Using joint and constrained inversions as hypothesis testing tools

Max Moorkamp

19/10/2021

The great tragedy of science - the slaying of a beautiful hypothesis by an ugly fact.

Thomas Henry Huxley

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If the facts don't fit the theory, change the facts.

Albert Einstein

Formal integration methods

Combine different geophysical data through

Joint inversion: Combine all geophysical datasets in one inversion algorithm

Constrained inversion: Extract features from one model to steer inversion of another dataset

Post-inversion analysis: Use algorithms to identify matching structures in individual inversion results

A typical inversion approach

Coupling between different methods, is a crucial aspect

$$\Phi(\quad \sigma) = \Phi_{MT}(\sigma) + \qquad\qquad \lambda\Phi_{reg}(\quad \sigma)$$

Structural coupling

- Assumes coincident boundaries of features
- Cross-gradient (Gallardo and Meju, 2003) commonly used
- Widely applicable

Parameter relationship

- Directly relate velocity and resistivity
- Relationship can come from borehole data, theoretical models
- Provides strong coupling between methods, but also prone to artefacts

A typical inversion approach

Coupling between different methods, is a crucial aspect

$$\Phi(v,\sigma) = \Phi_{MT}(\sigma) + \Phi_{seis}(v) + \kappa\Phi_{coup}(v,\sigma) + \lambda\Phi_{reg}(v,\sigma)$$

Structural coupling

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Parameter relationship

- Directly relate velocity and resistivity
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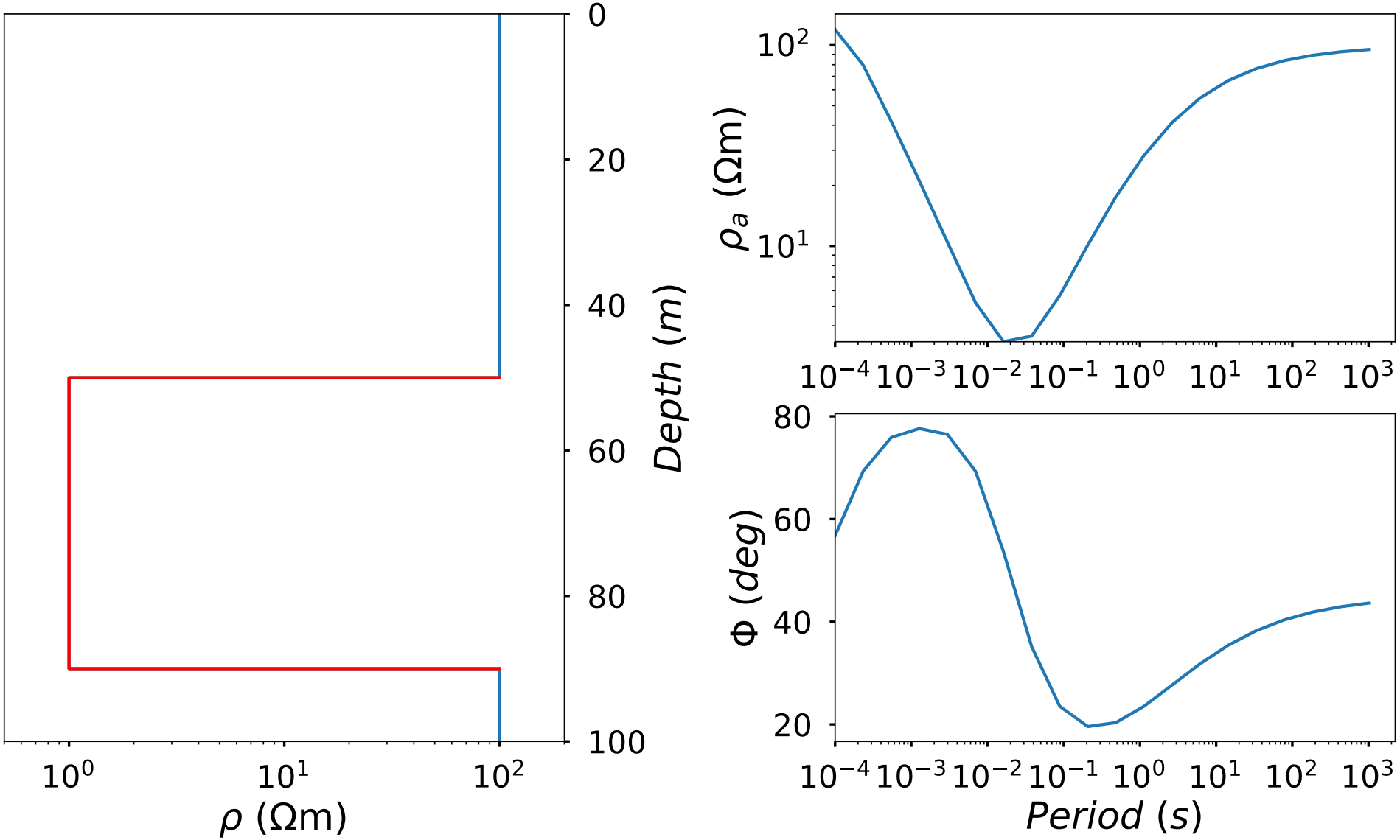
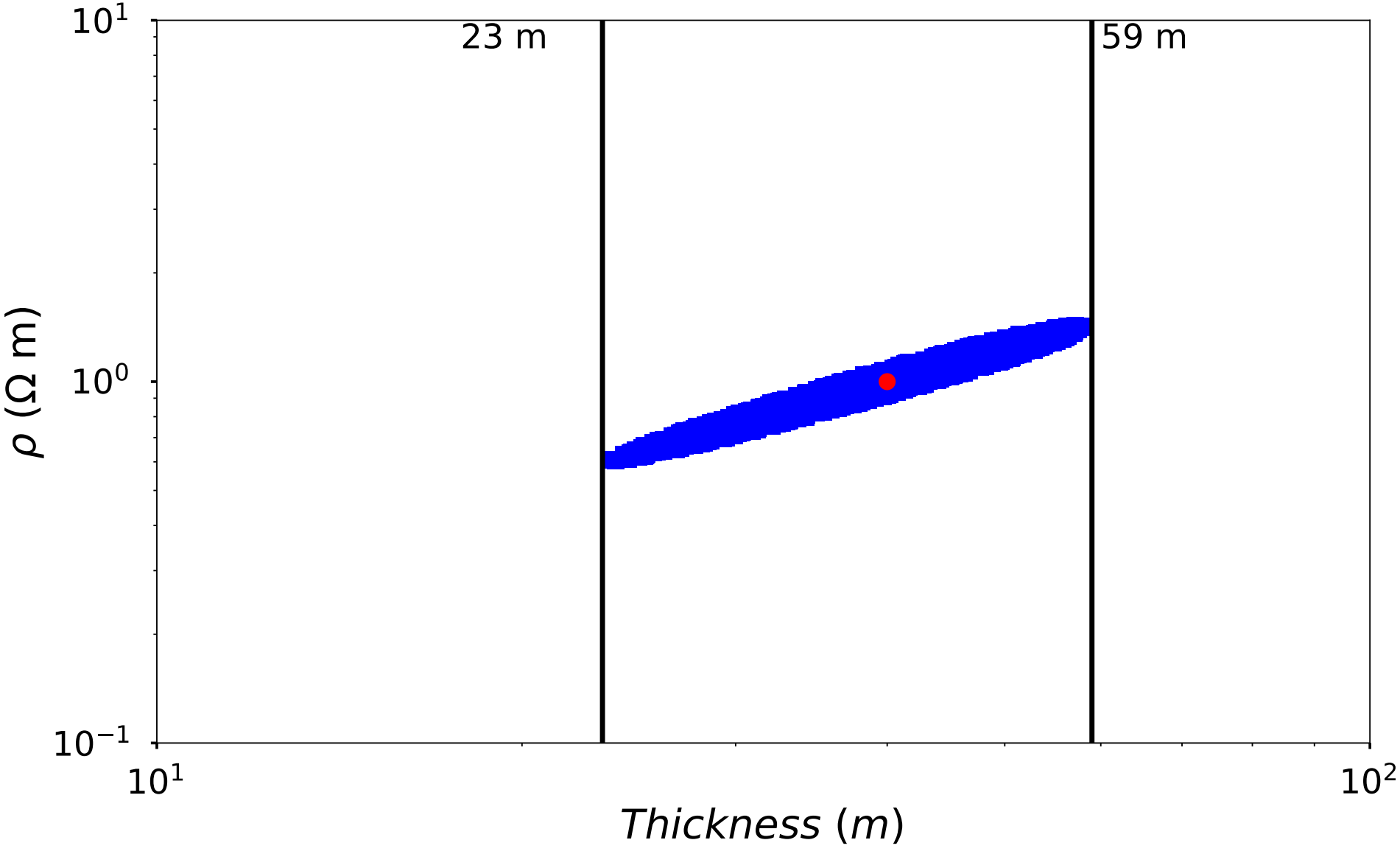
A simple test model



A simple three-layer model, find layer thickness and resistivity

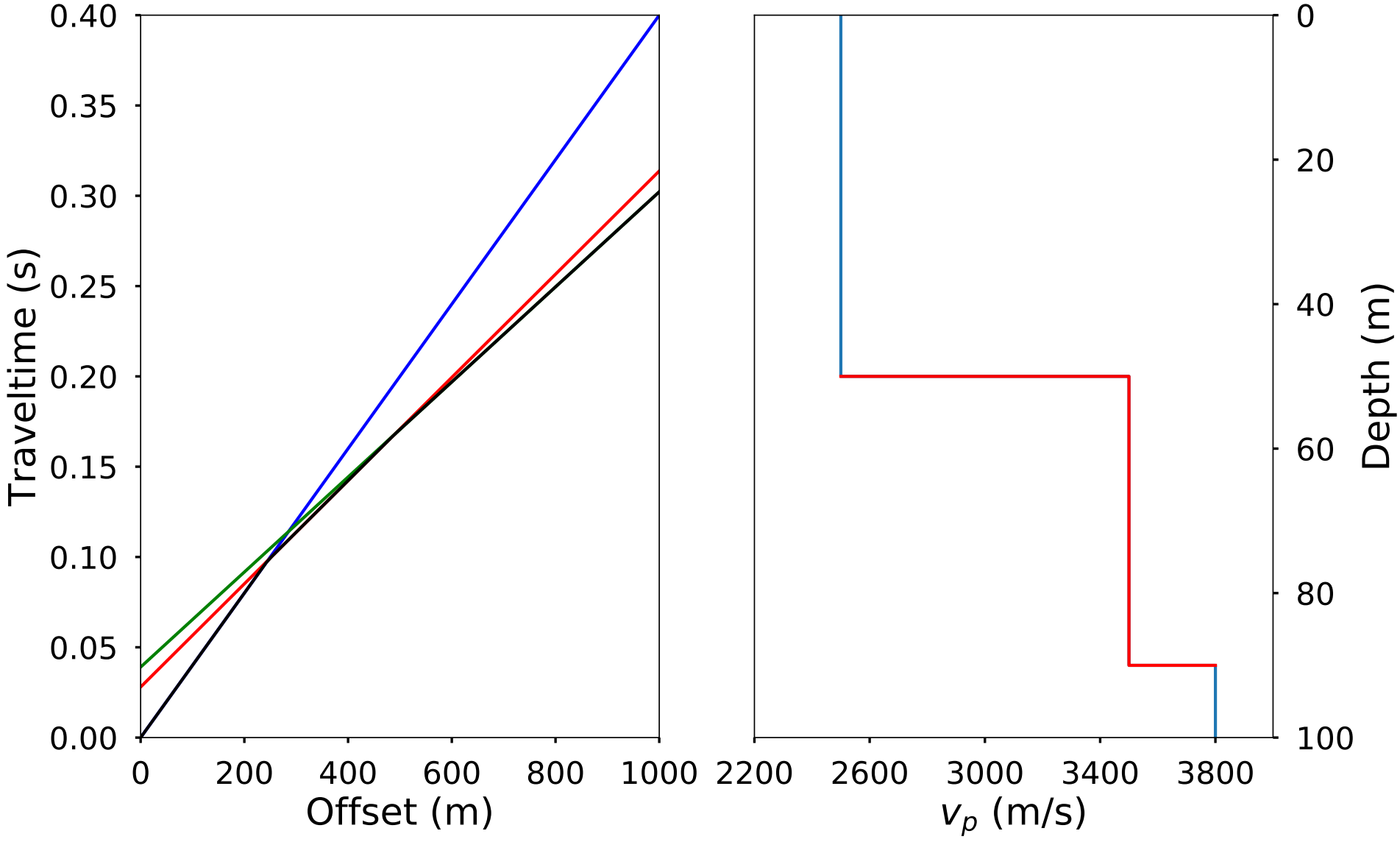
(Moorkamp, 2017)

Acceptable models for MT



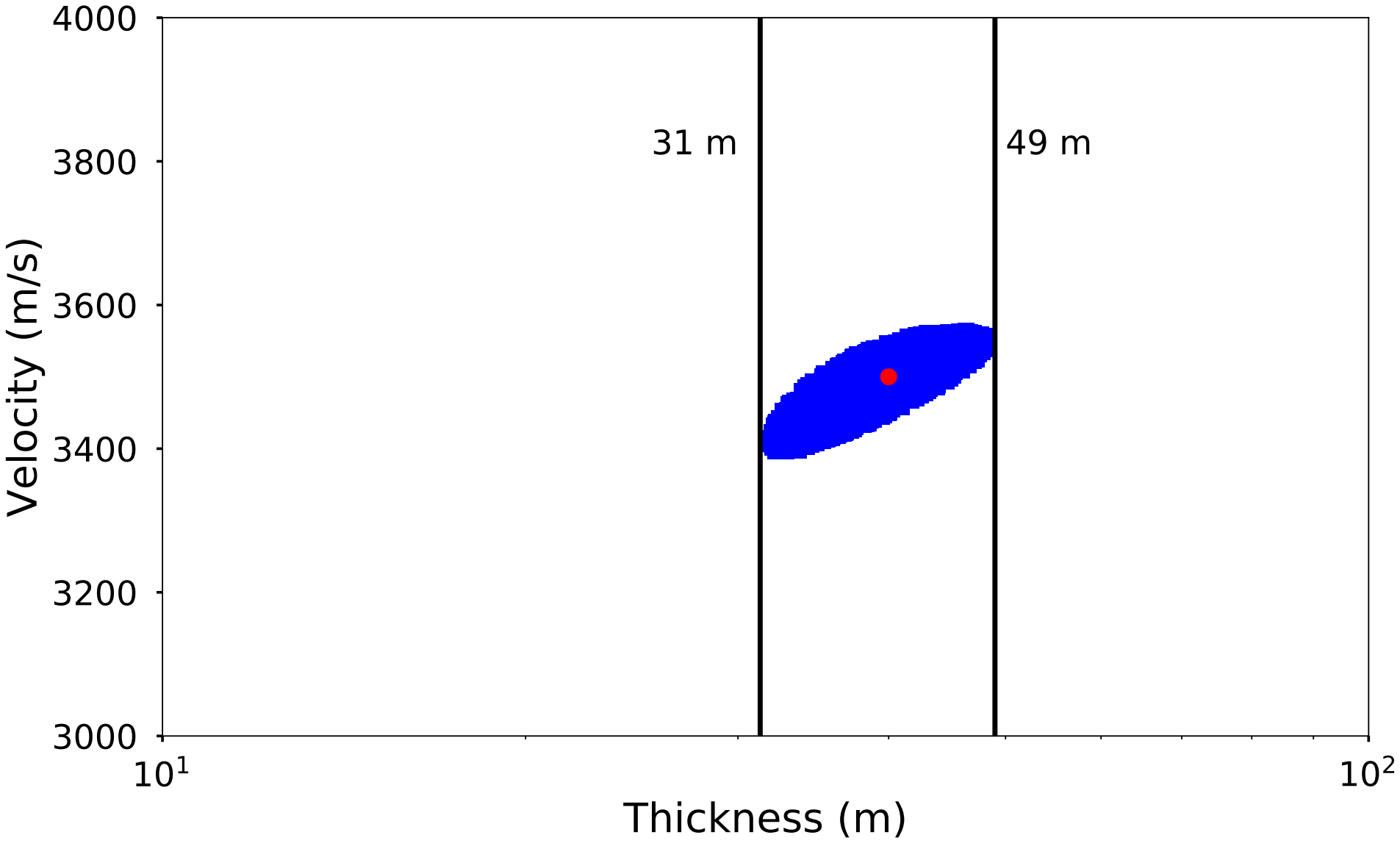
Cannot resolve thickness and resistivity independently, but conductance

A matching seismic model



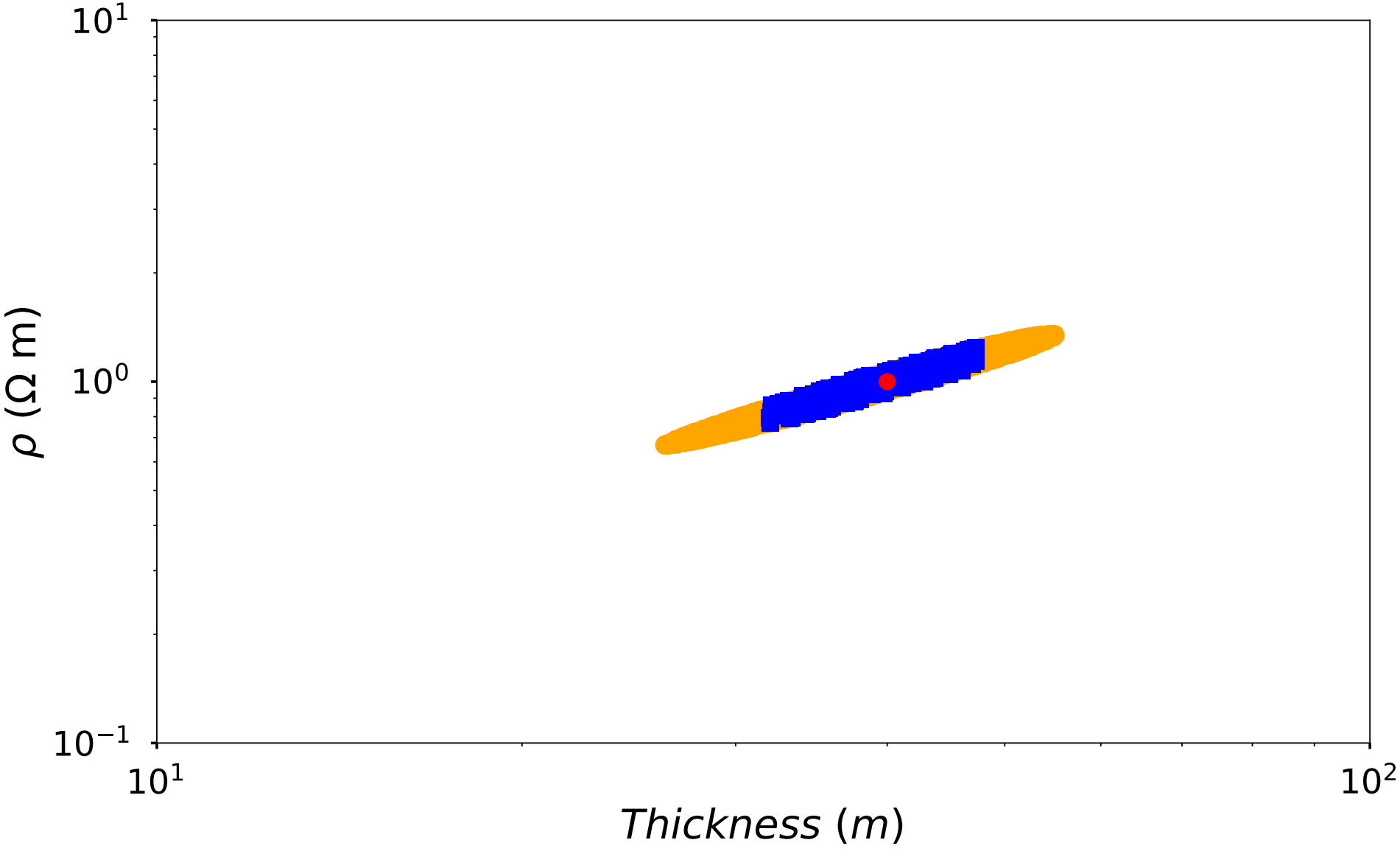
A seismic model with nearly "hidden" second layer

Acceptable models for seismics



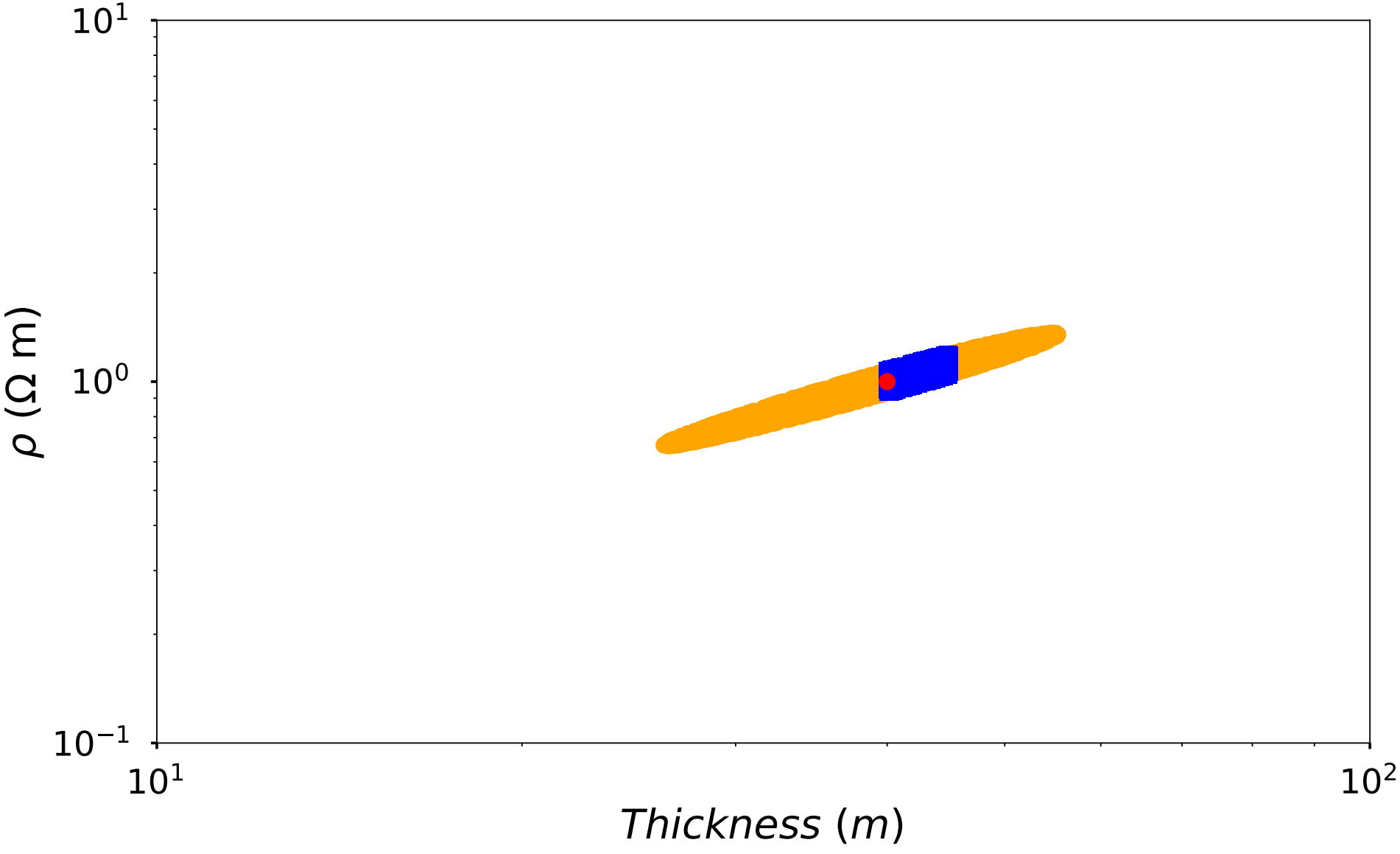
Range of acceptable thicknesses is smaller than for MT.

Joint inversion with matching layers - MT



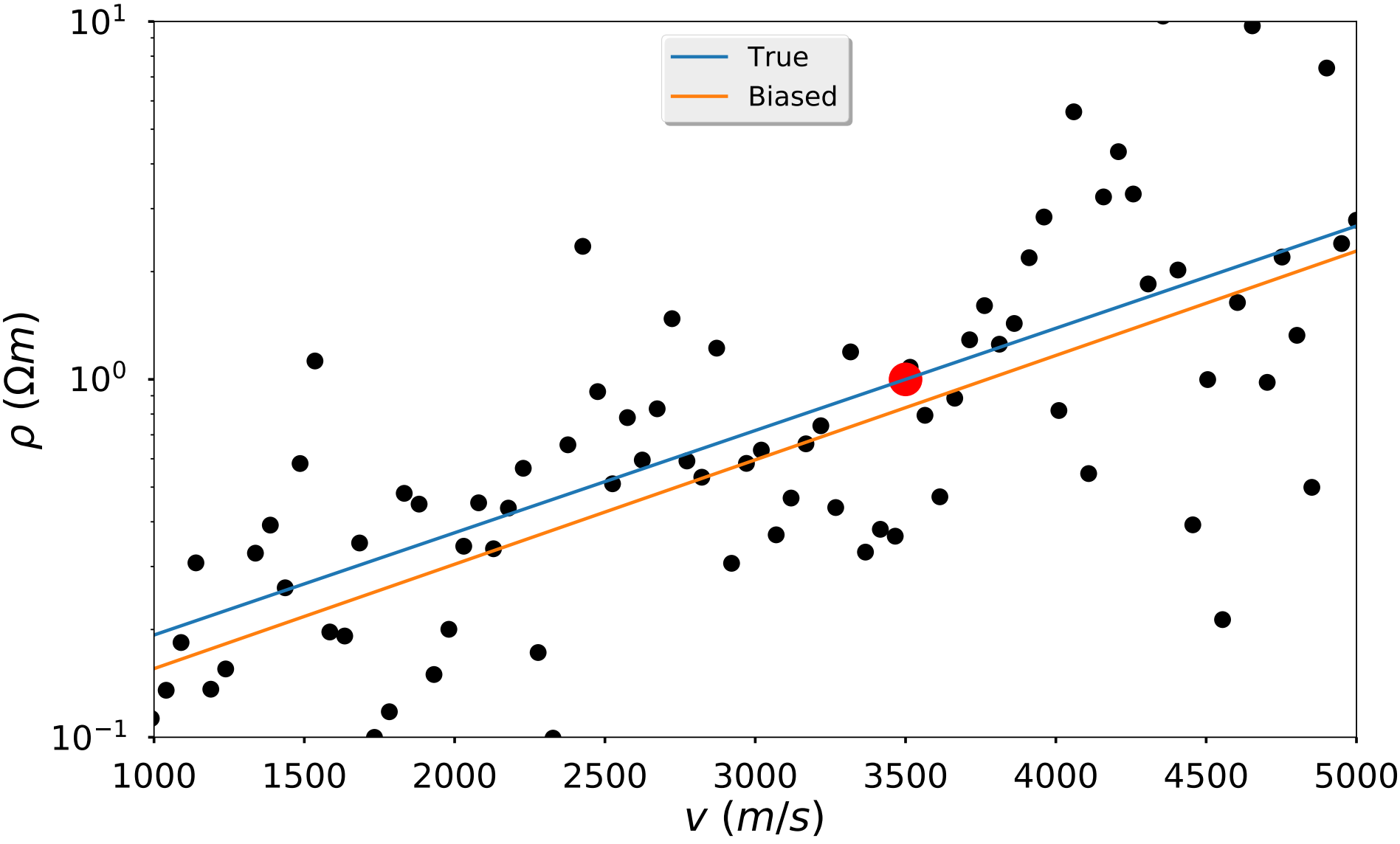
Range of thicknesses determined by seismics

MT inversion constrained by seismics



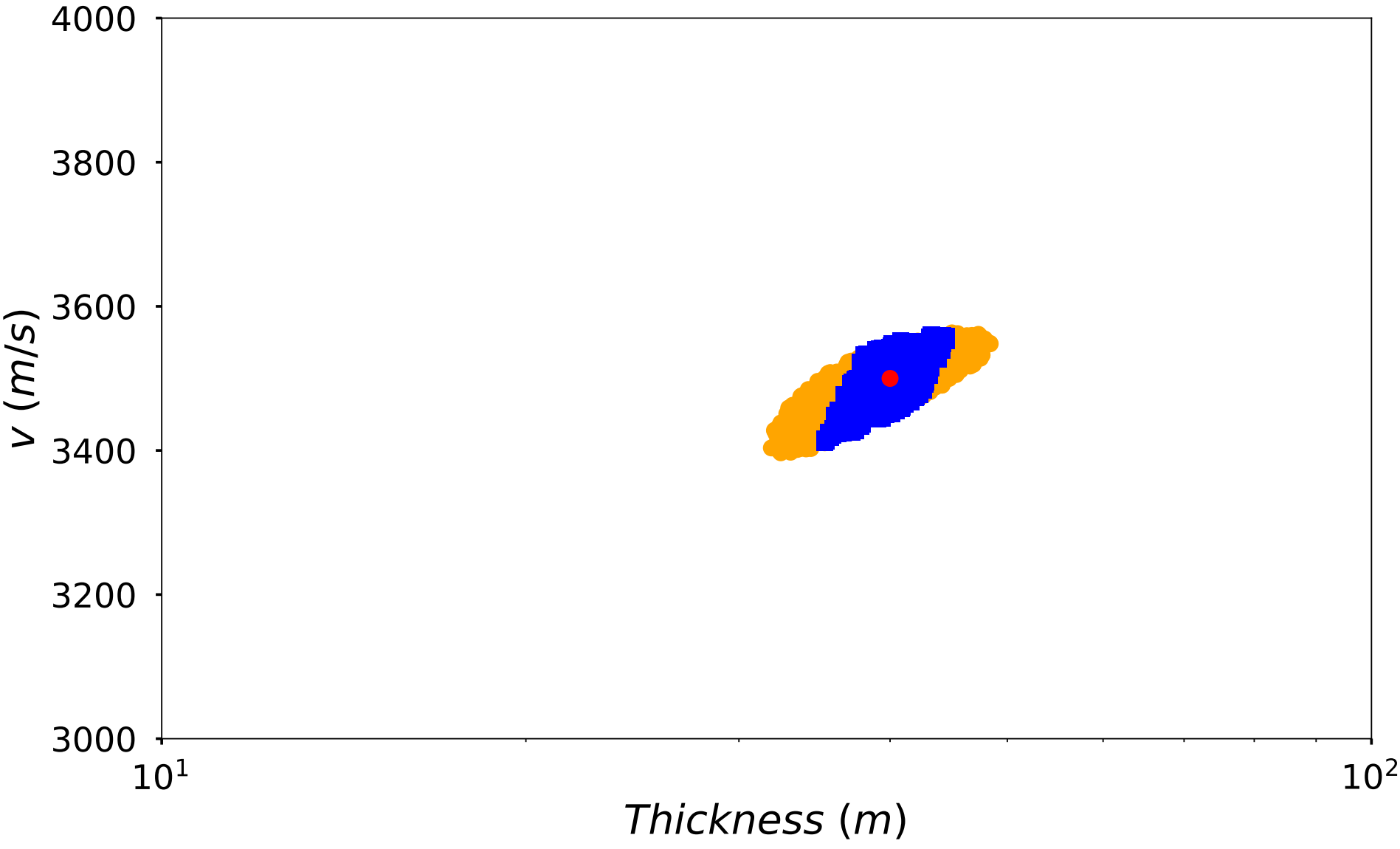
Picking a best seismic model as a constraint, restricts models further.

Using a parameter relationship



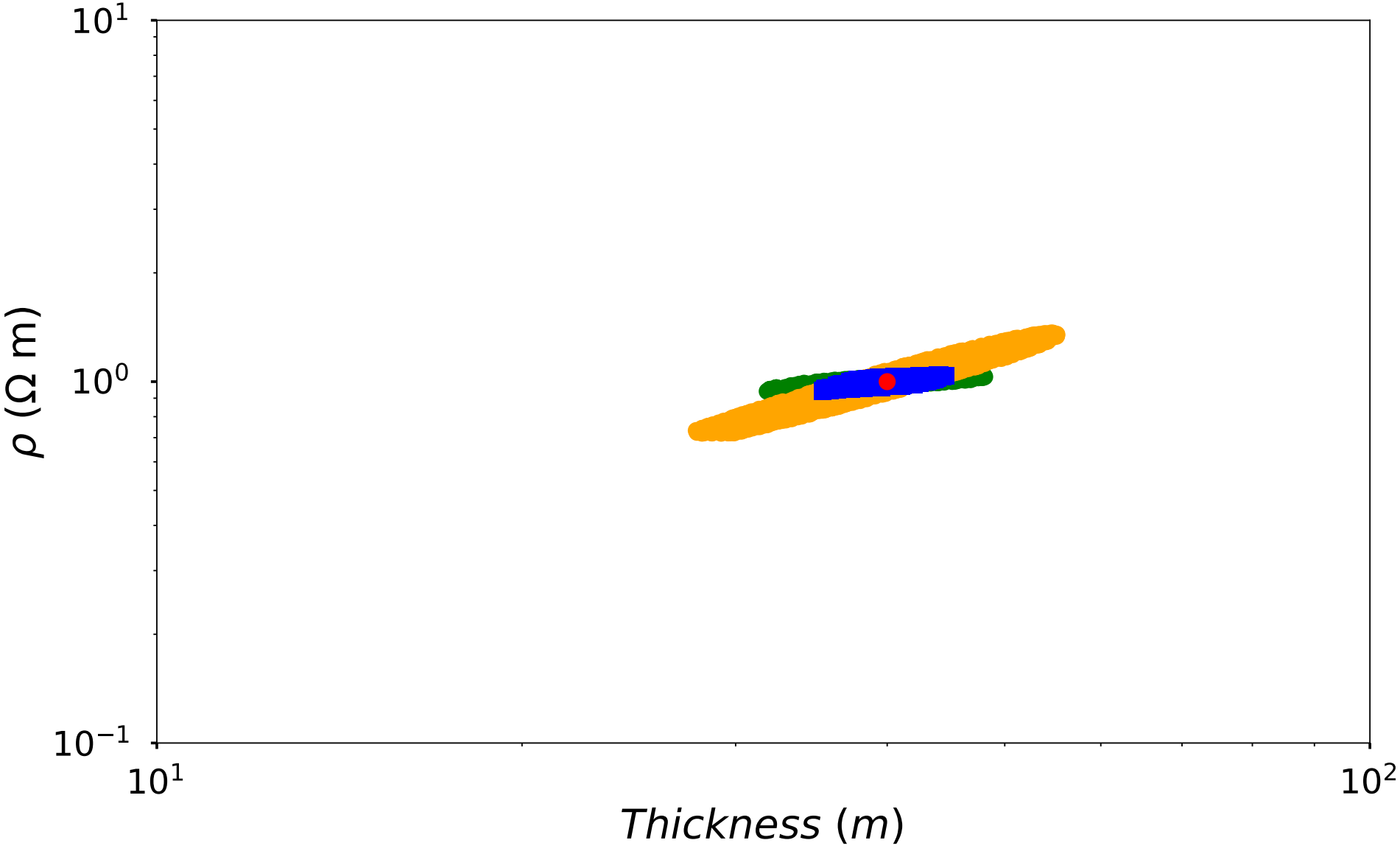
Estimating a parameter relationship can be difficult.

Inverting with true relationship



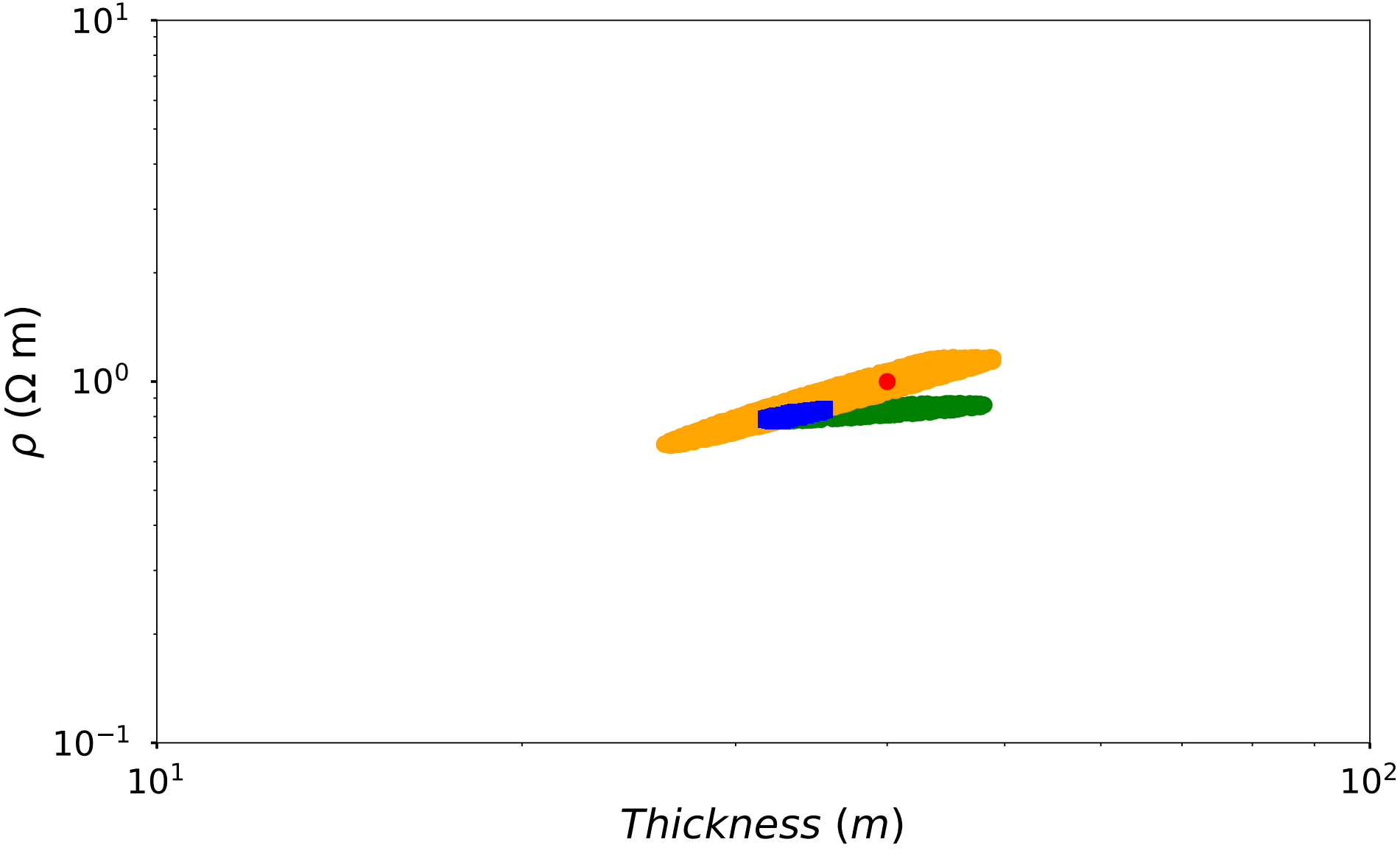
Parameter relationship restricts both seismics and MT if considered exact.

Inverting with true relationship



Parameter relationship restricts both seismics and MT if considered exact.

Inverting with biased relationship



Biased relationship has strong impact on accepted models

Consequences

- Joint and constrained inversion restrict space of acceptable models
- Constrained inversion provides stronger restrictions
- Coupling method is crucial, has strong effect on results

Two possible views

Explorative: Need to specify accurate/true relationship then models will be accurate/true

Hypothesis based: Specifying a coupling is equivalent to formulating a hypothesis that we want to test.

Falsification

In so far as a scientific statement speaks about reality, it must be falsifiable: and in so far as it is not falsifiable, it does not speak about reality.

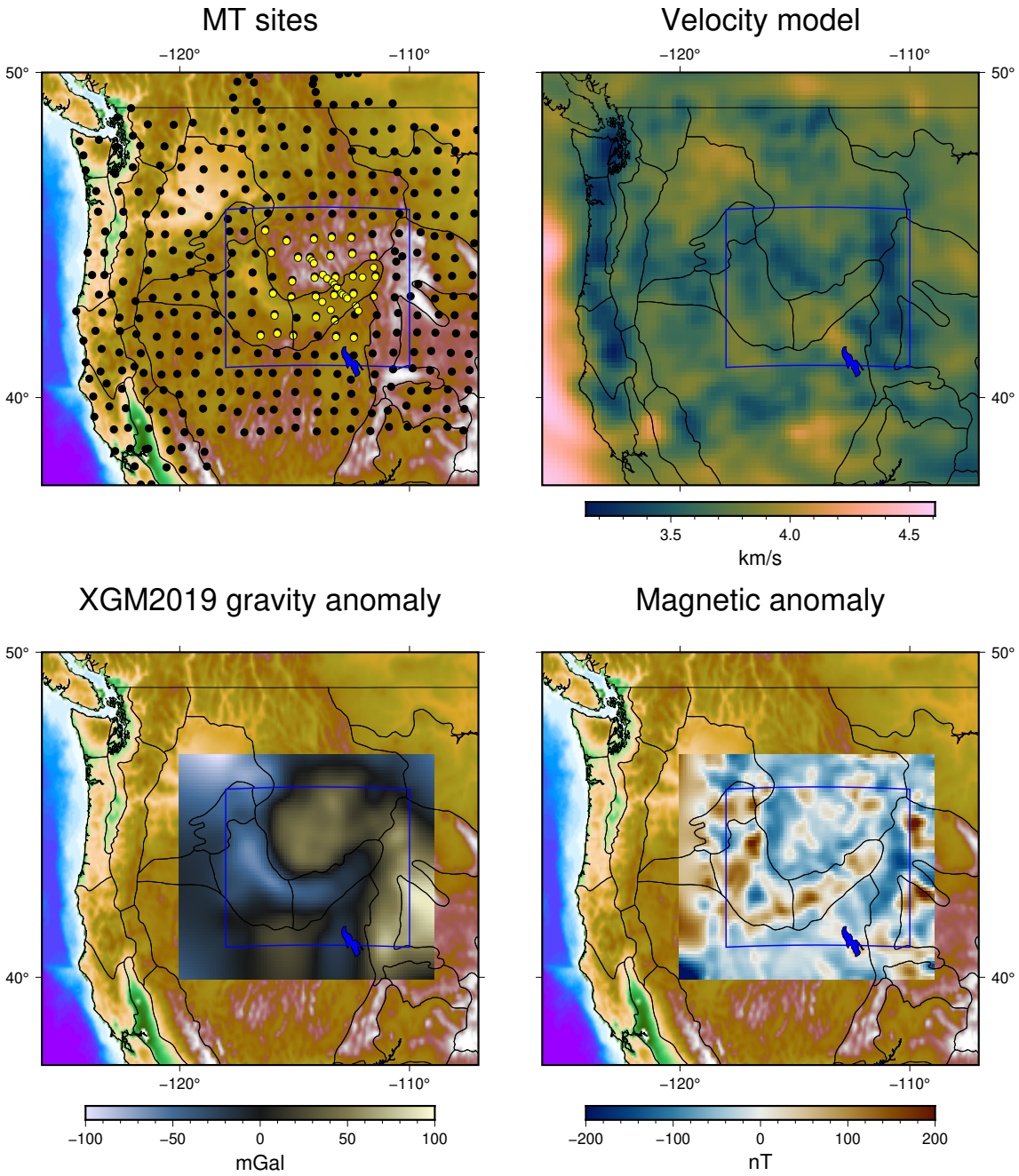
Karl R. Popper, The Logic of Scientific Discovery

When you have eliminated the impossible, whatever remains, however improbable, must be the truth.

Sherlock Holmes

- Successful joint inversion (fit the data, good coupling between models) corroborates hypothesis
- Problem, other possibilities might exist, might not be representative of "truth"
- "Unsuccesful" joint inversion (cannot fit data with chosen coupling) falsifies hypothesis
- Problem, cannot test all possibilities, many trivially false hypotheses

Map



- Western United States shows interesting tectonic features and high-quality data
- Magnetotelluric data from USArray
- Gravity from combined satellite/land data
- Magnetic field measurement database
- Seismic velocity model (Gao et al., 2014)

Variation of Information (VI)

- VI measures amount of information contained in variable X about variable Y (e.g. conductivity and density)
- Definition

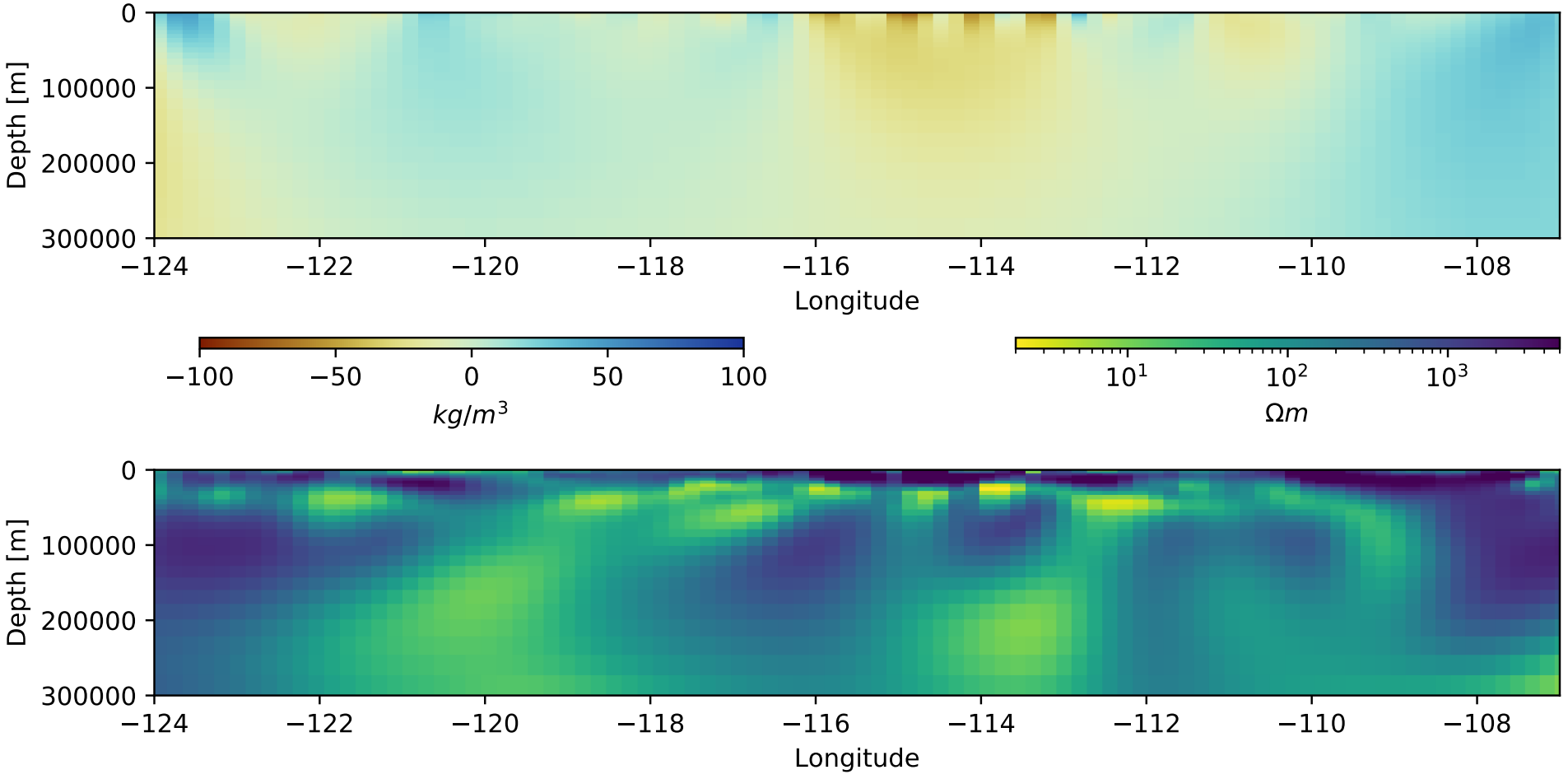
$$VI(\mathrm{X},\mathrm{Y}) = 2H(\mathrm{X},\mathrm{Y}) - H(\mathrm{X}) - H(\mathrm{Y}).$$

  where

$$H(X) = -\sum_{i=1}^{N} P(X_i) \log P(X_i)$$

  is the Shannon Entropy.
- Minimize Variation of Information as part of the inversion

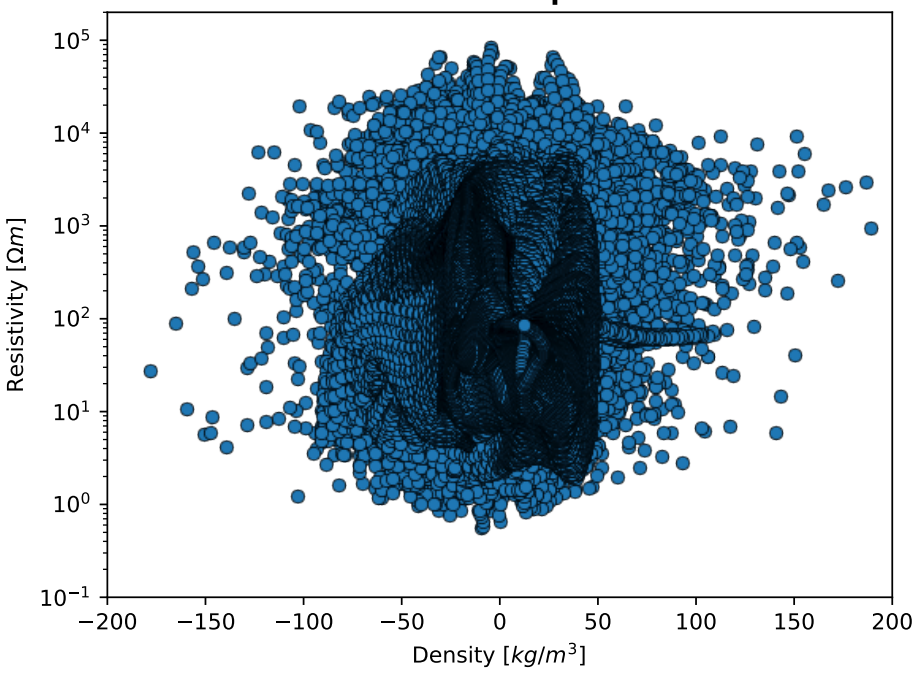
Mutual information – illustration



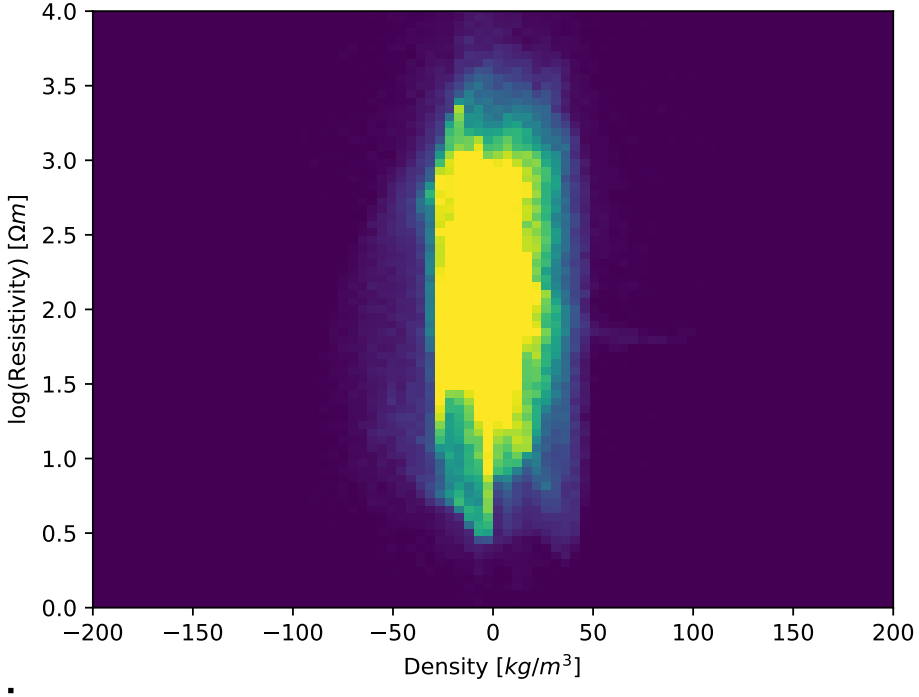
Individual models do not show similar features.

Mutual information – illustration

Parameter plot



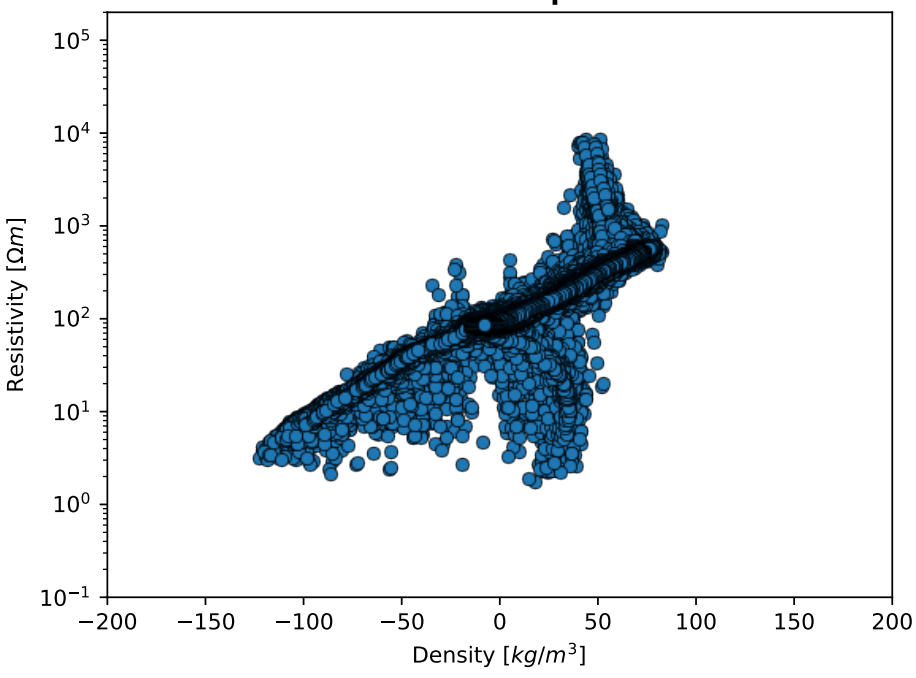
Histogram



No relationship $\rightarrow$ low mutual information

Mutual information – illustration

Parameter plot



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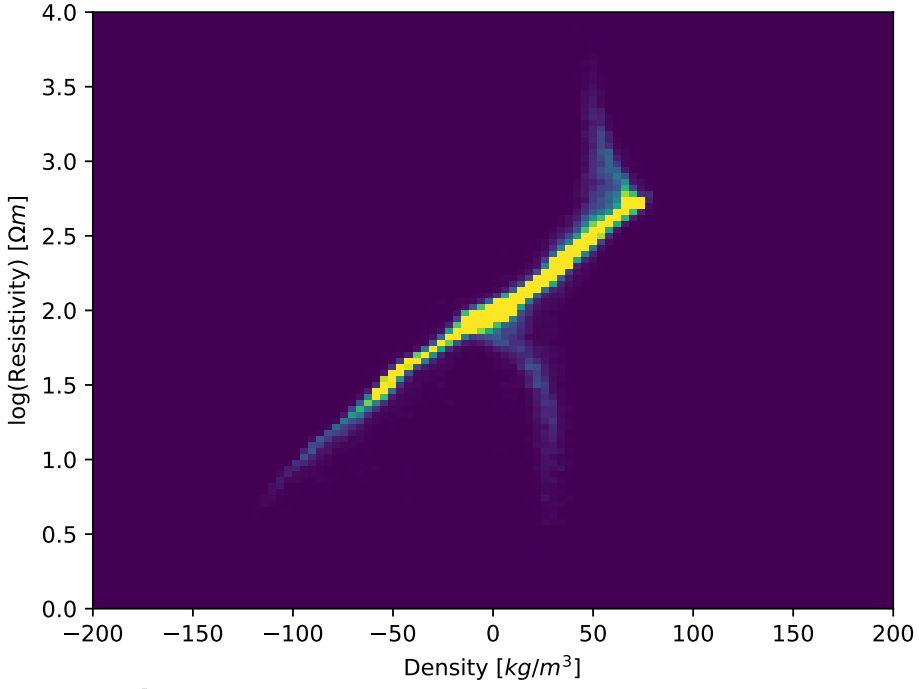
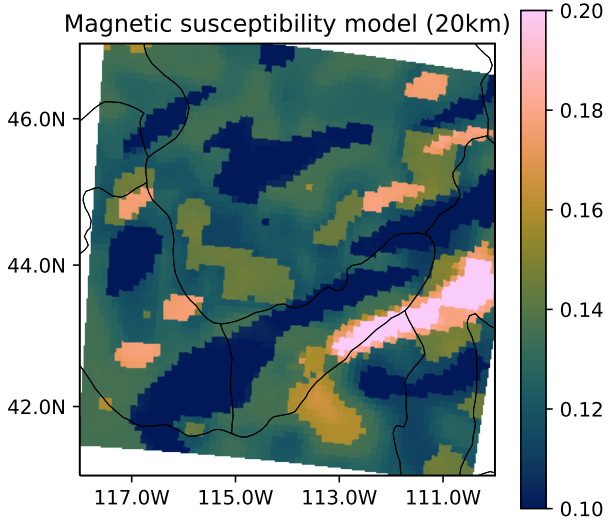


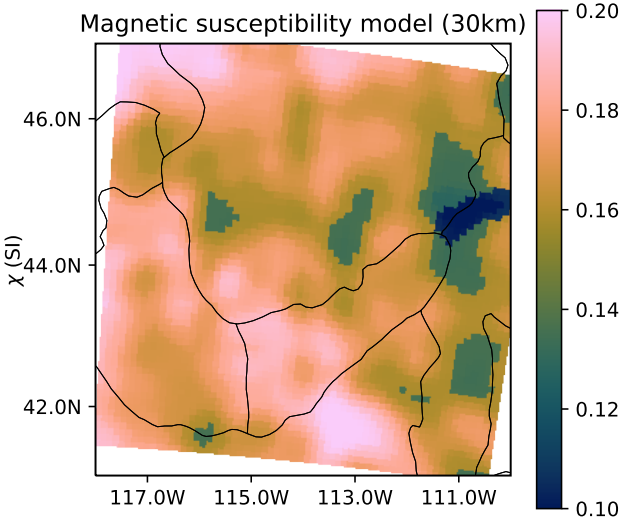
Noisy relationship $\rightarrow$ higher mutual information

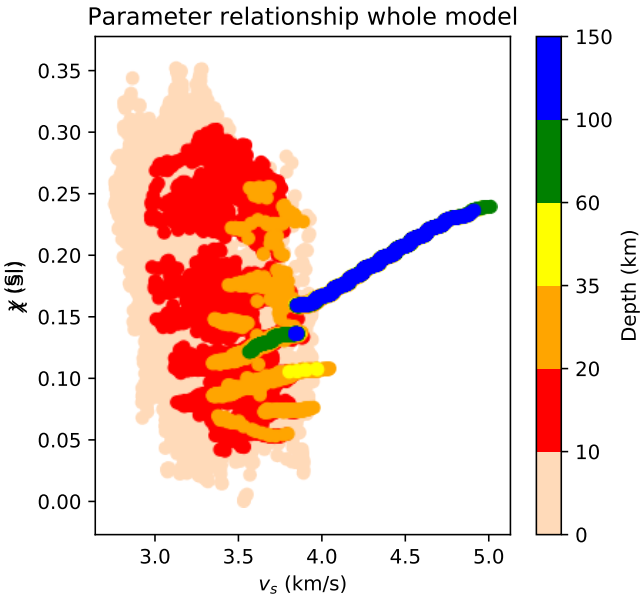
Joint and constrained inversion recipe

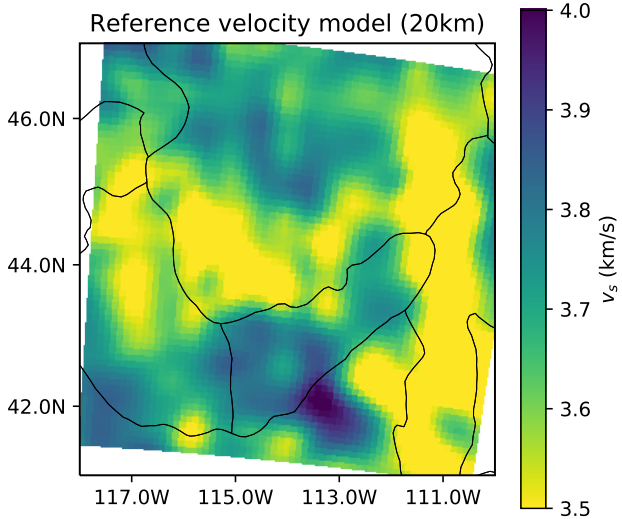
- Run individual inversions of each dataset to determine technical parameters (discretization, regularization) and establish appropriate data fit
- Run joint inversion/constrained with high coupling weight until no further progress is made
- Reduce coupling and iterate until data misfit of individual inversion is reached
- Result should be model with highest possible coupling

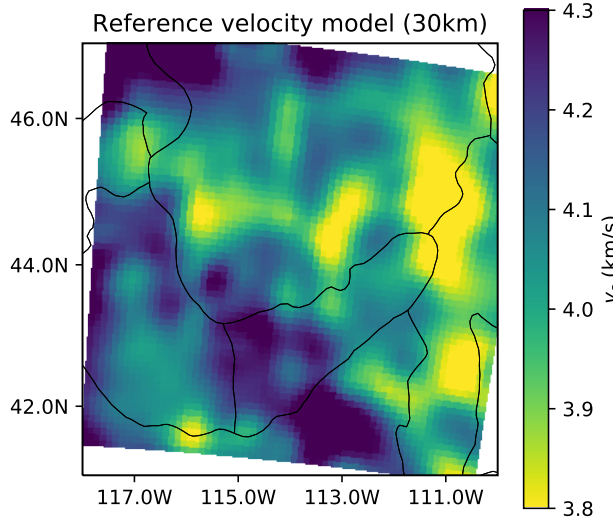
Constrained magnetic inversion

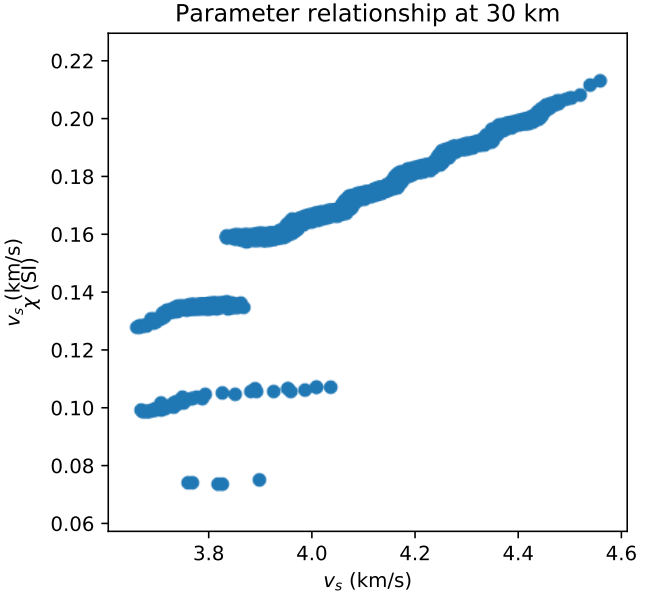




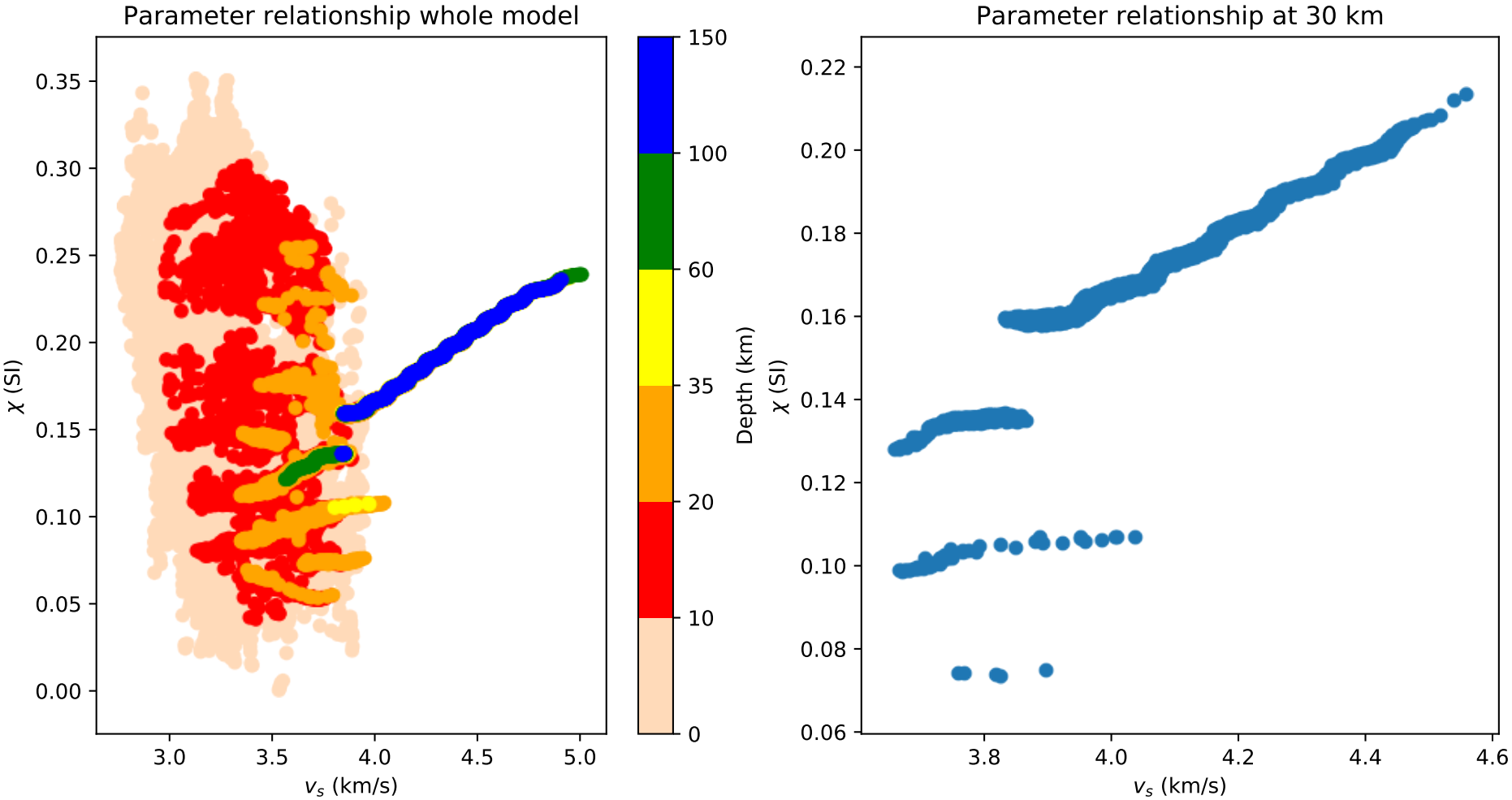






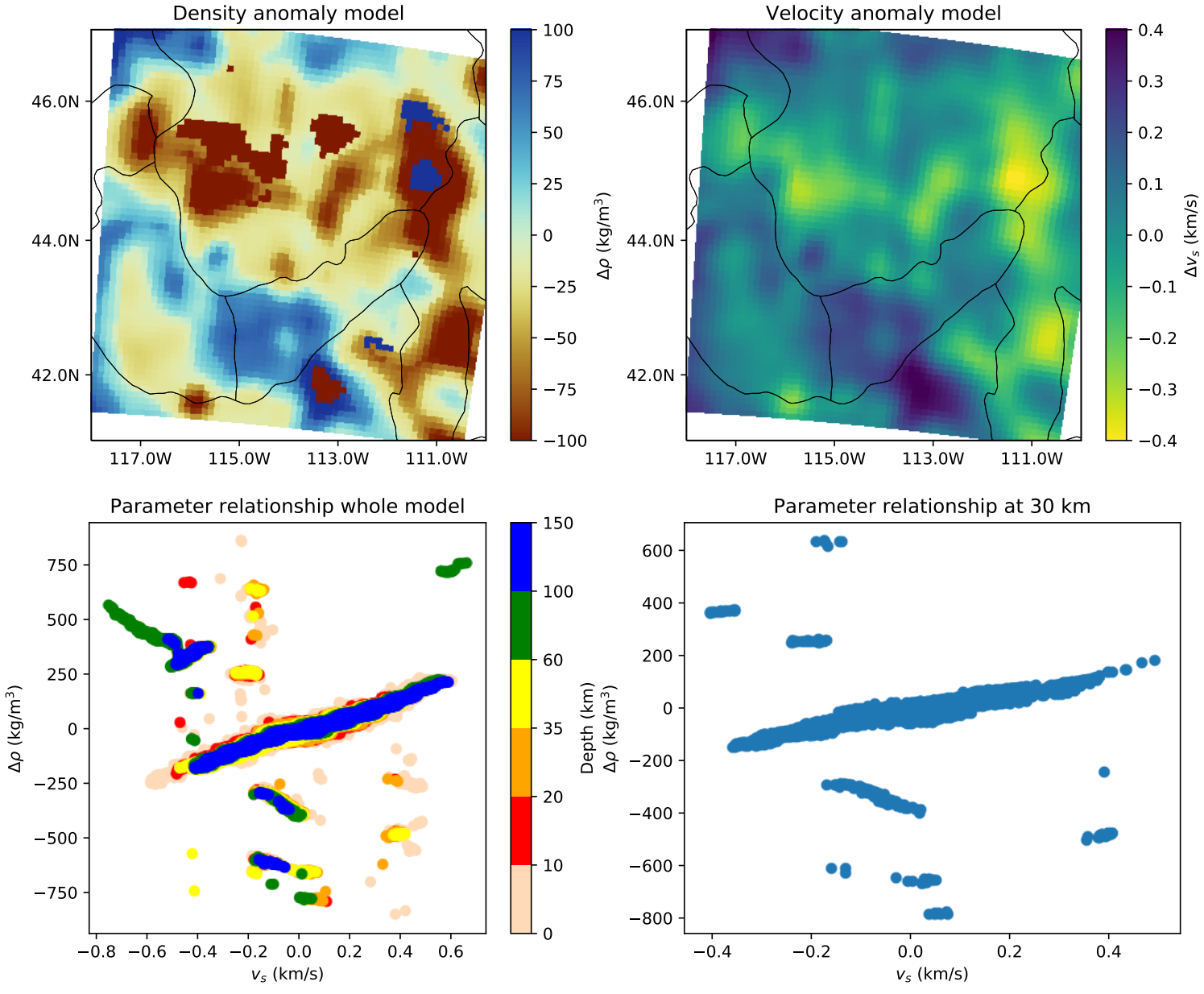


Parameter relationship

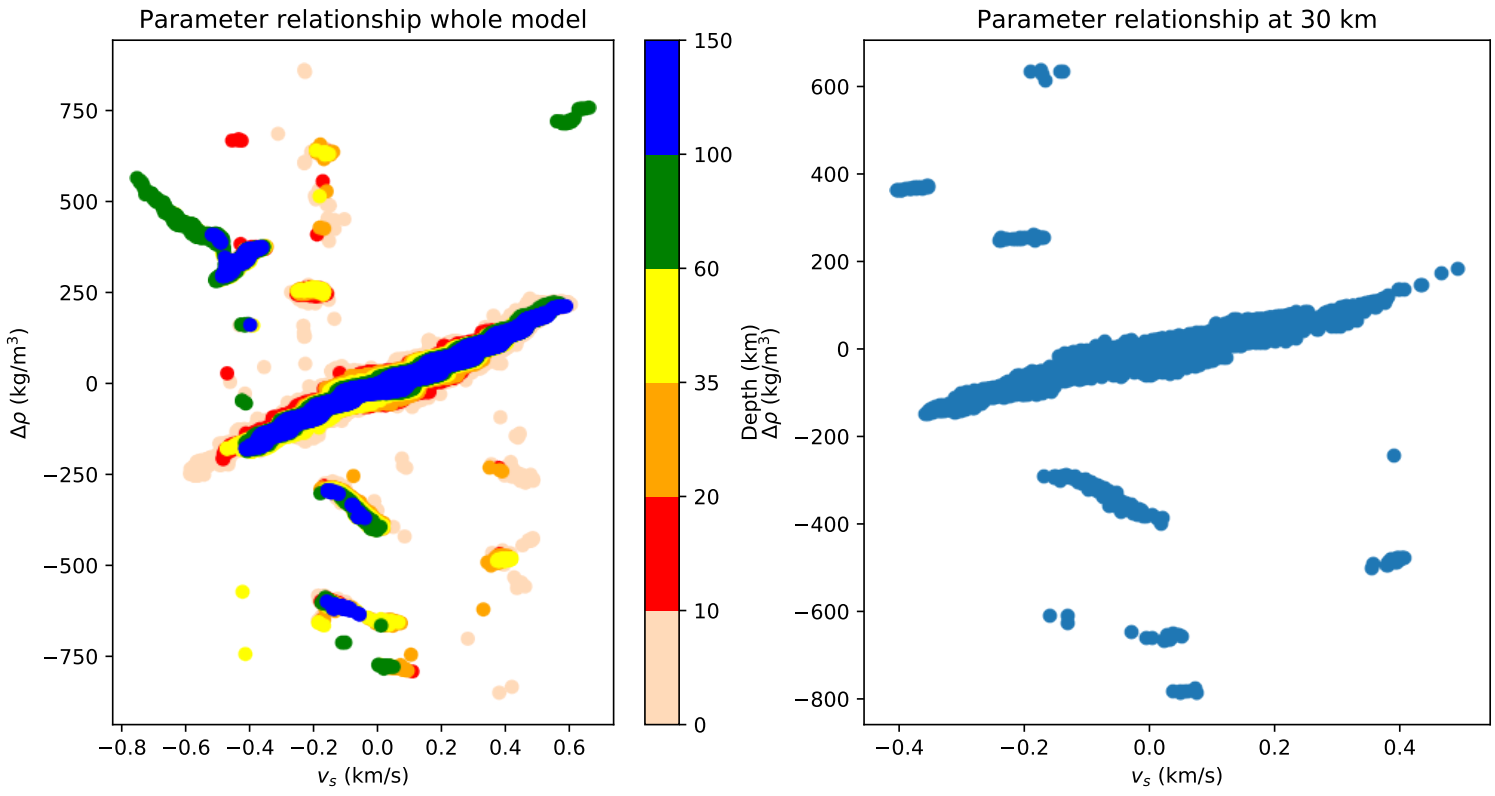


- No identifiable relationship in the crust
- No sensitivity of magnetic data at depth
- Simple one-to-one relationship between susceptibility and this model falsified

Constrained gravity inversion

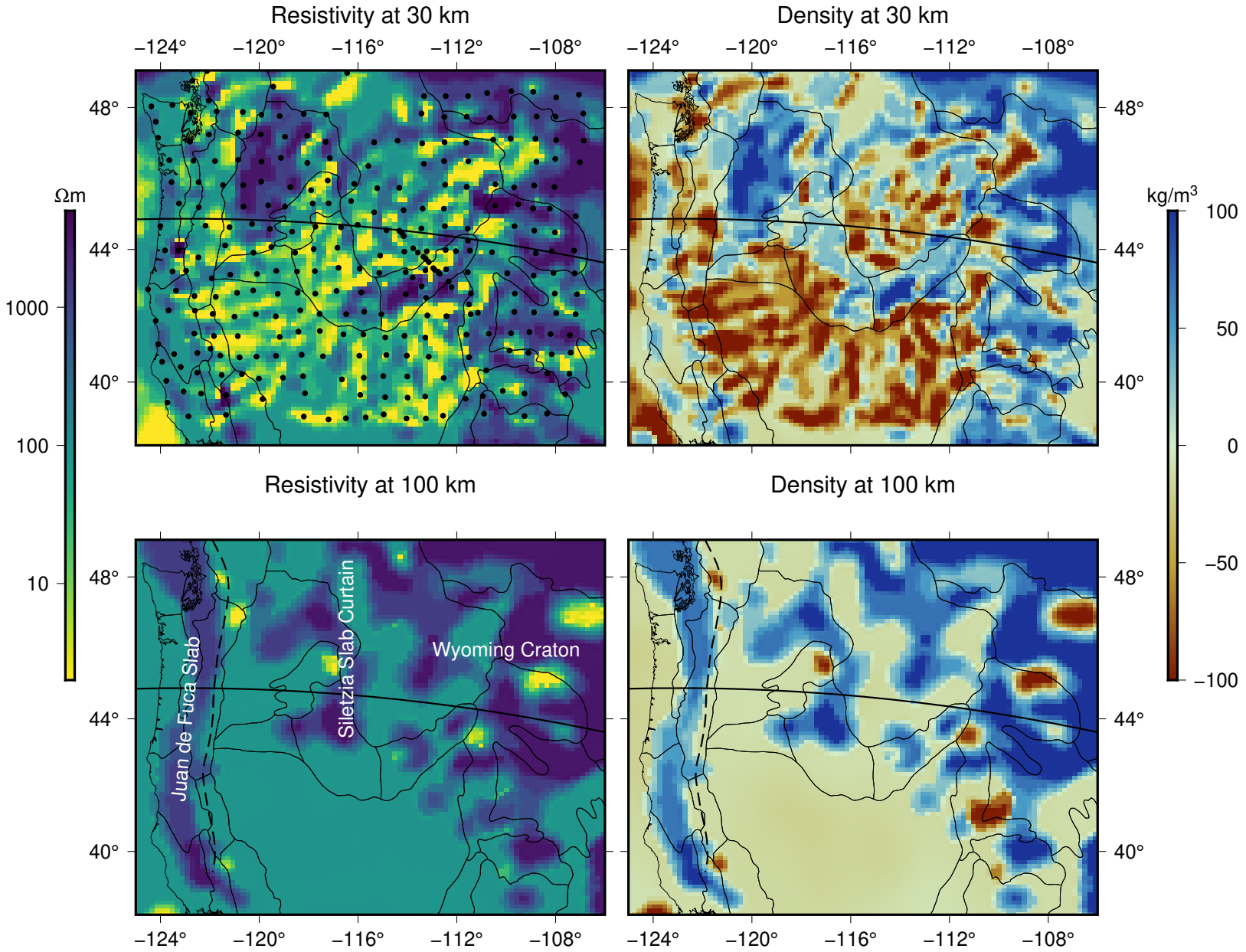


Constrained gravity inversion



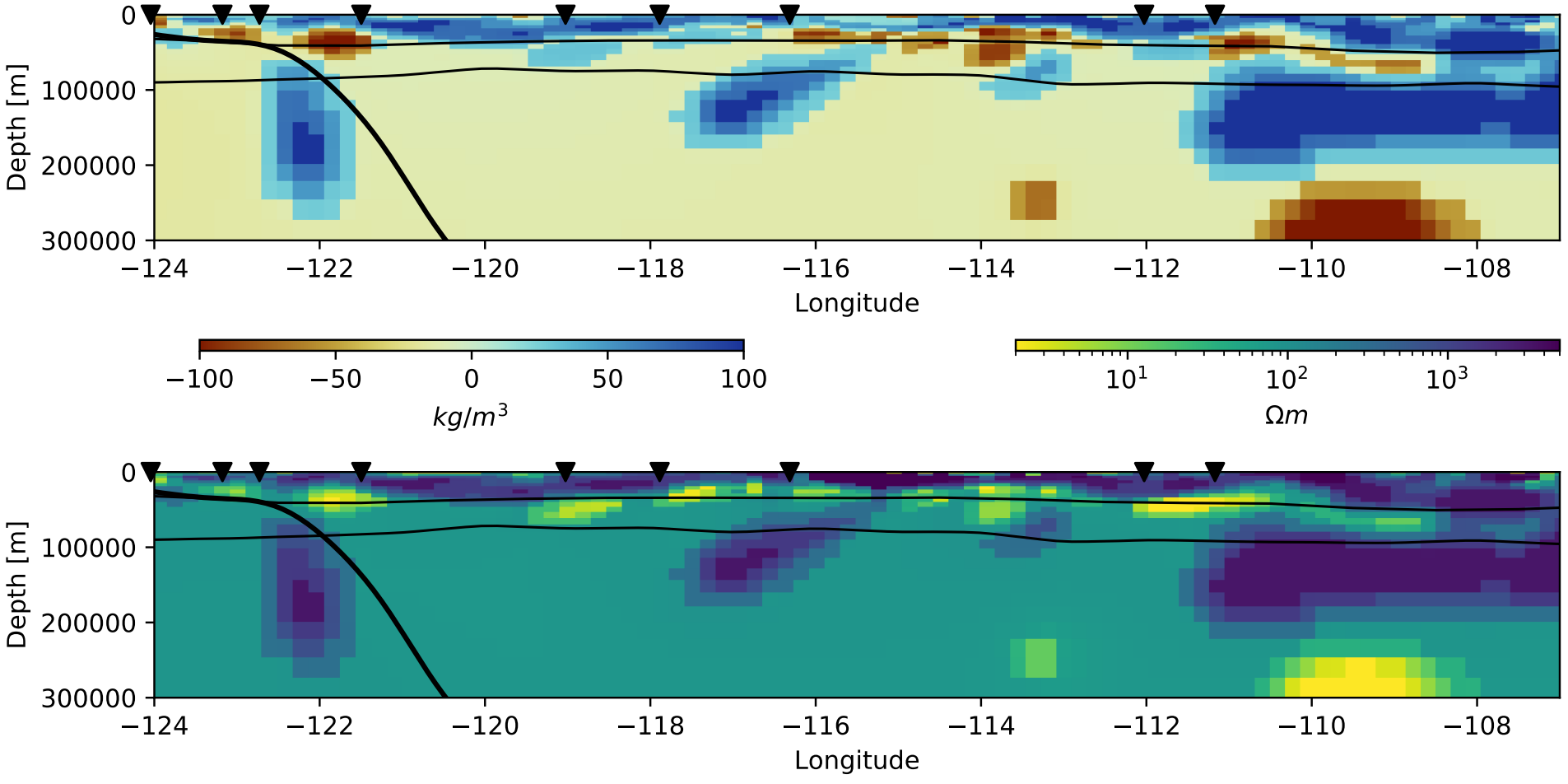
- Appears to show relationship, but strong scatter in some parts
- Relationship strongest for extreme ends of velocity anomaly
- Low velocity anomaly association with positive density anomaly suspicious

Joint inversion with MI

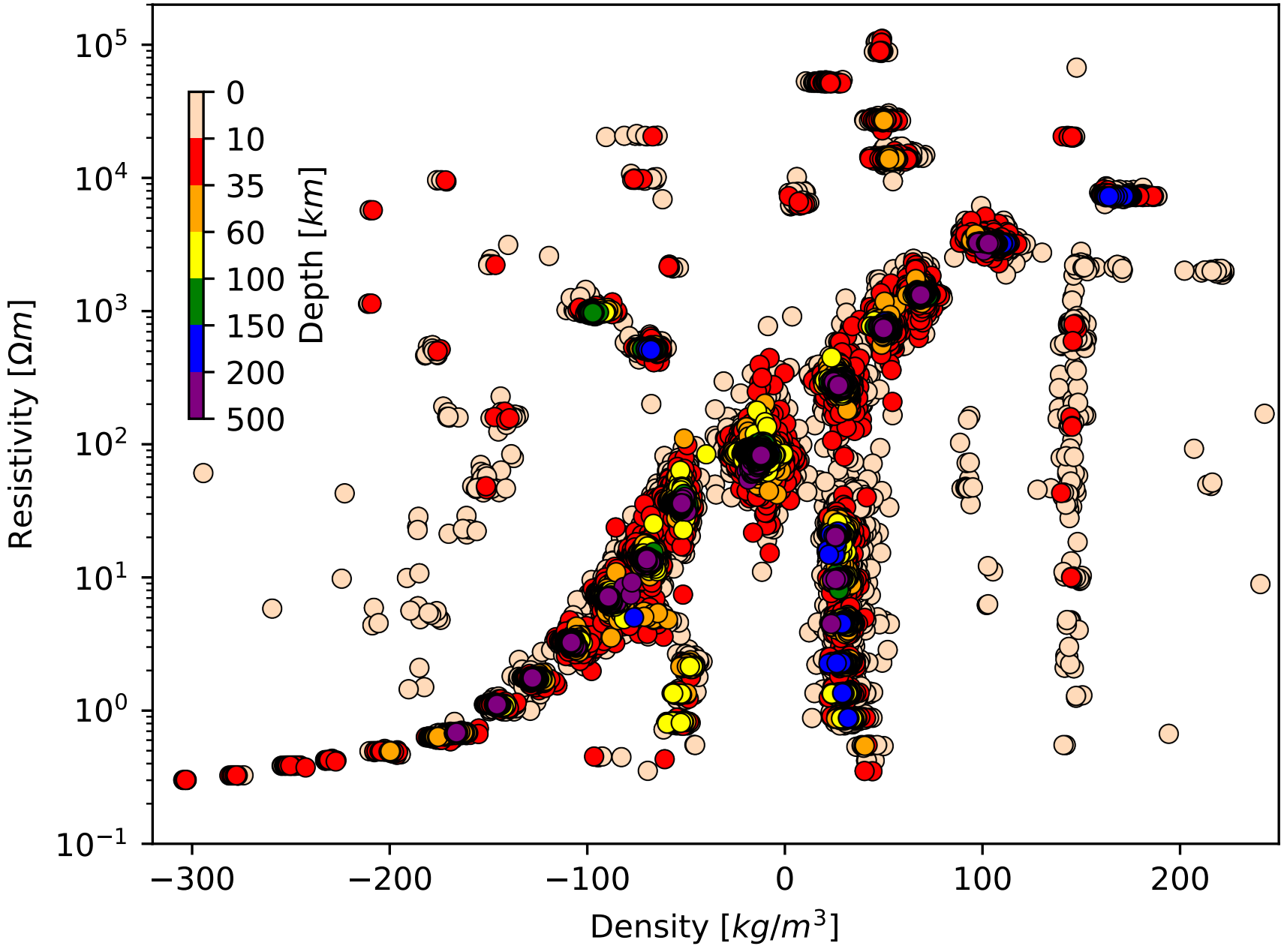


(Moorkamp, 2017)

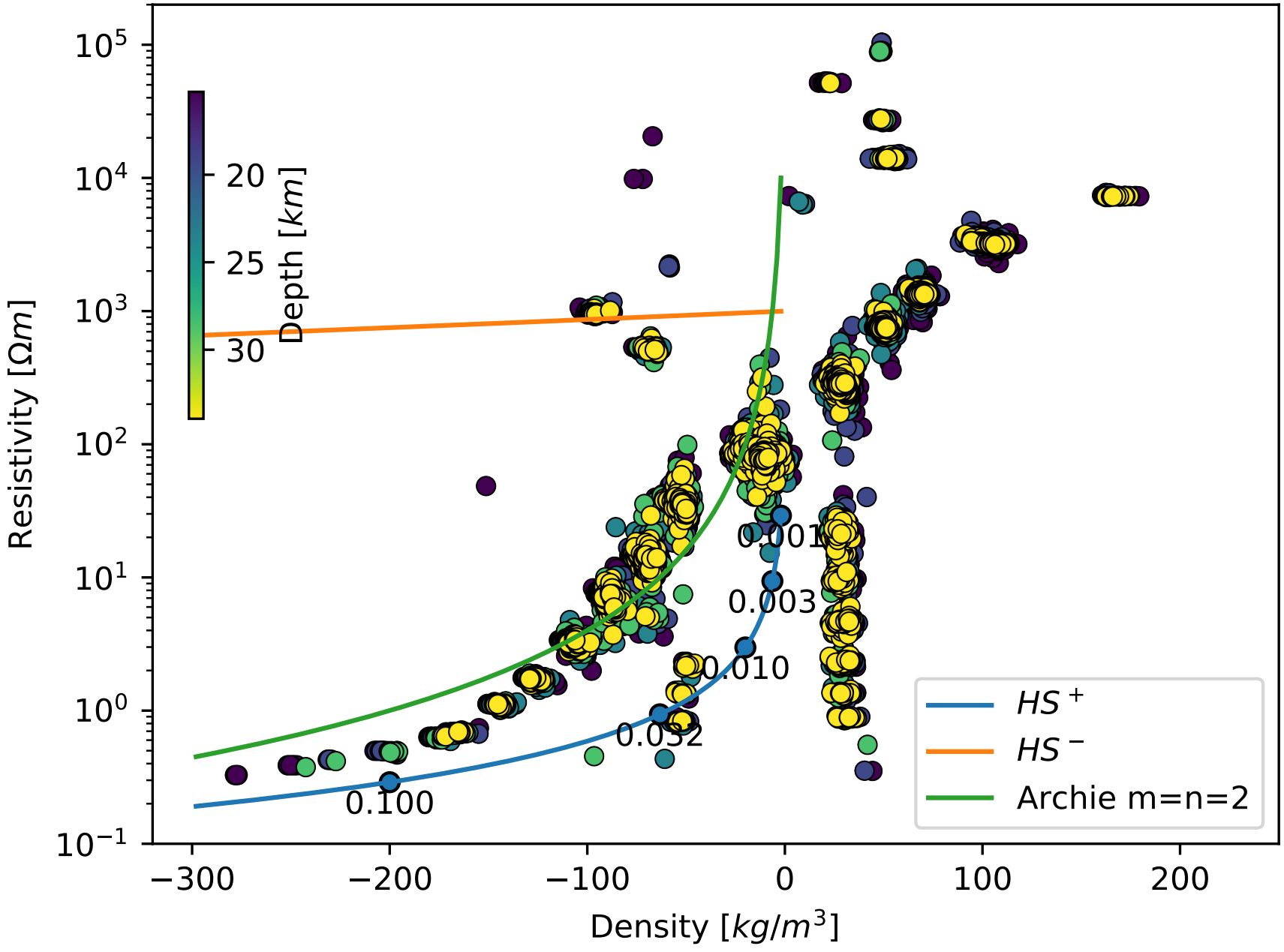
Joint inversion with MI – vertical profile



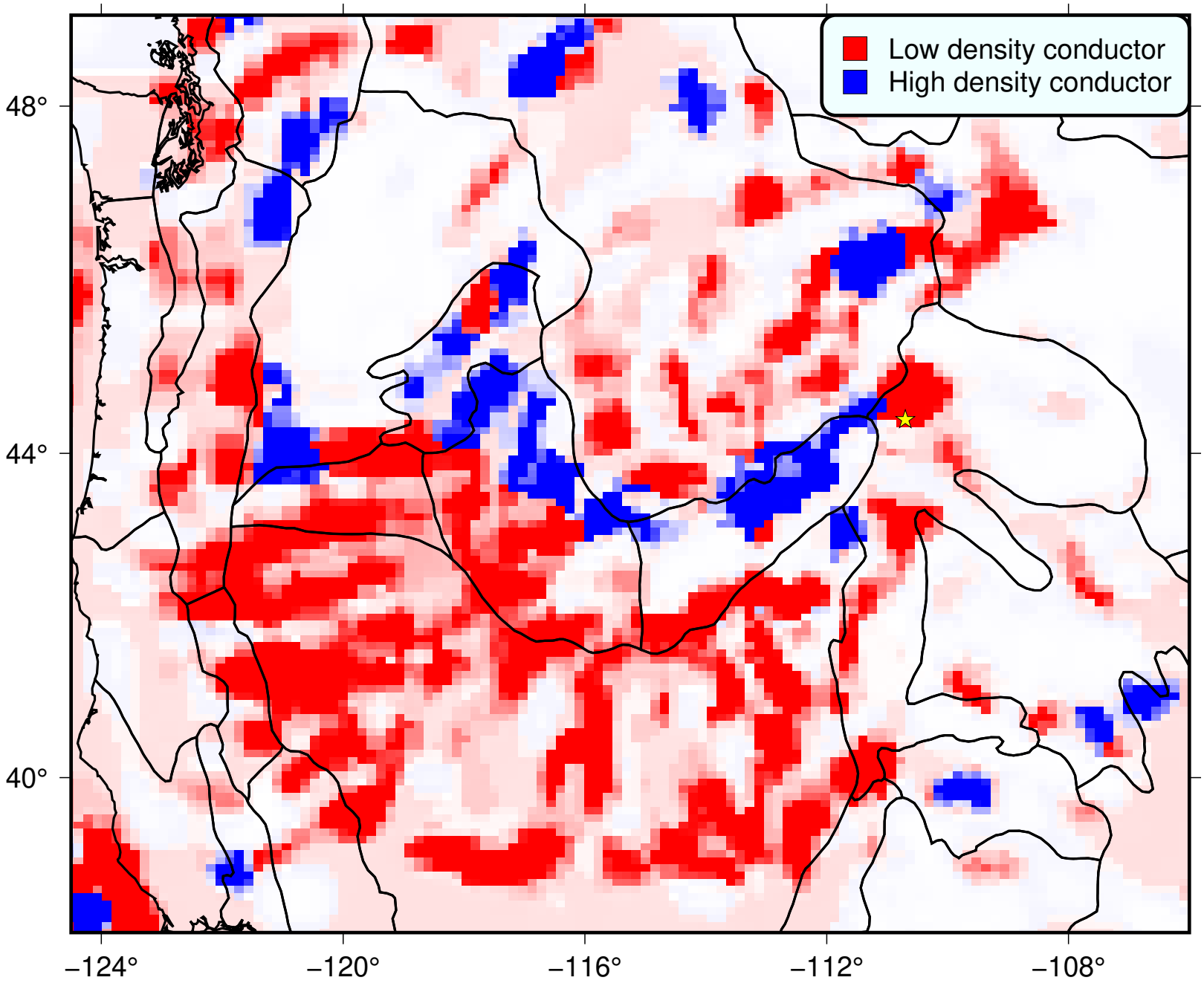
Parameter relationship –whole model



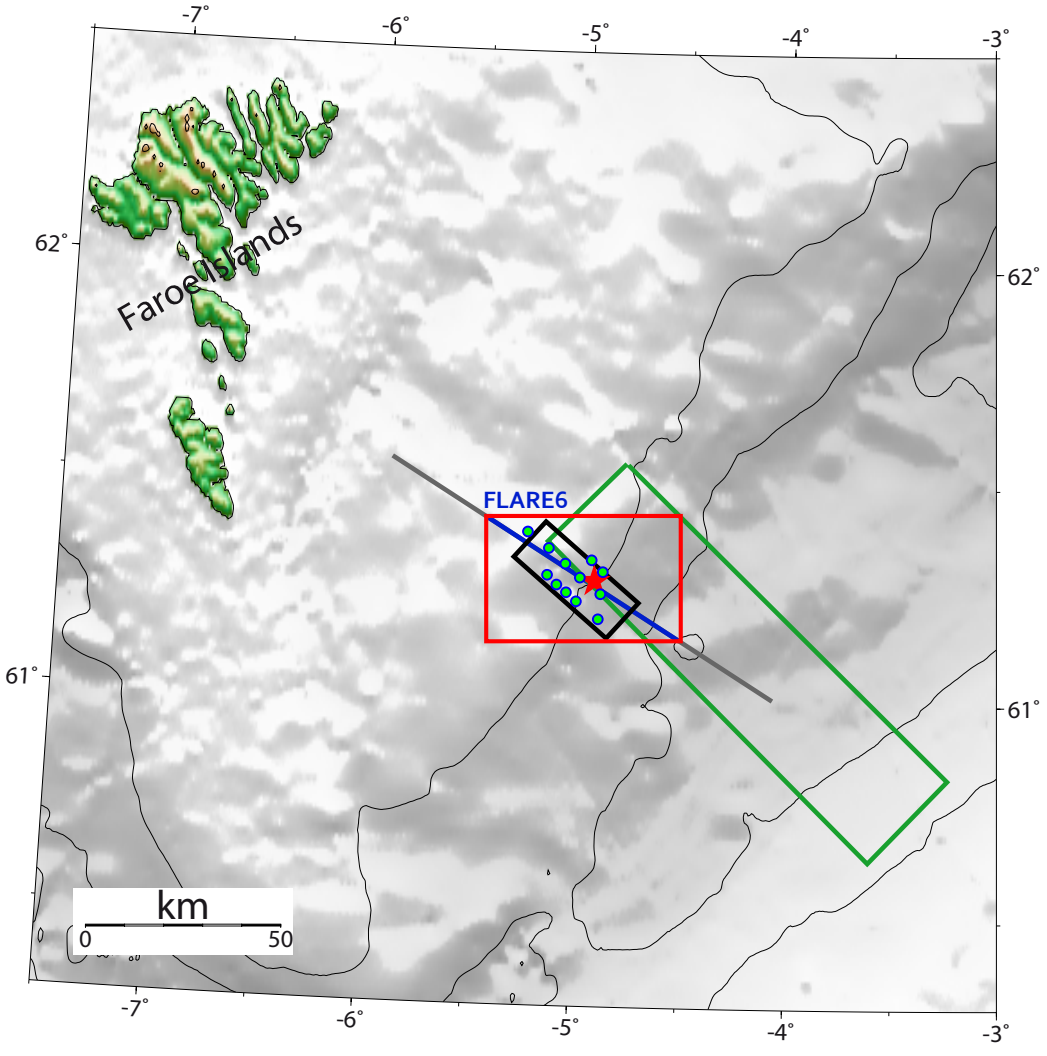
Parameter relationship –crust only



Crustal conductors

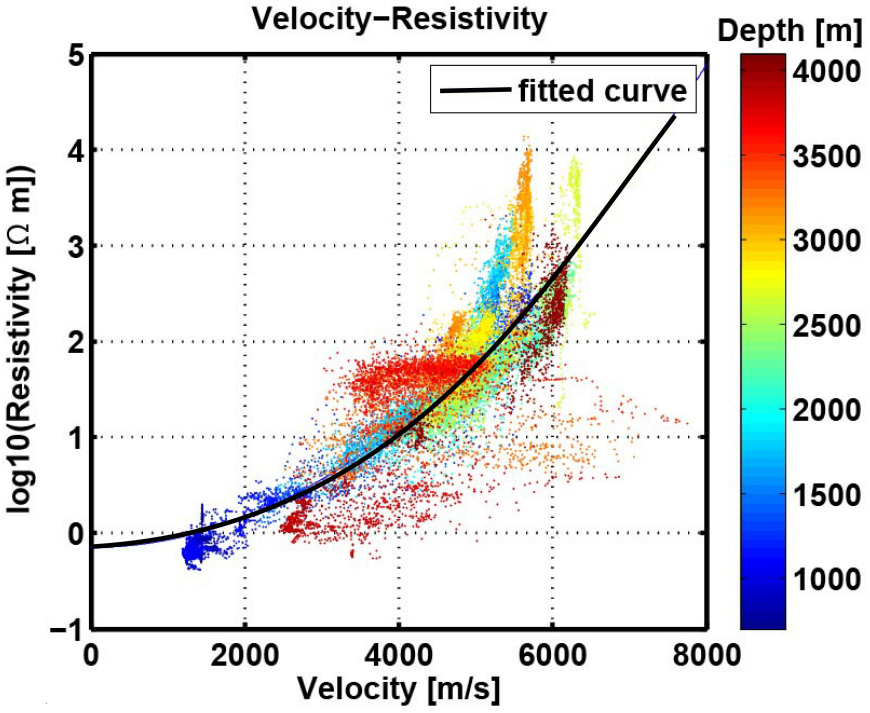


Sub-basalt imaging

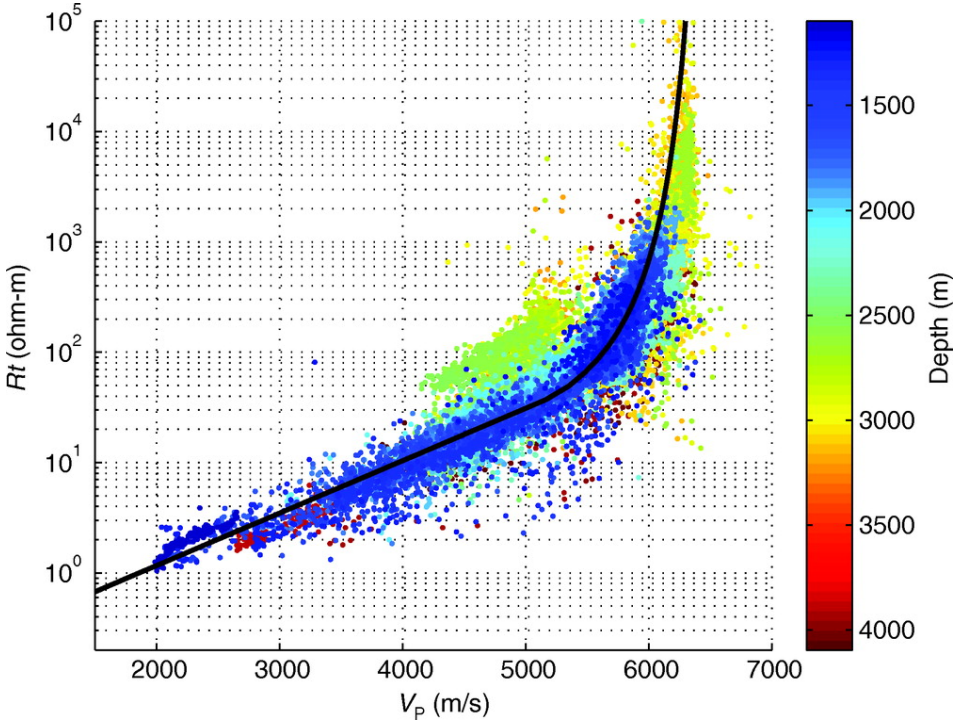


- Hydrocarbon exploration in sub-basalt sediments
- Two studies using similar data and approaches:
  Heincke et al. 2016, MT, seismics and gravity
  Panzner et al. 2016, MT, seismics and CSEM

Constructing a parameter relationship



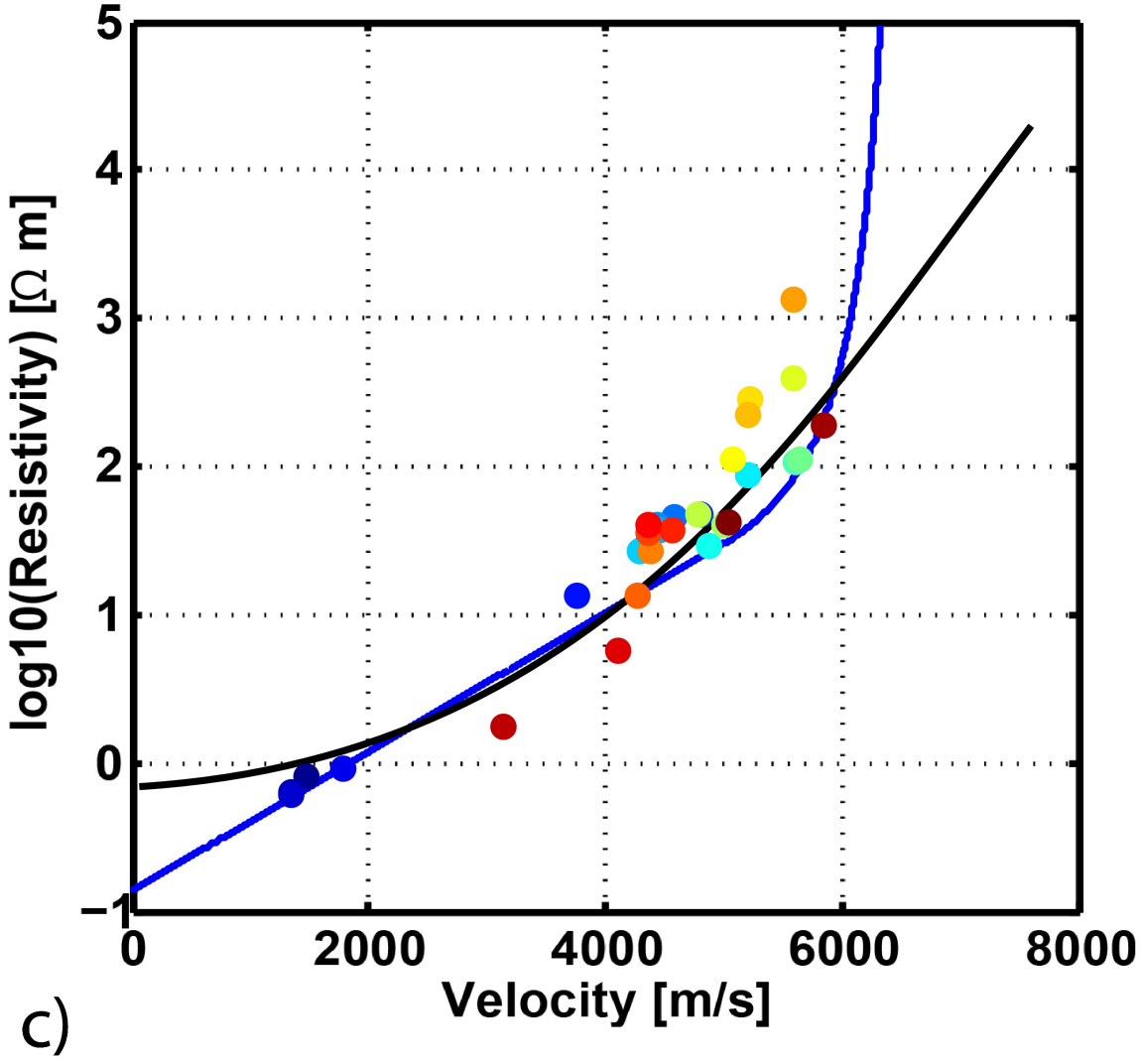
Heincke et al. (2017)



Panzner et al. (2016)

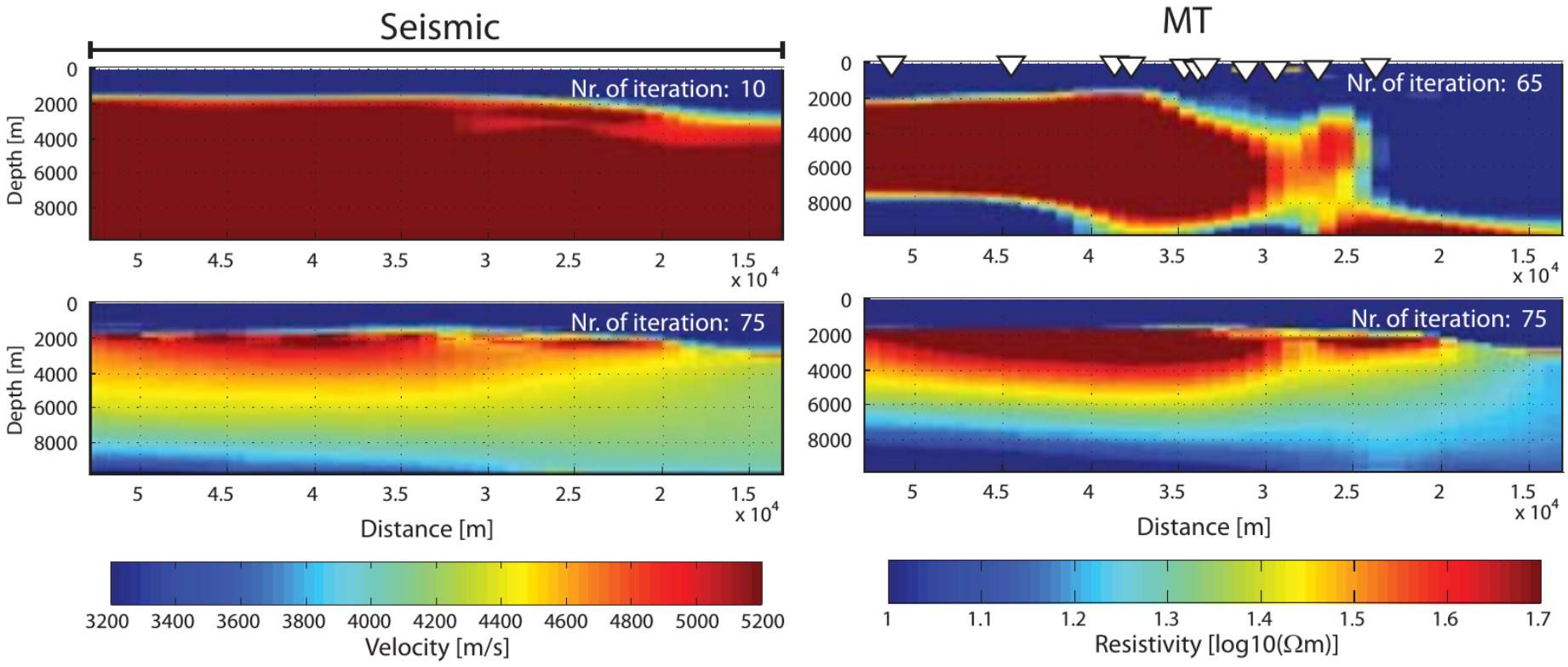
Borehole data are highly scattered, different studies assume different relationships

Averaged borehole log



When averaged to scale of inversion cells, scatter is strongly reduced

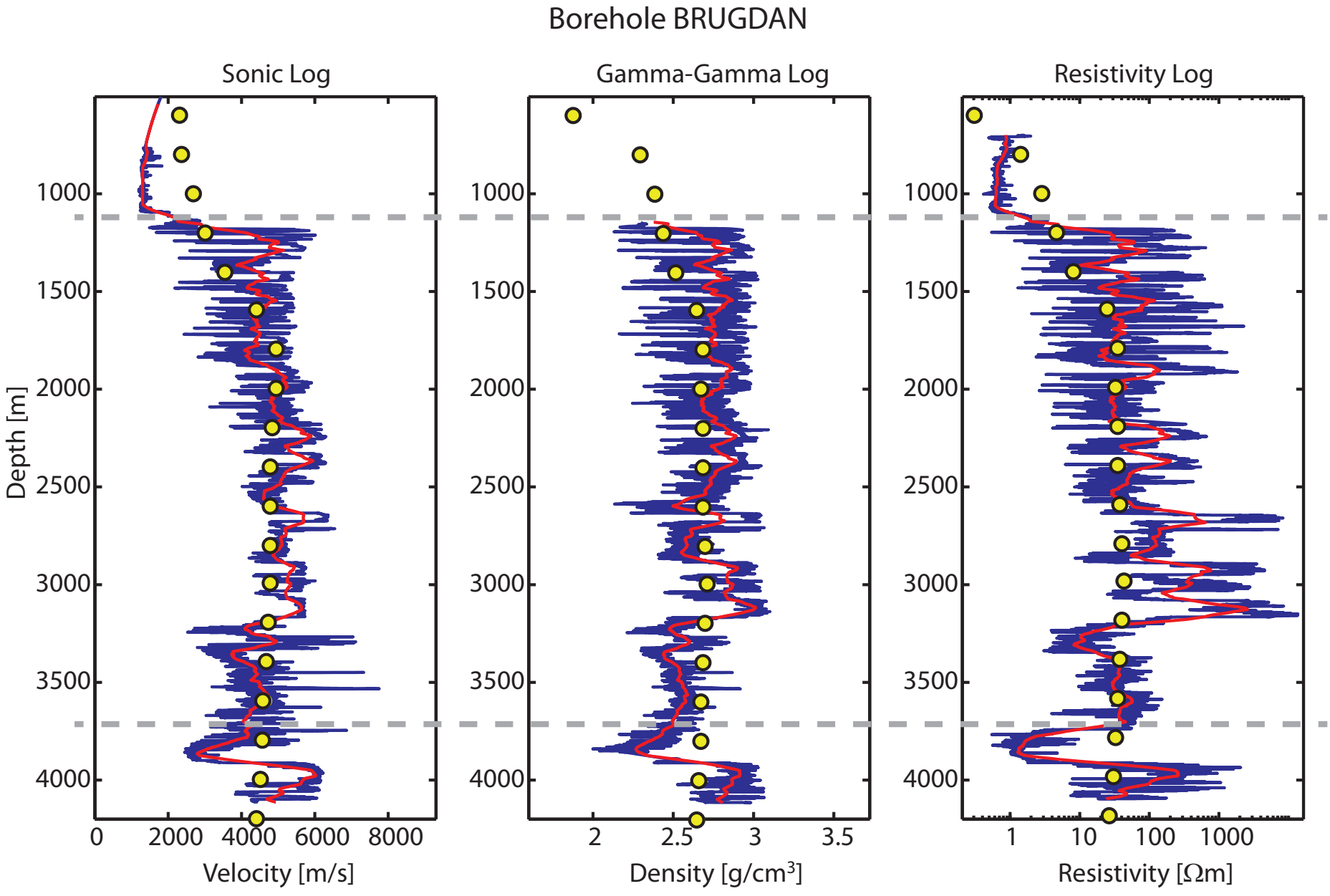
Joint vs. individual inversions



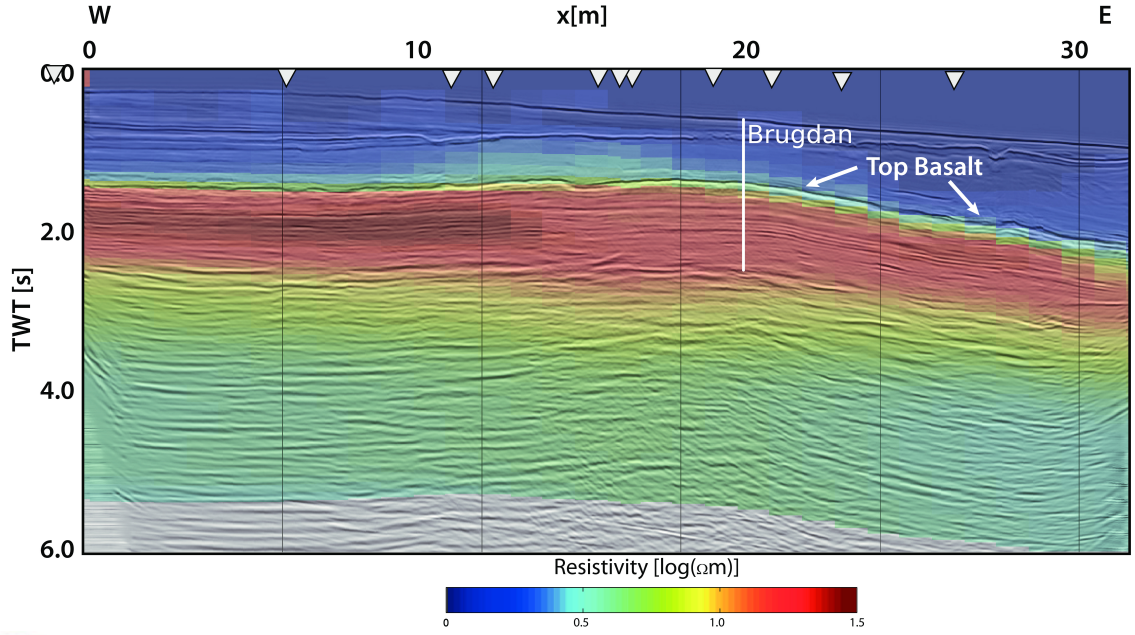
Heincke et al. (2016)

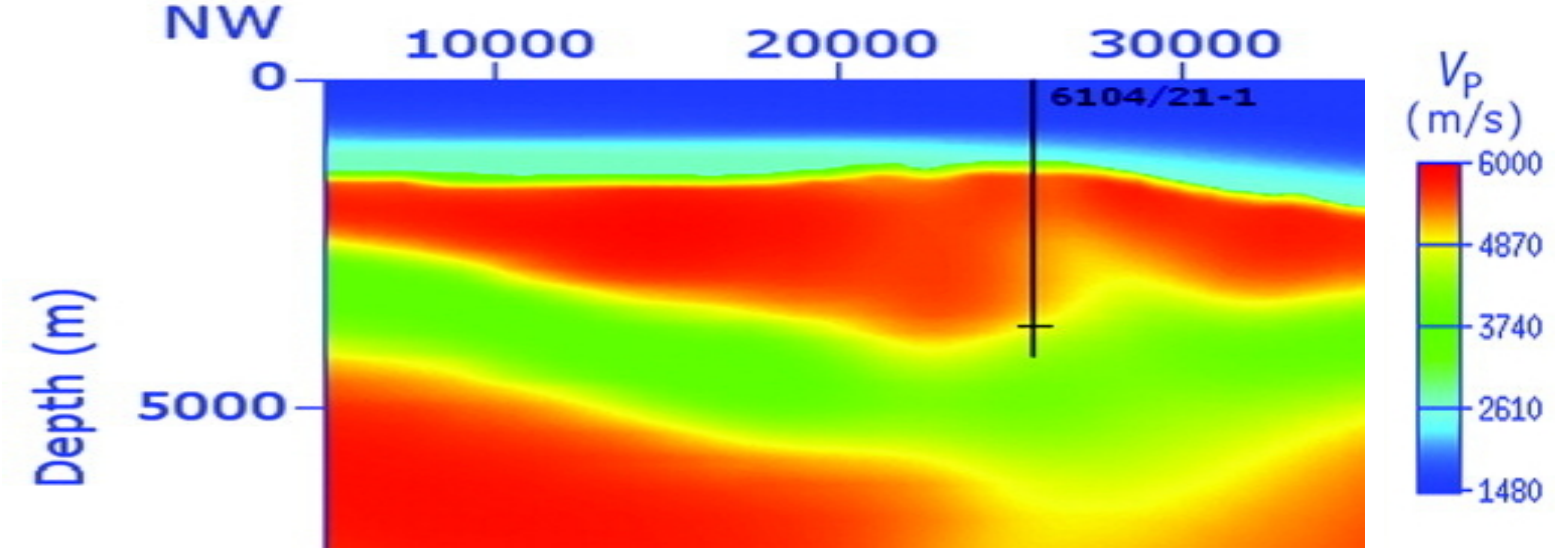
- Individual inversions resolve different aspects of structures
- Joint inversion combines strengths of the methods

Borehole vs. inversion result – Heincke et al.



Comparing joint inversion results





Summary

- Joint and constrained inversions are useful tools to investigate the Earth
- Coupling is a crucial element of JI as it describes assumed connection
- Can approach JI from two perspectives
- Explorative: If all ingredients are representative resulting models have merit
- Hypothesis based: Specifying a coupling means formulating a hypothesis
- Hypothesis based approach most powerful when we can refute hypothesis