The Abdus Salam International Centre for Theoretical Physics

Trieste Algebraic Geometry Summer School (TAGSS) 2021 - Hyperkähler and Prym varieties: classical and new results | (SMR 3609)

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T01

Tropical Prym varieties

I will discuss a tropical counterpart of Prym varieties. Given a double cover of algebraic curves, we obtain a double cover of graphs by degenerating the source and the target at the same time. As a result, we may study Prym varieties by playing a certain combinatorial game on the vertices of the graphs. As I will show, this approach leads to new results in Brill--Noether theory of Prym varieties.

T02

Involutions of a Cubic Fourfold

Cubic fourfolds are interesting objects to study in algebraic geometry due to their relation with constructing compact hyper-K\"ahler manifolds. Cubic fourfolds are Fano manifolds whose middle cohomology is of level 2, with $h^{3,1}=1$; up to a Tate twist, the cohomology resembles that of a K3 surface. In a similar way to K3 surfaces, we are able to study automorphisms of cubic fourfolds via the action on cohomology. In this talk, we will focus on recent work in preparation using lattice theoretic methods to classify involutions of a cubic fourfold. In particular, we compute the sublattice of algebraic classes for a general fourfold with a specific involution. In doing so, we will also exhibit a ten dimensional family of cubic fourfolds contained in the intersection of all Hassett divisors in the moduli space of smooth cubic fourfolds.

T03

GEOMETRY OF PRYM SEMICANONICAL PENCILS

ANDRÉS ROJAS

In the moduli space $\mathcal{R}_g$ of double étale covers of curves of a fixed genus g, the locus of covers of curves with a semicanonical pencil (i.e. curves with an even theta-characteristic admitting nonzero global sections) decomposes as the union of two irreducible divisors $\mathcal{T}_g^o$ and $\mathcal{T}_g^e$.

When restricted to these divisors, the Prym map yields substantial differences between $\mathcal{T}_g^o$ and $\mathcal{T}_g^e$. In this talk I will describe the geometry of these restricted Prym maps, with special focus on the case of $\mathcal{T}_5^o$ (initially considered by Izadi), where cohomological methods lead to enumerative properties of cubic threefolds. This is joint work with M. Lahoz and J.C. Naranjo.

T04

Hitchin fibration and Prym varieties

Let X be a compact Riemann surface of genus $g>1$. A Higgs bundle on X is a pair (E,ϕ) where E is a vector bundle and $\phi : E \to E \otimes K_X$ is a morphism, where K_X is the canonical bundle. There is a surjective morphism, called the \textit{Hitchin fibration}, from the moduli of G-Higgs bundles (where $G = Sp(2m,\mathbb{C})$ or $SO(n,\mathbb{C})$) to a vector space. The generic fibers of this fibration are Prym varieties.\\ Let $D \subset X$ be a set of finitely many points. A parabolic bundle is a vector bundle together with a weighted flag over the fiber for each point in D. In this talk, we will prove that the generic fibers of the Hitchin fibration for the moduli of parabolic G-Higgs bundles (where $G = Sp(2m,\mathbb{C})$ or $SO(n,\mathbb{C})$) are Prym varieties of spectral curves.

T05

On Ramified Prym Maps and Their Fibers

As we will see, Prym varieties are polarized abelian varieties associated with double covers of irreducible genus g curves. As such they have been studied for many years, starting from the classical case of unramified coverings. Only quite recently the ramified case, i.e. the one concerning Prym varieties associated with ramified double covers, has started being investigated. In this talk, we will overview some results in this direction. In particular, we will introduce ramified Prym maps and we will focus on the cases where the generic fiber is positive dimensional, providing a geometric description of it. This is joint work with P. Frediani and J. C. Naranjo.

T06

Kirchhoff’s theorem and semi-canonical representatives for the tropical Prym variety

The Jacobian Jac(G) is a finite group associated to a finite graph G, and Kirchhoff’s celebrated matrix tree theorem computes the order of Jac(G) as the number of spanning trees of G. The Jacobian Jac(G) of a metric graph G is a real torus of dimension equal to b_1(G), and a weighted version of Kirchhoff’s theorem expresses the volume of Jac(G) as a weighted sum over all spanning trees of G. A recent paper of An, Baker, Kuperberg, and Shokrieh gives a geometric interpretation of the weighted matrix-tree theorem of a metric graph G, based on an earlier result of Mikhalkin and Zharkov. Namely, each element of Jac(G) is represented by a unique (up to translation) so-called break divisor. The type of break divisor defines a canonical cellular decomposition of Jac(G), and the individual terms in the volume formula for Jac(G) are the volumes of the cells. I will state and prove analogous results for the tropical Prym variety associated to a double cover of a metric graph G, as defined by Jensen, Len, and Ulirsch. The volume of the Prym variety is calculated as a weighted sum over certain spanning cycles on the target graph G, generalizing a similar result of Zaslavsky, Reiner and Tseng for ordinary graphs. I will then give a geometric interpretation of the volume formula in terms of a semi-canonical representability result for Prym divisors. I will discuss possible applications to the problem of resolving the Prym-Torelli map. Joint work with Yoav Len.