Data analysis and modeling in the evolution of the solar cycle period

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October 25, 2021

Plan

- A brief but exciting description of the Sun as a complex system

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- The problem of sudden de-synchrnoziation of the components of the solar magnetic field

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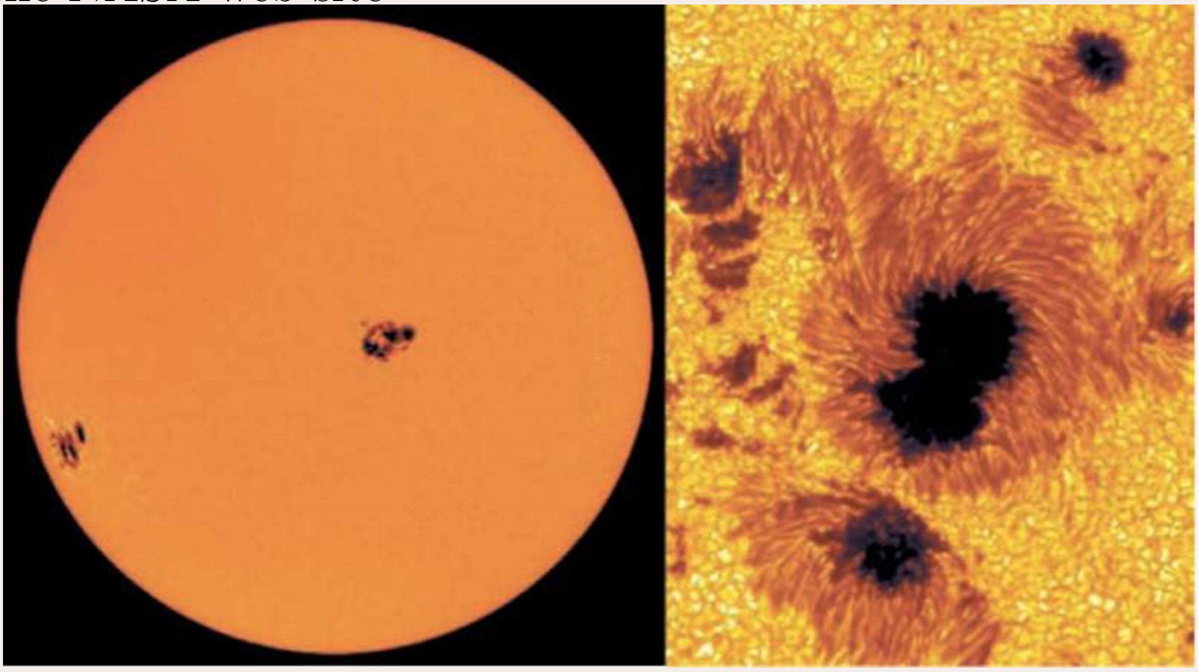
- A brief but exciting description of the Sun as a complex system
- The problem of sudden de-synchrnoziation of the components of the solar magnetic field
- It is tackled through the reconstruction of the coupling between poloidal and toroidal magnetic fields found with the Kuramoto model of coupling oscillators

The Sun as a complex system

The Sun as an example of heterogeneity

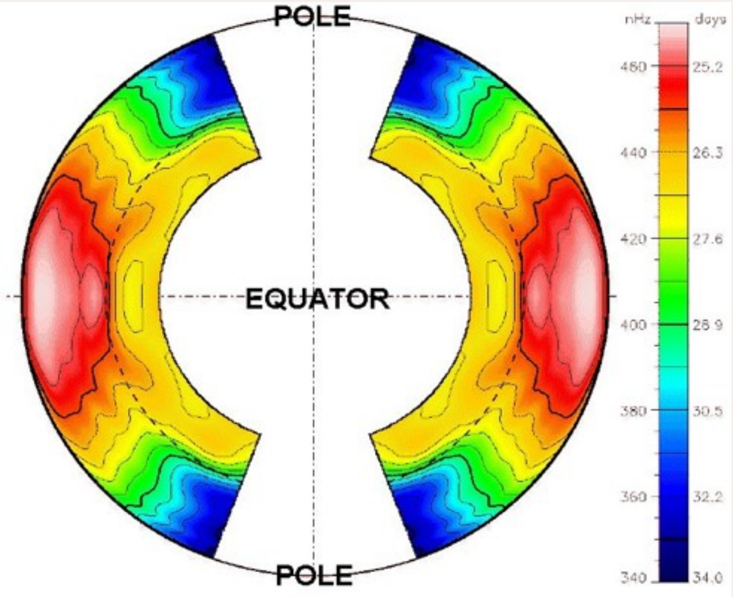
From the NASA web-site



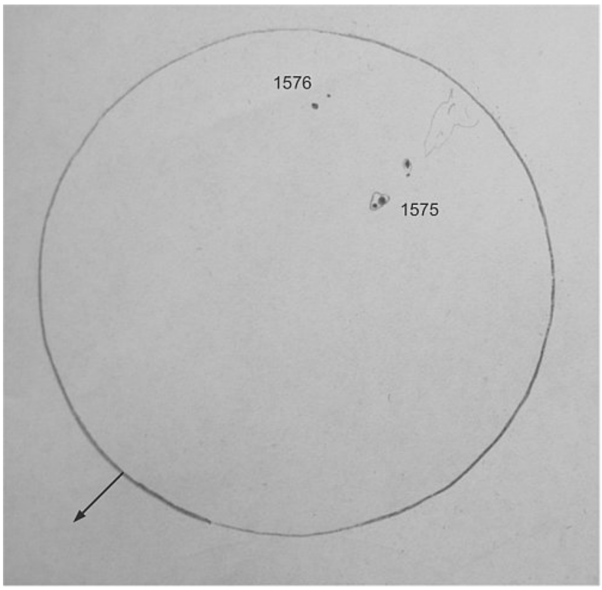
Heterogeneity at different scales

The Sun as an example of regularity breaking

The Sun exhibits differential rotation, where the solar rotation period varies from 25 to 32 days depending on the latitude. The average period is approximately 27 days
From the NASA web-site; D. Hathaway



The Sun as an example of heterogeneous obsrevations



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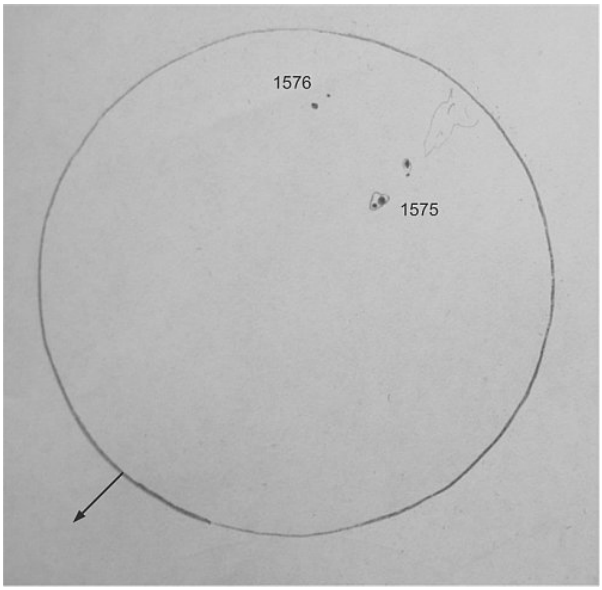
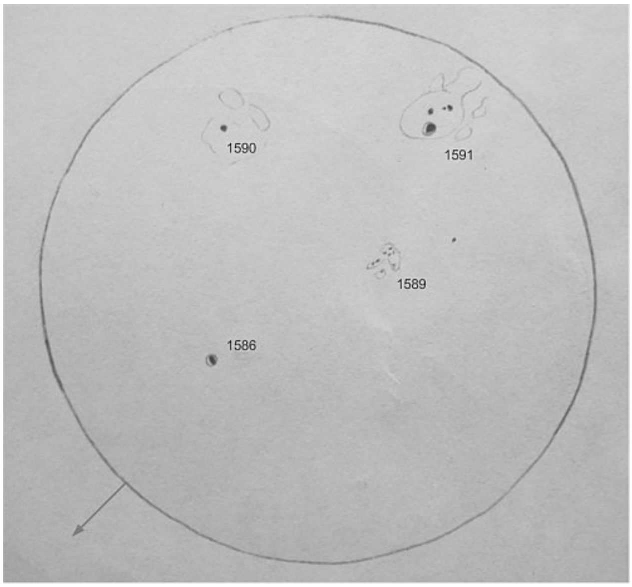


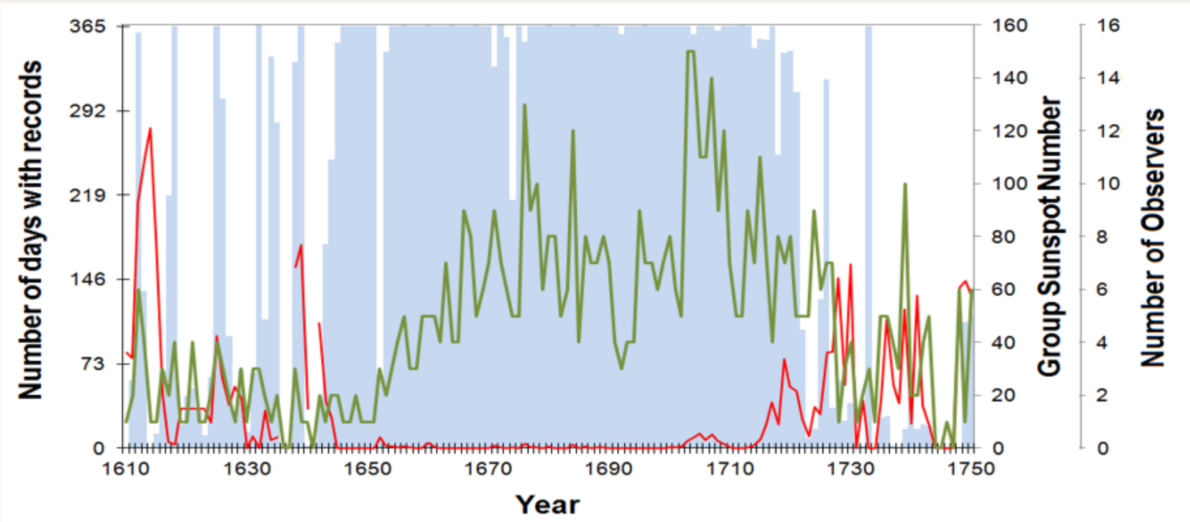


Observatory in Moscow; 22.09.2012 and 14.10.2012
The Wolf numbers defined as $R = k(10G + s)$,
where G is the number of the groups, s is the number of the sunspots within the group, and k is so called observer coefficient

Historical note

- Louis XIV, the Sun King, declared to observe sunspots
- and the sunspots immediately disappeared. The epoch called the Maunder minimum started

The Sun as an example of numerous observations

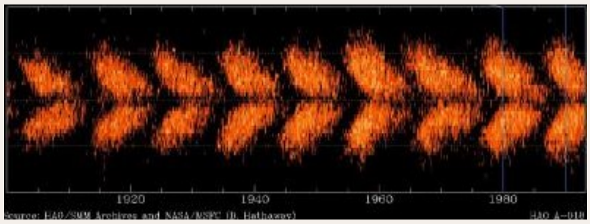


Clette et al 2014
Coverage of the original GN series in the time period 1610–1750: number of days with records per year (blue bars), number of sunspot observers per year (green line) and GN (red line).

The Sun as an example of cyclicity

https://www2.hao.ucar.edu/Education/Sun/butterfly-diagram

- The number of sunspots along the latitude



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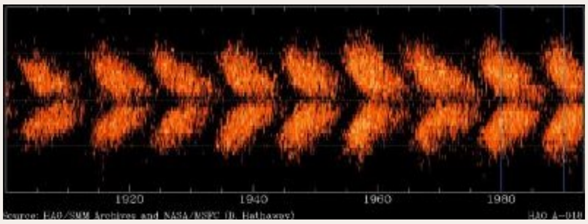

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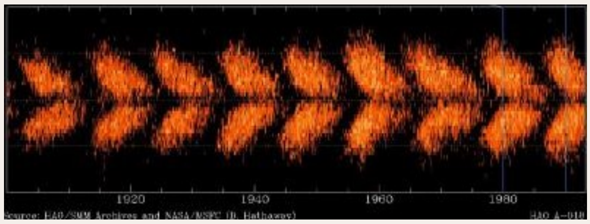

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- Equatorward drift of sunspots as the cycle proceeds North–South asymmetry, 20th cycle, 1965–1976

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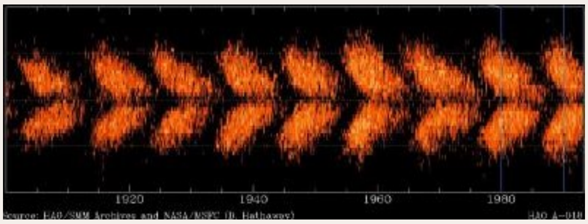

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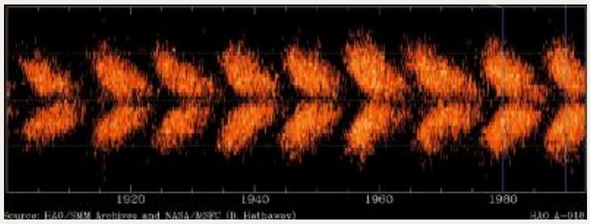

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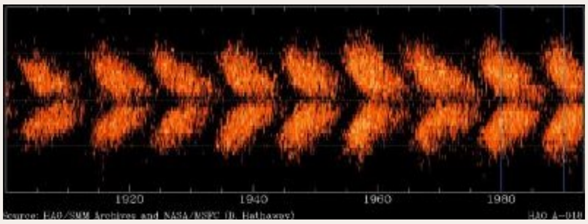

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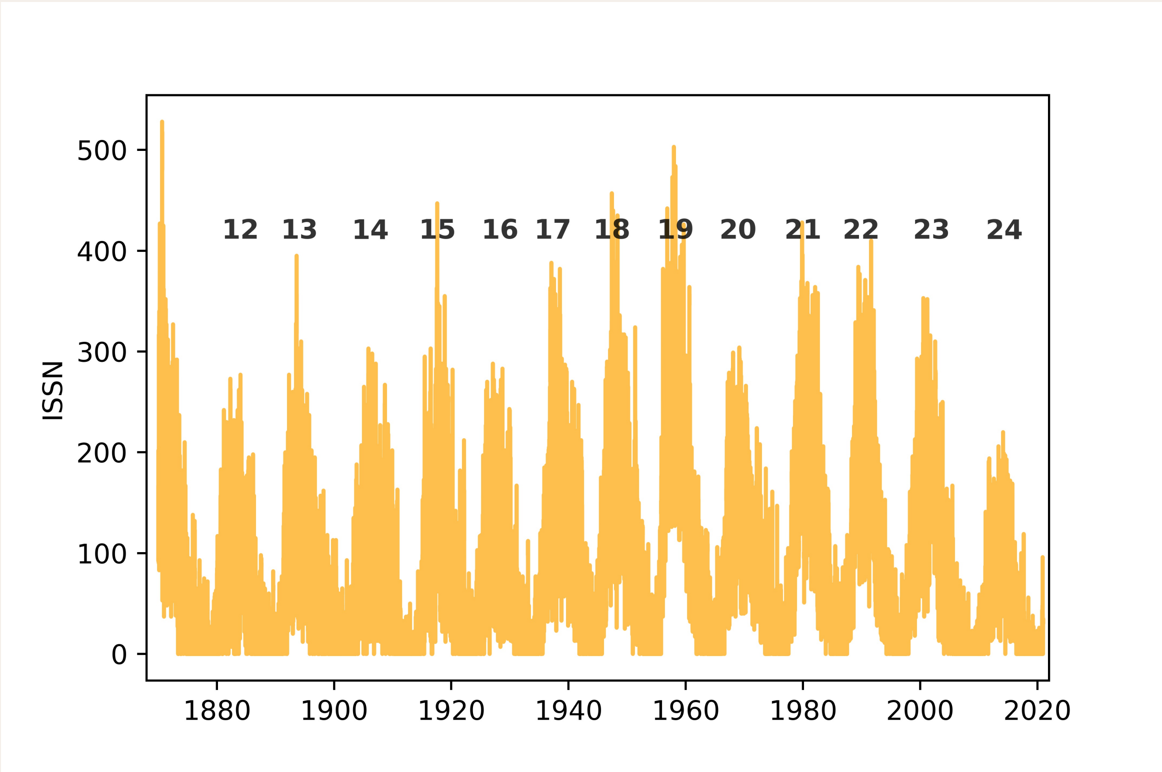
- Butterfly diagram
- $\sim$ 11-year solar cycle, $\sim$ 22-year Hale cycle = 2 solar cycles

The Sun as an example of cyclicity

Sunspot numbers: Index ISSN



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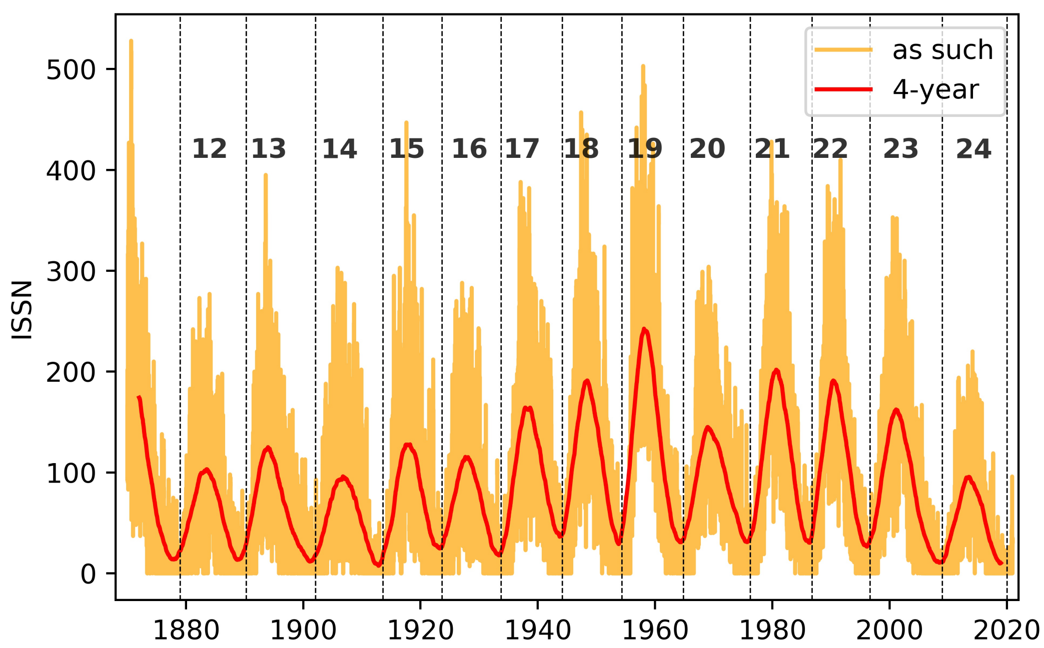

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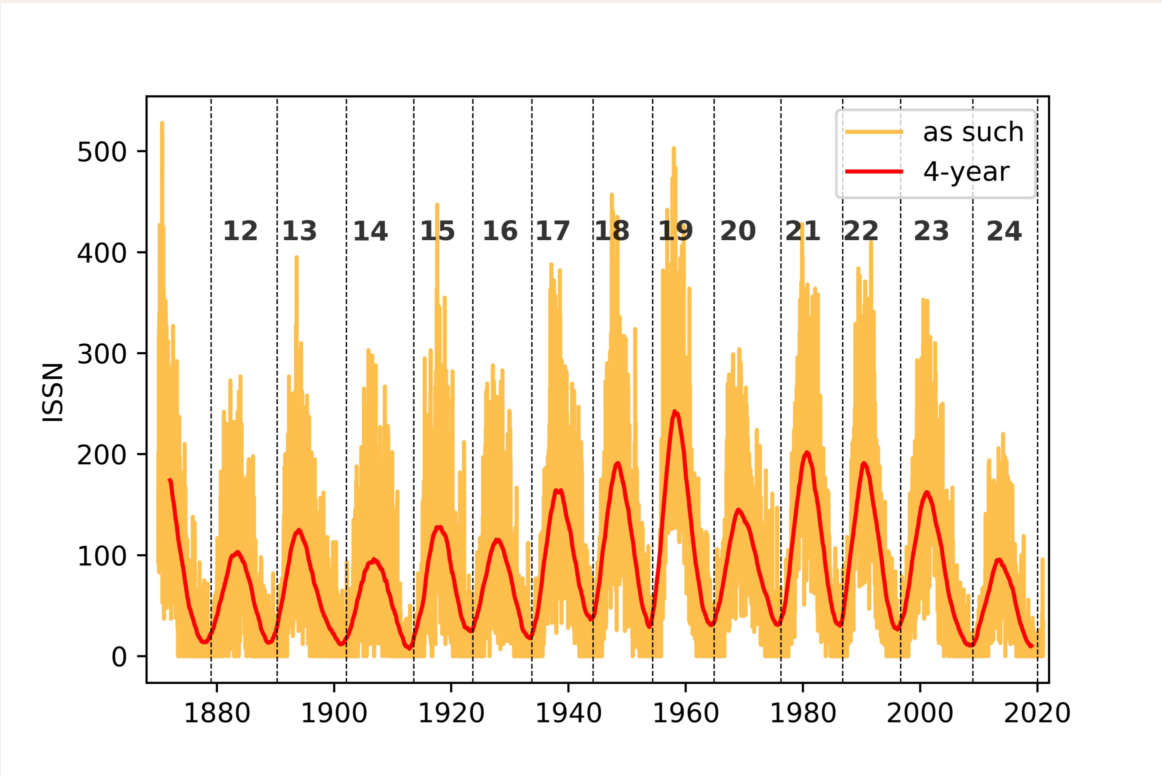
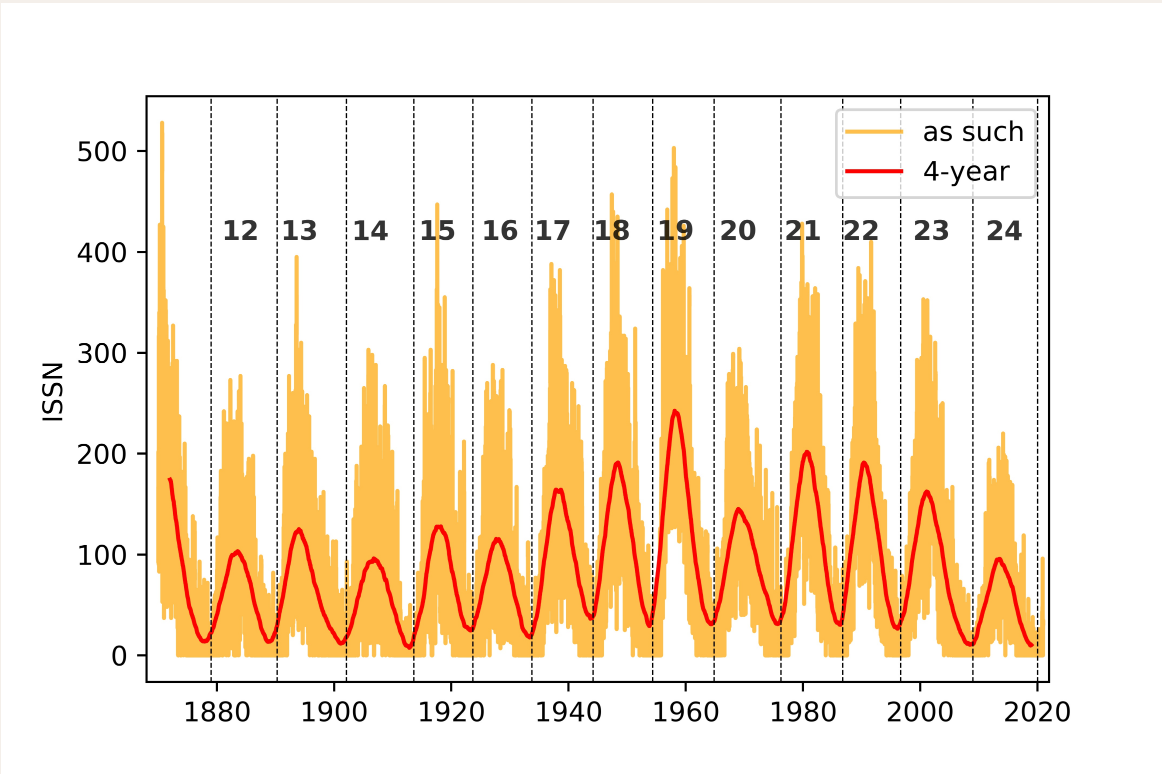


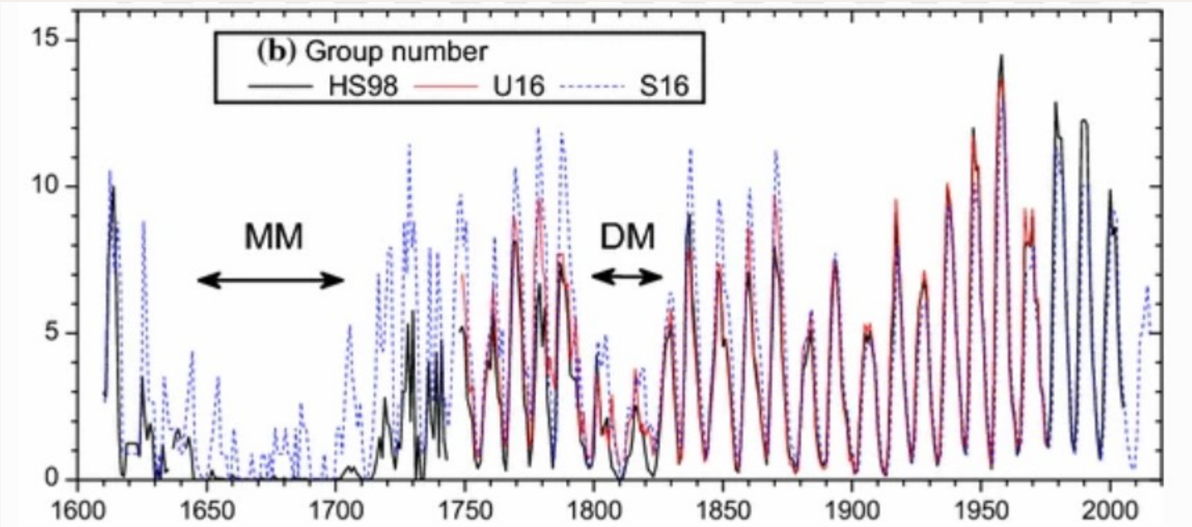
- Approximately 11-year solar cycle

The Sun as an example of cyclicity



- Both period and amplitude vary from cycle to cycle

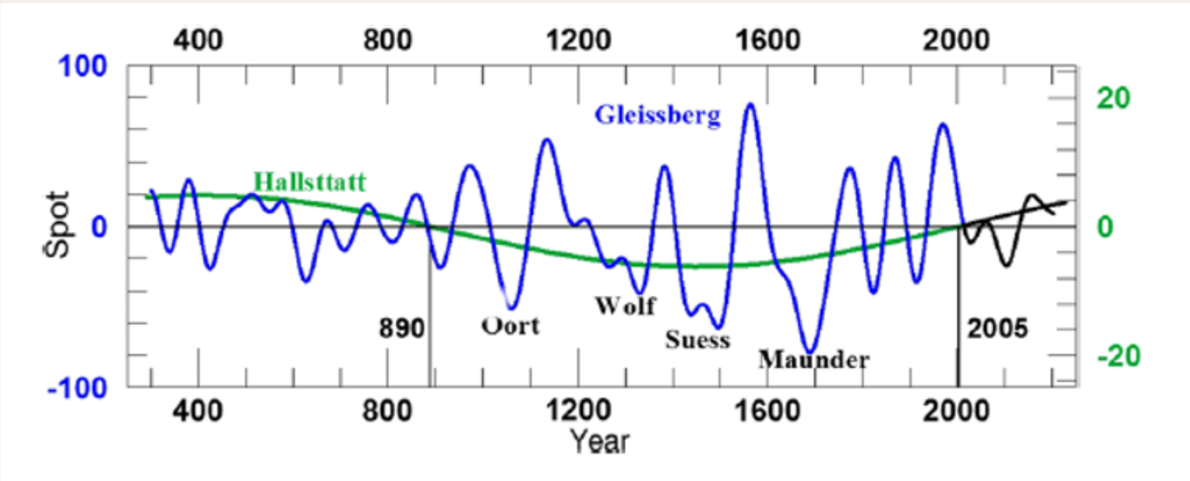
The Sun as an example of various scales



Usoskin 2016 and 2017

- Secular Gleissberg cycle, 2000–2400 year Hallstatt cycle

The Sun as an example of and low-dimensional chaos



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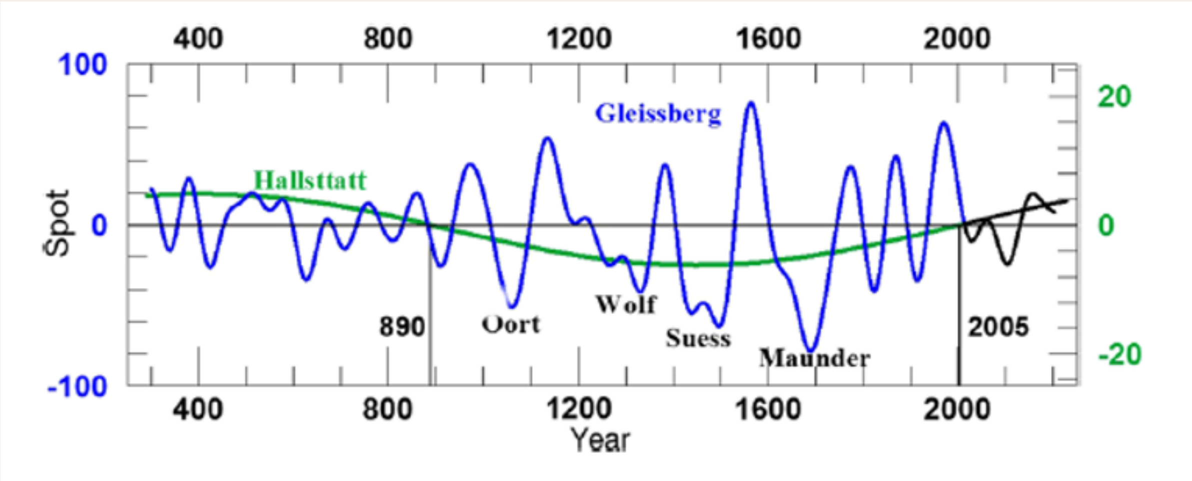

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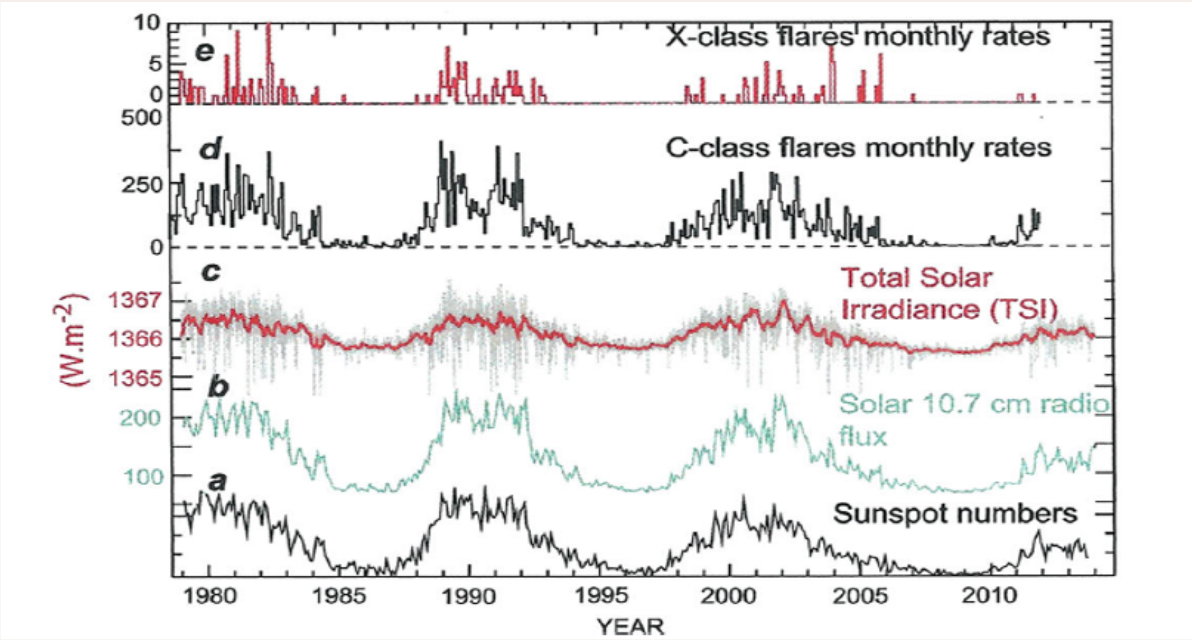
Carbon-14 is a weakly radioactive isotope of Carbon; also known as radiocarbon, it is an isotopic chronometer

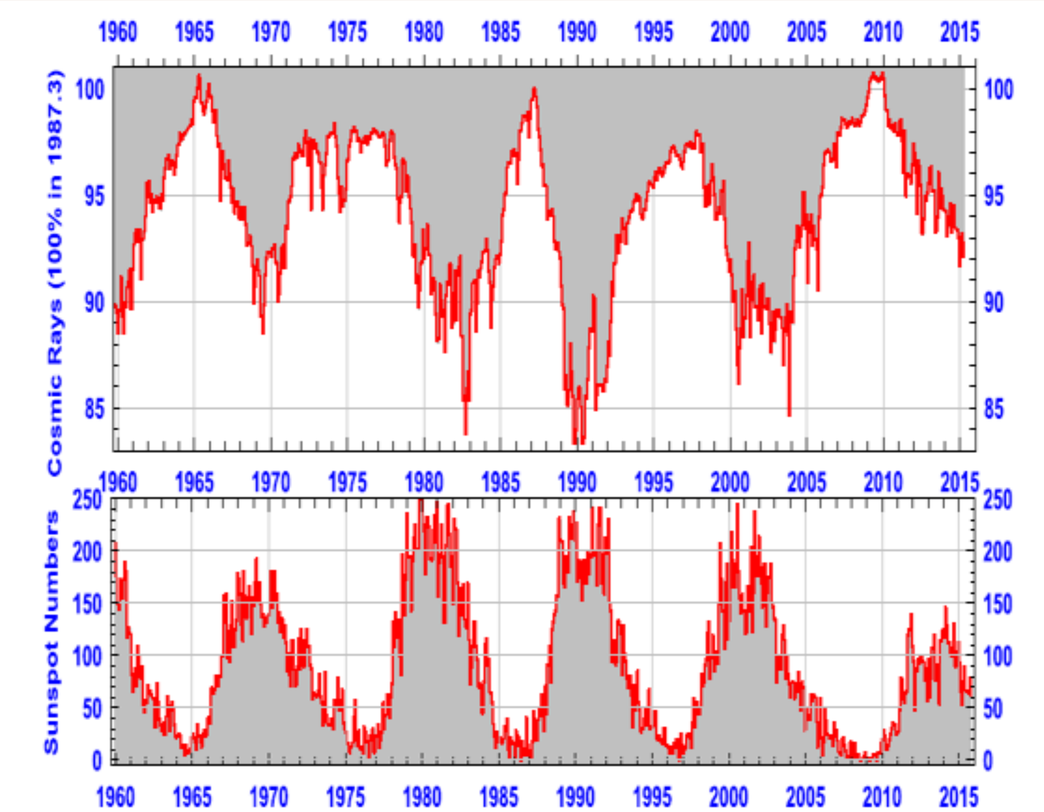
The Sun as an example of synchronization



De Jager 2016

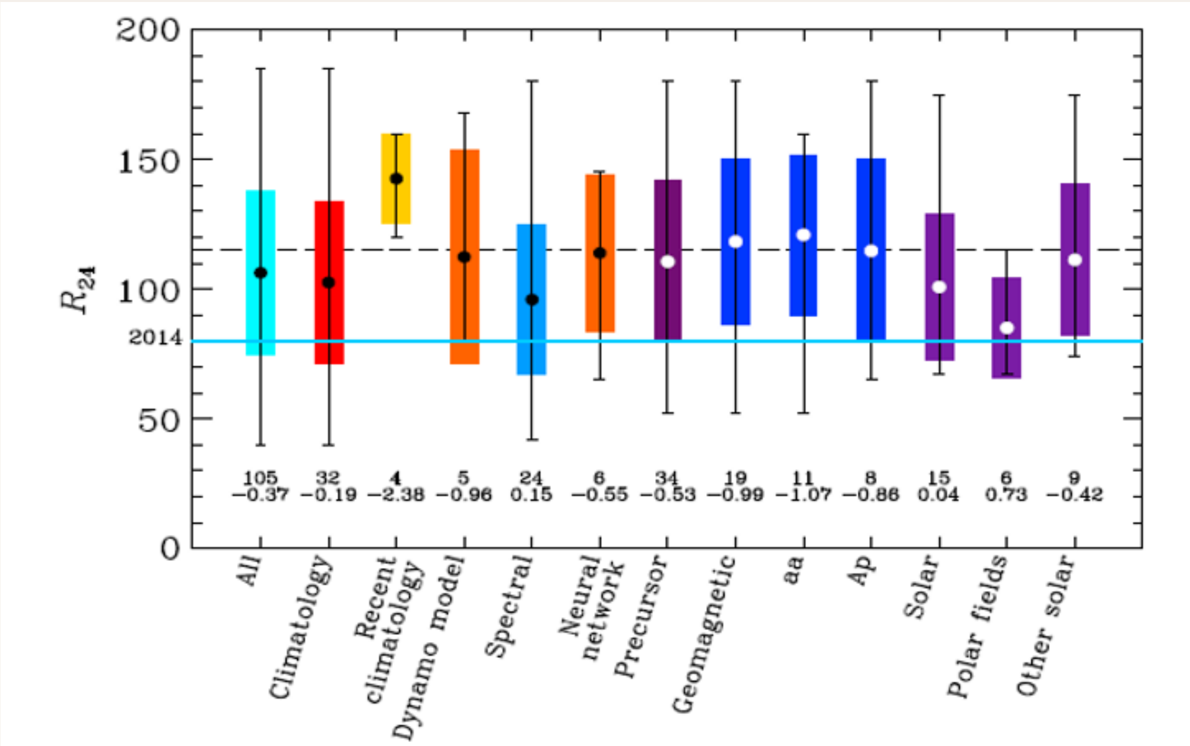
Global synchronization (anti-correlation) but various losses of synchronization with intervals of several years

The Sun as an example of de-synchronization



De Jager 2016

The Sun as an example of various predictions



Pesnel (2016) collected numerous prediction of the maximum of cycle 24
Large deviation of the majority of predictions to the observed value (blue level)

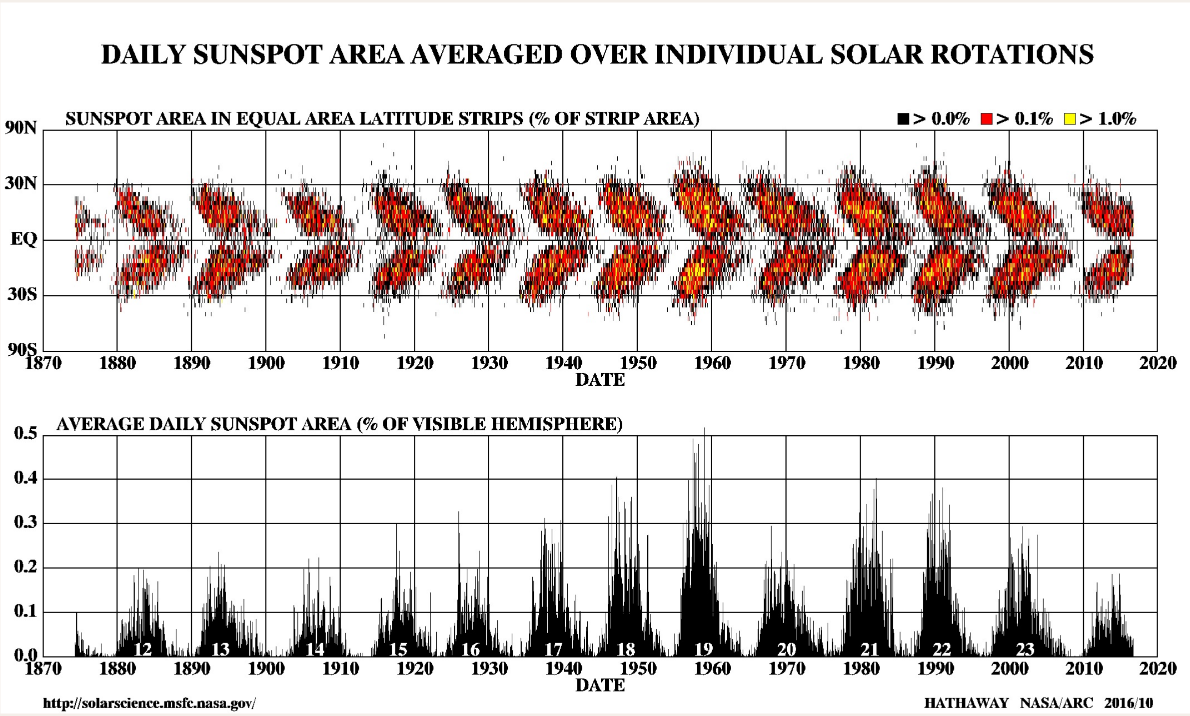
Conclusion: The Sun is a simple natural example

of a complex system with

- Continuous development of knowledge
- Huge data volume
- Non-linear modeling
- Non-trivial prediction

Identification of de-synchronization

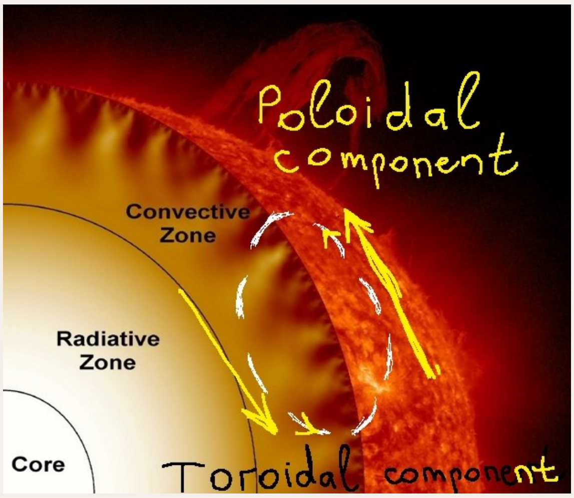
Butterfly diagram



Meridional flow

https://solarscience.msfc.nasa.gov/dynamo.shtml

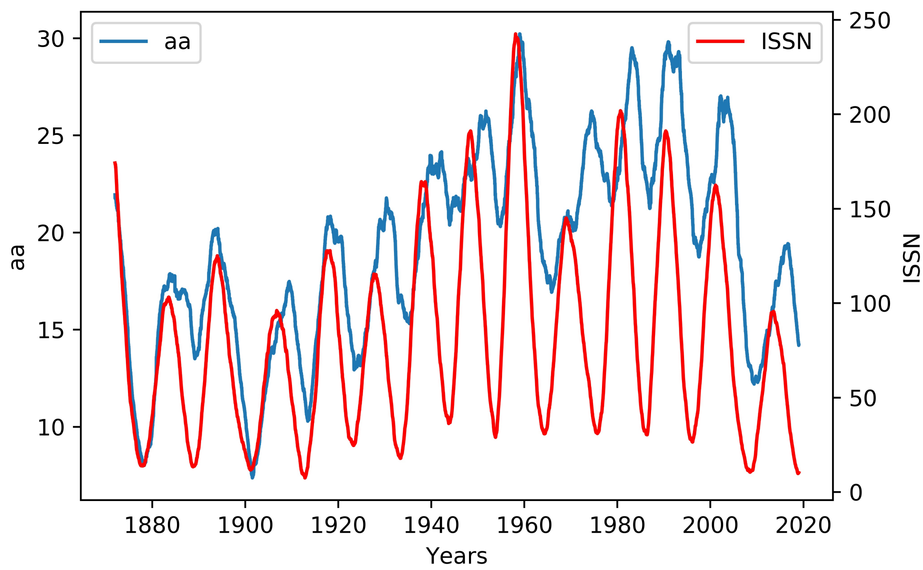
At the surface this flow is a slow 20 m/s (40 mph) but the return flow toward the equator inside the Sun where the density is much higher must be much slower still from 1 to 2 m/s (2 to 4 mph). This slow return flow would carry material from the mid-latitudes to the equator in about 11 years.



One can use Sunspot Number and aa index as proxy to the toroidal and poloidal components of the solar magnetic field

Variation of the instantaneous solar cycle period

4-year smoothing of ISSN and aa



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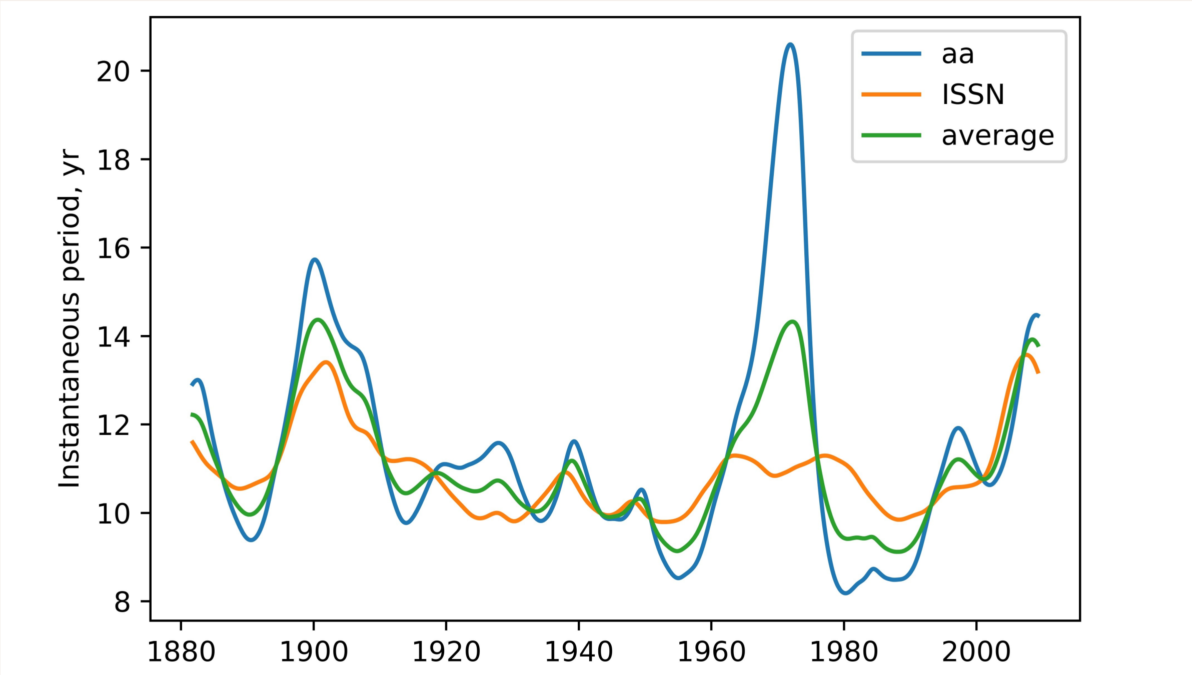

- Oscillations around 11 years corresponding to the solar cycle
- Anomalies centered at $\sim$ 1905, 1975, 2015

Research question

Will anomalous growth in the instantaneous period results in de-syhcnronization of toroidal and poloidal components of the solar magnetic field?

Methods

- MHD: Partial differential equations
- Measurements “inside” the Sun are difficult to perform; required to determine the coefficients
- Huge simplification of MHD equations are required to solve them numerically
- Examples: P. Charbonneau, Dynamo models of the solar cycle, Living Reviews in Solar Physics 4 (2020); G. Hazra, Recent advances in the 3d kinematic Babcock–Leighton solar dynamo modeling, Journal of Astrophysics and Astronomy 42 (2) (2021) 354 1–21.

Methods

We tackle the problem

with the Kuramoto model of coupling oscillators

Two Kuramoto Oscillators: Theoretical Foundations

Definition

$$\dot{\theta}_1 = \omega_1 + \frac{k_1}{2}\sin(\theta_2 - \theta_1)$$
$$\dot{\theta}_2 = \omega_2 + \frac{k_2}{2}\sin(\theta_1 - \theta_2)$$

where ω_1 and ω_2 are natural frequencies,
θ_1 and θ_2 are phases
k_1 and k_2 represent the coupling coefficients, which are here different and therefore the mutual relationship is not symmetrical.

Two Kuramoto Oscillators: Theoretical Foundations

Notation

$$\theta(t) = \theta_1(t) - \theta_2(t) \quad \text{phase difference}$$

the average frequency and the difference of the natural frequencies:

$$\Omega = \frac{\omega_1 + \omega_2}{2} \qquad \Delta\omega = \frac{\omega_1 - \omega_2}{2}$$

symmetrical and asymmetrical component of the coupling:

$$k = \frac{k_1 + k_2}{2} \qquad \Delta k = \frac{k_1 - k_2}{2}$$

Two Kuramoto Oscillators: Theoretical Foundations

Model: to recall

$$\dot{\theta}_1 = \omega_1 + \frac{k_1}{2}\sin(\theta_2 - \theta_1)$$
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Equivalent form of the model

$$\frac{\dot{\theta}_1 + \dot{\theta}_2}{2} = \Omega - \frac{\Delta k}{2}\sin\theta(t) \quad (1)$$
$$\dot{\theta}(t) = 2\Delta\omega - k(t)\sin\theta(t) \quad (2)$$

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Synchronization

Theorem

If the symmetrical component k of the coupling is large enough, then the phase difference $\theta = \theta_1 - \theta_2$ tends to a limit.

Two Kuramoto Oscillators: Theoretical Foundations

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Direct problem

- Given k_1, k_2, ω_1, and ω_2,

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- Given k_1, k_2, ω_1, and ω_2,
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- It is trivial

Two Kuramoto Oscillators: Inverse Problem

- $\frac{\dot\theta_1+\dot\theta_2}{2} = \Omega - \frac{\Delta k}{2}\sin\theta(t); \quad \dot\theta(t) = 2\Delta\omega - k(t)\sin\theta(t)$

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- Solution is also turned to algebraic equations

Let's see the triviality

$$\frac{\dot{\theta}_1 + \dot{\theta}_2}{2} = \Omega - \frac{\Delta k}{2} \sin\theta(t) \tag{3}$$

$$\dot{\theta}(t) = 2\Delta\omega - k\sin\theta(t) \tag{4}$$

Let's see the triviality

$$\varphi = \Omega - \frac{\Delta k}{2} \sin \theta \tag{3}$$

$$0 = 2\Delta\omega - k \sin \theta(t) \tag{4}$$

Relation to the Sun

- What is the input of any model?

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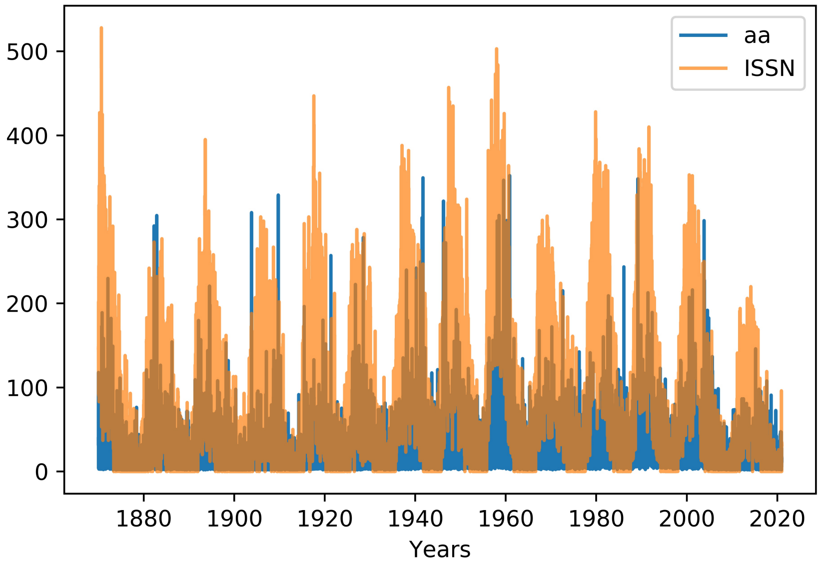

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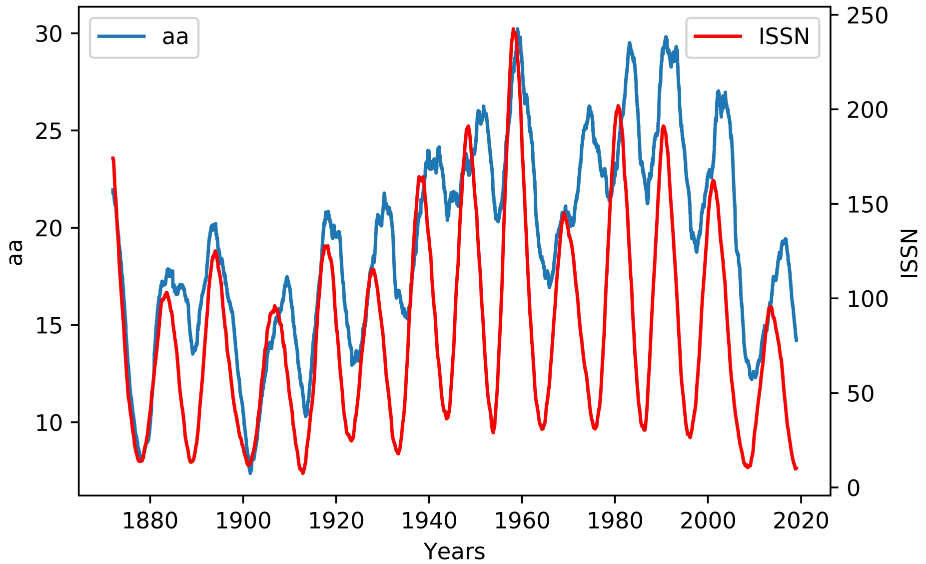

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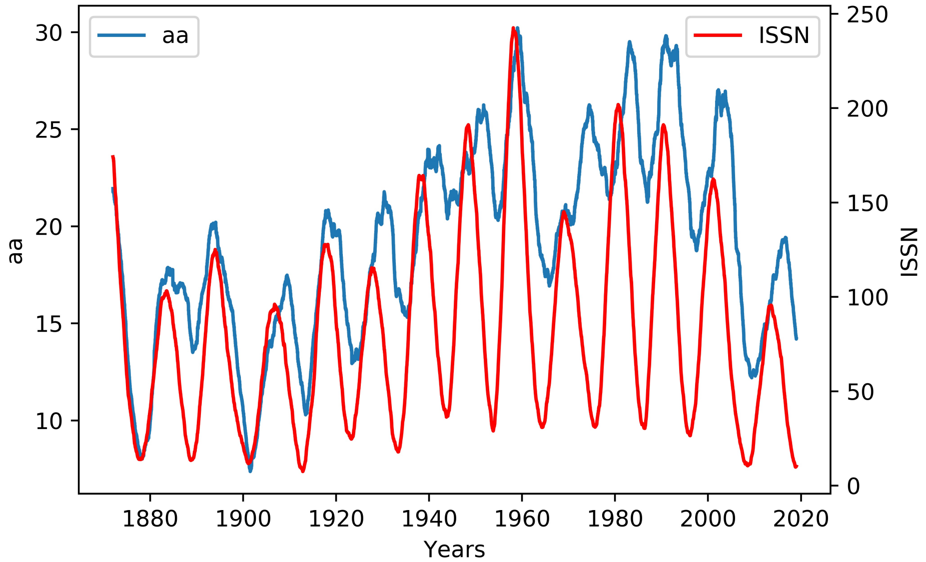

- 4-year average of ISSN goes as a (sine?) wave; that of aa is less regular
- The correlation between the two waves determines the phase difference

Correlation and the phase difference

- With the phases θ_1 and θ_2 given by the equations

$$\dot{\theta}_1 = \Omega + \Delta\omega + \frac{k_1}{2}\sin(\theta_2 - \theta_1)$$
$$\dot{\theta}_2 = \Omega - \Delta\omega + \frac{k_2}{2}\sin(\theta_1 - \theta_2)$$

we associate two oscillators

$$X_0(t) = \sin\left(\Omega_m t - \frac{\varphi}{2}\right), \quad Y_0(t) = \sin\left(\Omega_m t + \frac{\varphi}{2}\right),$$

where $\Omega_m = \frac{\dot{\theta}_1 + \dot{\theta}_2}{2}$, $\varphi = \theta_1(t) - \theta_2(t)$ in the synchronized state

Correlation and the phase difference

- $$X_0(t) = \sin\left(\Omega_m t - \frac{\varphi}{2}\right), \quad Y_0(t) = \sin\left(\Omega_m t + \frac{\varphi}{2}\right),$$
- The correlation between the oscillators $X_0(t)$ and $Y_0(t)$ computed over the period T is

$$\rho = \frac{\frac{1}{T}\int_s^{s+T} X_0(t) Y_0(t)\,dt}{\frac{1}{T}\left(\int_s^{s+T} X_0^2(t)\,dt \int_s^{s+T} Y_0^2(t)\,dt\right)^{1/2}}$$

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- The direct computations of the integrals give evidence that the numerator is $\frac{\cos\varphi}{2}$, the denominator is $\frac{1}{2}$, and the correlation is

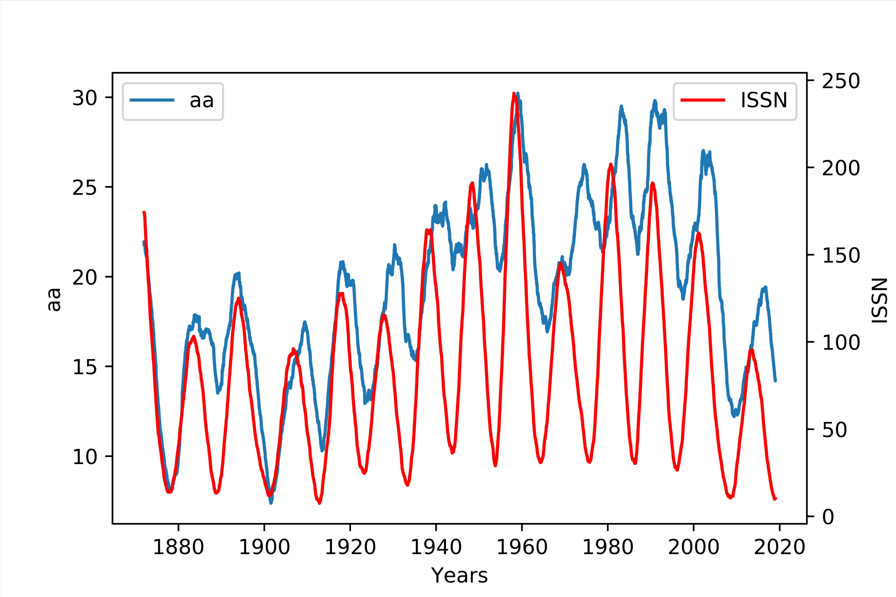
$$\rho = \cos\varphi$$

Real and Ideal Oscillators

We relate ideal oscillators

$$X_0(t) = \sin\left(\Omega_m t - \frac{\varphi}{2}\right), \quad Y_0(t) = \sin\left(\Omega_m t + \frac{\varphi}{2}\right),$$

to smoothed solar proxies



Inverse Problem: a step to specification

- Given smoothed ISSN and `aa`, compute the correlation ρ between them over the solar cycle: the period T is assigned to 10.75 years

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- Given smoothed ISSN and aa, compute the correlation ρ between them over the solar cycle: the period T is assigned to 10.75 years
- $\theta = \arccos\rho$ is served as a proxy to the phase difference in the Kuramoto equations

$$\frac{\dot\theta_1 + \dot\theta_2}{2} = \Omega - \frac{\Delta k}{2}\sin\theta(t)$$
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- No. Earlier we assumed that the phases are synchronized: $\dot\theta = \dot\theta_1 - \dot\theta_2 = 0$. Now, $\theta(t) = \arccos\rho(t)$ is time-dependent.

Inverse Problem: Quasi-Synchronization

Mathematics

Assumption: The phase sum $\theta_1 + \theta_2$ and difference $\theta = \theta_1 - \theta_2$ vary slowly as a generalization of the synchronization described by a constant sum and difference of the phases.

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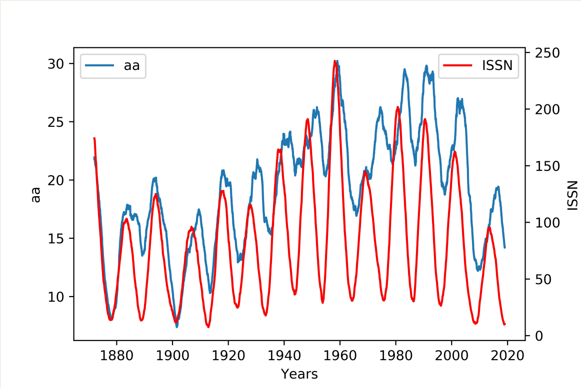

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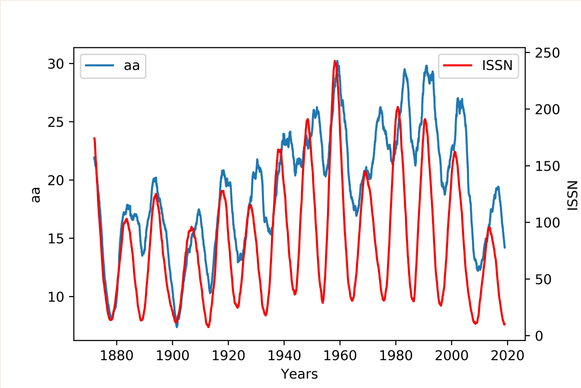

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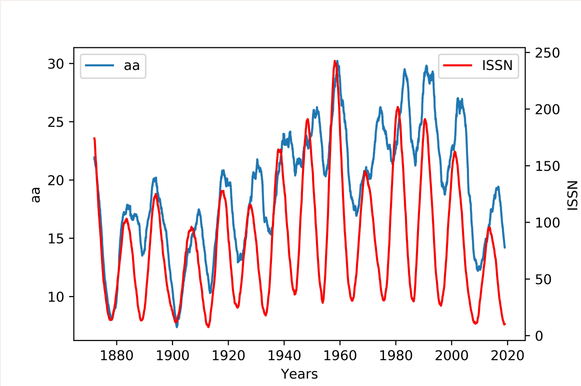

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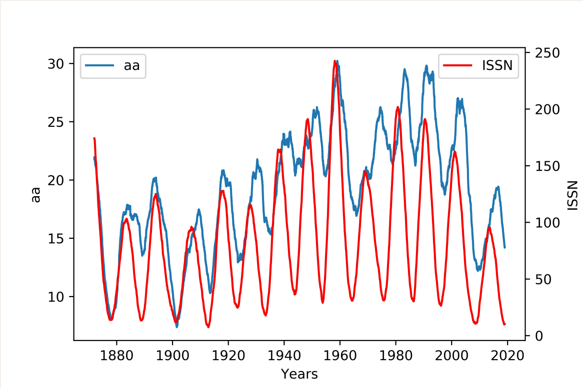

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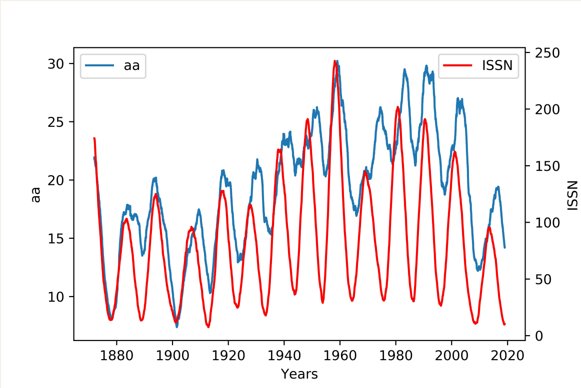
- Clear exceptions: 1870–1905, 1960–1970; NOWADAYS?



Inverse Problem: Known Left Hand-Side

Instead of constants in the left-hand side

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Technical details

- Find the Fourier series $\frac{f_0}{2} + f_1 e^{-ix\xi T/2\pi} + f_2 e^{-2ix\xi T/2\pi} + \ldots$
- Define the instantaneous phases $\theta_1(t)$ and $\theta_2(t)$ as $\arg(f_1)$ computed for the data over the time windows $(t - T/2, t + T/2)$

Inverse Problem: Values of the Natural Frequncies

$\Omega - \Delta\omega$ and $\Omega + \Delta\omega$

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$2\pi/11 \approx 0.57$ years^{-1}

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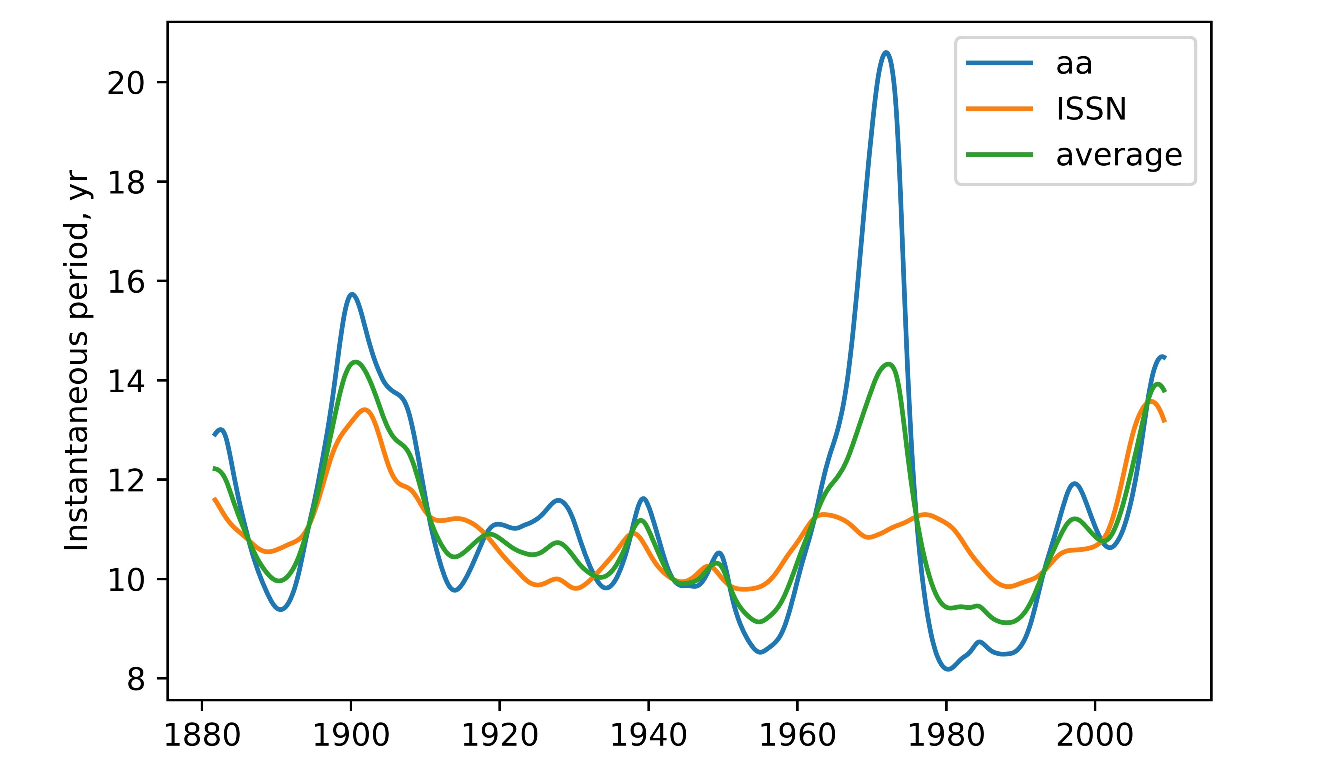
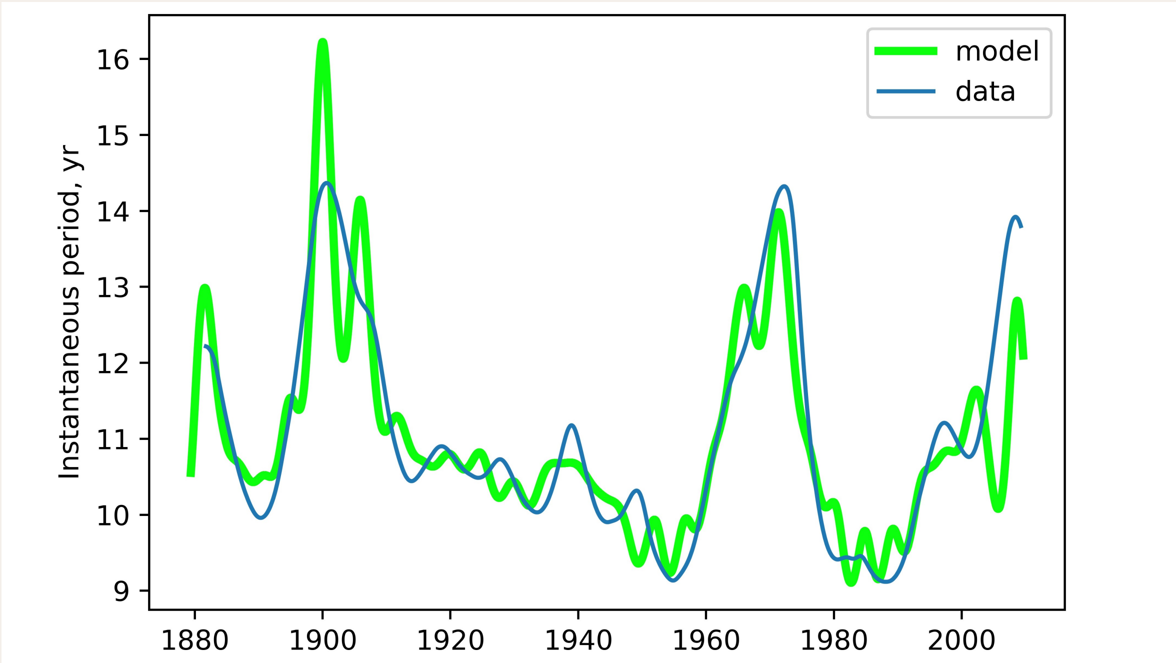
- We focus on the first case and then discuss the differences

Instantaneous Period: Data



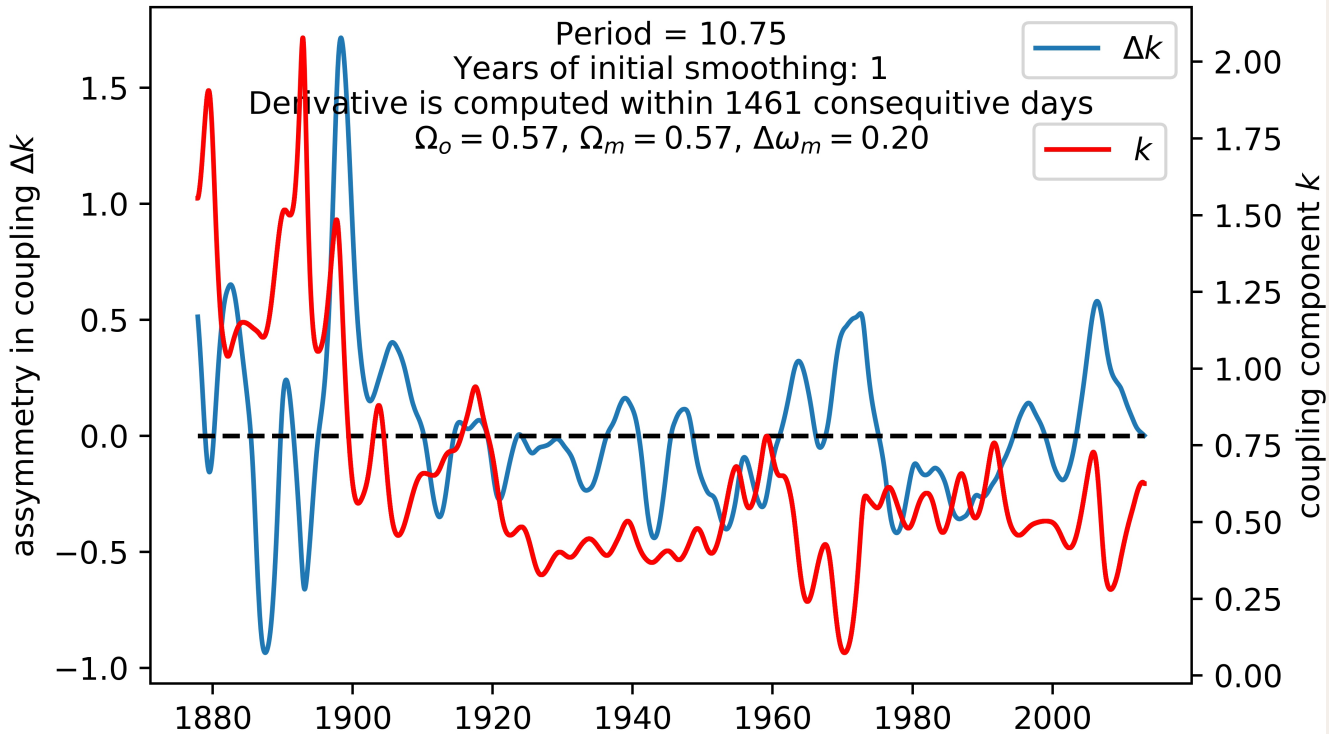
Instantaneous Period: Data and Model



- Agreement between the data and the model
- A more smoothed curve represents the model
- Anomaly at the right: the model blue curve demonstrates more extreme values than the green curve. A model prediction?

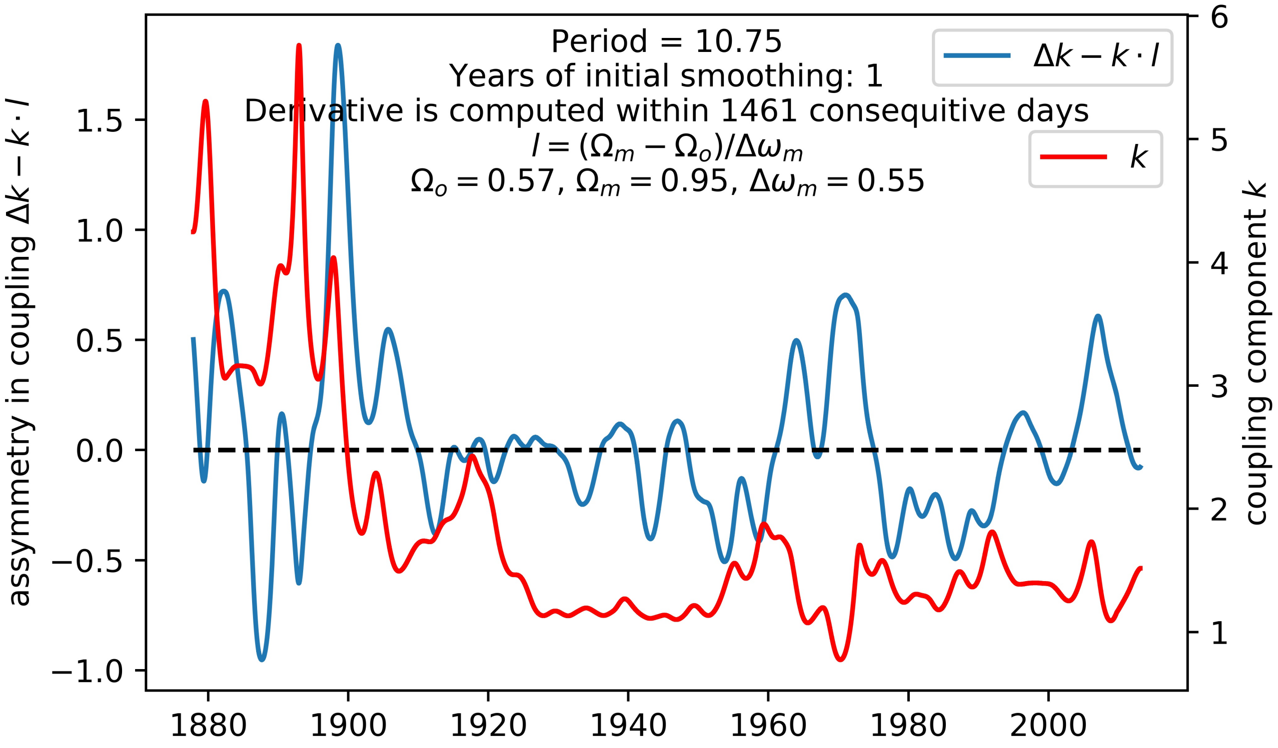
Solution of the Inverse Problem: Coupling

Natural frequency is equal to the observed average frequency



Solution of the Inverse Problem: Coupling

Natural frequency is larger than the observed average frequency



Reconstructed Coupling

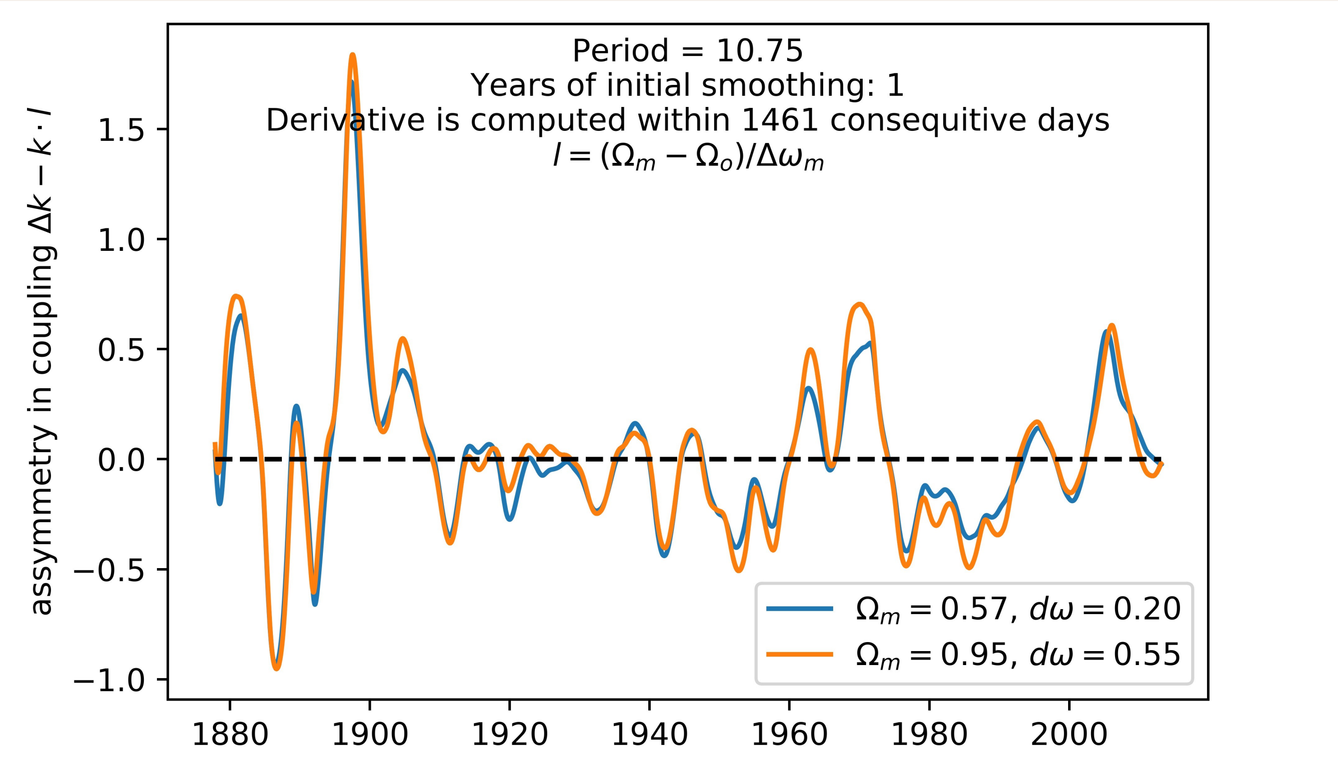
Anomalies

- 1880–1910: the absence of the constant phase shift between ISSN and `aa`; in particular this shift is close to 0 in 1880–1890
- 1965–1975 (anomalous 20th cycle): de-synchronization of the components of the solar magnetic field (technically, k and Δk anti-correlate and Δk attains anomalous values)
- Nowadays: Δk attains anomalous values and there are traces of the anti-correlation between Δk and k

Technical details

- The expression $\Delta k - kl$, where $l = (\Omega_m - \Omega)/\Delta\omega$, normalizes the asymmetrical component of the coupling
- The expression $k/\delta\omega$ normalizes the symmetrical component of the coupling

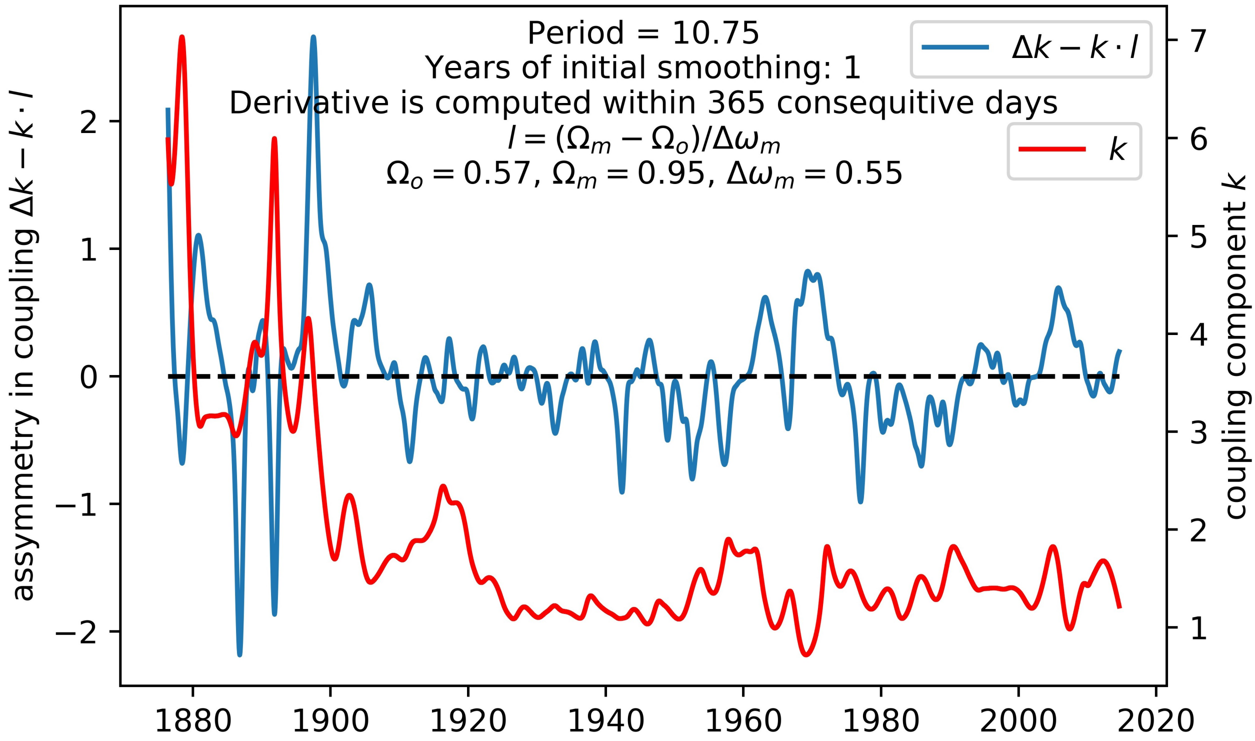
Choice of natural frequencies does not matter



Current Anomaly(?): We need additional half-cycle

Will blue and red curves continue their movement

upward and downward respectively?



Conclusion

- The Kuramoto model successfully reproduces the variation of the solar cycle duration
- It describes the loss of the synchronization in the 20th cycle
- It uncovers traces of the current de-synchronization but requires additional data for a definite conclusion

T H A N K   Y O U!

Questions and requests for Python codes can be addressed to `abshapoval @ gmail.com`