On the thermodynamical formalism for expanding measures

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Let $\mathbb{X}$ be a separable metric space and $\mathcal{C}$ a closed subset of $\mathbb{X}$ with empty interior. Let $f : \mathbb{X} \setminus \mathcal{C} \to \mathbb{X}$ be a locally bi-Lipchitz map. Given $p \in \mathbb{X}$ and $n \in \mathbb{N}$, we say that n is a (α, δ)-*expanding time* for p with respect to f if there exists an open neighborhood $V_n(p)$ of p such that $f^n : V_n(p) \to B(f^n(p), \delta)$ is a homeomorphism that extends continuously to the boundary, and

$$d(f^j(x), f^j(y)) \leq \alpha^{n-j} d(f^n(x), f^n(y)),$$

$\forall\, 0 \leq j \leq n-1$ and $x, y \in V_n(p)$.

Letting $\mathbb{N}_p(\alpha, \delta, f)$ be the set of all (α, δ)-expanding times to p with respect to f, a set $\Lambda \subset \mathbb{X}$ is called an ***expanding set for*** f if there is $0 < \alpha < 1$ and $\delta > 0$ such $\limsup_{n\to+\infty} \frac{1}{n} \#\{1 \leq j \leq n \,;\, j \in \mathbb{N}_p(\alpha, \delta, f)\} > 0$ for every $p \in \Lambda$. A f-invariant probability μ is called an ***expanding measure for*** f if there exists $\ell \in \mathbb{N}$ and an expanding set Λ for f^ℓ such that $\mu(\Lambda) = 1$. Let $\mathcal{E}(f)$ be ***the set of all expanding measure for*** f.

Given a continuous potential $\varphi : \mathbb{X} \to \mathbb{R}$, we say that an f-invariant probability μ is an ***expanding equilibrium state for*** f if $\mu \in \mathcal{E}(f)$ and

$$h_\mu(f) + \int \varphi d\mu = sup\left\{ h_\nu(f) + \int \varphi d\nu \,;\, \nu \in \mathcal{E}(f) \right\}.$$

We say that f is ***strongly transitive*** if $\alpha_f(x) = \mathbb{X}\ \forall\, x \in \mathbb{X} \setminus \mathcal{C}$ and f is called ***weak topologically mixing*** if $f \times f$ is transitive, where $\alpha_f(x)$ is the α-limit set of x.

Theorem A. Suppose that f is strongly transitive and weak topologically mixing.

(1) Then f has at most one expanding equilibrium state for $\varphi \equiv 0$.
(2) If f has an expanding equilibrium state for $\varphi \equiv 0$ then f has one and only one expanding equilibrium state μ_ψ for any given Hlder potential ψ close enough to φ.

Corollary B. A Viana map has one and one equilibrium state for every Hlder potential with small variation.

This talk is based on a joint work with Paulo Varandas.