



## CIRM-ICTP Complex Analysis and Geometry - XXV (smr 3601)

The CIRM "Meeting on Complex Analysis and Geometry" has now reached its 25th edition and is well known to be an ideal occasion to share among experts and young researchers the most interesting advances in these fields.

**5 online mini-courses with discussion sessions will be broadcasted online.**

### 1. Jean-Pierre Demailly (Grenoble) - $L^2$ extension theorems and applications to algebraic geometry

[Abstract:

The lectures will present  $L^2$  extension theorems for holomorphic sections of bundles and sheaves over complex spaces, under suitable positivity and pseudoconvexity assumptions. The first incarnation of such results is the famous Ohsawa-Takegoshi theorem, proved in 1987. Recently, several generalizations have been obtained, for instance new extension-interpolation theorems over non necessarily reduced complex subvarieties.

General plan of the lectures:

1. General setup and main goals. Technical preliminaries: fundamental a priori estimates, twisted Bochner technique
2. Proof of the main statements
3. Applications to algebraic and analytic geometry

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### 2. Valentino Tosatti (McGill) - Introduction to the Kähler-Ricci flow

[Abstract:

The Ricci flow is an evolution equation which deforms a Riemannian metric in the direction of its Ricci tensor. If the underlying manifold is complex and the initial metric is Kähler, then so are the evolved metrics, and the flow is called the Kähler-Ricci flow. When the manifold is also compact, the flow becomes intimately related to the complex structure of the manifold, and there is a conjectural picture due to Song-Tian for the behavior of the flow in arbitrary dimensions.

In these lectures I will give an introduction to the Kähler-Ricci flow, and present some results which fit in this framework. The topics to be covered include: short-time existence of the flow and characterization of the maximal existence time, the formation of singularities in finite time, and the long-time behavior of immortal solutions.

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### **3. Anna Fino (Torino) - Interplays of Complex and Symplectic Geometry**

[Abstract:

I will present - mostly using examples - how complex and symplectic structures relate on compact manifolds.

I will show how symplectic forms taming complex structures are intimately related to a special type of Hermitian metrics,

known in the literature also as "pluriclosed" metrics. Then I will present some results on the symplectic Calabi-Yau problem in dimension 4

and on balanced metrics in relation to the Strominger-Hull system. A tentative schedule:

Pluriclosed metrics

Symplectic Calabi-Yau problem

Balanced metrics and the Strominger-Hull system

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### **4. Vincent Guedj (Toulouse) - Canonical metrics on mildly singular compact Kaehler varieties**

[Abstract:

The aim of this series of lectures is to present a few motivating problems on the classification of compact Kaehler varieties. We shall review the construction of some canonical Kaehler metrics on mildly singular varieties and discuss the construction and long term behavior of weak Kaehler-Ricci flows.

The precise plan is as follows:

Lecture 1: A panoramic view of the singular Kaehler-Einstein problem

Lecture 2: A new proof of a key uniform a priori estimate for degenerate complex Monge-Ampère equations

Lecture 3: Song-Tian program and construction of weak Kaehler-Ricci flows.

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### **5. Franc Forstneric (Ljubljana) - Minimal Surfaces from a Complex Analytic Viewpoint**

[Abstract:

In the first introductory lecture, I shall explain the basics about minimal surfaces in real Euclidean spaces  $\mathbb{R}^n$  ( $n \geq 3$ ) and their connection to complex analysis. The second lecture will be focused on a new Schwarz-Pick lemma and distance estimates for minimal surfaces in the ball, and the resulting hyperbolicity theory for domains in  $\mathbb{R}^n$  based on minimal surfaces. In the third lecture I will survey a few other recent results including the Runge-Weierstrass approximation and interpolation theorems, the existence of minimal surfaces with a given Gauss map, and new results on the Calabi-Yau conjecture for minimal surfaces. Several of these results are explained in more detail in the recently published monograph with the same title by Alarcón, Forstnerič, and López (Springer Monograph in Mathematics, 2021, [url{https://www.springer.com/gp/book/9783030690557}](https://www.springer.com/gp/book/9783030690557)).

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