The Hochschild cohomology of gentle algebras

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Homological methods provide important information about the structure of associative algebras, revealing sometimes hidden connections amongst them. The Hochschild homology, cohomology - together with its graded algebra structure and its Gerstenhaber structure - of unital associative algebras over a field are invariants preserved by derived equivalences. The Lie algebra of derivations modulo inner derivations -that is, the forst Hochschild cohomology of the algebra- is particularly interesting.

The family of gentle algebras has attracted the attention of many authors in recent times. These algebras were introduced by Assem and Skowroński in the '80s as a generalization of iterated tilted algebras of type A_n, and affine type $\tilde{A}_n$. They are connected to many other areas of mathematics such as dimer models, Lie algebras, cluster theory, and homological mirror symmetry.

We will give a complete description of the structure of the Hochschild cohomology ring of a gentle algebra A both as a graded commutative algebra and as Gerstenhaber algebra. Furthermore, we will show how these structures are encoded in the geometric surface model of the bounded derived category associated to a gentle algebra via its ribbon graph. Moreover, we will explain how the first Hochschild cohomology acts on the category of A-modules and on its derived category.

This is a joint work with Cristian Chaparro Acosta, Sibylle Schroll and Mariano Suárez-Álvarez.