Introduction to Ionospheric Modeling

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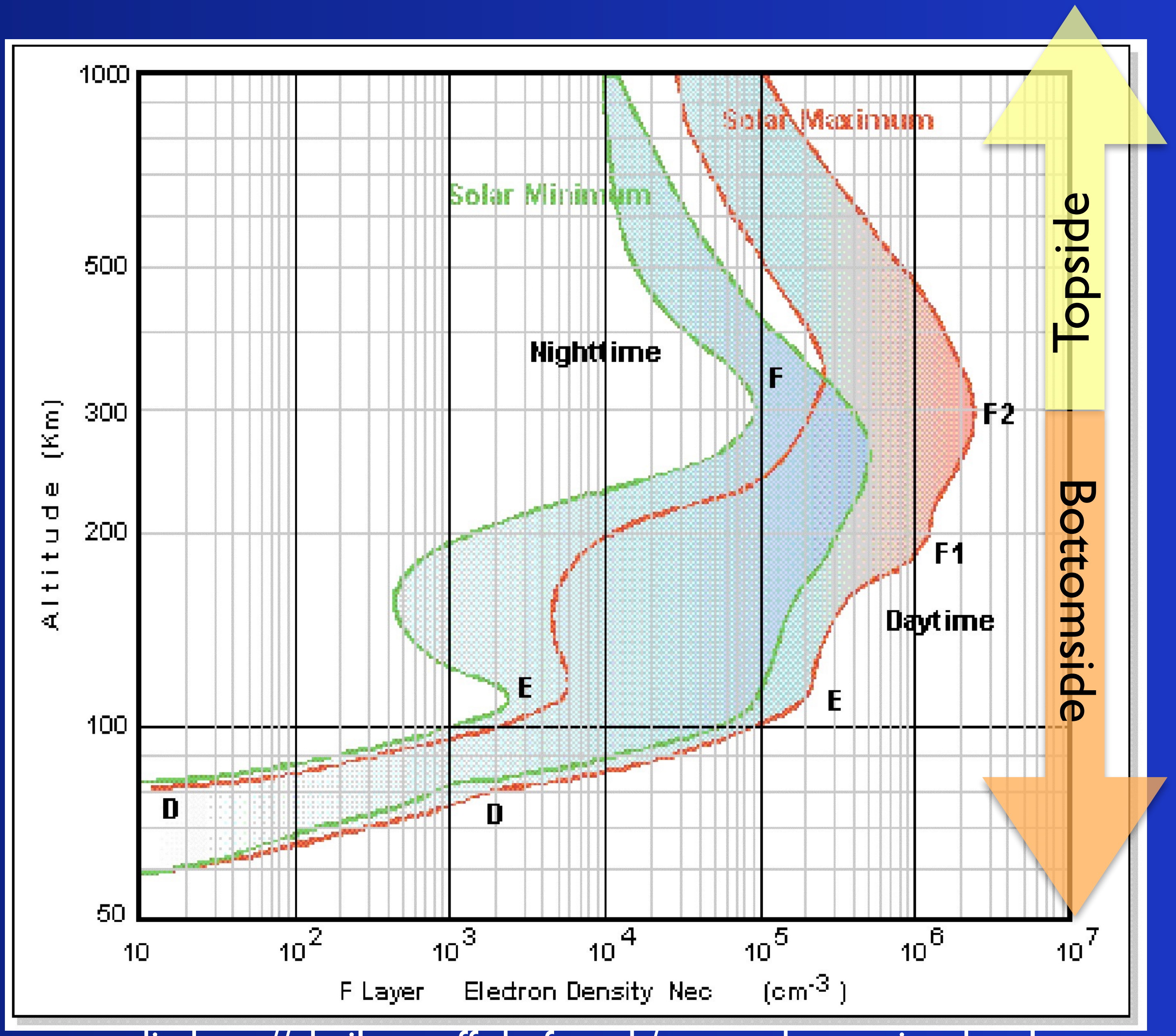
Ionospheric modelling

The understanding of the behaviour of the ionosphere and its effects on human activity is determined by the ability to model at least the height, geographical and time distributions of the electron density.



credit: http://gbailey.staff.shef.ac.uk/researchoverview.html

Types of models

Two main different types of models can be identified, each of them having implicit limitations and advantages.

- Physics-based models
- Empirical (or semi-empirical) models

The additional category of data assimilation (or assimilative) models is commonly introduced when specific mathematical techniques are used to incorporate experimental data into physics-based or empirical background models.

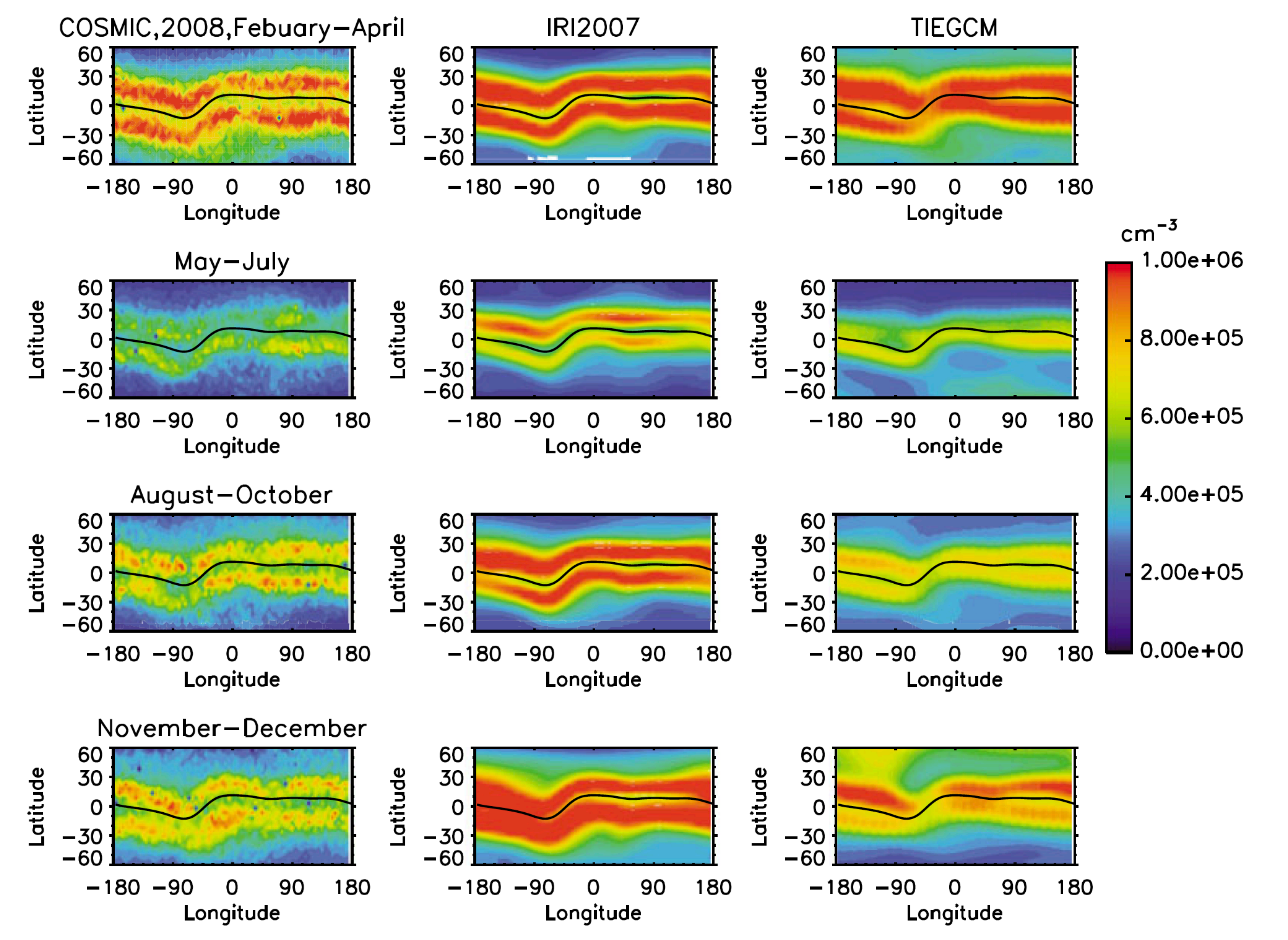
Physics-based models

- In these models conservation (continuity, momentum, energy, etc.) equations are solved numerically as a function of spatial and time coordinates to calculate plasma densities, temperatures and flow velocities.
- They require solar, magnetospheric and atmospheric input parameters and their accuracy depend on the quality of the input data and on the completeness of the physics and chemistry included in the models.
- They can be powerful tools to understand the physical and chemical processes of the upper atmosphere

Physics-based models (TIE-GCM)

- The Thermosphere-Ionosphere-Electrodynamics General Circulation Model (TIE-GCM), is developed at the National Center for Atmospheric Research (NCAR) High-Altitude Observatory (HAO).
- It is a global 3-D numerical model that simulates the coupled thermosphere/ionosphere system from ~97 to ~600 km altitude.
- The TIE-GCM self-consistently solves the fully coupled, nonlinear, hydrodynamic, thermodynamic, and continuity equations of the neutral gas, the ion and electron energy and momentum equations, the ion continuity equation, and neutral wind dynamo.
- It is an open-source community model and is also available for runs-on-request at the NASA Community Coordinated Modeling Center (CCMC).

Physics-based models (TIE-GCM)



NmF2 observed by COSMIC, IRI, TIE-GCM, during 2008. NmF2 is averaged over 10:00–13:00 LT and over the months shown in each panel; from: Qian et al. (2014).

Empirical models

- They are based based on an analytical description of the ionosphere with functions obtained from experimental data or adapted from physical models.

- These types of models are able to give a “climatological” description of the ionosphere (median models).

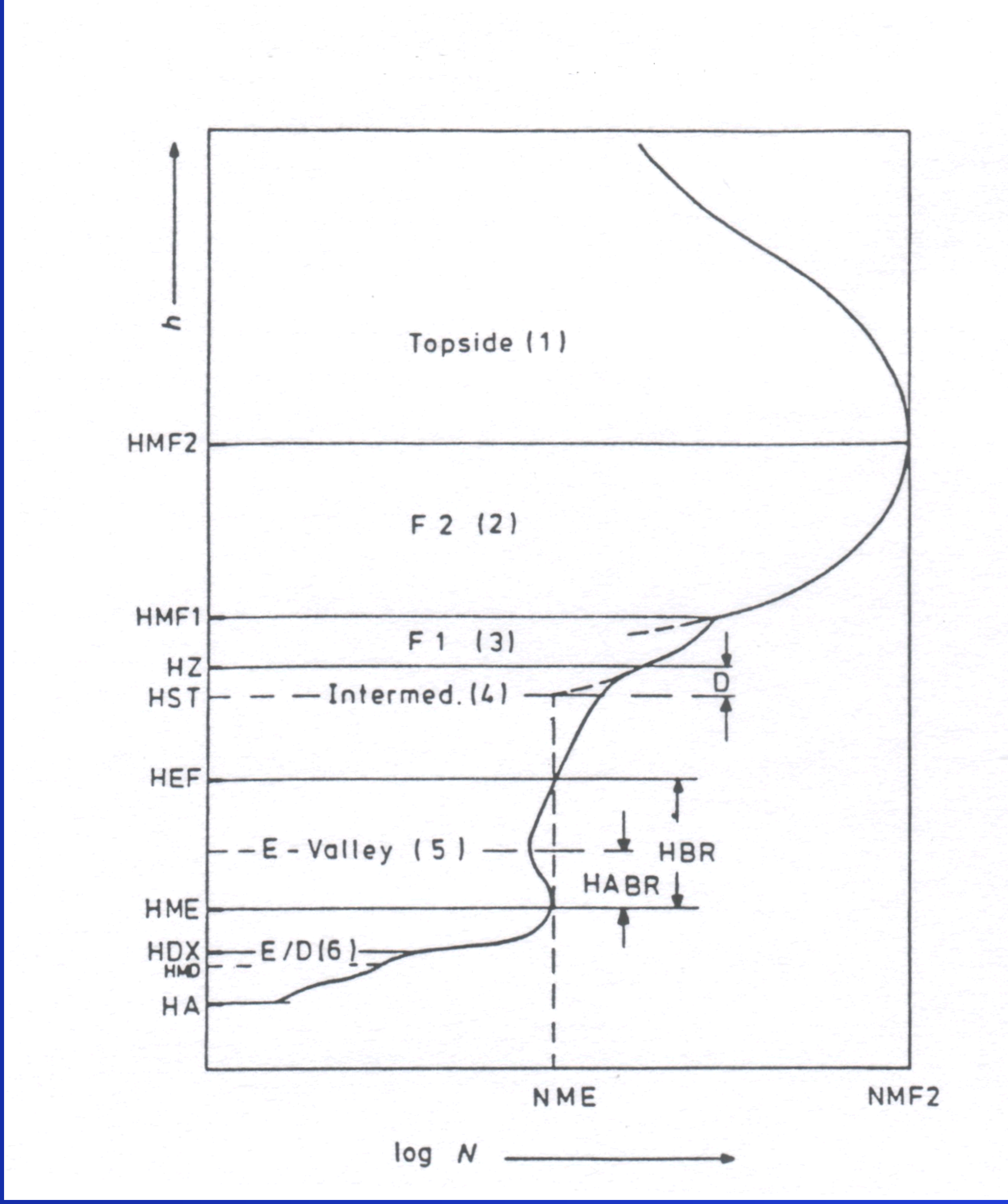
- Examples: the International Reference Ionosphere (IRI), Semi-Empirical Low-Latitude Ionospheric Model (SLIM), Fully Analytical Ionospheric Model (FAIM), Parameterised Real-time Ionospheric Specification Model (PRISM), Bent model, NeQuick, COSTProf, NeUoG-plas, the more recently developed regional Empirical Canadian High Arctic Ionospheric Model (E-CHAIM) and Neustrelitz Plasmasphere Model (NPSM).

IRI

- The International Reference Ionosphere (IRI) is an international project sponsored by the Committee on Space Research (COSPAR) and the International Union of Radio Science (URSI). These organizations formed a Working Group (members) in the late sixties to produce an empirical standard model of the ionosphere, based on all available data sources.
- For given location, time and date, IRI describes the electron density, electron temperature, ion temperature, and ion composition in the altitude range from about 50 km to about 2000 km; and also the electron content.
- IRI is updated periodically and has evolved over a number of years. (Bilitza, D. The International Reference Ionosphere - Status 2013, Advances in Space Research, 55, 8, (2015), Pages 1914–1927).

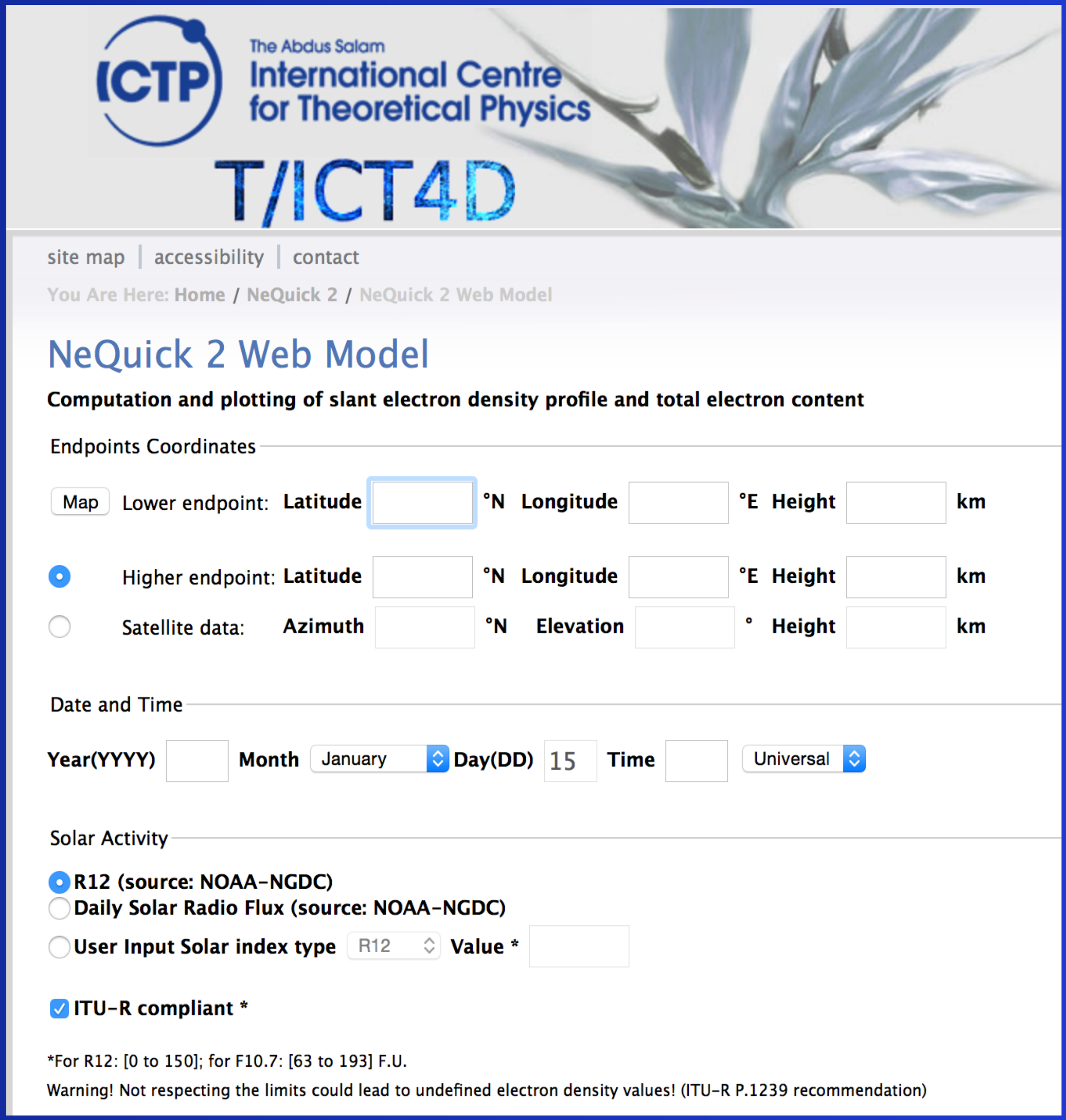
IRI

- The IRI electron density profile is divided in six sub-regions: the topside, the F2 bottomside, the F1 layer, the intermediate region, the E region valley, the bottomside E and D region. The boundaries are defined by the presence of characteristic points that include the F2, F1 and E peaks.
- The shape of the IRI topside electron density profile was based on the descriptive compilation of Alouette topside sounder data and Epstein functions.



The NeQuick model

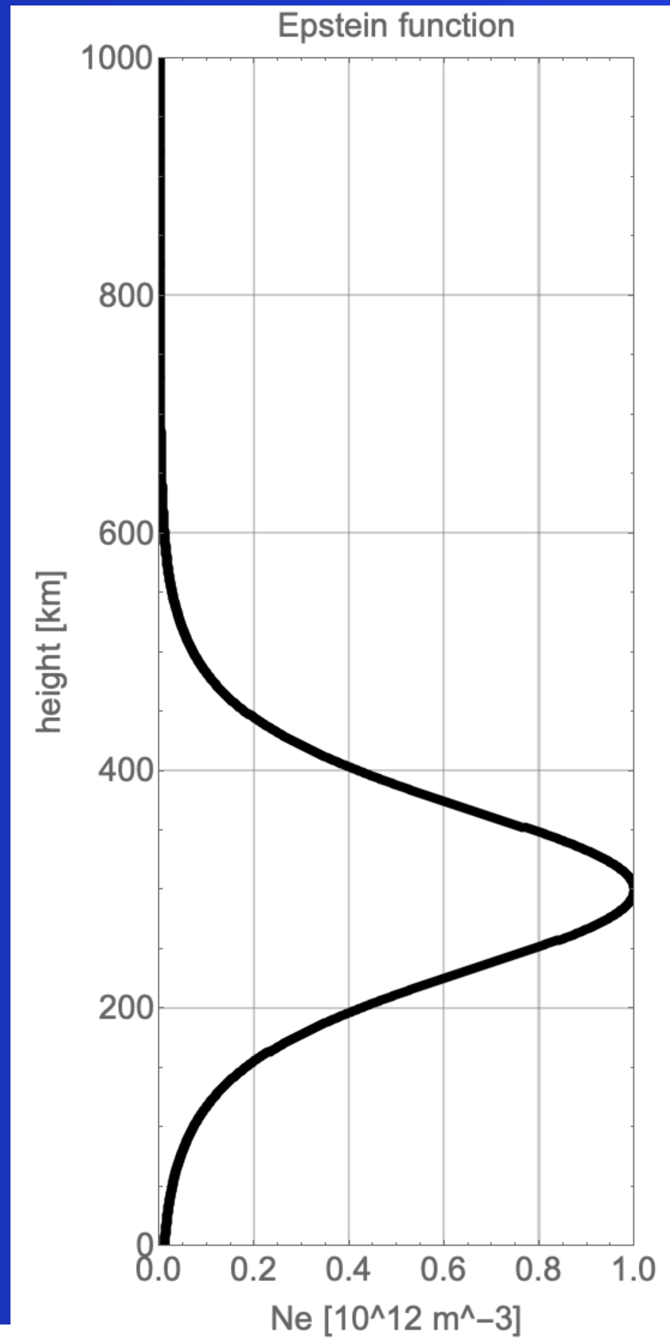
- The NeQuick is an ionospheric electron density model developed at the T/ICT4D Laboratory of The Abdus Salam International Centre for Theoretical Physics (ICTP), Trieste, Italy, and at the Institute for Geophysics, Astrophysics, and Meteorology (IGAM) of the University of Graz, Austria.
- It is a quick–run empirical model particularly designed for trans–ionospheric propagation applications, conceived to reproduce the median behavior of the ionosphere.
- It is based on the DGR "profiler" proposed by Di Giovanni and Radicella [1990] and subsequently modified by other co-authors (Leitinger, Zhang, Coïsson, Nava).



https://t-ict4d.ictp.it/nequick2/

NeQuick 2

- The model profile formulation includes 6 semi-Epstein layers with modeled thickness parameters and is based on anchor points defined by foE, foF1, foF2 and M(3000)F2 values.



$$N_E(h) = \frac{4Nm^*E}{\left(1+\exp\left(\frac{h-hmE}{BE}\xi(h)\right)\right)^2}\exp\left(\frac{h-hmE}{BE}\xi(h)\right)$$

$$N_{F1}(h) = \frac{4Nm^*F1}{\left(1+\exp\left(\frac{h-hmF1}{B1}\xi(h)\right)\right)^2}\exp\left(\frac{h-hmF1}{B1}\xi(h)\right)$$

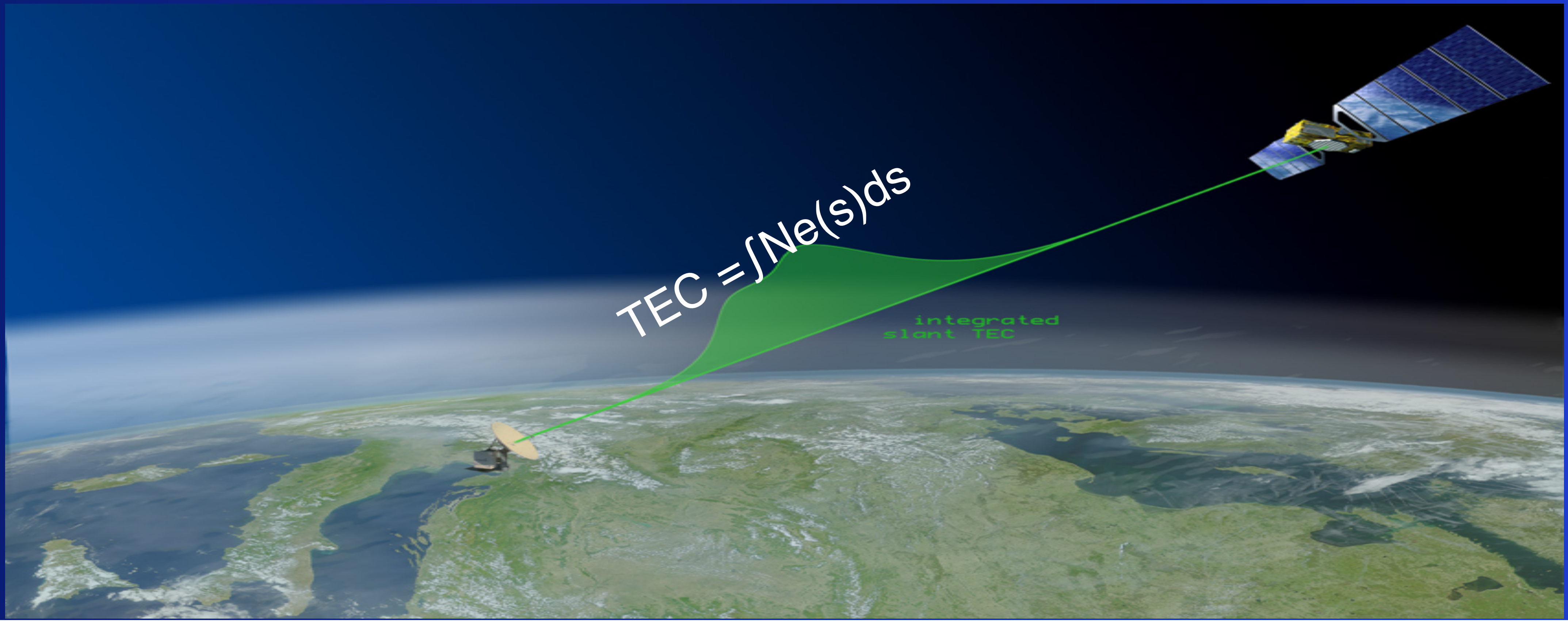
$$N_{F2}(h) = \frac{4NmF2}{\left(1+\exp\left(\frac{h-hmF2}{B2}\right)\right)^2}\exp\left(\frac{h-hmF2}{B2}\right)$$

where

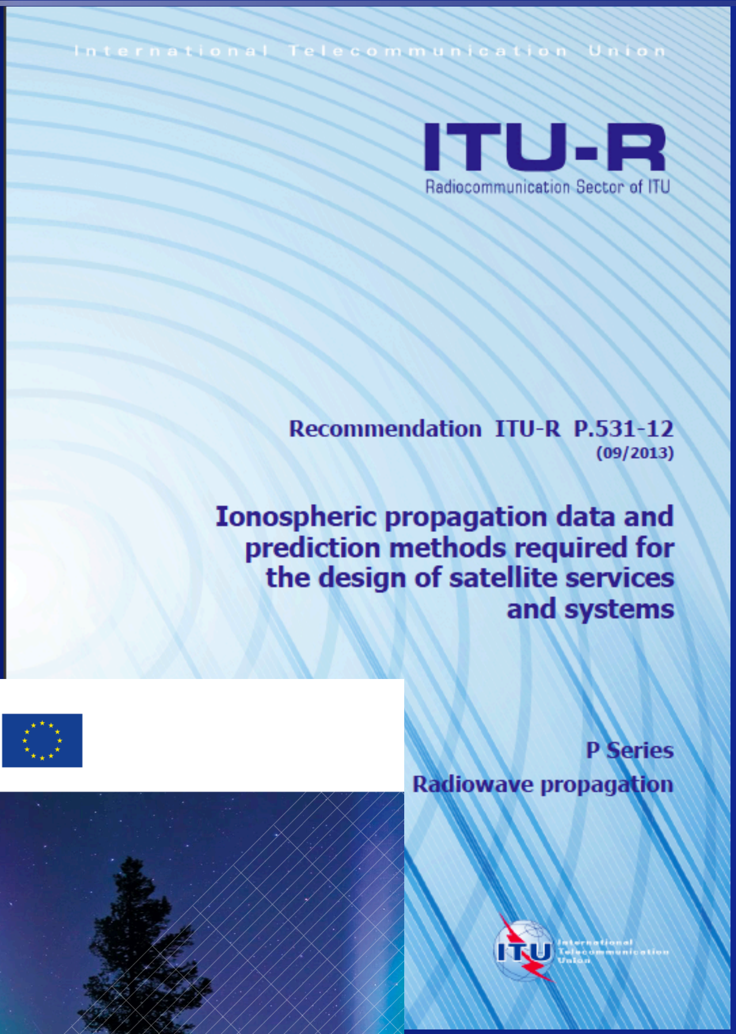
$$\xi(h) = \exp\left(\frac{10}{1+1|h-hmF2|}\right)$$

NeQuick 2

- These values can be modeled (e.g. ITU-R coefficients for foF2, M(3000)F2 or experimentally derived.
- NeQuick inputs are: position, time and solar flux; the output is the electron concentration at the given location and time.
- NeQuick package includes routines to evaluate the electron density along any "ground-to-satellite" ray-path and the corresponding Total Electron Content (TEC) by numerical integration.



NeQuick developments



- The NeQuick (v1) has been adopted by Recommendation ITU-R P. 531 as a procedure for estimating TEC.
- Subsequently, the NeQuick 2 has substituted the NeQuick (v1) and it is the one currently recommended by ITU (ITU-R Recommendation P.531-12).
- A specific version of NeQuick (NeQuick G, implemented by ESA) has been adopted as Galileo Single-Frequency Ionospheric Correction algorithm and its performance has
- been confirmed during In-Orbit Validation (Roberto Prieto-Cerdeira et al.; GPS World, June 2014).
- More recently Montenbruck and González Rodríguez (2020) have demonstrated that NeQuick G can as well be used for ionospheric correction of single-frequency observations from spaceborne platforms.

ICA

- GPS uses a simple ionospheric model, the Ionospheric Correction Algorithm (ICA), which gives a representation of the mean vertical delay at L1, for given geomagnetic location and local time (Klobuchar, 1987).

- The diurnal variation of vertical delay is modeled by a cosine function, centered at 14 LT. During night-time the vertical ionospheric delay is approximated to a constant value: 5 ns.

- The amplitude and period of the cosine are represented in the model by 3rd order polynomials, which coefficients are broadcast in GPS navigation message. These coefficients were derived from numerical output of Bent model, determining a 370 sets of coefficients for the different conditions of the ionosphere.

ICA Iono delay (T_{iono} in s @ L1)

$$T_{iono} = F\left(5\times10^{-9} + \sum_{n=0}^{3}\alpha_n\Phi_m^n\left(1-\frac{x^2}{2}+\frac{x^4}{24}\right)\right) \quad \text{if } |x| \le 1.57$$

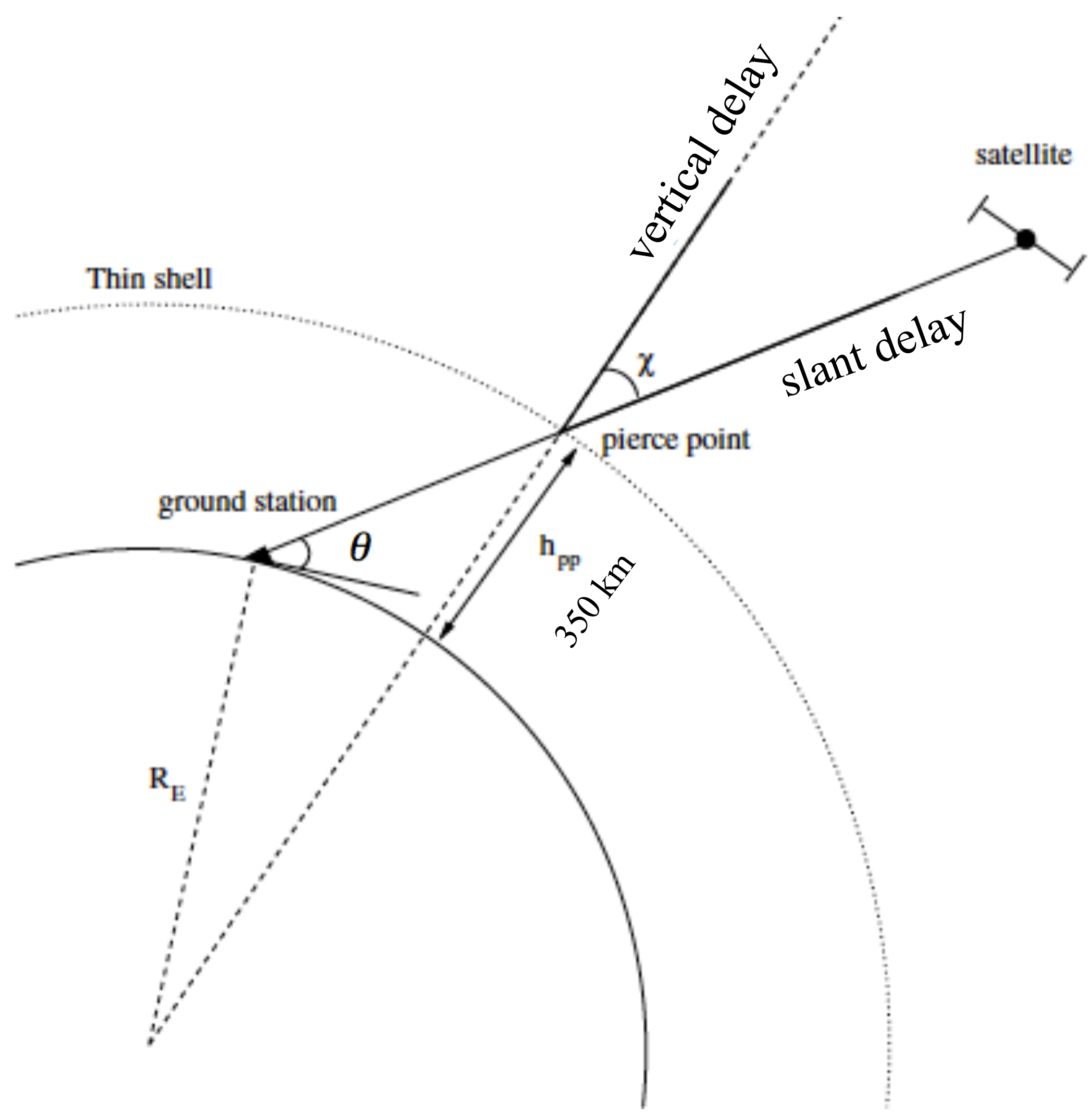
$$T_{iono} = 5\times10^{-9}F \quad \text{if } |x| > 1.57$$

with

$$x = \frac{2\pi(t-50400)}{\sum_{n=0}^{3}\beta_n\Phi_m^n}$$

$$F = 1+16(0.53-\theta)^3$$

α_n , β_n	broadcast coefficients
t	local time
Φ_m	geomagnetic latitude of the pierce point
F	obliquity factor
θ	satellite elevation



NeQuick for Galileo

- The model will be driven by an "effective ionisation level" Az, valid for the whole world and applicable for a period of typically 24 hours.

$$Az(\mu) = a_0 + a_1\mu + a_2\mu^2$$

$$\tan\mu = \frac{I}{\sqrt{\cos\varphi}}$$

μ = **modip**

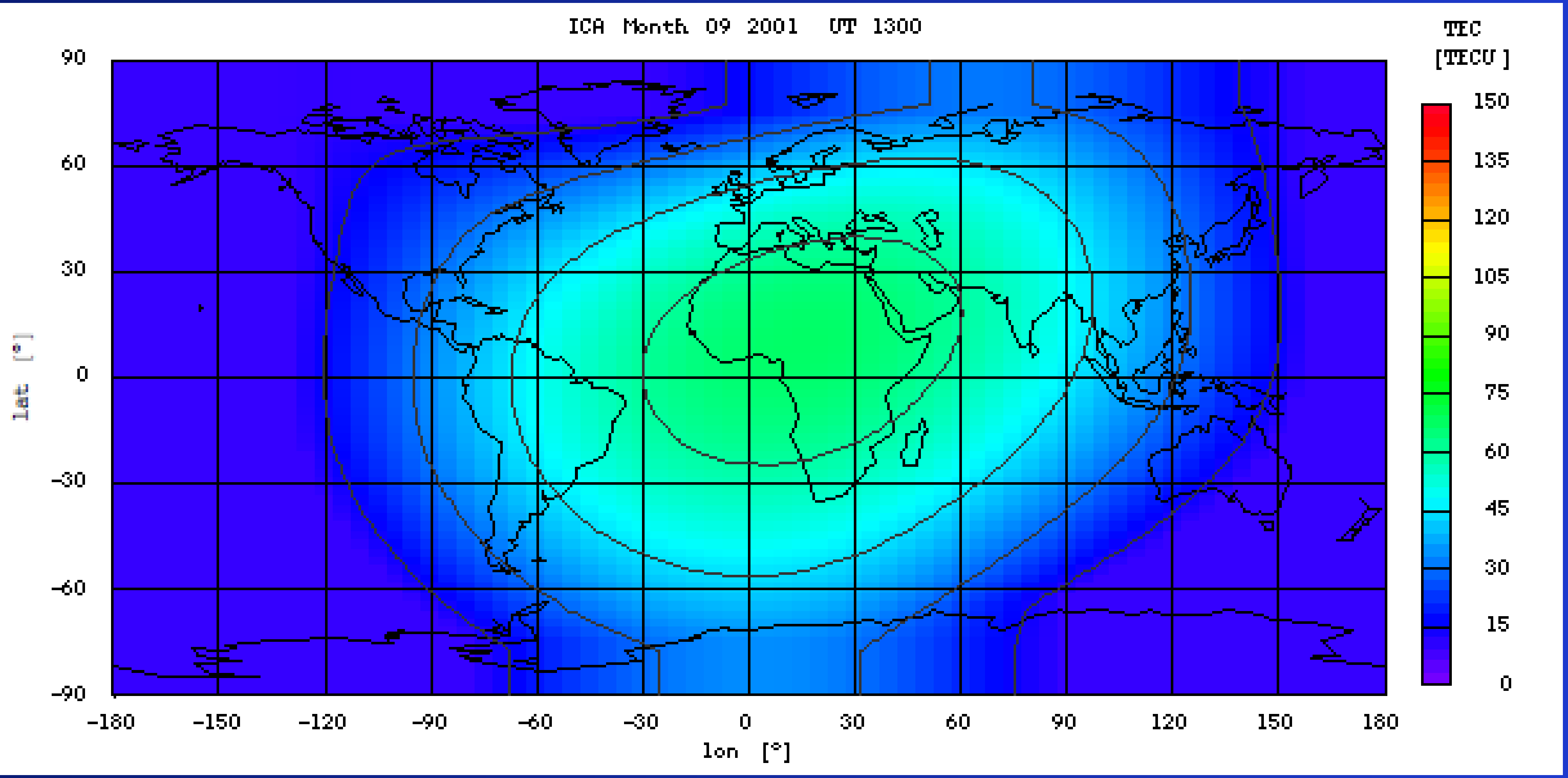
I magnetic inclination at 300 km of height

φ geographic latitude

- The coefficients a_0, a_1, a_2 are broadcast to the user to allow Az calculation at any wanted location.

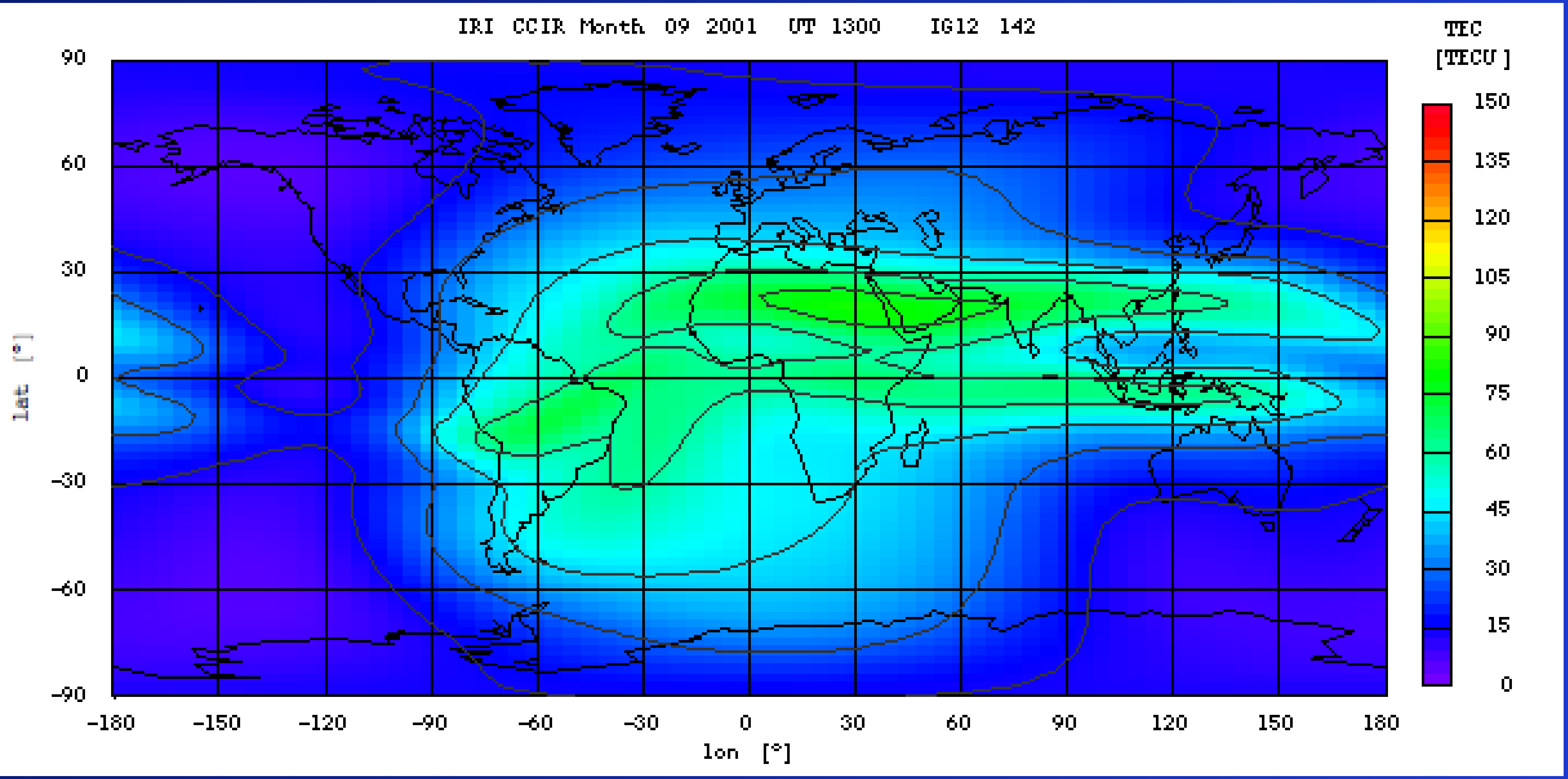
- Az coefficients broadcasted and used for one day are computed at System level using TEC data of the previous day.

ICA vTEC



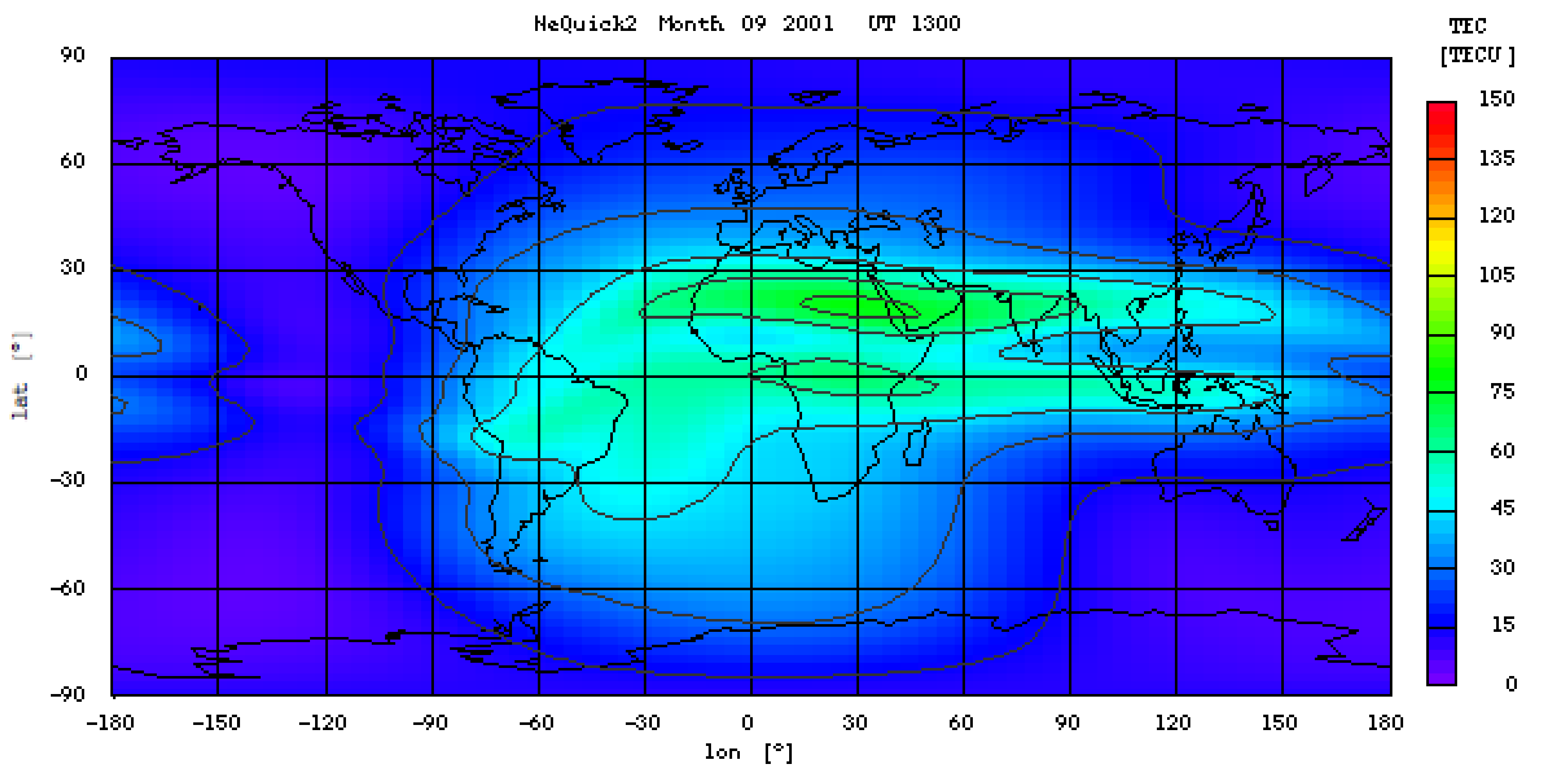
Global map of vertical TEC using ICA.

IRI vTEC



Global map of vertical TEC using IRI (with NeQuick topside).

NeQuick 2 vTEC



Global map of vertical TEC using NeQuick 2.

Data ingestion/assimilation

- Empirical models like IRI and NeQuick have been developed as climatological models, able to reproduce the typical median condition of the ionosphere.

- For research purposes and practical applications, in order to pass from “climate” to “weather”, there is a need to have models able to reproduce the current conditions of the ionosphere.

- Considering that there is an increasing availability of experimental data even in real time (ground and space-based GNSS, ionosondes), several assimilation schemes have been developed. They are of different complexity and rely on different kinds of data.

Assimilation schemes (example)

- Utah State University (USU) Global Assimilation of Ionospheric Measurements (GAIM) [Schunk et al., 2004] or the Jet Propulsion Laboratory (JPL)/University of Southern California (USC) Global Assimilative Ionospheric Model (GAIM) [Wang et al., 2004], or [Schunk et al., 2014], for example, are based on assimilation of data originating from different sources and imply the use of first principle models.

GAIM-band limited (BL)	Midlatitude to Low-Latitude Ionosphere
GAIM-Gauss Markov (GM)	Midlatitude to Low-Latitude Ionosphere
GAIM-4DVAR	Midlatitude to Low-Latitude Ionosphere with Drivers
GAIM-full physics (FP)	Midlatitude to Low Latitude Ionosphere-Plasmasphere with Drivers
Middle-low electro-DA	Midlatitude to Low Latitude Ionosphere with Drivers
IDED-DA	High-Latitude Ionosphere with Drivers
GTM-DA	Global Thermosphere Model-Data Assimilation

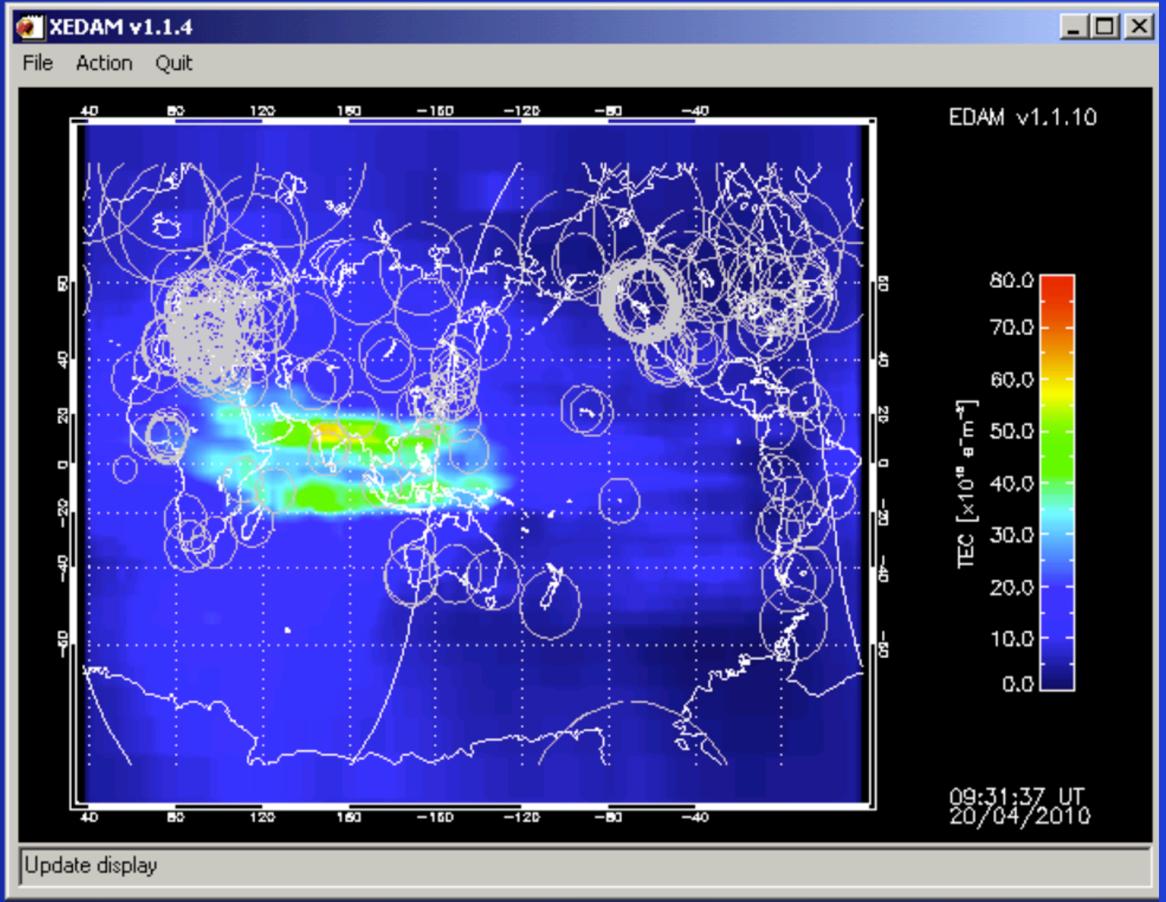
Ionosphere	Electrodynamics	Thermosphere
Ground-based GPS-TEC	Ground magnetometers	Satellite UV emissions
Satellite-based GPS occultation	DMSP cross-track velocities	In situ neutral densities and winds
Ionosonde and digisonde	SuperDARN line-of-sight velocities	Satellite accelerometer and drag
In situ N_e	Iridium magnetometers	FPI winds
911 Å, 1356 Å, limb, disk (UV)	ACE interplanetary magnetic field, *Dst*	ISR neutral parameters
Solar UV, EUV	Solar UV, EUV	Solar UV, EUV

Assimilation schemes (example)

- The Electron Density Assimilative Model (EDAM) [Angling and Khattatov, 2006; Angling, M. J., and N. K. Jackson-Booth, 2011] provides a mean to assimilate ionospheric measurements into a background ionospheric model.

- Assimilated data are: ground-based and space-based GPS-derived TEC, ionosondes-derived parameters
  - Currently IRI is used as a background model (electron density only)

- Extended, localised Gauss Markov Kalman Filter
  - BLUE + time evolution of the differences between the measurements and the background ionosphere
  - Model variances are propagated
  - Covariance are estimated as required



Assimilation schemes (example)

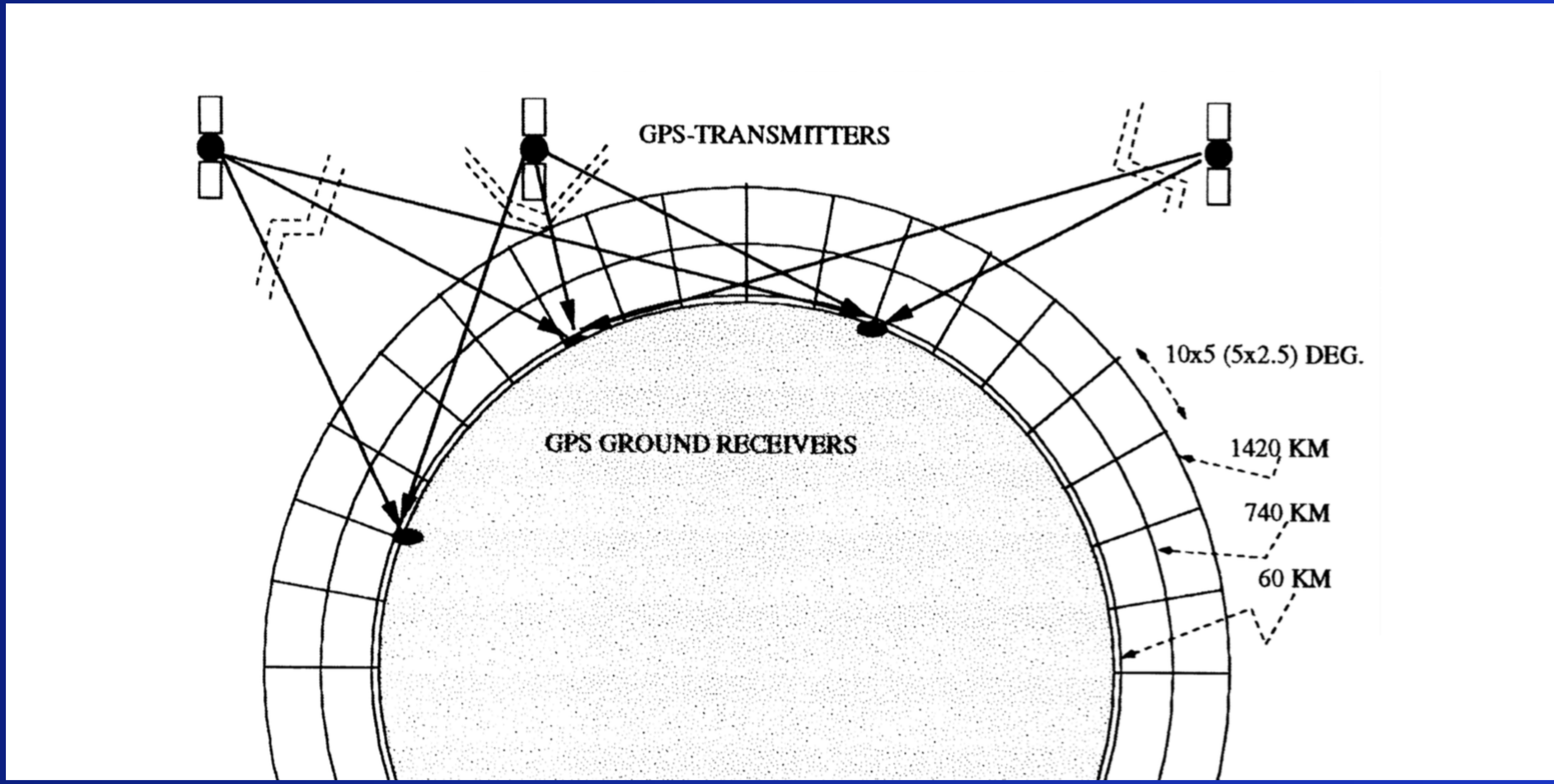
- The Multi Instrument Data Analysis System (MIDAS) [Mitchell C. N. and Spencer P. S. 2003] is a tomographic approach where TEC data are inverted to evaluate the distribution and time evolution of electron concentration.

  - Orthonormal basis functions and SVD are used to solve the inverse problem.

- Review paper: Bust, G. S., and C. N. Mitchell (2008), “History, current state, and future directions of ionospheric imaging, Rev. Geophys., 46,RG1003, doi:10.1029/2006RG000212.

Assimilation schemes (example)

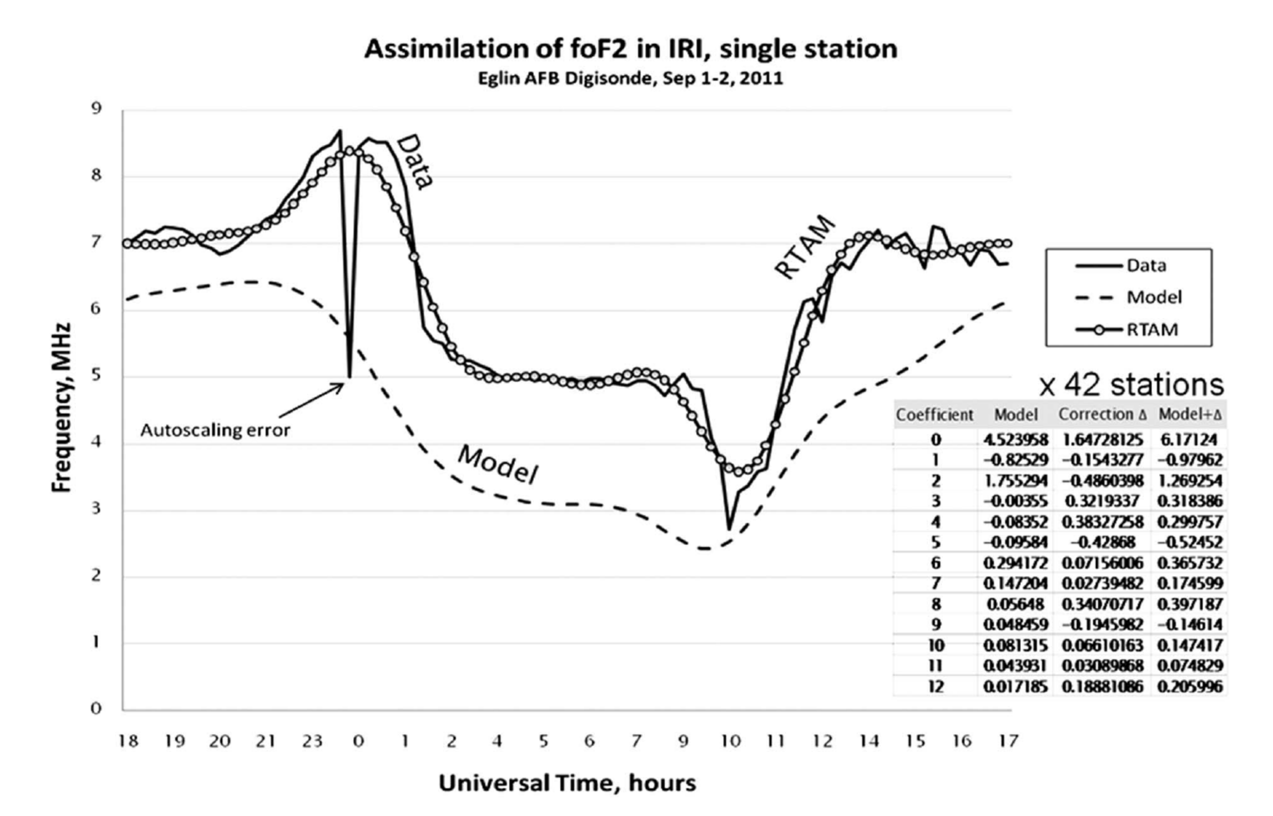
- TOMographic IONosphere model (TOMION), [Hernández-Pajares, M. et al., 1999] generates Global Ionospheric Maps (GIMs) of vertical TEC starting from ground based dual-frequency GNSS measurements. The ionosphere is represented by two or more layers of voxels and in each voxel the electron density is assumed to be constant. It includes an interpolation module relying on the Kriging technique to obtain the relevant vertical TEC at the grid-points [Orús et al., 2005]. No background model is used.



Assimilation schemes (example)

- IRI Real Time Assimilative Model (IRTAM) [Galkin, I. A., et al. 2012], has been developed to assimilate Global Ionosphere Radio Observatory (GIRO) data (foF2, hmF2) in order to "update" the IRI electron density distribution, while preserving the IRI's typical ionospheric feature representations.

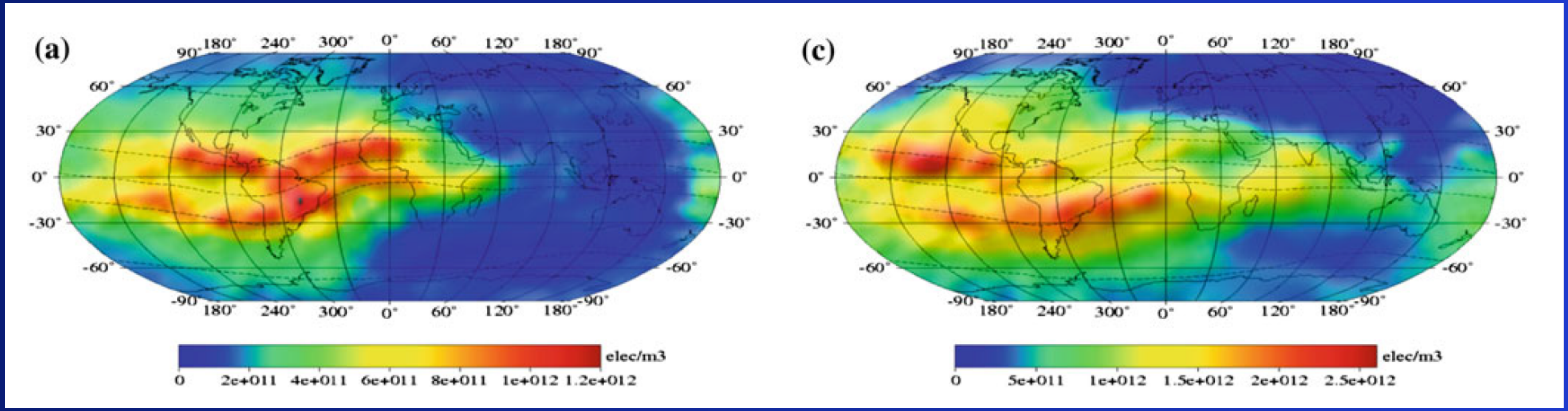
- The technique calculates the corrected coefficients for the spherical/diurnal expansion used by the CCIR-67/URSI-88 model to specify the global foF2 maps, and similarly the maps for all other IRI profile parameters.



Coefficient	Model	Correction Δ	Model+Δ
0	4.523958	1.64728125	6.17124
1	−0.82529	−0.1543277	−0.97962
2	1.755294	−0.4860398	1.269254
3	−0.00355	0.3219337	0.318386
4	−0.08352	0.38327258	0.299757
5	−0.09584	−0.42868	−0.52452
6	0.294172	0.07156006	0.365732
7	0.147204	0.02739482	0.174599
8	0.05648	0.34070717	0.397187
9	0.048459	−0.1945982	−0.14614
10	0.081315	0.06610163	0.147417
11	0.043931	0.03089868	0.074829
12	0.017185	0.18881086	0.205996

Assimilation schemes (example)

- A similar approach has been used by Brunini et al., [2013] in order to update the ITU-R database using radio occultation (COSMIC) electron density profiles.
- For this purpose the La Plata Ionospheric Model (LPIM) (after linearisation) is adjusted by Least Squares to every RO profile available for the time period of interest.



Global representation of the NmF2 estimated value within the 18–20 UT interval
a) NmF2 for the 2007 September equinox c) NmF2 for the 2011 December solstice

Assimilation schemes (example)

- The Ionospheric Data AssimilationThree-Dimensional (IDA3D), [Bust et al., 2004] uses a three-dimensional variational data assimilation technique (3DVAR).
- It is capable of incorporating ground based and space based GPS-TEC measurements and electron density measurements from radars and satellites.
- The background specification is based upon empirical ionospheric models, but IDA3D is capable of using any global ionospheric specification as a background. IDA3D produces a spatial analysis of the electron density distribution at a specified time. A time series of these specifications can be created using past specifications to determine the background for the current analysis.

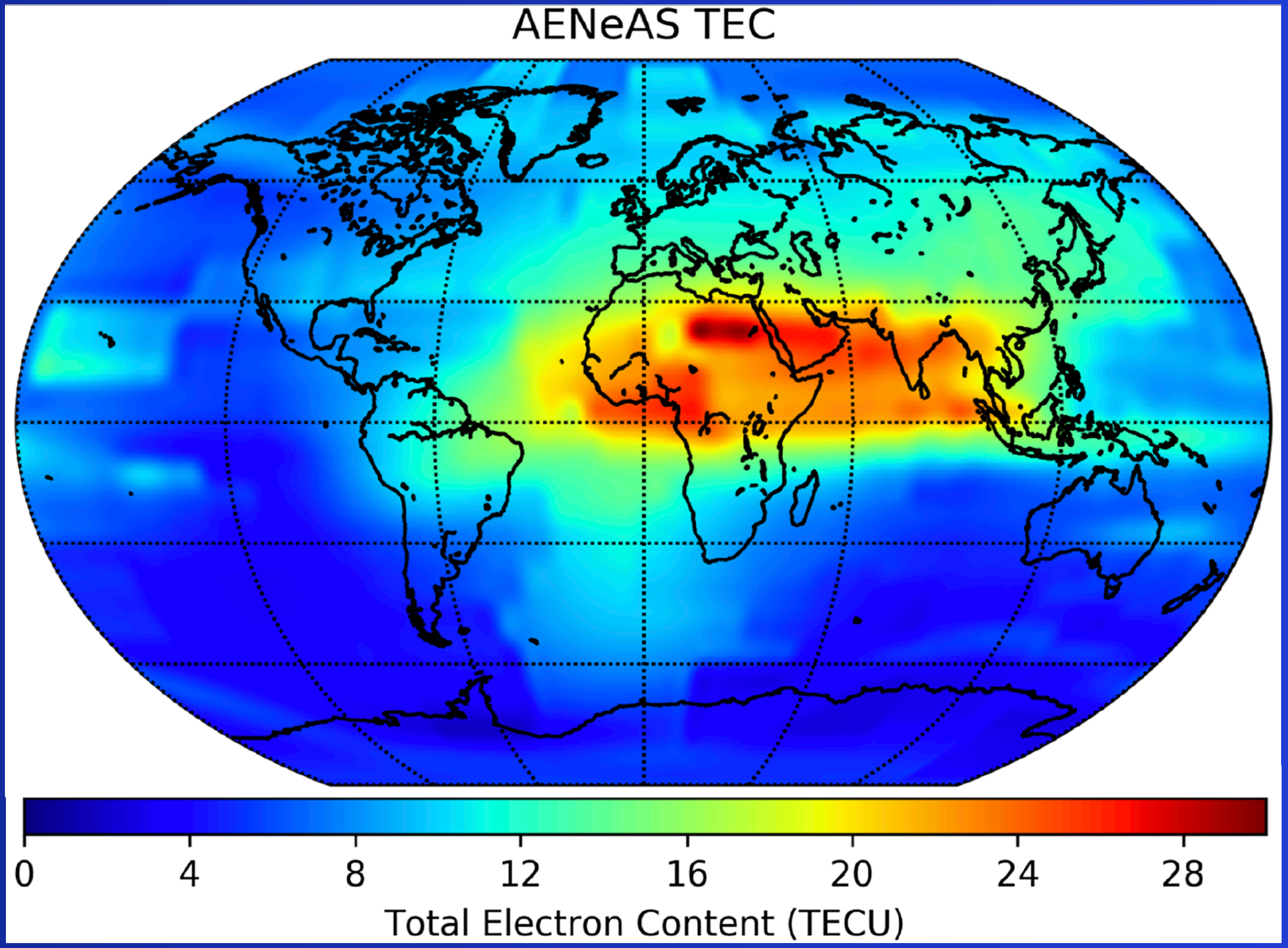
Assimilation schemes (example)

AENeAS - The Advanced Ensemble electron density (Ne) Assimilation System

- AENeAS is a physics-based DA model of the ionosphere/ thermosphere
    - It uses the local ensemble transform Kalman filter (LETKF) for the assimilation scheme.

- Background model
    - Thermosphere Ionosphere Electrodynamics General Circulation Model (TIE-GCM)

- The current ionospheric topside model used by AENeAS is NeQuick (to extend electron density grids to 25000 km)



TEC map for June 5th at 1230 from AENeAS (from: Elvidge and Angling, 2019)

Use of effective parameters

Vertical TEC maps data ingestion

Pignalberi, A., Pezzopane, M., Rizzi, R. Galkin, I., "Effective Solar Indices for Ionospheric Modeling: A Review and a Proposal for a Real-Time Regional IRI"; Surv Geophys (2018) 39: 125. https://doi.org/10.1007/s10712-017-9438-y.

vTEC map data ingestion

At a given epoch

One vTEC map

↓

Minimize the mismodelings

$|vTECexp_i - vTECmod(az)_i|$

↓

Az (effective F10.7) grid

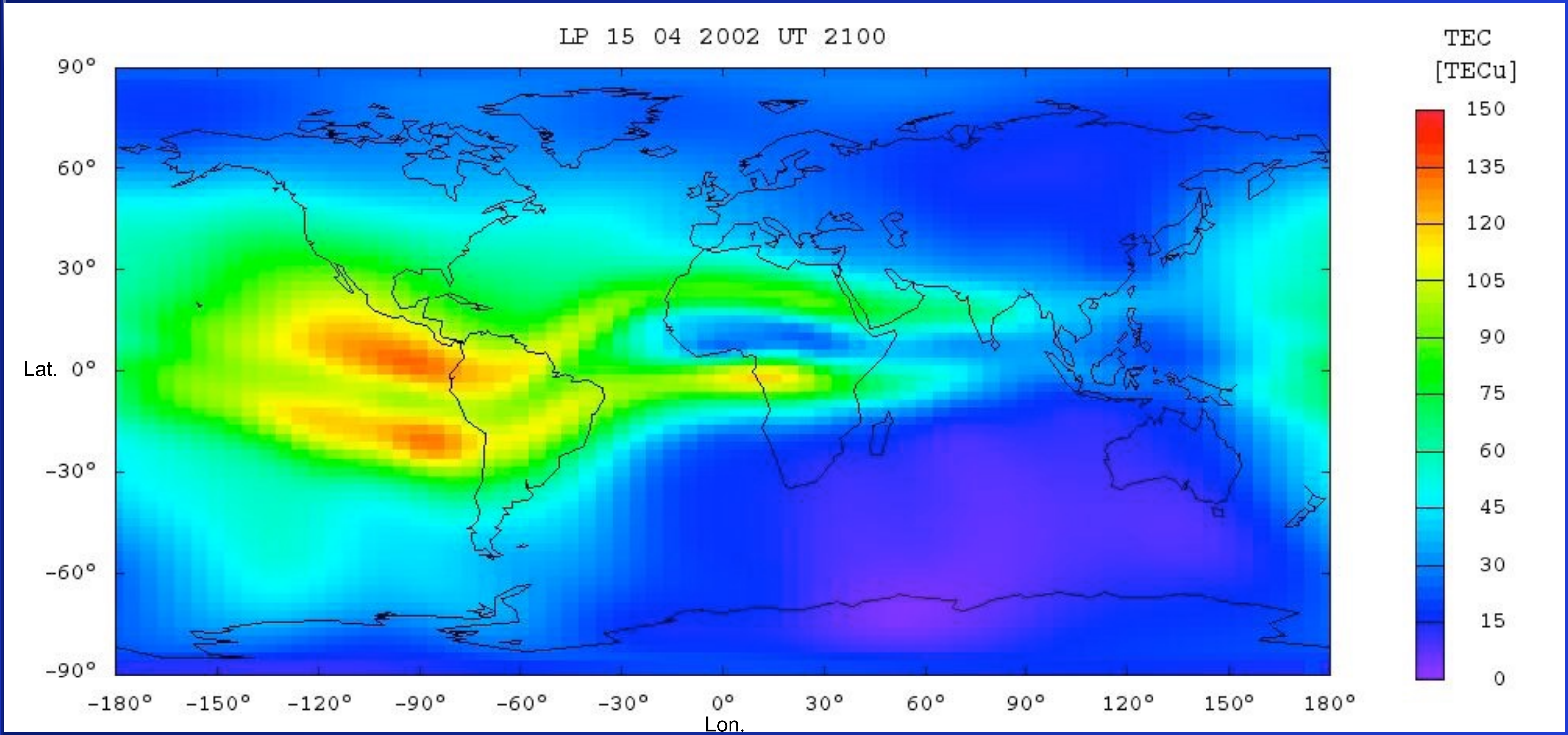
↓

Use NeQuick to reconstruct the 3D electron density of the ionosphere that reproduces the starting vTEC map

↓ Reconstruct sTEC along any ray-path

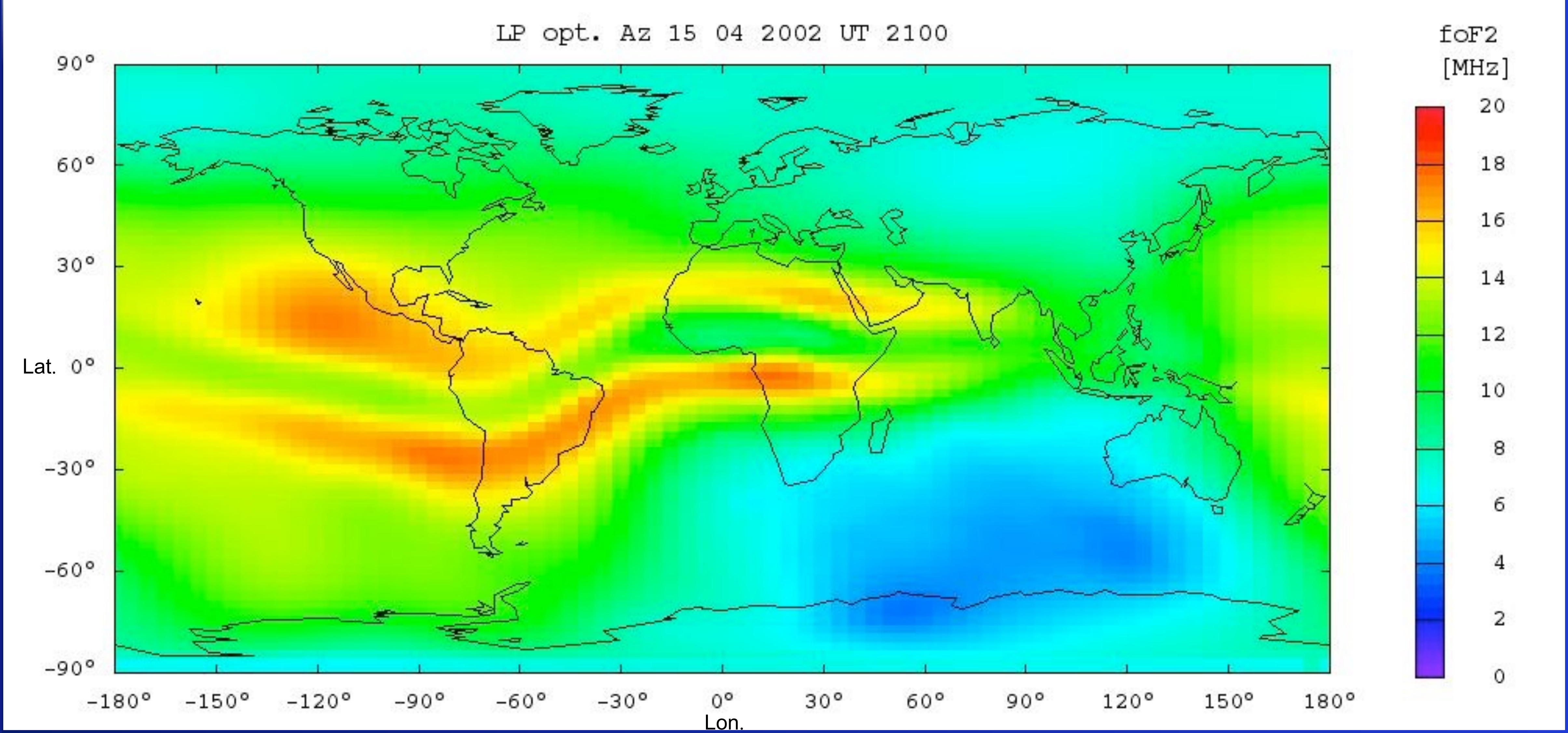
↓ Reconstruct foF2 maps

vTEC map



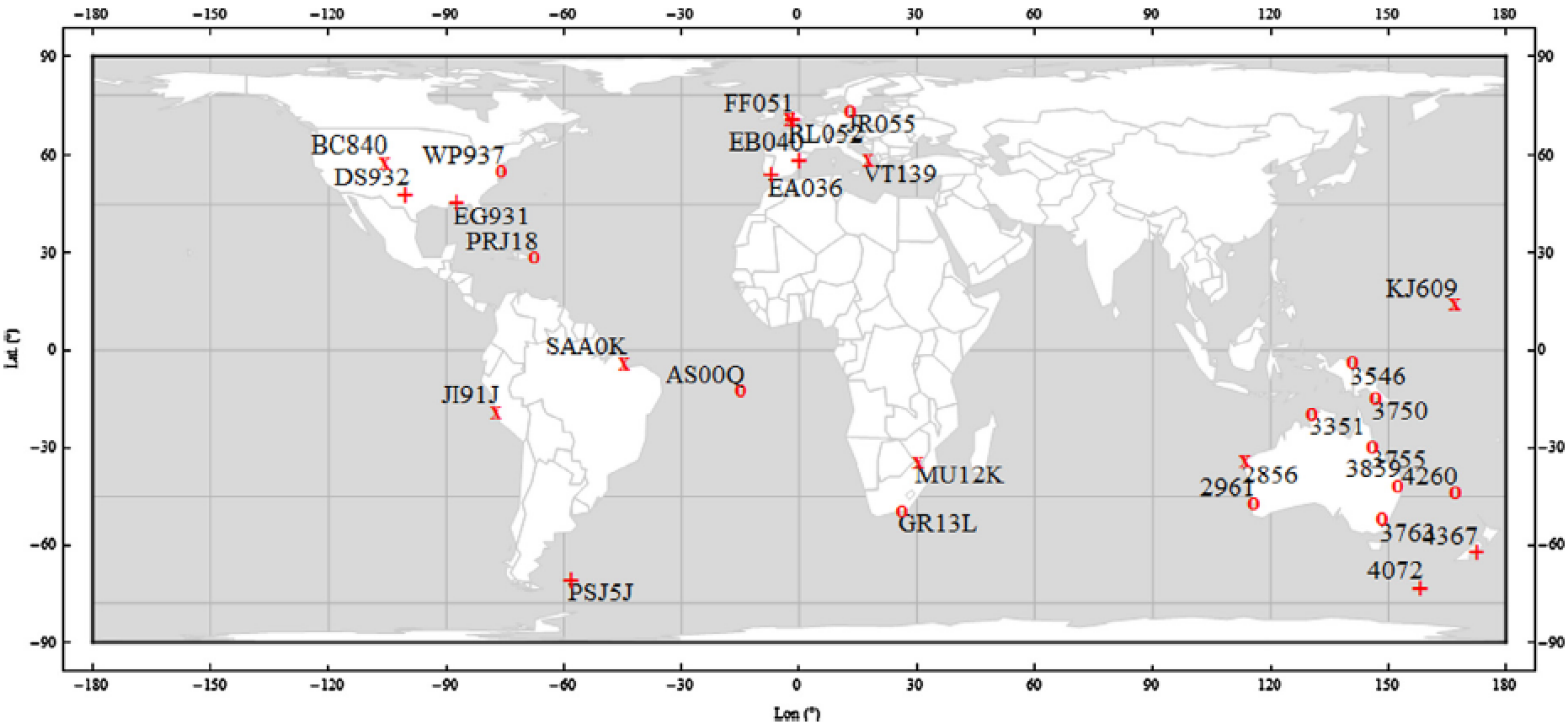
grid points:
lat.=-90°, 90° step 2.5°
lon.=-180°, 180° step 5°

Reconstructed foF2 map



grid points:
lat.=-90°, 90° step 2.5°
lon.=-180°, 180° step 5°

VTEC ingestion: validation example



Locations of the ionosonde stations used for the validation

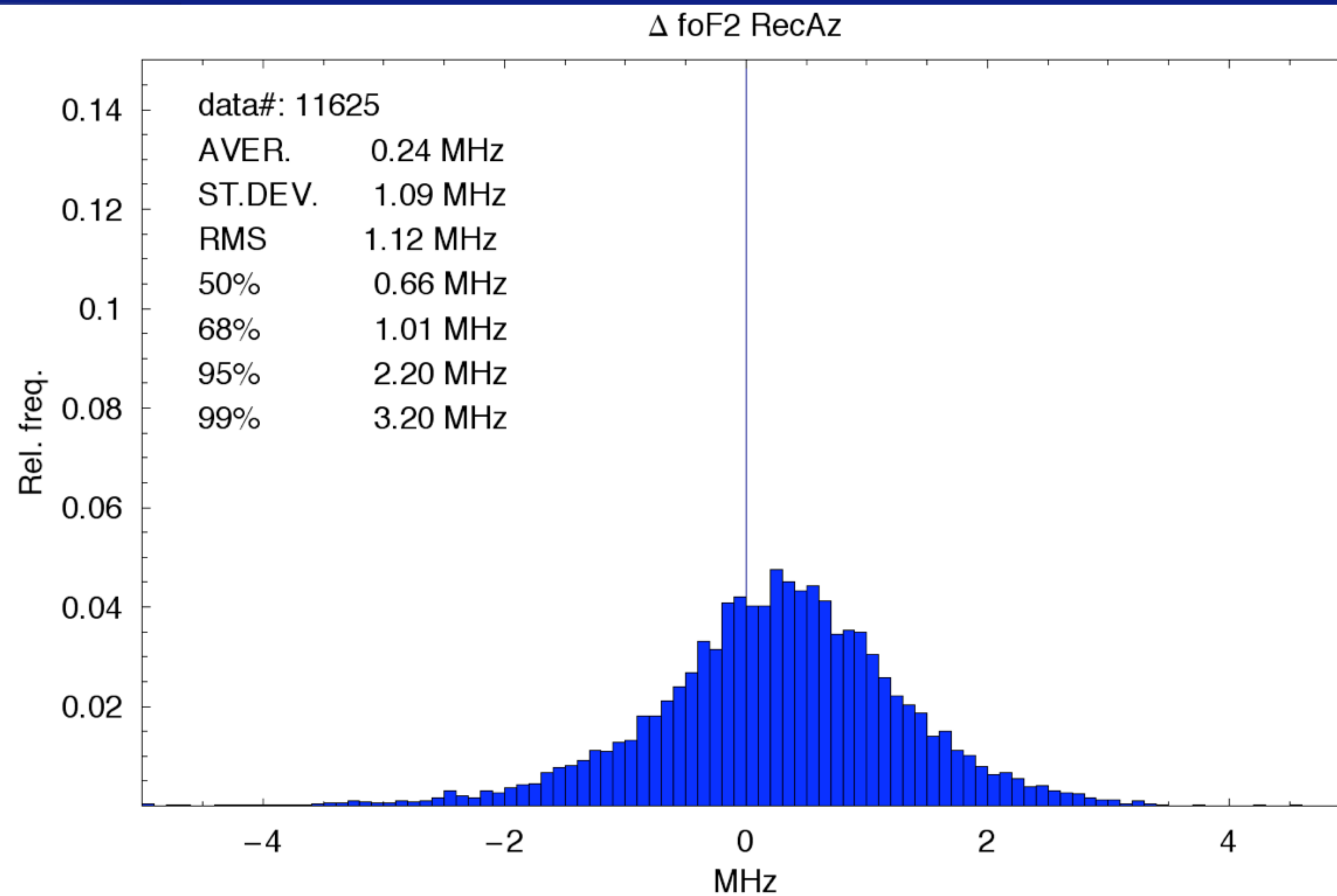
\+ April 2000

x September 2006

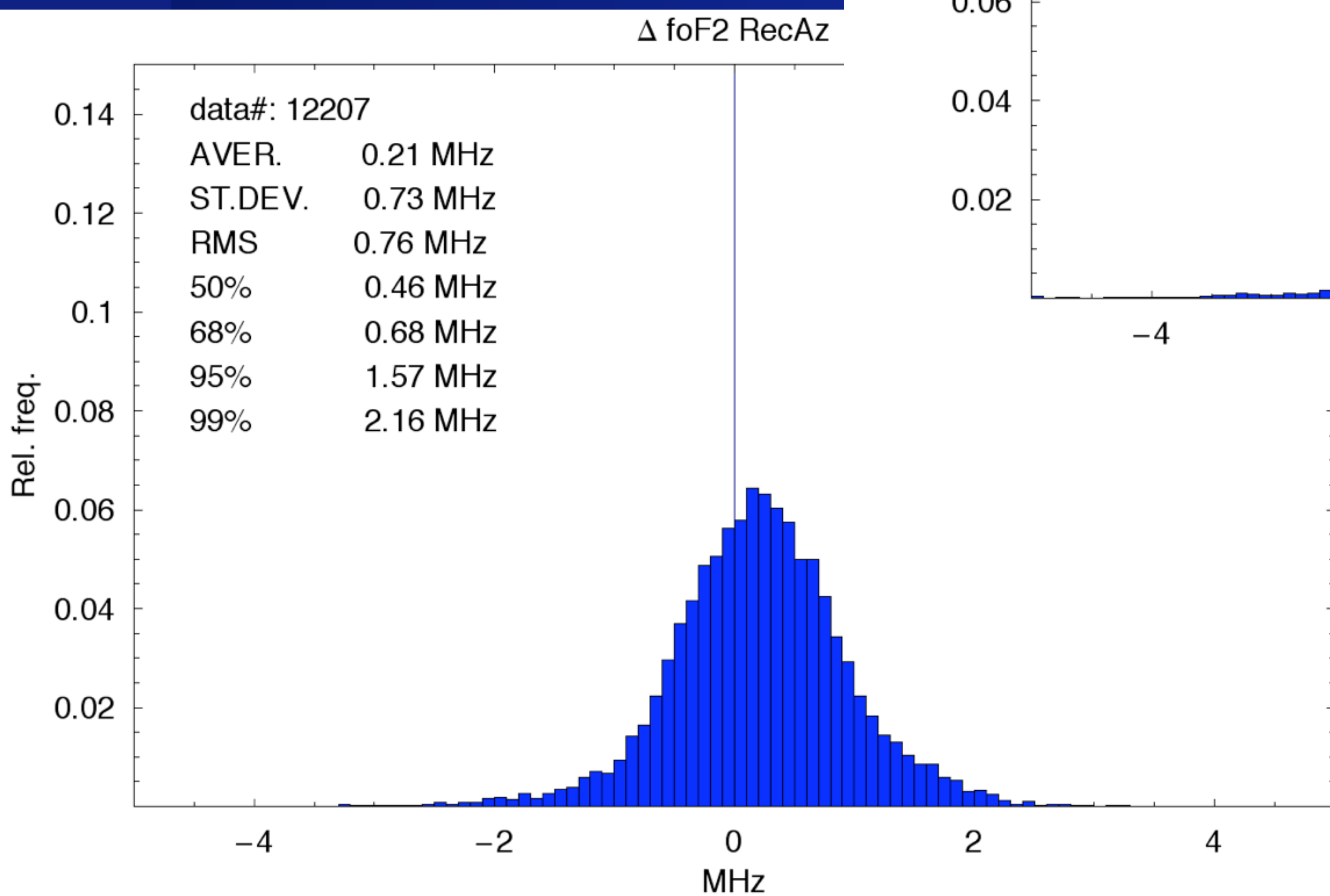
o April 2000 & September 2006

NeQuick2: validation results (effective F10.7)

Apr. 2000



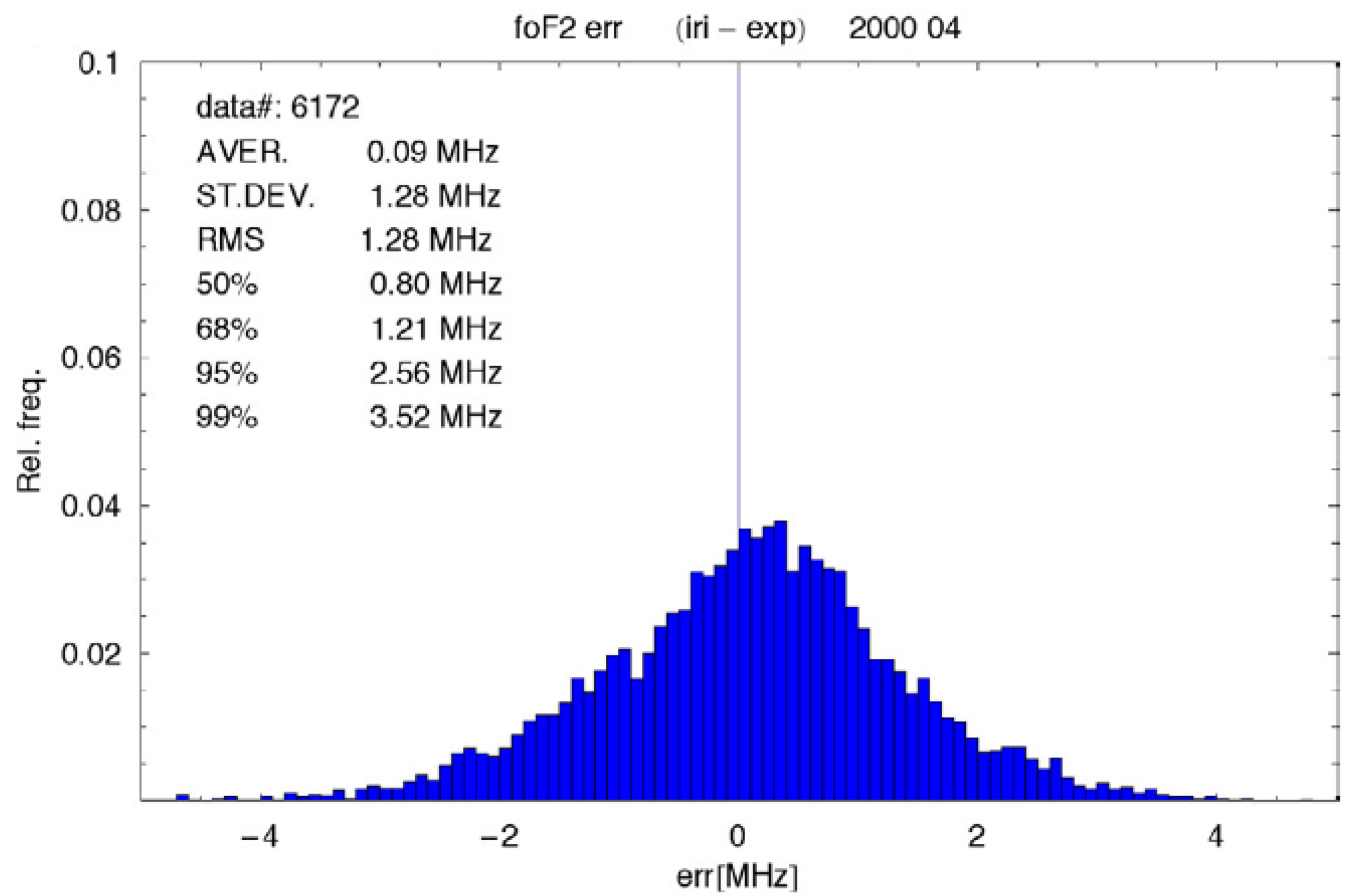
Sep. 2006



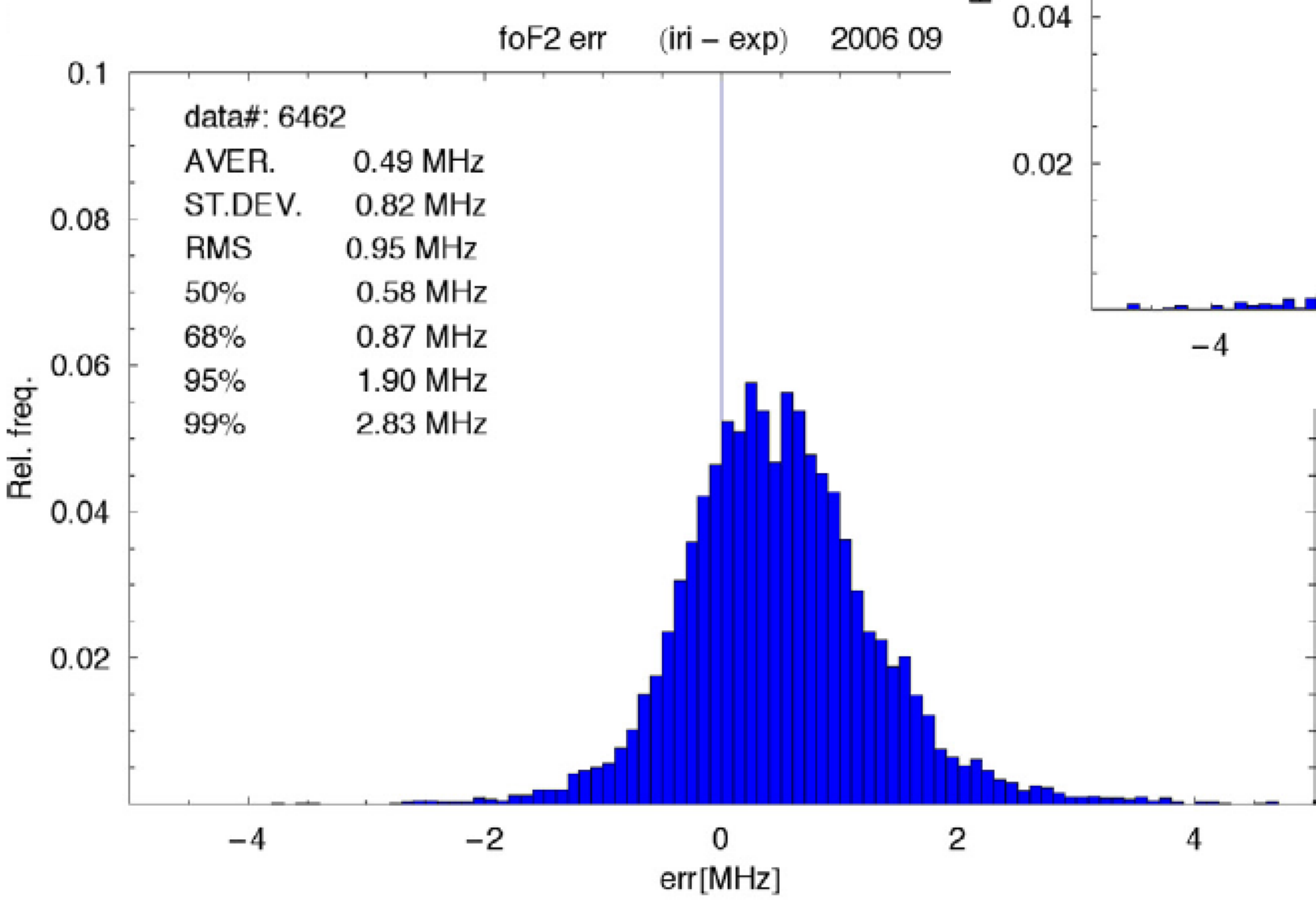
Nava, B., S. M. Radicella and F. Azpilicueta, "Data ingestion into NeQuick 2", (2011), Radio Sci., 46, RS0D17, doi:10.1029/2010RS004635

IRI 2012: validation results (effective F10.7)

Apr. 2000



Sep. 2006



Migoya Orue,Y., Nava, B., Radicella, S., Alazo, K., "GNSS derived TEC data ingestion into IRI 2012", (2015), Advances in Space Research 55, 1994–2002

Least Square Estimation

Least Square Estimation

To further improve the NeQuick performance in retrieving the 3D electron density of the Ionosphere, a minimum variance least-squares estimation has also been utilised to assimilate ground and space-based TEC data into NeQuick 2, considered as a background (like in e.g. Minkwitz et al., 2018).

Best Linear Unbiased Estimator (BLUE)*

y vector of observations

$\mathbf{x}_b$ background model state

$\mathbf{x}_a$ analysis model state

H observation operator

R covariance matrix of observation errors

B covariance matrix of background errors

A covariance matrix of analysis errors

*http://www.ecmwf.int/newsevents/training/rcourse_notes/DATA_ASSIMILATION/ASSIM_CONCEPTS/Assim_concepts2.html#962570

Least Square Estimation

- The following hypotheses are assumed:
  - *Linearized observation operator*: the variations of the observation operator in the vicinity of the background state are linear.
  - *Non-trivial errors*: B and R are positive definite matrices.
  - *Unbiased errors*: the expectation of the background and observation errors is zero.
  - *Uncorrelated errors*: observation and background errors are mutually uncorrelated.
  - *Linear analysis*: we look for an analysis defined by corrections to the background which depend linearly on background observation departures.
  - *Optimal analysis*: we look for an analysis state which is as close as possible to the true state in an r.m.s. sense (i.e. it is a minimum variance estimate).

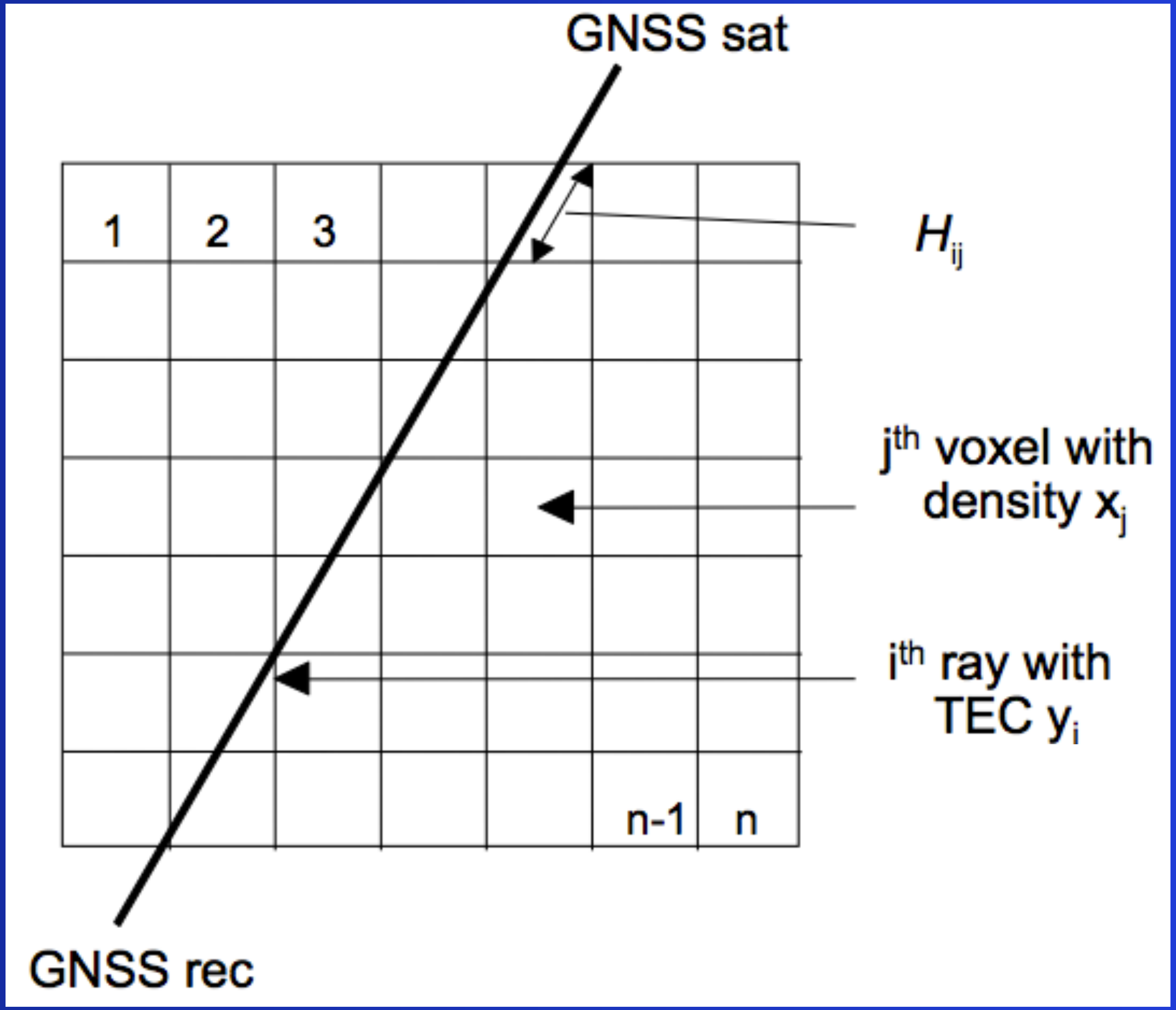
Least Square Estimation

The optimal least-square estimator (BLUE analysis) is defined by

$$\mathbf{x_a} = \mathbf{x_b} + \mathbf{K}\,(\mathbf{y} - \mathbf{Hx_b})$$

$$\mathbf{K} = \mathbf{BH^T(HBH^T + R)^{-1}}$$

$$\mathbf{A} = \mathbf{(I\text{-}KH)B}$$



K is called *gain* of the analysis

In our case:

y = TEC
$\mathbf{x_a}$ = retrieved electron density
$\mathbf{x_b}$ = background electron density
H -> "crossing lengths" in "voxels"

e.g. bckg_TEC = $\mathbf{Hx_b} = \sum_j H_{ij}\, x_{bj}$

Least Square Estimation

Notice:

The BLUE analysis is equivalently obtained as a solution to the variational optimization problem:

$$
\begin{aligned}
\mathbf{x}_a &= \mathrm{Arg}\ \min J \\
J(\mathbf{x}) &= (\mathbf{x}-\mathbf{x}_b)^T \mathbf{B}^{-1} (\mathbf{x}-\mathbf{x}_b) + (\mathbf{y}-H[\mathbf{x}])^T \mathbf{R}^{-1} (\mathbf{y}-H[\mathbf{x}]) \\
&= J_b(\mathbf{x}) + J_o(\mathbf{x})
\end{aligned}
$$

where J is called the cost function of the analysis

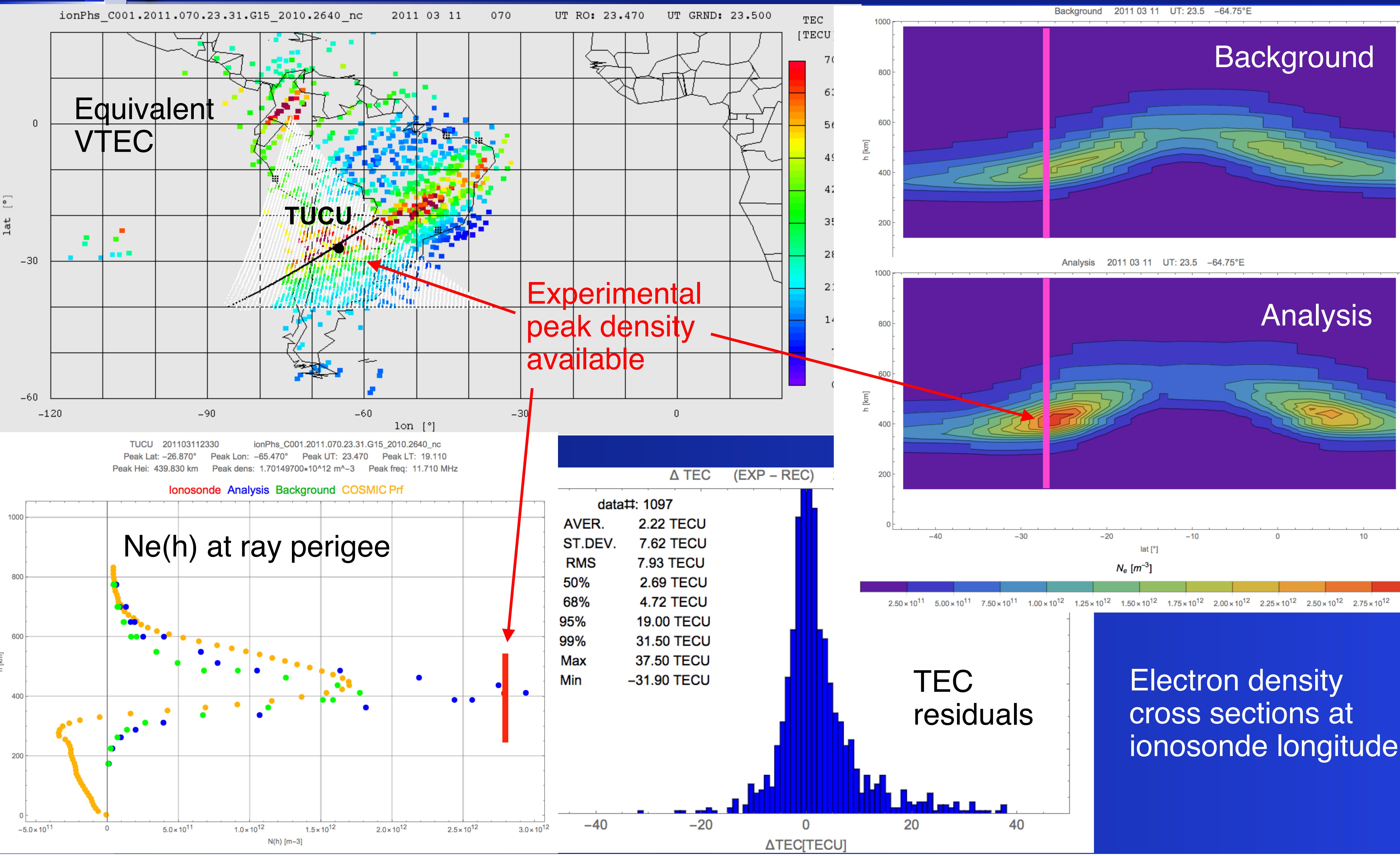
J_b is the background term

J_o is the observation term

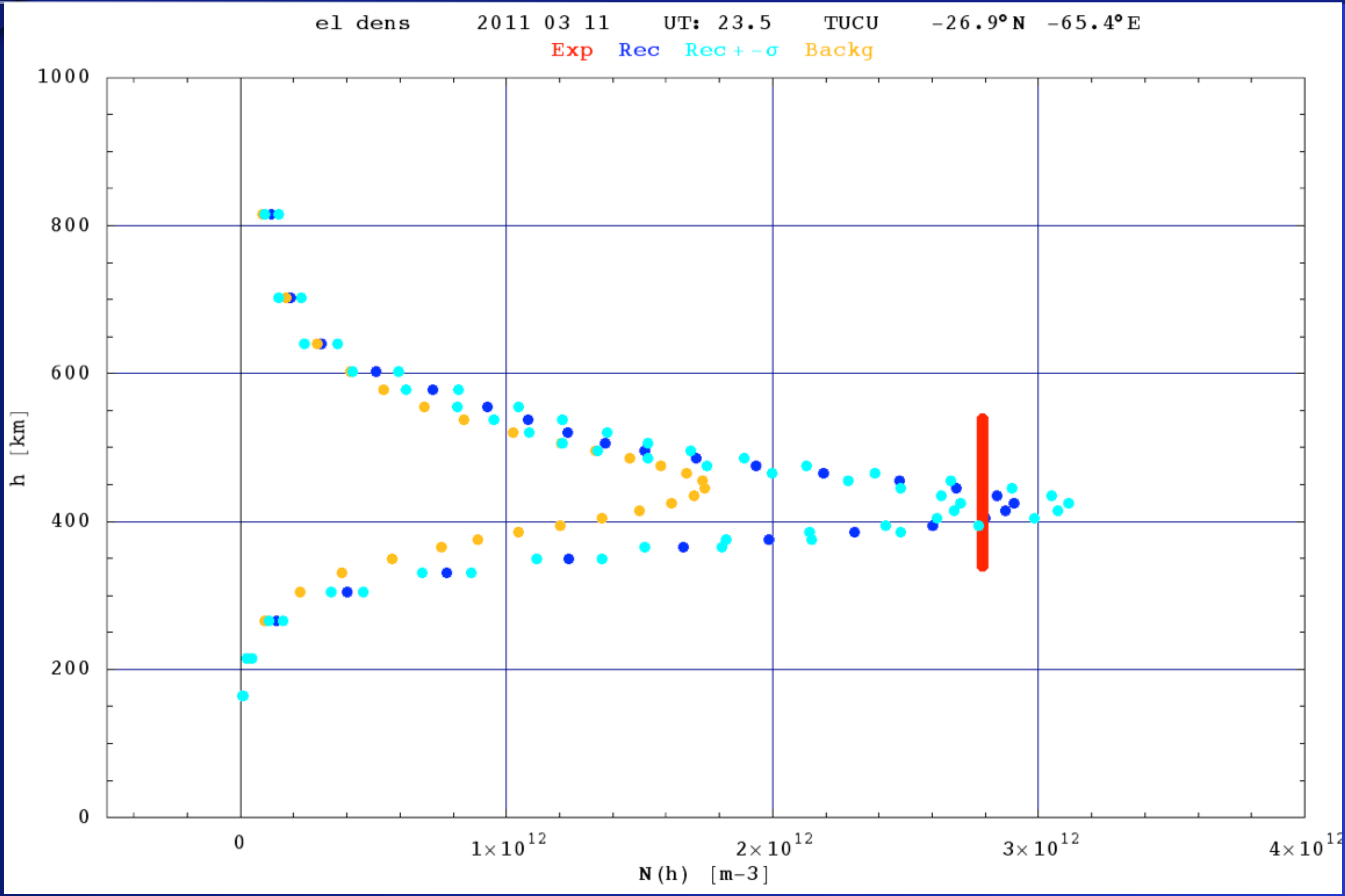
Data used (test case)

- For the assimilation
  - Ground-based GPS-derived slant TEC data provided by the Low Latitude Ionospheric Sensor Network (LISN)
  - Radio-Occultation-derived TEC data obtained by COSMIC (calibrated TEC values along the LEO−to−GPS link below the LEO orbit)
- For the validation
  - Manually scaled foF2 data obtained from the Tucuman Ionosonde
  - JRO electron density profiles

TEC DA into NeQuick

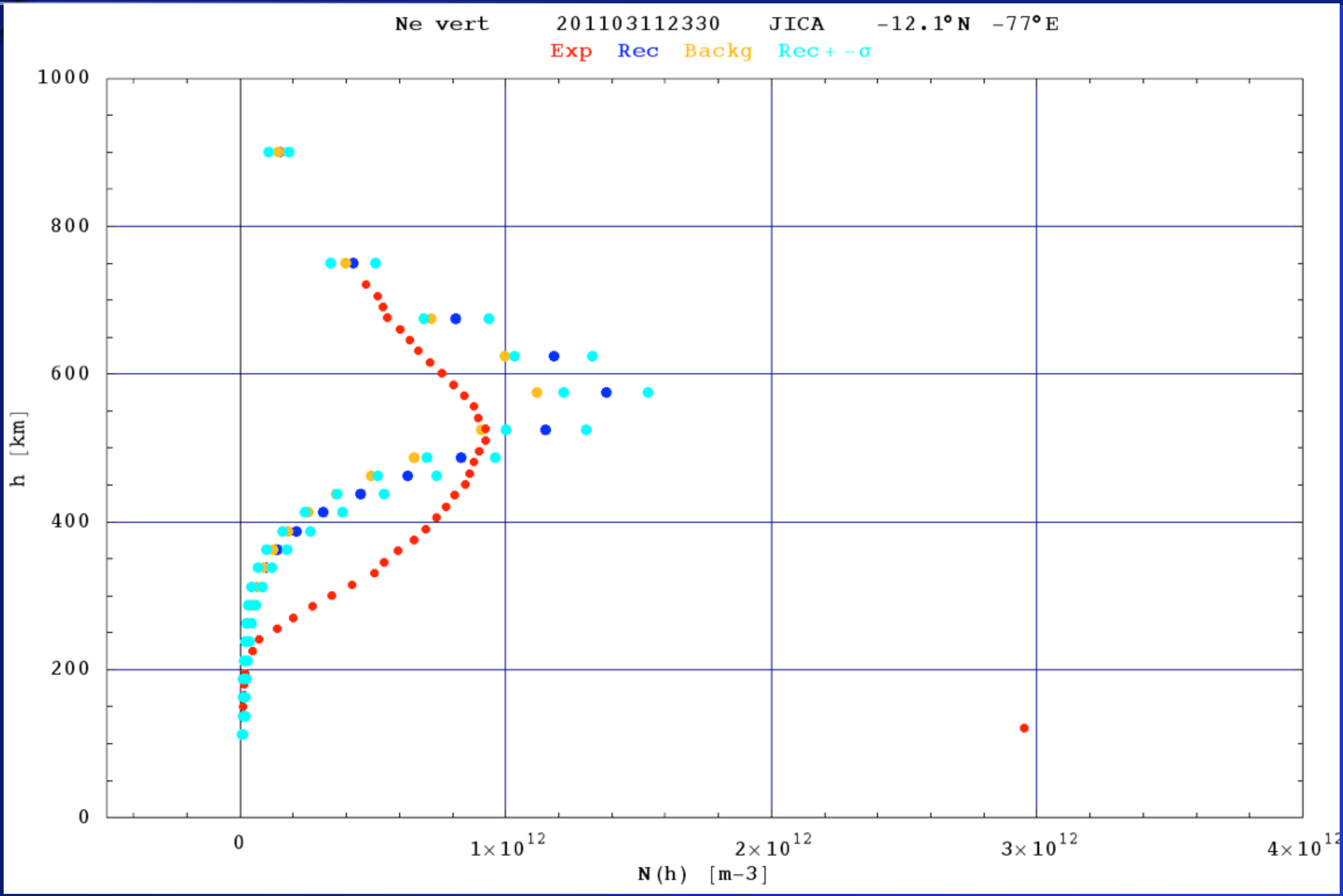


Method validation



Electron density profiles at Ionosonde location

Method validation



Electron density profiles at JRO location

Thank you for your attention