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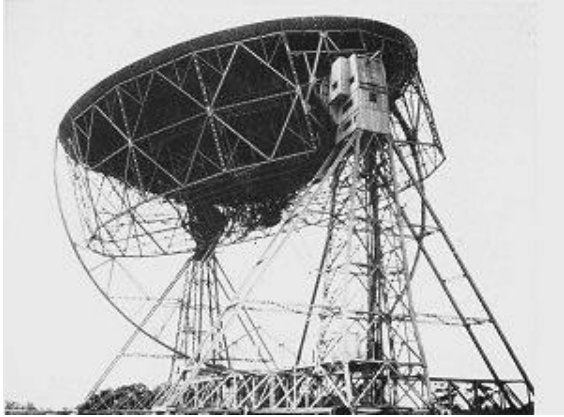
Sounding the ionosphere with GNSS - TEC calibration

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**Eastern Africa GNSS and
Space Weather Capacity Building Workshop (online)**

Kenya Space Agency, Pwani University, UNOOSA-ICG, Boston College, INGV
Trieste, Italy, 21 - 25 June 2021

How all started: The prehistory



Mark I Radiotelescope
(1957)



Sir Bernard Lovell

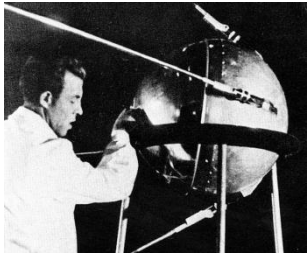
The first “satellite” used to study the ionosphere was
the moon!

- Pulsed radar transmissions at a frequency of 120 MHz reflected off the surface of the moon as a feasible technique for studying the ionosphere

=> I. C. Browne J. V. Evans, J. K. Hargreaves, W. A. S. Murray, **1956-57**.

- Slow signal amplitude fading on two closely spaced frequencies due to the ionospheric ***Faraday effect***

=> *integrated electron density* along the propagation path when the moon was in transit over Jodrell Bank Observatory near Manchester, England.



Sputnik 1



Sputnik 3

Space Age ~ 6 decades ago

The launch of artificial satellites starting with Sputnik 1 in September **1957** allowed the use of radio beacon transmissions from an spacecraft.

Geostationary satellites

In **1961** the US NASA started the SYNCOM (synchronous communication satellite) program with the launch of the first geostationary communication satellites.

Syncom 3 launched in August **1964** allowed the use of radio beacon on board of the spacecraft for the study of **TEC**, mainly by making use of the Faraday effect due to the presence of the ionospheric plasma in the presence of the geomagnetic field.

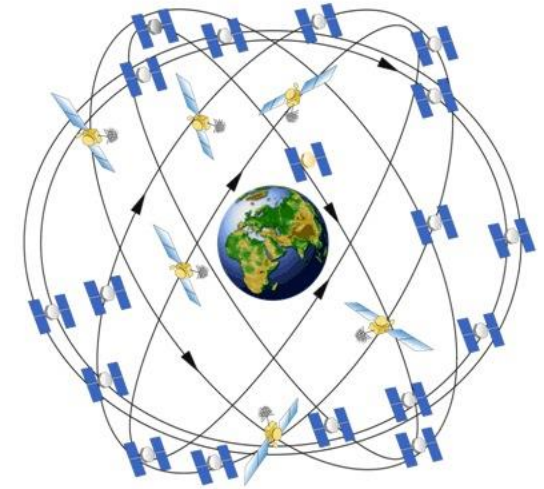
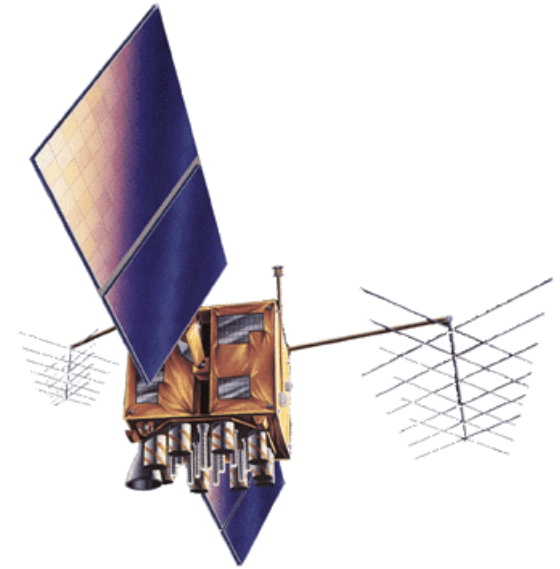


GPS (Global Positioning System)

- The project was launched by U.S. Department of Defense for **military use** (1973)

NAVSTAR (Navigation System with Timing And Ranging)

- eleven satellites in orbit (1978 - 1985)
- A full 24 satellite constellation was operational (1995)
- It was allowed for **civilian use** in the 1980s
- "Selective Availability" (signal quality degraded) **is off** (May 2000)
- precise, circular orbits at 20 200 km
- L1 = 1,575.42 MHz L2 = 1,227.60 MHz



the era of GNSS



- Globalnaya Navigazionnaya Sputnikovaya Sistema, "Global Navigation Satellite System" in English (**GLONASS**). **Declared operational in 1993 and brought to its optimal status of 24 operational satellites in 1995.**
- The People Republic of China **BeiDou** (Big Dipper or Ursa Major main stars in Chinese). Beidou is currently centred on the Asia Pacific region where provides positioning, navigation, timing, and short-message communication service capabilities. **The system is designed to give global coverage - 2020.**
- The European Commission and European Space Agency joined forces to build **Galileo**, a European global system under civilian control. **Galileo is fully operational, - 2020, with ~ 30 MEO satellites at an altitude of 23,222 km.**

“Constellation” of 32 + satellites

GPS NAVSTAR Global Positioning System

<http://www.gps.gov/technical/icwg/>

GLONASS *Globalnaya Navigatsionnaya Sputnikovaya Sistema*

<http://www.glonass-ianc.rsa.ru/en/>

<http://www.glonass-center.ru/en/>

GALILEO European Global Navigation Satellite System

<http://galileognss.eu/>

BeiDou China Navigation Satellite System

<http://en.beidou.gov.cn/>

NAVIC Navigation Indian Constellation

Former IRNSS Indian Regional Navigation Satellite System

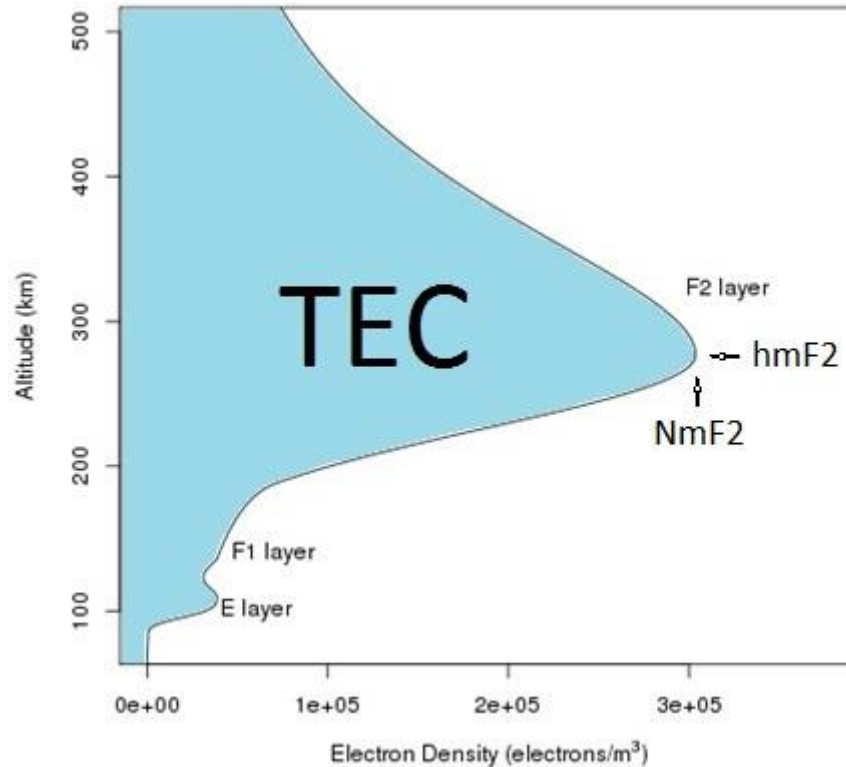
<http://www.isro.gov.in/>

Ionosphere and GNSS problem solving

Satellite derived TEC for ionospheric research

- How TEC is derived from GNSS
- preprocessing
- assumptions for modeling
- Sat by Sat vs Arc by Arc solution
- combining different constellations
- Multiconstellation TEC *single station solution*

Idealized Ionospheric profile



the total number of electrons in a given column of unit surface
along a path between two points

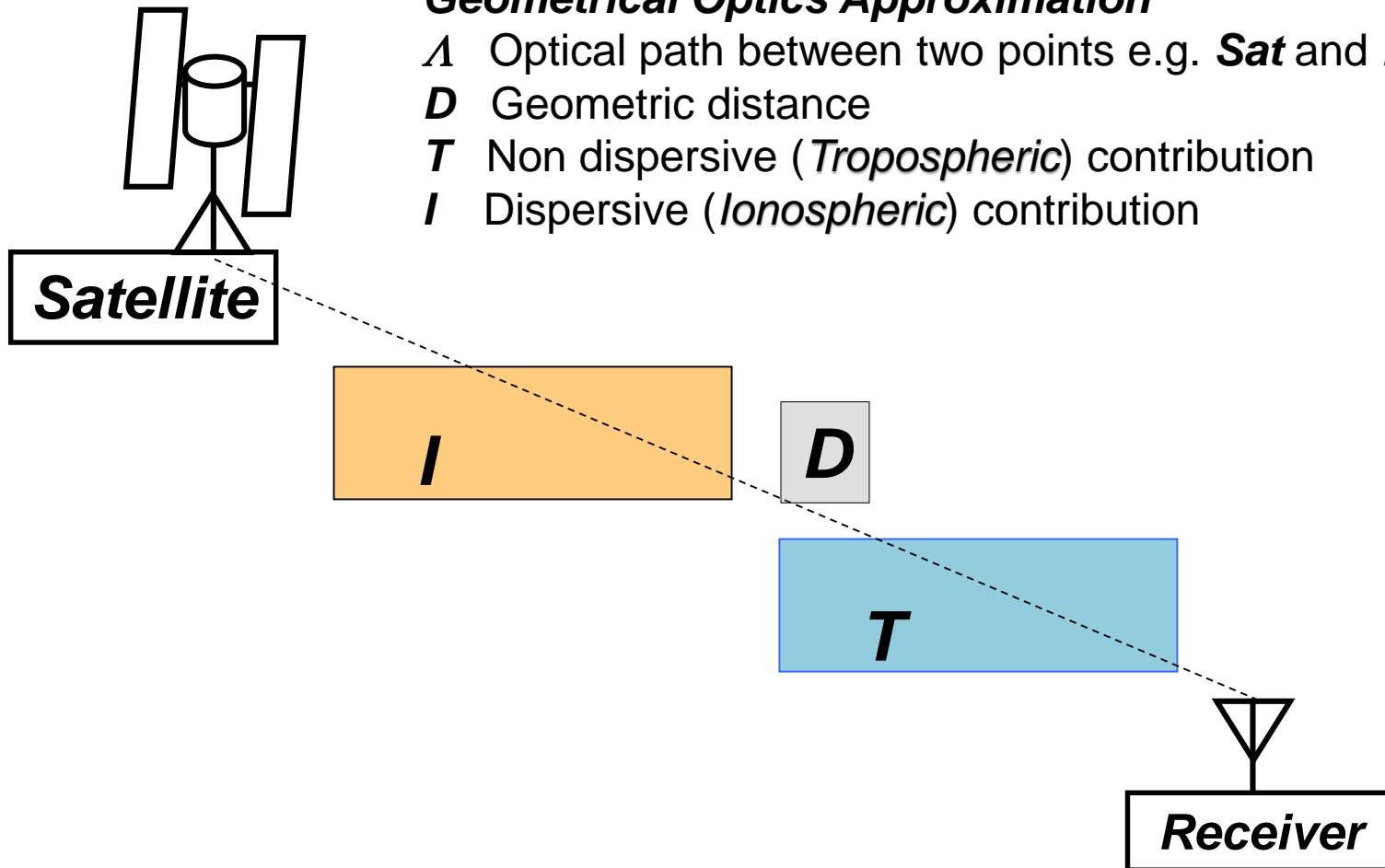
$$10^{16} \text{ electrons/m}^2 = 1 \text{ TEC unit (TECu)}$$

Propagation delays in the Optical path

$$\Delta = D + T + I$$

Geometrical Optics Approximation

- Δ Optical path between two points e.g. **Sat** and **Rec**
- D Geometric distance
- T Non dispersive (*Tropospheric*) contribution
- I Dispersive (*Ionospheric*) contribution



Propagation contribution to optical path L :

Refractivity $R = n - 1$, n Index of Refraction

$$I = \int R_{\text{iono}}(s) ds$$

$$TEC = \int N_e(s) ds,$$

after Magneto-Ionic Theory &

1st order Appleton-Hartree Formula

$$R_{\text{iono}} = -\frac{40.3 \cdot N_e}{f^2}$$

$$I = -\frac{40.3 \cdot TEC}{f^2}$$

➤ group velocity delay

$$L = \frac{D + T + I}{\lambda} = \frac{f}{c} (D + T) - \frac{40.3 TEC}{cf}$$

➤ carrier phase advance

$$G = \frac{dL}{df} = \frac{D + T}{c} + \frac{40.3 TEC}{cf^2}$$

How TEC is derived from GNSS

slants TEC from dual frequency combination

Differential delays in the optical path -> Geometry free combination

$$\Phi_{Ll} = \Phi_{L1} - \Phi_{L2} \Rightarrow$$

$$S_L = \alpha_{f12} (\phi_{L1} c/f_1 - \phi_{L2} c/f_2)$$

$$S_L = sTEC + \Omega$$

$$P_{Ll} = P_{L2} - P_{L1} \Rightarrow$$

$$S_c = \alpha_{f12} (\rho_{L2} - \rho_{L1})$$

$$S_c = sTEC + \beta + \gamma + n + m$$

$\alpha_f = 40.3d16 / f^2$ conversion factor
between the integrated electron density
along the ray path $sTEC$ and the signal
delay at frequency f

$$\alpha_{f12} = 1.0 / (40.3d16 * (1/f_2^2 - 1/f_1^2))$$

Ω differential offset

β, γ differential biases receiver, satellite

n, m noise and multipath

Preprocessing

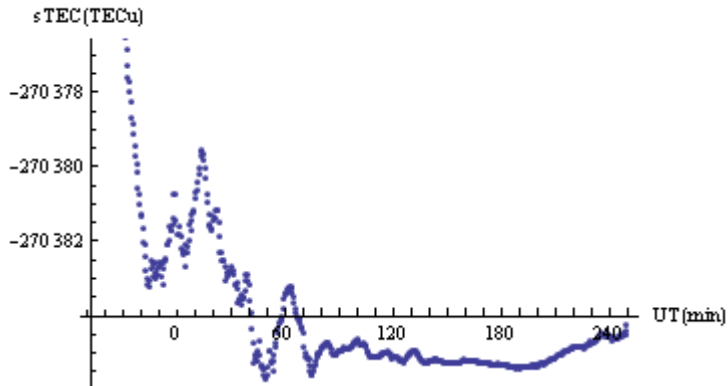
A series of statistical/mathematical considerations are applied to the RINEX data in order to correct for phase jumps and cycle slips.

A NOT AVOIDABLE stage for all GPS data processing that uses the phase observable; in particular a PRE-step to any GPS-TEC calibration method TO OBTAIN A HIGH QUALITY RESULT

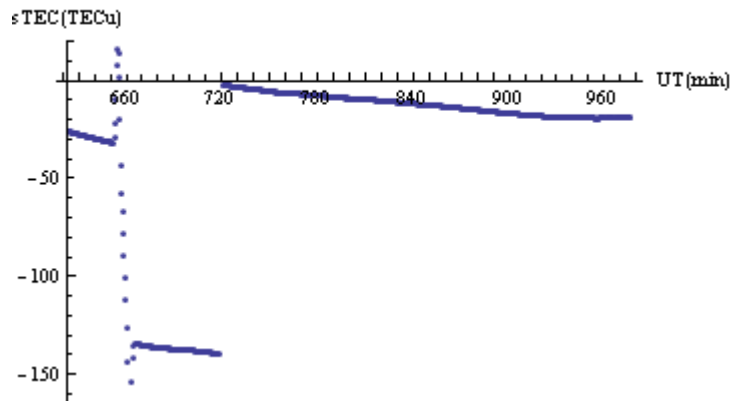
Cycle slips

Discontinuities of an integer number of cycles in the measured carrier phase resulting from a **temporary loss of lock** in the carrier tracking loop of a GPS receiver.

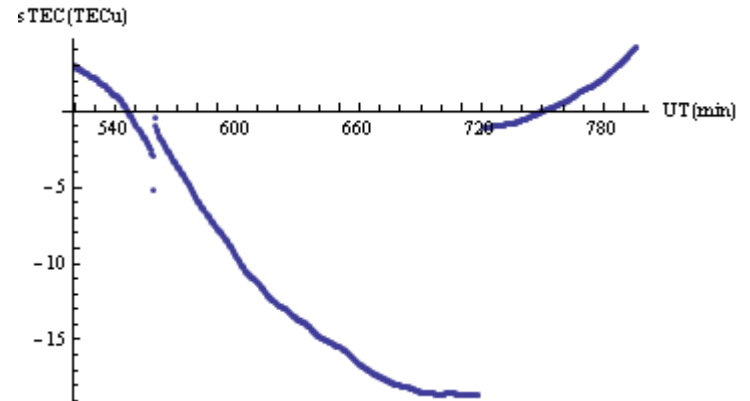
272/2004 PRN 19



245/2004 PRN 24



245/2004 PRN 7



Causes

*Spatial and temporal Ionospheric irregularities that causes rapid GPS phase and pseudorange variations, i.e. ionospheric scintillation.

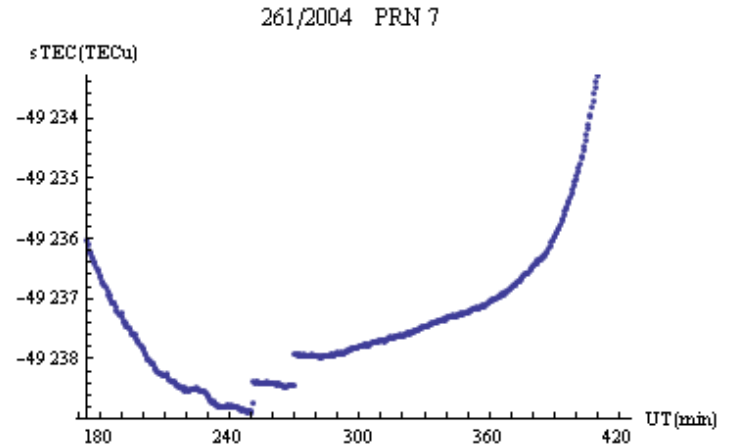
*Relative strong multipath environment of the receiver, obstruction of the satellite signal by physical obstacles.

*A low signal to noise ratio (SNR) or alternatively carrier to noise power density ratio (C/No) due to disturbed ionosphere, low satellite elevation angles or multipath.

*Receiver software malfunctioning, etc

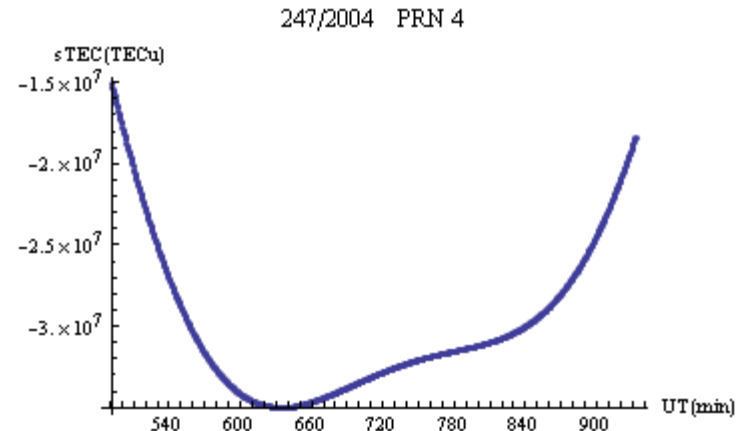
What should any cycle slip correction technique do

- * It must be correctly detected and identified
=> **location** of the jump and their **size**.
- * It must be removed or corrected by another value
=> **estimate** the number of L1/L2 frequency cycles contained by the jump and then **correct** the phase cycle by these integer estimates



What to do in practice with these errors

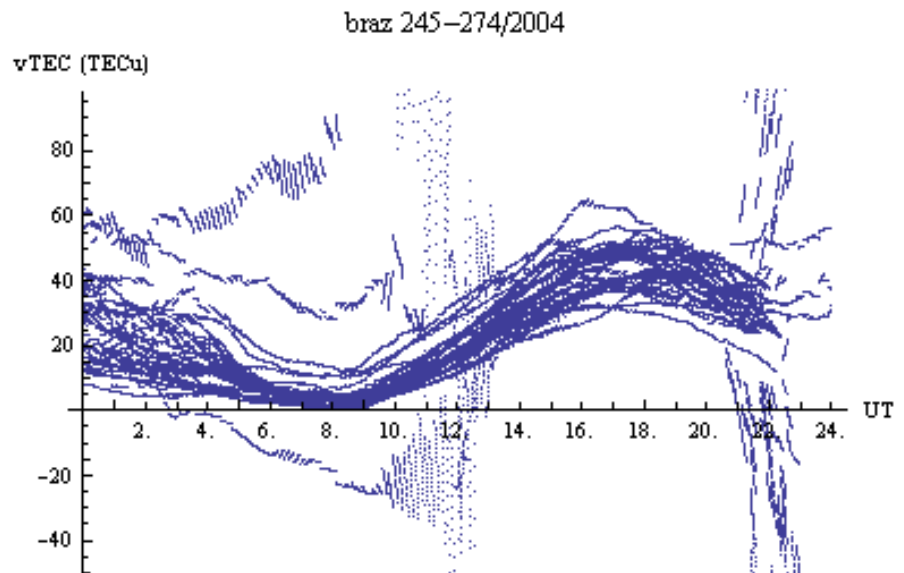
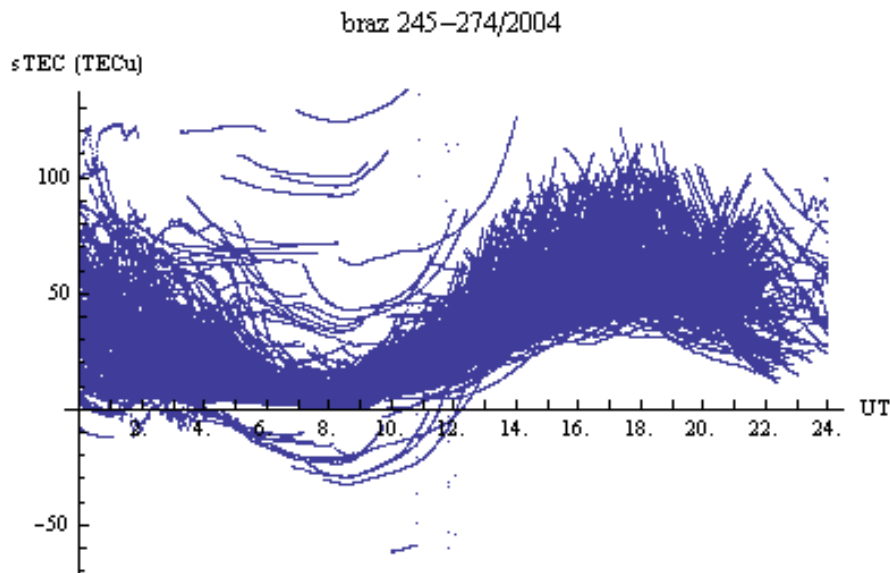
- * Remember that any/several errors may be present together.
- * Select a method for detecting and identifying the discontinuities in the data arcs.
- * Combine 1+ methods



We suggest that bad multipath conditions and high cycle slip rates are better addressed by an appropriate selection of field equipment and sites than by data processing techniques.
Blewitt G., (1990). An automatic editing algorithm for GPS data. Geophysical Res. Lett., 17:3, pp 199-202.

What we must particularly be concerned is the data arc quality

- *** arc continuity, i.e. no cycle slips or phase jumps
- *** arc time extent : to have an appropriate spatial and temporal representation of the un-calibrated slant TEC by such arc
- *** to control the size of time gaps in the arc : otherwise the recovery cycle slip algorithm would have more possibility to fail



some SV excluded from preprocessing

How TEC is derived from GNSS

calibrating slants TEC

sTEC=MappingFunction(vTEC)
vTEC=vTEC(Geometry)

Leveling errors
usually disregarded

$$S_{ijt} = sTEC_{ijt} + \beta_i + \gamma_j + (\lambda_{Arc}) + \epsilon_{ijt}$$

uncalibrated sTEC
from GNSS observables

assumed as
constants

residuals

Assumptions

- Ionosphere is horizontally homogeneous, locally.
- Ionosphere could be represented by a **Thin Shell Model**. The THM altitude is around the F2 peak height. The satellites position are represented in the Ionospheric Pierce Points. THM of (1-n) layers.
- **Ionosphere is slowly varying**. The vTEC could be represented by a function during a "Refreshing Interval" \sim 10 min to 2 hours.
- **Receiver and satellite Biases are geometry independent**. The vTEC could be represented by a 2-D (3-D) function of geometry of the IPPs.

$$S = TEC + B\beta_{arc}$$

$$\beta_{iarc} = \beta_i + \gamma_j + (\lambda_{Arc})$$

Refreshing interval

Refreshing interval = 10 minutes

10 minutes => 20 epochs
4 satellites => 4 arcs

$\sim 4 * 20 \Rightarrow 80$ equations

r_1 interval

$$\left. \begin{aligned} usTEC_1 &= sTEC_1 + \beta_1 \\ usTEC_2 &= sTEC_2 + \beta_2 \\ usTEC_3 &= sTEC_3 + \beta_3 \\ usTEC_4 &= sTEC_4 + \beta_1 \\ usTEC_5 &= sTEC_5 + \beta_2 \\ usTEC_6 &= sTEC_6 + \beta_3 \\ usTEC_7 &= sTEC_7 + \beta_1 \\ usTEC_8 &= sTEC_8 + \beta_2 \\ usTEC_9 &= sTEC_9 + \beta_3 \end{aligned} \right\}$$

r_t interval

$$\left. \begin{aligned} \dots \\ usTEC_i &= sTEC_i + \beta_1 \\ usTEC_{i+1} &= sTEC_{i+1} + \beta_2 \\ usTEC_{i+2} &= sTEC_{i+2} + \beta_3 \\ usTEC_{i+3} &= sTEC_{i+3} + \beta_4 \\ usTEC_{i+4} &= sTEC_{i+4} + \beta_2 \\ usTEC_{i+5} &= sTEC_{i+5} + \beta_3 \\ usTEC_{i+6} &= sTEC_{i+6} + \beta_4 \end{aligned} \right\}$$

r_R last interval

$$\left. \begin{aligned} \dots \\ usTEC_{78} &= sTEC_{78} + \beta_2 \\ usTEC_{79} &= sTEC_{79} + \beta_3 \\ usTEC_{80} &= sTEC_{80} + \beta_4 \end{aligned} \right\}$$

$$S = TEC + B\beta_{arc}$$

$$\beta_{iarc} = \beta_i + \gamma_j + (\lambda_{Arc})$$

$$TEC = A \times C$$

$$vTEC = vTEC(\text{geometry})$$

e.g. 2-D THM

$$vTEC = c_0 + c_1 dx + c_2 dy$$

$$dx = \text{lat}_{IPP} - \text{lat}_0$$

$$dy = \text{lon}_{IPP} - \text{lon}_0$$

$(\text{lat}_0, \text{lon}_0)$ Station coordinates

r_t interval

$$\begin{bmatrix} sTEC_1 \\ sTEC_2 \\ sTEC_3 \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ sTEC_l \\ sTEC_{l+1} \\ sTEC_{l+2} \\ sTEC_{l+3} \\ sTEC_{l+4} \\ \cdot \\ \cdot \\ sTEC_{L-2} \\ sTEC_{L-1} \\ sTEC_L \end{bmatrix} = A \times \begin{bmatrix} c_{10} \\ c_{11} \\ c_{12} \\ c_{20} \\ c_{21} \\ c_{22} \\ \cdot \\ \cdot \\ \cdot \\ c_{t0} \\ c_{t1} \\ c_{t2} \\ \cdot \\ \cdot \\ \cdot \\ c_{T0} \\ c_{T1} \\ c_{T2} \end{bmatrix} + B \times \beta$$

coefficients matrix **C**

$$S = TEC + B\beta_{arc}$$

$$\beta_{iarc} = \beta_i + \gamma_j + (\lambda_{Arc})$$

mapping function

e.g. $vTEC = sTEC \cos \chi$

$$sTEC_r = \begin{bmatrix} sTEC_1 & sat_1_at_epoch=t \\ sTEC_2 & sat_2_at_epoch=t \\ sTEC_3 & sat_3_at_epoch=t \\ sTEC_4 & sat_1_at_epoch=t+30sec \\ sTEC_5 & sat_2_at_epoch=t+30sec \\ sTEC_6 & sat_3_at_epoch=t+30sec \\ \vdots & \vdots \\ sTEC_N & all_sats_at_R_r=10min \end{bmatrix}$$

uncalibrated sTEC matrix **S**

$$A_r = \begin{bmatrix} 1/\cos \chi_1 & dx/\cos \chi_1 & dy/\cos \chi_1 \\ 1/\cos \chi_2 & dx/\cos \chi_2 & dy/\cos \chi_2 \\ 1/\cos \chi_3 & dx/\cos \chi_3 & dy/\cos \chi_3 \\ 1/\cos \chi_4 & dx/\cos \chi_4 & dy/\cos \chi_4 \\ 1/\cos \chi_5 & dx/\cos \chi_5 & dy/\cos \chi_5 \\ 1/\cos \chi_6 & dx/\cos \chi_6 & dy/\cos \chi_6 \\ \vdots & \vdots & \vdots \\ 1/\cos \chi_N & dx/\cos \chi_N & dy/\cos \chi_N \end{bmatrix}$$

mapping function matrix **A**

$$\begin{bmatrix} sTEC_{r=1} \\ sTEC_{r=2} \\ sTEC_{r=3} \\ \vdots \\ \vdots \\ sTEC_{R-2} \\ sTEC_{R-1} \\ sTEC_R \end{bmatrix} = \begin{bmatrix} A_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & A_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & A_3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & A_{R-2} & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & A_{R-1} & \vdots & \vdots \\ 0 & 0 & 0 & 0 & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & A_R & \vdots \end{bmatrix} \times C + B \times \beta$$

$$S = TEC + B\beta_{arc}$$

$$\beta_{iarc} = \beta_i + \gamma_j + (\lambda_{Arc})$$

10 min

$$\begin{bmatrix} usTEC_1 \\ usTEC_2 \\ usTEC_3 \\ usTEC_4 \\ usTEC_5 \\ usTEC_6 \\ usTEC_7 \\ usTEC_8 \\ usTEC_9 \\ usTEC_{10} \\ usTEC_{11} \\ usTEC_{12} \\ usTEC_{13} \\ usTEC_{14} \\ usTEC_{15} \\ \vdots \\ usTEC_{L-2} \\ usTEC_{L-1} \\ usTEC_L \end{bmatrix} = A \times C + \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \cdot & \cdot & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \cdot & \cdot & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \cdot & \cdot & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & \cdot & \cdot & \cdot & \cdot & \cdot & 0 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & 0 \\ 0 & 1 & 0 & 0 & \cdot & \cdot & \cdot & \cdot & \cdot & 0 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & 0 \\ 0 & 0 & 1 & 0 & \cdot & \cdot & \cdot & \cdot & \cdot & 0 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & 0 \\ 1 & 0 & 0 & 0 & \cdot & \cdot & \cdot & \cdot & \cdot & 0 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & 0 \\ 0 & 1 & 0 & 0 & \cdot & \cdot & \cdot & \cdot & \cdot & 0 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & 0 \\ 0 & 0 & 1 & 0 & \cdot & \cdot & \cdot & \cdot & \cdot & 0 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & 0 \\ 0 & 0 & 0 & 1 & \cdot & \cdot & \cdot & \cdot & \cdot & 0 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & 0 \\ 0 & 1 & 0 & 0 & \cdot & \cdot & \cdot & \cdot & \cdot & 0 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & 0 \\ 0 & 0 & 1 & 0 & \cdot & \cdot & \cdot & \cdot & \cdot & 0 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & 0 \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ 0 & 0 & 0 & 0 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \cdot & \cdot & 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} \beta_1 \\ \beta_2 \\ \beta_3 \\ \beta_4 \\ \beta_5 \\ \beta_6 \\ \beta_7 \\ \beta_8 \\ \beta_9 \\ \beta_{10} \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \beta_{narcs-2} \\ \beta_{narcs-1} \\ \beta_{narcs} \end{bmatrix}$$

arcs matrix **B**

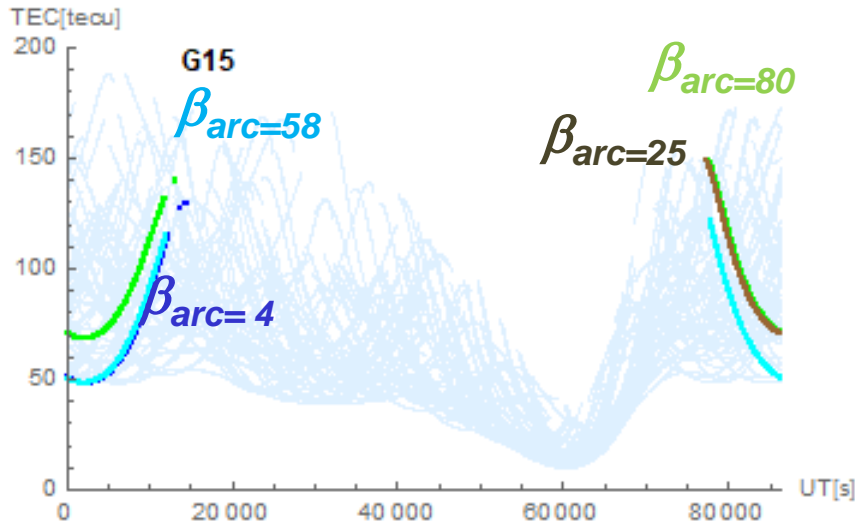
$$S = TEC + B\beta_{arc}$$

$$\beta_{iarc} = \beta_i + \gamma_j + (\lambda_{Arc})$$

ARC dependent biases

$$(\lambda_{Arc}) \neq 0$$

ftna.GREC 100-101-102/2018



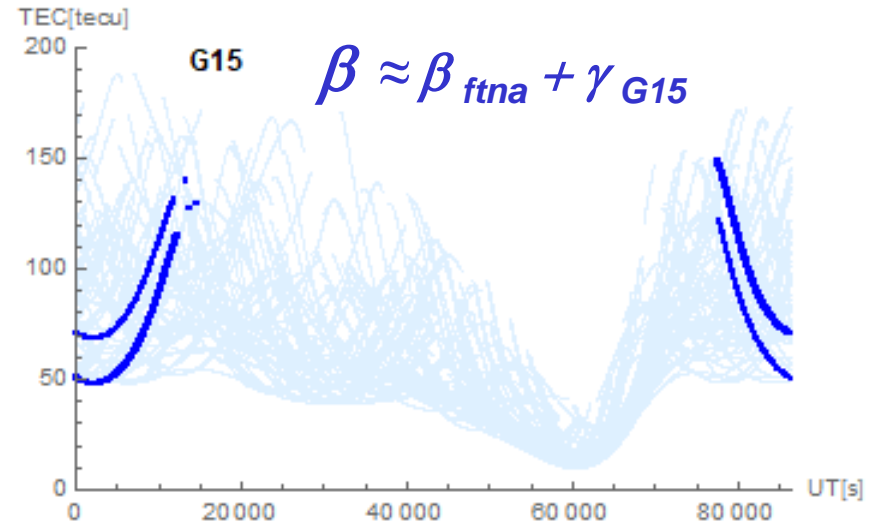
$$\beta_{arc=4} \neq \beta_{arc=25} \neq \beta_{arc=58} \neq \beta_{arc=80}$$

Treated as independent variables

Rec + Sats dependent biases

$$(\lambda_{Arc}) = 0$$

ftna.GREC 100-101-102/2018



$$\begin{aligned} \beta_{arc=4} &= \beta_{ftna} + \gamma_{G15} + (\lambda_{arc=4}) \\ \beta_{arc=25} &= \beta_{ftna} + \gamma_{G15} + (\lambda_{arc=25}) \\ \beta_{arc=58} &= \beta_{ftna} + \gamma_{G15} + (\lambda_{arc=58}) \\ \beta_{arc=80} &= \beta_{ftna} + \gamma_{G15} + (\lambda_{arc=80}) \end{aligned}$$

$$\beta \approx \beta_{arc=4} \approx \beta_{arc=25} \approx \beta_{arc=58} \approx \beta_{arc=80}$$

Treated as a same variable

One Constellation(GPS)

$$S = TEC + B\beta_{arc}$$

$$\beta_{iarc} = \beta_i + \gamma_j + (\lambda_{Arc})$$

ARC dependent biases

$$S = A \times C + B \times$$

$$\begin{bmatrix} \beta_1 \\ \beta_2 \\ \beta_3 \\ \beta_4 \\ \beta_5 \\ \beta_6 \\ \beta_7 \\ \beta_8 \\ \beta_9 \\ \beta_{10} \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \beta_{narcs-2} \\ \beta_{narcs-1} \\ \beta_{narcs} \end{bmatrix}$$

Rec + Sats dependent biases

$$S = A \times C + B \times$$

$$\begin{bmatrix} \beta_i + \gamma_1 \\ \beta_i + \gamma_2 \\ \beta_i + \gamma_3 \\ \beta_i + \gamma_1 \\ \beta_i + \gamma_2 \\ \beta_i + \gamma_3 \\ \beta_i + \gamma_1 \\ \beta_i + \gamma_2 \\ \beta_i + \gamma_3 \\ \beta_i + \gamma_4 \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \beta_i + \gamma_{J-2} \\ \beta_i + \gamma_{J-1} \\ \beta_i + \gamma_J \end{bmatrix}$$

$\rightarrow \gamma_j \equiv 0$

Single-station solution

- arc-by-arc - solve for β_{arc}
- sat-by-sat - precomputed γ, β
 - precomputed γ , solve for β
 - solve for β_{sat}

Multi-station solution

- arc-by-arc - solve for β_{arc}
- sat-by-sat - precomputed γ, β
 - precomputed γ , solve for β
 - SOME satellite a zero-reference bias and SOME receiver a zero-reference bias (A zero-mean condition)

Multi-constellation Solution

Potential advantages of GPS + ...

“Constellation” of 32 + satellites

- Larger quantity of observations
- Better geometry due to spatial distribution of visible satellites
- Improved numerical solutions for algorithms of positioning, navigation, etc...also TEC.
- Better performance on areas of restricted visibility
- GLONASS orbits have better coverage in high latitudes N or S

Potential disadvantages

Combination of errors from different systems / technologies ?

Dealing with different: Time Standards, Datum, constellation almanac: approximate ephemerides for all satellites, Orbits computation, Navigation files format

Multiconstellation

$$S = TEC + B\beta_{arc}$$

$$\beta_{iarc} = \beta_i + \gamma_j + (\lambda_{Arc})$$

Rec + Sats dependent biases

$$S = A \times C + B \times$$

$$\begin{bmatrix} \beta_{receiver}^{satsys} + \gamma_1 \\ \beta_{receiver}^{satsys} + \gamma_2 \\ \beta_{receiver}^{satsys} + \gamma_3 \\ \beta_{receiver}^{satsys} + \gamma_1 \\ \beta_{receiver}^{satsys} + \gamma_2 \\ \beta_{receiver}^{satsys} + \gamma_3 \\ \beta_{receiver}^{satsys} + \gamma_1 \\ \beta_{receiver}^{satsys} + \gamma_2 \\ \beta_{receiver}^{satsys} + \gamma_3 \\ \beta_{receiver}^{satsys} + \gamma_4 \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \beta_{receiver}^{satsys} + \gamma_{J-2} \\ \beta_{receiver}^{satsys} + \gamma_{J-1} \\ \beta_{receiver}^{satsys} + \gamma_J \end{bmatrix}$$

For each constellation

- A receiver biases
- A zero-mean condition for satellites biases

$$\rightarrow \begin{matrix} GPS - \gamma_j \equiv 0 \\ \beta_{receiver}^G \end{matrix}$$

$$\rightarrow \begin{matrix} GALILEO - \gamma_j \equiv 0 \\ \beta_{receiver}^E \end{matrix}$$

$$\rightarrow \begin{matrix} BeiDou - \gamma_j \equiv 0 \\ \beta_{receiver}^C \end{matrix}$$

$$\rightarrow \begin{matrix} GLONASS - \gamma_j \equiv 0 \\ \beta_{receiver}^R \end{matrix}$$

SOME receiver a zero-reference bias

Highlights for TEC processing

The addition of data implies that:

- the number of unknowns arc offsets to be solved in the calibration procedure is increased
- also the number of equations is increased, as the numbers of satellites is increased
- having a same set of expansion coefficients for V_{eq} .

As explicit: Considering a one hour period, a refreshing interval of 10 minutes for V_{eq} , a polynomial expansion on 6 coefficients,

- 5 GPS SVs/epoch, each 30 sec:

$$120*5 \text{ satellites} = 600$$

number of equations

$$5 \text{ arc offsets} + 6*6 \text{ coefficients} = 41$$

number of unknowns

- The addition of 3 GLONASS SVs:

$$120*8 \text{ satellites} = 960$$

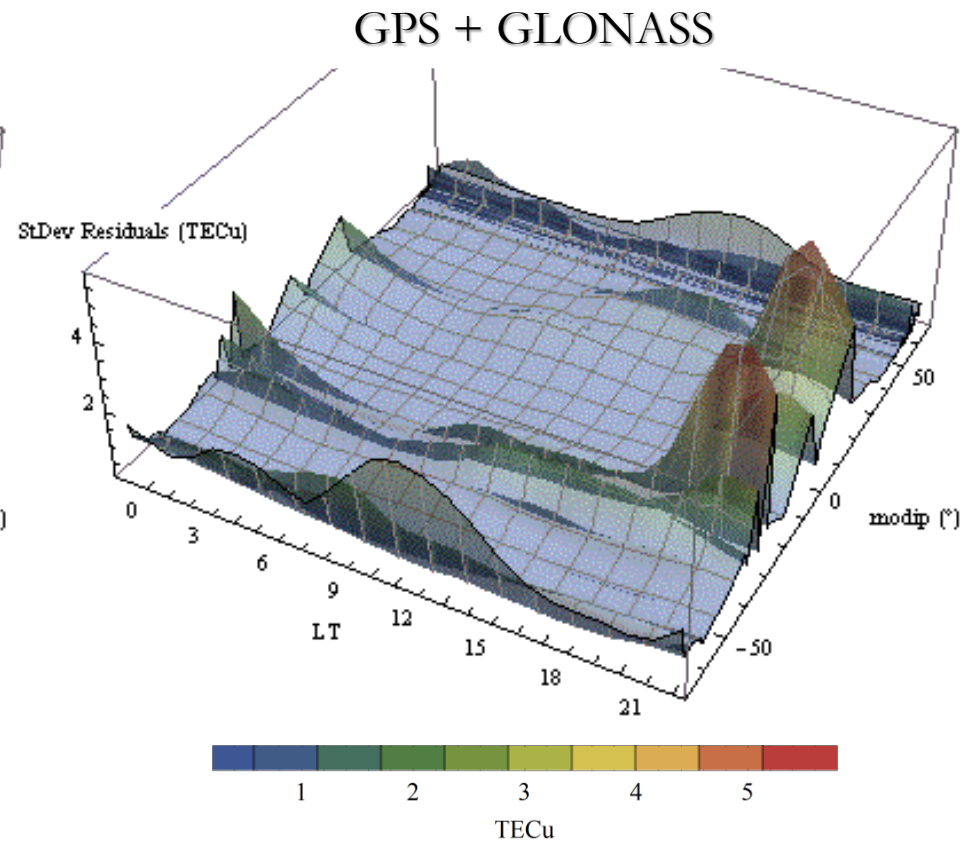
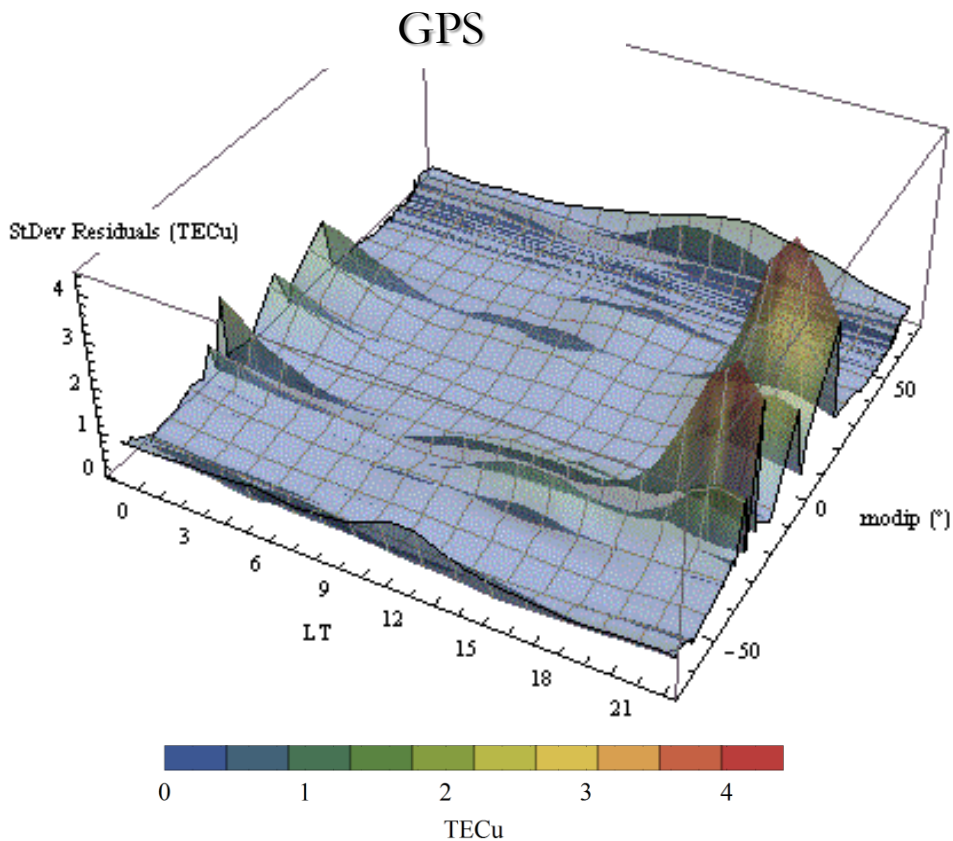
number of equations

$$8 \text{ arc offsets} + 6*6 \text{ coefficients} = 44$$

number of unknowns

Adding Constellations-> the observations/unknowns budget is more robust²⁸₁₀

Calibration residuals Related to arcoffsets determination



	MARKER	modip(°)	geo.lat (°)	geo.lon(°)
StDev > 2 TECu	cas1	-66.1	-66.1	110.5
	maw1	-63.0	-67.5	62.9
	ufpr	-31.3	-25.3	-49.2
	savo	-25.3	-12.9	-38.4
	salu	-3.2	-2.6	-44.2
	kour	18.7	5.2	-52.8

~100 GPS/GLONASS receiving stations

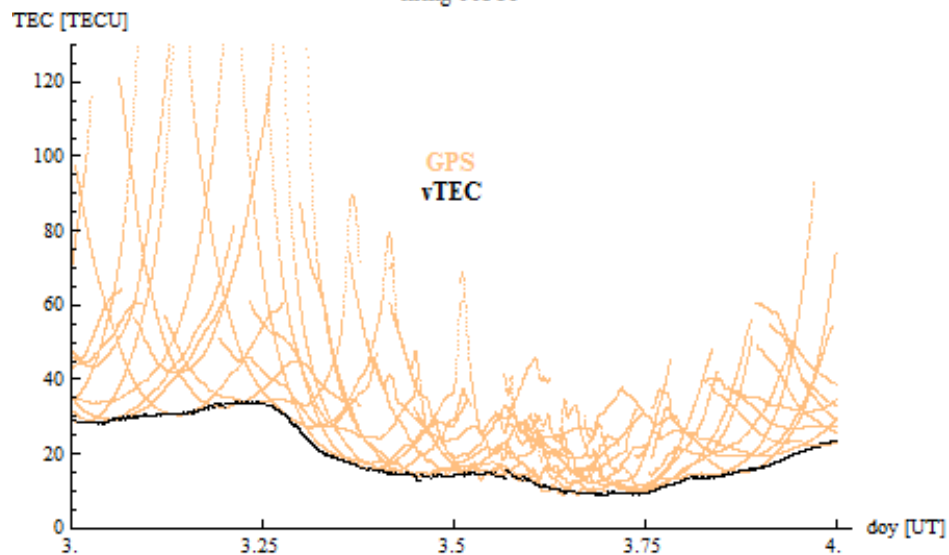
<http://igscb.jpl.nasa.gov/>

<http://www.ngs.noaa.gov/CORS/>

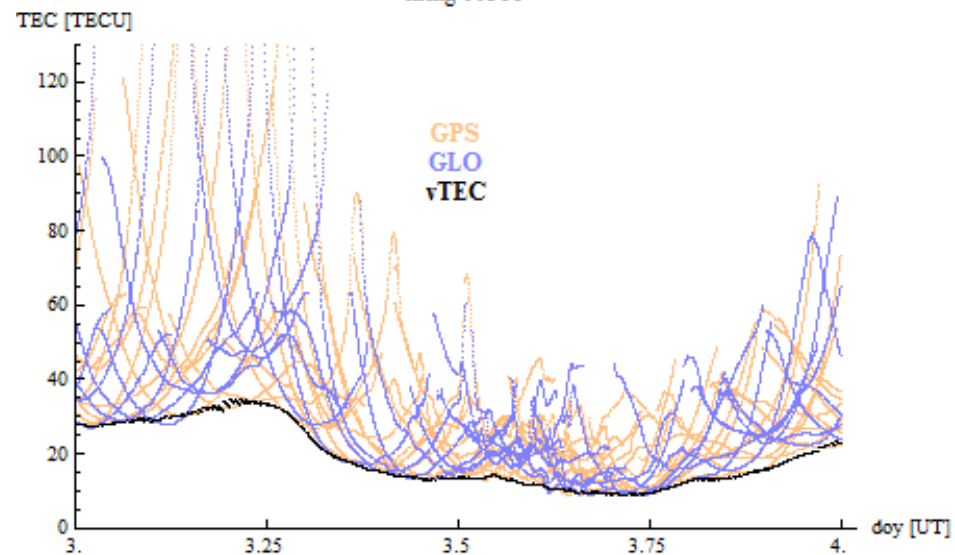
1 to 31 January, 2012. -> low-middle solar activity period with no significant disturbed geomagnetic conditions

Arc-offsets Multi-constellation Solution

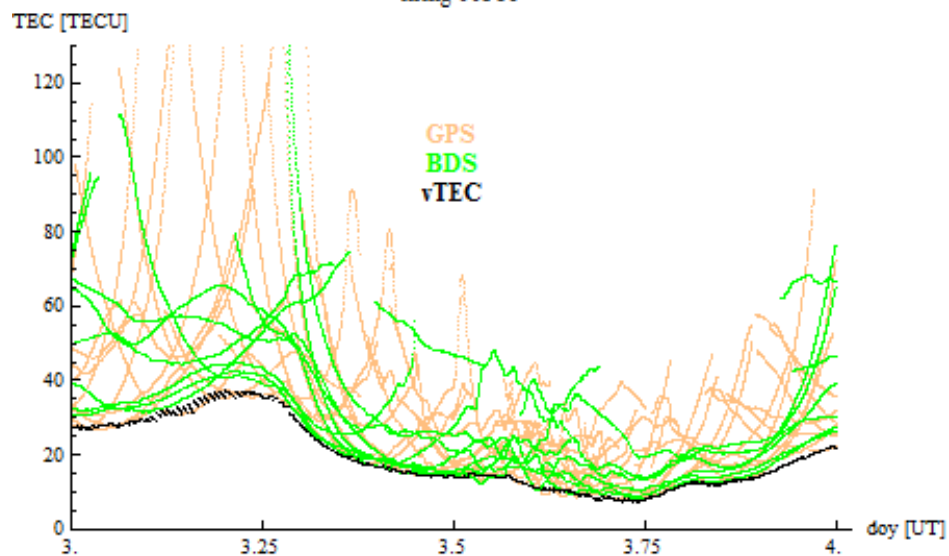
nrmg 00316



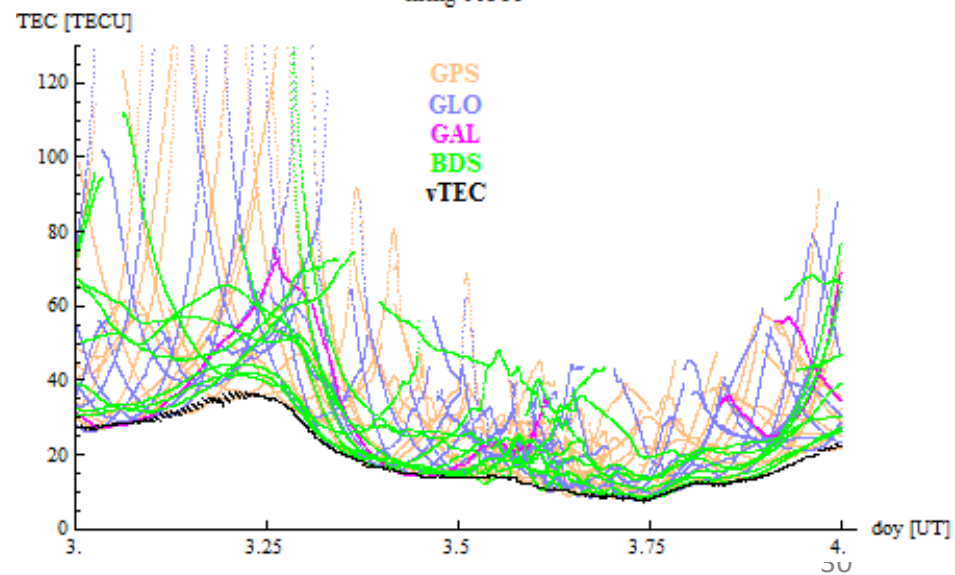
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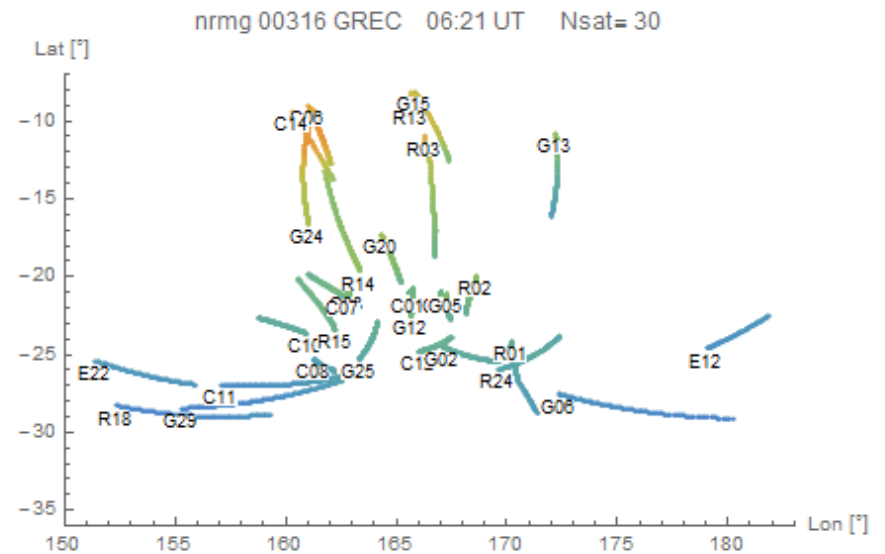
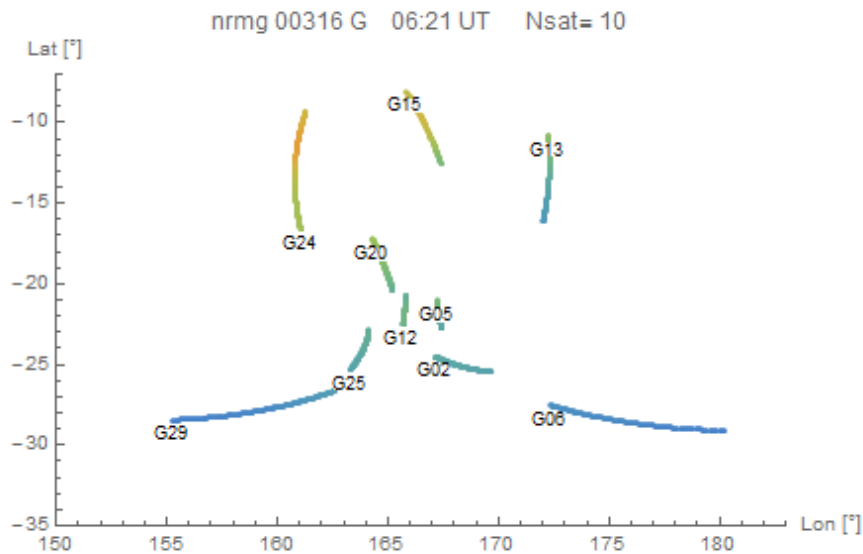
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Arc-offsets Multi-constellation Solution



One hour traces
06:21 UT is the central time of the trace

Conclusions

- The combined processing of GNSS measurements, over the processing of only GPS measurements, provides a better definition on the continuity of the solution and the sensitivity of the intra-day variability of the estimated vertical TEC.
- Handling calibrated TEC from mixed constellations could be a benefit for later uses, as example those interested in TEC data ingestion into ionospheric models. (In detriment of the number of observations but as an advantage in the quality of the procedure, it is possible to discard those data that seem unreasonable, being on preprocessing or after TEC calibration).
- In the mixed constellation the *Vertical Equivalent TEC* model residuals are increased around fractions of TECu.
- The residuals increase could be interpreted as following: The processing of a higher quantity of GNSS observations brings a more realistic representation of the complexity of the real ionospheric conditions in comparison to the considered (simple but effective) model.