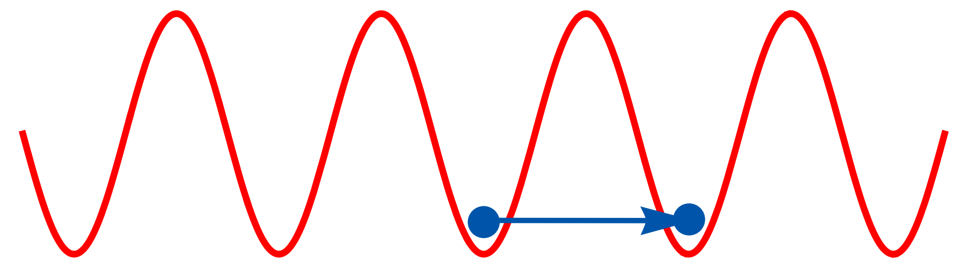
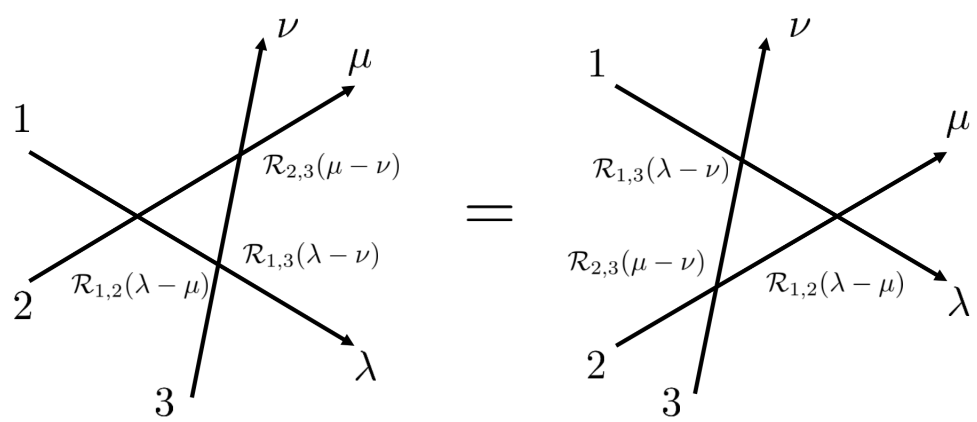
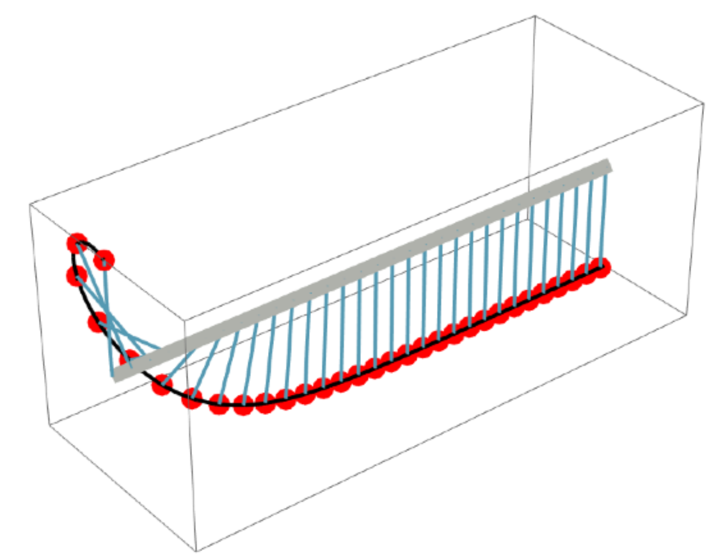
- Non-trivial field topology
- Solitons:
  - the field interpolates between different minima of the cosine potential
  - stable under collisions (integrability)
  - form breathers: multi-soliton bound states







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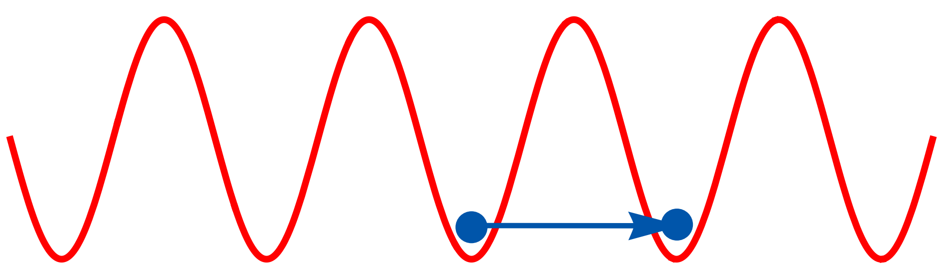

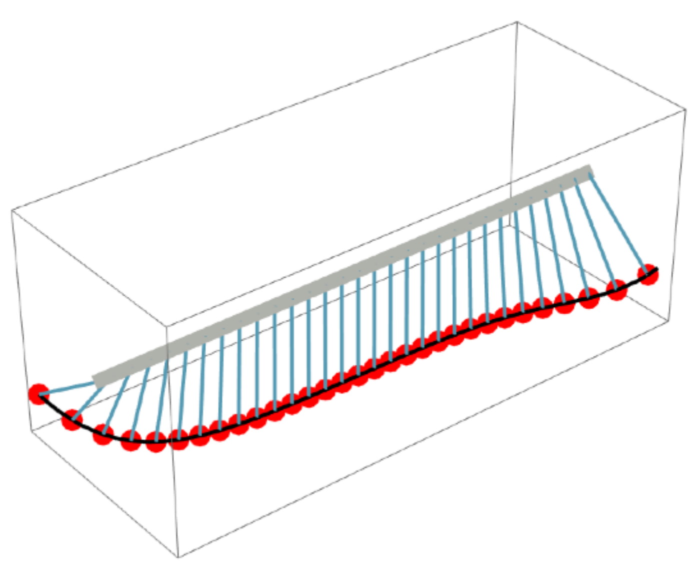

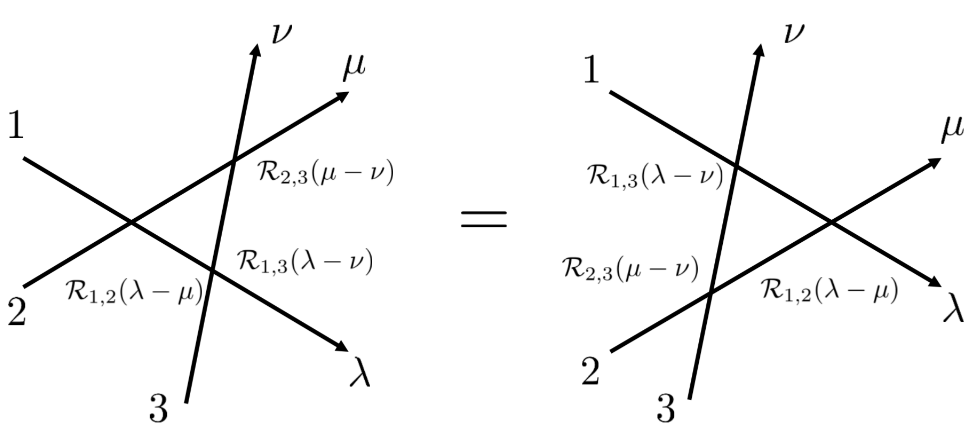

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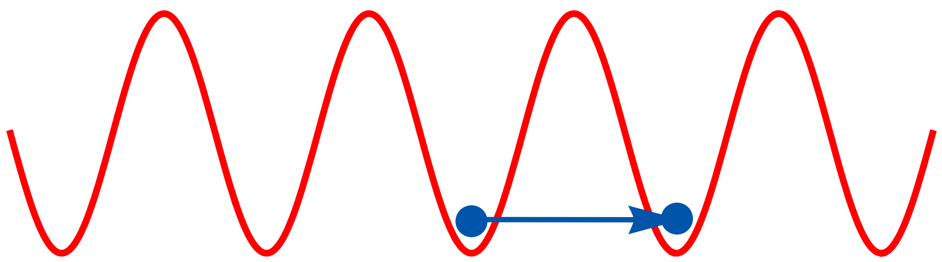

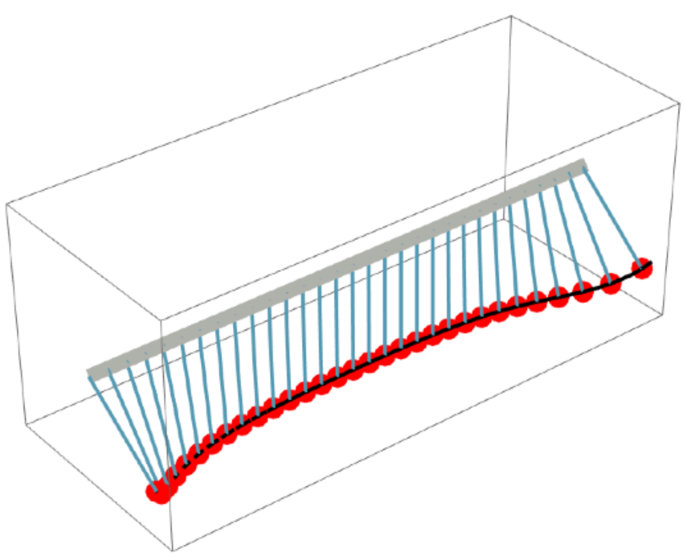

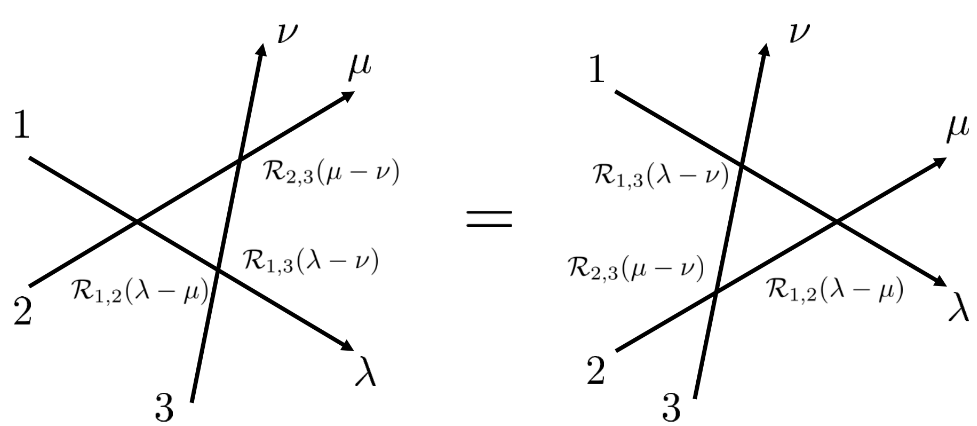

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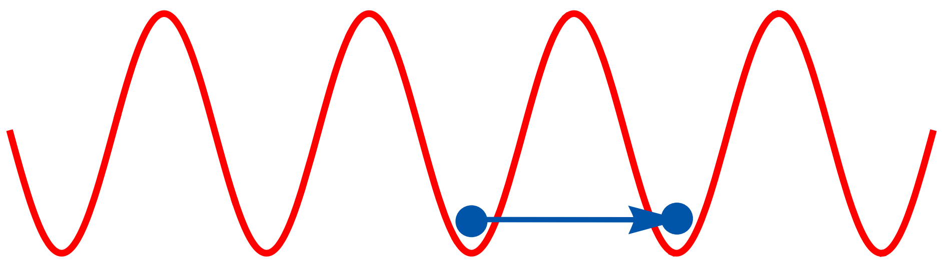
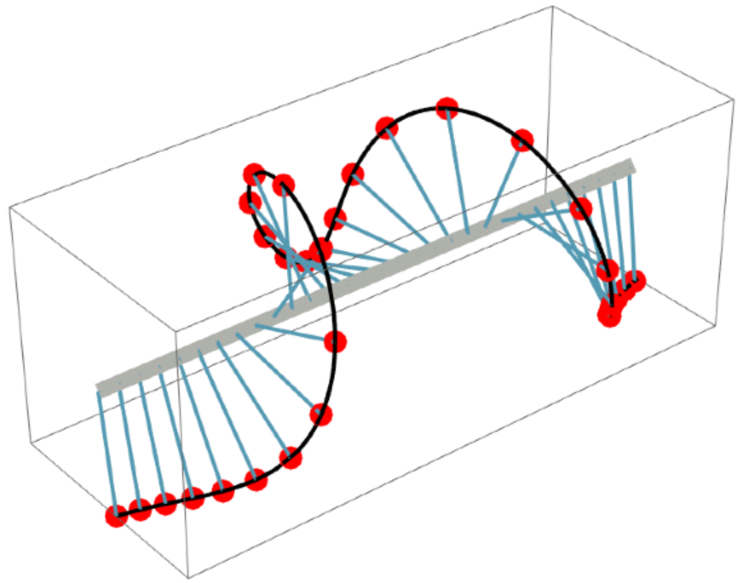
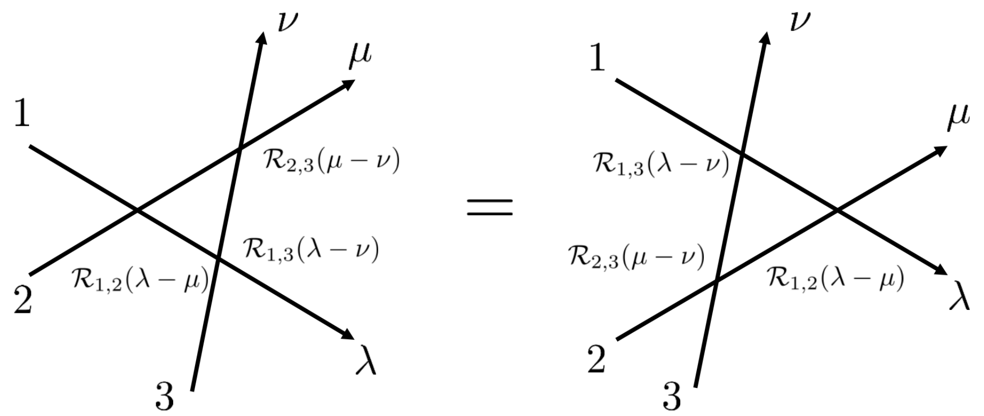
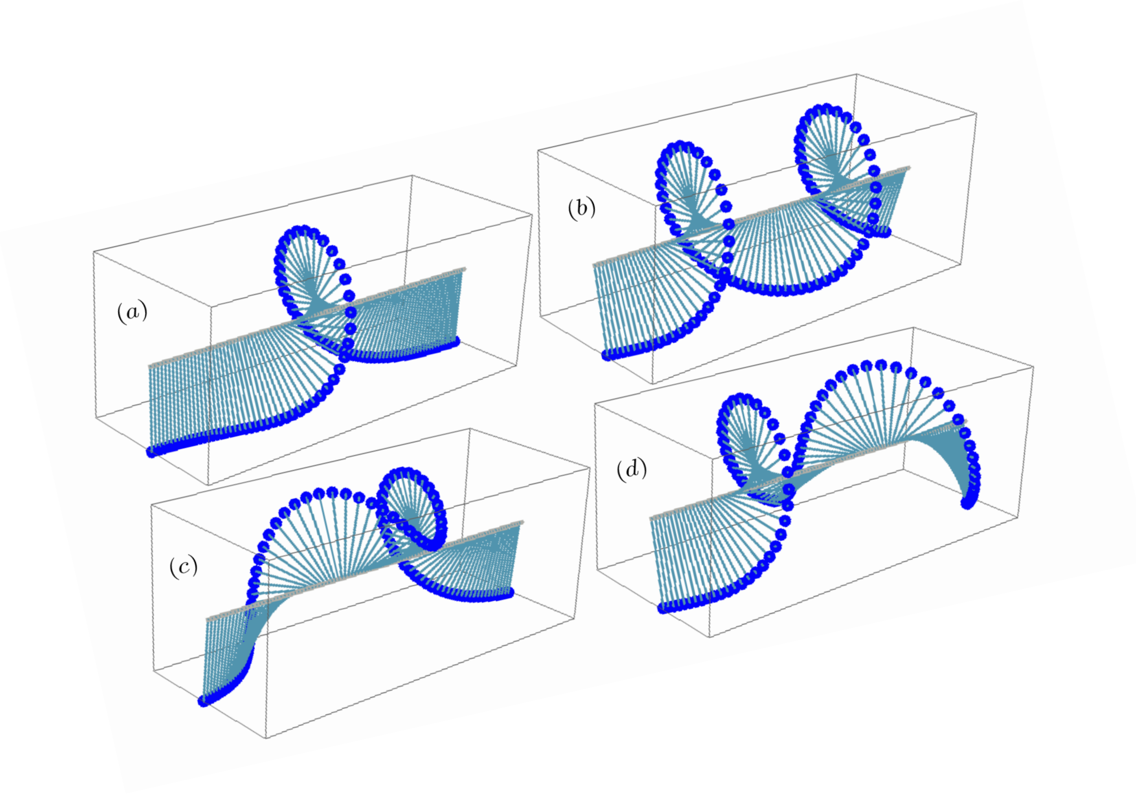
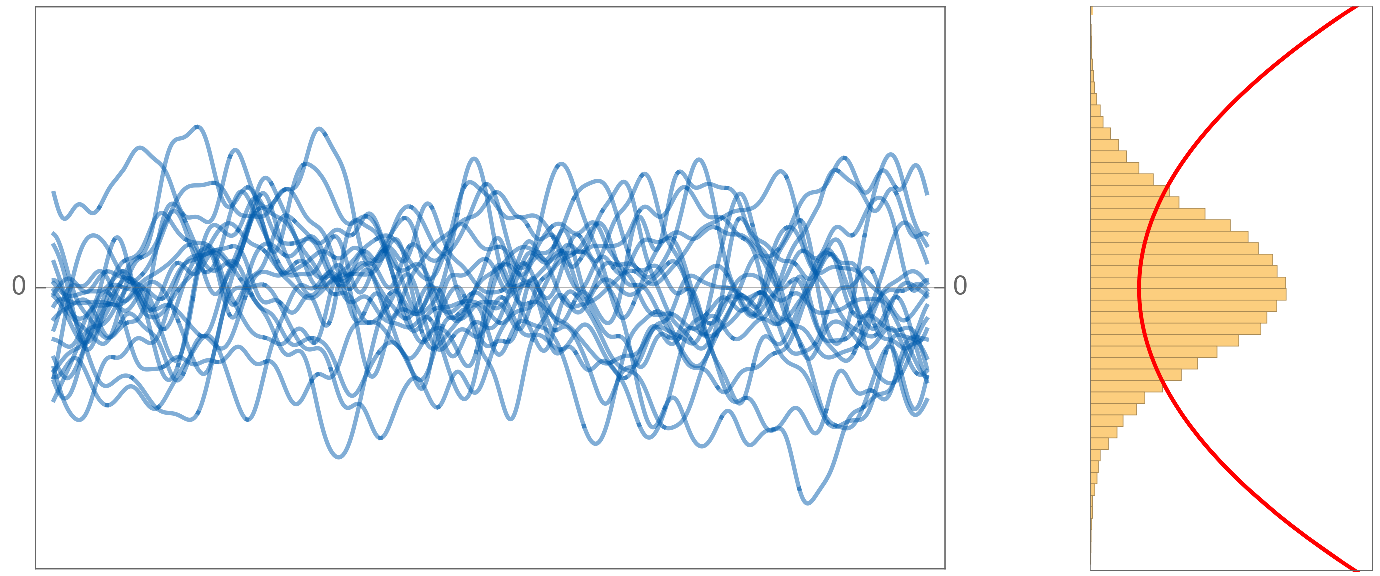


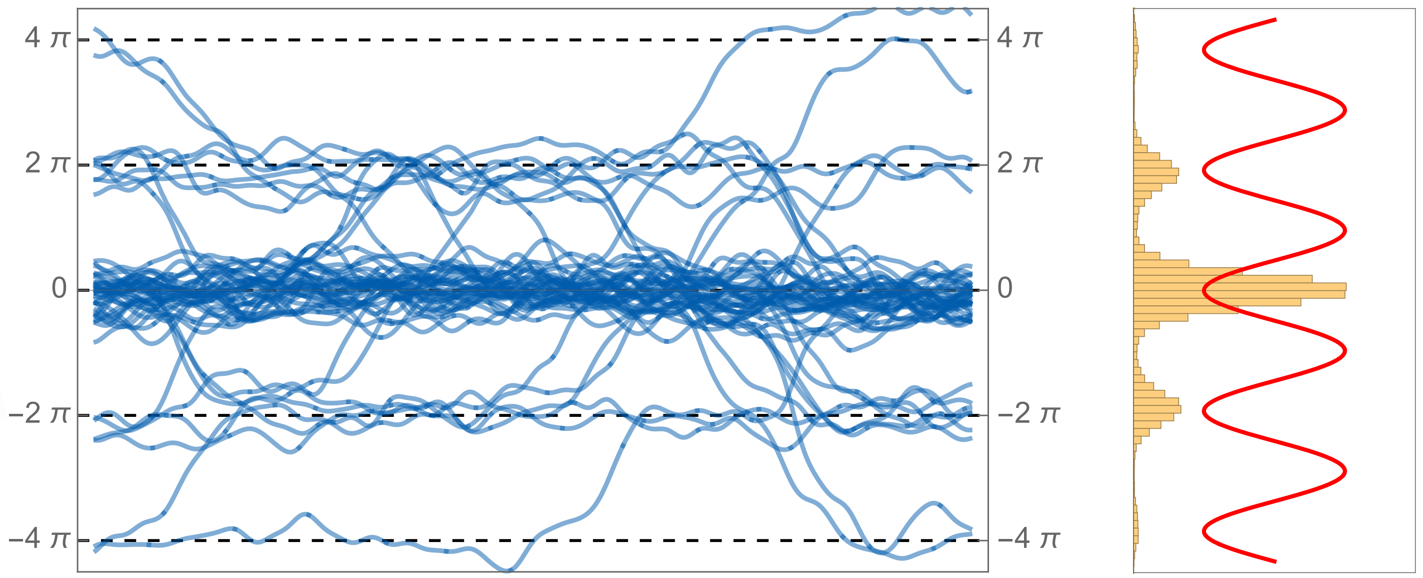




Equilibrium







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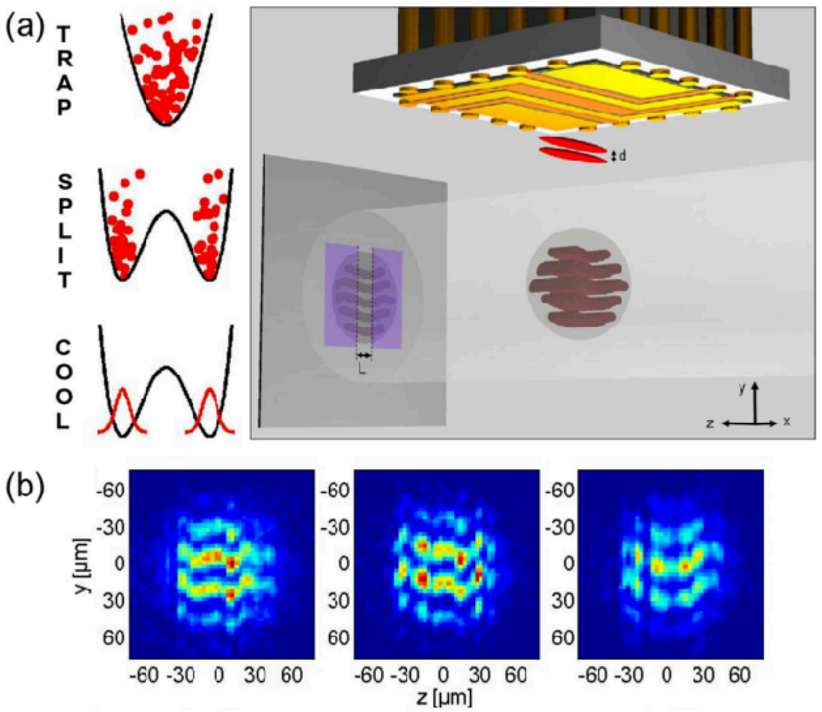

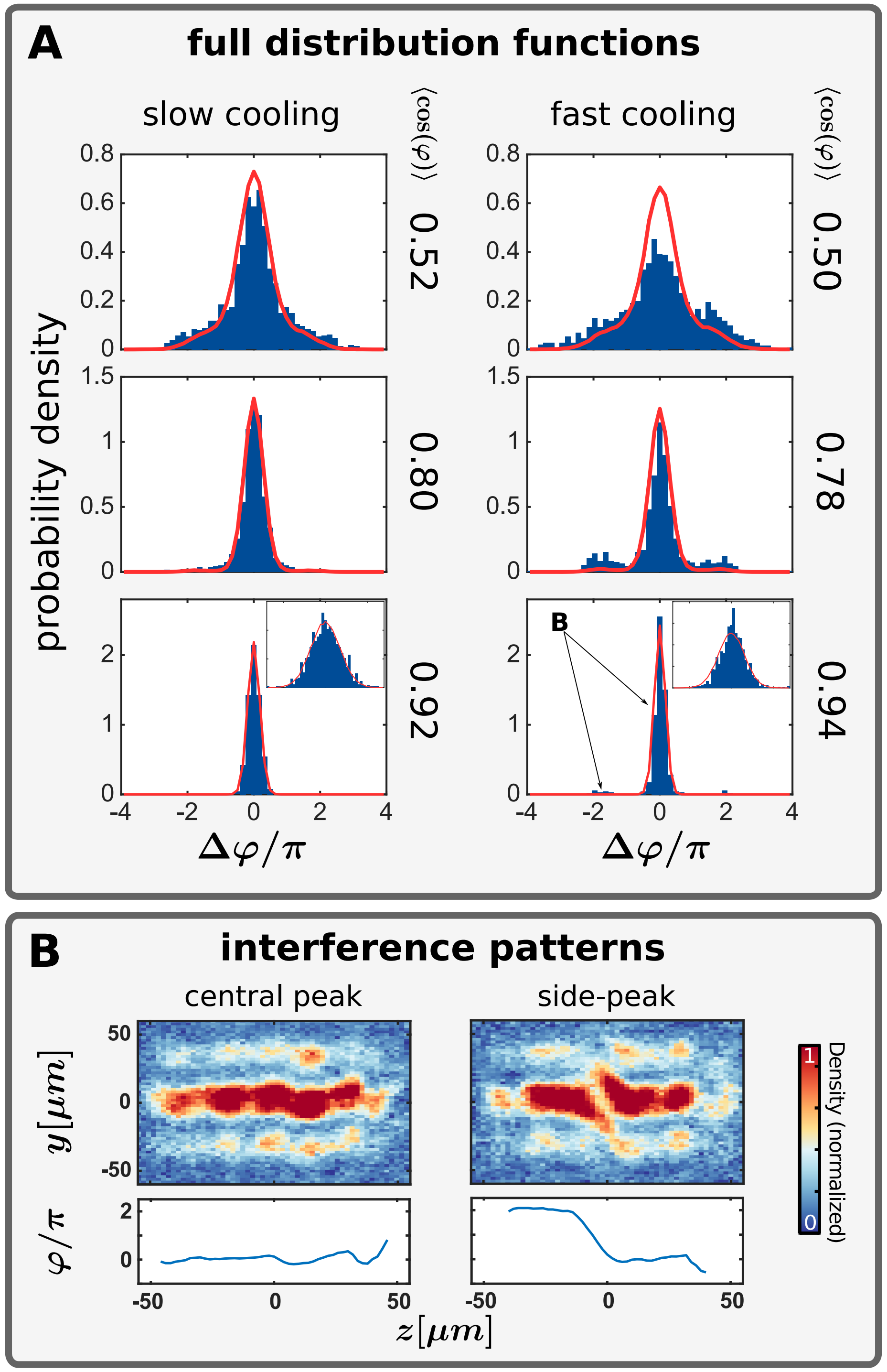

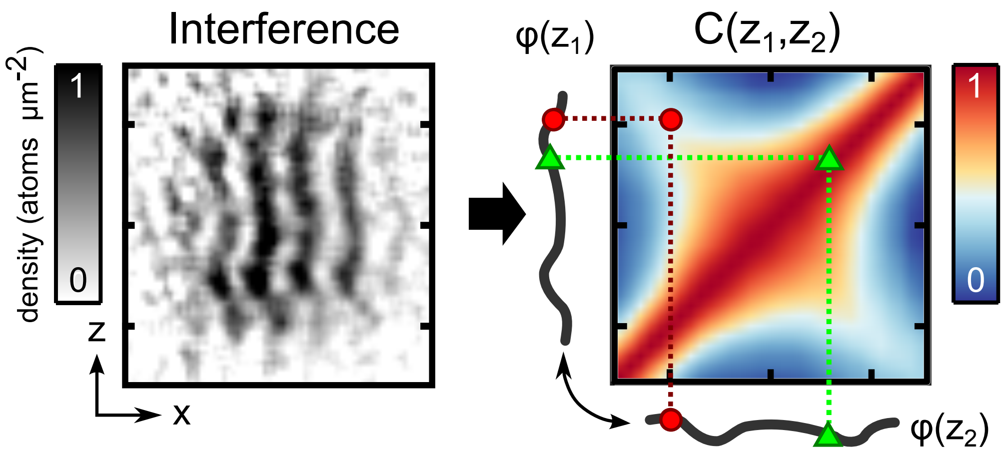

Schweigler et al., Nature (2017)

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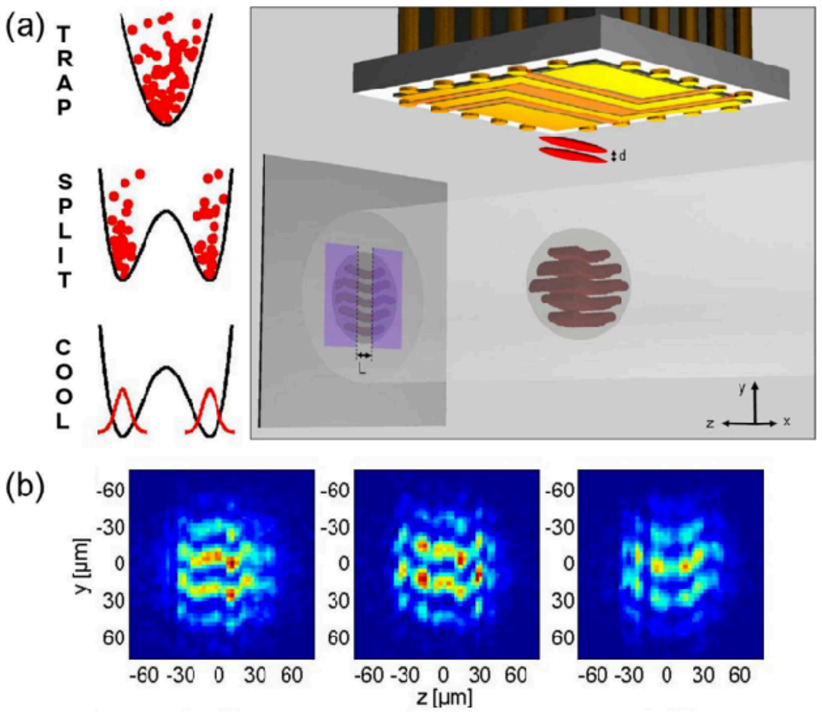
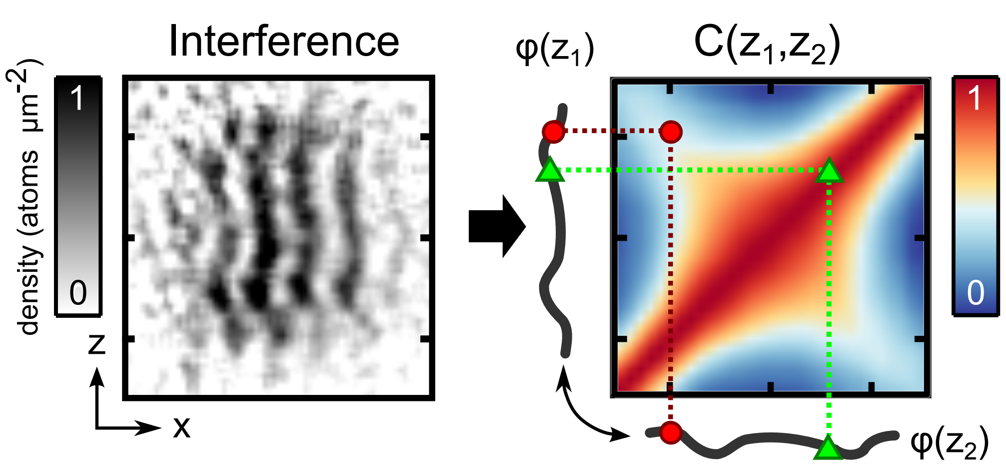
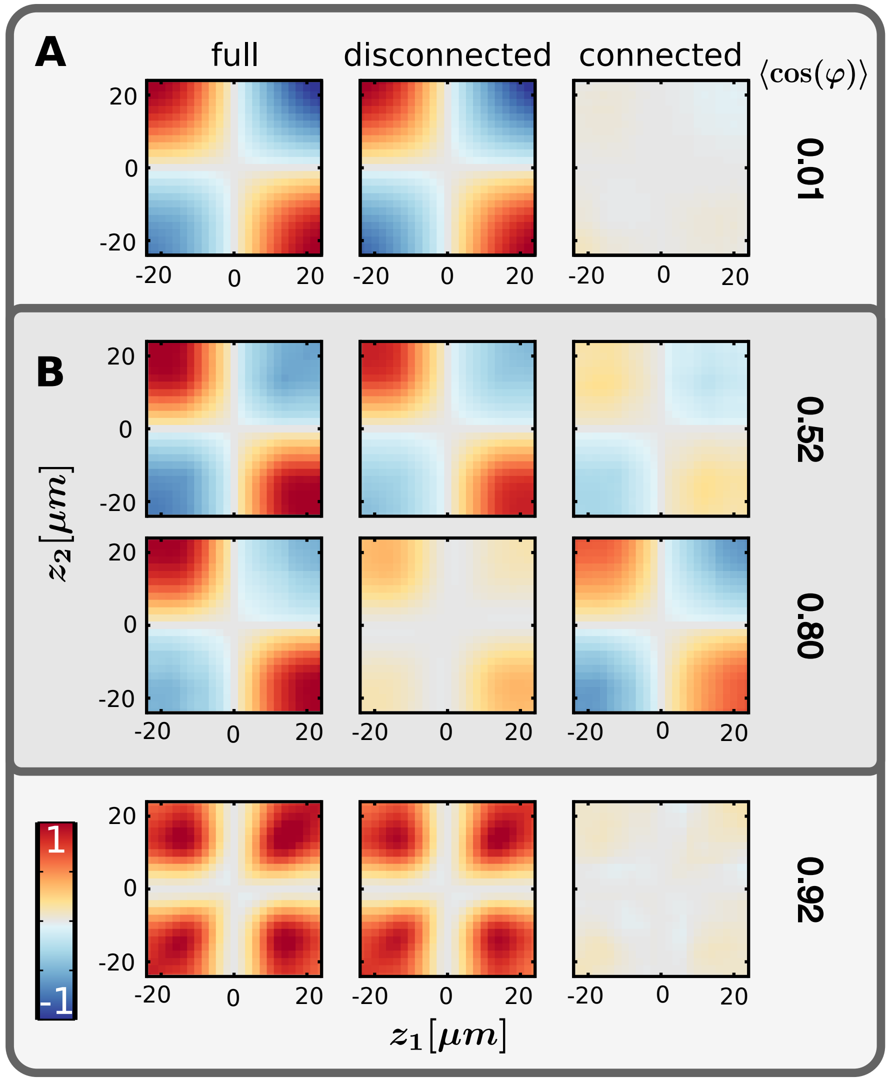


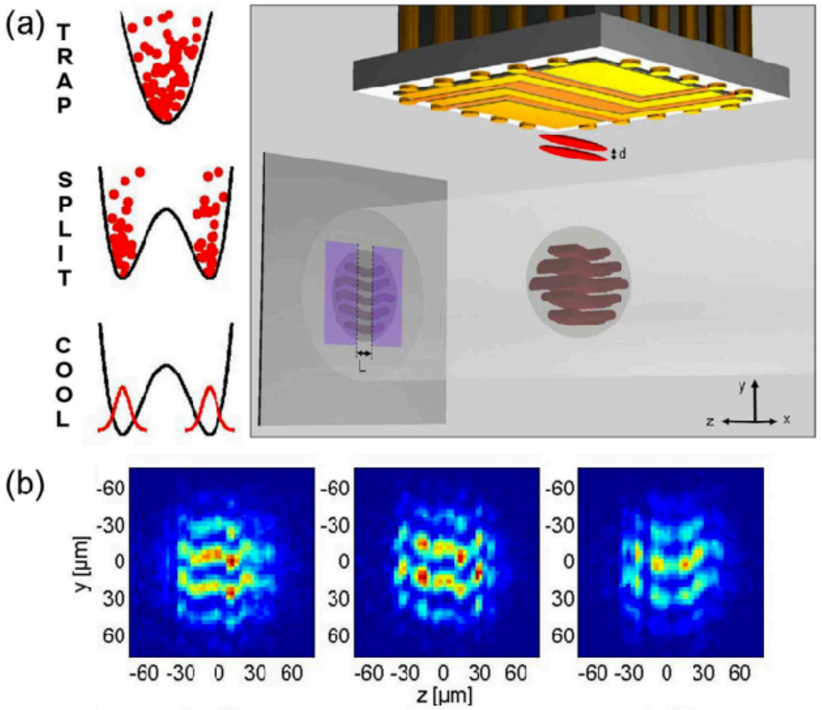


2-point correlation functions



Langen et al., Science (2015)

Dynamics



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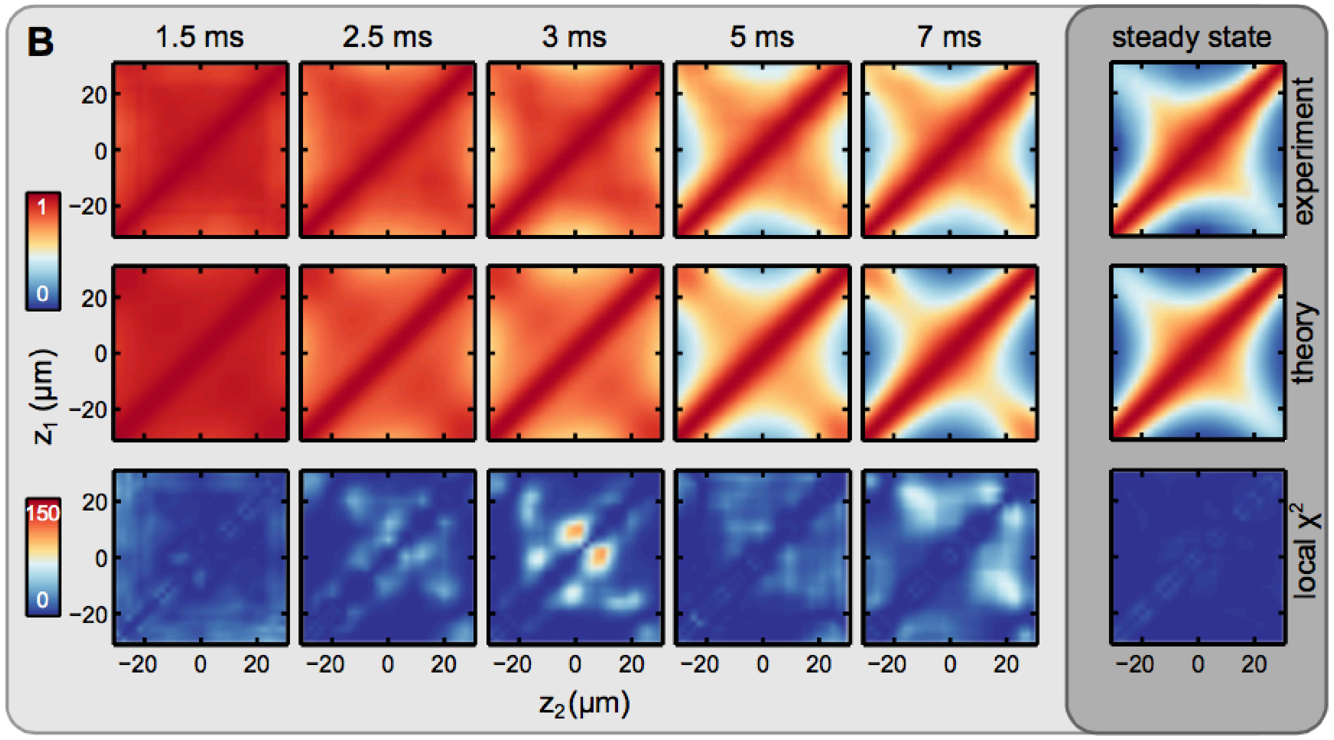

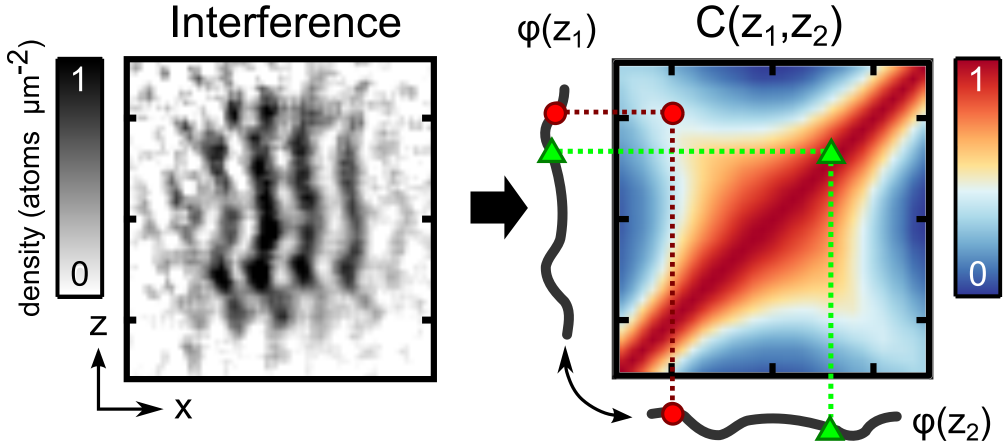

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Classical “simulation” of a quantum simulator

a numerical RG method for QFT

Theoretical problem

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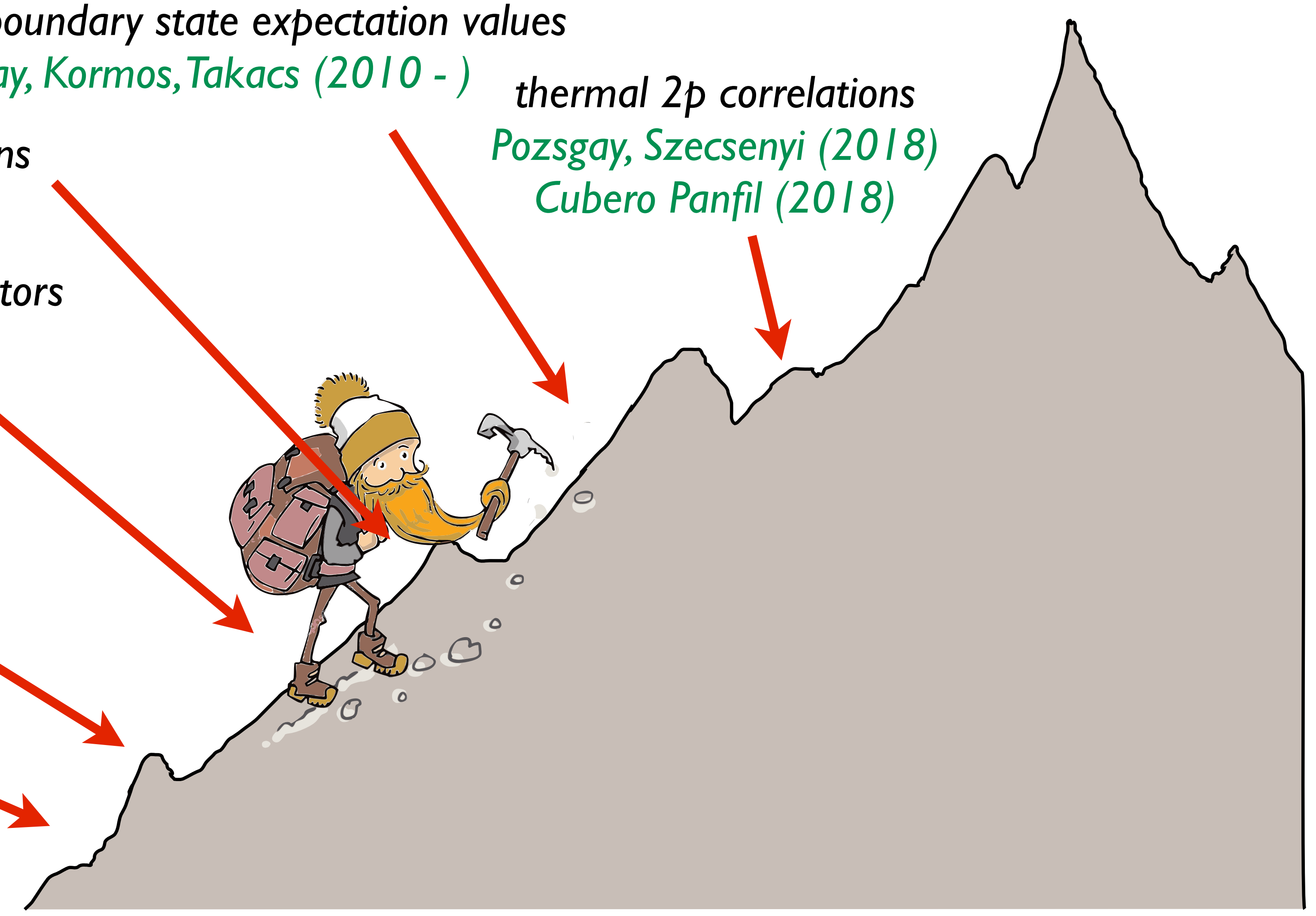
- Integrable, yet correlation functions hard to calculate



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GOAL:
predict values of observables in the quantum sine-Gordon model

thermal + boundary state expectation values
Pozsgay, Kormos, Takacs (2010 -)

thermal 2p correlations
Pozsgay, Szecsenyi (2018)
Cubero Panfil (2018)

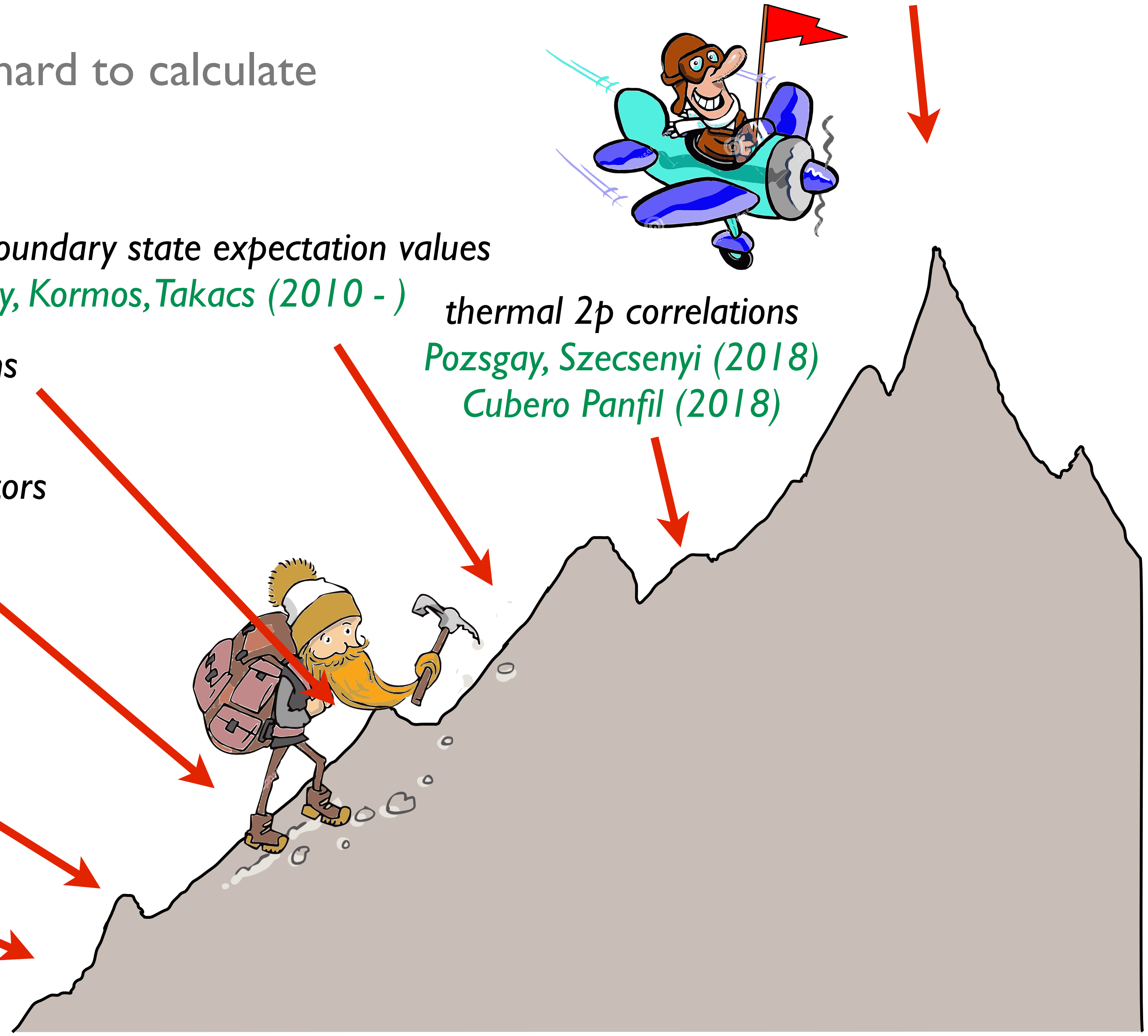
ground state 2p correlations
Essler, Konik (2005)

ground state expectation values of vertex operators
Lukyanov Zamolodchikov (1997)

form factors of local operators
Smirnov (1992)

S-matrix
Zamolodchikov (1977)
Zamolodchikov Zamolodchikov (1979)

mass spectrum
Korepin Faddeev (1975)
Dashen Hasslacher Neveu (1975)



Truncated Conformal Space Approach

- Numerical method for the study of continuous (1+1)D QFT (integrable or non-integrable)
- Based on Renormalisation Group and Conformal Field Theory
- In contrast to tensor network methods that work in 1d lattice systems, TCSA is one of the few methods applicable to continuous systems (1+1D or even higher)
- Positive:
  Captures efficiently *non-perturbative* effects
- Negative:
  does not solve the "curse of dimensionality" problem
- Introduced by: *Yurov & Zamolodchikov (1991)*
  later applied to sG by: *Feverati, Ravanini, Takacs (1998-99)*
  and in sG dynamics by: *Kukuljan Sotiriadis Takacs (2018), (2019)*

Truncated Conformal Space Approach

- **Problem:**
  Find the spectrum of a (continuous) QFT in finite volume

- Express it as $H = H_0 + \lambda\,\Delta H$
  where H_0 : known spectrum and eigenstates $E_\alpha, |\Psi_\alpha\rangle$
  and ΔH : known matrix elements in eigenstates of H_0

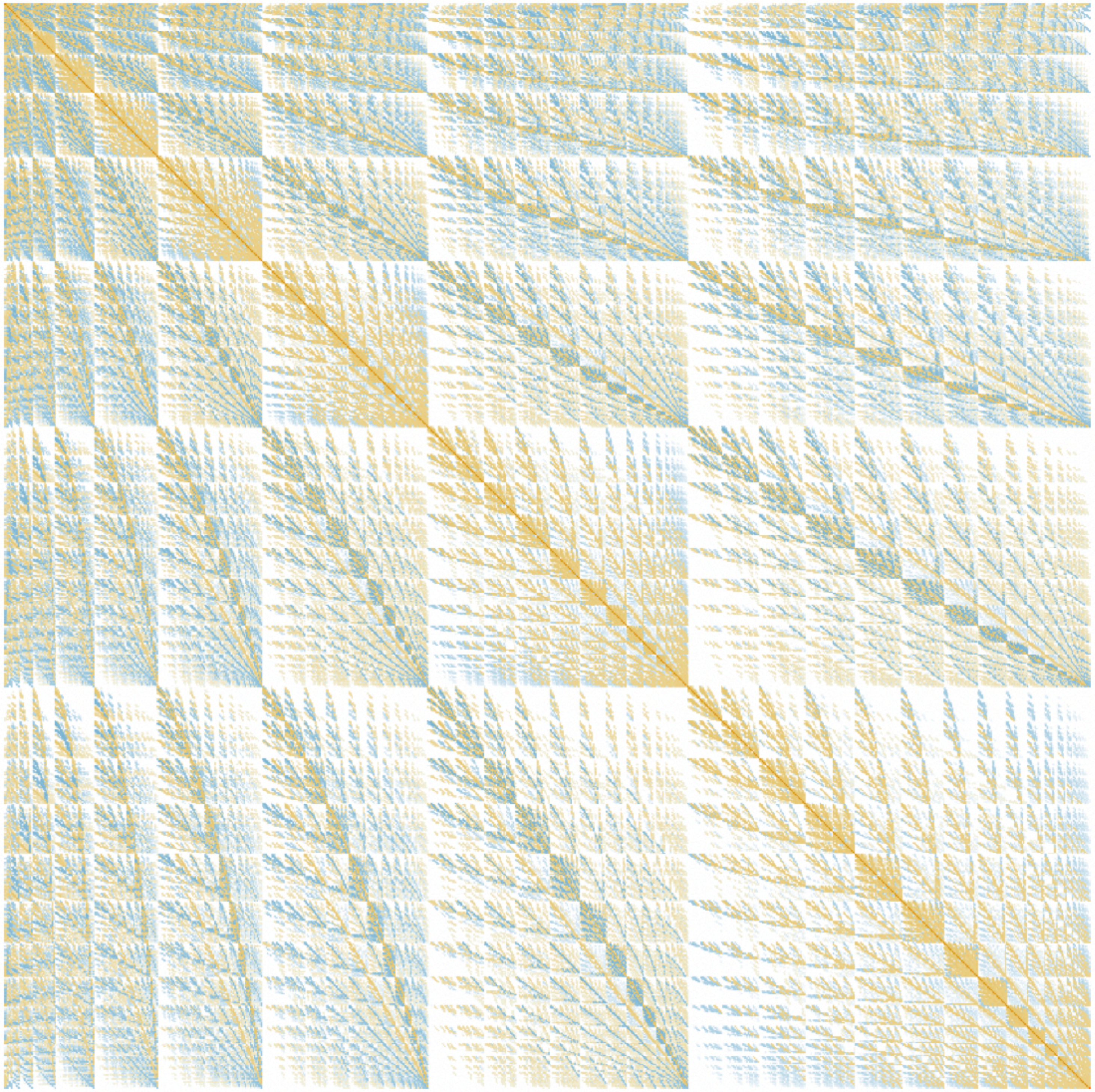
$$\Delta H_{\alpha\beta} = \langle \Psi_\alpha | \Delta H | \Psi_\beta \rangle$$

  - finite volume → discrete spectrum
  - apply *high-energy cutoff* → finite truncated Hilbert space

- Diagonalise numerically the truncated Hamiltonian matrix

$$H_{SGM} = \int \left(\frac{1}{2}(\partial_t \phi)^2 + \frac{1}{2}(\partial_x \phi)^2 - \frac{m^2}{\beta^2}\cos\beta\phi \right) dx$$

Truncated Conformal Space Approach



truncation cutoff	number of states
5	19
10	139
15	684
20	2714
25	9296
30	28629
35	81156
40	215308
45	540635
50	1295971

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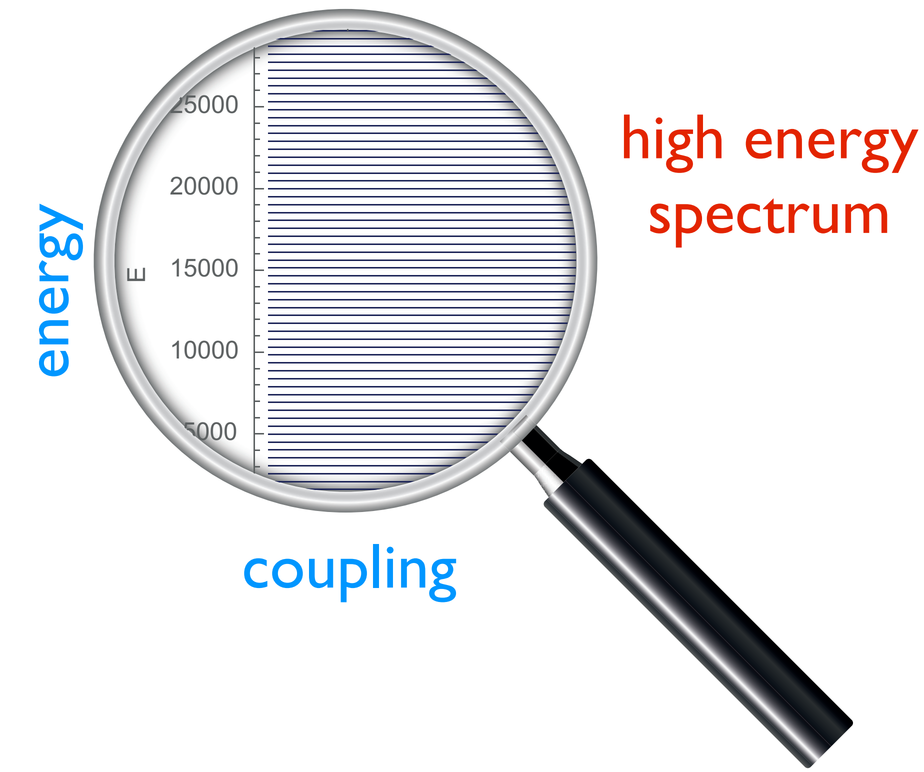

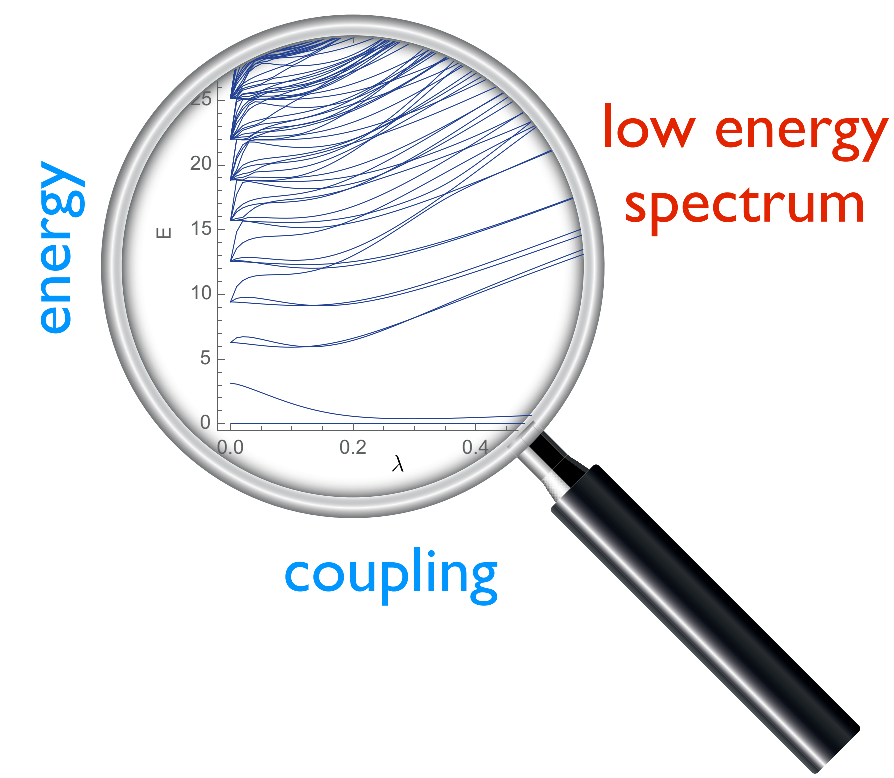

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Quantum Chaos
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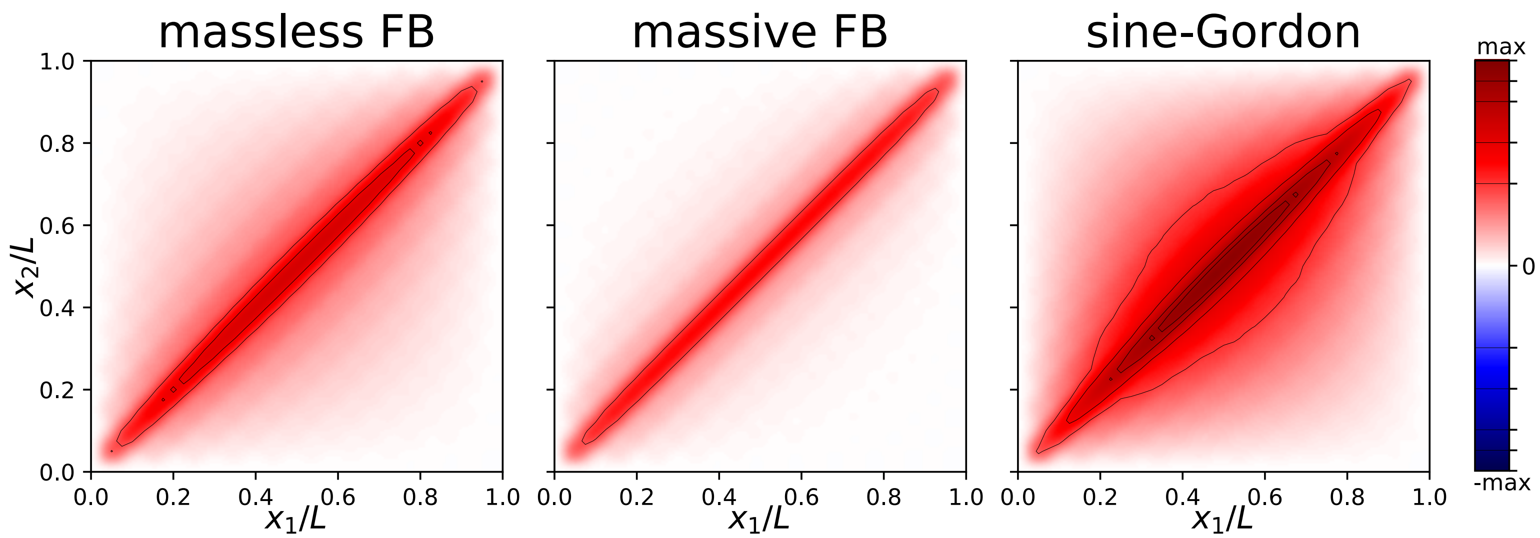
solitons and non-locality

SG ground state correlations

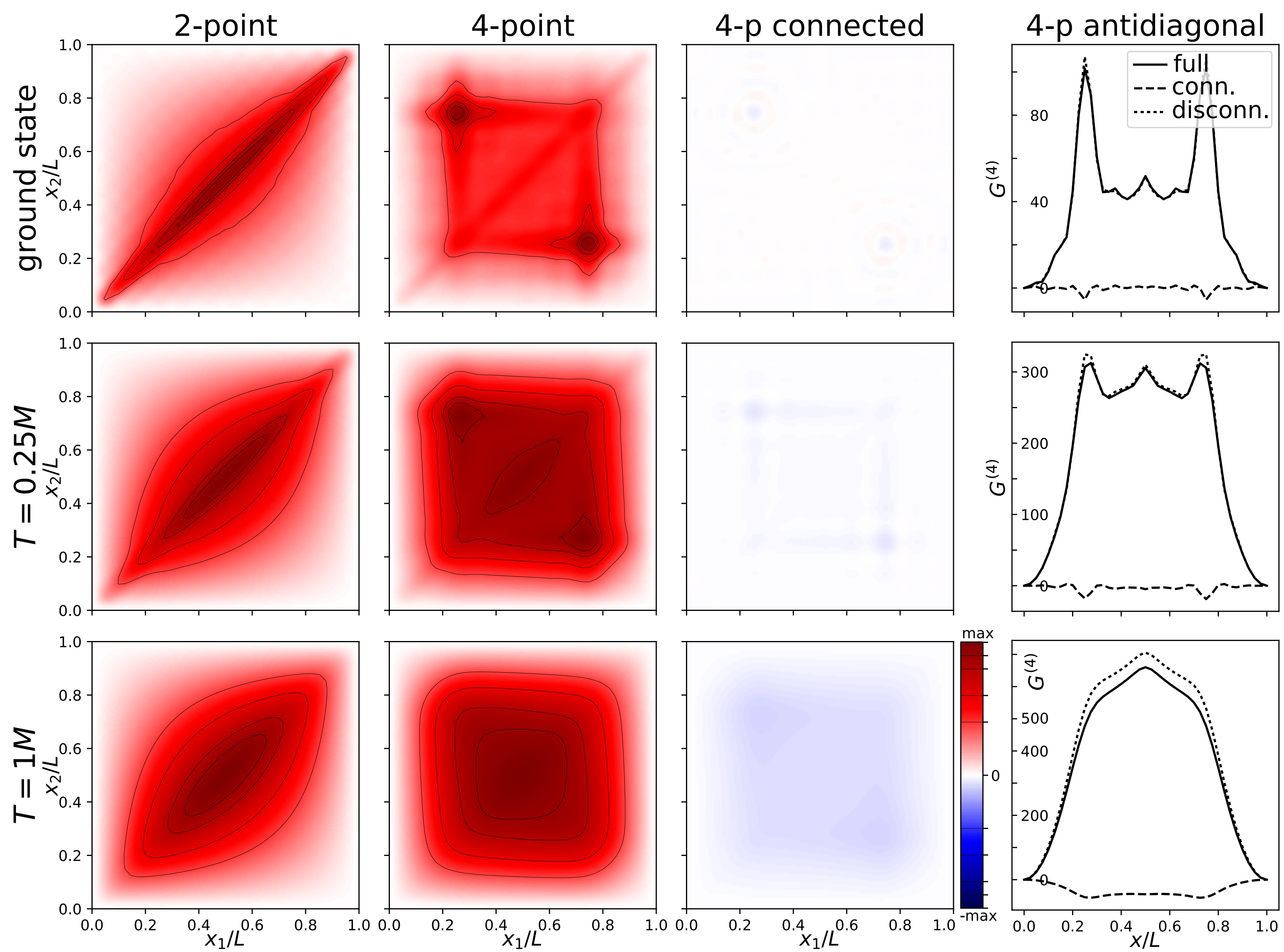
- 2p correlations in free massless boson ground state: algebraically decaying
- In free massive boson (Klein-Gordon) ground state: exponentially decaying
- In sG ground state: much more extended than those of Klein-Gordon ground state at mass equal to lightest breather mass



Kukuljan Sotiriadis Takacs, PRL (2018)

SG thermal states

- 4p conn. correlations: almost vanishing in ground state
- increase with temperature, but still relatively small compared to 2p
- Analysis of interaction / temperature effects on correlations

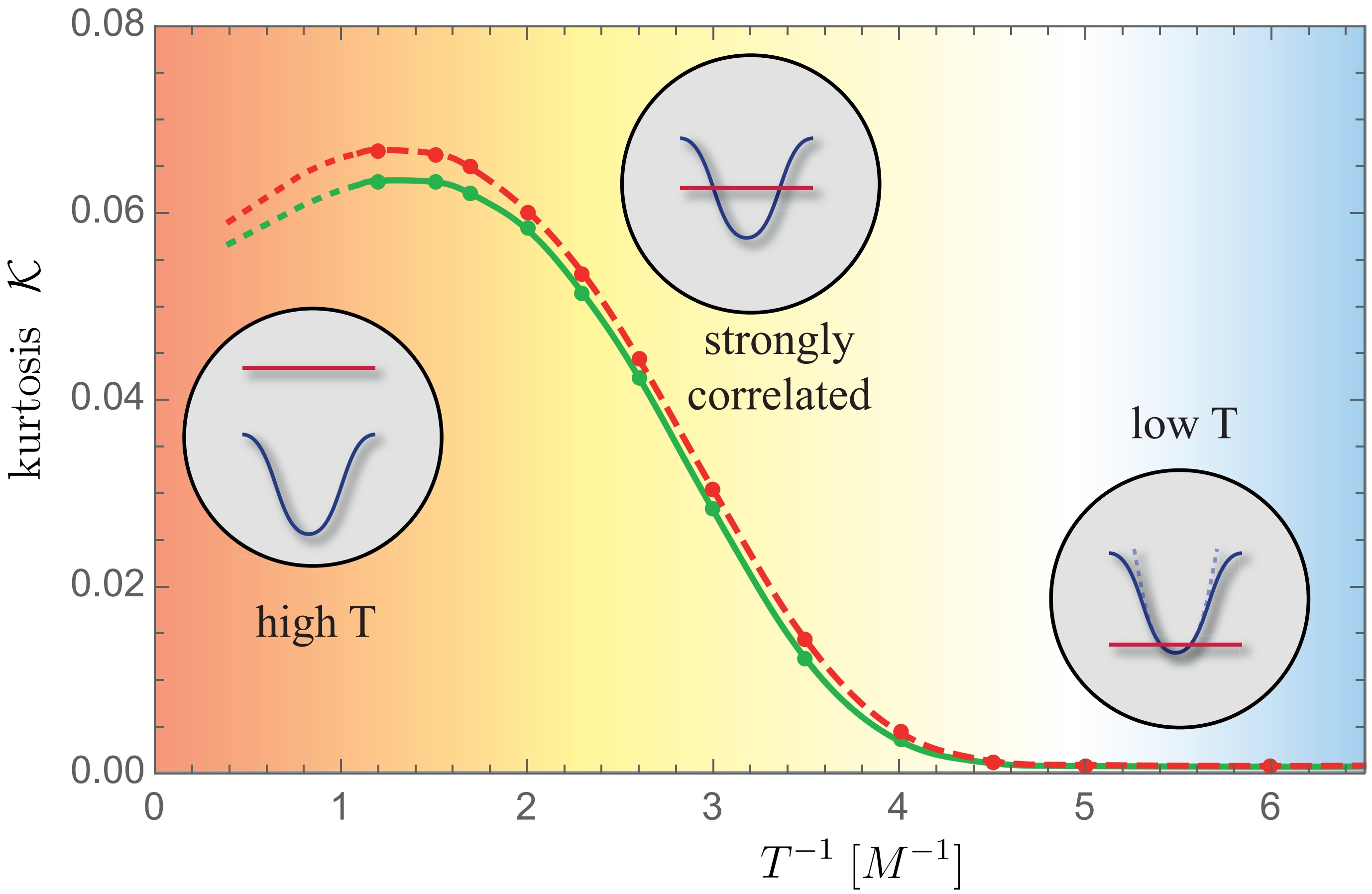


Kukuljan Sotiriadis Takacs, PRL (2018)

Deviations from Gaussianity

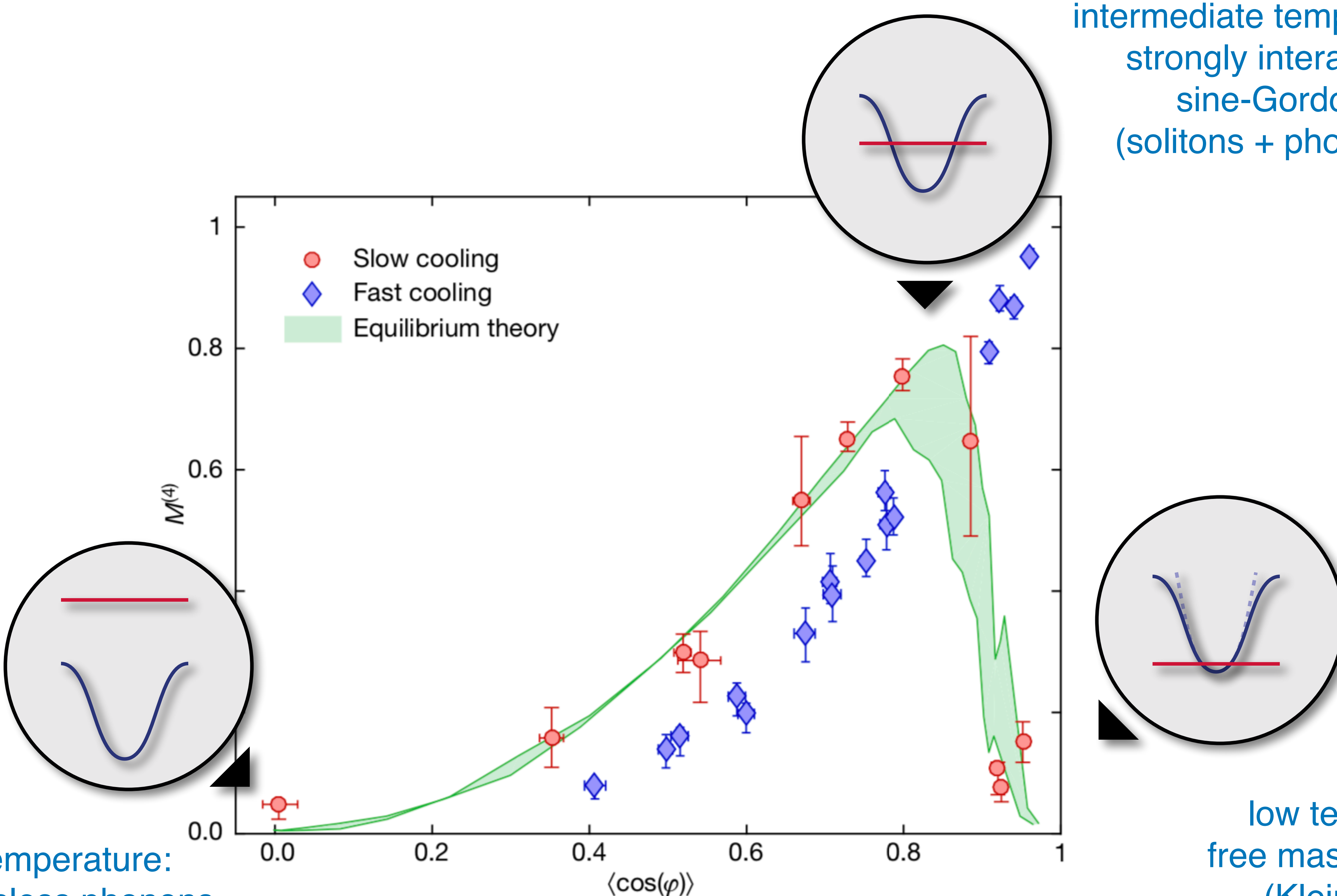
- Numerical calculation of kurtosis (experimental measure of non-Gaussianity) in sine-Gordon ground and thermal states
- Identification of experimentally observed regimes

$$\mathcal{K} := \frac{\int d^4x \left| G^{(4)}_{\rm con}(x_1, x_2, x_3, x_4) \right|}{\int d^4x \left| G^{(4)}(x_1, x_2, x_3, x_4) \right|}$$



Kukuljan Sotiriadis Takacs, PRL (2018)

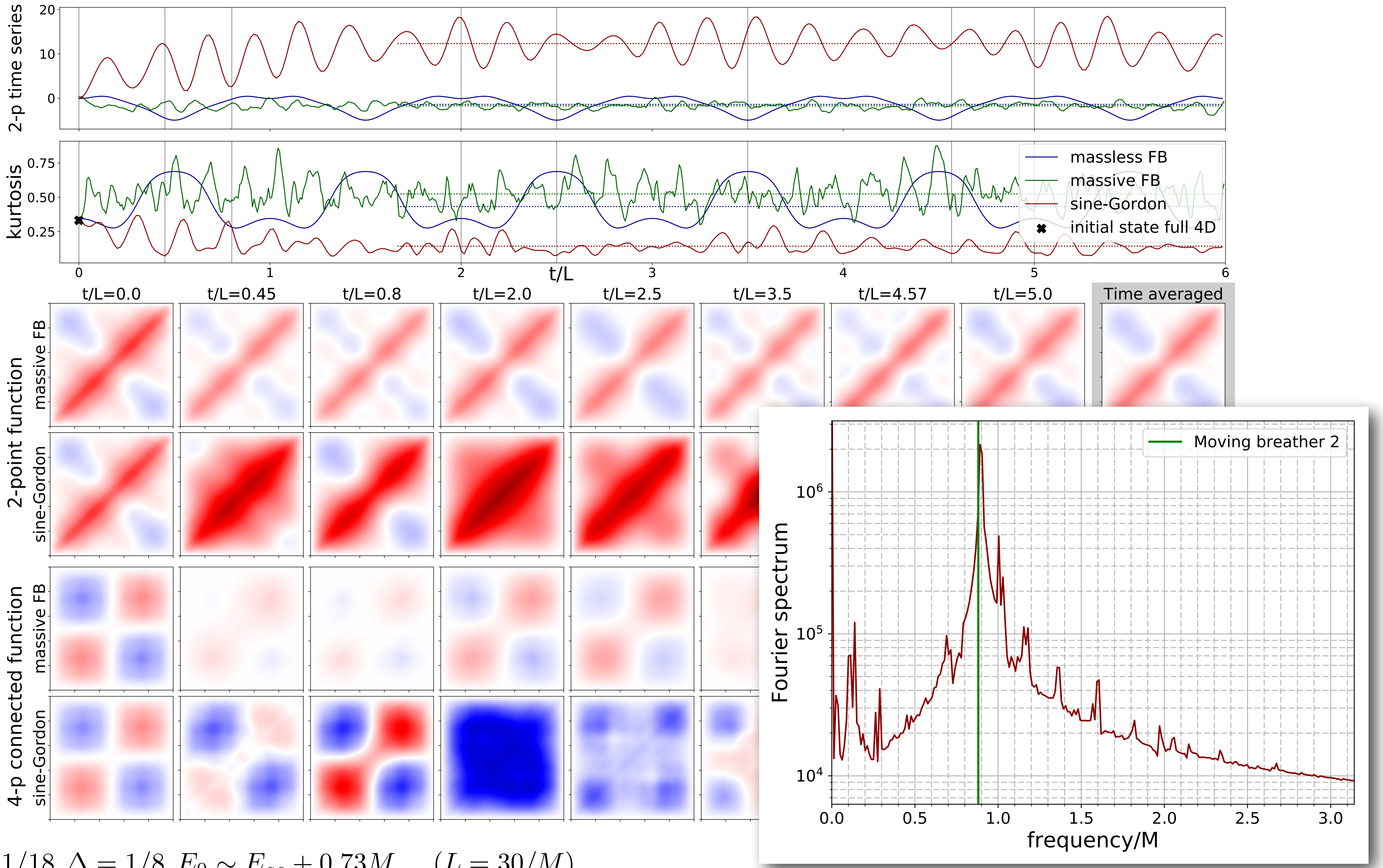
Kurtosis of sG thermal states



Schweigler et al., Nature (2017)

Quantum sine-Gordon: dynamics

- Quench dynamics



$\Delta_0 = 1/18, \Delta = 1/8, E_0 \sim E_{gs} + 0.73M, \quad (L = 30/M)$

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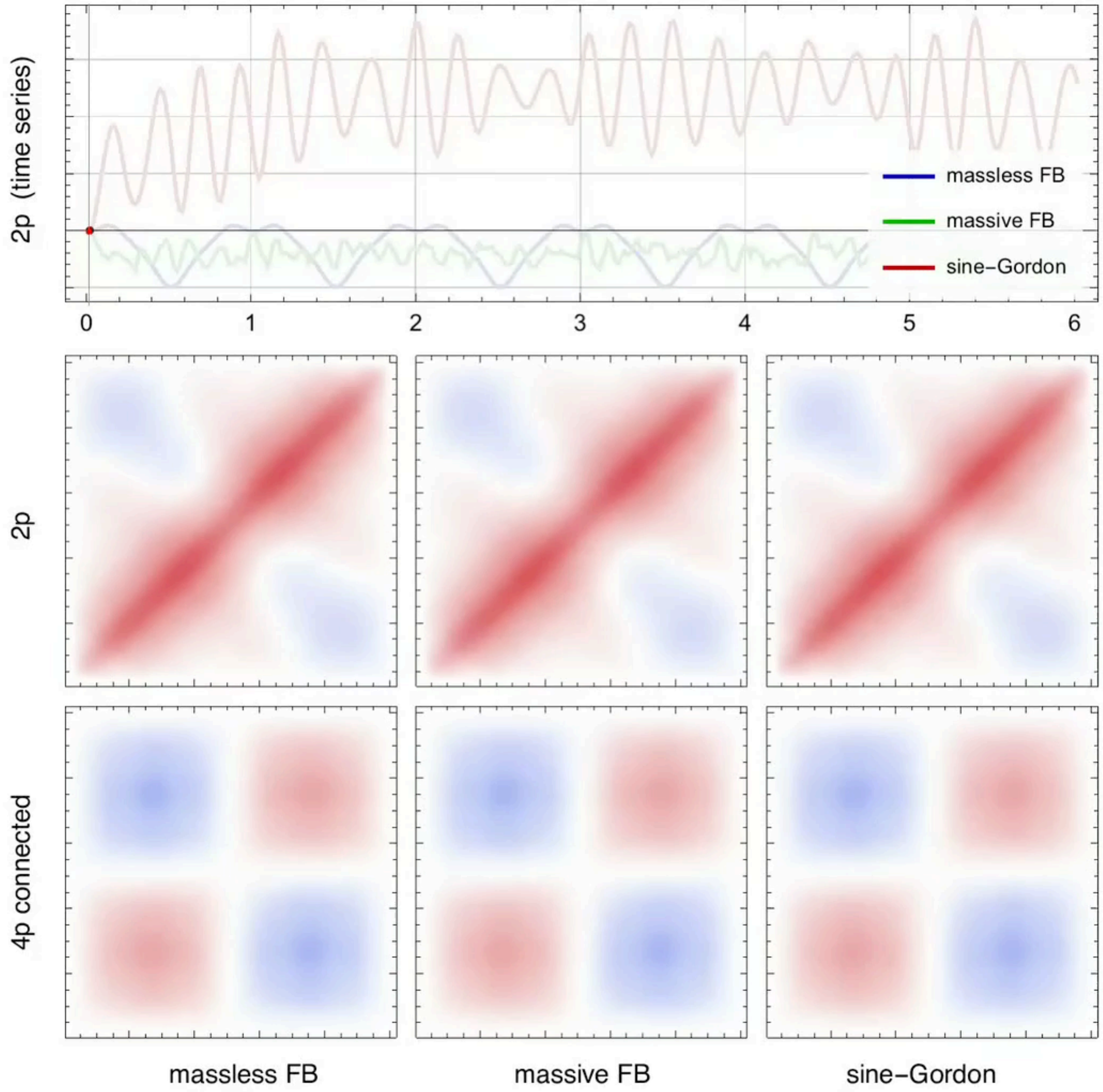

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Violation of the Horizon effect

after a quantum quench in sine-Gordon

Horizon Effect

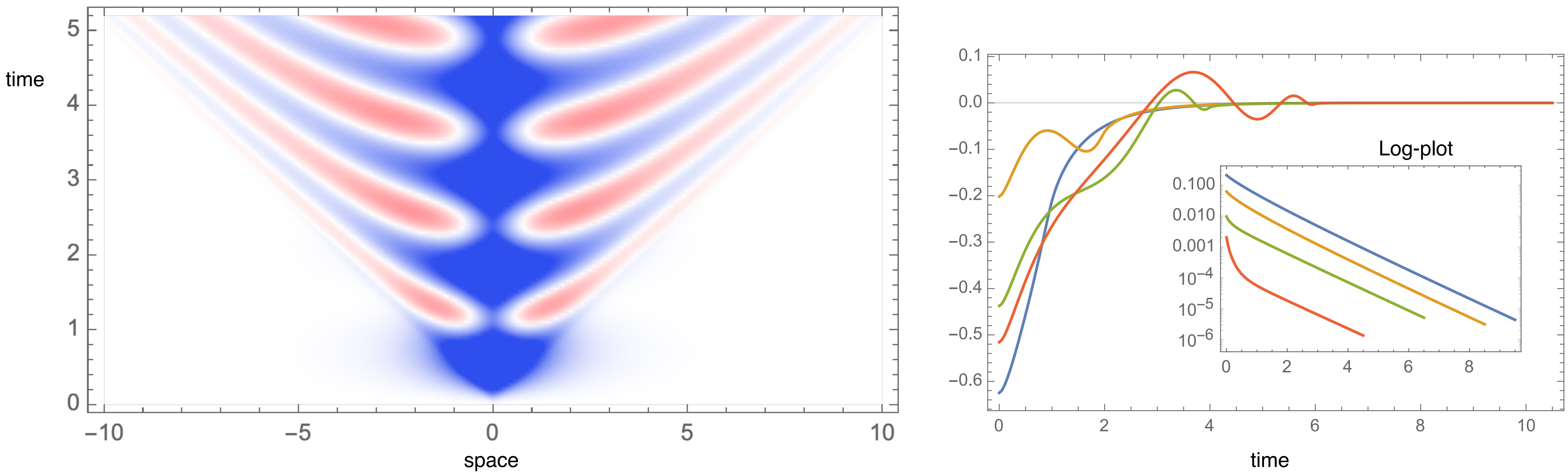
- Dynamics in relativistic Quantum Field Theory
- Example: quantum quench of the mass in Klein Gordon model

$$\hat{H}_{KG} = \int \left(\frac{1}{2}\hat{\pi}^2 + \frac{1}{2}(\partial_x \hat{\phi})^2 + \frac{1}{2} m^2 \hat{\phi}^2 \right) \mathrm{d}x$$

- Correlation functions exhibit "light-cone spreading": connected correlations are restricted within the region $|r| < 2ct$ and exponentially suppressed outside

Calabrese Cardy, PRL (2006)

$$C(x-y,t) = \left\langle \hat{\phi}(x,t)\hat{\phi}(y,t) \right\rangle$$



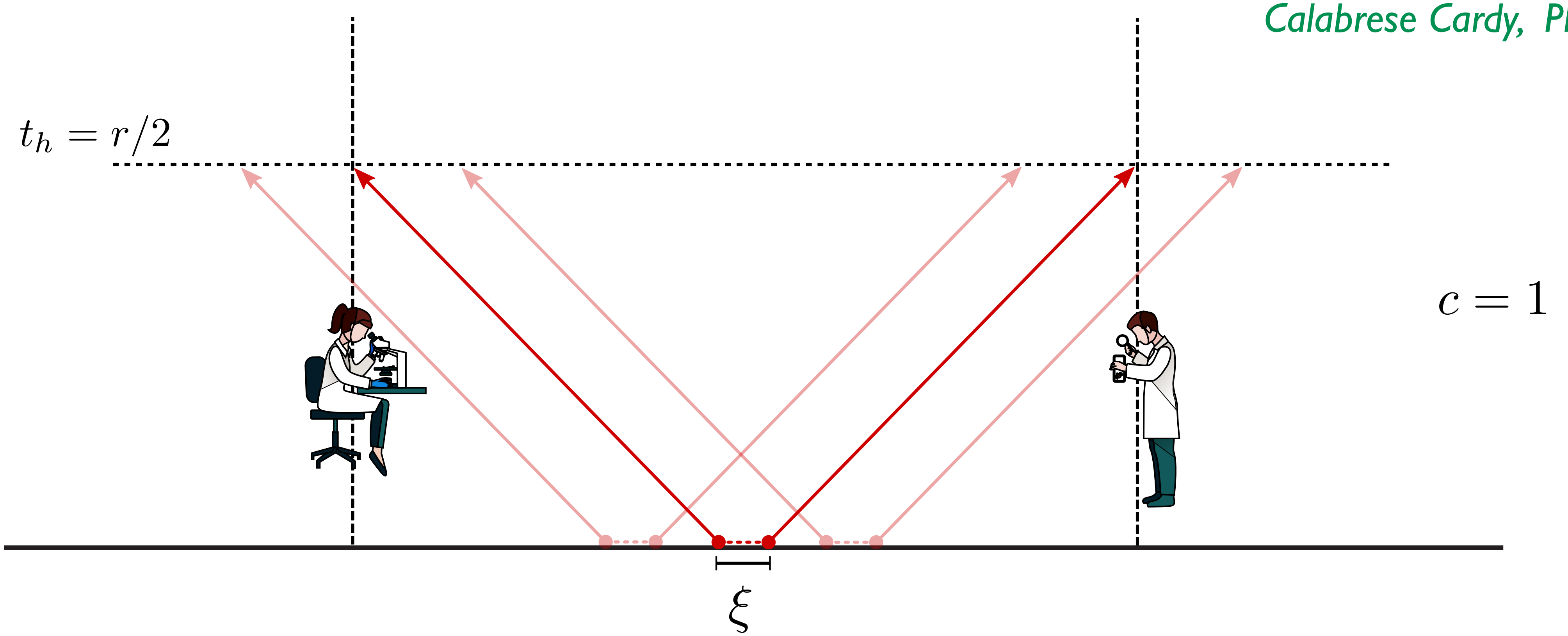
Horizon Effect

- Horizon effect:
  measurements at distant points remain uncorrelated until time $t_h = r/2$ when the fastest pair of entangled particles originating from the middle reach both observers

- Connected correlations are exponentially small outside the horizon

$$|\langle \hat{\mathcal{O}}(x,t)\hat{\mathcal{O}}(y,t)\rangle - \langle \hat{\mathcal{O}}(x,t)\rangle\langle \hat{\mathcal{O}}(y,t)\rangle| < A\mathrm{e}^{-(|x-y|-2t)/\xi} \quad \text{for all } |x-y| > 2t$$

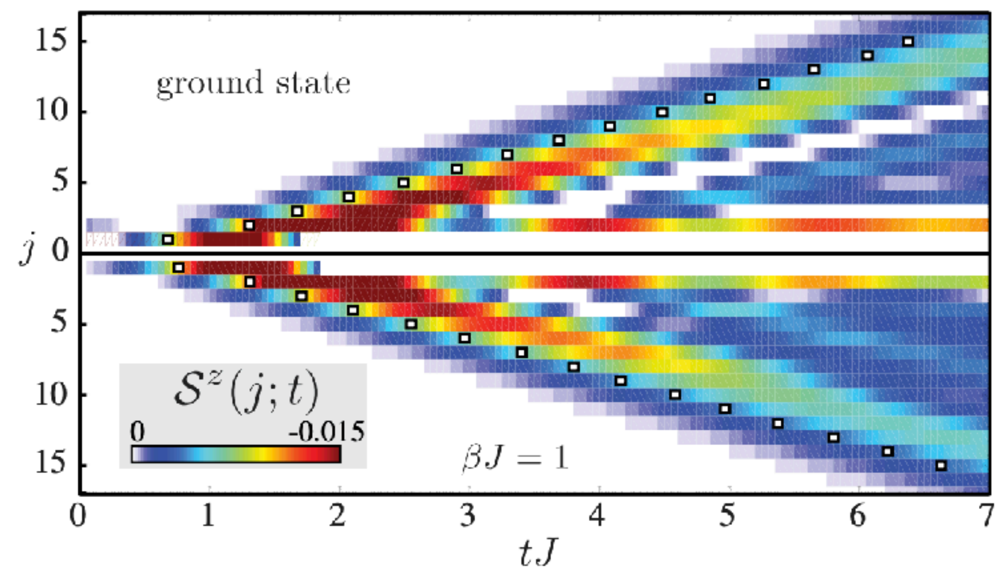
Calabrese Cardy, PRL (2006)



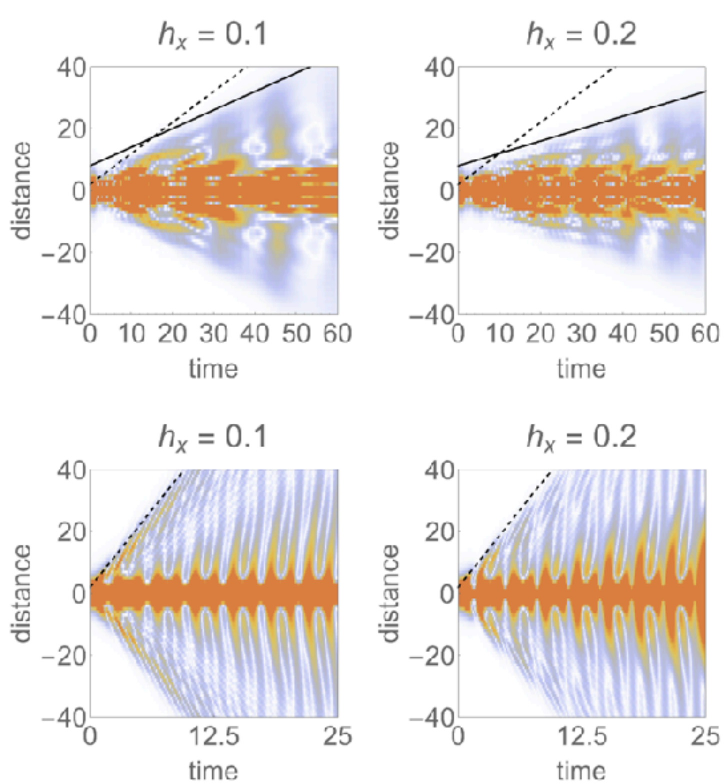
Examples

- Lieb-Robinson bounds
  (lattice systems with local interactions)

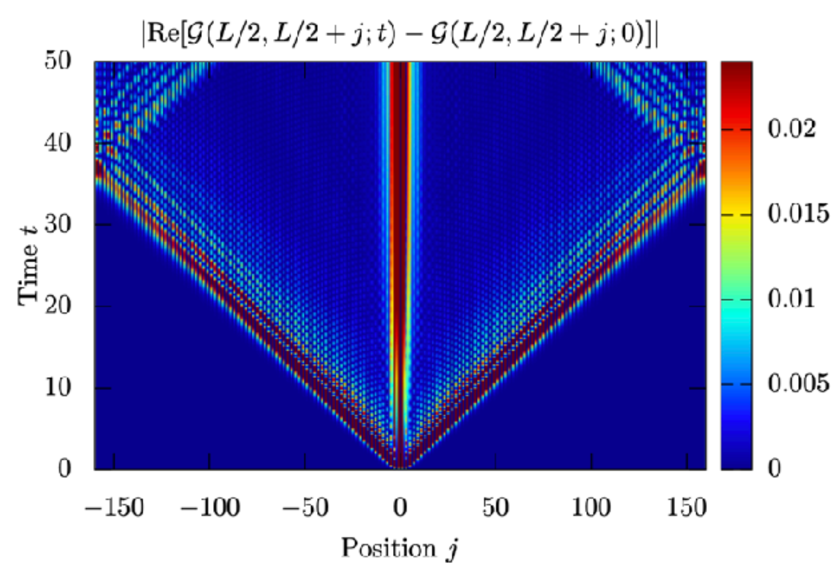
$$||[\mathrm{e}^{+\mathrm{i}Ht} A \mathrm{e}^{-\mathrm{i}Ht}, B]|| < \mathrm{e}^{-\lambda(r_{AB} - vt)}$$



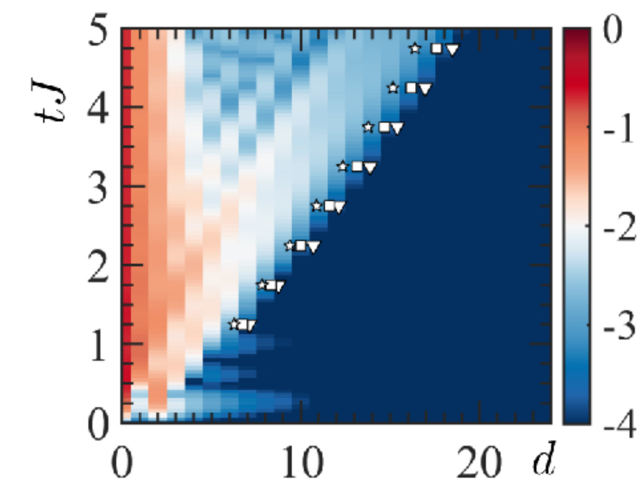
Bonnes Essler Läuchli, PRL (2014)



Kormos Collura Takács Calabrese, Nature Phys (2017)



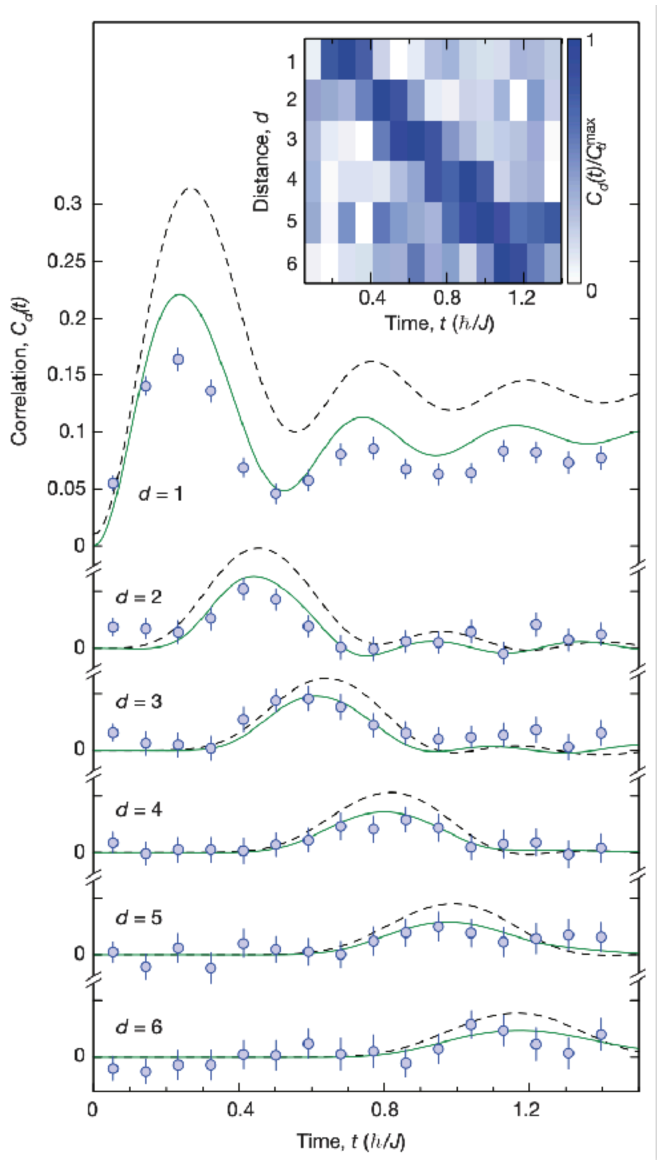
Bertini Essler Groha Robinson, PRB (2016)



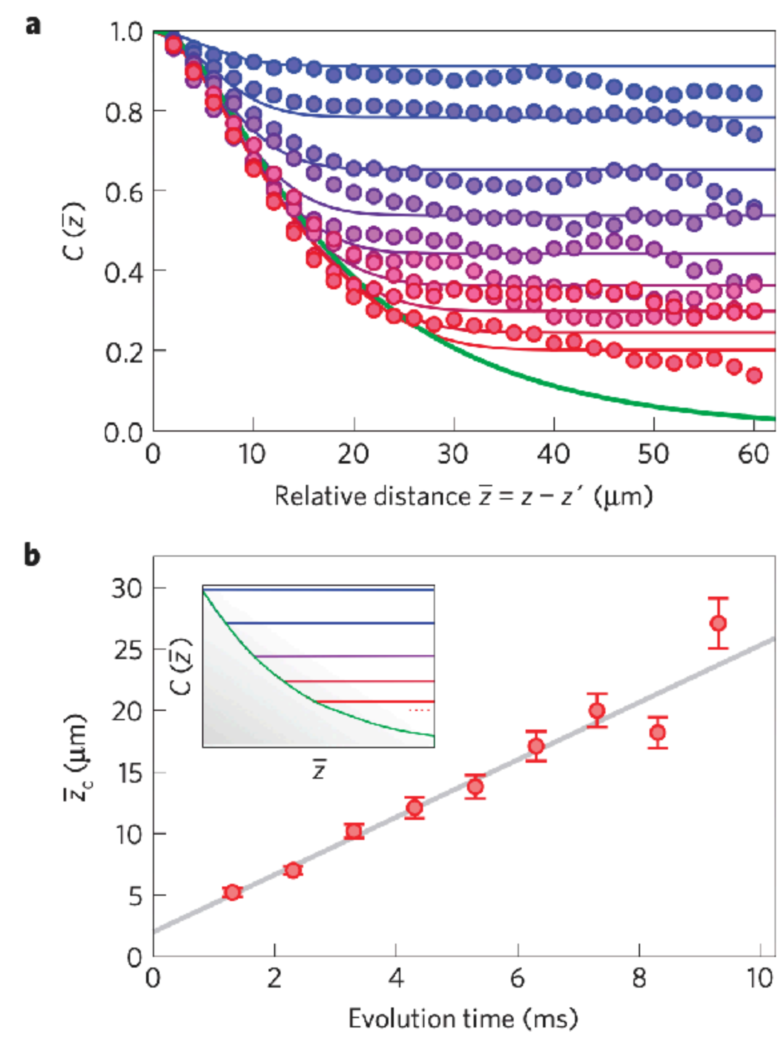
Buyskikh et al, PRA (2016)

Examples

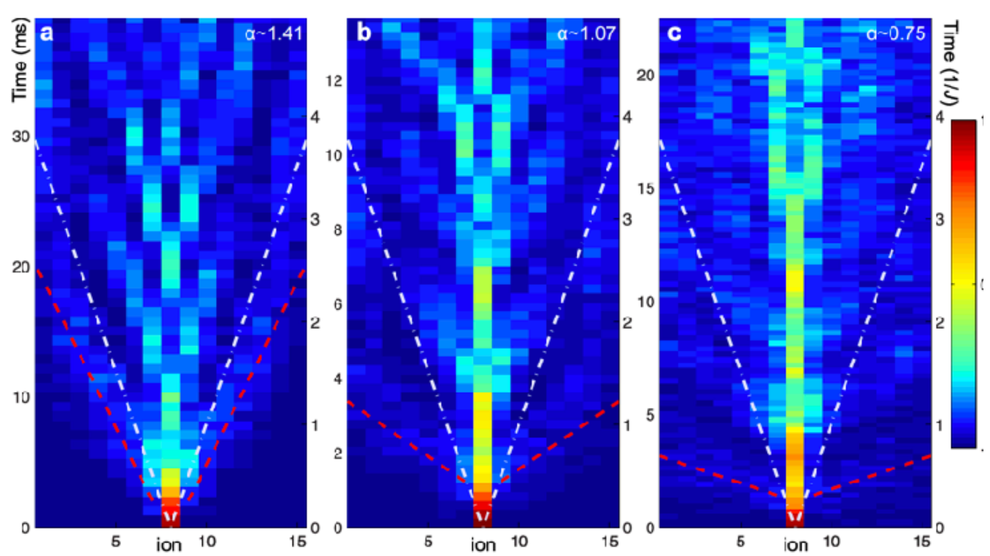
- Experimental observation in cold-atom systems (lattice & continuous)



Cheneau et al, Nature (2012)



Langen et al, Nature Phys (2013)



Jurcevic et al, Nature (2014)

What happens under sine-Gordon dynamics?

- Initial state $|\Omega\rangle$: ground state of Klein Gordon model

$$\hat{H}_{KG} = \int \left(\frac{1}{2}\hat{\pi}^2 + \frac{1}{2}(\partial_x \hat{\phi})^2 + \frac{1}{2} m_0^2 \hat{\phi}^2 \right) \mathrm{d}x$$

- Time evolve under SGM Hamiltonian $\hat{H}_{SG}$

- Calculate dynamics of connected correlation functions of $\hat{\phi}$, $\partial_x \hat{\phi}$ and $\Pi = \partial_t \hat{\phi}$
  Due to field compactification $\hat{\phi}$ is not a well-defined local field - should be defined through spatial integration of $\partial_x \hat{\phi}$ which is local, starting from some reference point

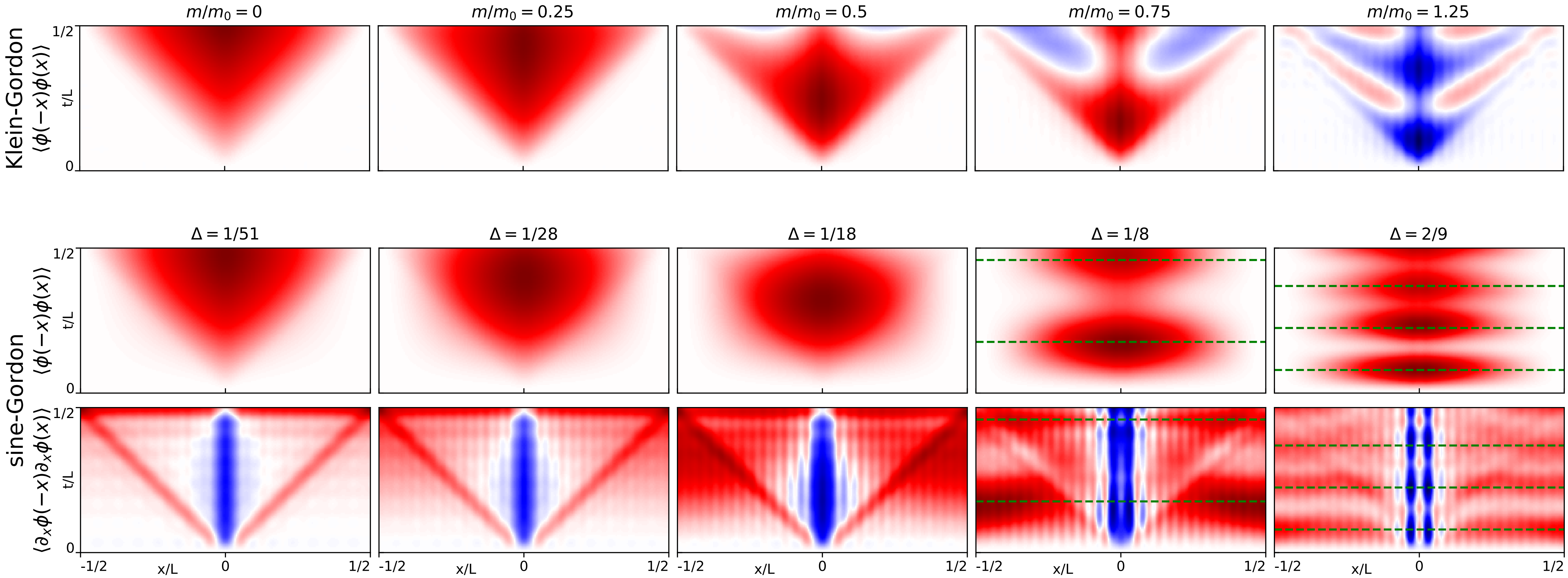
$$C_{\mathcal{O}}(x,y,t) = \langle \mathcal{O}(x,t)\mathcal{O}(y,t) \rangle$$

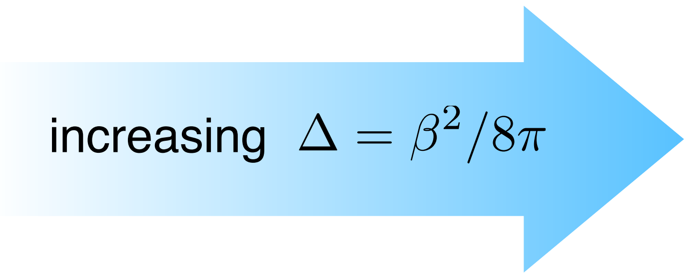
- Despite integrability, calculation of out-of-equilibrium correlation functions not possible yet

- Use numerical simulation: Truncated Conformal Space Approach

Correlation spreading after a quantum quench

$$C_\phi(x,y,t) = \langle \hat{\phi}(x,t)\hat{\phi}(y,t)\rangle \qquad C_{\partial\phi}(x,y,t) = \langle \partial\hat{\phi}(x,t)\partial\hat{\phi}(y,t)\rangle$$





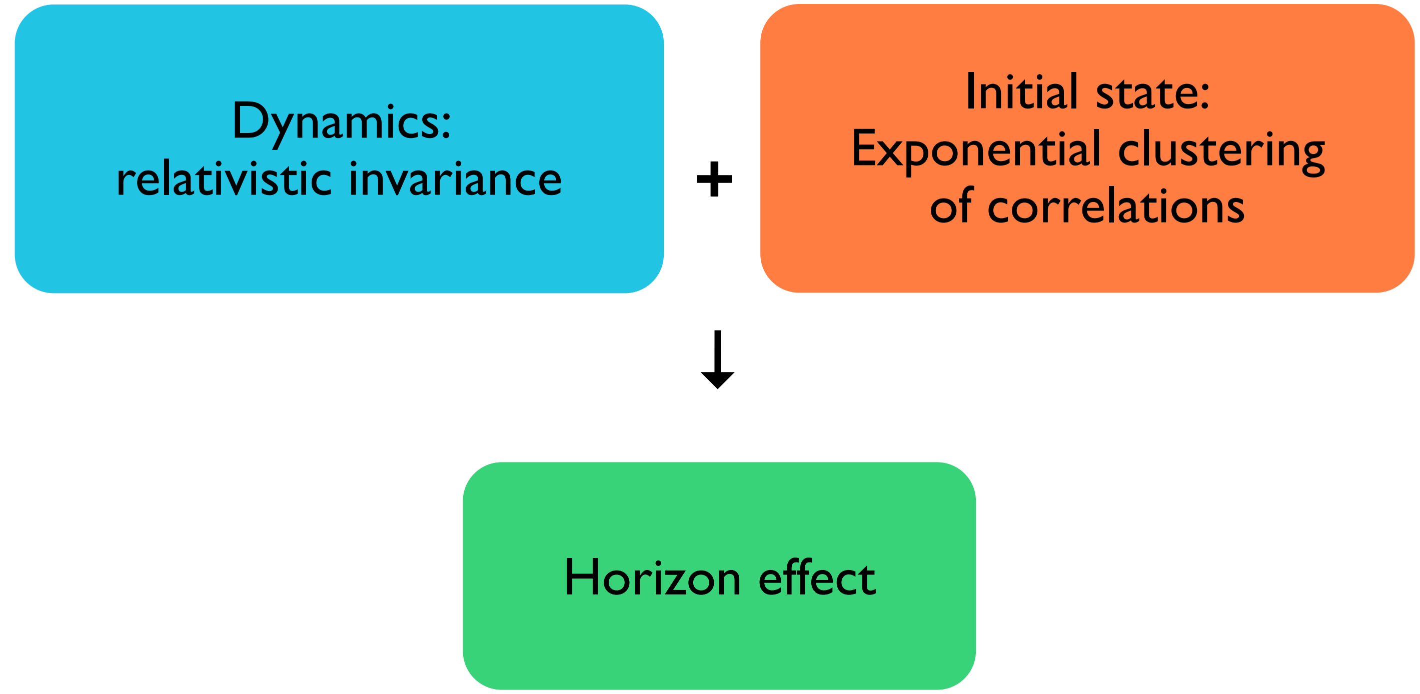
Violation of the "horizon effect"

Kukuljan Sotiriadis Takacs, JHEP (2020)

Explanation based on soliton non-locality

- Relativistic dynamics alone does not guarantee presence of horizon

- For short-range initial state and free dynamics: initial correlations between free particles decay with distance



- But for sine-Gordon dynamics quasiparticles are solitons: non-local fields

$$\Psi_{\pm}(x) = \mathcal{N} : \exp\left[i\frac{2\pi}{\beta} \int_{-\infty}^{x} dx' \pi(x') \pm i\frac{\beta}{2}\phi(x) \right]$$

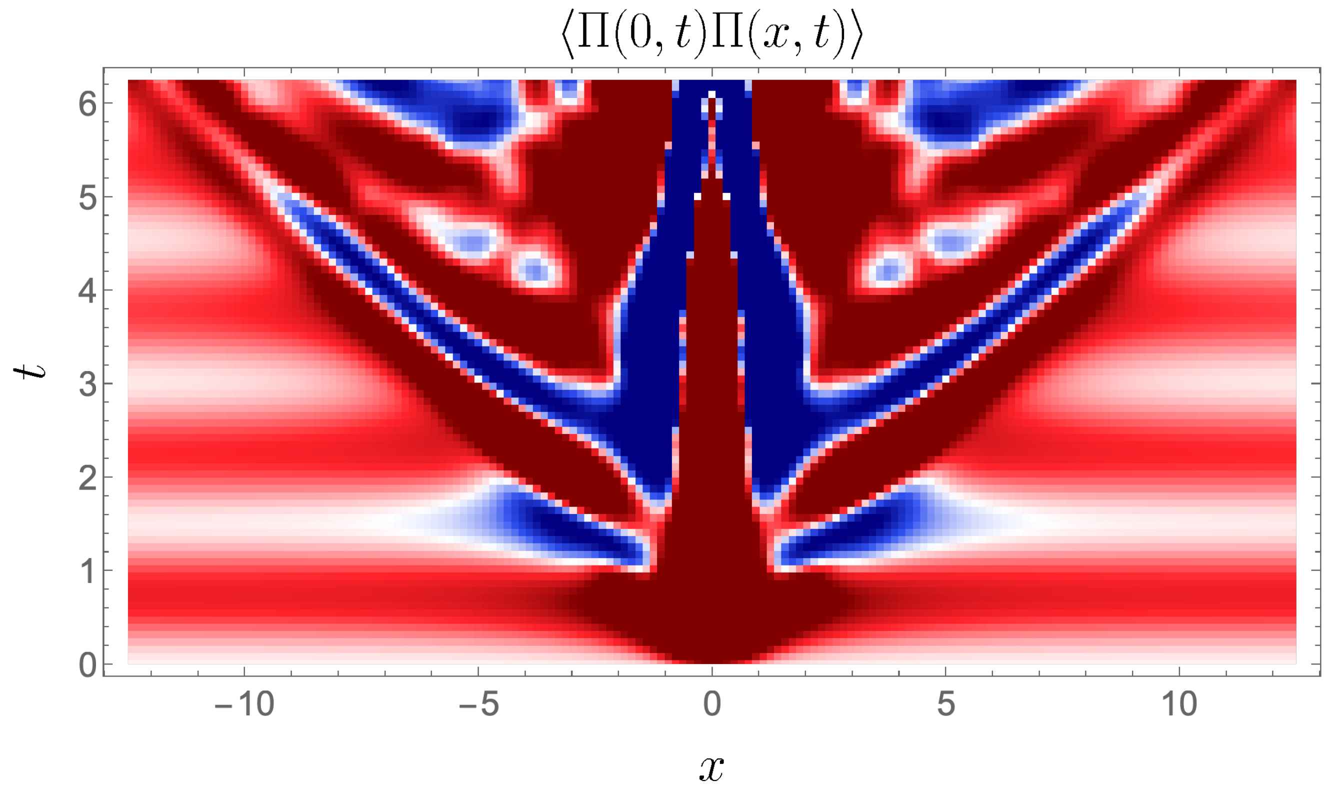
Mandelstam (1975)

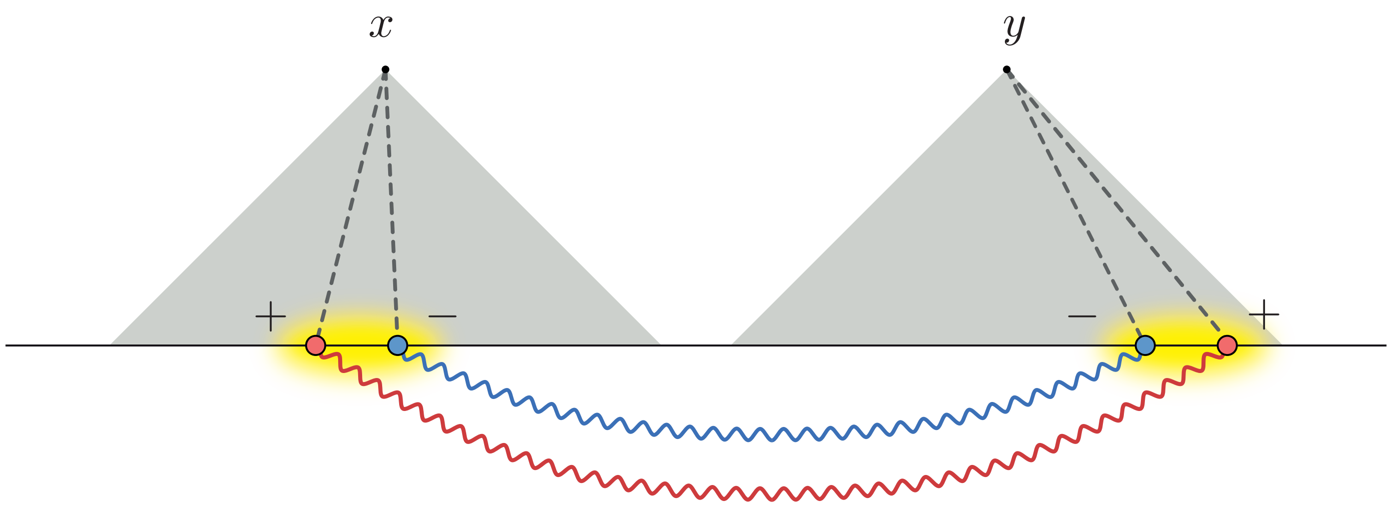
- Decay of quasiparticle correlations not guaranteed, even for short-range initial states

- Test scenario by means of analytical calculation exploiting Duality between sine-Gordon & massive Thirring model

Analytical verification

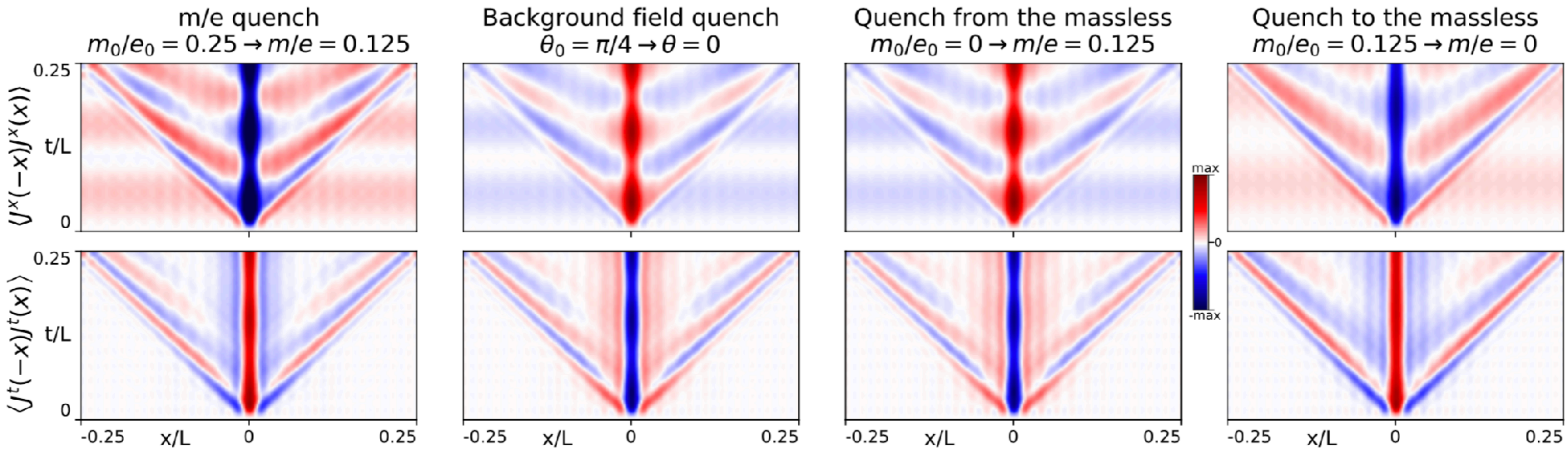
- Exact dynamics at the Luther-Emery point exploiting the sine-Gordon — massive Thirring model duality
- No violation of relativistic invariance: Green's functions supported only inside past light-cone
- Even short-range correlated initial states exhibit infinite-range correlations between soliton fields due to their non-locality (cluster decomposition not valid).

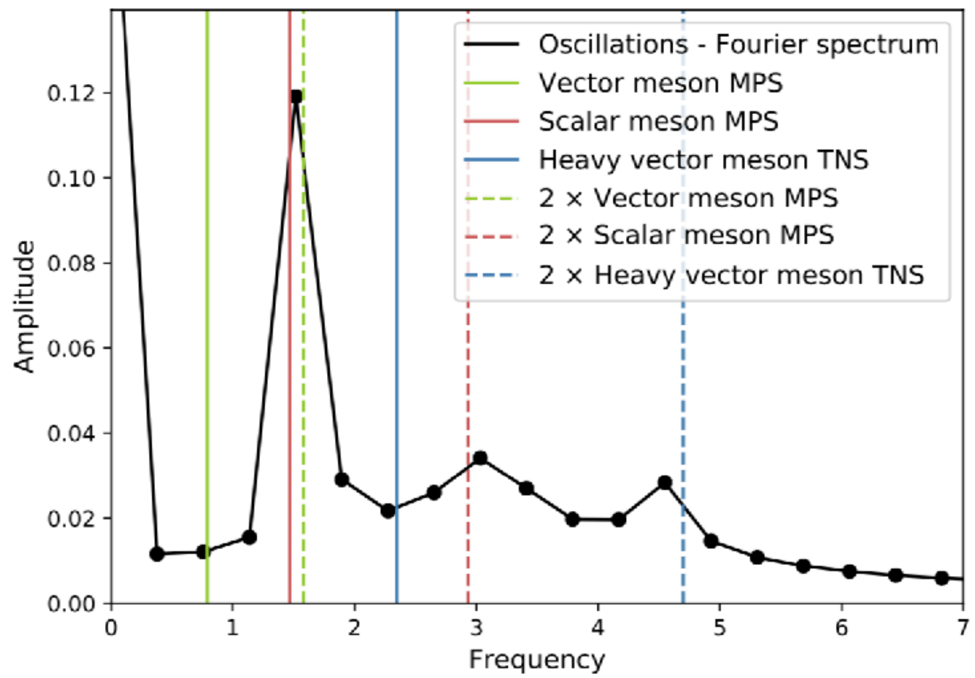




Kukuljan Sotiriadis Takacs, JHEP (2020)

Further results: Horizon Violation in 1+1D QED





- Schwinger model

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \bar{\Psi}\left(i\gamma^\mu\partial_\mu - e\gamma^\mu A_\mu - m\right)\Psi,$$

Kukuljan (2021) arXiv: 2101.07807

Quantum equilibration and recurrences

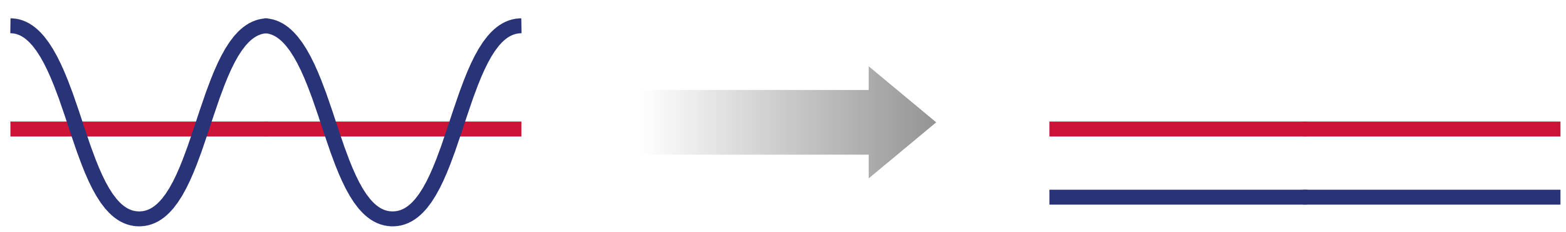
a quantum central limit theorem

A Quantum Quench Experiment

$$H_{sG} = \int \left(\frac{1}{2}\pi^2 + \frac{1}{2}(\partial_x \phi)^2 - \frac{\mu^2}{\beta^2} \cos \beta\phi \right) \mathrm{d}x$$

$$\downarrow$$

$$H_{LL} = \int \left(\frac{1}{2}\pi^2 + \frac{1}{2}(\partial_x \phi)^2 \right) \mathrm{d}x$$

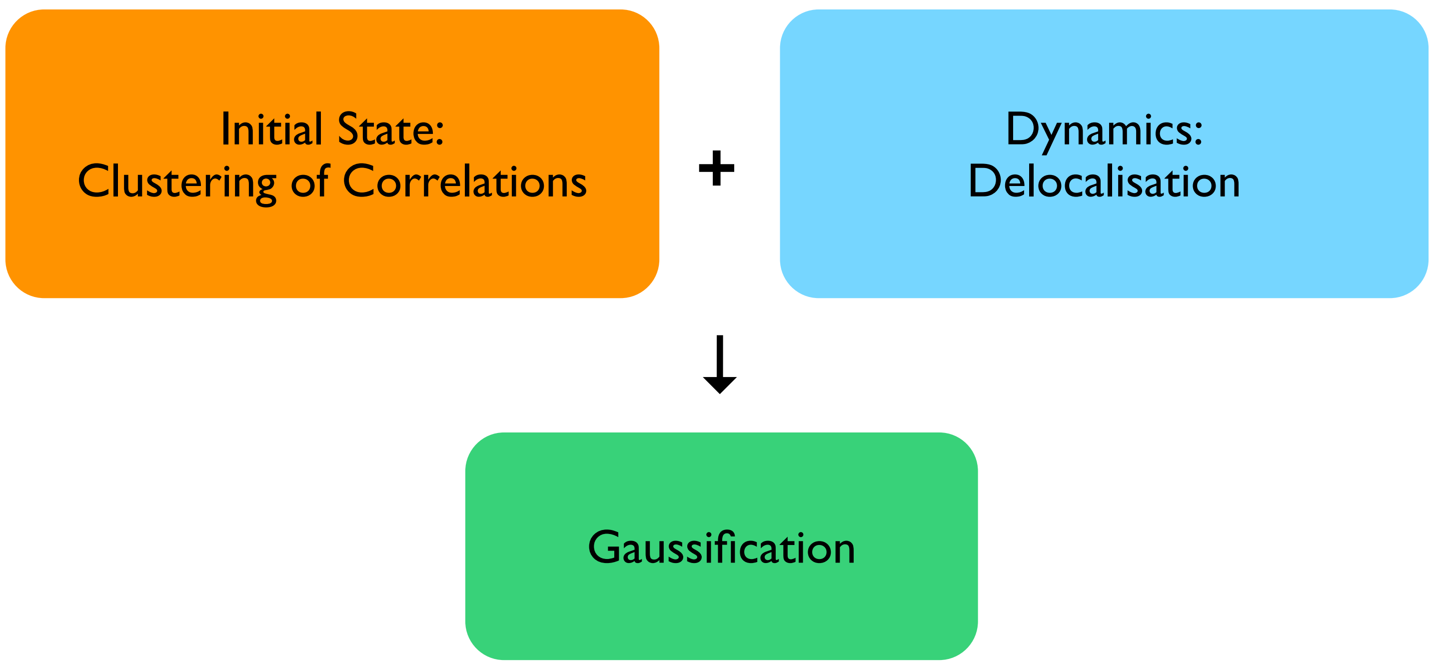


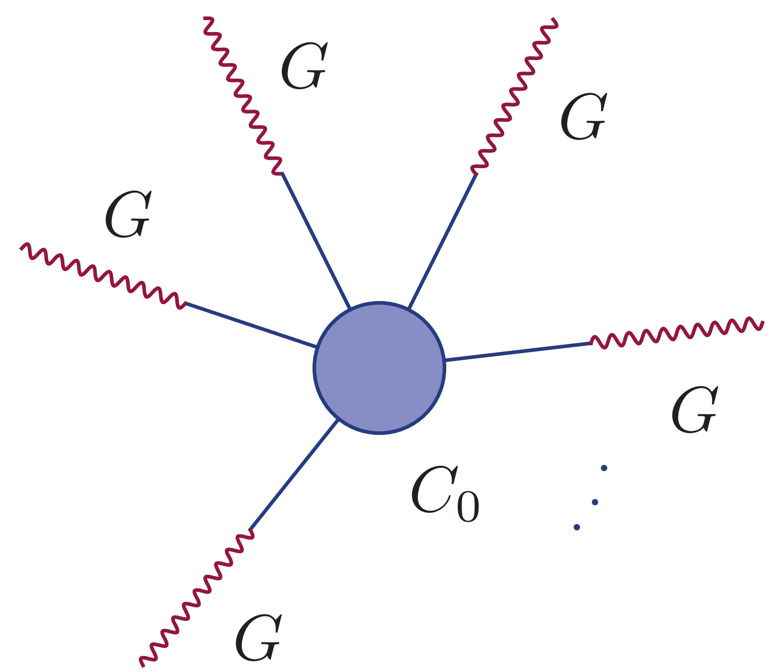
- Non-Gaussian initial state, Gaussian dynamics

Theoretical prediction: Gaussification

"A quantum quench from a general interacting to a non-interacting Hamiltonian results in relaxation to a Gaussian non-thermal steady state, under two conditions:
1. *clustering of initial correlations*
2. *delocalising dynamics."*

Cramer Eisert (2010), Sotiriadis Calabrese (2014), Sotiriadis (2016), (2017), Doyon (2017), Gluza et al. (2016), Murthy Srednicki (2018)

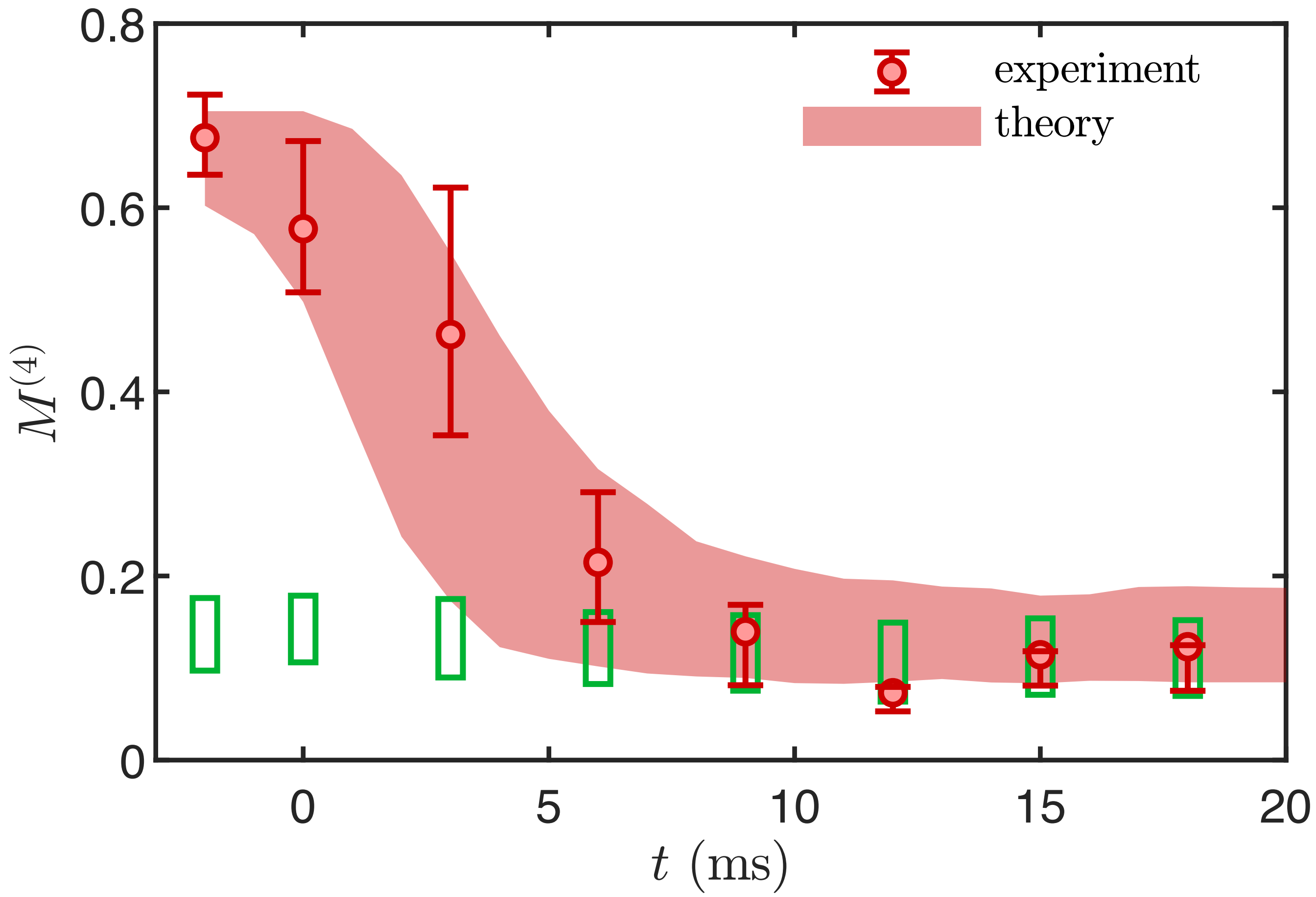




- *None* of these conditions satisfied in the experimental quench!

A Quantum Quench Experiment

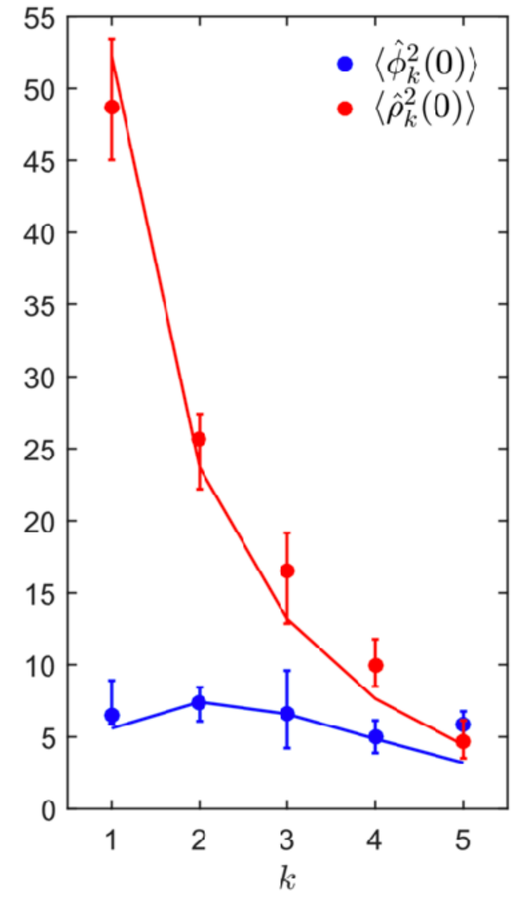
$$M^{(4)}(t) = \frac{\sum_{\boldsymbol{z}} \left| G^{(4)}_{\text{con}}(\boldsymbol{z}, t) \right|}{\sum_{\boldsymbol{z}} \left| G^{(4)}(\boldsymbol{z}, t) \right|} = \frac{S^{(4)}_{\text{con}}(t)}{S^{(4)}(t)}$$

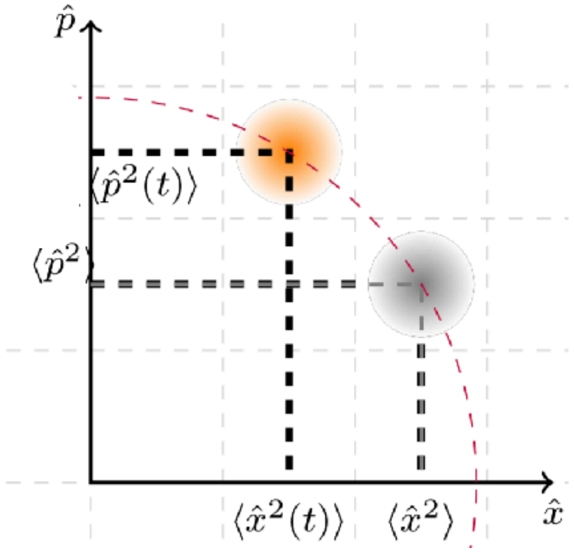


- Twist:
  The experimental system *does* relax to a Gaussian state!

Schweigler et al. (2020)

A Quantum Quench Experiment





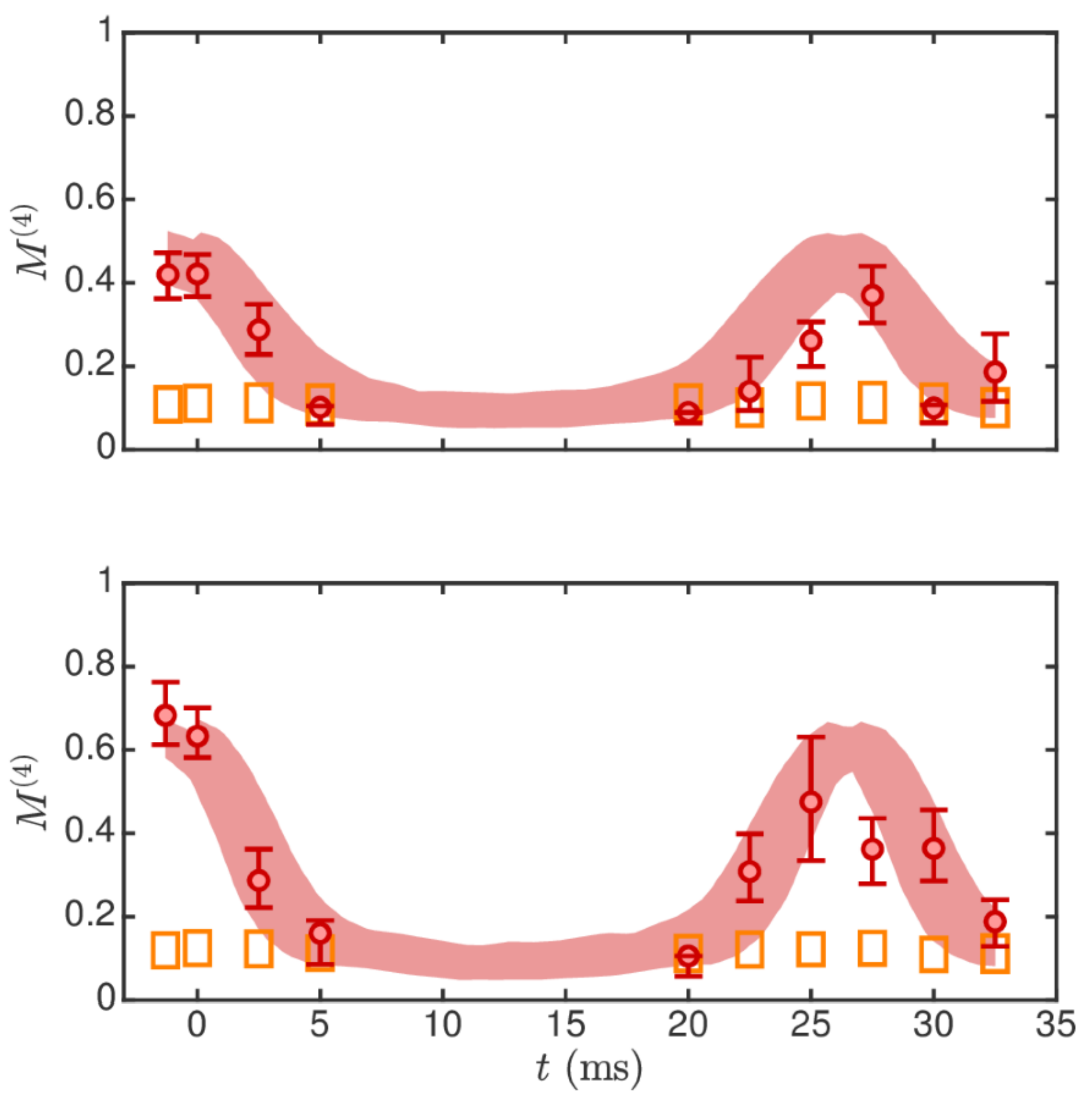
- New information-scrambling mechanism:
  based on dominance of momentum π over ϕ field fluctuations in the initial state and phase-space rotation under dynamics

- Conjugate momentum fluctuations play the role of Gaussian bath

Schweigler et al. (2020)

A Quantum Quench Experiment

$$M^{(4)}(t) = \frac{\sum_{\boldsymbol{z}} \left| G^{(4)}_{\text{con}}(\boldsymbol{z}, t) \right|}{\sum_{\boldsymbol{z}} \left| G^{(4)}(\boldsymbol{z}, t) \right|} = \frac{S^{(4)}_{\text{con}}(t)}{S^{(4)}(t)}$$



- Quantum revivals also observed
- Initial state information scrambled but fully preserved: recurs at revival time

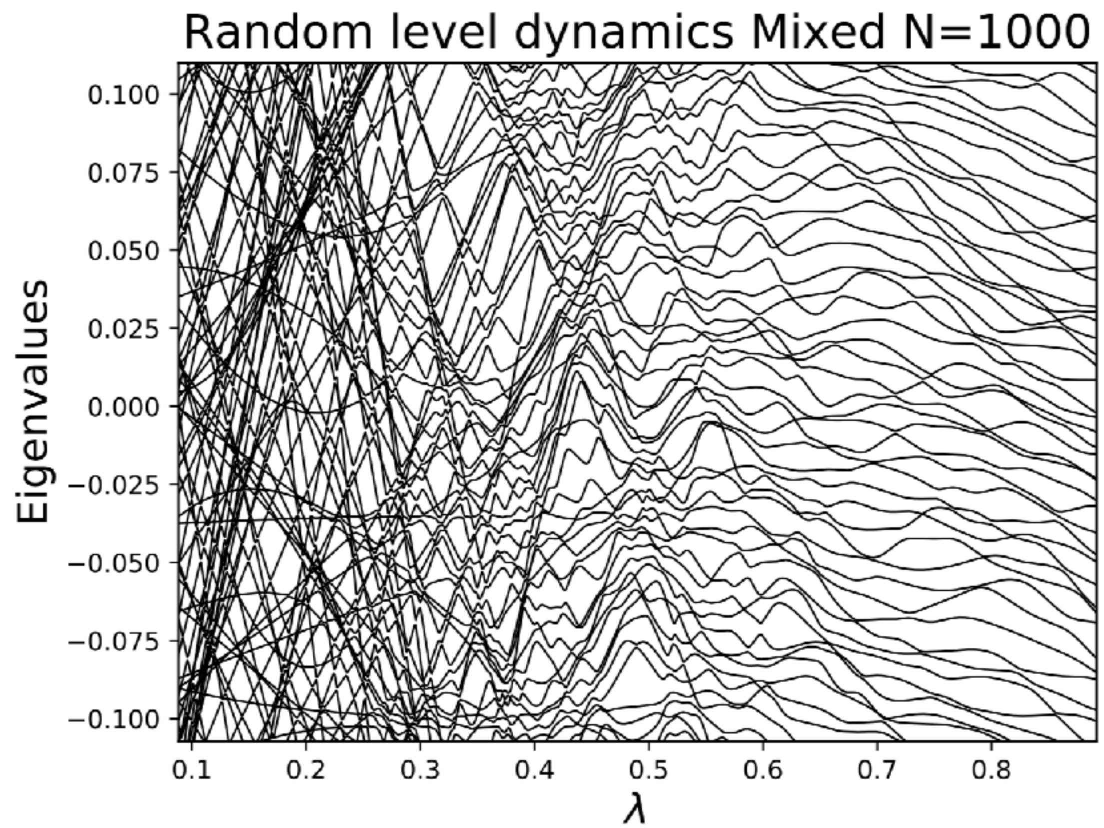
Schweigler et al. (2020)

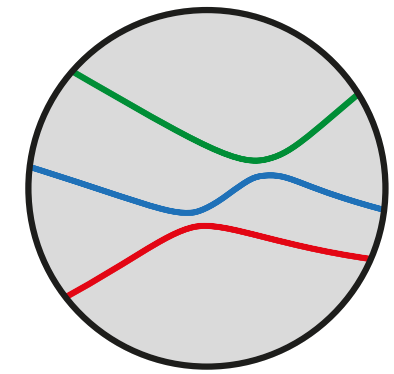
Quantum Chaos

level spacing & eigenvector statistics

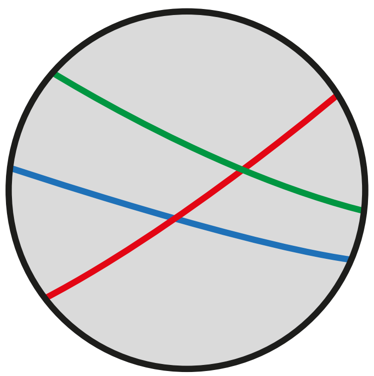
Level spacing statistics

$$H(\lambda) = H_0 + \lambda H_1$$





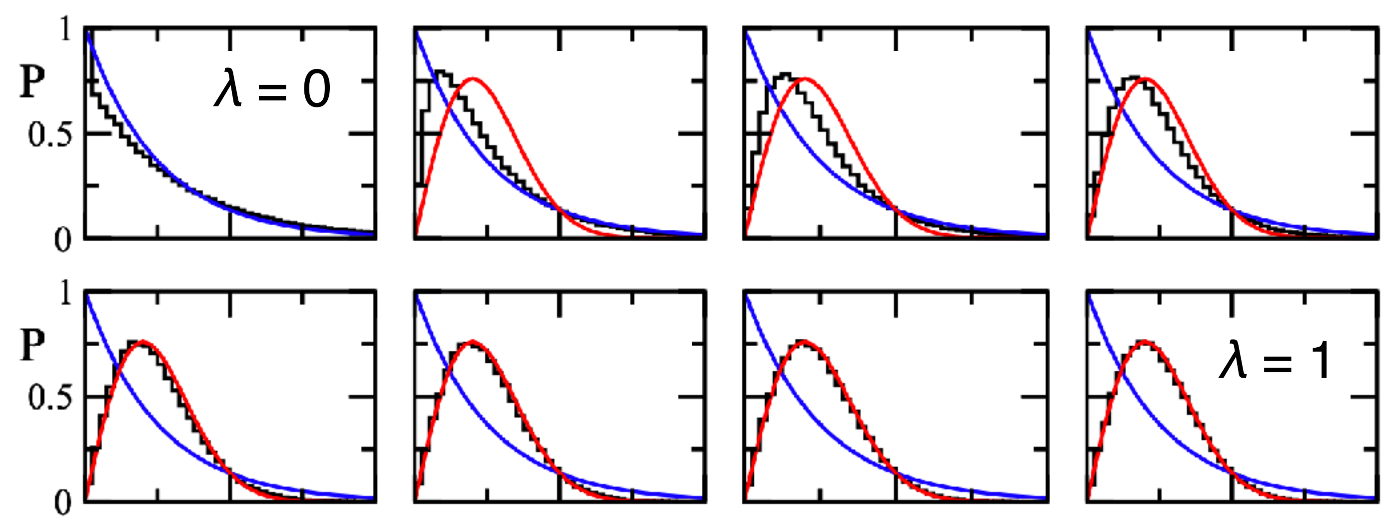
non-integrable:
level repulsion



integrable:
level crossings

distribution of energy level spacings

Poisson



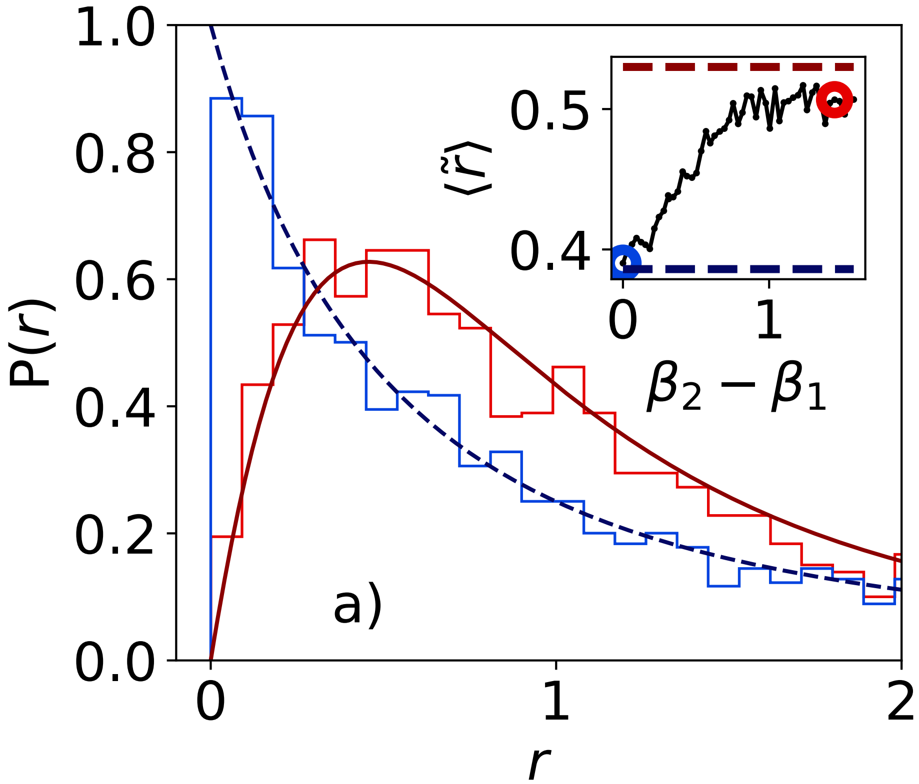
RMT

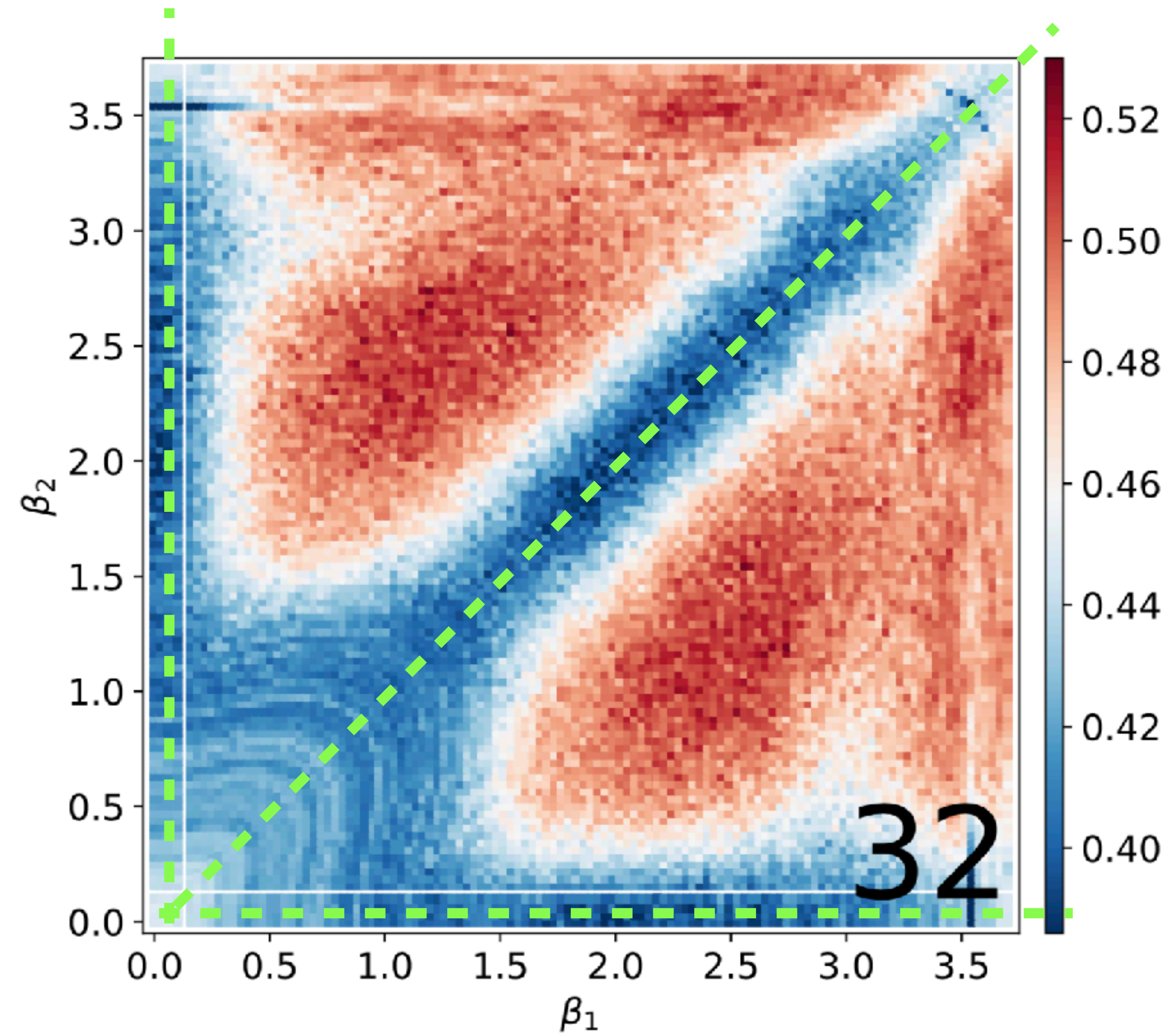
Level spacing statistics

double sine-Gordon model

$$H_{DSGM} = \frac{1}{2} \int \left[(\partial_t \phi)^2 - (\partial_x \phi)^2 - \mu_1 \cos \beta_1 \phi - \mu_2 \cos \beta_2 \phi \right] \, \mathrm{d}x$$

RMT: GOE



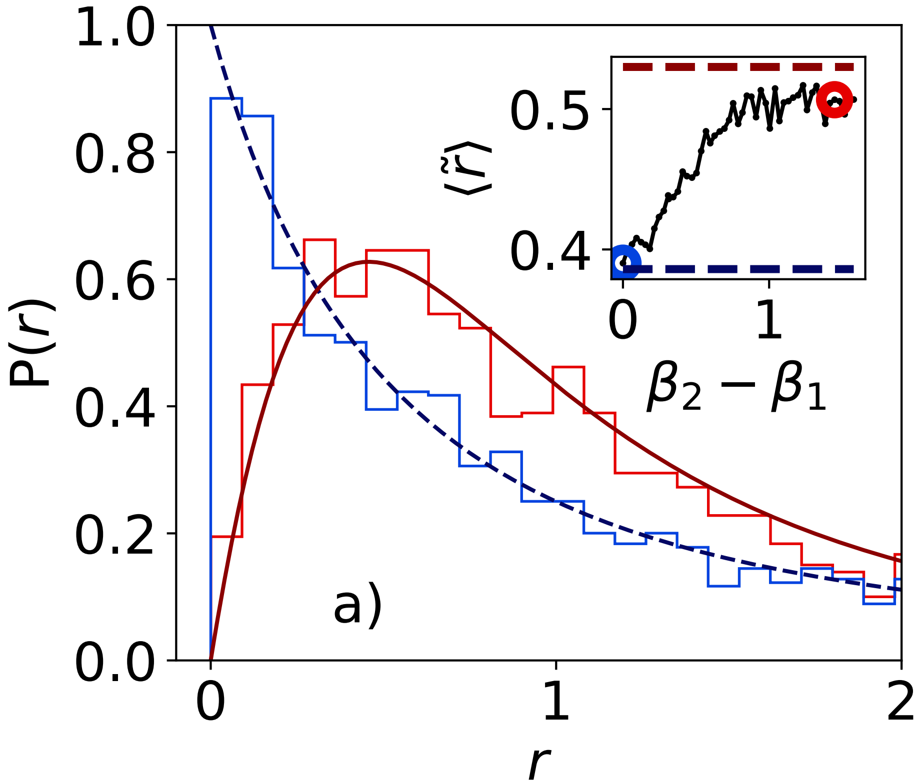


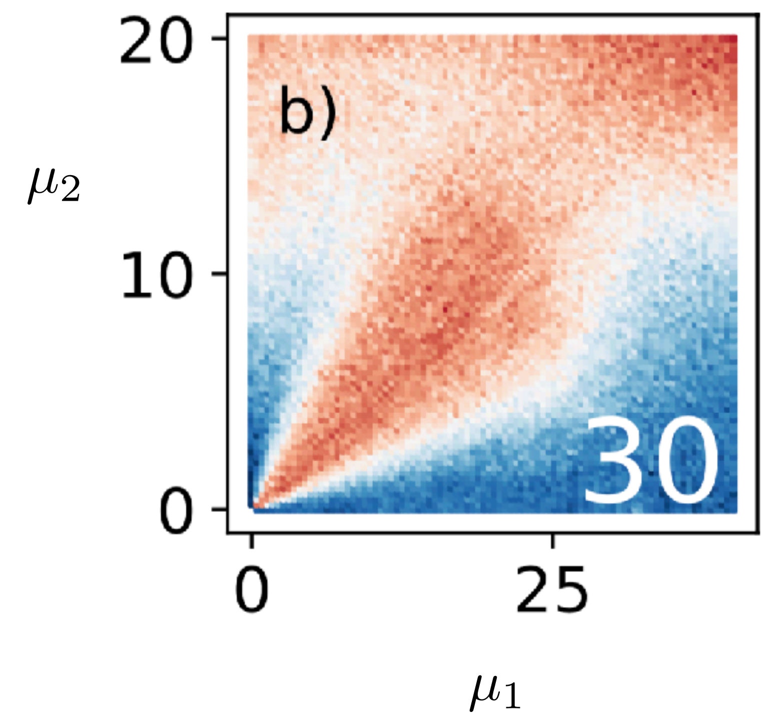
Poisson

Level spacing statistics

double sine-Gordon model

$$H_{DSGM} = \frac{1}{2} \int \left[(\partial_t \phi)^2 - (\partial_x \phi)^2 - \mu_1 \cos \beta_1 \phi - \mu_2 \cos \beta_2 \phi \right] \, \mathrm{d}x$$



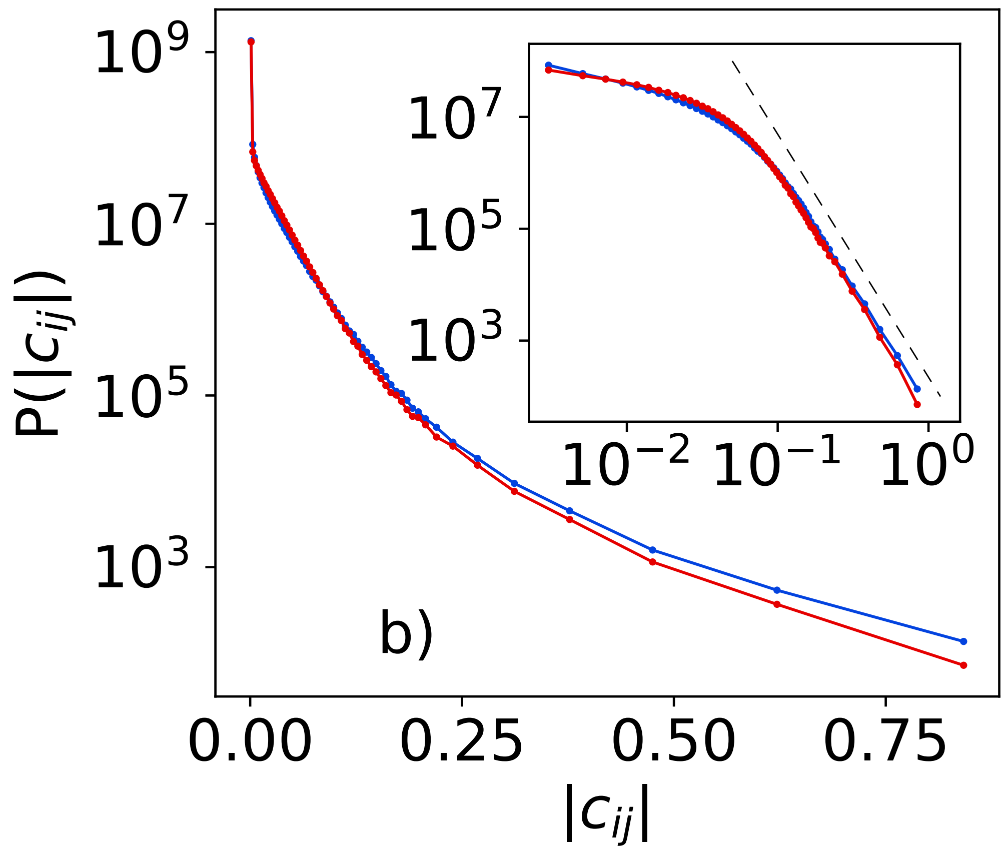


- Note:
  RMT behaviour observed even in the *weakly perturbative* regime!
  (in contrast to quantum chaos intuition)

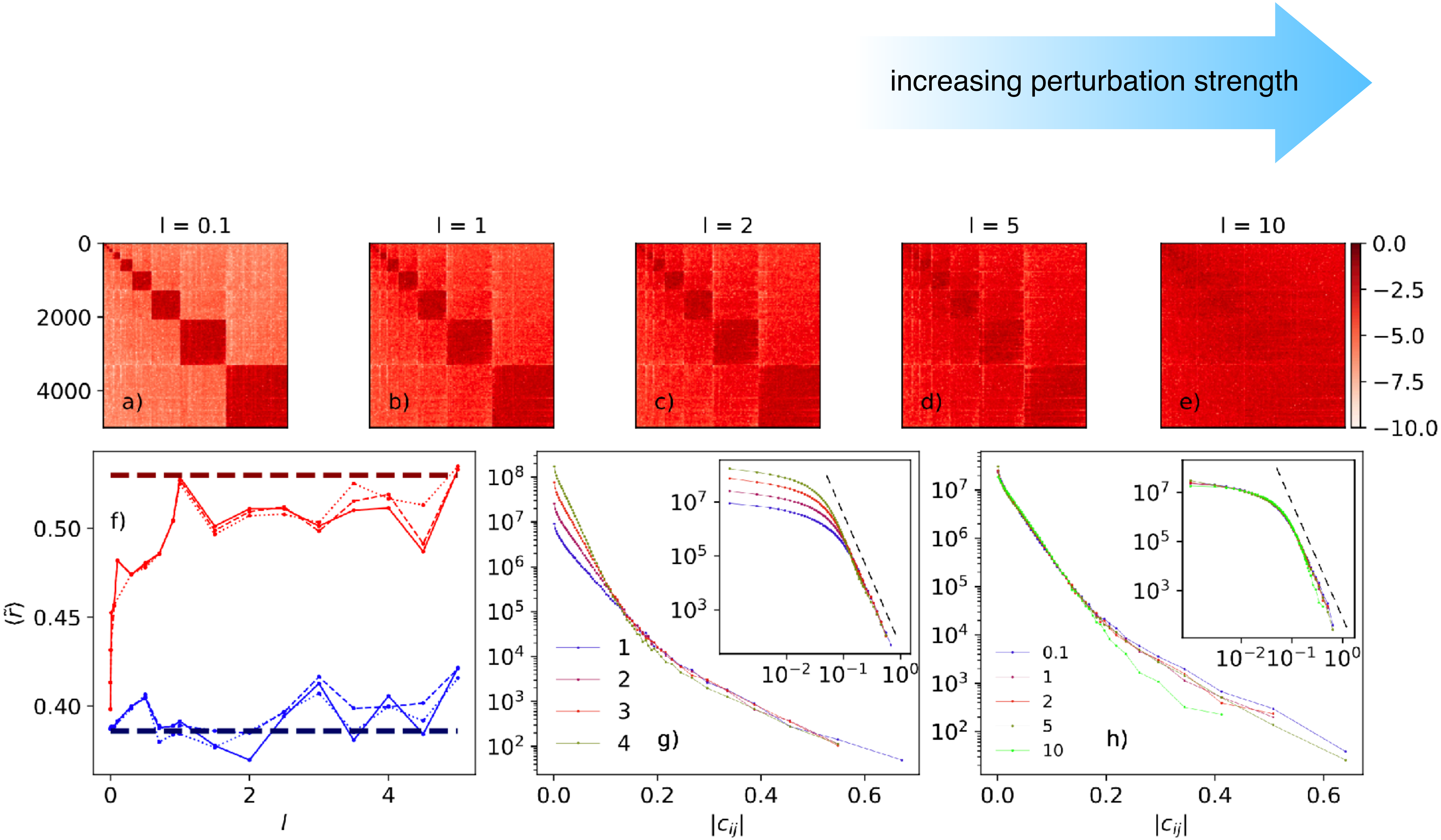
Eigenvector component statistics

- Surprise:
  Even though level statistics obey RMT, eigenvector component statistics *do not follow* the RMT prediction!
  (long-tails instead of Gaussian distribution)

- Gaussianity of eigenvector components statistics crucial for validity of the Eigenstate Thermalisation Hypothesis (ETH)
  → challenges thermalisation in DSG



Level spacing & eigenvalue component stats



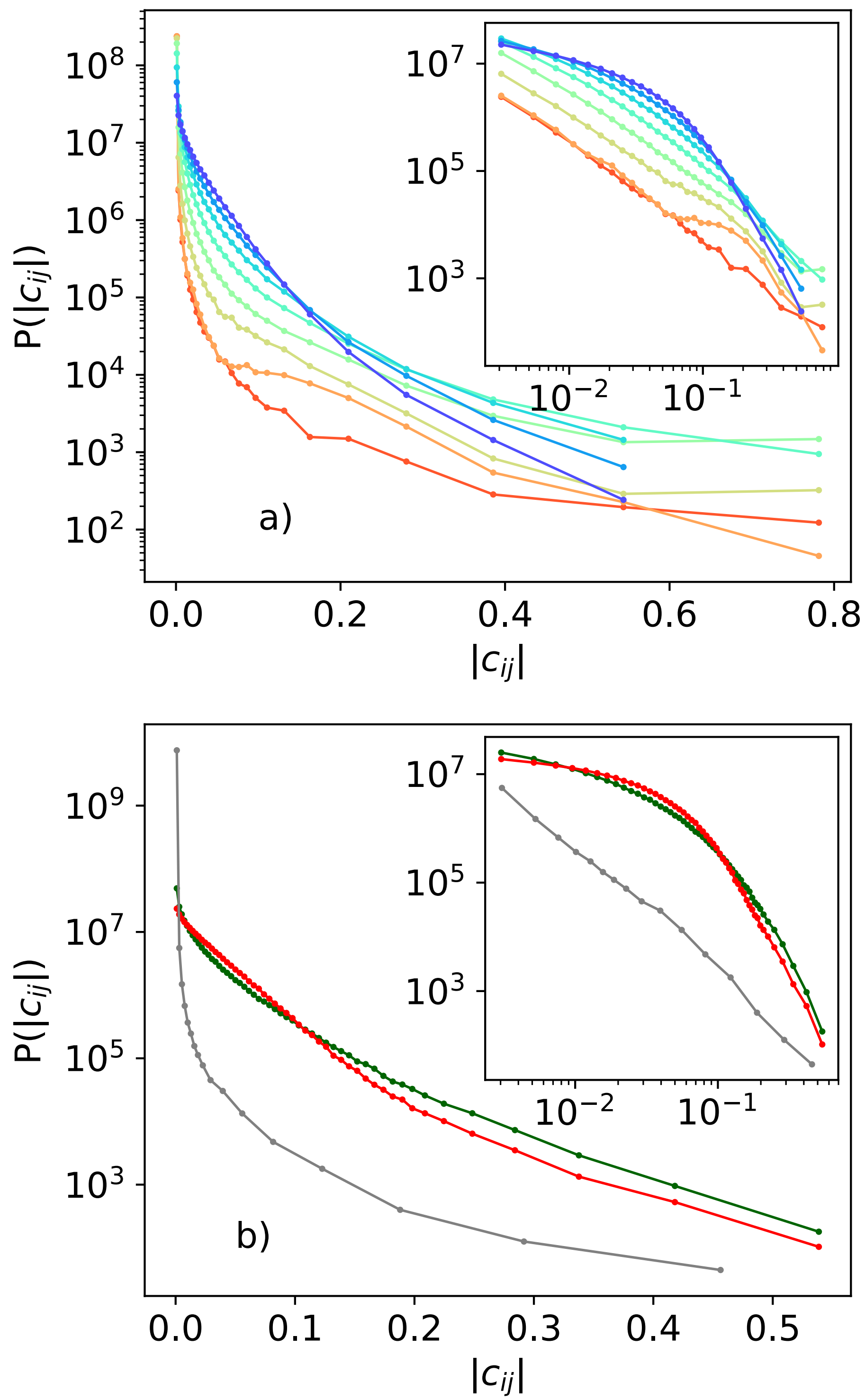
- Long-tail distribution unchanged from weak to intermediate perturbation strength

Eigenvector component statistics

- Surprise:
  Even though level statistics obey RMT, eigenvector component statistics *do not follow* the RMT prediction!
  (long-tails instead of Gaussian distribution)

- Gaussianity of eigenvector components statistics crucial for validity of the Eigenstate Thermalisation Hypothesis (ETH)
  → challenges thermalisation in DSG

- Same observations in (1+1)D ϕ^4 model



Outline

Introduction

motivation

quantum many-body dynamics

why one spatial dimension?

An analog quantum field simulator

how cold-atom experiments can help us solve the mysteries of QFT

Classical "simulation" of a quantum simulator

a numerical RG method for QFT

Effects of topological excitations in and out of equilibrium

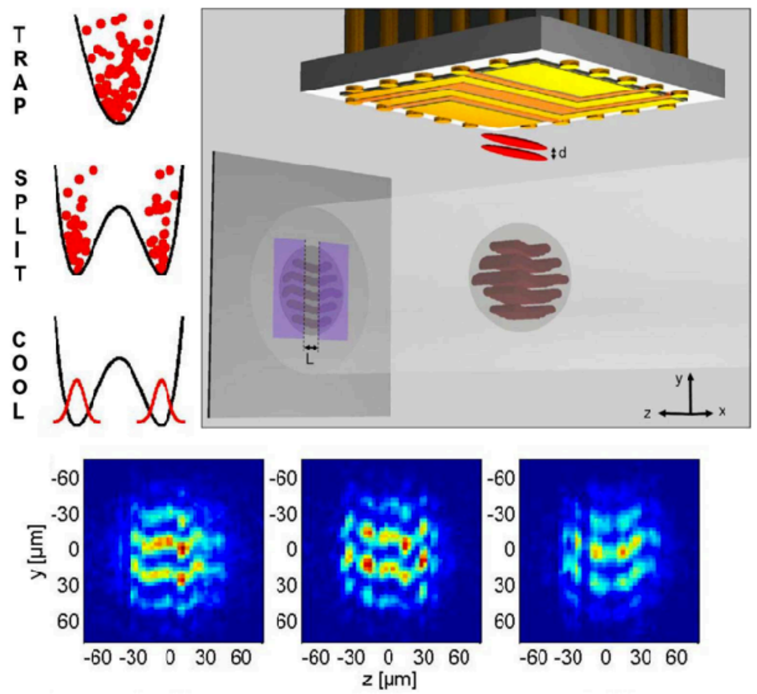
solitons and non-locality

Quantum equilibration and recurrences

a quantum central limit theorem

Quantum Chaos

level spacing & eigenvector statistics



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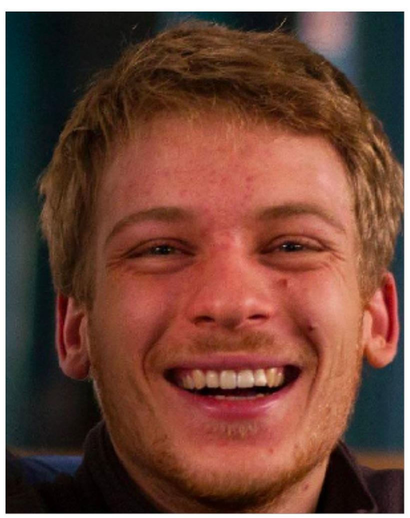

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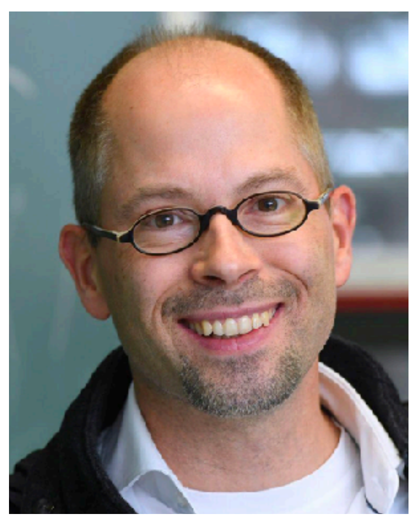

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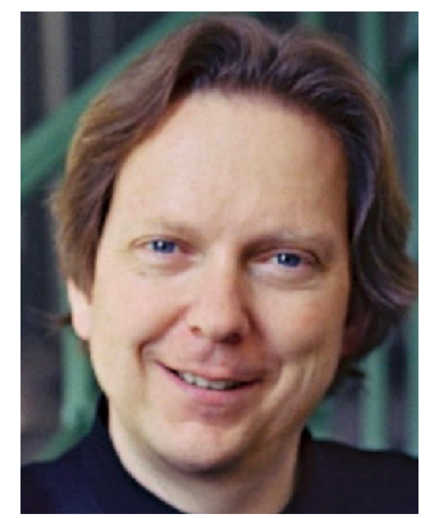

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Thank you for your attention!