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**International Centre  
for Theoretical Physics**

[www.ictp.it](http://www.ictp.it)  
Trieste, Italy



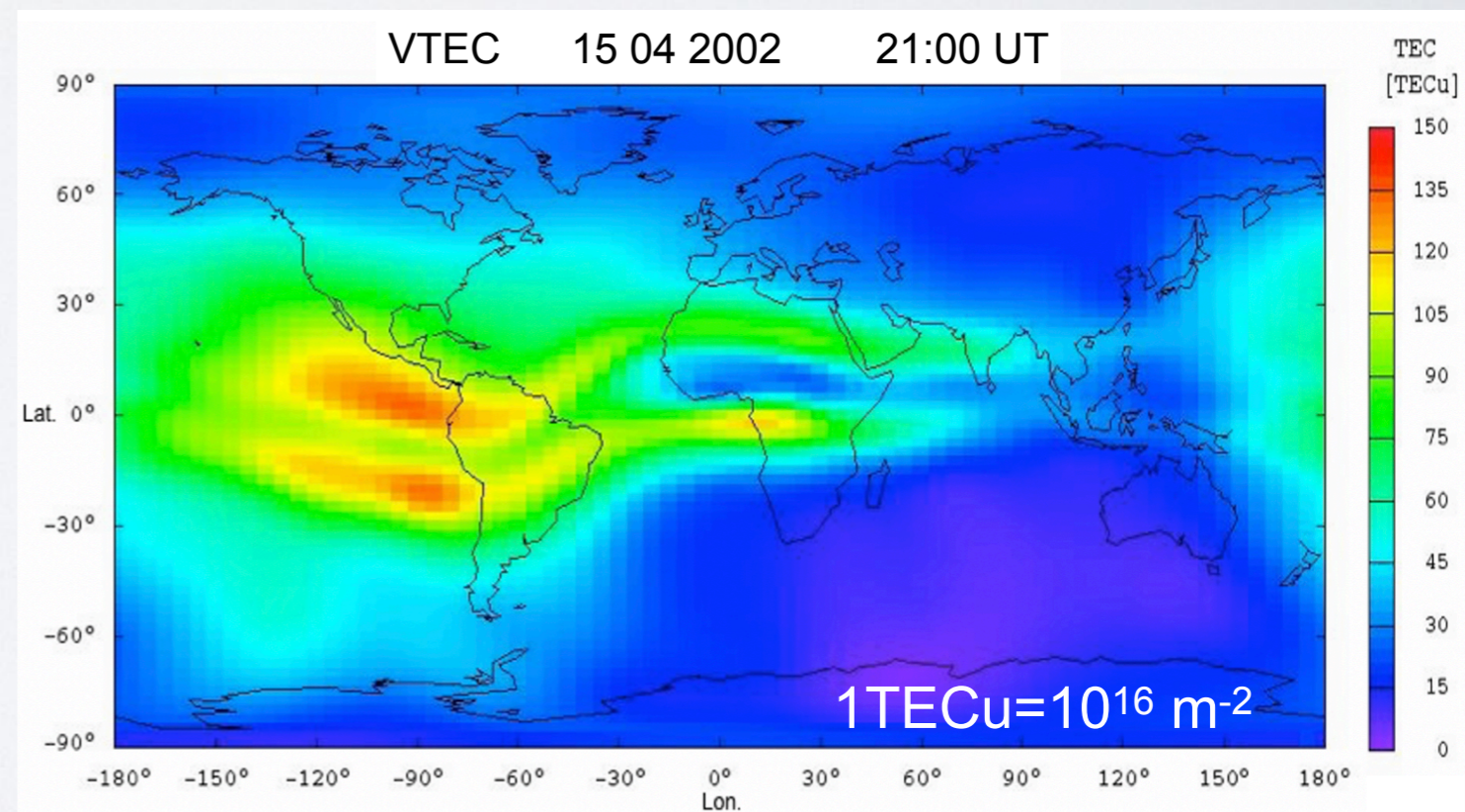
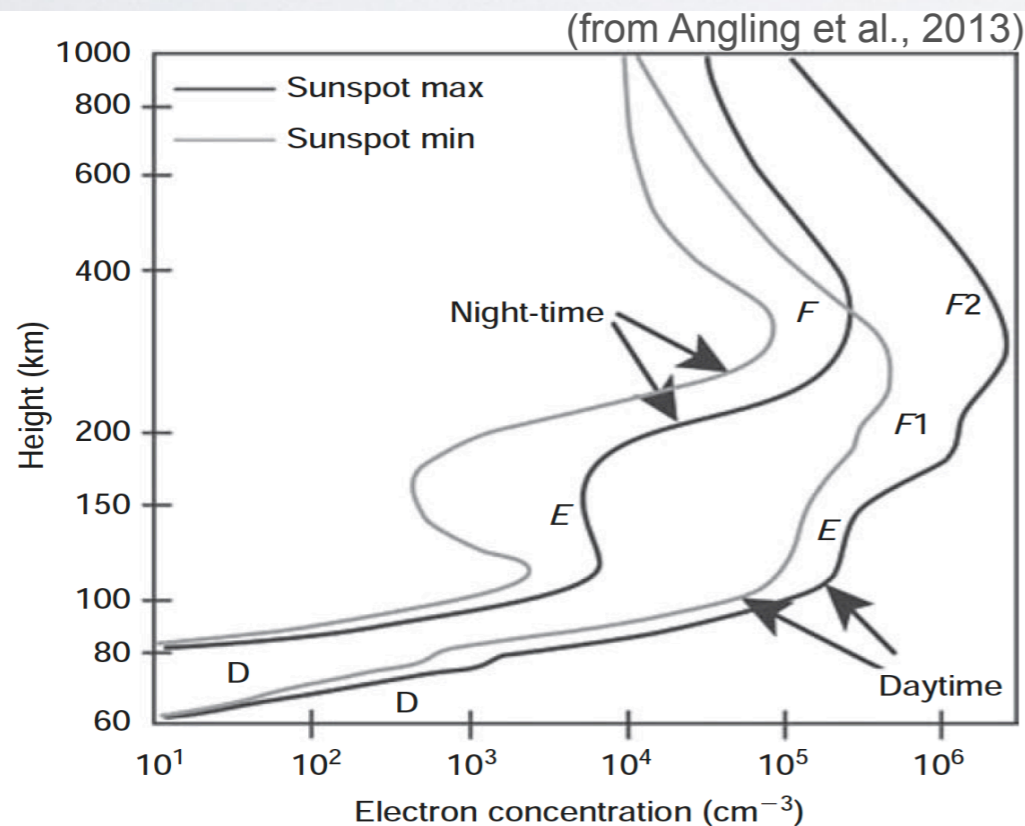
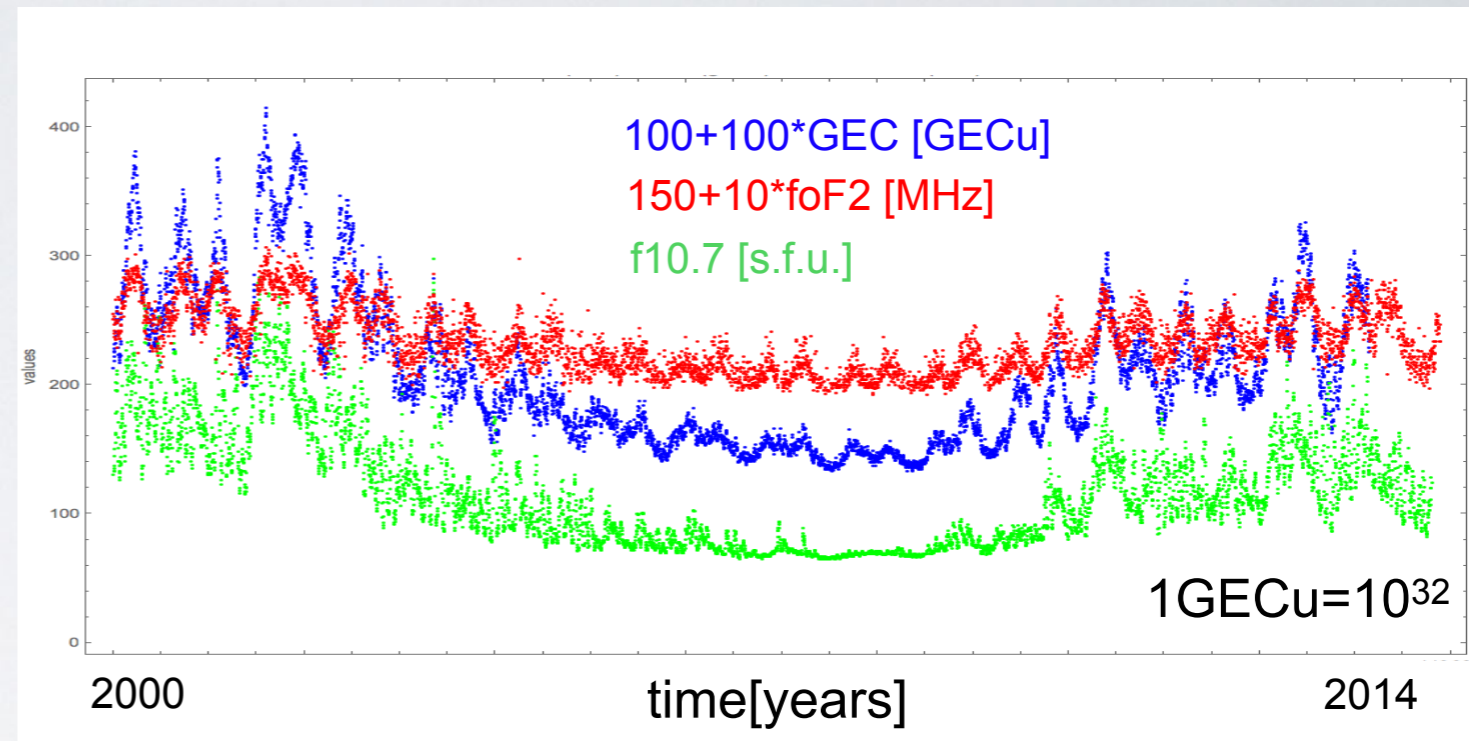
# Introduction to Ionospheric Modeling

**Bruno Nava**  
ICTP, Trieste, Italy

Regional Workshop on GNSS and Space Weather  
Rabat, 9 - 13 May 2021

# The Earth's Ionosphere

- Part of the upper atmosphere where sufficient free electrons and ions exist to affect the propagation of radio waves
- Between 80-1000 km height (Plasmasphere > 1000 km)
- Produced by solar X, UV radiation
- $\text{NO}^+$ ,  $\text{O}_2^+$ ,  $\text{O}^+$ ,  $\text{H}^+$
- Stratified in D, E, F layers
- Strong variability (daily, seasonal, solar activity); geomagnetic field





# Ionospheric models

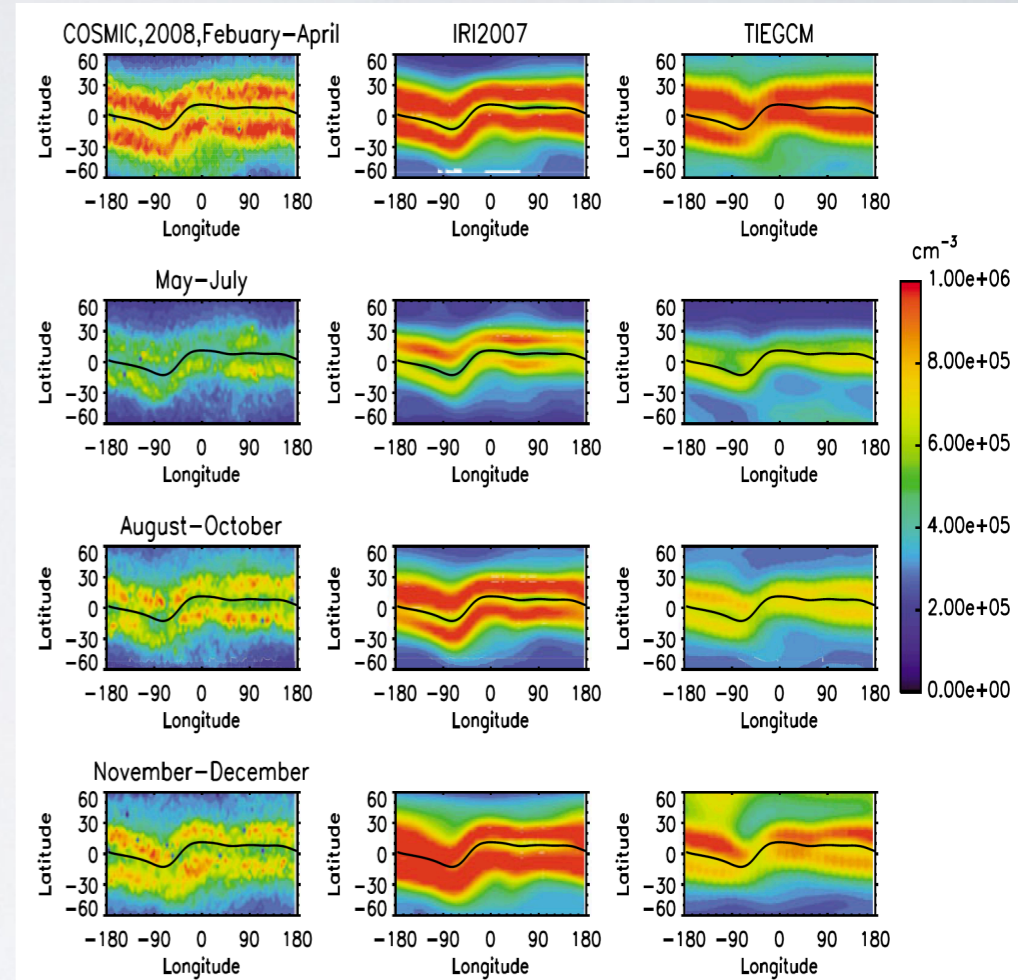
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The understanding of the ionosphere behaviour is determined by the ability to model at least the height, geographic and time distributions of the electron density.

- **Physics-based models**
  - Conservation (continuity, momentum, energy, etc.) equations are solved numerically as a function of spatial and time coordinates to calculate plasma densities
  - Highly demanding in terms of implementation and computational costs
  - Effective for understanding the physical processes of the upper atmosphere
- **Empirical models**
  - Based on an analytical description of the ionosphere with functions obtained from experimental data or adapted from physics-based models
  - Relatively simple
  - Very good performance in reproducing the “climatological” behaviour of the ionosphere (median models)
- **Data assimilation models**
  - Use specific mathematical techniques to incorporate experimental data into physics-based or empirical background models

# TIE-GCM

- The Thermosphere-Ionosphere-Electrodynamics General Circulation Model (TIE-GCM), is developed at the National Center for Atmospheric Research High-Altitude Observatory.
- It is a global 3-D numerical model that simulates the coupled thermosphere/ionosphere system from 95 to 600 km altitude.
- The TIE-GCM self-consistently solves the fully coupled, nonlinear, hydrodynamic, thermodynamic, and continuity equations of the neutral gas, the ion and electron energy and momentum equations, the ion continuity equation, and neutral wind dynamo.
- It is an open-source community model and is also available for runs-on-request at the NASA Community Coordinated Modeling Center.

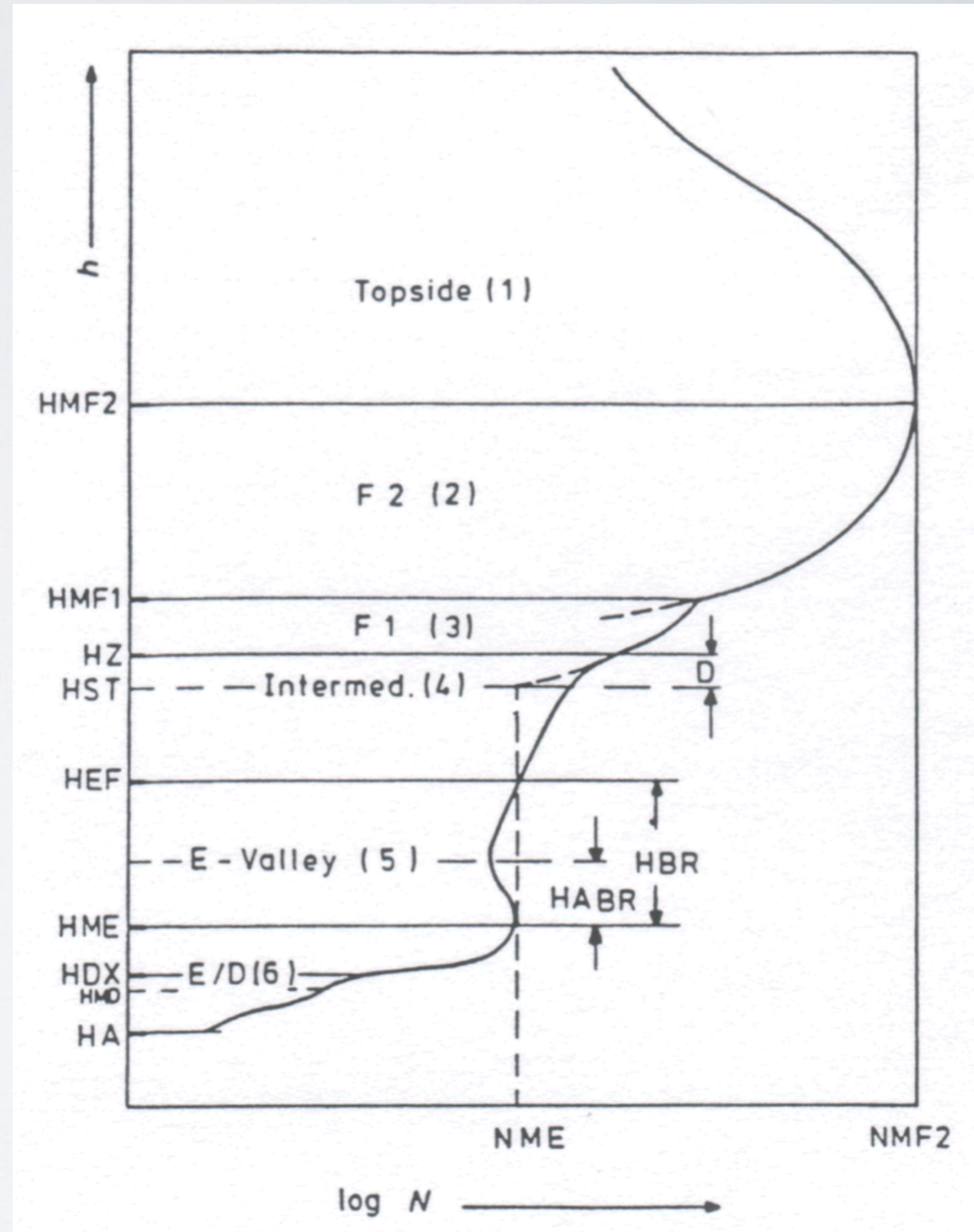


NmF2 observed by COSMIC, IRI, TIE-GCM, during 2008. NmF2 is averaged over 10:00–13:00 LT and over the months shown in each panel; from: Qian et al. (2014).



# IRI

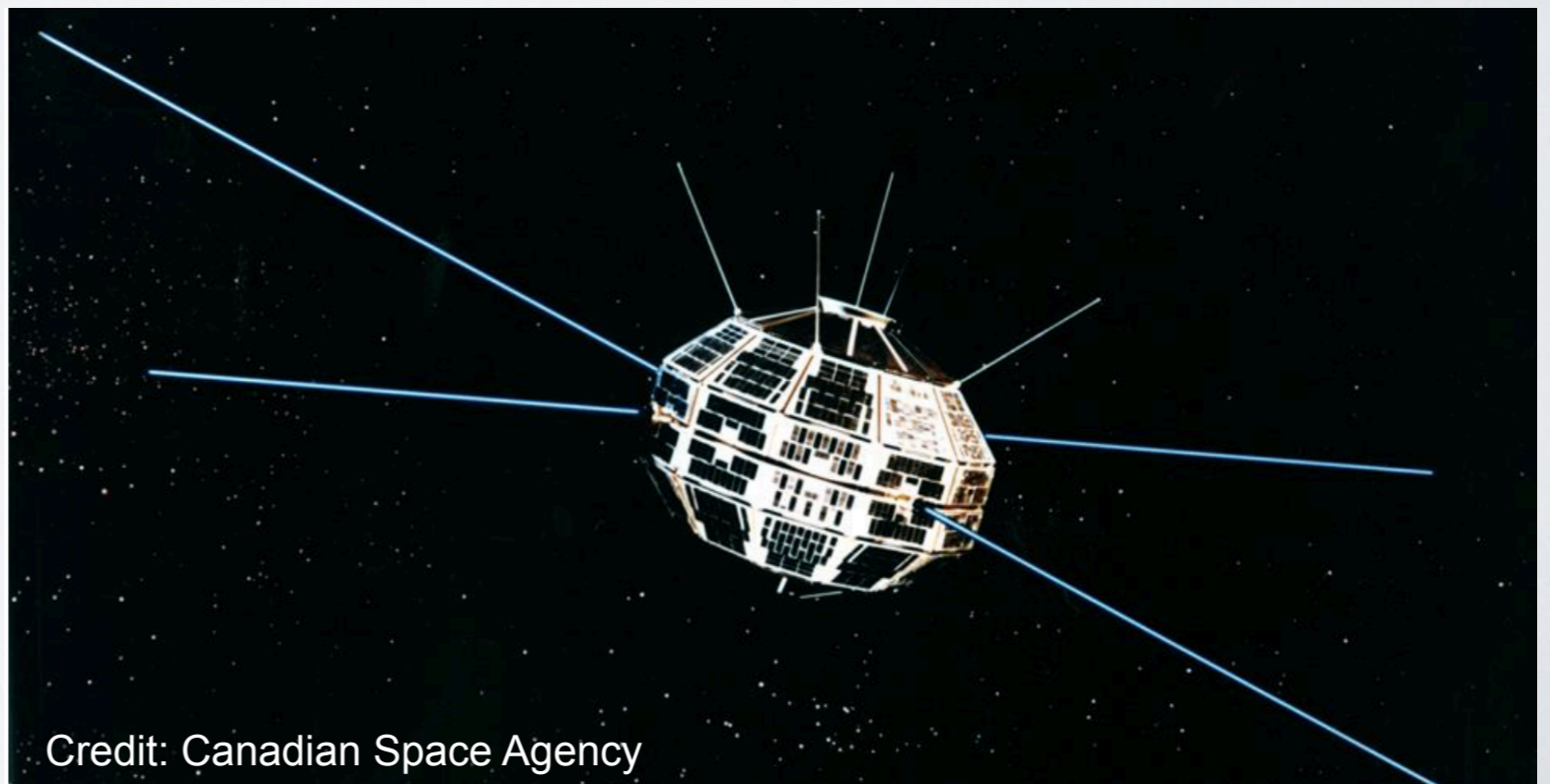
- The International Reference Ionosphere (IRI; Bilitza et al., 2001; Bilitza, 2015) is an empirical model of the ionosphere, obtained as the results of an international project sponsored by the Committee on Space Research and the International Union of Radio Science.
- For a given location and time, IRI provides the electron density, electron temperature, ion temperature and ion composition ( $O^+$ ,  $H^+$ ,  $He^+$ ,  $N^+$ ,  $NO^+$ ,  $O_2^+$ ) in the altitude range 50-2000km and also the relevant TEC.
- The IRI electron density profile is divided in six sub-regions: the topside, the F2 bottomside, the F1 layer, the intermediate region, the E region valley, the bottomside E and D region.



# IRI

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- The boundaries are defined by the presence of characteristic points that include the F2, F1 and E peaks.
- The shape of the IRI topside electron density profile was based on the descriptive compilation of Alouette topside sounder data and Epstein functions.
- Two parameters, B0 and B1, determine the bottomside thickness and shape, respectively.
- IRI includes a model to describe ionospheric storm-time conditions and has adopted, as an option, the NeQuick 2 model topside formulation.
- The latest version of IRI, IRTAM, has also the capability to describe real-time ionospheric weather conditions based on the ingestion of real-time measurements like ionosonde-derived peak parameter values.



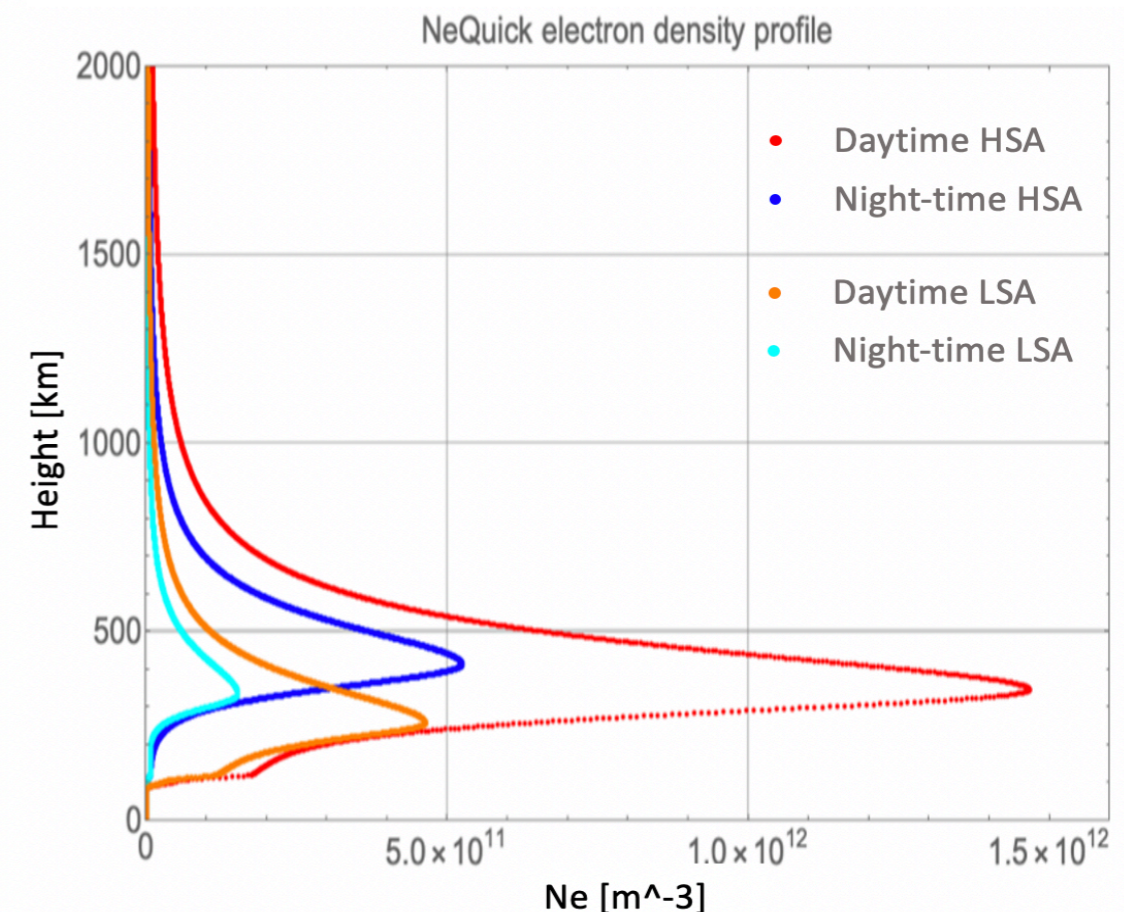
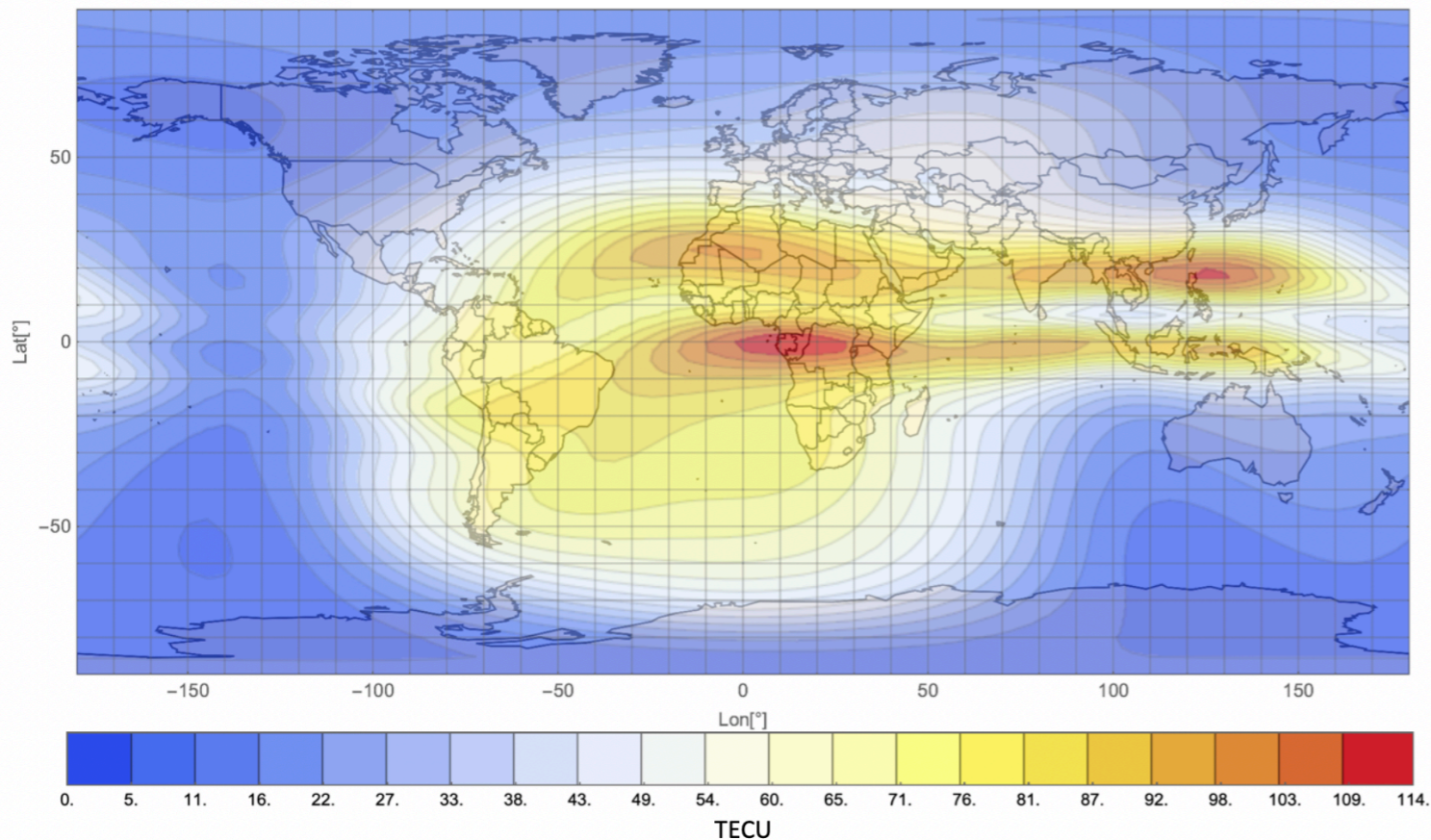
Credit: Canadian Space Agency



# NeQuick

- The **NeQuick 2** is an ionospheric electron density model developed at the former ARPL of ICTP, Trieste, Italy, in collaboration with Institute for Geophysics, Astrophysics and Meteorology of the University of Graz, Austria.
- It is a quick-run empirical model particularly designed for trans-ionospheric propagation applications, conceived to reproduce the median behavior of the ionosphere.
- NeQuick inputs are: position, time and solar flux; the output is the electron concentration at the given location and time.

NeQuick VTEC    month: 4    UT: 14:00    F10.7: 190 s.f.u.



# NeQuick

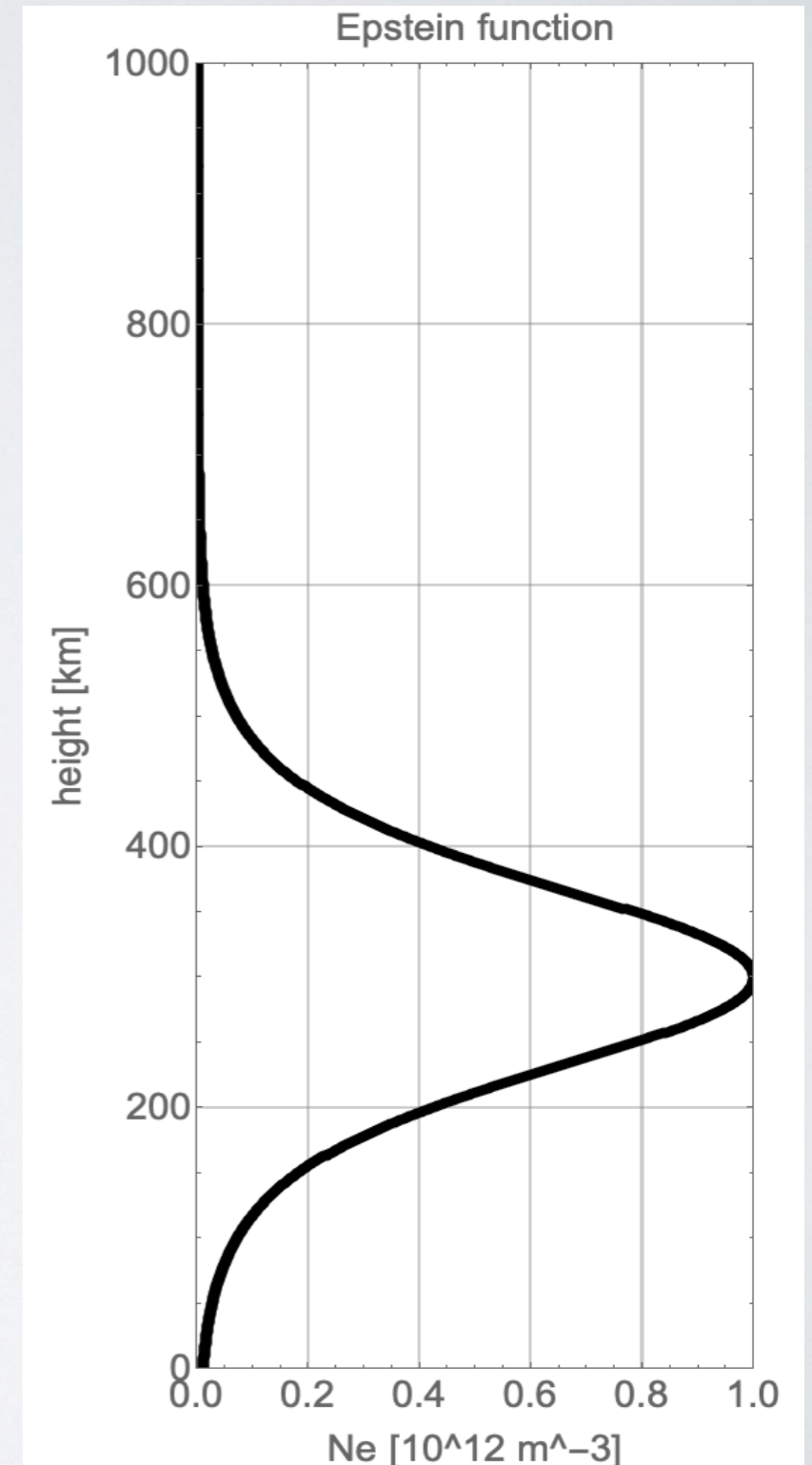
- The model profile formulation includes 6 semi-Epstein layers with modeled thickness parameters and is based on anchor points defined by foE, foF1, foF2 and M(3000)F2 values.
- These values can be modeled (ITU-R coefficients for foF2, M(3000)F2) or experimentally derived.

$$N_E(h) = \frac{4Nm^*E}{\left(1 + \exp\left(\frac{h-hmE}{BE}\xi(h)\right)\right)^2} \exp\left(\frac{h-hmE}{BE}\xi(h)\right)$$

$$N_{F1}(h) = \frac{4Nm^*F1}{\left(1 + \exp\left(\frac{h-hmF1}{B1}\xi(h)\right)\right)^2} \exp\left(\frac{h-hmF1}{B1}\xi(h)\right)$$

$$N_{F2}(h) = \frac{4NmF2}{\left(1 + \exp\left(\frac{h-hmF2}{B2}\right)\right)^2} \exp\left(\frac{h-hmF2}{B2}\right)$$

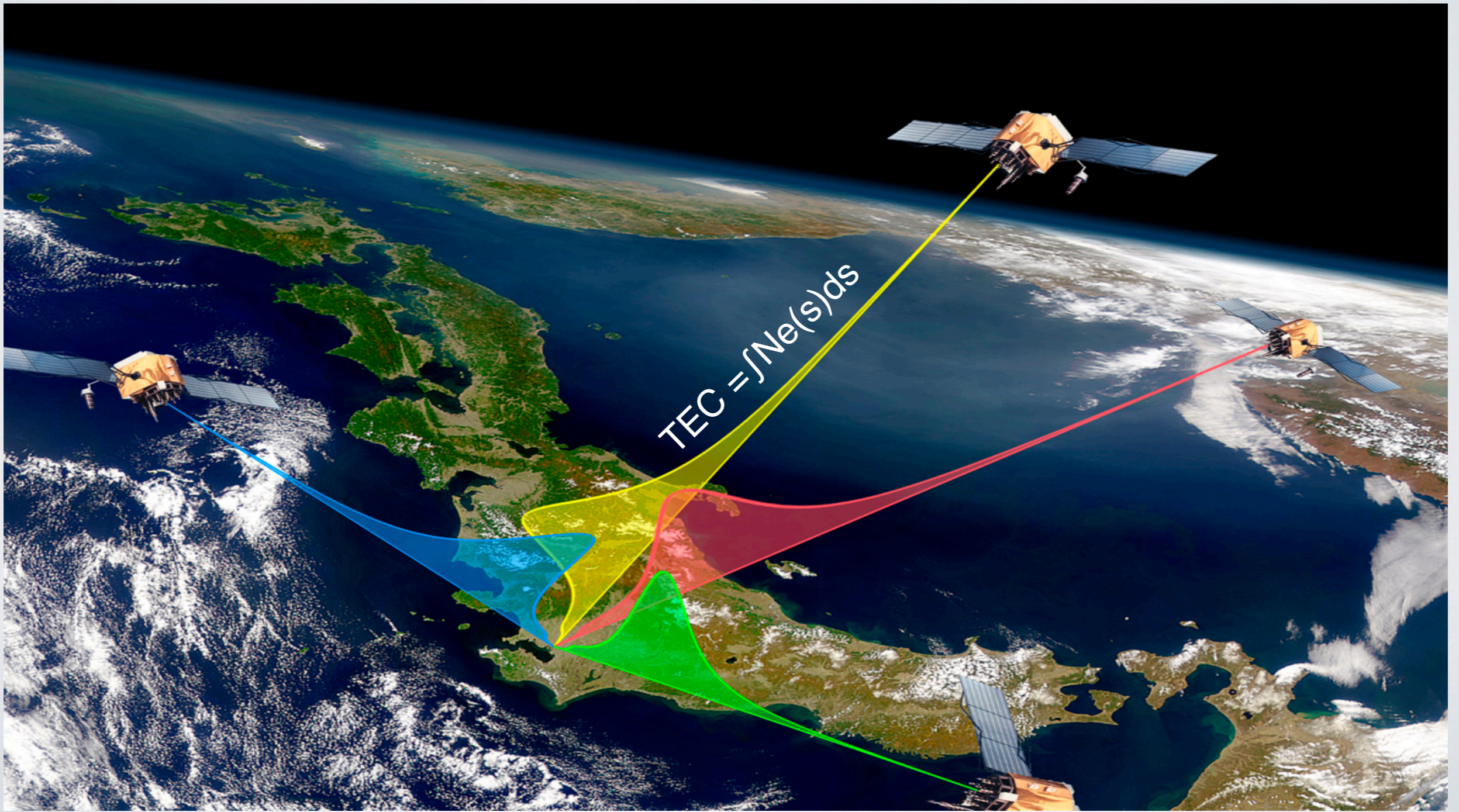
where  $\xi(h) = \exp\left(\frac{10}{1 + 1|h - hmF2|}\right)$





# NeQuick

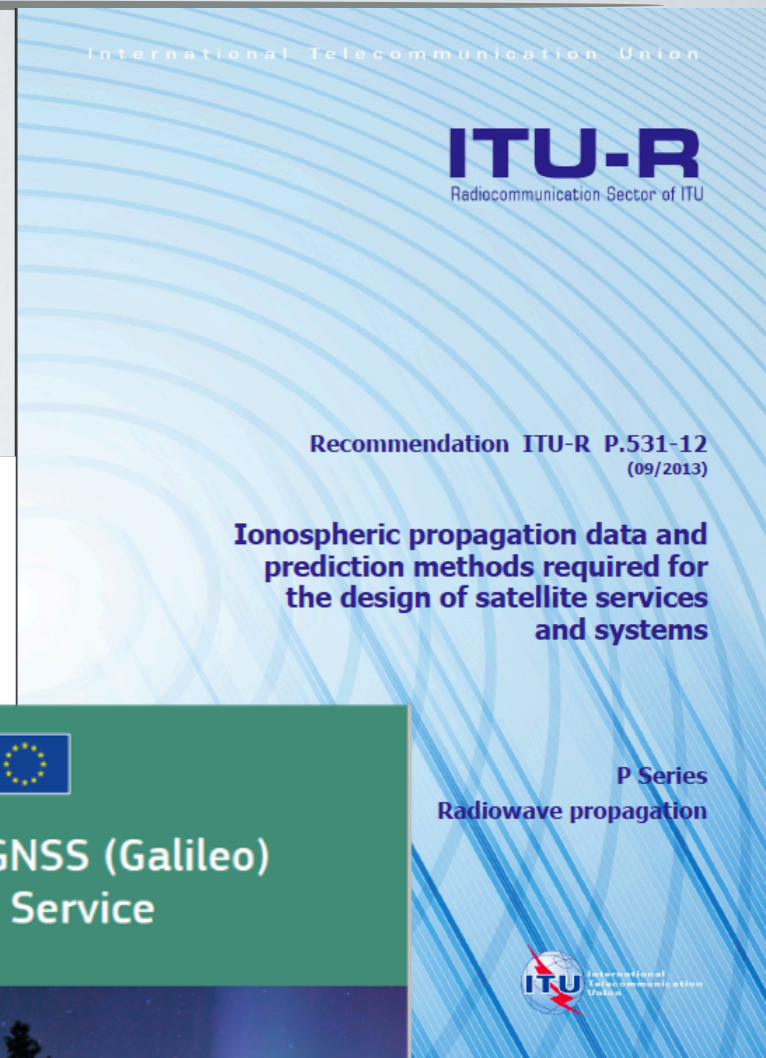
- NeQuick package includes routines to evaluate the electron density along any “ground-to-satellite” ray-path and the corresponding Total Electron Content (TEC) by numerical integration.





# NeQuick implementations

- The NeQuick (v1) has been adopted by Recommendation ITU-R P. 531 as a procedure for estimating TEC.
- Subsequently, the NeQuick 2 has substituted the NeQuick (v1) and it is the one currently recommended by ITU (ITU-R Recommendation P.531-12).
- A specific version of NeQuick (NeQuick G, implemented by ESA) has been adopted as Galileo Single-Frequency Ionospheric Correction Algorithm and its performance has been confirmed during In-Orbit Validation.
- ESA has also included NeQuick 2 in to Space Environment Information System.





# ICA

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- GPS uses a simple ionospheric model, the Ionospheric Correction Algorithm (ICA), which gives a representation of the mean vertical delay at L1, for given geomagnetic location and local time (Klobuchar, 1987).
- The diurnal variation of vertical delay is modeled by a cosine function, centered at 14 LT. During night-time the vertical ionospheric delay is approximated to a constant value: 5 ns.
- The amplitude and period of the cosine are represented in the model by 3rd order polynomials, which coefficients are broadcast in GPS navigation message. These coefficients were derived from numerical output of Bent model, determining a 370 sets of coefficients for the different conditions of the ionosphere.



# ICA

$$T_{iono} = F \left( 5 \times 10^{-9} + \sum_{n=0}^3 \alpha_n \Phi_m^n \left( 1 - \frac{x^2}{2} + \frac{x^4}{24} \right) \right) \quad \text{if } |x| \leq 1.57$$

$$T_{iono} = 5 \times 10^{-9} F \quad \text{if } |x| > 1.57$$

with

$$x = \frac{2\pi(t - 50400)}{\sum_{n=0}^3 \beta_n \Phi_m^n}$$

$$F = 1 + 16(0.53 - \theta)^3$$

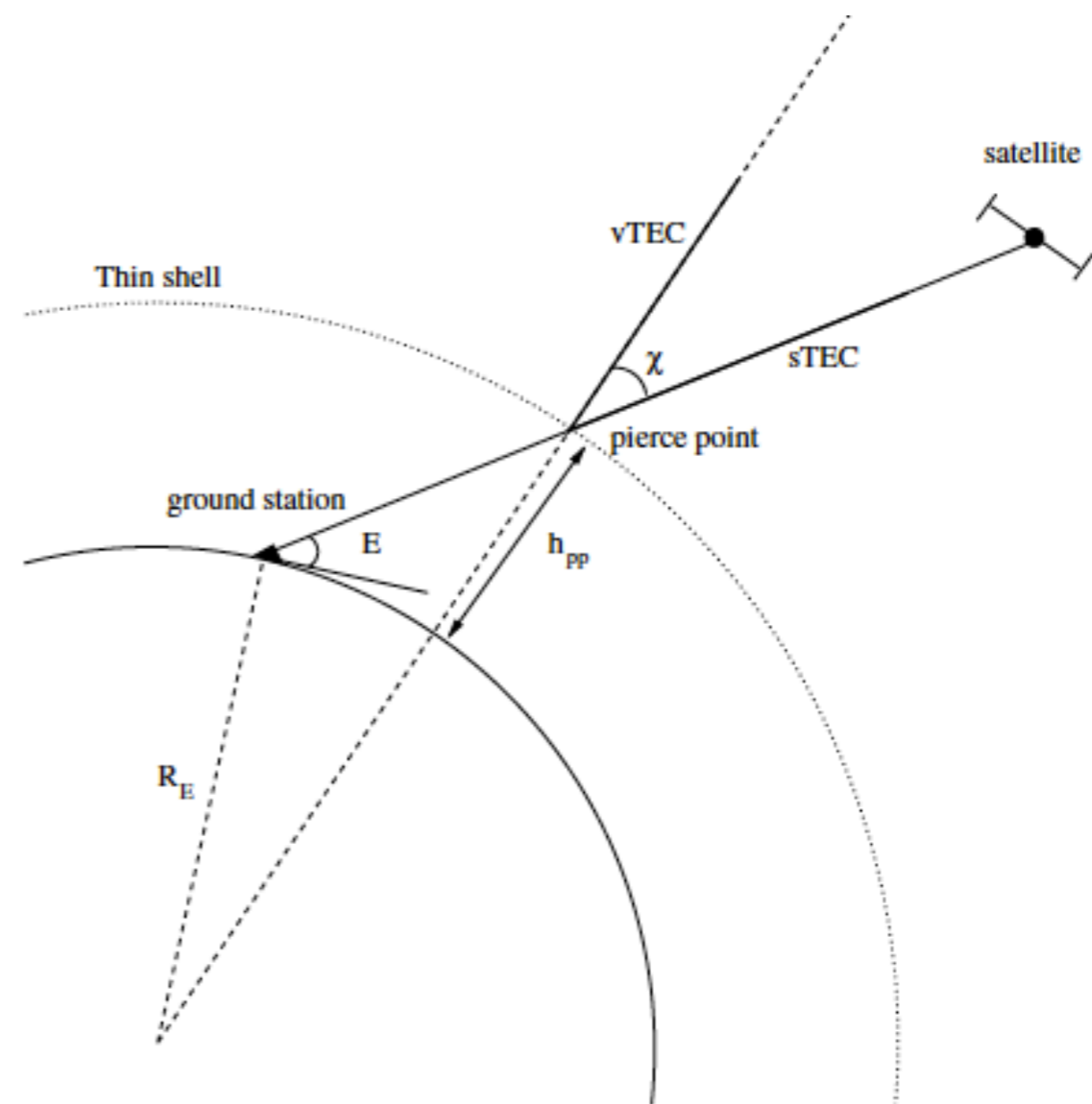
$\alpha_n$   $\beta_n$  broadcast coefficients

$t$  local time

$\Phi_m$  geomagnetic latitude  
of the pierce point

$F$  obliquity factor

$\theta$  satellite elevation





# NeQuick G

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- The model will be driven by an "effective ionisation level"  $A_z$ , valid for the whole world and applicable for a period of typically 24 hours.

$$A_z(\mu) = a_0 + a_1\mu + a_2\mu^2$$

$$\mu = \text{modip}$$

$$\tan \mu = \frac{I}{\sqrt{\cos \varphi}}$$

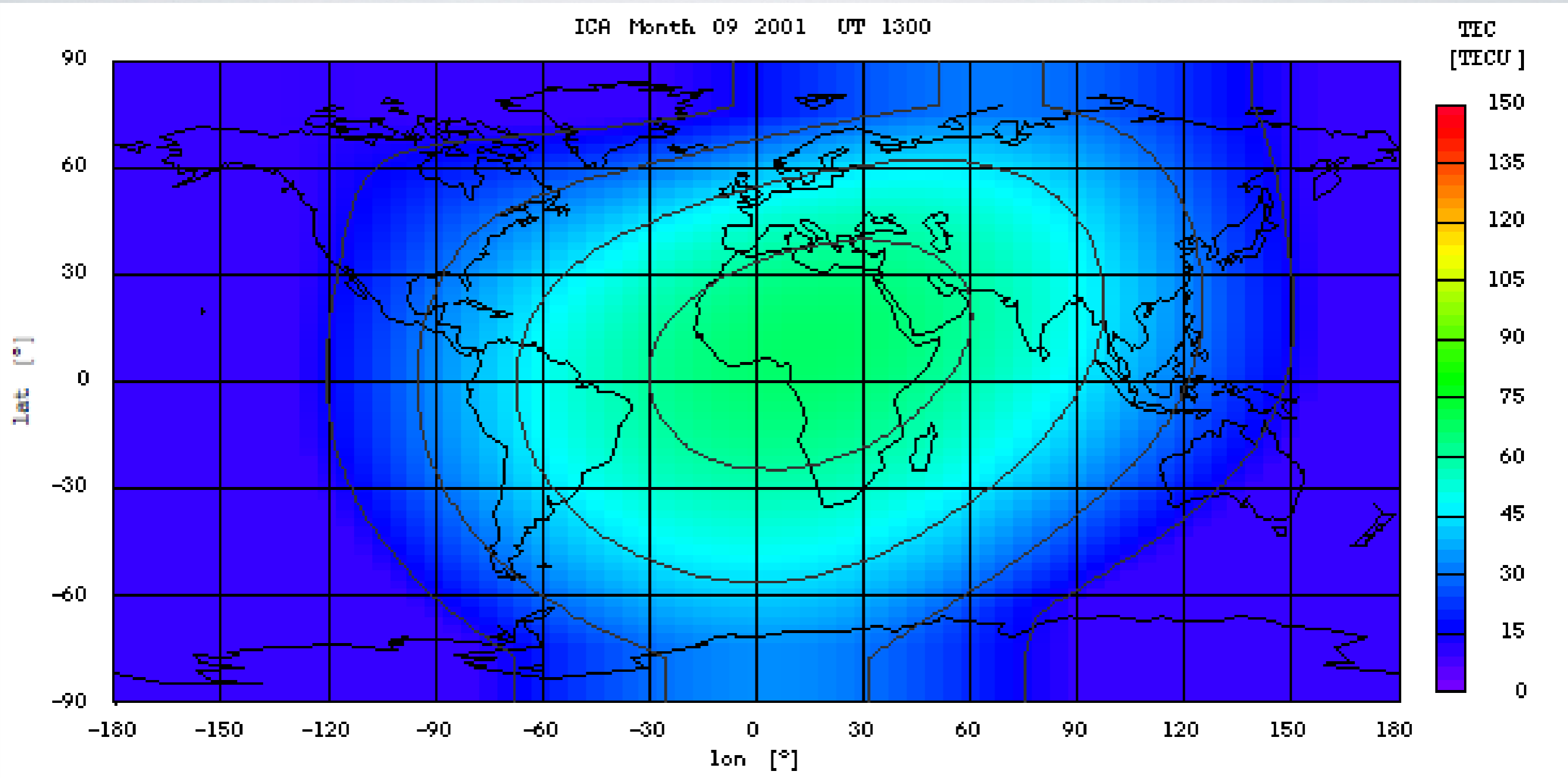
$I$  magnetic inclination at 300 km of height

$\varphi$  geographic latitude

- The coefficients  $a_0$ ,  $a_1$ ,  $a_2$  are broadcast to the user to allow  $A_z$  calculation at any wanted location.
- $A_z$  coefficients broadcasted and used for one day are computed at System level using TEC data of the previous day.

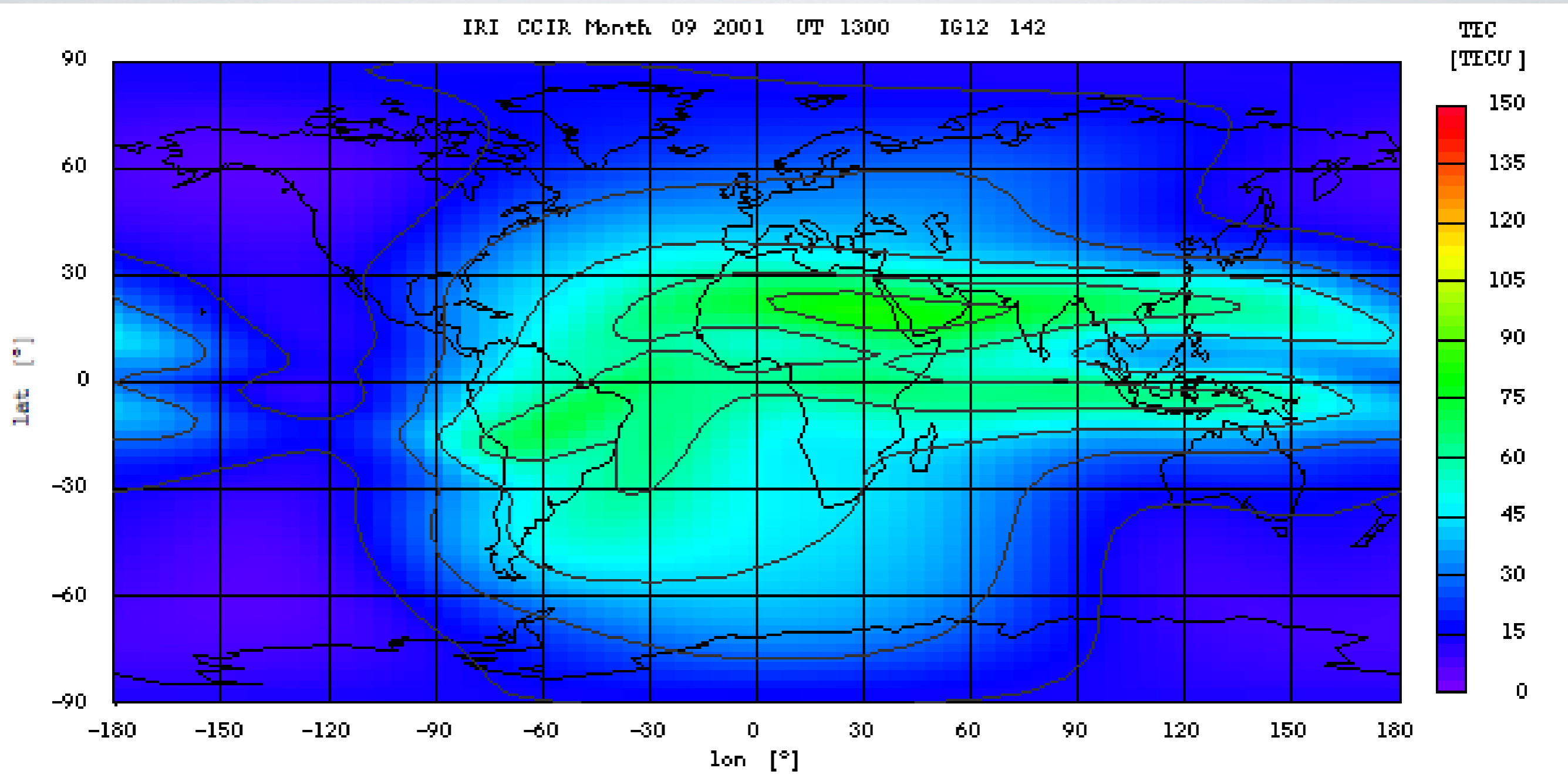


# ICA vTEC



Global map of vertical TEC using ICA

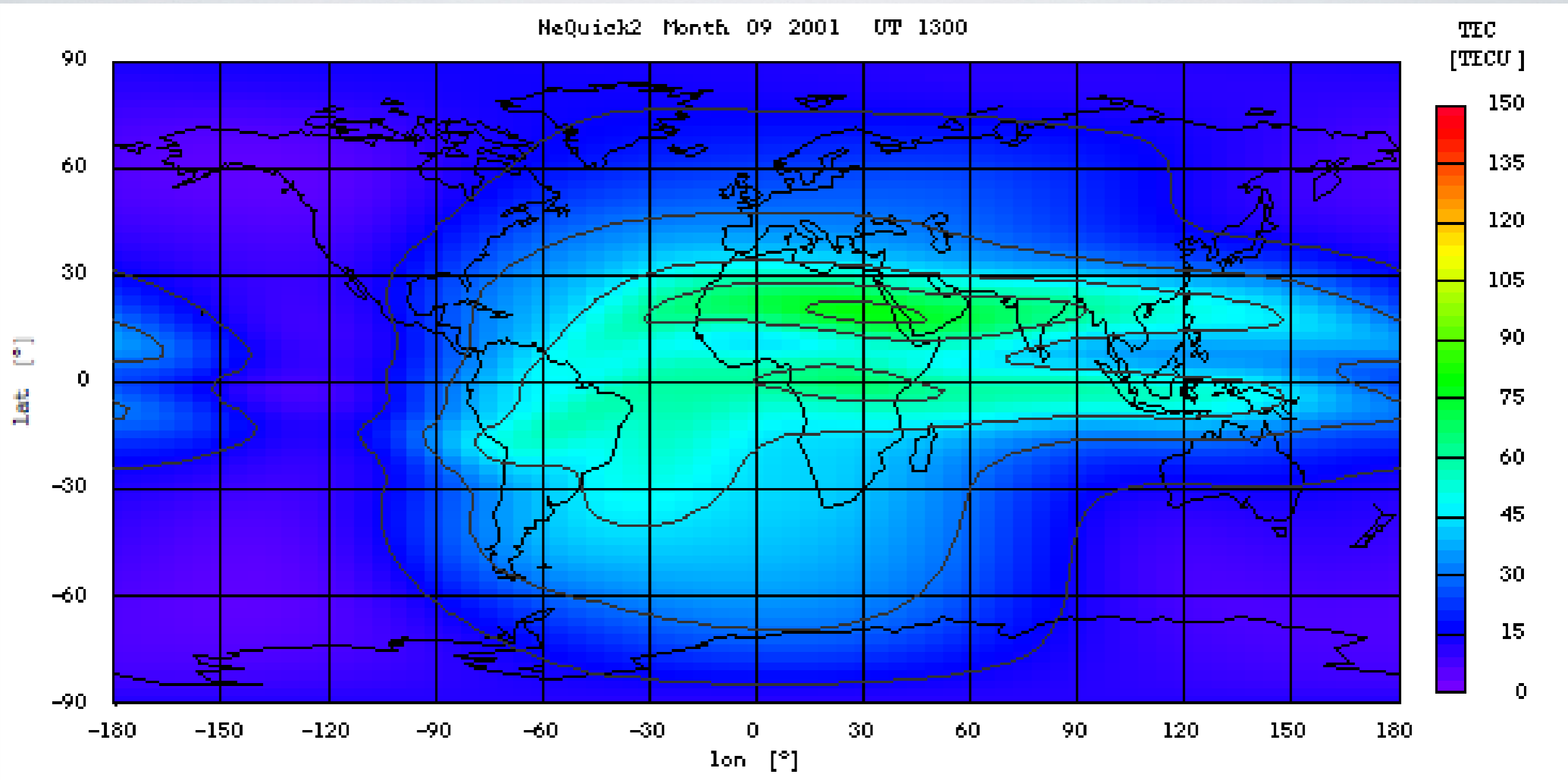
# IRI vTEC



Global map of vertical TEC using IRI



# NeQuick 2 vTEC



Global map of vertical TEC using NeQuick 2

# Other empirical models

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- Semi-Empirical Low-Latitude Ionospheric Model (SLIM)
- Fully Analytical Ionospheric Model (FAIM)
- Parameterised Real-time Ionospheric Specification Model (PRISM)
- Bent Model
- NeQuick, COSTProf, NeUoG-plas (Trieste-Graz family)
  
- The more recently developed regional Empirical Canadian High Arctic Ionospheric Model (E-CHAIM) and the Neustrelitz Electron Density Model (NEDM2020).



# Data Assimilation (DA)

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- "DA is an analysis technique in which the observed information is accumulated into the model state by taking advantage of consistency constraints with laws of time evolution and physical properties" (Bouttier and Courtier, 1999).
- DA and Data Ingestion techniques are usually associated with each other and not clearly distinguished (Aa et al, 2018).
- Ionospheric DA schemes are based on different mathematical techniques:
  - Variational techniques
    - 3D-VAR
    - 4D-VAR
  - Kalman filters
    - extended Kalman filter (EKF)
    - ensemble Kalman filter (EnKF)
    - local ensemble transform Kalman filter (LETKF)
- Data are a critical element of a DA.

# GAIM

- Utah State University (USU) Global Assimilation of Ionospheric Measurements (GAIM) [Schunk et al., 2004] or the Jet Propulsion Laboratory (JPL)/University of Southern California (USC) Global Assimilative Ionospheric Model (GAIM) [Wang et al., 2004], or [Schunk et al., 2014], for example, are based on assimilation of data originating from different sources and imply the use of first principle models.

GAIM-band limited (BL)  
 GAIM-Gauss Markov (GM)  
 GAIM-4DVAR  
 GAIM-full physics (FP)  
 Middle-low electro-DA  
 IDED-DA  
 GTM-DA

Midlatitude to Low-Latitude Ionosphere  
 Midlatitude to Low-Latitude Ionosphere  
 Midlatitude to Low-Latitude Ionosphere with Drivers  
 Midlatitude to Low Latitude Ionosphere-Plasmasphere with Drivers  
 Midlatitude to Low Latitude Ionosphere with Drivers  
 High-Latitude Ionosphere with Drivers  
 Global Thermosphere Model-Data Assimilation

## Ionosphere

Ground-based GPS-TEC  
 Satellite-based GPS occultation  
 Ionosonde and digisonde  
 In situ  $N_e$   
 911 Å, 1356 Å, limb, disk (UV)  
 Solar UV, EUV

## Electrodynamics

Ground magnetometers  
 DMSP cross-track velocities  
 SuperDARN line-of-sight velocities  
 Iridium magnetometers  
 ACE interplanetary magnetic field, *Dst*  
 Solar UV, EUV

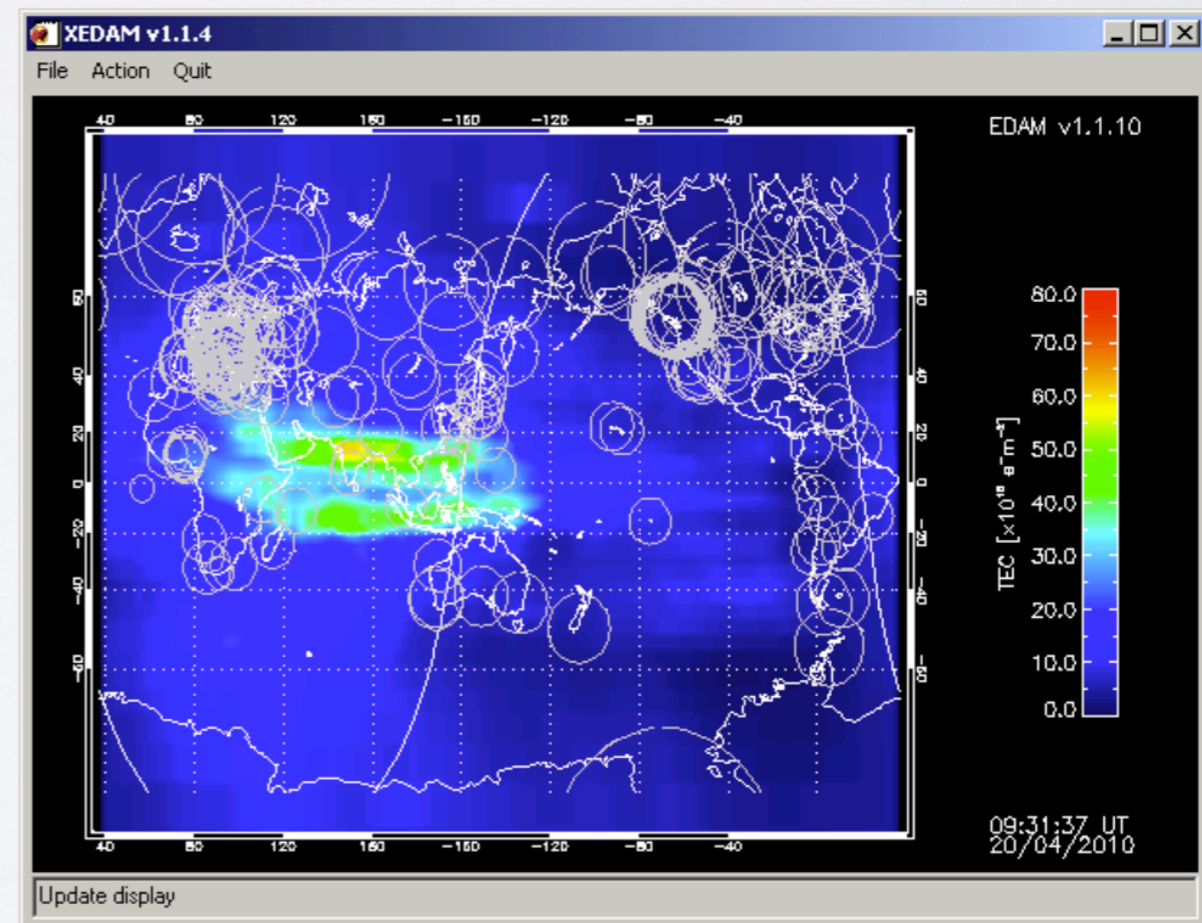
## Thermosphere

Satellite UV emissions  
 In situ neutral densities and winds  
 Satellite accelerometer and drag  
 FPI winds  
 ISR neutral parameters  
 Solar UV, EUV



# EDAM

- The Electron Density Assimilative Model (EDAM) [Angling and Khattatov, 2006; Angling, M. J., and N. K. Jackson-Booth, 2011] provides a mean to assimilate ionospheric measurements into a background ionospheric model.
- Assimilated data are: ground-based and space-based GPS-derived TEC, ionosondes-derived parameters.
- Currently IRI is used as a background model (electron density only).
- Extended, localised Gauss Markov Kalman Filter
  - Time evolution of the differences between the measurements and the background ionosphere
  - Model variances are propagated
  - Covariances are estimated as required



# MIDAS

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- The Multi Instrument Data Analysis System (MIDAS) (Mitchell C. N. and Spencer P. S., 2003) is a tomographic approach where TEC data are inverted to evaluate the distribution and time evolution of electron concentration. Orthonormal basis functions and Singular Value Decomposition (SVD) are used to solve the inverse problem. Additional information on MIDAS and other assimilation techniques can be found in the review paper by Bust, and Mitchell (2008).

# IDA3D

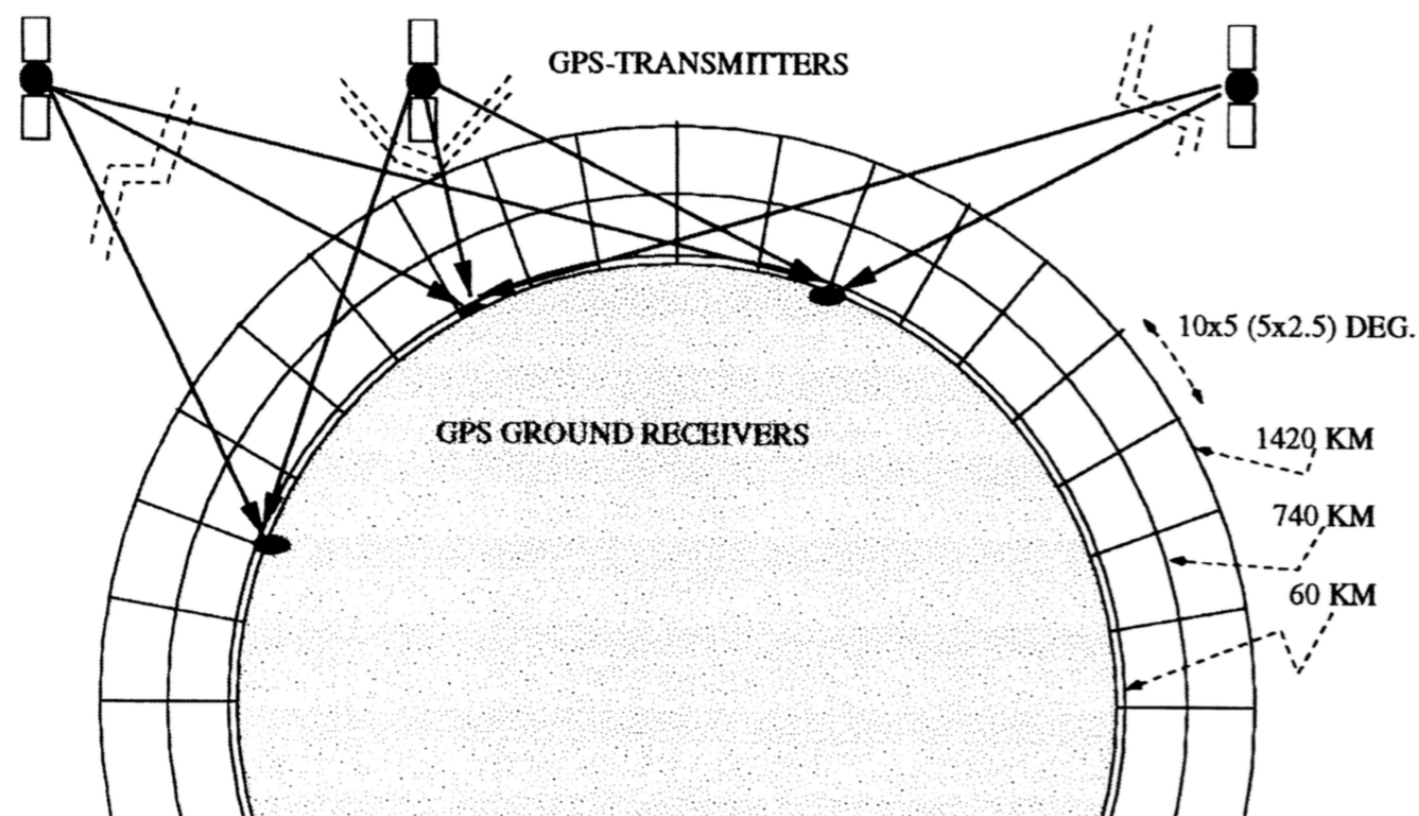
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- The Ionospheric Data Assimilation Three-Dimensional (IDA3D), (Bust et al., 2004) uses a 3D-VAR data assimilation technique. It is capable of incorporating ground based and space based GNSS-TEC measurements and electron density measurements from radars and satellites. The background 3D electron density is based upon empirical ionospheric models, but IDA3D is capable of using any global ionospheric specification as a background to produce a spatial analysis of the electron density distribution at a specified time.



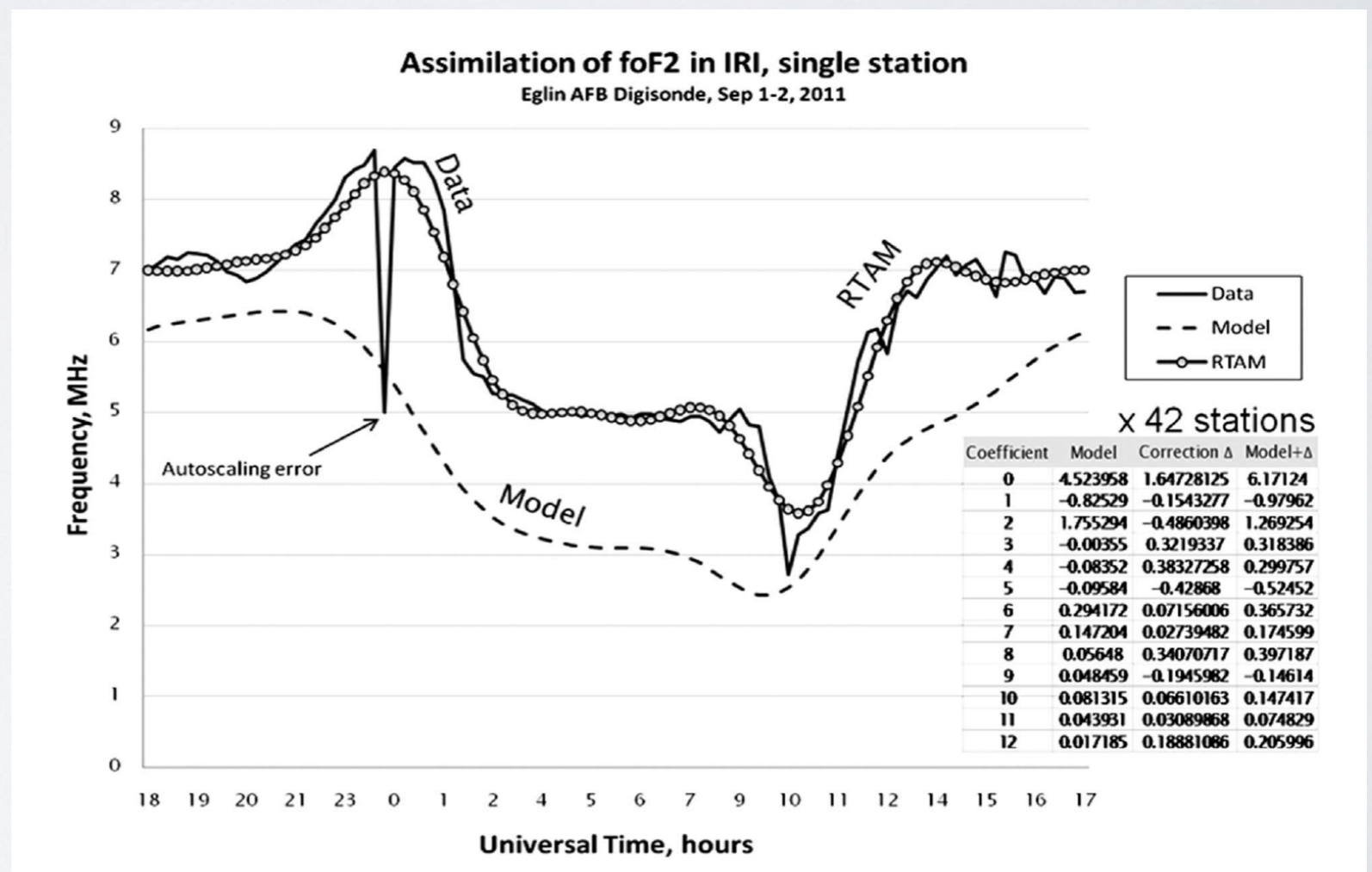
# TOMION

- The TOMographic IONosphere model (TOMION), (Hernández-Pajares, M. et al., 1999) generates Global Ionospheric Maps (GIMs) of vertical TEC starting from ground based dual-frequency GNSS measurements. To overcome the assumption of a (single) thin shell approximation, in TOMION the ionosphere is represented by two or more layers of voxels, and in each voxel the electron density is assumed to be constant. It also includes an interpolation module relying on the Kriging technique to obtain the relevant vertical TEC at the grid-points (Orús et al., 2005). A remarkable feature of TOMION is that no background model is used.



# IRTAM

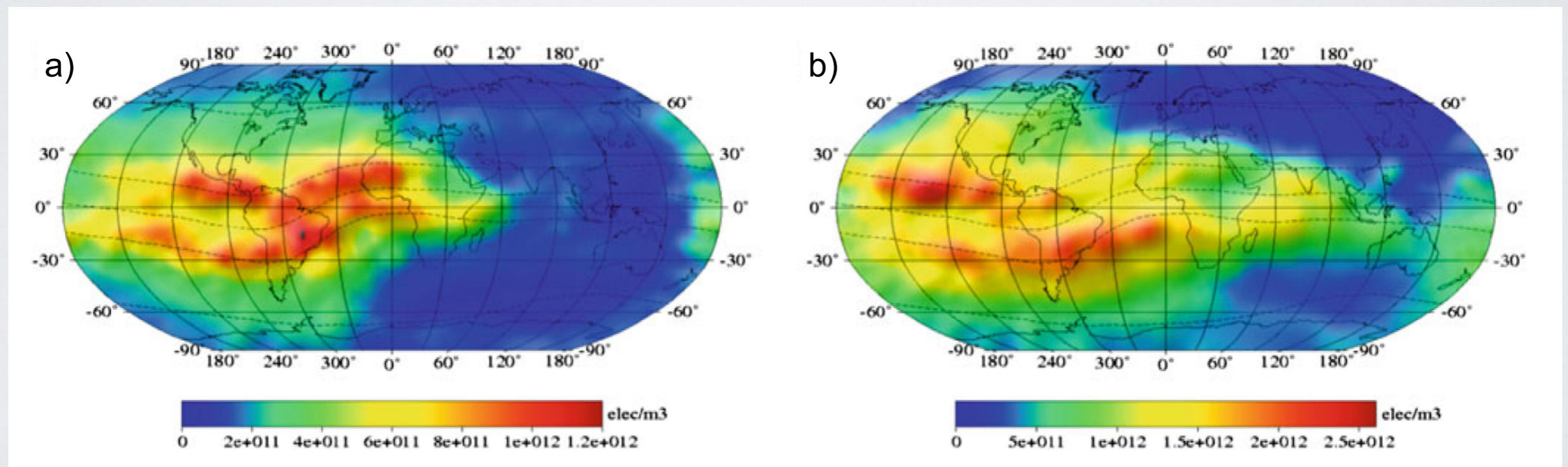
- IRI Real Time Assimilative Model (IRTAM) [Galkin, I. A., et al. 2012], has been developed to assimilate Global Ionosphere Radio Observatory (GIRO) data (foF2, hmF2) in order to “update” the IRI electron density distribution, while preserving the IRI’s typical ionospheric feature representations.
- The technique calculates the corrected coefficients for the spherical/diurnal expansion used by the CCIR-67/URSI-88 model to specify the global foF2 maps, and similarly the maps for all other IRI profile parameters.





# LPIM-AM

- A similar approach has been used by Brunini et al., [2013] in order to update the ITU-R database using radio occultation (COSMIC) electron density profiles.
- For this purpose the La Plata Ionospheric Model (LPIM) (after linearisation) is adjusted by Least Squares to every RO profile available for the time period of interest.



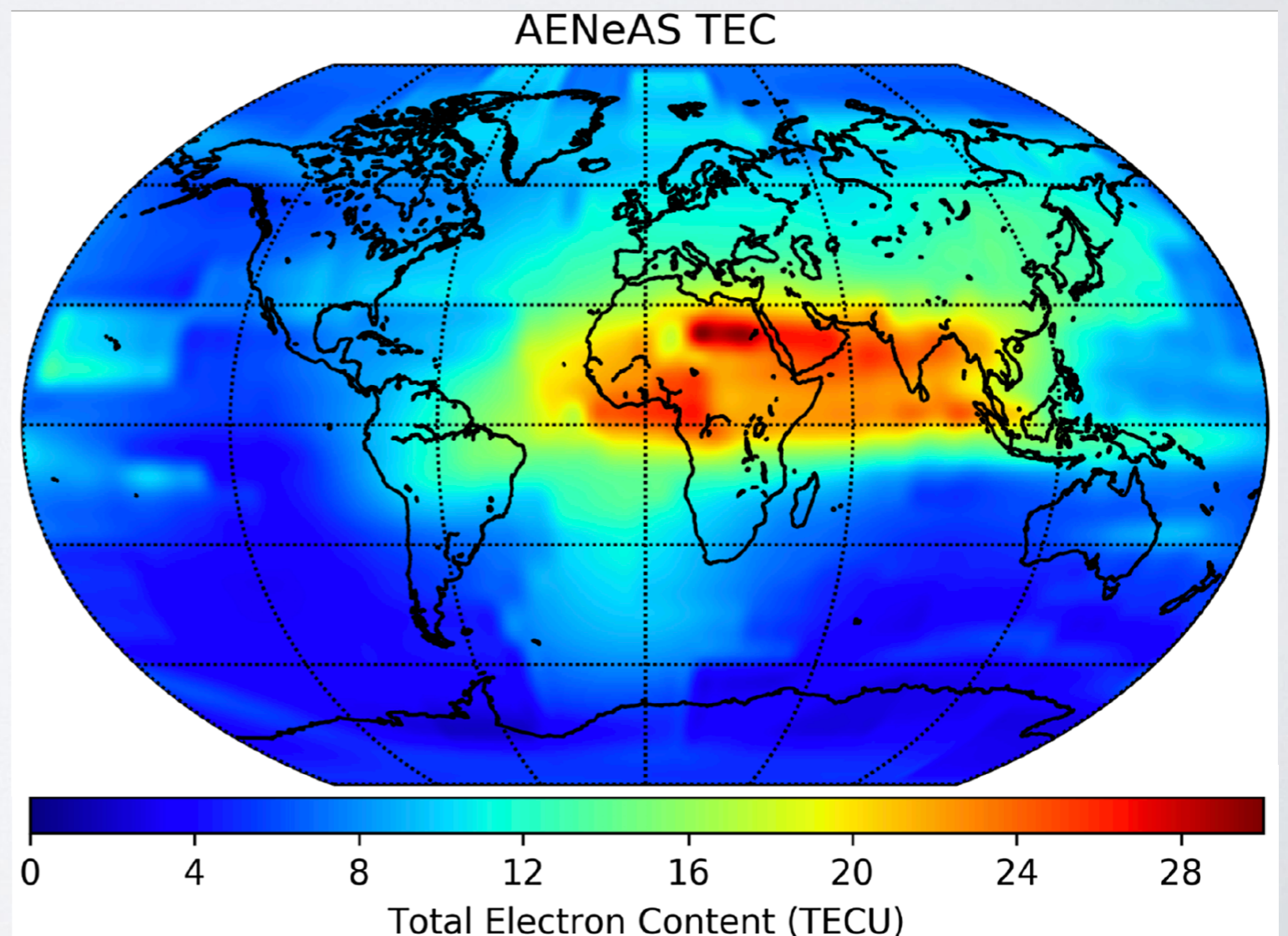
Global representation of the NmF2 estimated value within the 18–20 UT interval;  
a) NmF2 for the 2007 September equinox b) NmF2 for the 2011 December solstice.



# AENeAS

- The Advanced Ensemble electron density (Ne) Assimilation System (AENeAS; Elvidge and Angling, 2019) is a physics-based data assimilation model of the ionosphere/thermosphere.
- The background model is the TIE-GCM and the ionospheric data (mainly TEC) are assimilated by using the Local Ensemble Transform Kalman filter (LETKF).
- Since the TIE-GCM only reaches 600 km of height, in order to extend the electron density grids up to 25000 km, AENeAS currently adopts the NeQuick model topside.

AENeAS TEC map for June 5th at 1230 UT after slant TEC DA (from: Elvidge and Angling, 2019).





# Use of effective parameters

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- In order to incorporate experimental data into ionospheric models, methodologies relying on the use of effective parameters have also been implemented.
- During past years several solar indices have been developed to relate the response of the ionosphere to solar Extreme Ultraviolet (EUV) emissions.
- Indices like sunspot number (SSN), solar radio noise flux at 10.7 cm wavelength (F10.7) or smoothed sunspot numbers (R12) have become standard inputs for many ionosphere electron density models.
- Nevertheless, these solar indices are far from ideal proxies for the solar activity in the EUV part of the solar radiation spectrum.
- The difficulties found when applying these solar-based indices, led to the development of a number of "effective" indices, which are based on the use of models and experimental ionospheric data.
- One of the first effective parameter that has been proposed is the "effective sunspot number" (SSNe).
- More recently, other effective solar indices have been developed utilizing different kinds of models and observations.

# Use of effective parameters

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- The first works concerning the implementation of effective solar indices by means of electron density models adaptation to TEC data are described in the papers by
  - Komjathy et al. (1998), where IRI is used to infer an IG12 index by using GPS-derived vertical TEC maps;
  - Hernandez-Pajares et al. (2002), where IRI is fitted directly to slant TEC observations by tuning the SSN input.
- An example related to the use of TEC models is given by Center for Orbit Determination in Europe (CODE) of the University of Bern, where Klobuchar-style ionospheric coefficients ("effective" alphas and betas) are computed by fitting the CODE vertical TEC maps with the Klobuchar model.
- An exhaustive historical and critical review of the methodologies using effective solar indices to update climatological ionospheric models is given in Pignalberi et al. (2018).
- Some of these techniques will be considered in the following slides.....



# vTEC map data ingestion

At a given epoch

One vTEC map



Minimize the mismodelings  
 $|vTEC_{exp_i} - vTEC_{mod}(az)_i|$



Az (effective F10.7) grid



Use NeQuick to reconstruct the 3D electron density of the ionosphere that reproduces the starting vTEC map

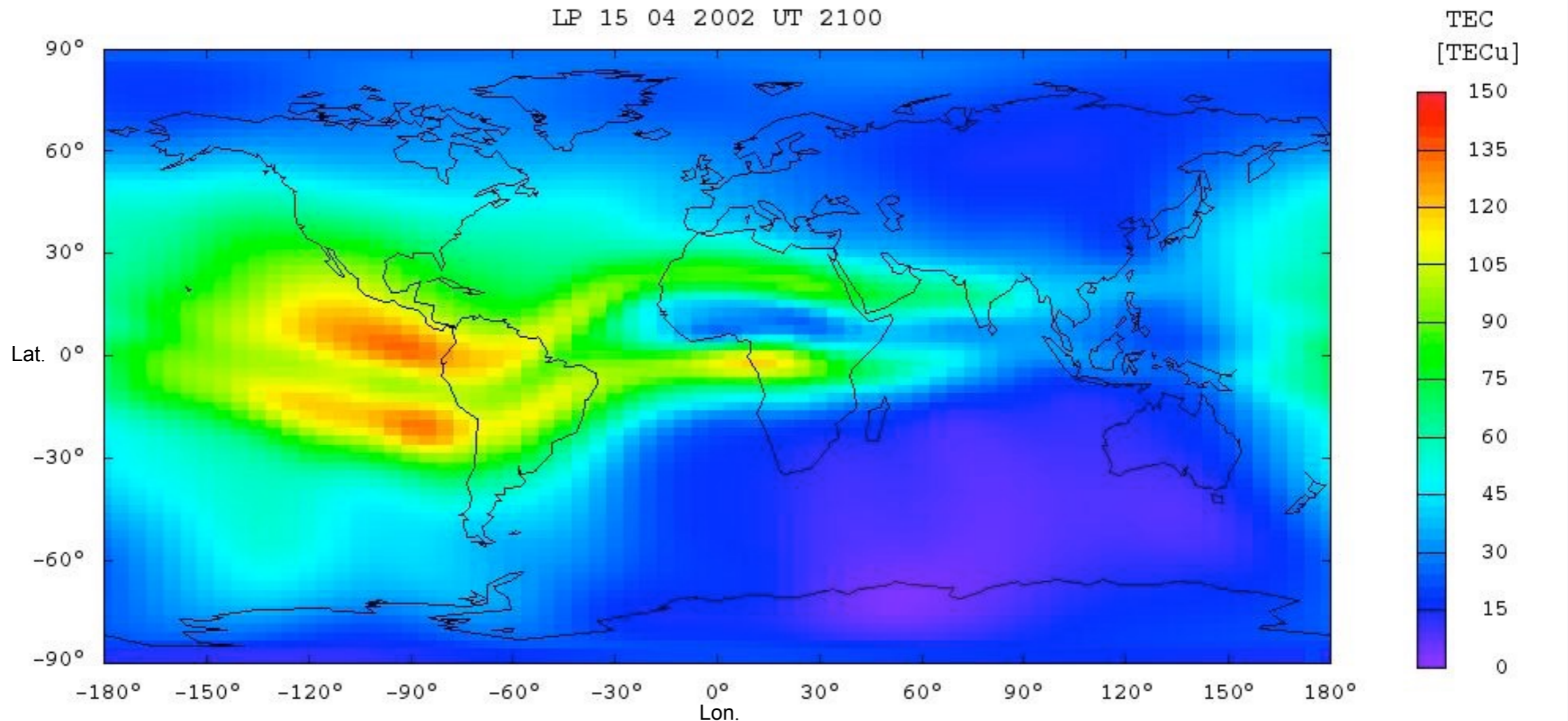


Reconstruct sTEC  
along any ray-path



Reconstruct  
foF2 maps

# vTEC map



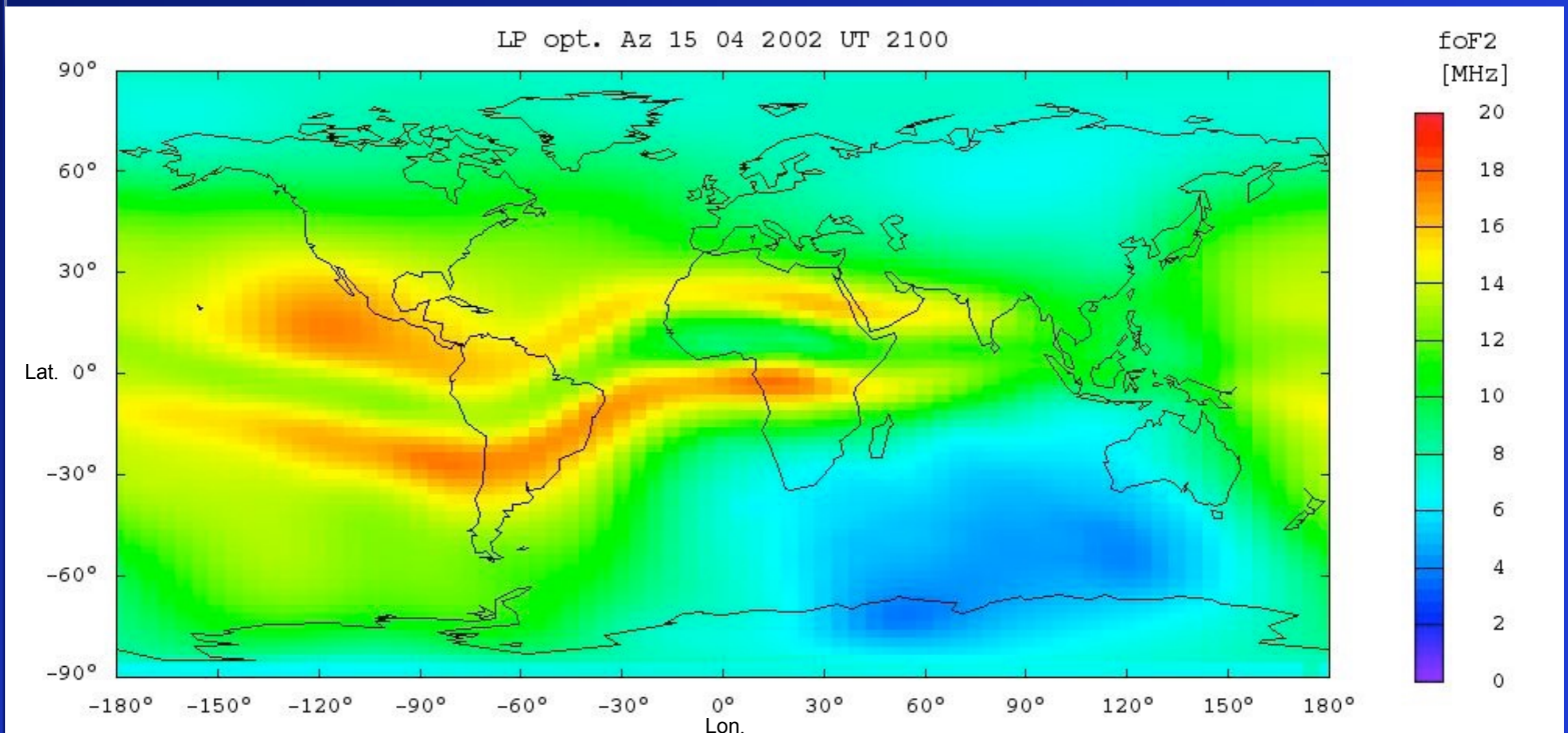
grid points:

lat. =  $-90^\circ$ ,  $90^\circ$  step  $2.5^\circ$

lon. =  $-180^\circ$ ,  $180^\circ$  step  $5^\circ$



# Reconstructed foF2 map



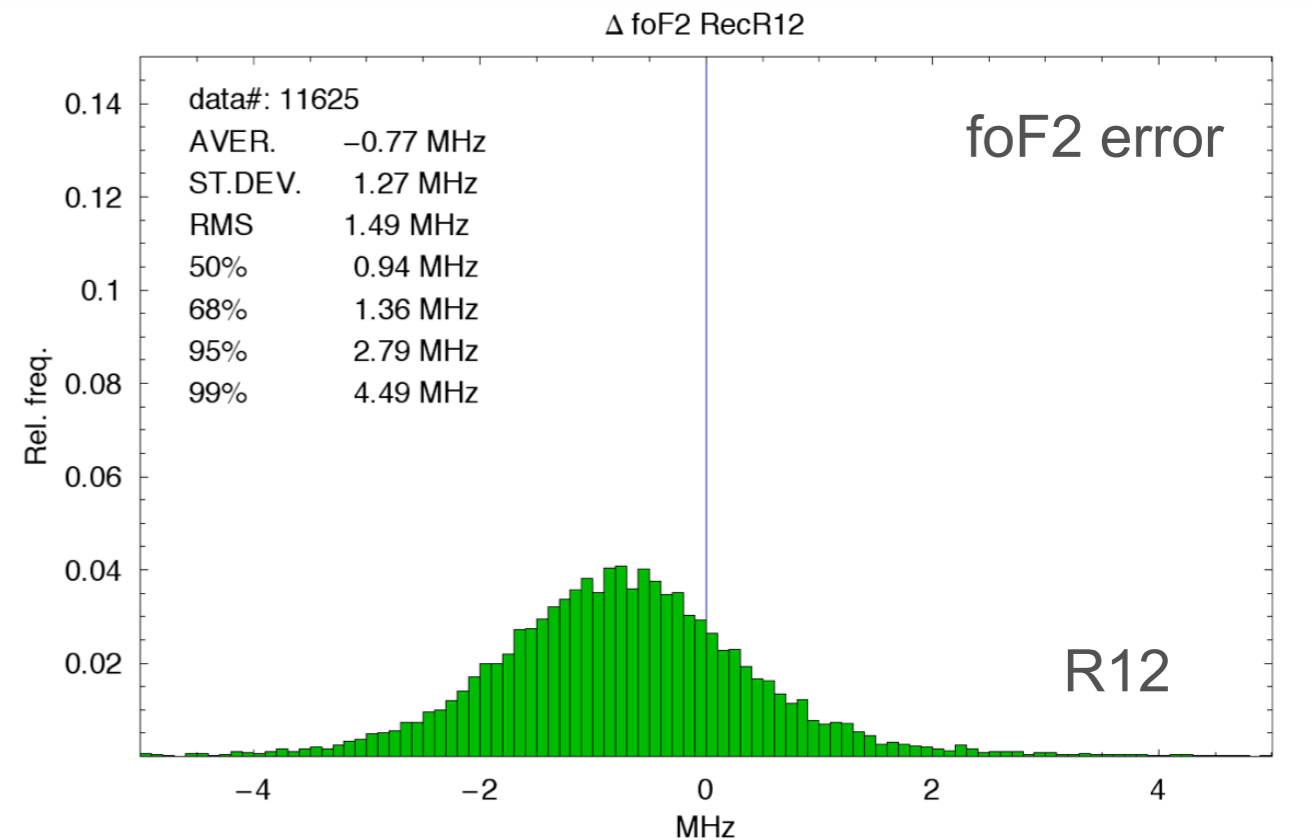
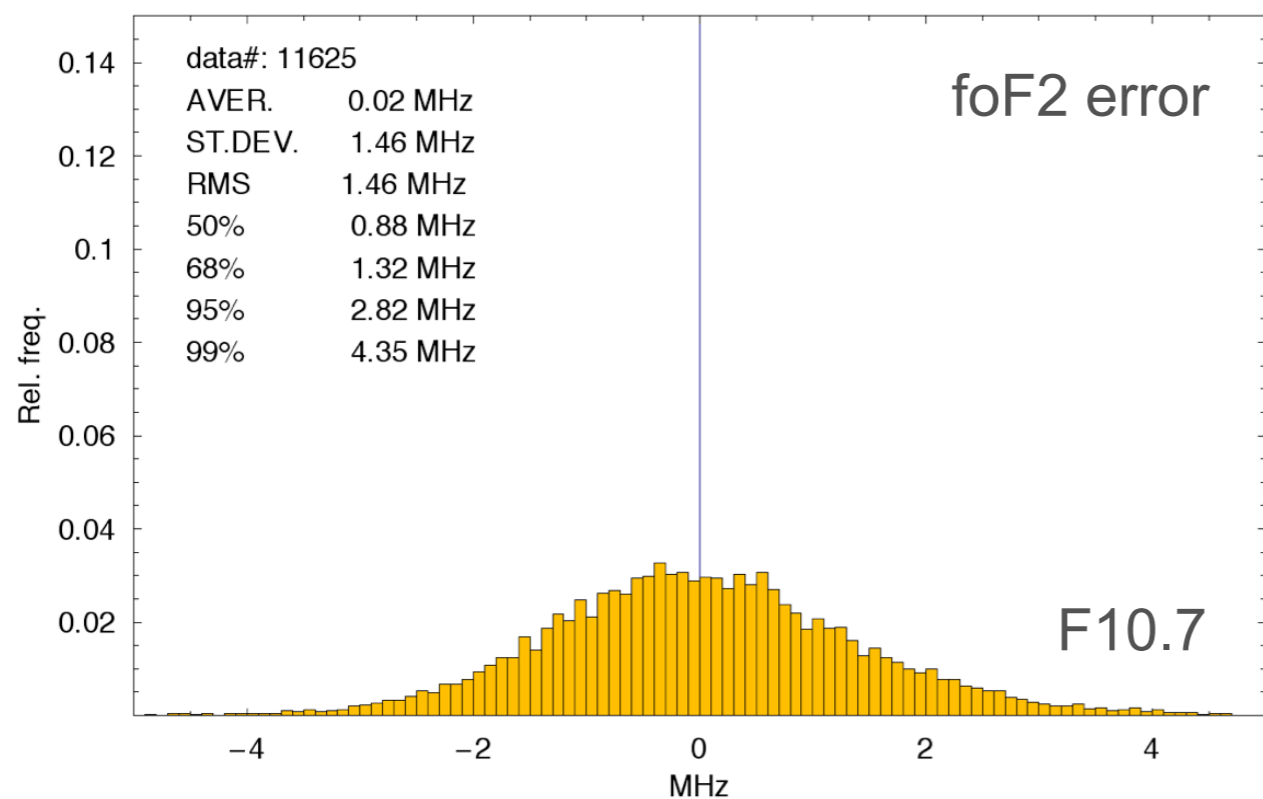
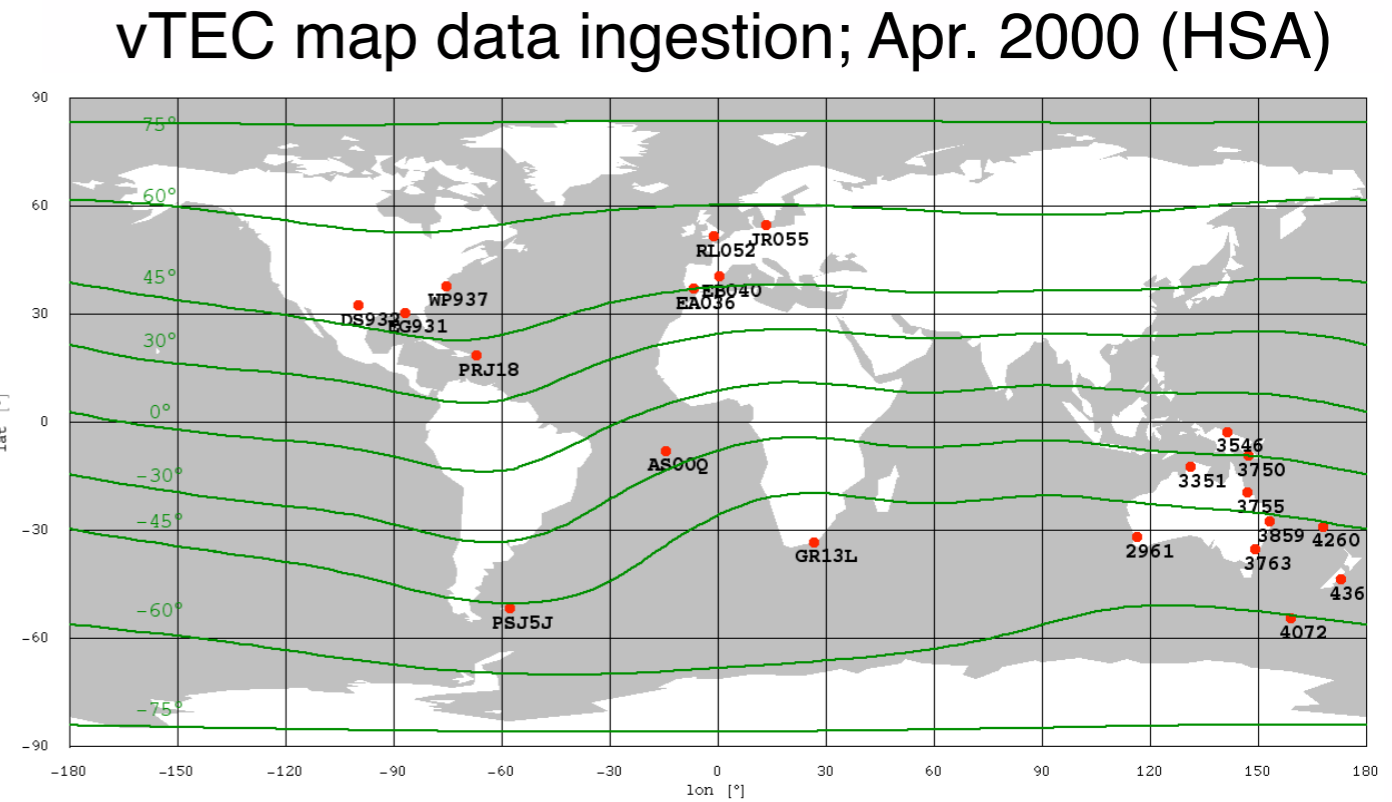
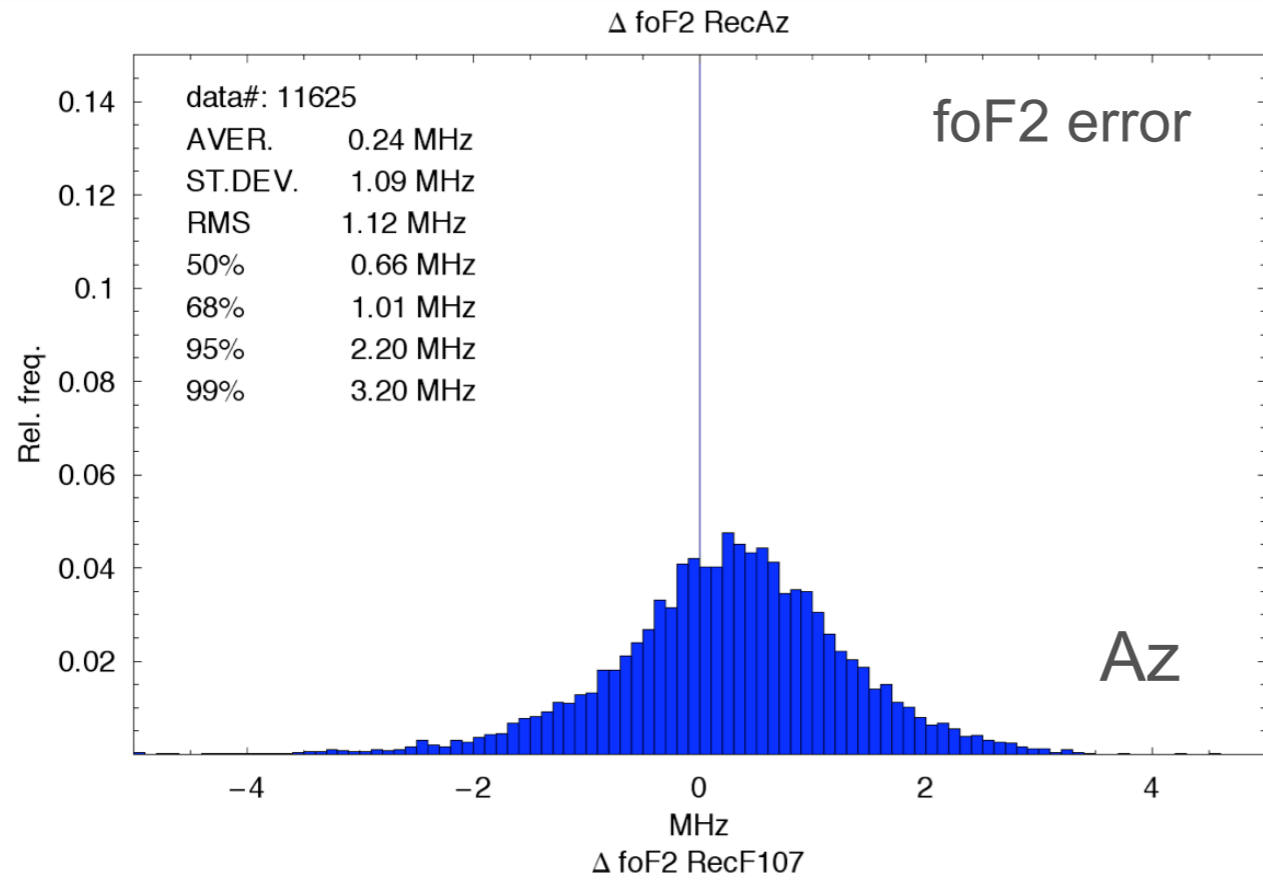
grid points:

lat. =  $-90^\circ$ ,  $90^\circ$  step  $2.5^\circ$

lon. =  $-180^\circ$ ,  $180^\circ$  step  $5^\circ$

# Vertical TEC map data ingestion

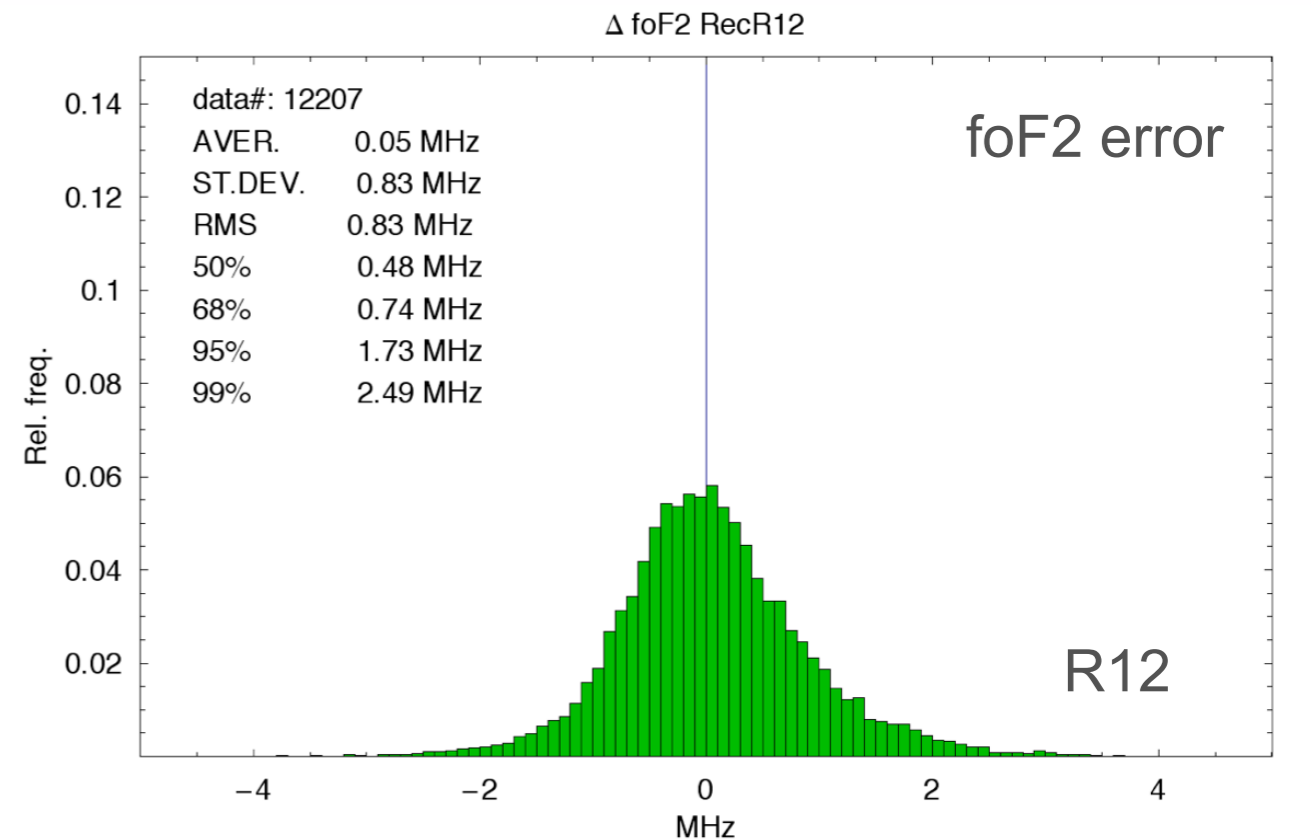
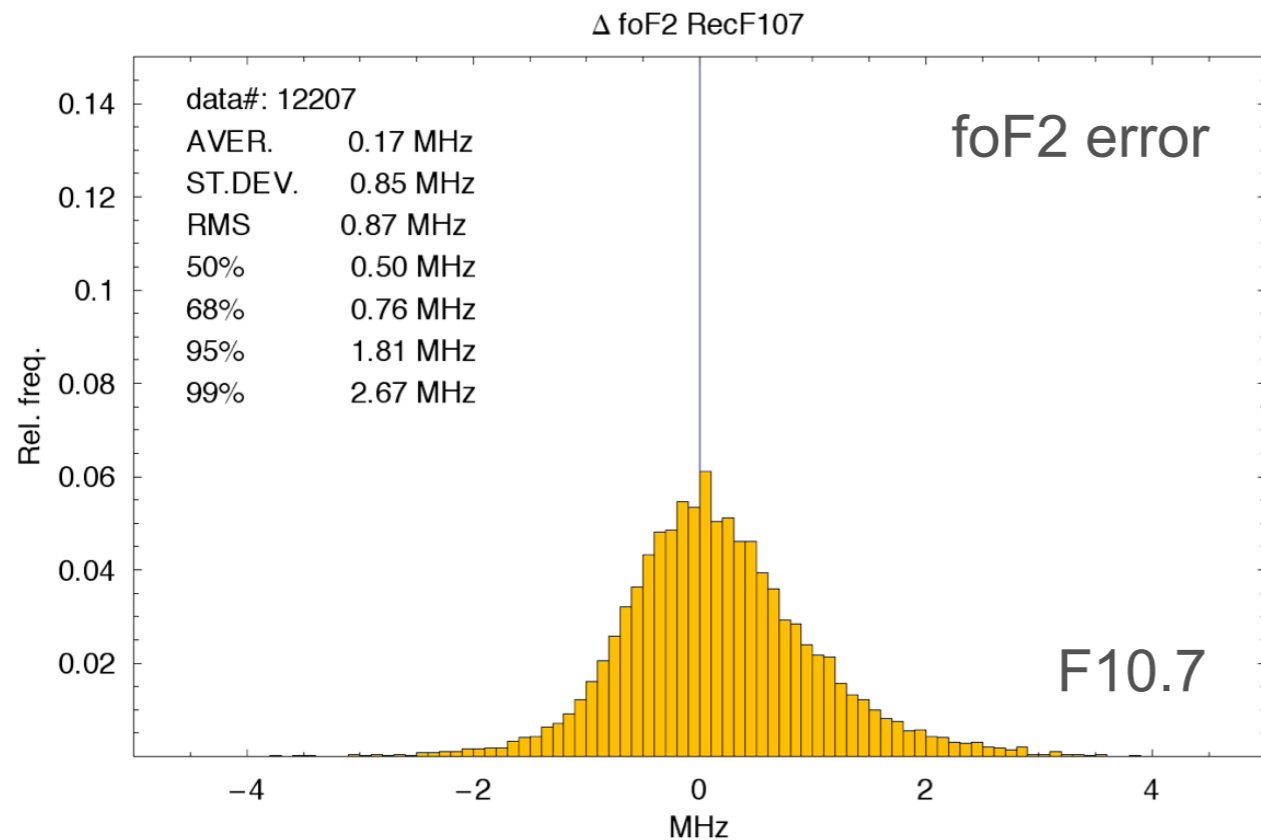
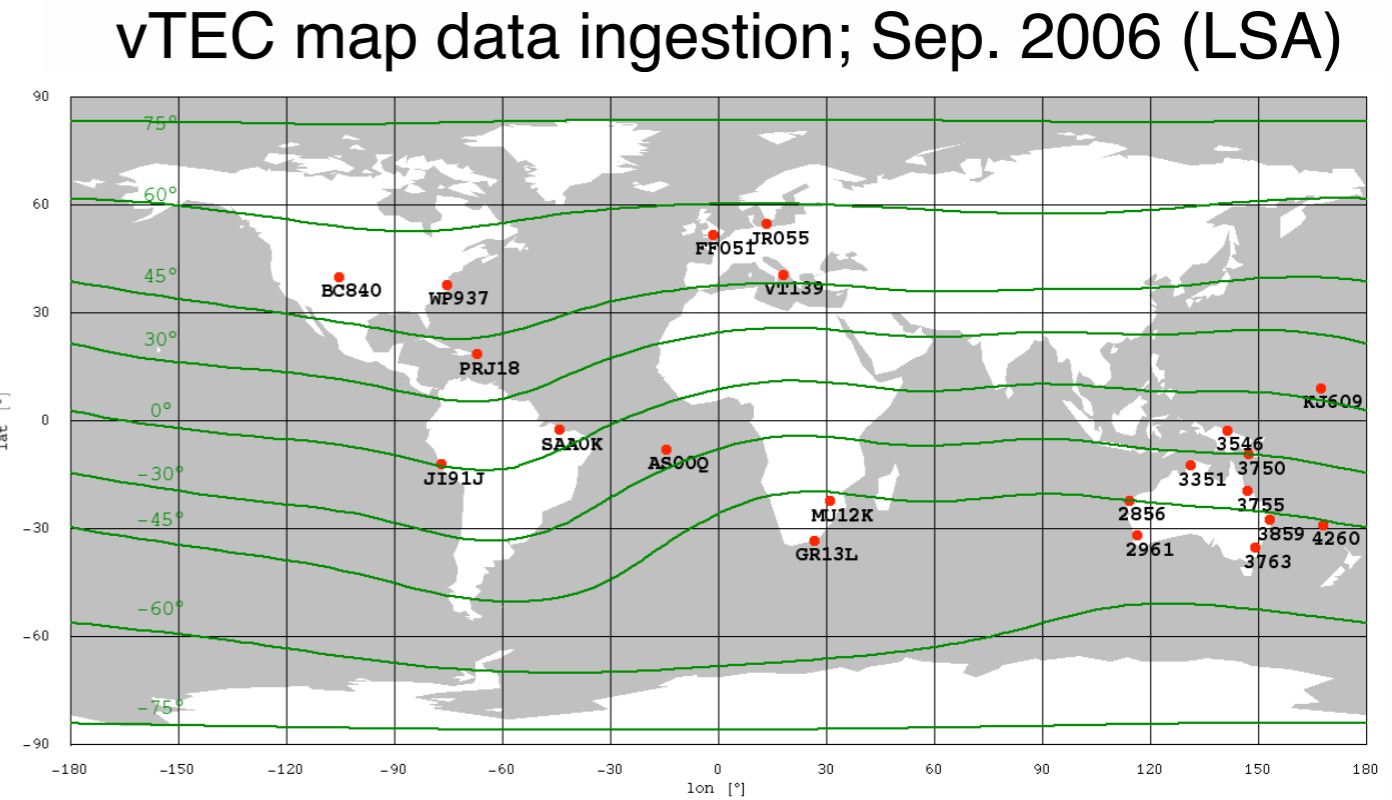
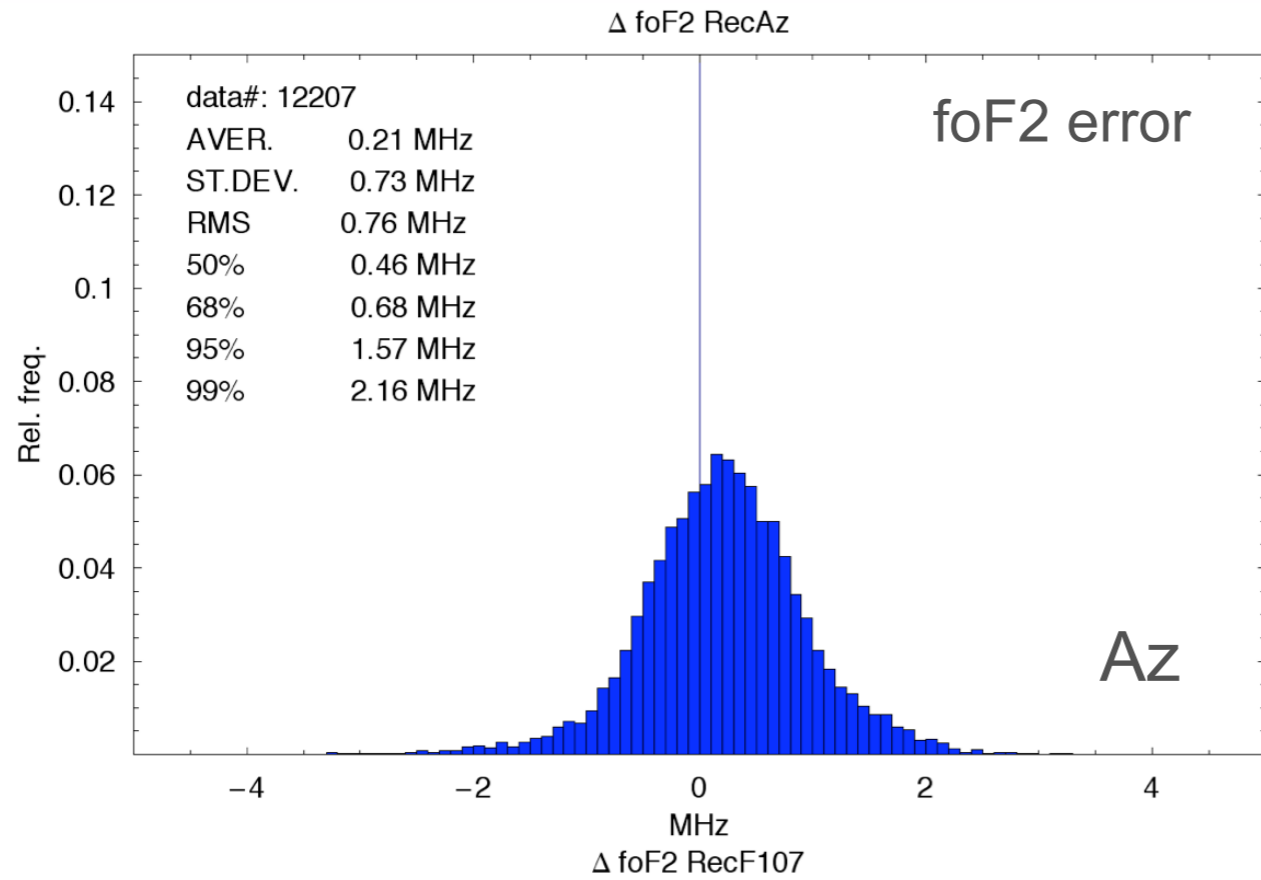
High Solar Activity



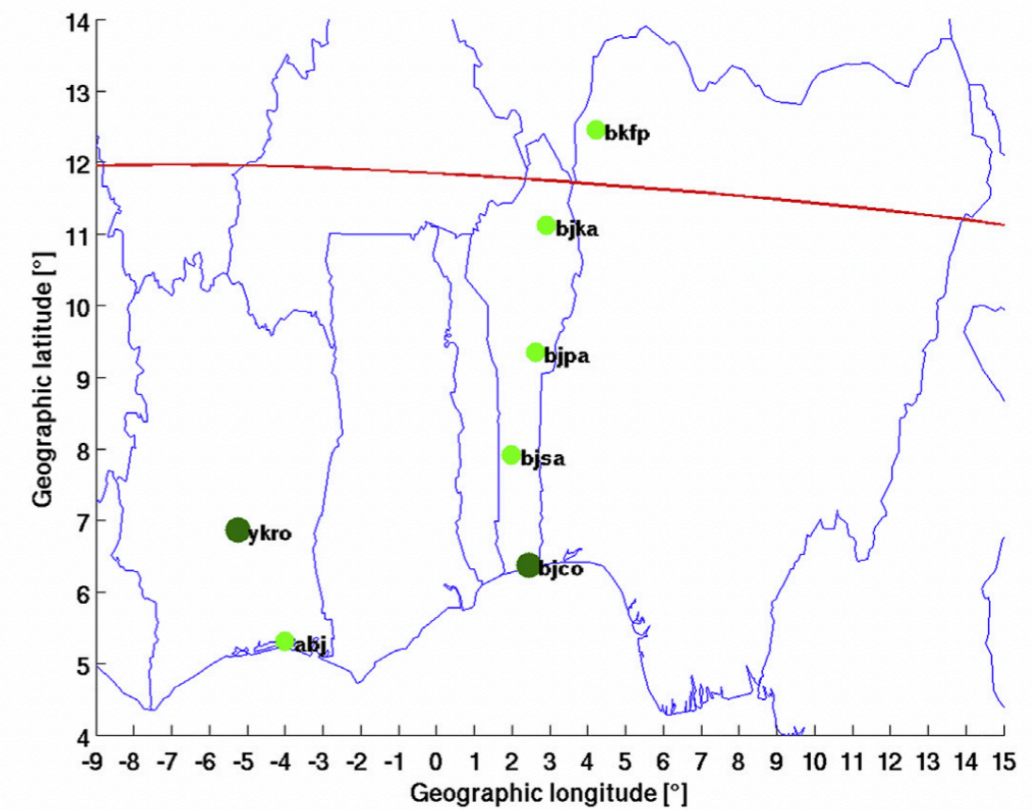
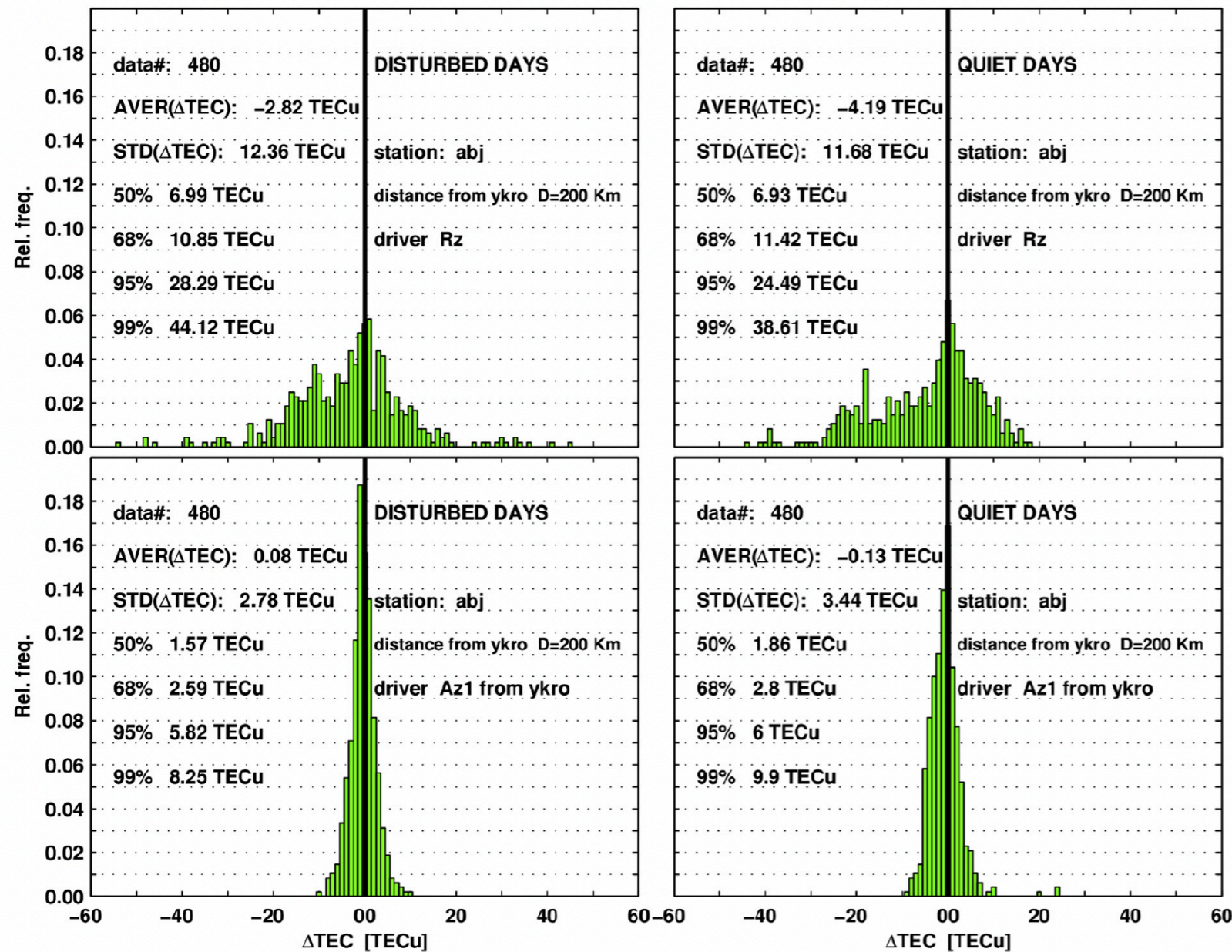


# Vertical TEC map data ingestion

Low Solar Activity



# vTEC ingestion; geomagnetically disturbed period

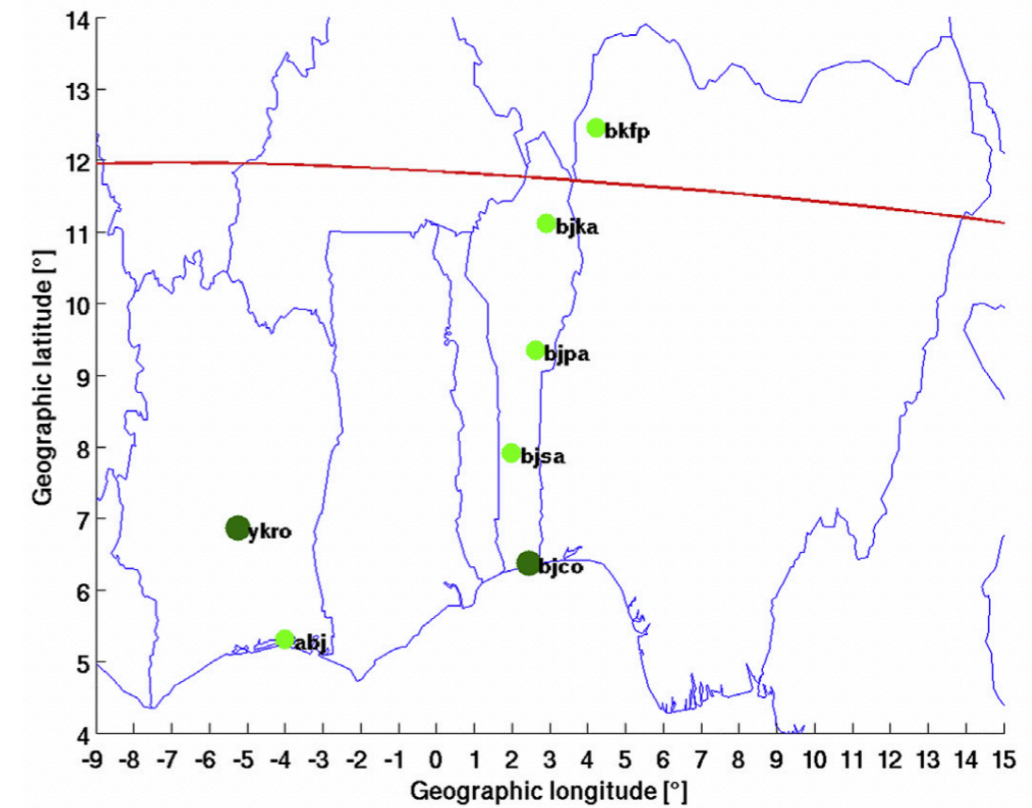
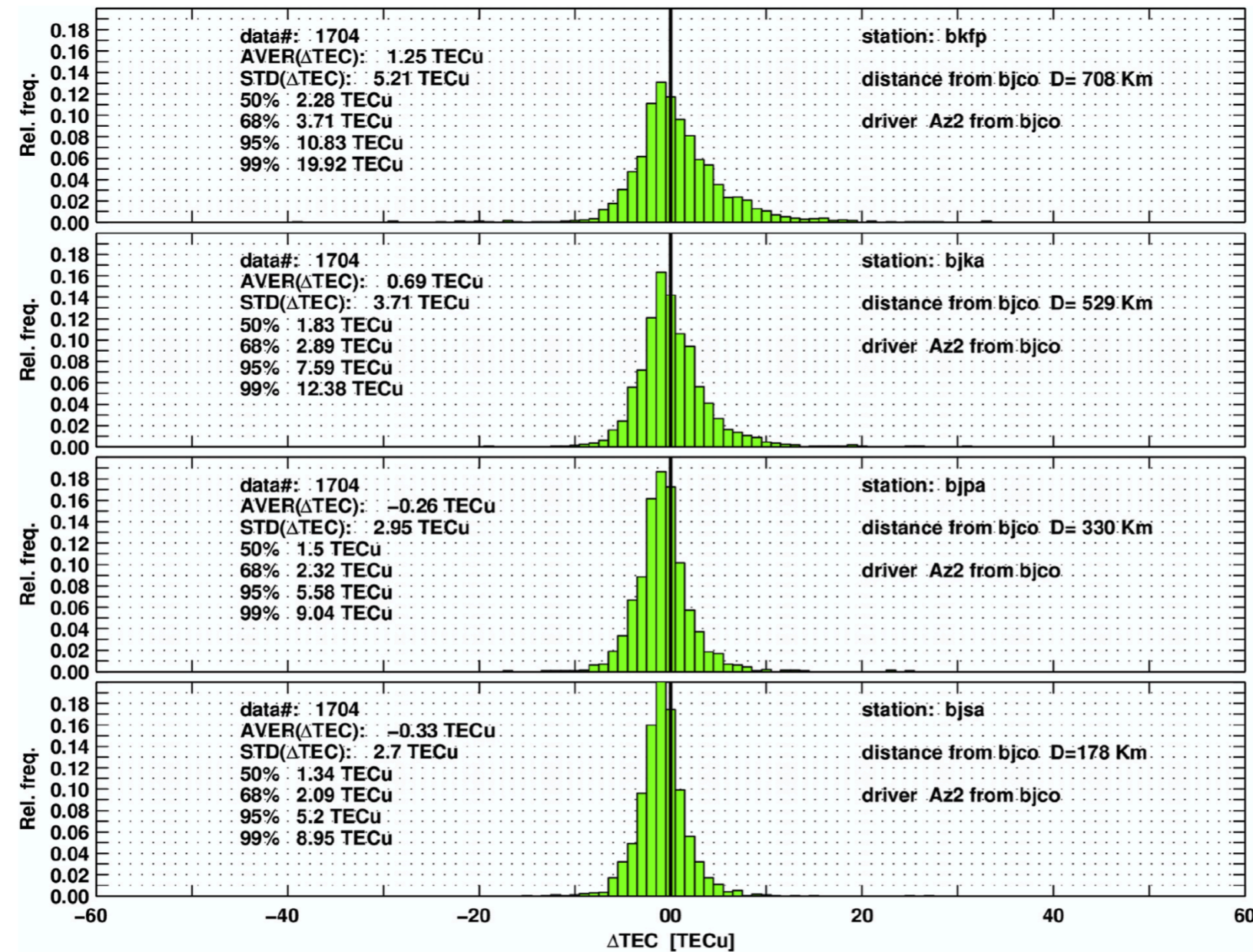


Yao et al., 2018

Distribution of the differences between modeled and experimental vertical TEC during geomagnetically disturbed days (left) and quiet days (right) in 2015 at abj station. NeQuick 2 is driven by the daily sunspot number Rz (top) and the effective solar flux Az1 as inferred from ykro data (bottom).



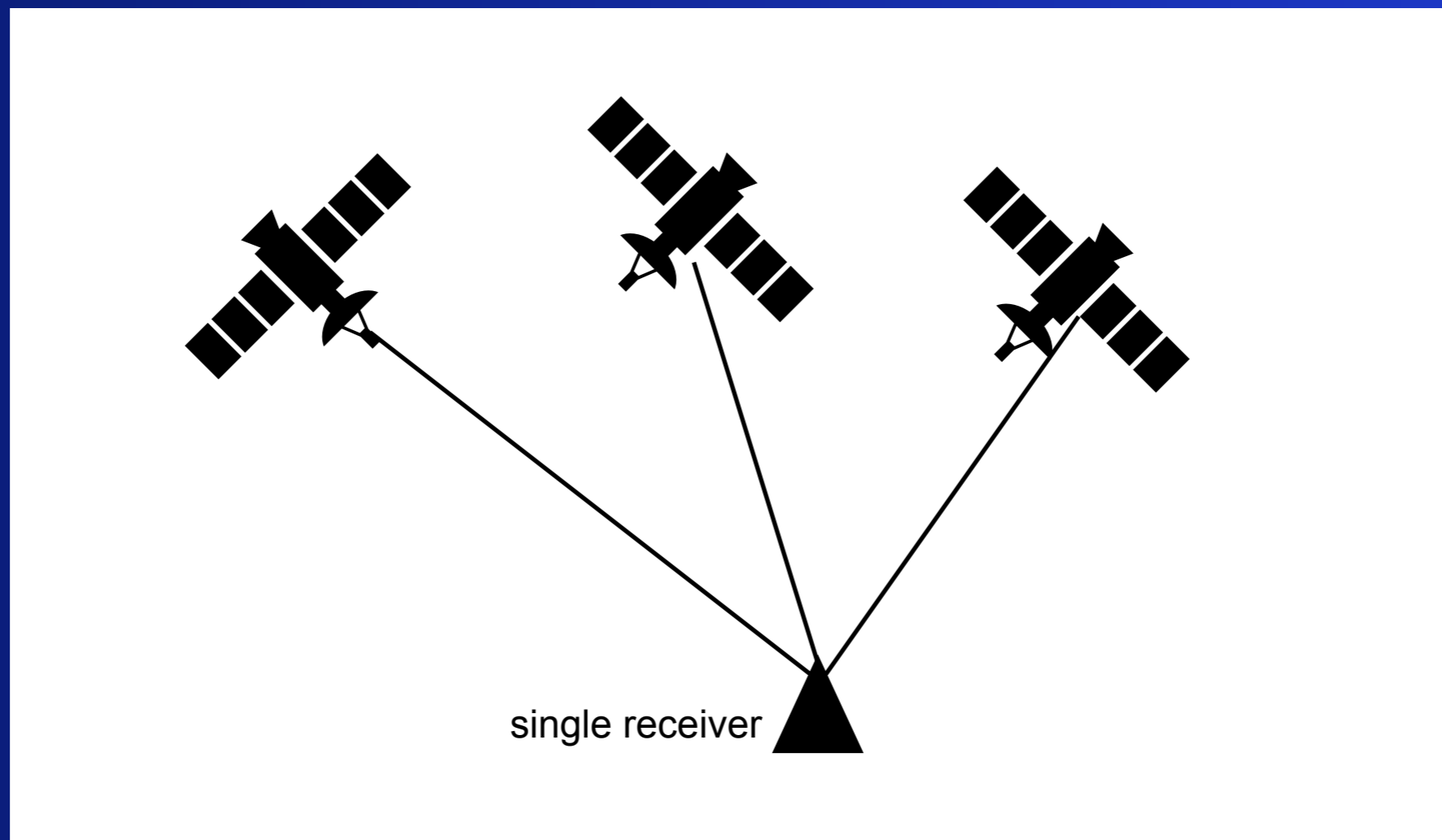
# vTEC ingestion; correlation with distance



Yao et al., 2018

Distribution of the differences between modelled and corresponding experimental vertical TEC when NeQuick 2 is driven by the effective solar flux Az2 as inferred from *bjco* data during the years 2014-2015.

# Adapting NeQuick model to experimental slant TEC data at a given location





# sTEC data ingestion, single stat.

At a given epoch

One station, n experimental sTEC (n satellites)



Minimize RMS of the TEC mismodelings  
as a function of (formally) F10.7



Az (effective F10.7) at the station, for the given epoch



Use NeQuick to retrieve (locally) the 3D  
electron density of the ionosphere

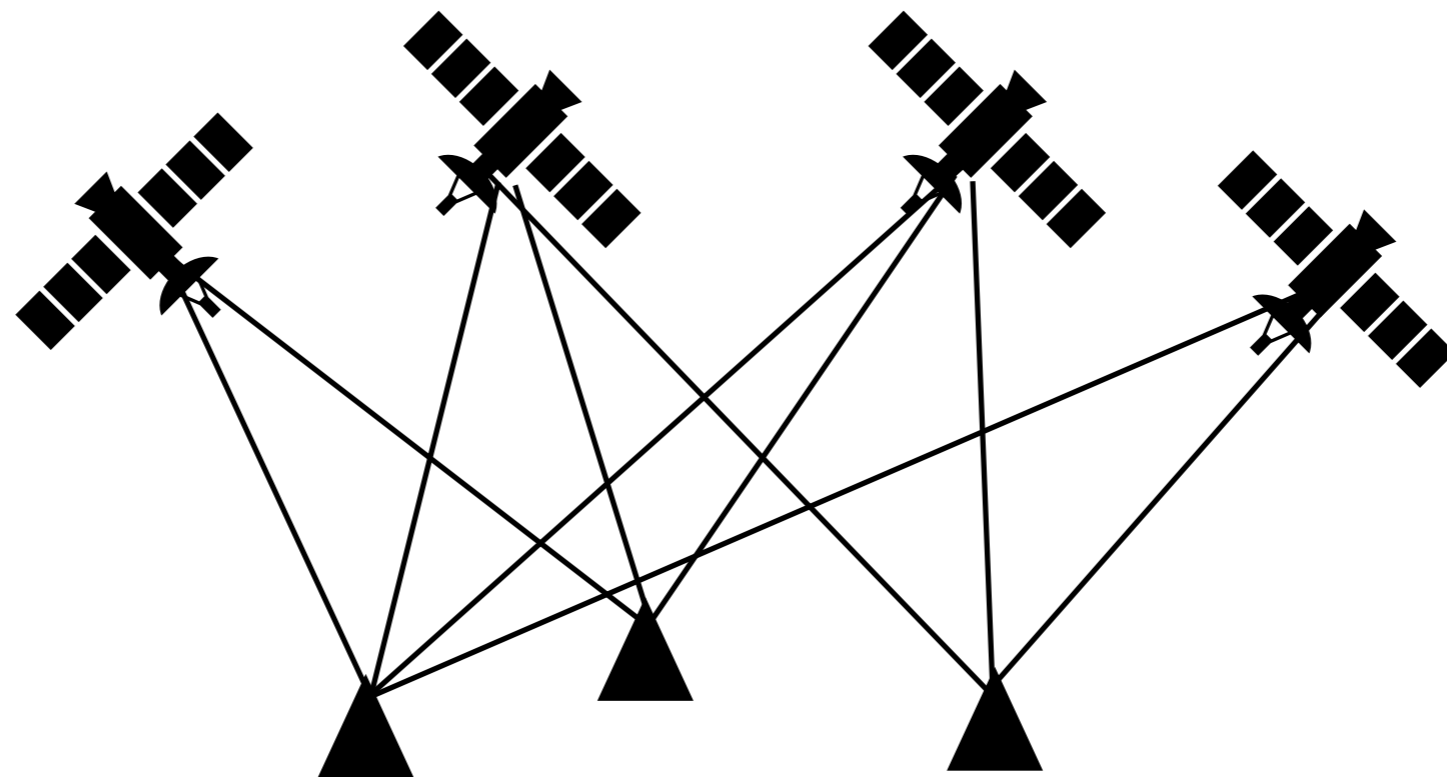


Reconstruct TEC along any  
Station-to-satellite ray-path



Retrieve the foF2 values  
at the Station

# Adapting NeQuick model to experimental slant TEC data at several locations



multiple receivers (Az grid)



# sTEC data ingestion, multi stat.

At a given epoch

$m$  sTECexp (several stations & satellites)



Minimize each mismodeling

$$|sTECexp_i - sTECmod(az)_i|$$



Scattered  $Az_i \rightarrow$  Interpolate to get regularly spaced grid



Use NeQuick to reconstruct the 3D electron density of the ionosphere

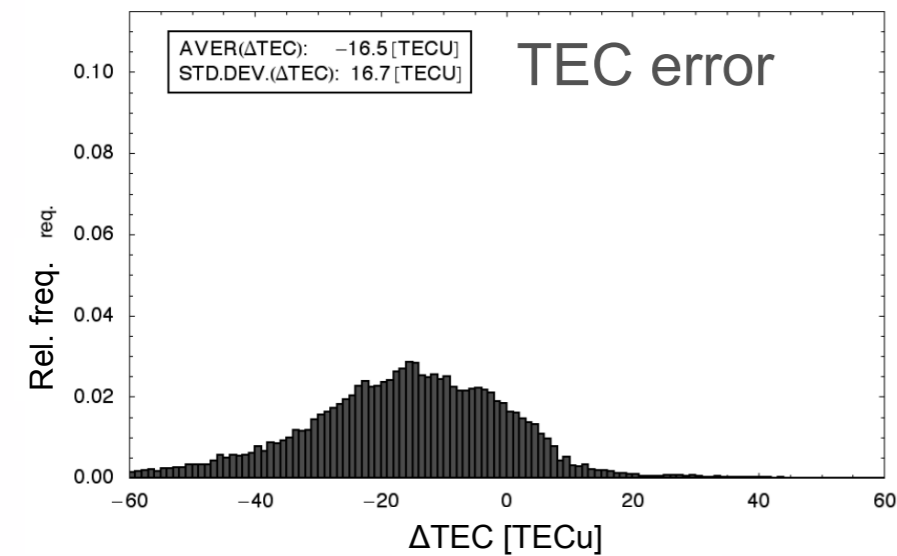


Reconstruct TEC along any given ray-path

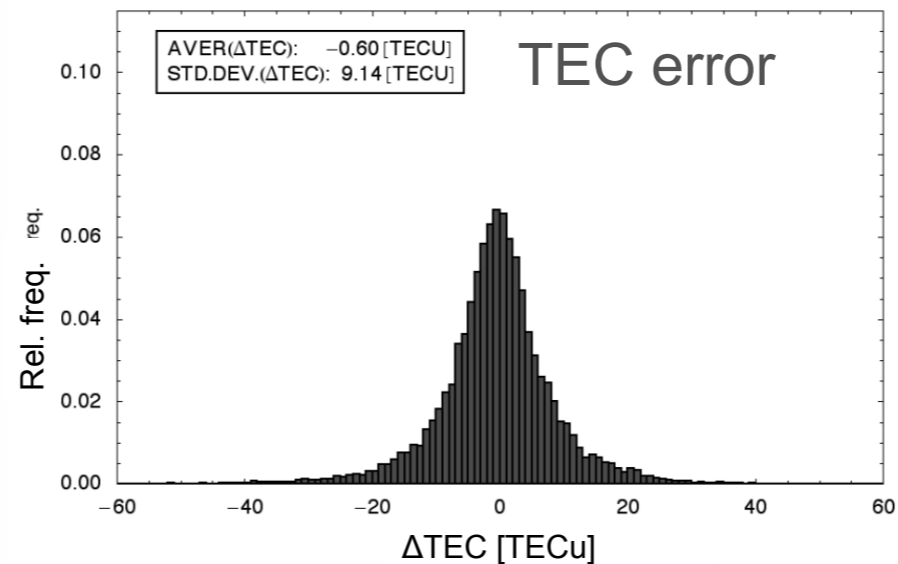


Reconstruct any foF2 value (foF2 map)

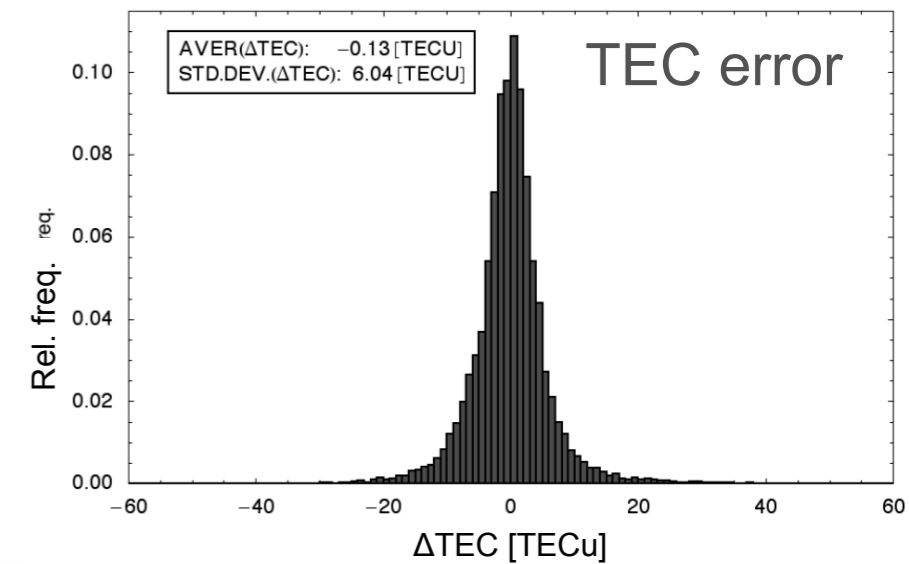
# Slant TEC data ingestion



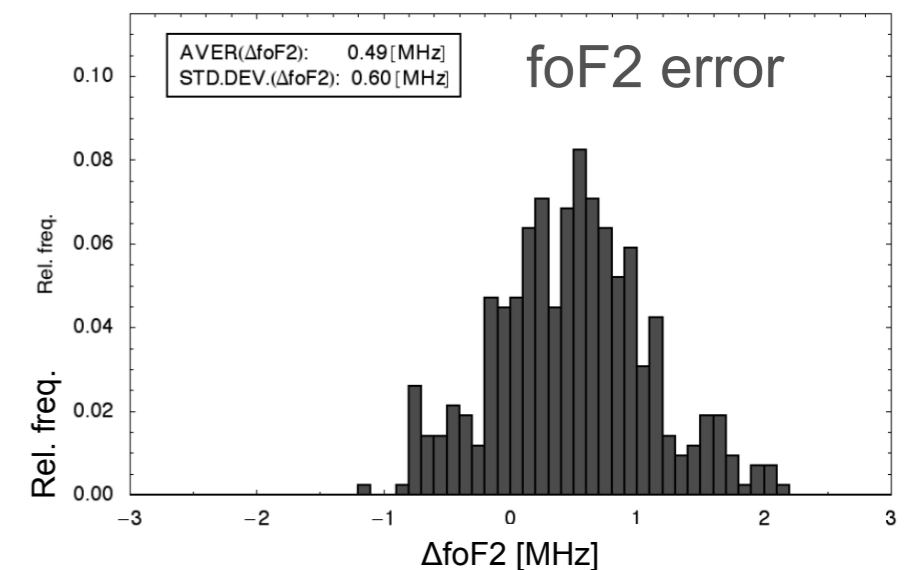
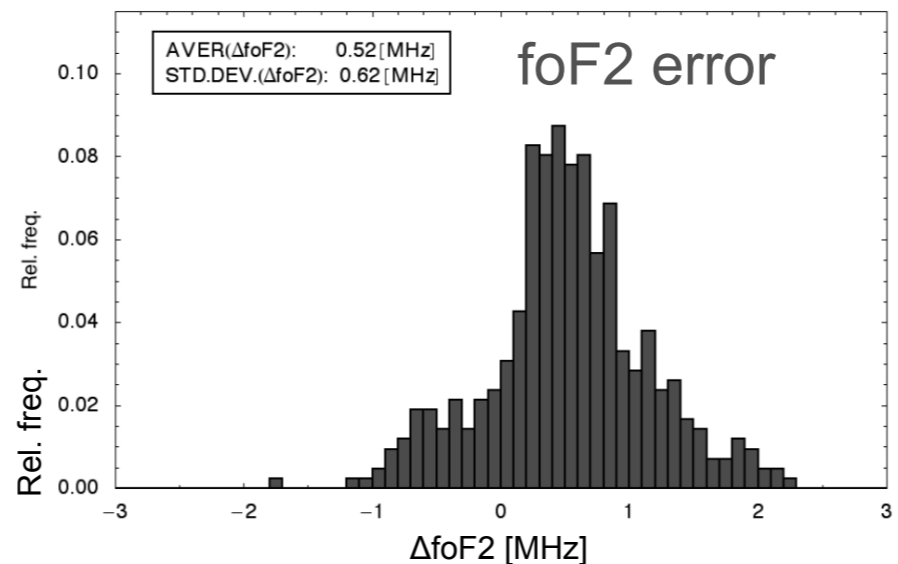
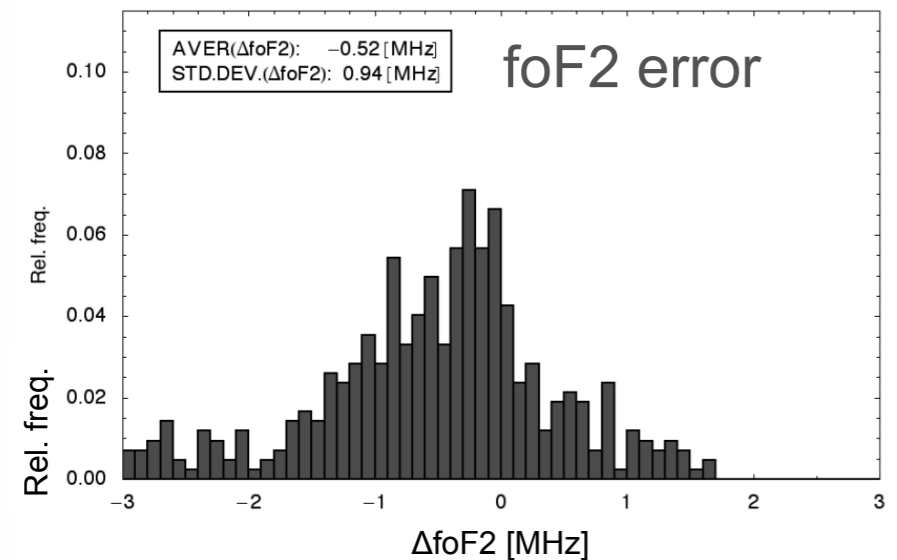
F10.7



Az (single)



Az (multiple)

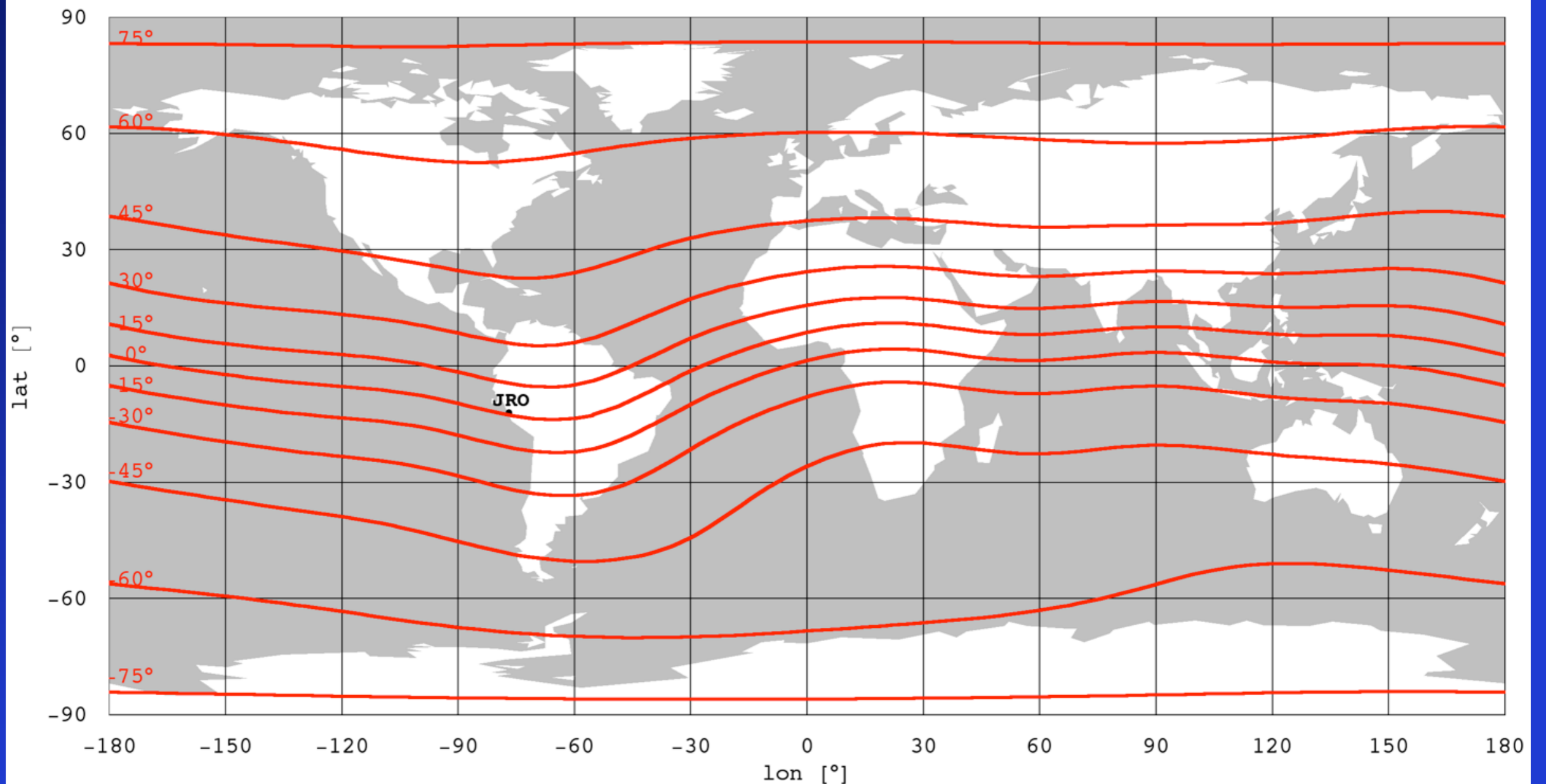


Distribution of the differences between modeled and experimental slant TEC data for 25 ground stations (top panels) and between modeled and experimental foF2 data for six ionosondes (bottom panels) when NeQuick is driven by F10.7 (first column), Az computed using the single (second column) and multiple (third column) technique.



# Adapting NeQuick model to experimental slant TEC and foF2 data at a given location

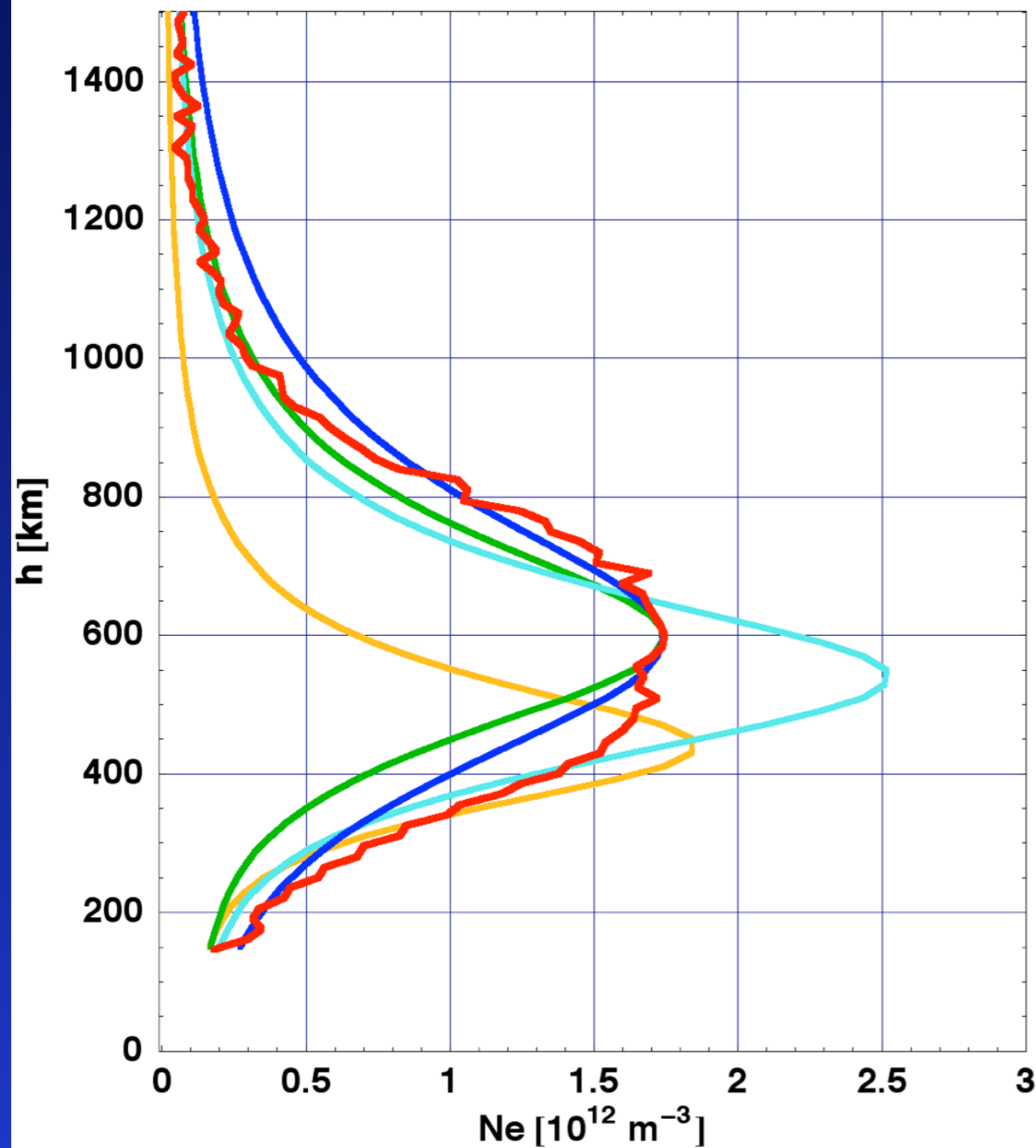
(Use of slab thickness to constrain the NeQuick profile shape parameter)



# Adaptation method validation

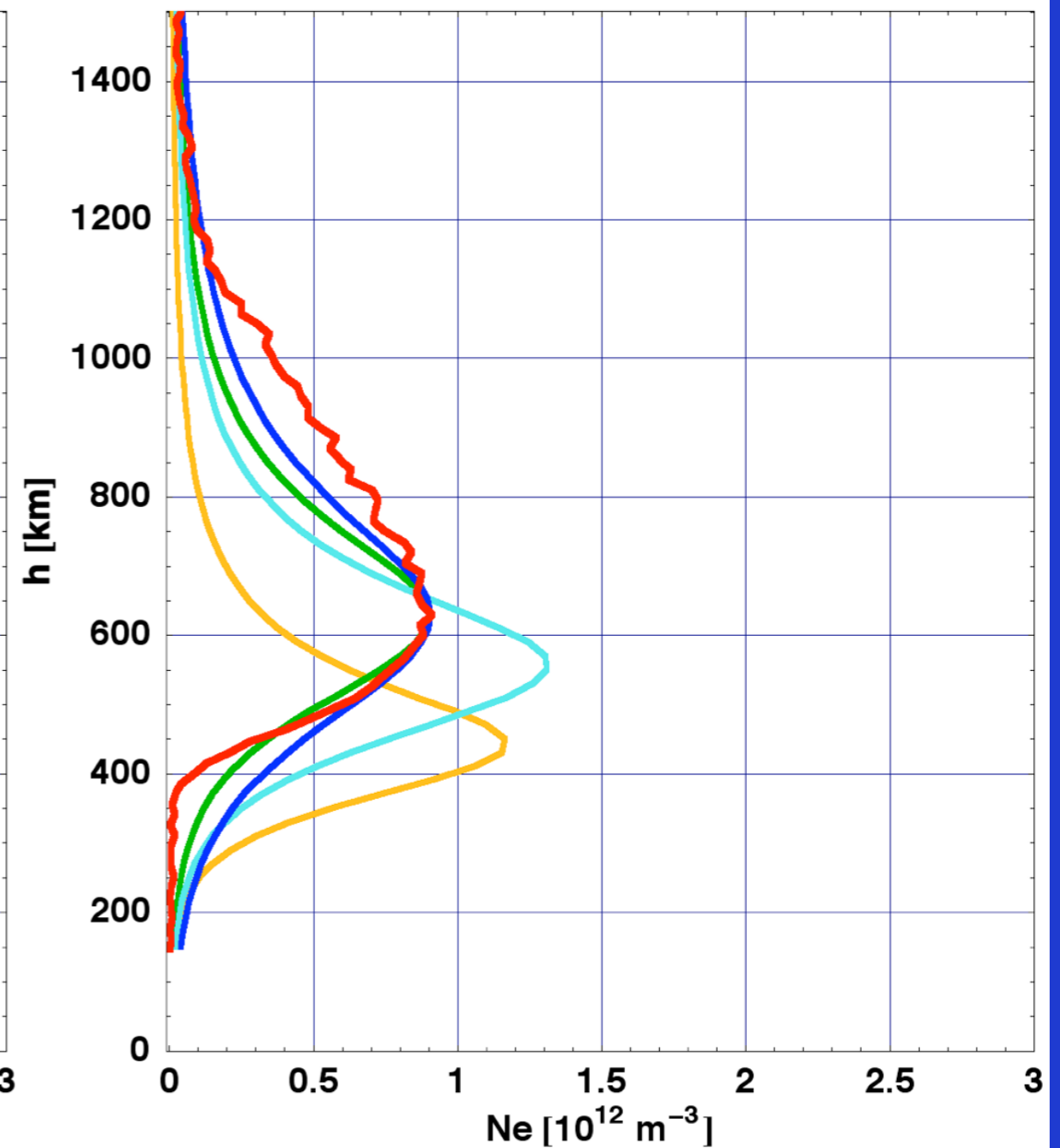
JRO 2000 11 09 UT: 20.83

Exp Std Tec F2 peak F2 peak & TEC



JRO 2000 11 11 UT: 0.03

Exp Std Tec F2 peak F2 peak & TEC



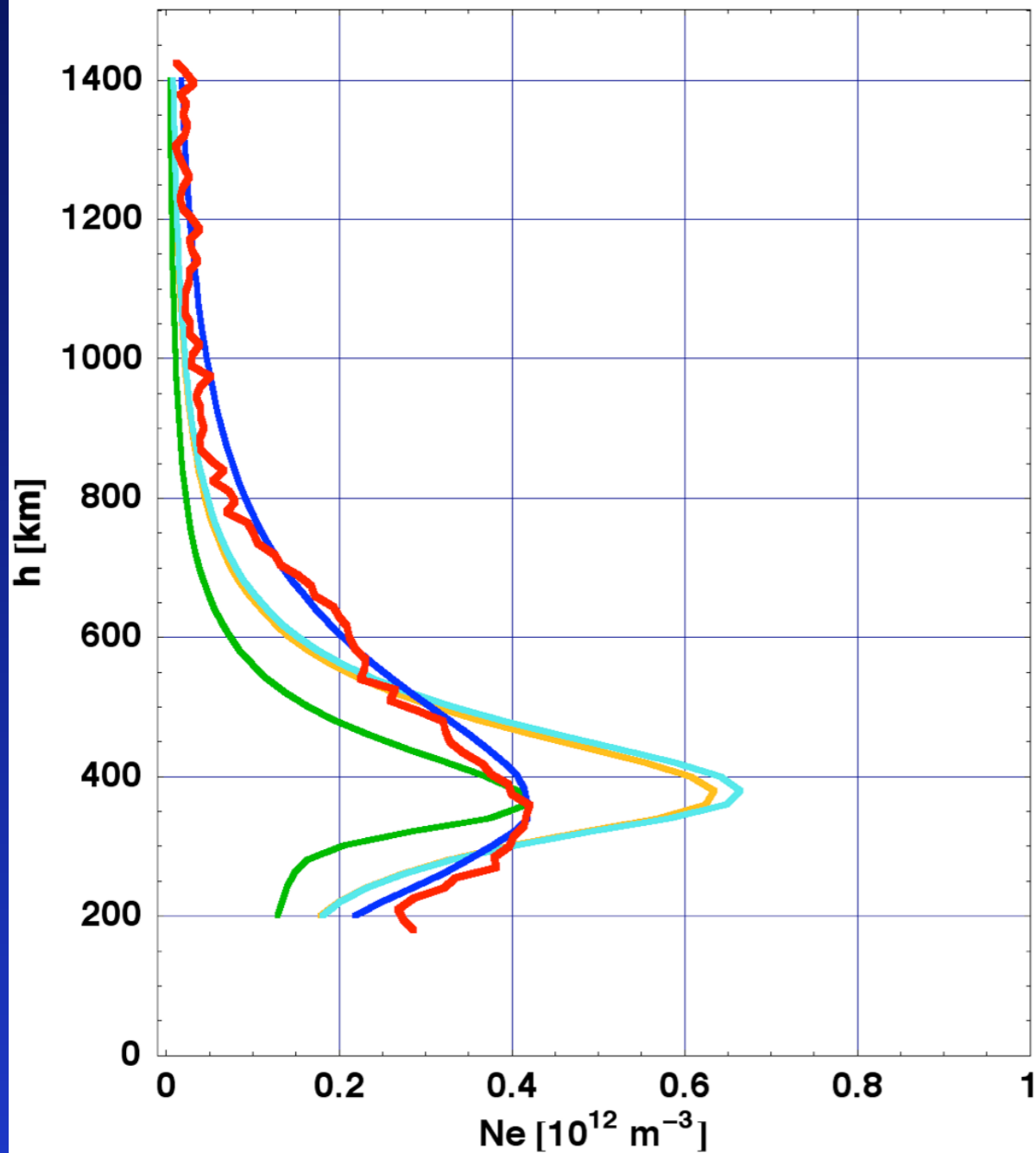
Model: NeQuick



# Adaptation method validation

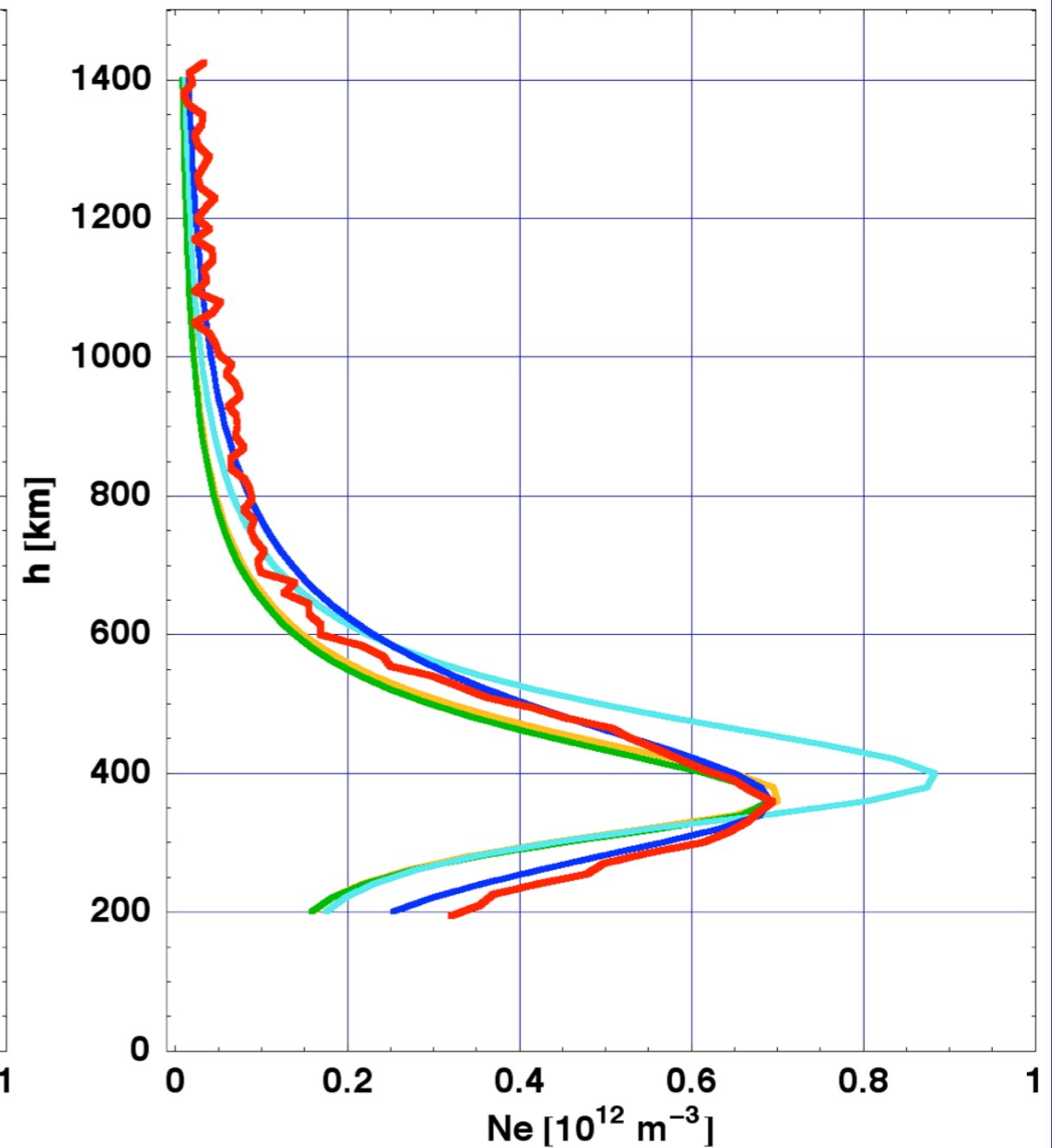
JRO 2006 09 20 UT: 16.18

Exp Std Tec F2 peak F2 peak & TEC



JRO 2006 09 20 UT: 19.43

Exp Std Tec F2 peak F2 peak & TEC

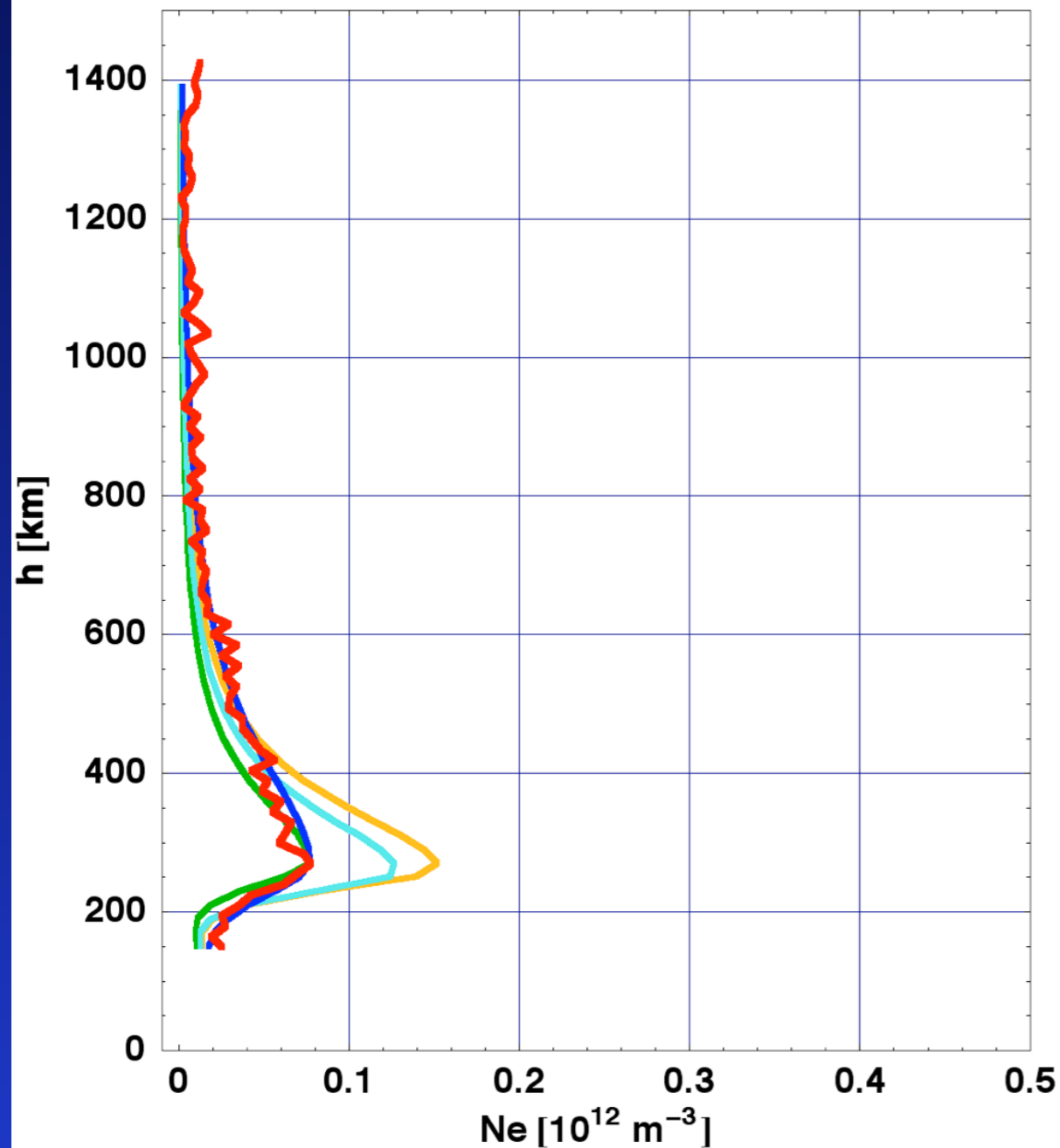


Model: NeQuick

# Adaptation method validation

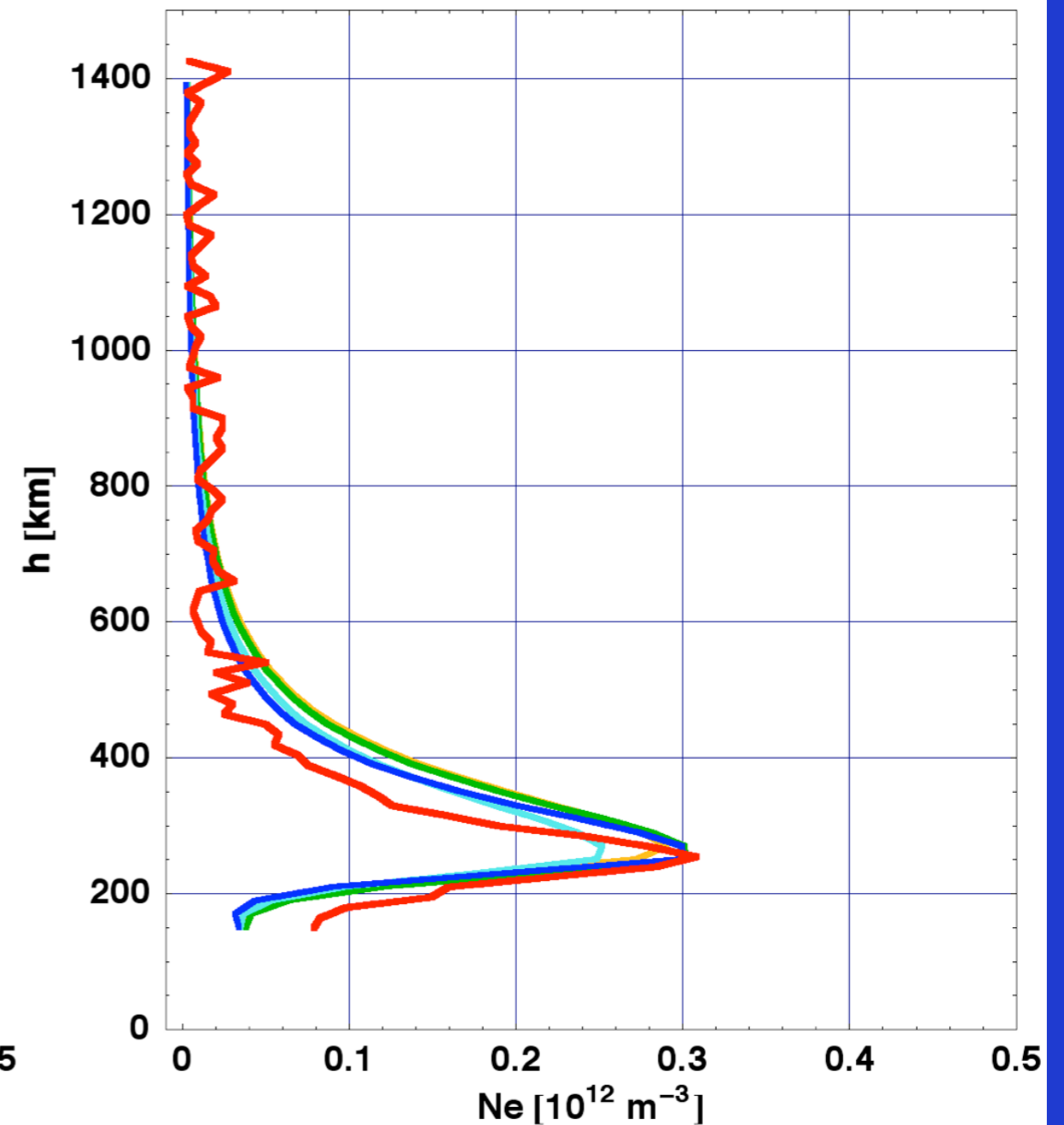
JRO 2006 09 22 UT: 10.80

Exp Std Tec F2 peak F2 peak & TEC



JRO 2006 09 22 UT: 11.55

Exp Std Tec F2 peak F2 peak & TEC



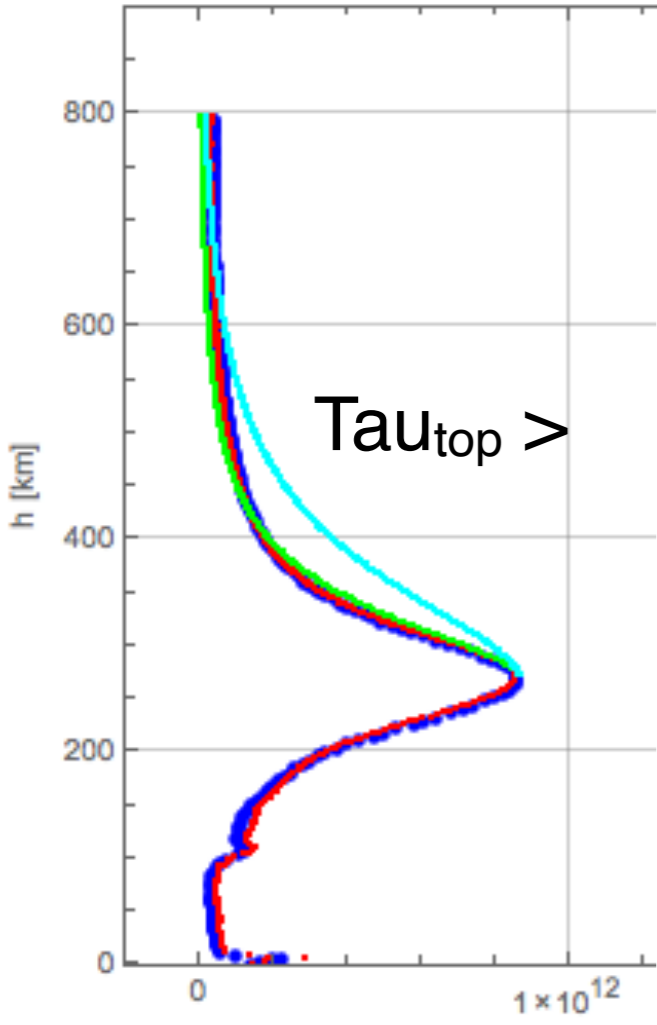
Model: NeQuick



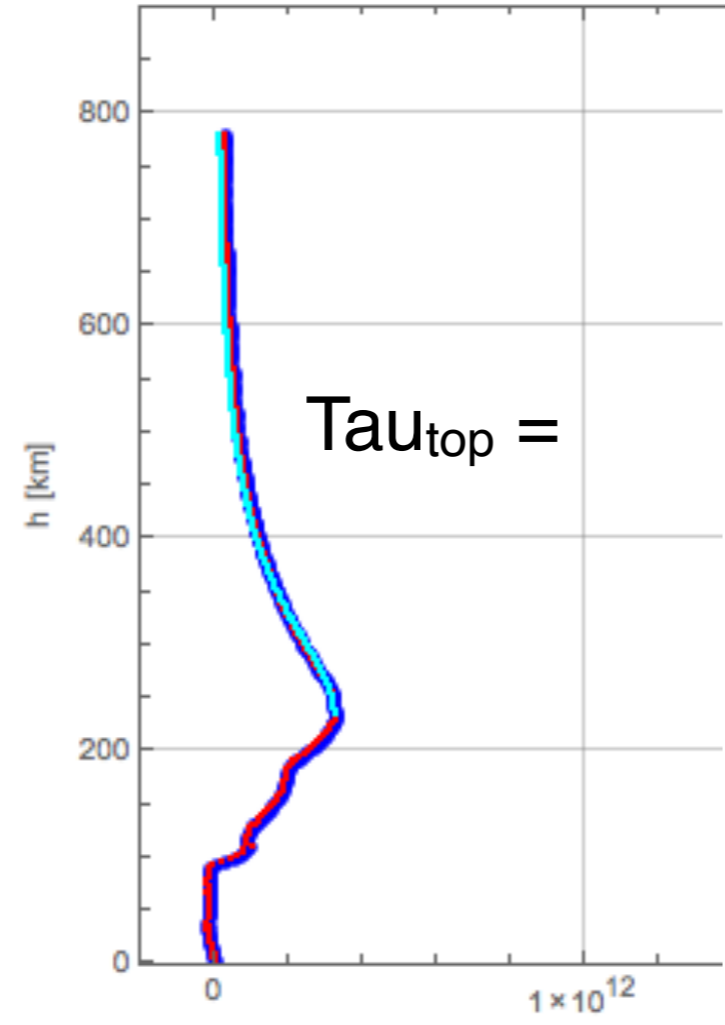
# NeQuick adaptation to foF2, hmF2 (and $\tau_{top}$ )

Matching the first 200 km above the peak

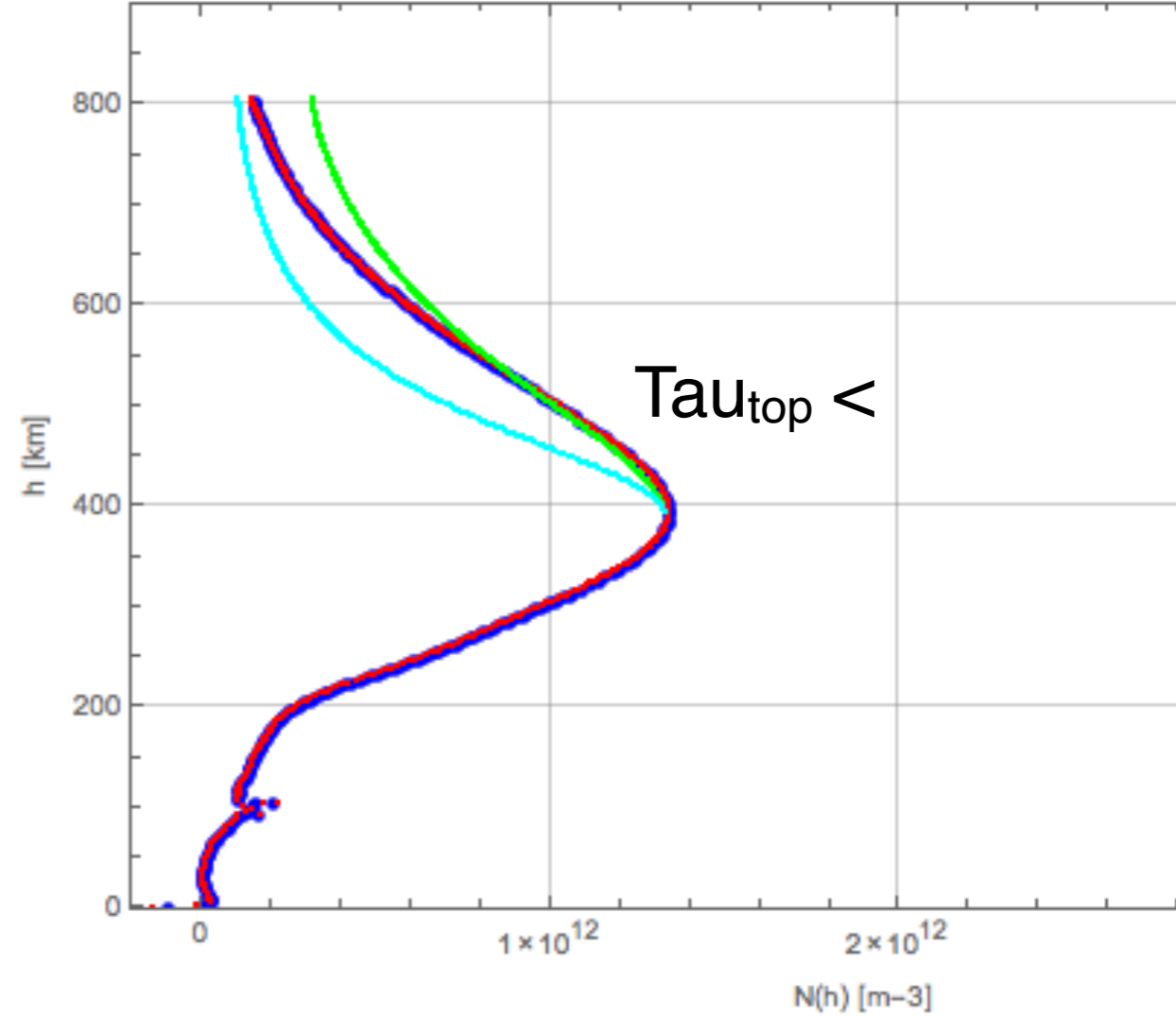
ionPhs\_C001.2011.072.13.28.G10\_2010.2640\_nc  
 Peak Lat: 2.064° Peak Lon: -89.526° Peak UT: 13.450 Peak LT: 7.486  
 Peak Hei: 269.810 km Peak dens:  $0.85707113 \times 10^{12} \text{ m}^{-3}$  Peak freq: 8.314 MHz



ionPhs\_C006.2017.156.00.46.G06\_2016.1120\_nc  
 Peak Lat: 49.750° Peak Lon: 100.910° Peak UT: 0.772 Peak LT: 7.499  
 Peak Hei: 235.160 km Peak dens:  $0.32578130 \times 10^{12} \text{ m}^{-3}$  Peak freq: 5.126 MHz



ionPhs\_C001.2011.072.21.03.G16\_2010...  
 Peak Lat: 0.007° Peak Lon: -48.494° Peak UT: 21.02...  
 Peak Hei: 394.280 km Peak dens:  $1.34063300 \times 10^{12} \text{ m}^{-3}$



- COSMIC
- ICTP
- NeQuick (ing. hmF2, foF2)
- NeQuick (ing. hmF2, foF2,  $\tau_{top}$ )



# Least Square Estimation



# Least Square Estimation

## Best Linear Unbiased Estimator (BLUE)\*

$\mathbf{x}_t$  true model state (dimension  $n$ )

$\mathbf{x}_b$  background model state (dimension  $n$ )

$\mathbf{x}_a$  analysis model state (dimension  $n$ )

$\mathbf{y}$  vector of observations (dimension  $p$ )

$H$  observation operator (dimension  $p \times n$ )

$B$  covariance matrix of background errors  $\boldsymbol{\varepsilon}_b = (\mathbf{x}_b - \mathbf{x}_t)$  (dimension  $n \times n$ )

$R$  covariance matrix of observation errors  $\boldsymbol{\varepsilon}_o = (\mathbf{y} - H[\mathbf{x}_t])$  (dimension  $p \times p$ )

$A$  covariance matrix of analysis errors  $\boldsymbol{\varepsilon}_a = (\mathbf{x}_a - \mathbf{x}_t)$  (dimension  $n \times n$ )

\*<https://www.ecmwf.int/sites/default/files/elibrary/2002/16928-data-assimilation-concepts-and-methods.pdf>

# Least Square Estimation

- The following hypotheses are assumed:
  - *Linearized observation operator*: the variations of the observation operator in the vicinity of the background state are linear.
  - *Non-trivial errors*: B and R are positive definite matrices.
  - *Unbiased errors*: the expectation of the background and observation errors is zero.
  - *Uncorrelated errors*: observation and background errors are mutually uncorrelated.
  - *Linear analysis*: we look for an analysis defined by corrections to the background which depend linearly on background observation departures.
  - *Optimal analysis*: we look for an analysis state which is as close as possible to the true state in an r.m.s. sense (i.e. it is a minimum variance estimate).



# Least Square Estimation

The optimal least-square estimator (BLUE analysis) is defined by

$$\mathbf{x}_a = \mathbf{x}_b + \mathbf{K} (\mathbf{y} - \mathbf{H}\mathbf{x}_b)$$

$$\mathbf{K} = \mathbf{B}\mathbf{H}^T(\mathbf{H}\mathbf{B}\mathbf{H}^T + \mathbf{R})^{-1}$$

$$\mathbf{A} = (\mathbf{I} - \mathbf{K}\mathbf{H})\mathbf{B}$$

$\mathbf{K}$  is called *gain* of the analysis

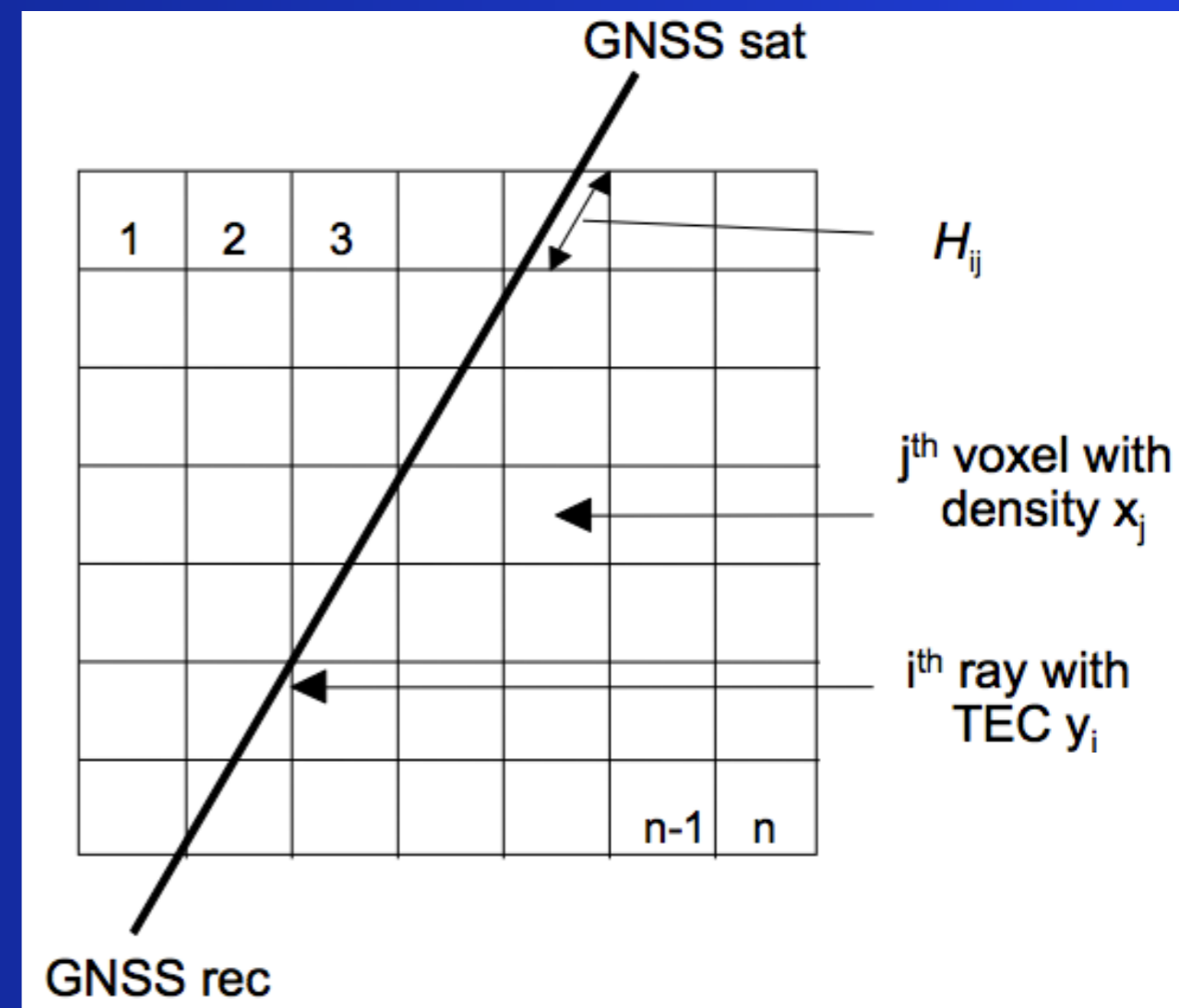
In our case:

$$\mathbf{y} = \text{TEC}$$

$\mathbf{x}_a$  = retrieved electron density

$\mathbf{x}_b$  = background electron density

$\mathbf{H}$  -> "crossing lengths" in "voxels"



$$\text{e.g. } \text{bckg\_TEC} = \mathbf{H}\mathbf{x}_b = \sum_j H_{ij} x_{bj}$$

# Least Square Estimation

$$R_{ij} = C_R \delta_{ij} y_i^2$$

(measurements are independent)

$$B_{ij} = C_B x_{bi} x_{bj} \text{Exp}[-(z_{ij}/L_z)^2] \text{Exp}[-(\alpha_{ij}/L_\alpha)^2]$$

(V & H correl. are separable)

$z_{ij} \sim$  height difference between voxels  $i$  and  $j$

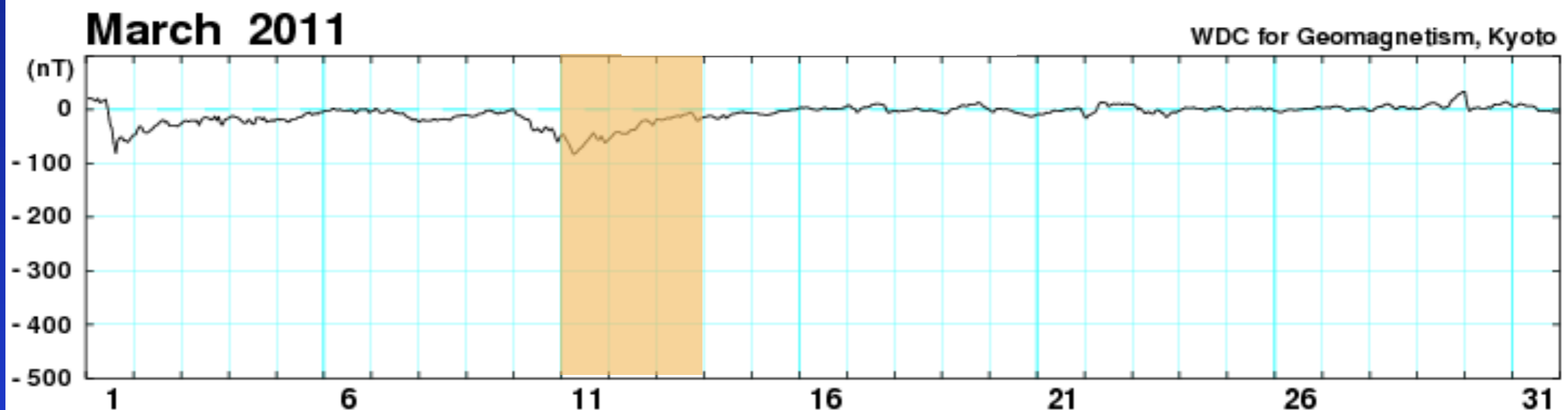
$\alpha_{ij} \sim$  angular (great circle) distance between voxels  $i$  and  $j$

$L_z \sim$  correl. distance in vert. direction (may depend on height)

$L_\alpha \sim$  correl. (angular) distance in hor. direction (may depend location,...)

# GNSS TEC DA - Example 1

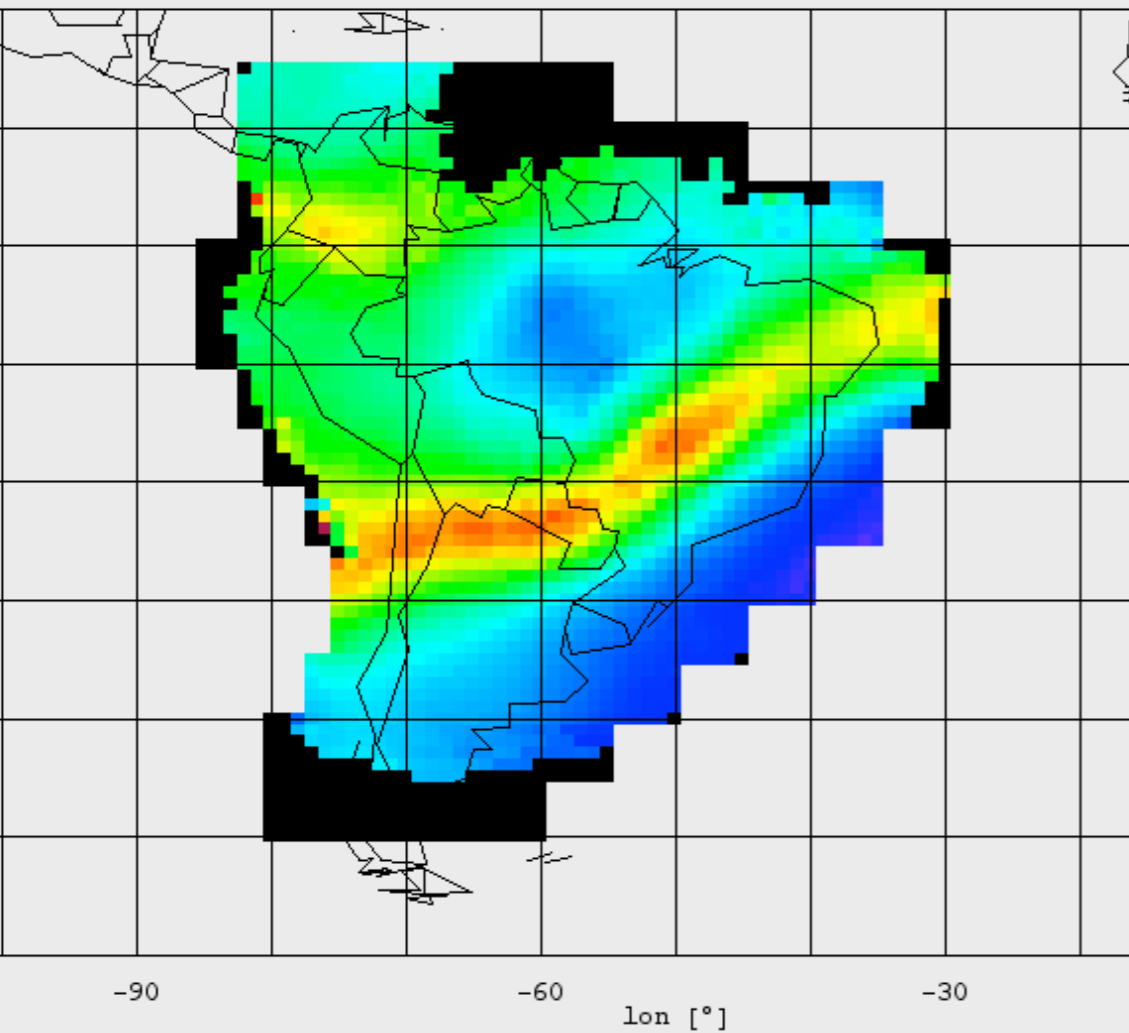
- For the assimilation
  - Calibrated ground-based GNSS-derived slant TEC data from about 150 receivers of the LISN network (C. Valladares), located in the South American region.
- The data correspond the period 11-13 March 2011





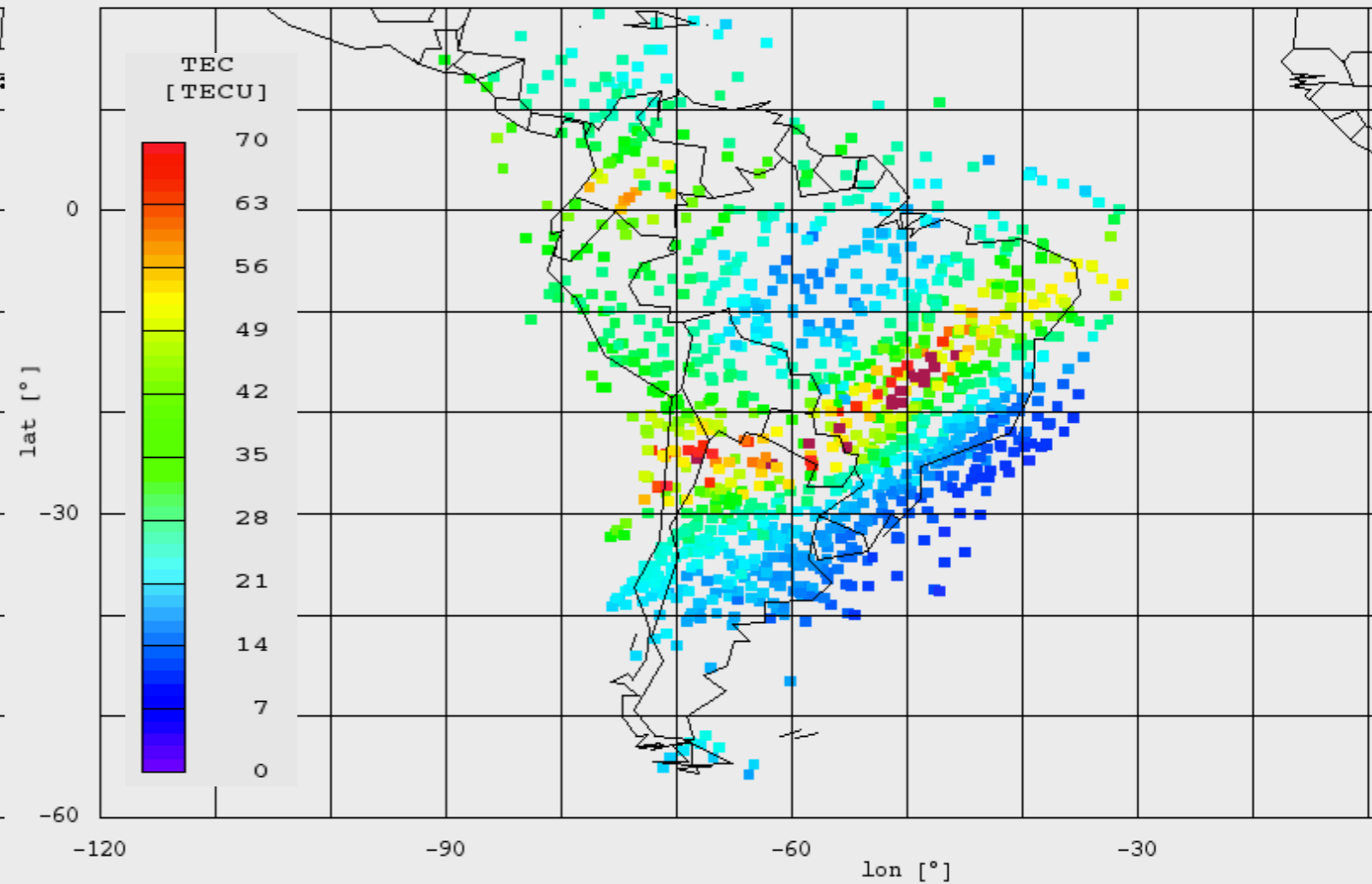
# LISN: 3 days data (2011/03/11-12-13)

lisn\_data\_exp\_20110311001000.txt doy: 070 UT: 0.167



Equivalent vertical TEC  
(LS adjustment)

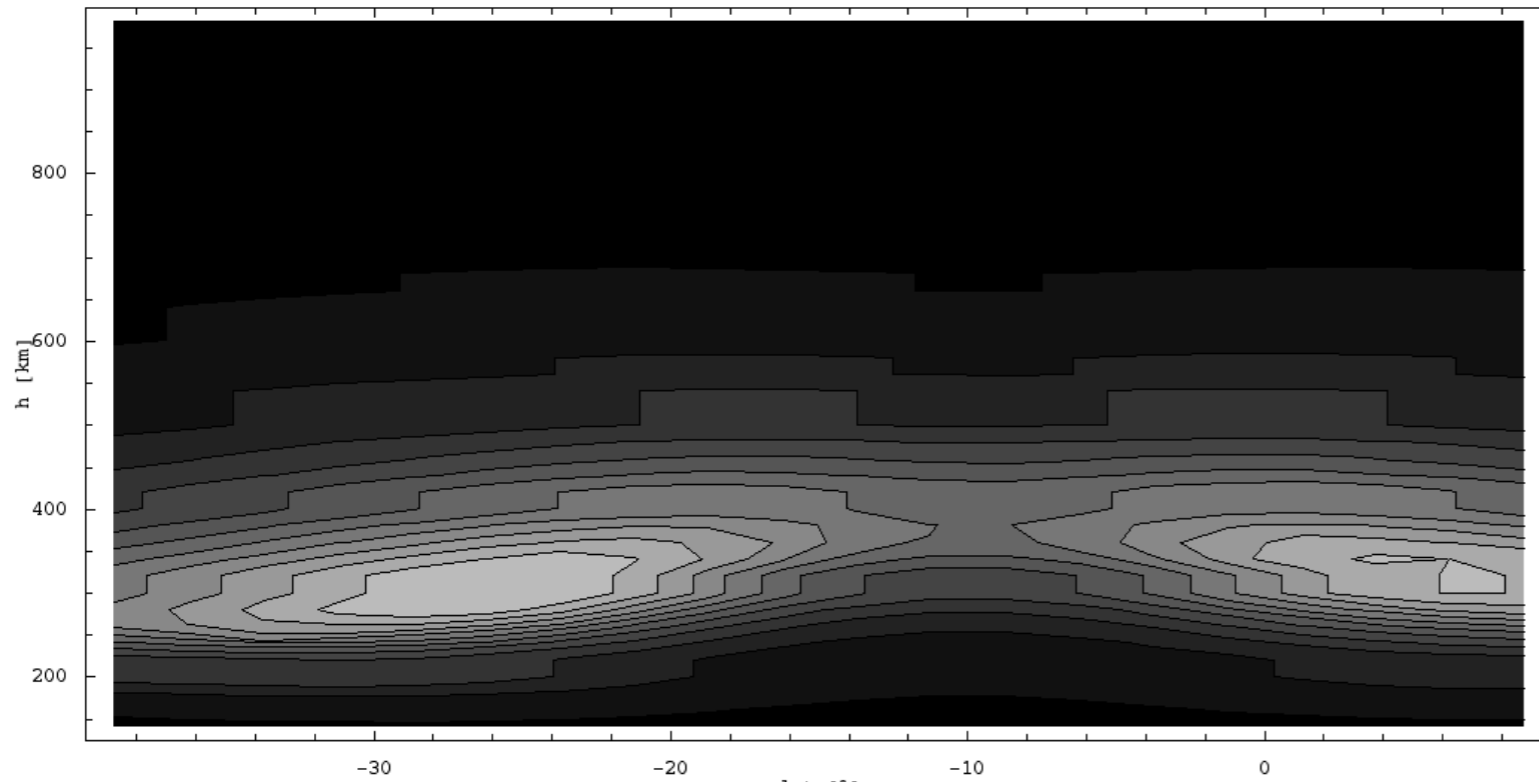
lisn\_data\_exp\_20110311001000.txt doy: 070 data UT: 0.167



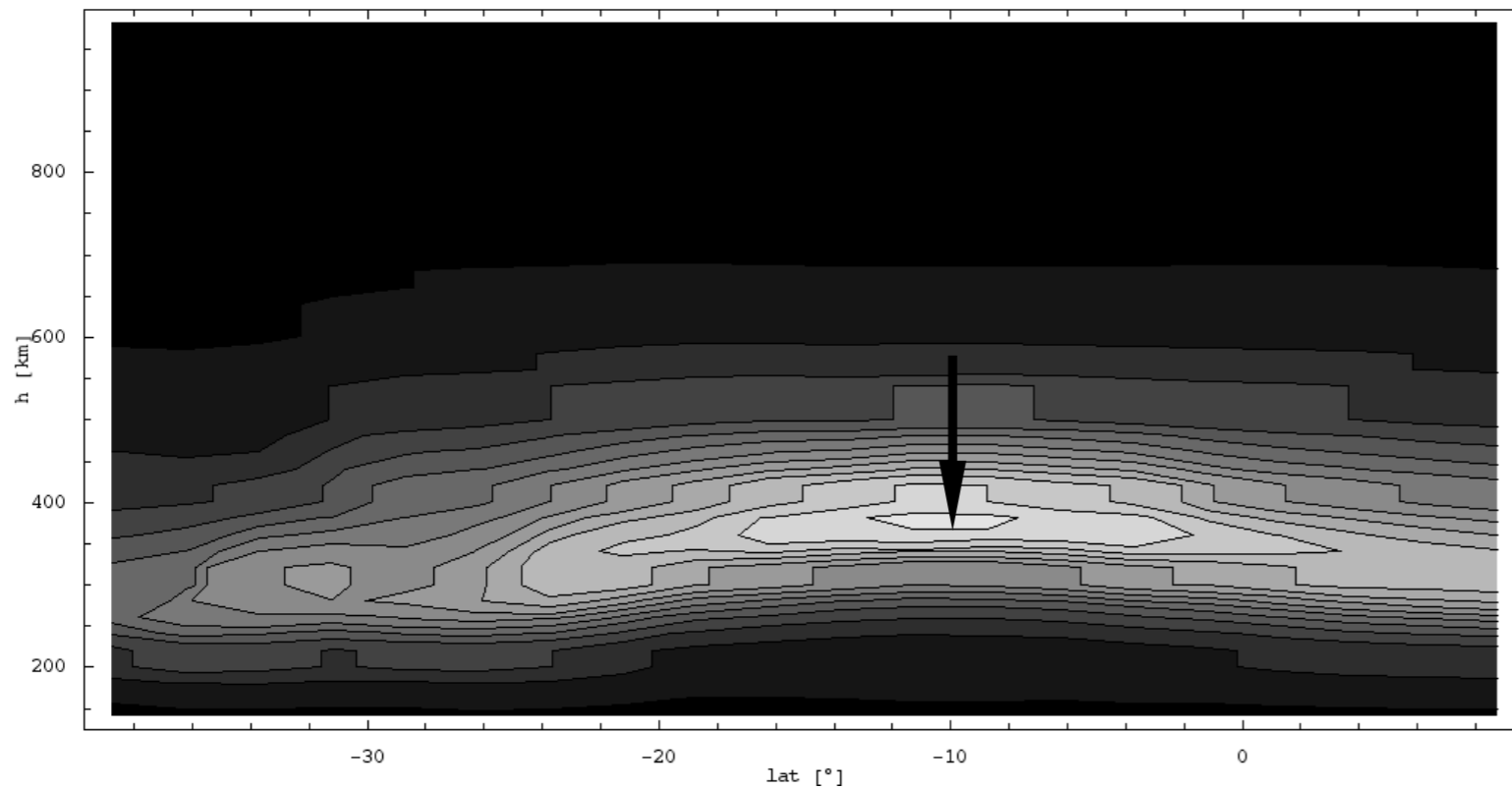
Equivalent vertical TEC  
at the pierce points

# Assimilation effect

Background 2011 03 11 UT: 19.3333 -64.75°E



Analysis 2011 03 11 UT: 19.3333 -64.75°E

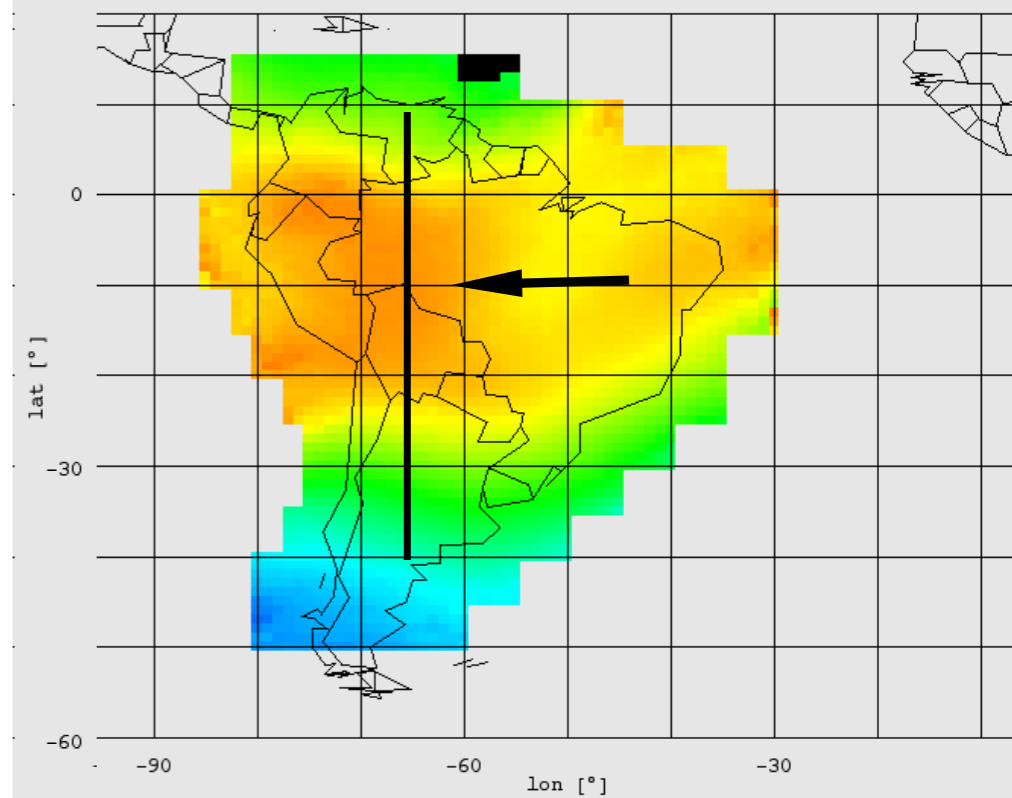


Background model

Cross section  
19:33UT; -64.75°E  
from -40°N to 10°N

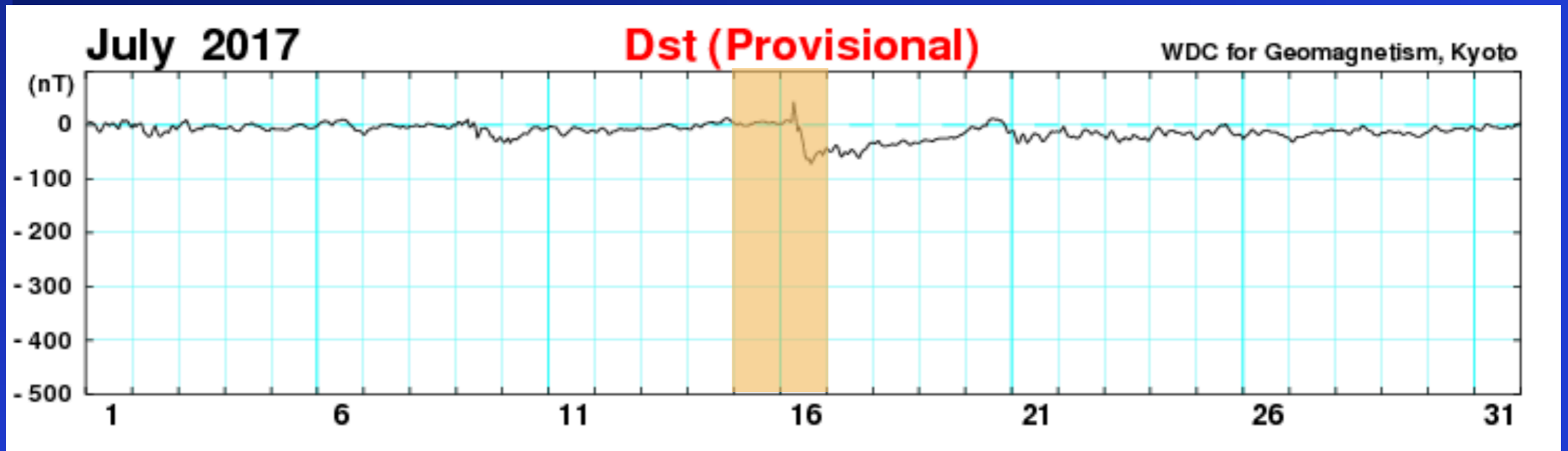
Analysis

lisn\_data\_exp\_20110311192000.txt doy: 070 UT: 19.330



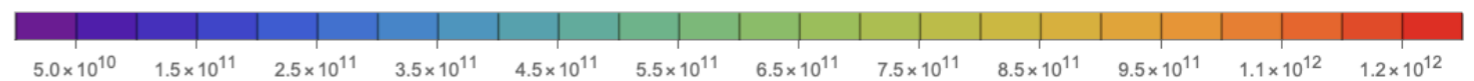
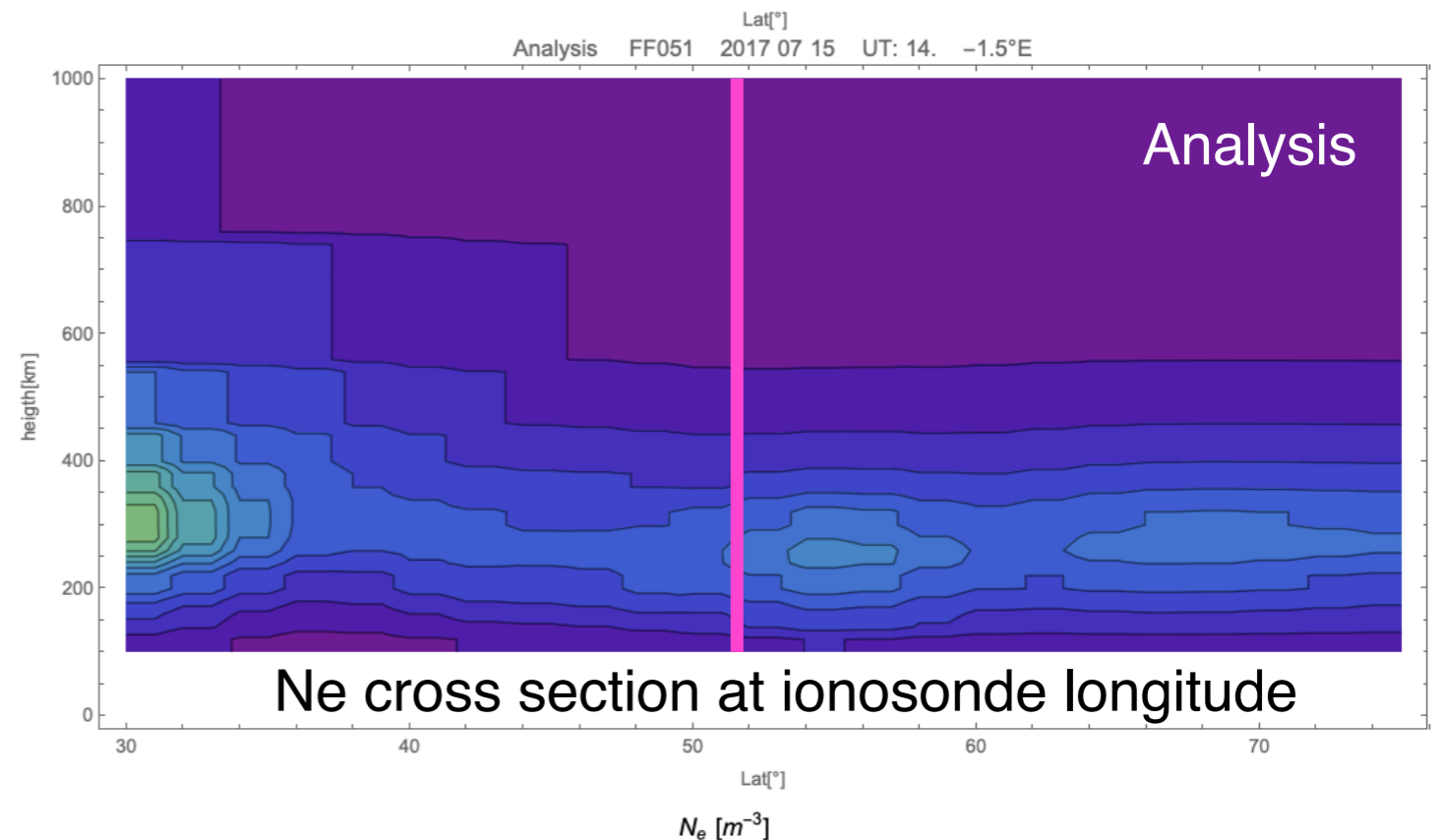
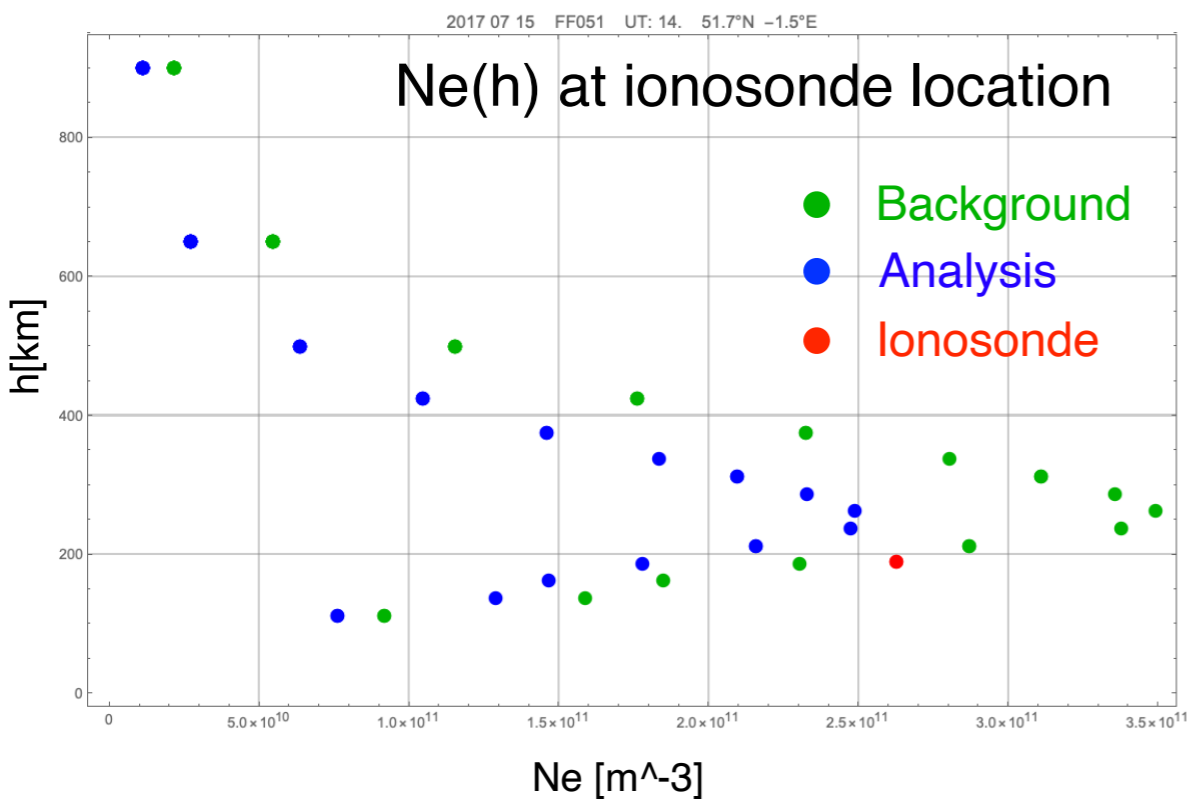
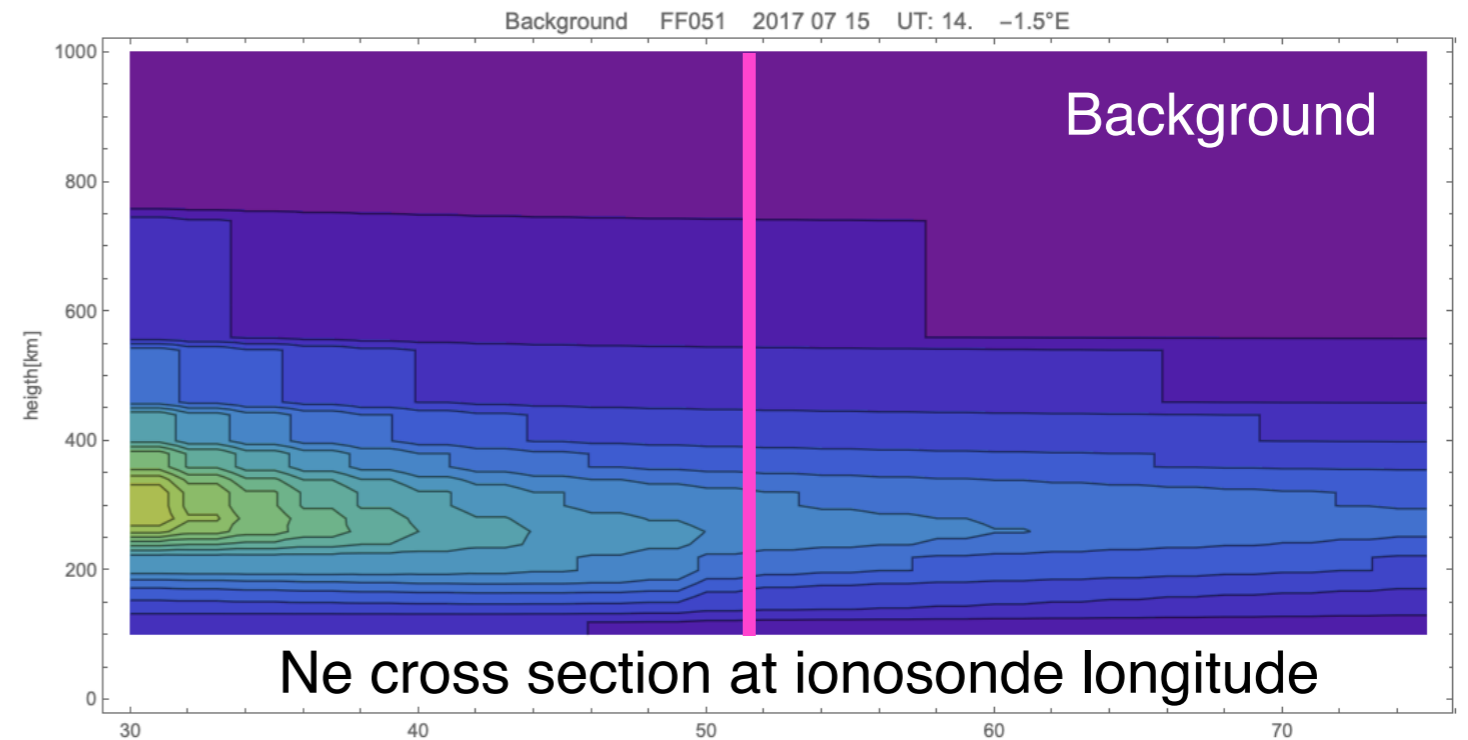
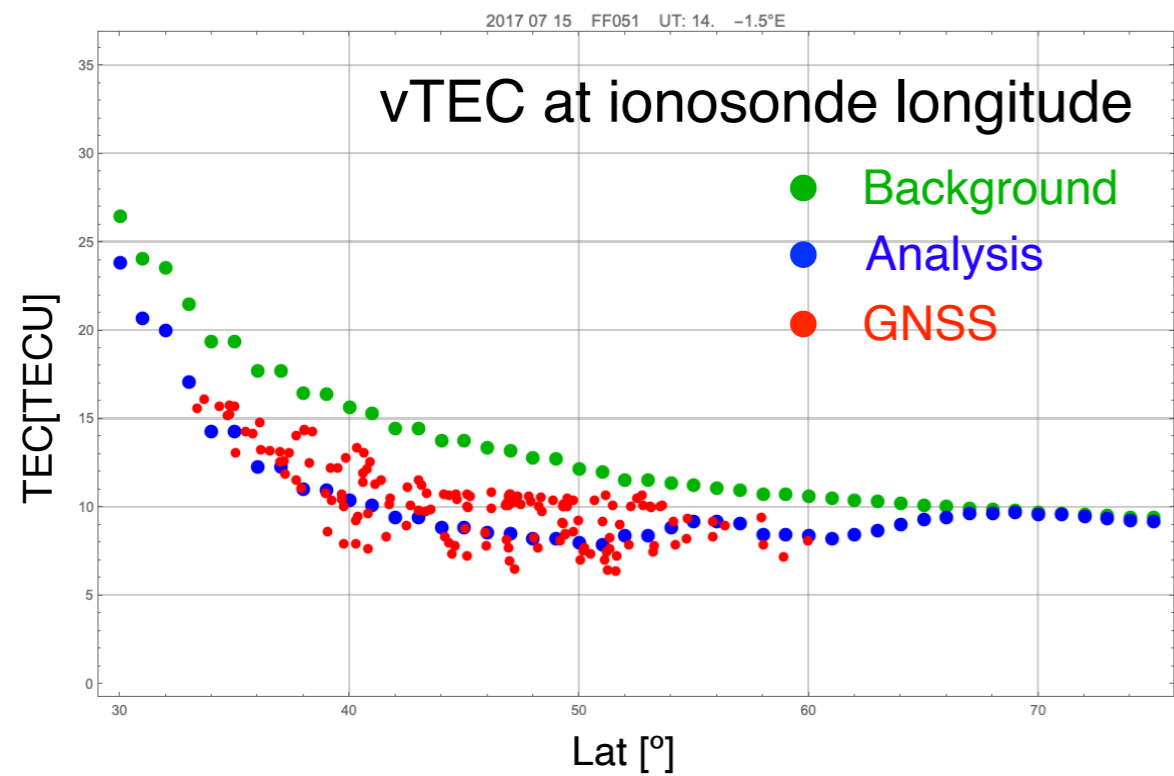
# GNSS TEC DA - Example 2

- For the assimilation
  - Calibrated (as in Themens et al. 2015) ground-based GNSS-derived slant TEC data from about 300 receivers located in the European region.
- For the validation
  - Manually scaled foF2 data obtained from Tromso (69.7°N, 9.0°E), Fairford (51.7°N, 1.5°W) and Juliusruh (54.6°N, 13.4°E) ionosondes at 1 hour time interval (only the result corresponding to Fairford will be illustrated).
- The data correspond the period 15-16 July 2017

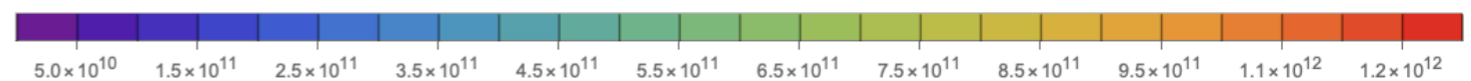
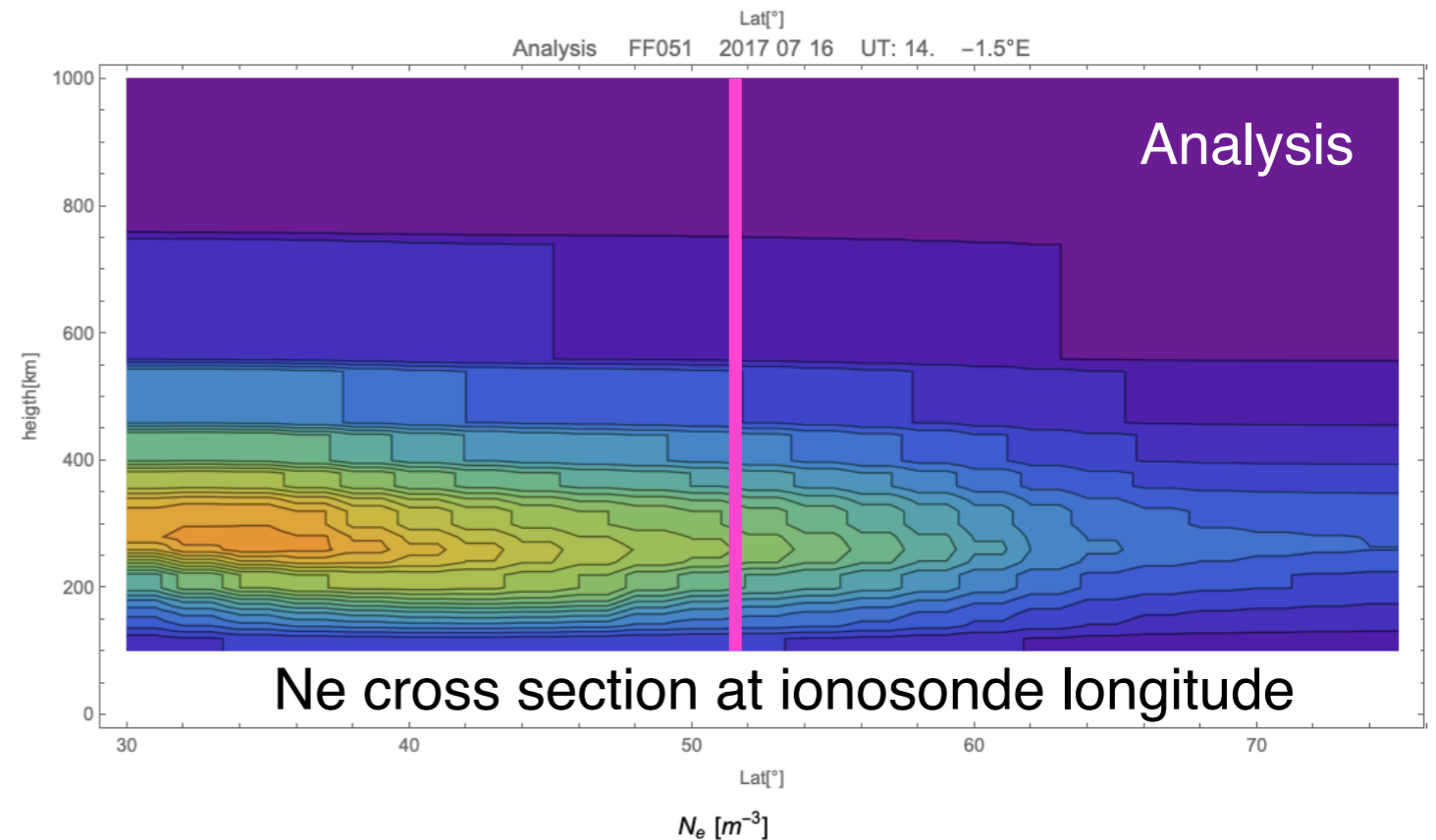
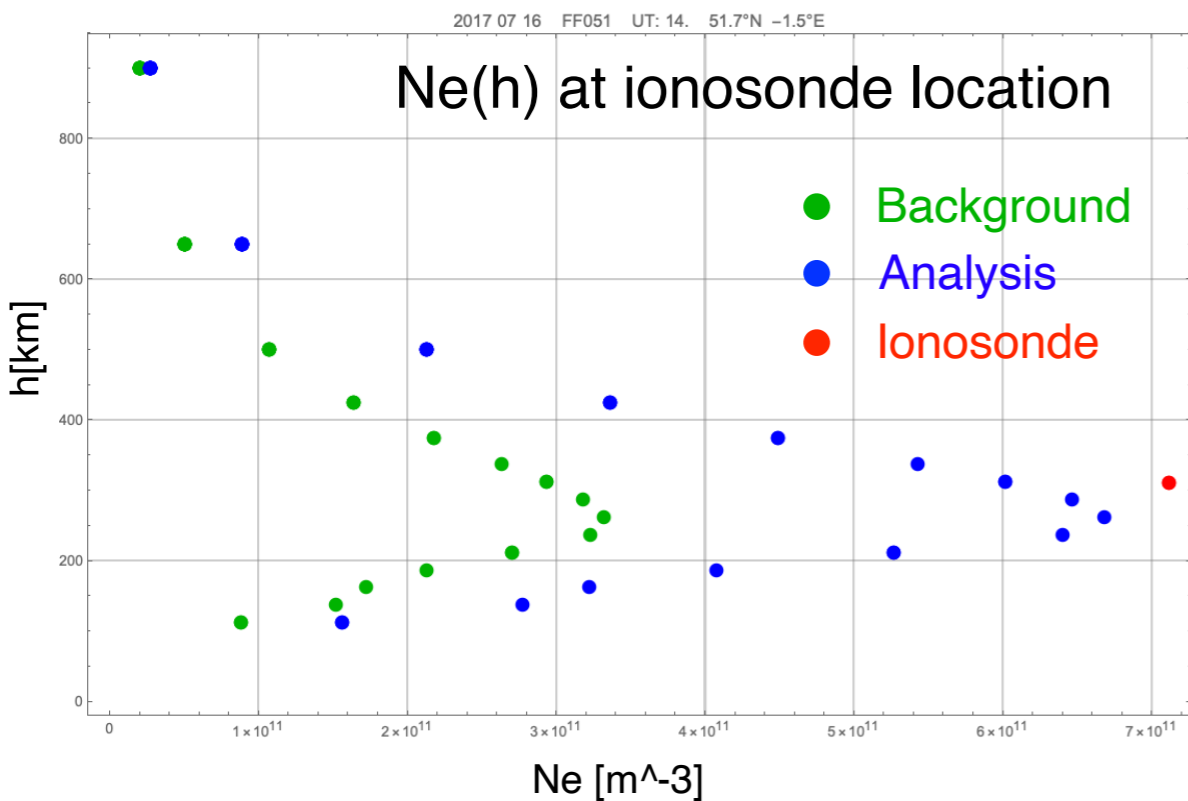
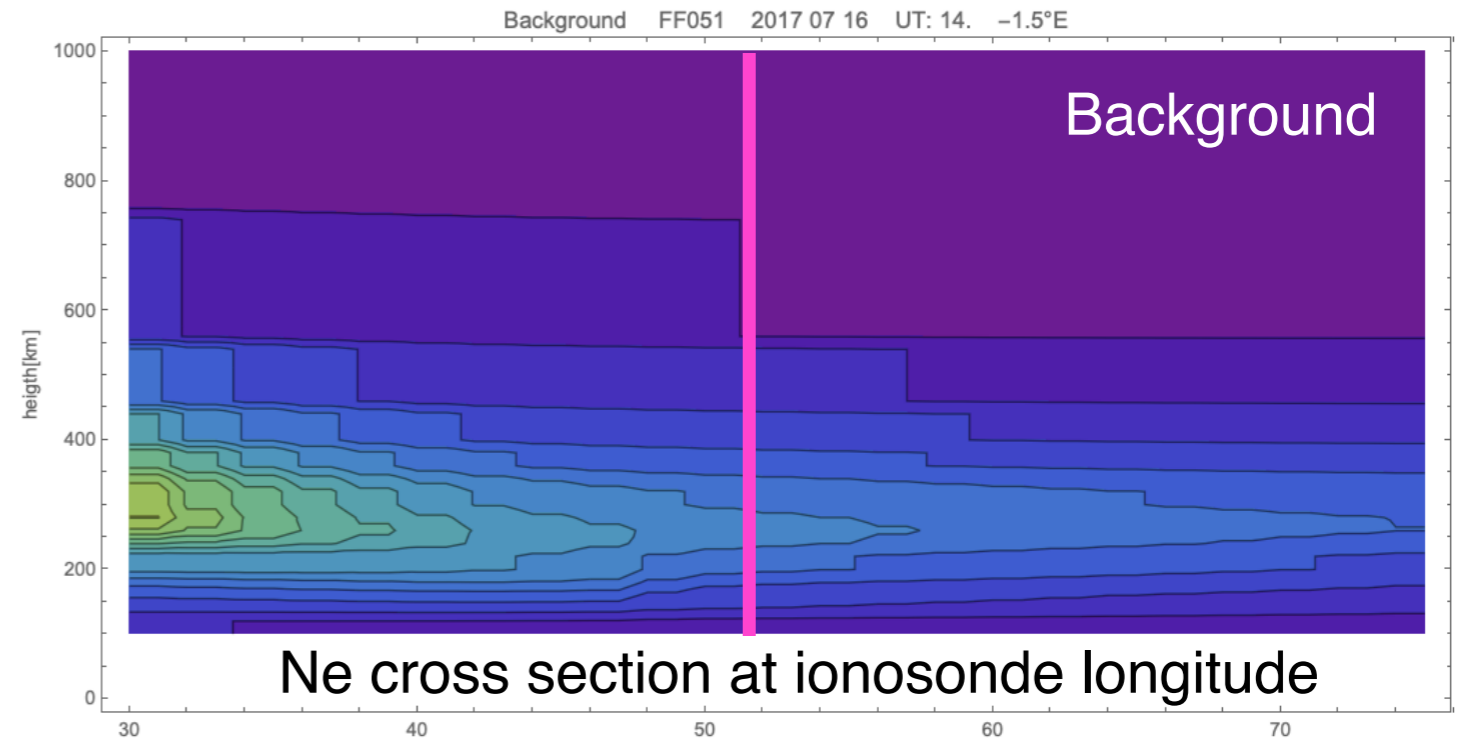
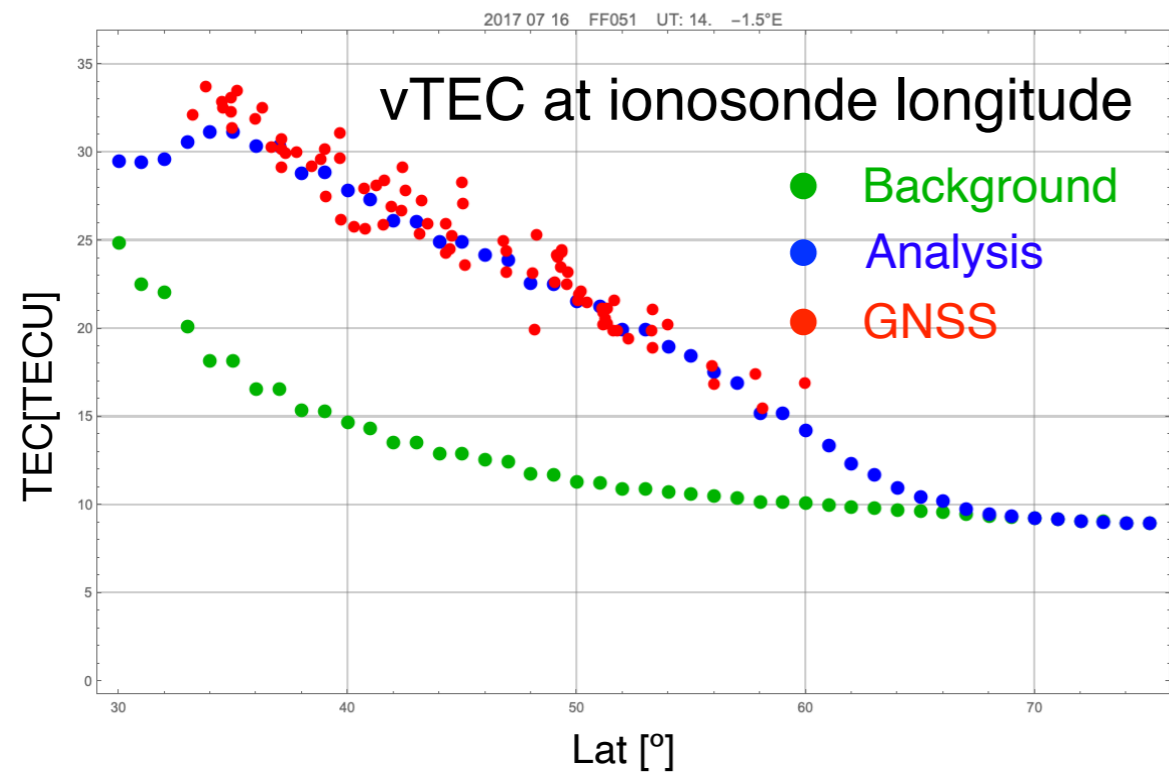




# Results: 15 July 2017; 14:00UT; FF051

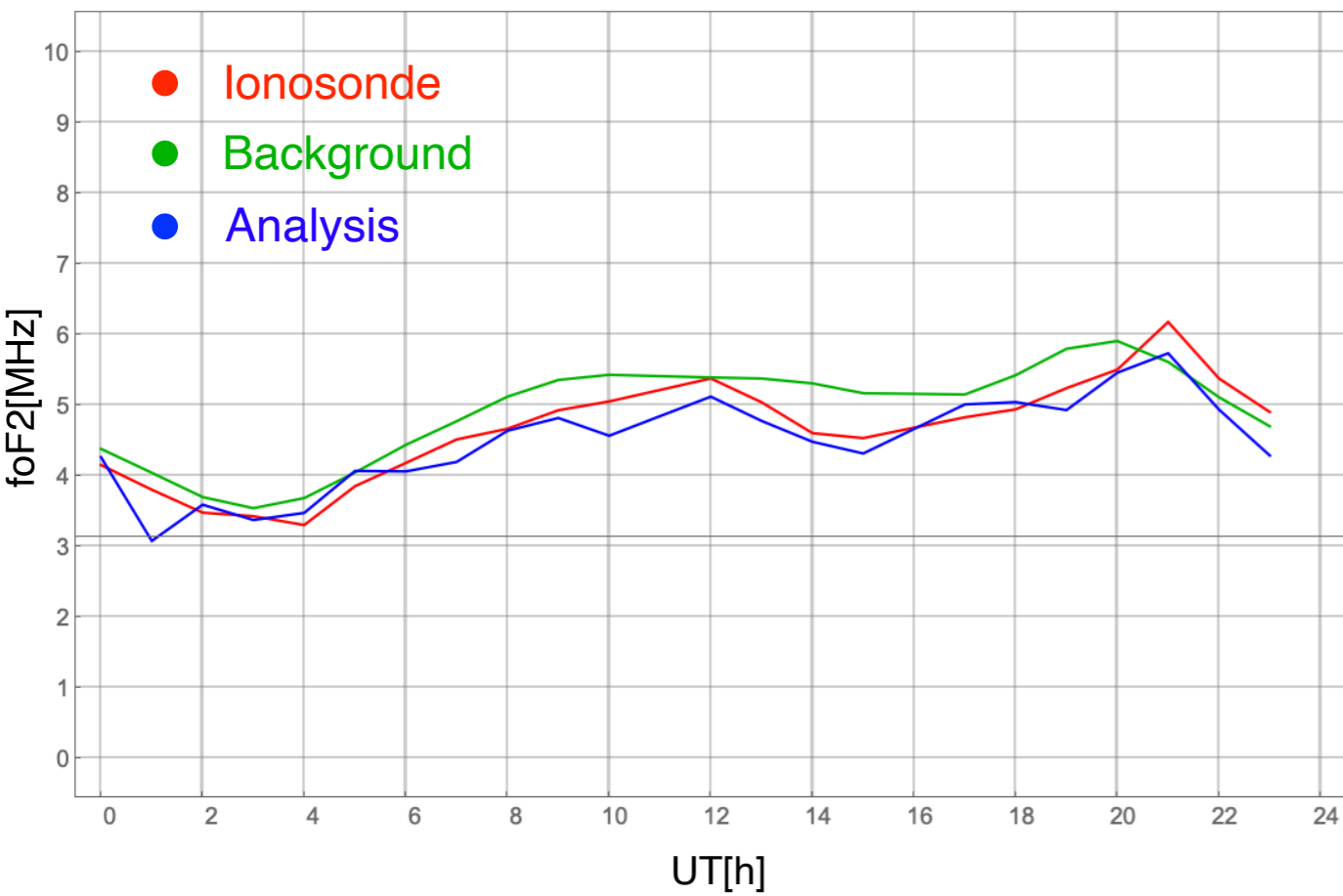


# Results: 16 July 2017; 14:00UT; FF051

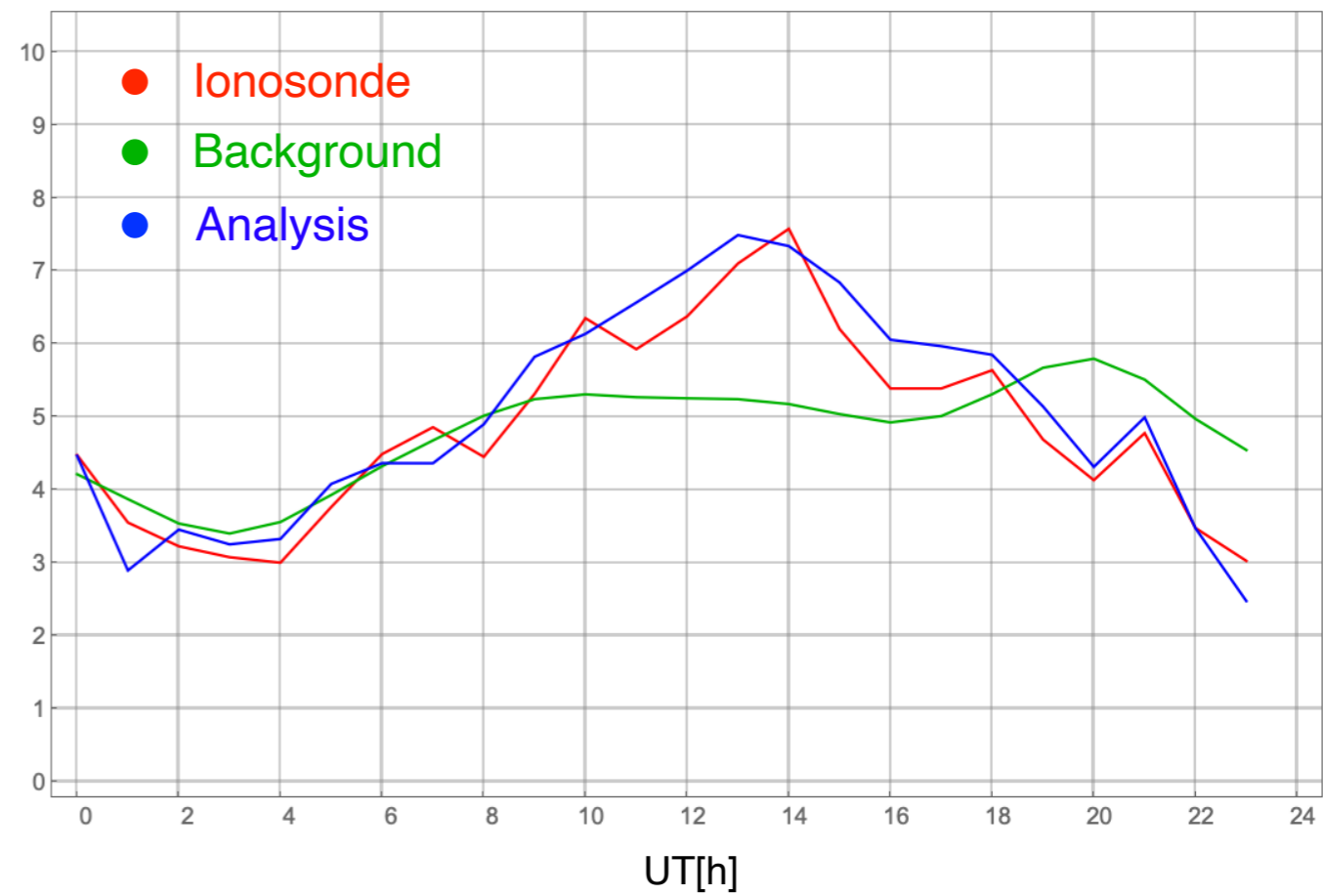


# Results: 15-16 July 2017 (sTEC DA)

FF051 2017-07-15



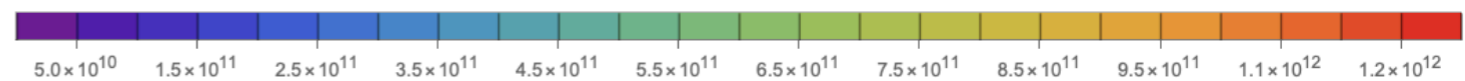
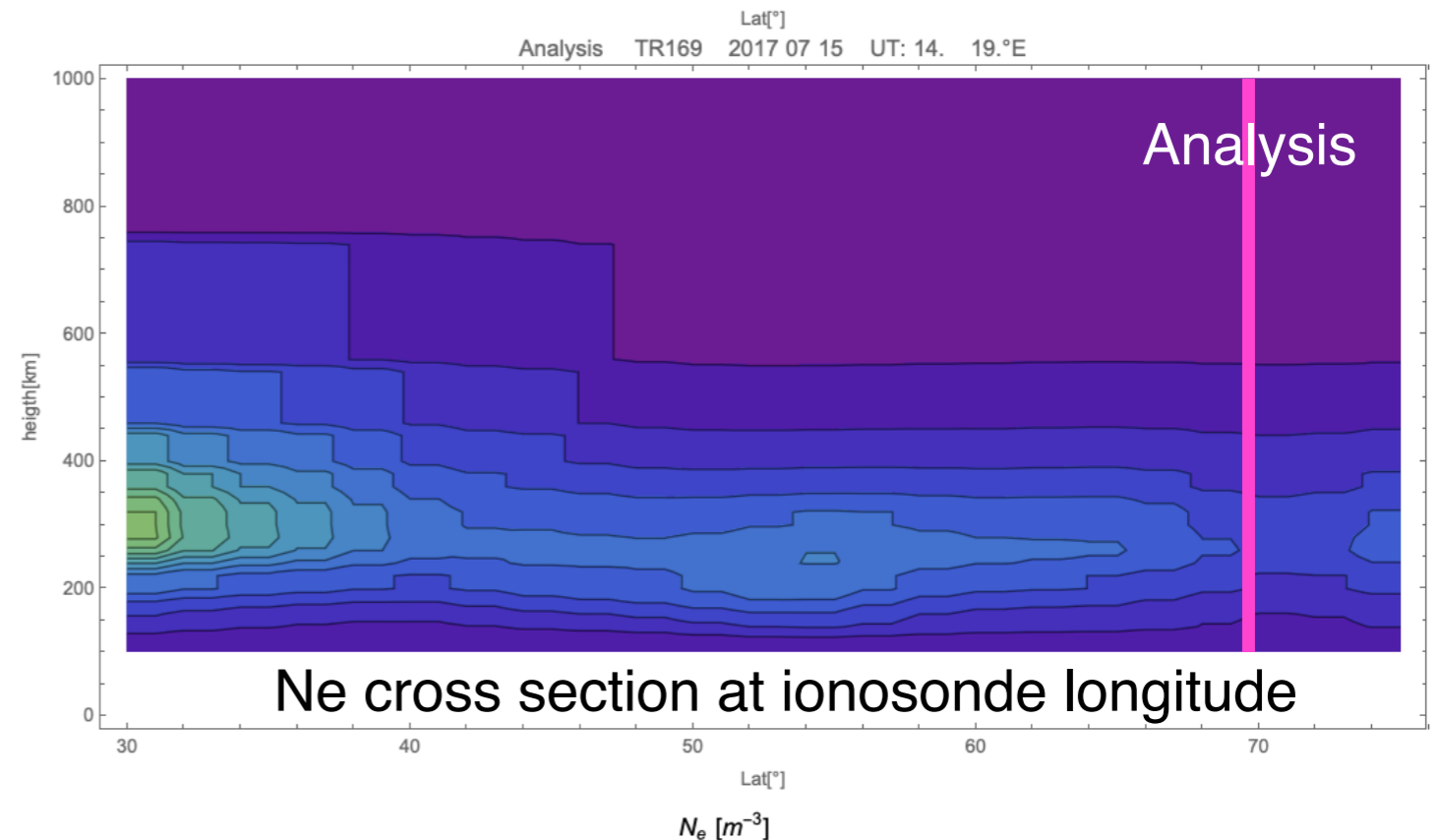
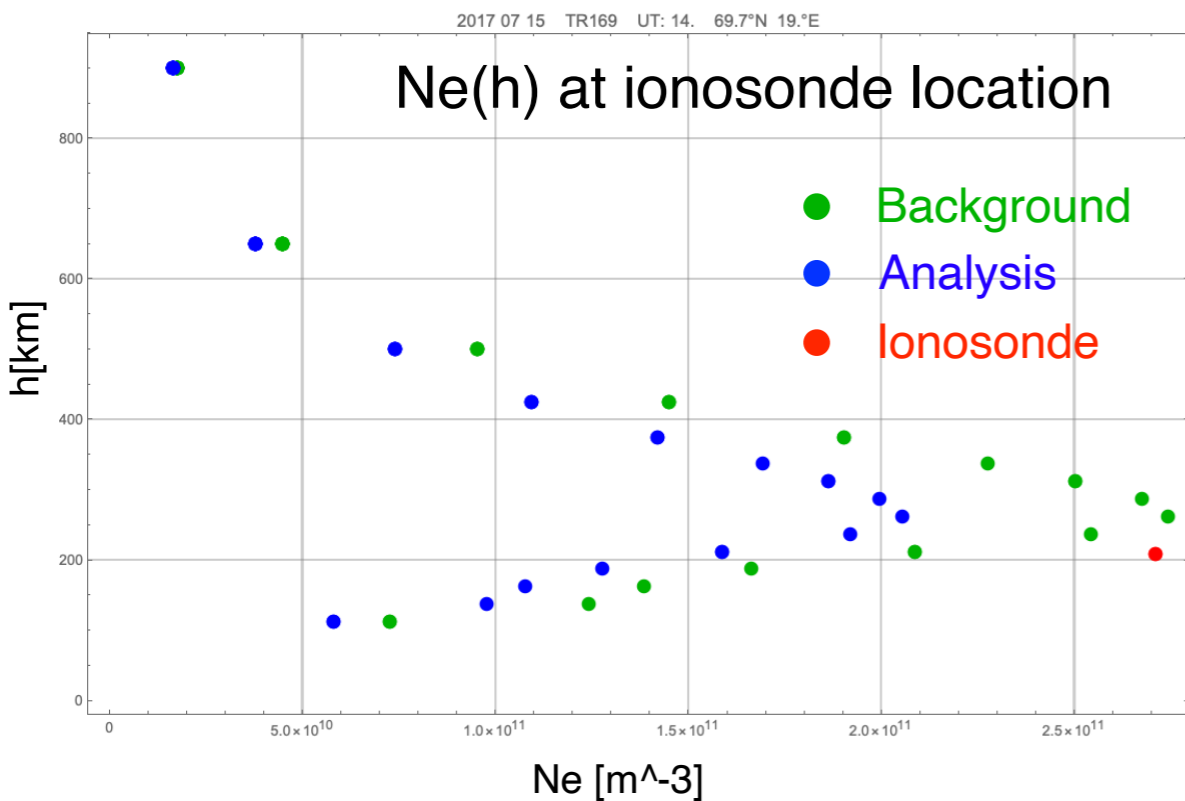
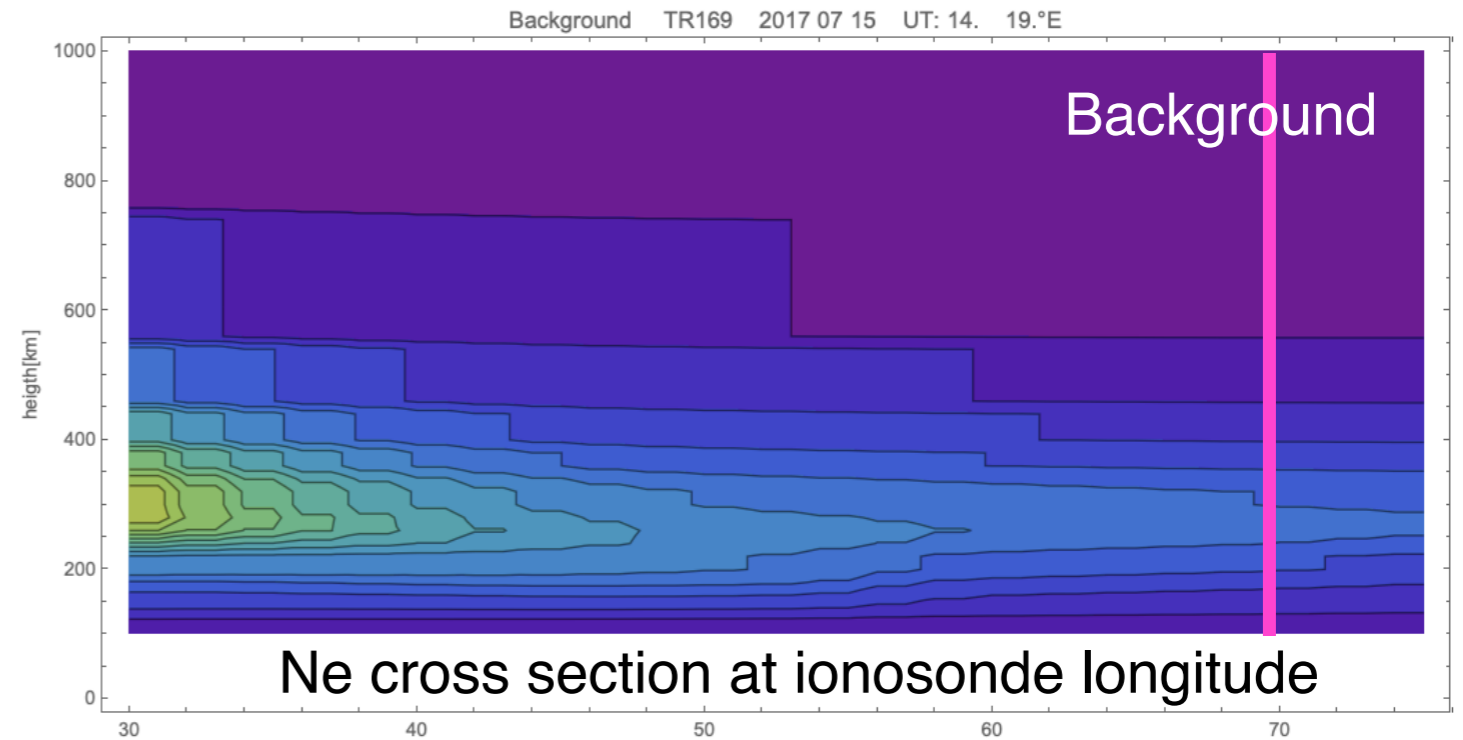
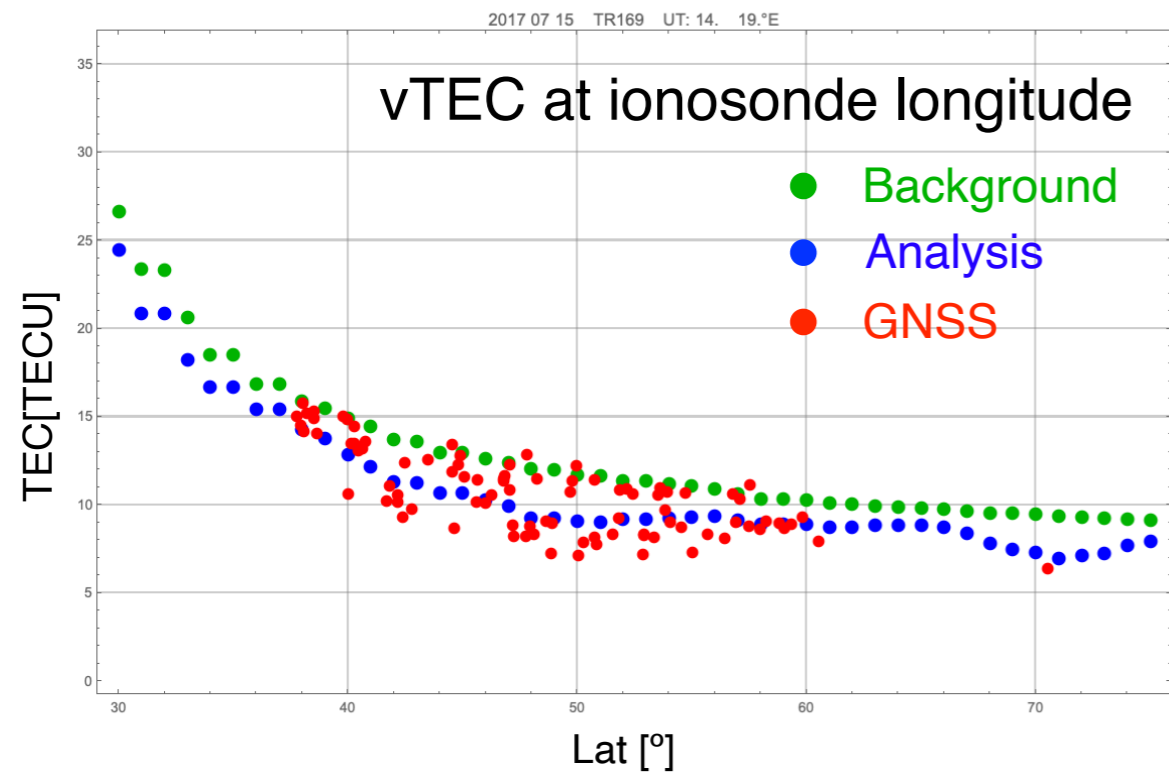
FF051 2017-07-16



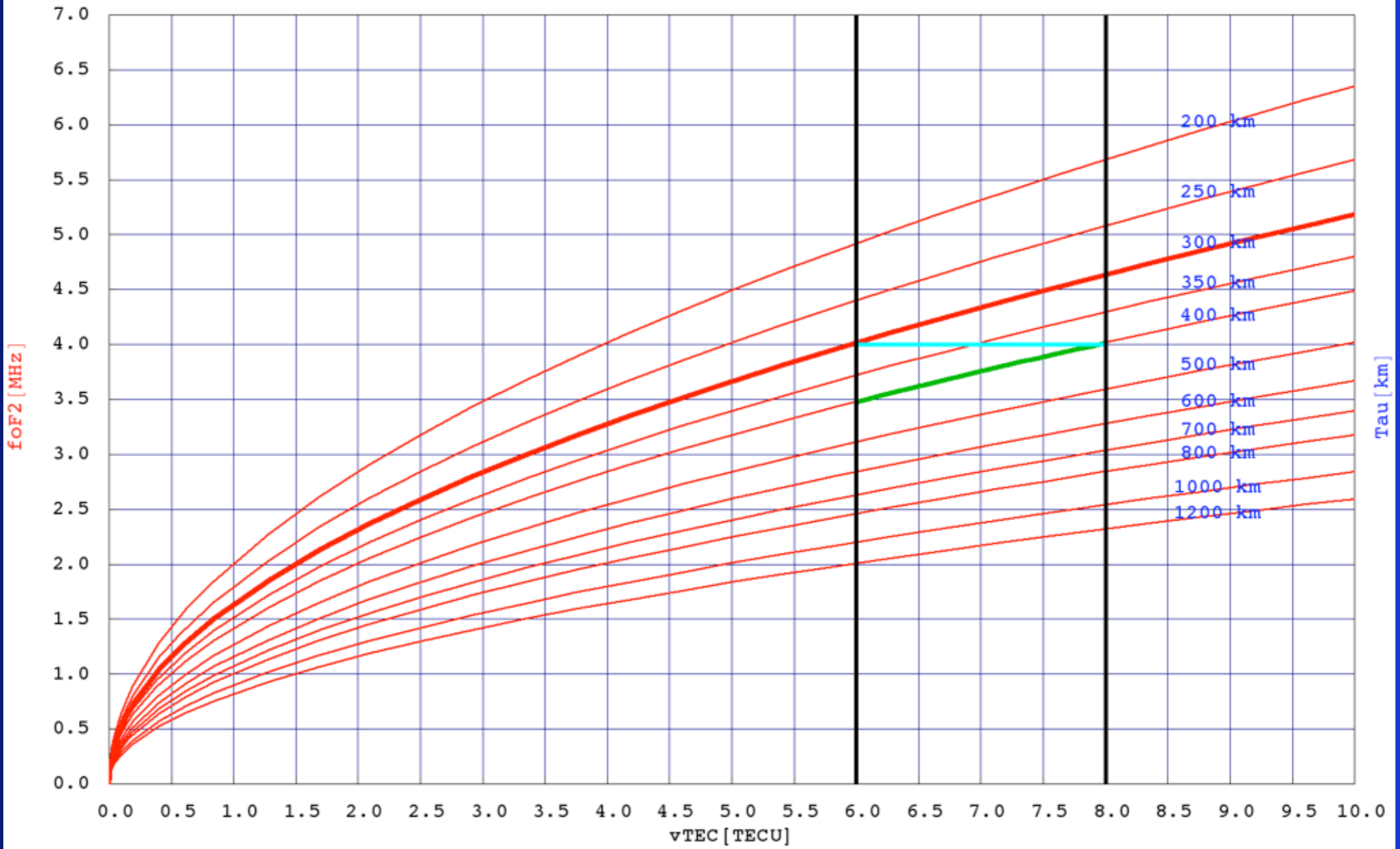
foF2 time evolution at ionosonde location



# Results: 15 July 2017; 14:00UT; TR169

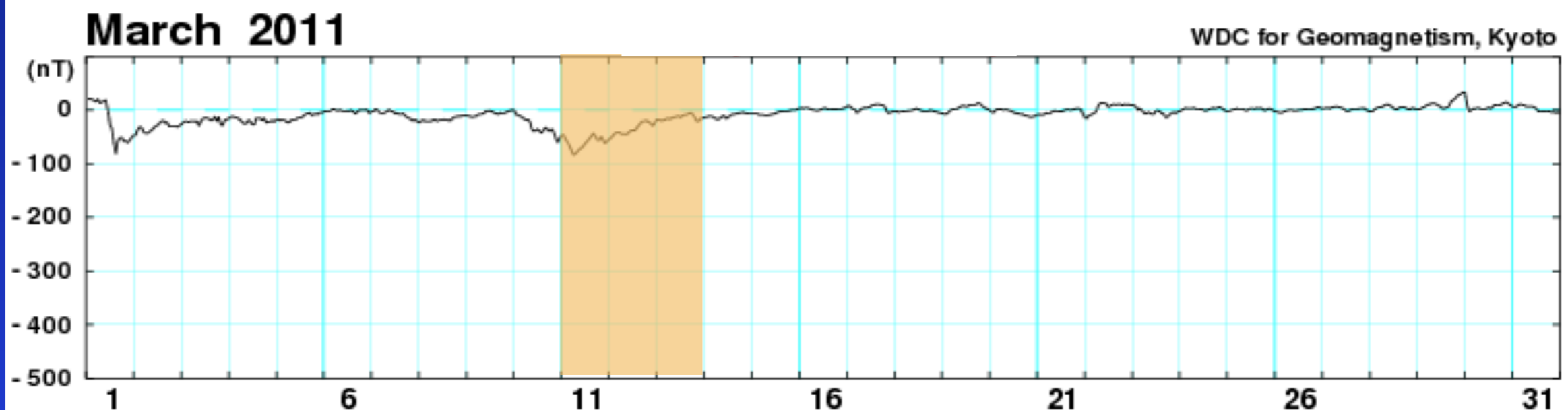


# Remark



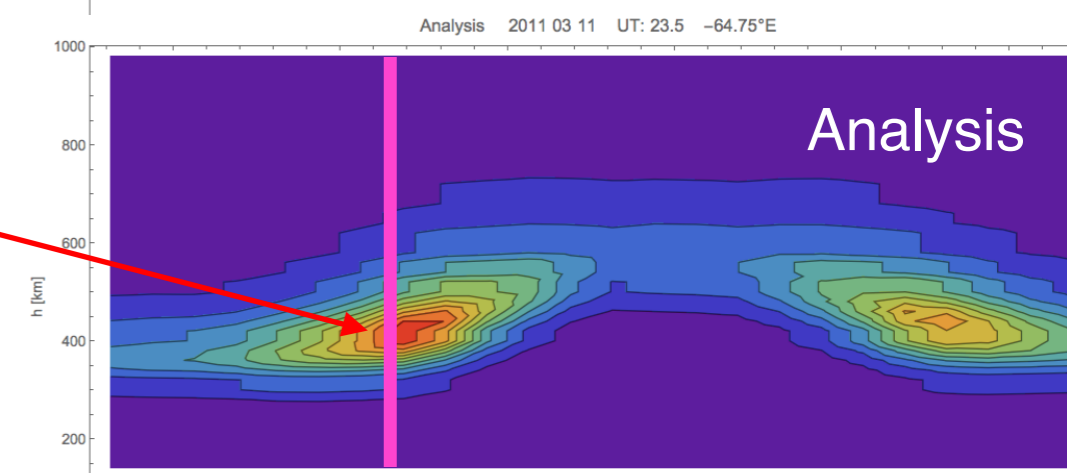
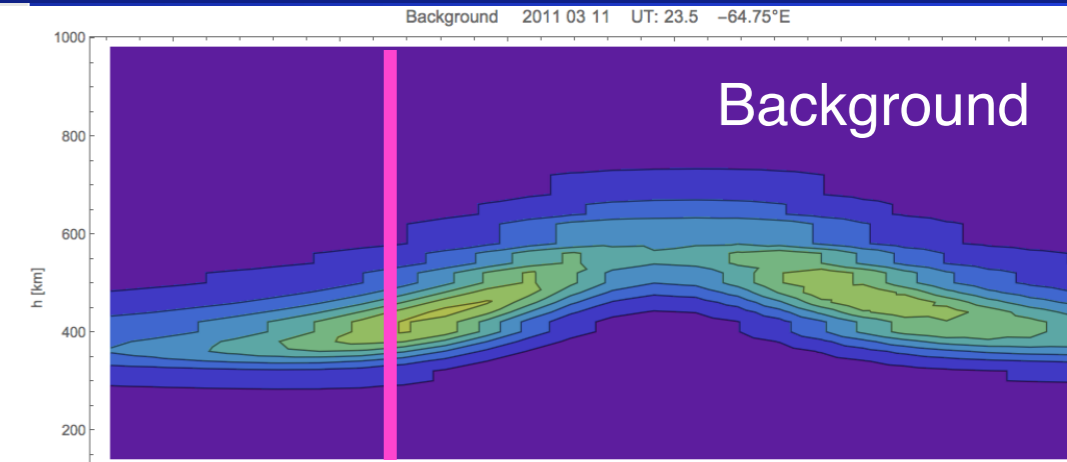
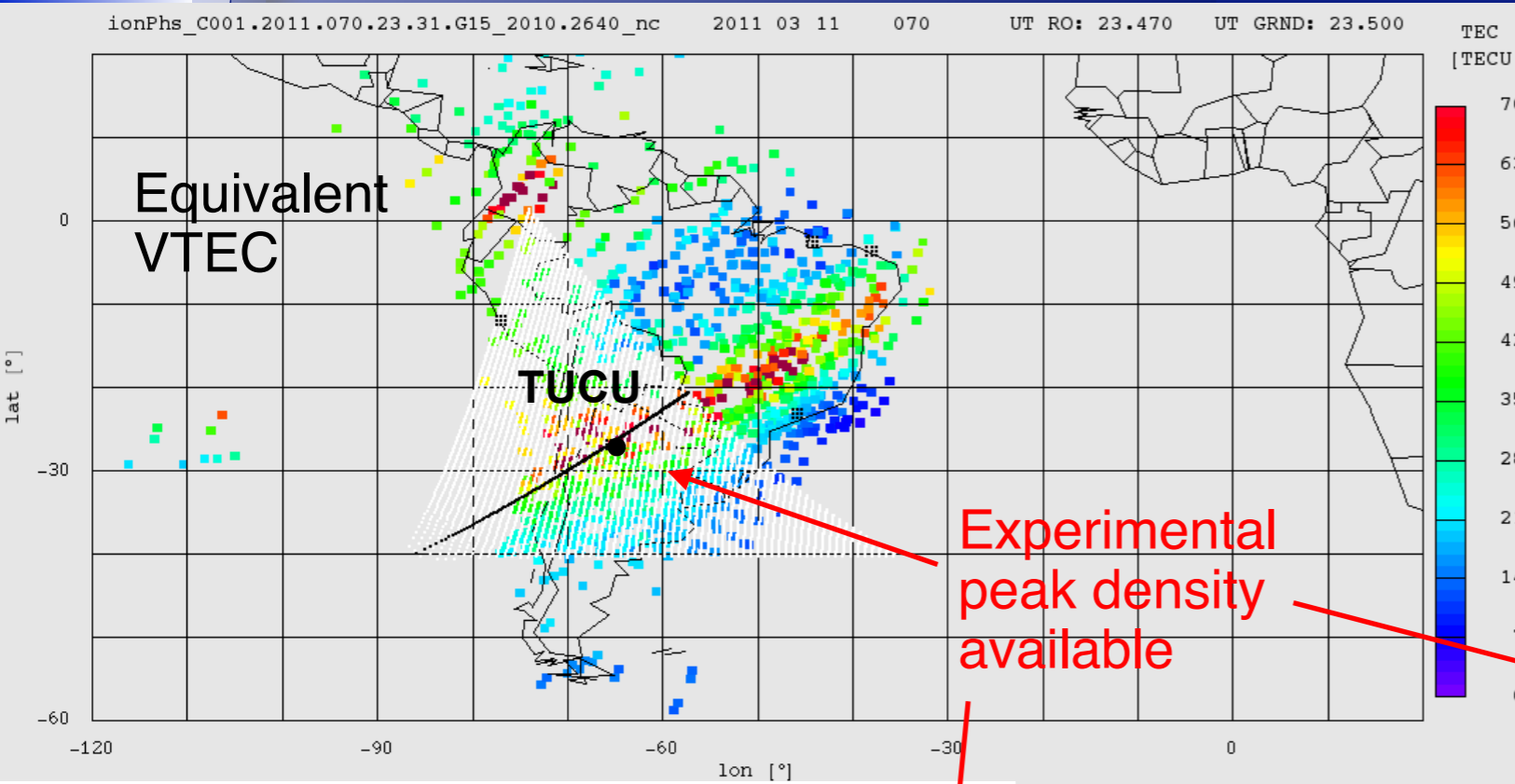
# GNSS TEC DA - Example 3

- For the assimilation
  - Ground-based GPS-derived slant TEC data provided by the Low Latitude Ionospheric Sensor Network (LISN)
  - Radio-Occultation-derived TEC data obtained by COSMIC (calibrated TEC values along the LEO-to-GPS link below the LEO orbit)
- For the validation
  - Manually scaled foF2 data obtained from the Tucuman Ionosonde
  - JRO electron density profiles
- The data correspond the period 11-13 March 2011



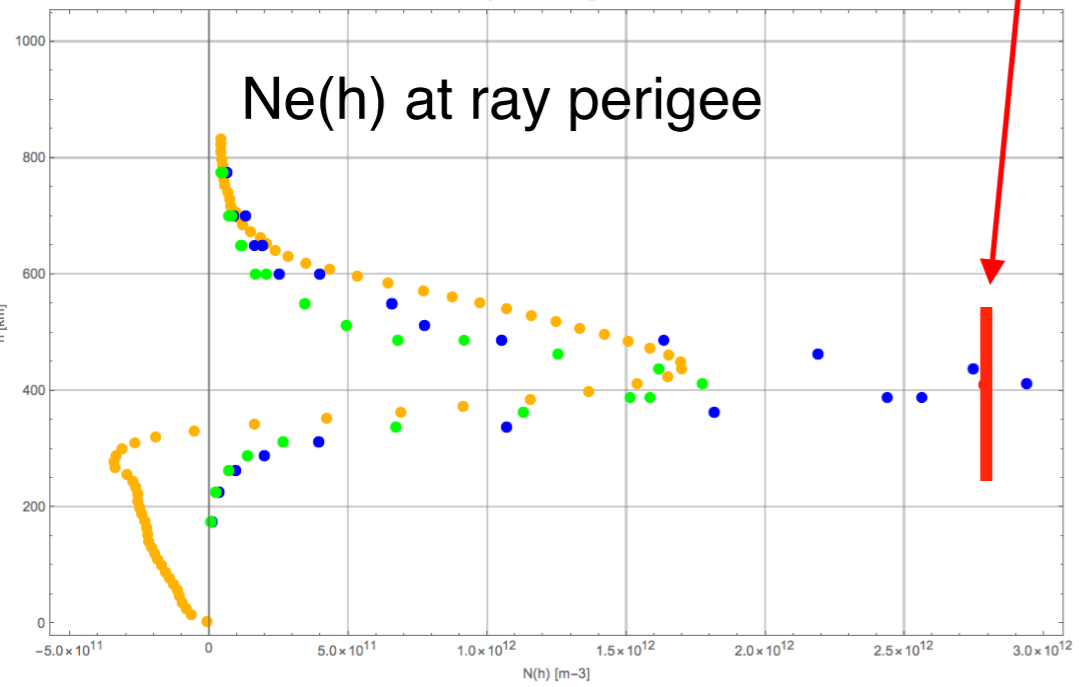


# TEC DA into NeQuick



TUCU 201103112330 ionPhs\_C001.2011.070.23.31.G15\_2010.2640\_nc  
 Peak Lat: -26.870° Peak Lon: -65.470° Peak UT: 23.470 Peak LT: 19.110  
 Peak Hei: 439.830 km Peak dens: 1.70149700\*10^12 m^-3 Peak freq: 11.710 MHz

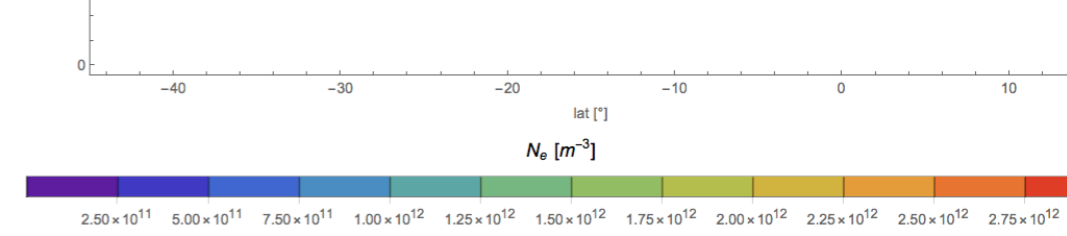
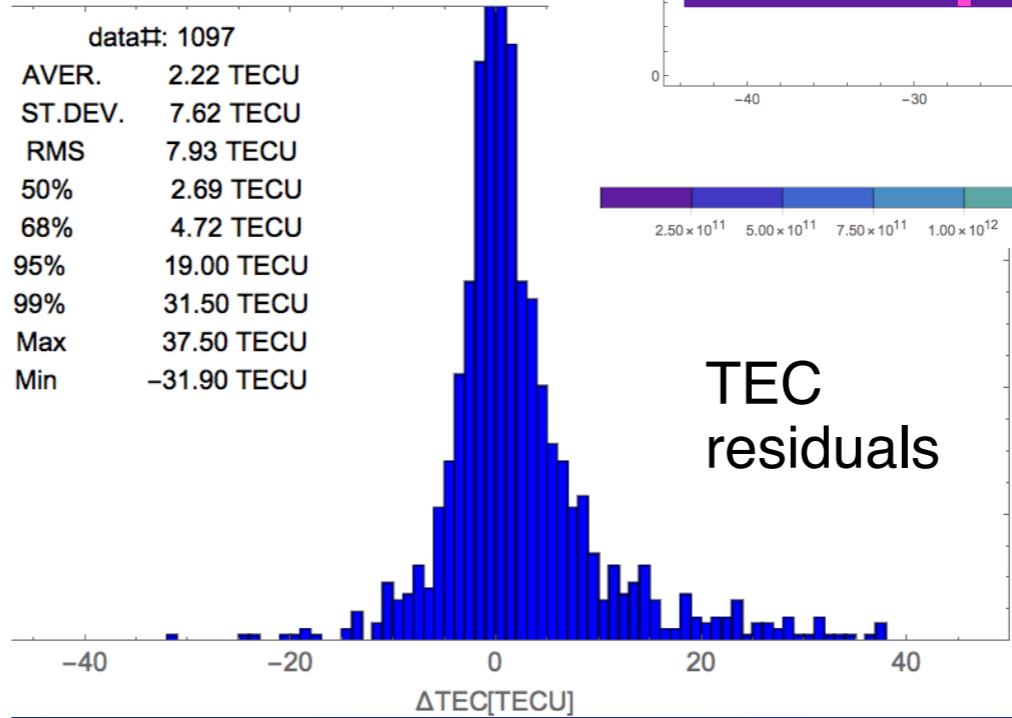
Ionosonde Analysis Background COSMIC Prf



$\Delta$  TEC (EXP - REC)

data#: 1097

AVER.	2.22 TECU
ST.DEV.	7.62 TECU
RMS	7.93 TECU
50%	2.69 TECU
68%	4.72 TECU
95%	19.00 TECU
99%	31.50 TECU
Max	37.50 TECU
Min	-31.90 TECU



Electron density cross sections at ionosonde longitude

Thank you for your attention

