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Non-Markovian Dynamics Far From Equilibrium | (SMR 3697)

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T01

A slowing down mechanism in relaxing wormlike micellar networks

Marco Baiesi[1,2], **Stefano Iubini**[3,4], **and Enzo Orlandini**[1,2]

[1] *Dipartimento di Fisica e Astronomia, Università di Padova, via Marzolo 8, I-35131 Padova, Italy*

[2] *INFN, Sezione di Padova, via Marzolo 8, I-35131 Padova, Italy*

[3] *Consiglio Nazionale delle Ricerche, Istituto dei Sistemi Complessi, via Madonna del Piano 10, I-50019 Sesto Fiorentino, Italy*

[4] *INFN, Sezione di Firenze, via G. Sansone 1, I-50019 Sesto Fiorentino, Italy*

Micellar networks are an important example of viscoelastic media with nontrivial relaxation and response properties. A mean-field kinetic model [1] supports a picture suggested earlier by numerical results [2]: the relaxation of a perturbed micellar network is a complex process in which branching (or cross-linking) first overshoots and then slowly reaches equilibrium, while the number of free end-caps remains in transient local equilibrium with branching. The kinetic equations of the model account for the scission and synthesis of wormlike and cross-linked portions of the surfactant network. They lead to the same mechanisms for the relaxation after abrupt thermal quenches and small mechanical perturbations, which indeed share some similarity in their phenomenologies. We speculate on possible underestimates of the typical time scales of viscoelastic fluids, due to the difficulty of detecting the subtle effects of the end-recombination dynamics.

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T02

Long-range effects in the Fick-Jacobs equation for diffusion in narrow channels

Guillermo Chacón-Acosta [1] **, Alejandro León-Ramírez**[2]

[1] *Applied Mathematics and Systems Department, Universidad Autónoma Metropolitana Cuajimalpa, Mexico City 5300, Mexico*

[2] *Natural Sciences and Engineering Graduate Studies, Universidad Autónoma Metropolitana Cuajimalpa, Mexico City 5300, Mexico*

In some situations such as hoarded media where there are high concentrations or when there are memory effects, the diffusive flux is not enough to describe the behavior of the system, so it is necessary to consider long-range terms. This can be achieved by introducing a biharmonic additional term in the Fick equation [1]. In this work, we study the effect of a biharmonic term on the diffusion of Brownian particles confined in a 2D narrow channel whose longitudinal coordinate is larger than the transverse coordinate [2]. A Fick-Jacobs type equation is found using the projection method [3], with a third-order entropic flux additional to the standard first-order flux, where, even at the lowest order, position-dependent modifications appear in the longitudinal diffusivity and in the drift term, involving an additional scale. Furthermore, higher order corrections to the corresponding diffusion and long-range coefficients, both dependent on the longitudinal coordinate, are obtained iteratively.

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T03

Measurement induced criticality from dissipative (non-)Markovian dynamics

It has recently been shown that phase transitions may be driven by the interaction of a quantum system with the environment and emerge both at the level of ordered-disordered phase (such as ferromagnetic vs paramagnetic) and at the level of the scaling law of entanglement entropy. In this talk, I present a study of these two classes of transition within the same setting - a Ising spin chain subject to unitary (from long-range interactions) and dissipative (from random projective measurements) dynamics. The two transitions are both present, but are distinct and occur at different interactions ranges. All the literature studies consider the case of Markovian projective measurements, but most of the environments have a non-Markovian structure. It seems therefore natural to also study the case of dissipative processes driven by a non-Markovian environment.

T04

Thermodynamics of the generalized Langevin equation and its applications

The generalized Langevin equation (GLE) is the non-Markovian extension of the equation introduced by Langevin to study the Brownian motion of particles immersed in a simple fluid. Indeed, if the particle is immersed in a solution containing for example long and complex polymers, a net separation of time scales relative to the colloidal particle and the fluid is not possible and memory effects occur. These effects are characteristic of viscoelastic fluids and, in this context, the GLE emerges as a very effective model to describe such kind of systems. Furthermore, the GLE can be used to model the motion of a reaction coordinate in a free energy landscape describing, for example, the zipping/unzipping dynamic of a DNA hairpin. By introducing a new method to deal with the large time behaviour of the solutions of the linear GLE, we will be able to highlight the differences as well as the similarities between the GLE and the Markovian Langevin equations in the context of stochastic thermodynamics. We will also show how some care must be taken when the non-Markovian extension of the overdamped Langevin equation is considered. The discussion will be also complemented by simple but at the same time relevant examples stemming from the field of biological Physics

T05

Different indicators for Markovian and non-Markovian dynamics

K. El Anouz [1] , A. El Allati [1,2] , and N. Metwally [3,4]

[1]*Laboratory of R&D in Engineering Sciences, Faculty of Sciences and Techniques Al-Hoceima, Abdelmalek Essaadi University, Tétouan, Morocco*

[2] *The Abdus Salam International Center for Theoretical Physics, Strada Costiera 11, Miramare-Trieste, Italy*

[3]*Math. Dept., College of Science, University of Bahrain, P.O. Box 32038, Bahrain*
[4] *Math. Dept., Faculty of Science, Aswan University, Aswan, Egypt*

The Markovianity/non-Markovianity of two different systems will be discussed by means of the quantum speed limit time and quantum Fisher information. The first system is described by a central mass particle that interacts locally with its surrounding particles, while the second and third systems consist of a single qubit that interacts with a non-detuning Lorentzian cavity and with a thermal reservoir, respectively. For the first model, the large distance between the central particle and the surrounding particles is a guarantee of a fixed quantum speed limit, while the driving time plays the central role in the fixed behavior of the quantum speed limit time. Due to the stable behavior of the quantum speed limit time and the quantum Fisher information, the exchange of information between the systems and their surroundings is limited. The distance between the central mass particle and its surrounding particles plays the main role in predicating the Markovianity/non-Markovianity. For the second system, driving time is an important parameter that controls the Markovianity/non-Markovianity behavior. Finally, the third system proves that non-Markovian dynamics may increase the speed and sensitivity of the open system.

[1] K. El Anouz, A. El Allati, N. Metwally, Phy. Lett. A. **384**, 126122 (2020)

T06

Non-Markovian Brownian dynamics in nonequilibrium baths

Gianmaria Falasco[1], **Klaus Kroy**[2]

[1] *Complex Systems and Statistical Mechanics, Physics and Materials Science Research Unit, University of Luxembourg, L-1511 Luxembourg, Luxembourg*
[2] *Institut für Theoretische Physik, Universität Leipzig, Postfach 100 920, D-04009 Leipzig, Germany*

The Brownian motion of a dense particle in a liquid at thermal equilibrium is the historical paradigm of non-Markovian dynamics—in which the slow decay of the velocity correlation function was first measured, questioned, and finally theoretically understood. The same dynamics taking place in a nonequilibrium solvent is arguably the simplest system to understand the interplay between long-time tails and energy dissipation. In my talk I will begin by considering a solvent under a thermal gradient. This case allows us to analytically work out the breaking of the equipartition and fluctuation-dissipation theorem starting from the fluctuating hydrodynamics of the liquid [2]. Then I will focus on a stirred (or active) solvent. To this aim I will introduce an approach based on nonequilibrium response theory able to unveil *inter alia* the breaking of the action-reaction law for suspended Brownian particles [3].

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T07

Entropy Production of Non-reciprocal Interactions

R. Garcia-Millan[1,2], **Z. Zhang**[3], **and G. Pruessner**[3]

[1]*Department of Applied Mathematics and Theoretical Physics, University of Cambridge, Cambridge, UK*
[2]*St John's College, University of Cambridge, Cambridge, UK*
[3]*Department of Mathematics, Imperial College London, London, UK*

Non-reciprocal interactions are very common in natural systems. They can be used to explain the emergence of certain patterns such as bird flocking [1]. Generally, systems with non-reciprocal interactions are out of equilibrium, although they can obey detailed balance under certain conditions [2]. In this talk, I will present a particle model with non-reciprocal pair interactions between two species of drift-diffusive particles, say dogs and sheep. Following a path integral approach, I will discuss the stationary two-point correlation function and the entropy production. Even in the absence of drift, detailed balance is broken by non-reciprocity except for a particular choice of pair interactions.

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[2] S. Loos, S. Klapp, New J. Phys. 22(12), 123051 (2020).

T08

First Passage Times for non-Markovian Gaussian processes

N Levernier[1], O Bénichou[2], R Voituriez[2,3], and T Guérin[4]

[1] *Aix Marseille Univ., Université de Toulon, CNRS, CPT, Turing Center for Living Systems, 13009 Marseille, France*

[2]*Laboratoire de Physique Théorique de la Matière Condensée, CNRS/UPMC, 4 Place Jussieu, 75005 Paris, France*

[3]*Laboratoire Jean Perrin, CNRS/UPMC, 4 Place Jussieu, 75005 Paris, France*

[4] *Laboratoire Ondes et Matière d'Aquitaine, University of Bordeaux, UMR 5798, CNRS, F-33400 Talence, France*

How much time does it take for a random walker to reach a target ? This question is recurrent in reactivity or search problems, and its answer is provided by the First Passage Time (FPT). First passage properties are now well understood for Markovian (memory-less) processes. However, as soon as a random walker interacts with other variables in its environment, its effective motion becomes non-Markovian and first passage properties are much less understood. I will present a formalism that provides the mean FPT of a Gaussian random walker to a target in a large confining volume [1]. The key aspect of the theory consists in analyzing the statistics of the paths in the future of the FPT, which can be determined self-consistently. The distribution of these paths is very different from stationary paths. The obtained results for the mean FPT can be very different from standard "pseudo-Markovian" approximations, but agree with simulations and are exact for weakly non-Markovian processes. Next, I will show how the same formalism can be adapted to characterize (i) the large time asymptotics of the FPT distribution without confinement [2], and (ii) the kinetics to reach a rarely visited configuration for a non-Markovian reaction coordinate (even non-Gaussian) [3].

[1] T Guérin, N Levernier, O Bénichou, R Voituriez, Nature **534**, 356-359 (2016).
[2] N Levernier, M Dolgushev, O Bénichou, R Voituriez, T Guérin, Nat. Com. **10**, 1-7 (2019).
[3] N Levernier, O Bénichou, R Voituriez, T Guérin, Phys. Rev. Res. **2**, 012057 (2020).

T09

Non-Arrhenius barrier-crossing dynamics of non-equilibrium non-Markovian systems

Motivated by in-vivo protein folding, we merge systems far from equilibrium and non-Markovian barrier-crossing dynamics and use a generic one-dimensional model that can be turned off equilibrium by choosing different friction and random relaxation times. We analytically derive a formula for the barrier-crossing time and favorably compare it with extensive non-equilibrium simulations. Non-equilibrium effects modify the basic law of reaction kinetics, the Arrhenius law, by changing the exponential dependence of the barrier-crossing time on the barrier energy, which is a very unexpected and fundamental result.

T10

Everlasting impact of inital perturbations on first-passage times of non-Markovian random walks

Persistence, defined as the probability that a fluctuating signal has not reached a threshold up to a given observation time, plays a crucial role in the theory of random processes. It quantifies the kinetics of processes as varied as phase ordering, reaction diffusion or interface relaxation dynamics. The fact that persistence can decay algebraically with time with non trivial exponents has triggered a number of experimental and theoretical studies. However, general analytical methods to calculate persistence exponents cannot be applied to the ubiquitous case of non-Markovian systems relaxing transiently after an imposed initial perturbation. Here, we introduce a theoretical framework that enables the non perturbative determination of persistence exponents of d-dimensional Gaussian non-Markovian processes with general non stationary dynamics relaxing to a steady state after an initial perturbation. Two prototypical classes of situations are analyzed: either the system is subjected to a temperature quench at initial time, or its past trajectory is assumed to have been observed and thus known. Altogether, our results reveal and quantify, on the basis of Gaussian processes, the deep impact of initial perturbations on first-passage statistics of non-Markovian processes.

T11

Abstract template for Non-Markovian Dynamics Far from Equilibrium

Ghost stochastic resonance in an asymmetric Duffing oscillator

A.F. Moyo Tala [1], Y.J. Wadop Ngouongo[1], G. Djuidjé Kenmoé[1,] and T.C. Kofané [1]

[1] Department of Physics, Faculty of Science, University of Yaounde I

We report the occurrence of ghost stochastic resonance (GSR) for a Brownian particle in three types of asymmetric potentials. The asymmetries are controlled by the asymmetry parameter and introduced in the potential by varying the left well. These variations are namely (i) the depth alone, (ii) the width alone and (iii) both depth and width. The properties of GSR in the asymmetric systems are different from the symmetric systems. It is demonstrated that the resonance observed at the ghost frequency is due to the existence of this frequency during the dynamics of the particle. We also examine numerically the effect of the asymmetry parameter on GSR phenomenon. It is found that the asymmetry parameter has a significant influence on the GSR and allows a precise control of the appearance of GSR.

T12

Enhancing spin-spin correlations in mixed-field Ising model with stochastic resetting

Vladimir Ohanesjan[1,2], **Irina Petreska**[2], **and Trifce Sandev**[2,3,4]

[1]*Instituut-Lorentz, Universiteit Leiden, P.O. Box 9506, 2300 RA Leiden, The Netherlands*
[2]*Institute of Physics, Faculty of Natural Sciences and Mathematics, Ss. Cyril and Methodius University in Skopje, Arhimedova 3, 1000 Skopje, Macedonia*
[3]*Research Center for Computer Science and Information Technologies, Macedonian Academy of Sciences and Arts, Bul. Krste Misirkov 2, 1000 Skopje, Macedonia*
[4]*Institute of Physics & Astronomy, University of Potsdam, D-14776 Potsdam-Golm, Germany*

In several recent works, it was suggested that stochastic resetting in quantum systems may lead to emergence of non-trivial correlations and effective creation of open systems by external interventions [1, 2]. In classical systems, problems of stochastic resetting have been extensively studied, while in this context, much less is known for quantum systems and many open questions are yet to be answered. Here, we are interested in the mixed-field Ising (MFI) model in presence of resetting protocols that have been previously applied to systems with classical dynamics [3, 4, 5]. We analyze analytically the transverse correlations $\sigma_i^x \sigma_j^x(t)$ for a particular MFI without nearest neighbour interactions when being reset to the all-up state $|\uparrow\rangle$ and find nontrivial resetting induced correlations, similar to what was reported in [6] for the longitudinal correlations. For a specific magnitude of the transverse field, the $\sigma_i^x \sigma_j^x(t)$ correlations reach a maximum and increasing the field further only decorrelates the system. In order to obtain a deeper insight into the maximum of the correlations, we have analyzed their dependence on both, the resetting rate and the external fields. Furthermore, using numerical techniques we study analytically inaccessible realizations of MFI, like the chaotic regime, and explore the effects of resetting on the internal unitary dynamics. Our results suggest that the $\sigma_i^x \sigma_j^x$ spin-spin correlations can be fine tuned by the stochastic resetting and we concluded that some optimal range exists for the resetting rate parameter, which enhances those correlations. Further investigations are needed to confirm if the stochastic resetting is an efficient approach to enable modulation of the order parameter and even affect the phase transitions [7].

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[6] M. Magoni, F. Carollo, G. Perfetto, I. Lesanovsky, arXiv:2202.12655 [quant-ph] (2022).
[7] M. Magoni, S. N. Majumdar, G. Schehr, Phys. Rev. Res. **2**, 033182 (2020).

T13

Abstract template for talk: Resetting quantum systems

Gabriele Perfetto [1], Federico Carollo [1], Matteo Magoni and Igor Lesanovsky [1,2]

[1] Institut für Theoretische Physik, Universität Tübingen, Auf der Morgenstelle 14, 72076 Tübingen, Germany

[2] School of Physics and Astronomy and Centre for the Mathematics and Theoretical Physics of Quantum Non-Equilibrium Systems, The University of Nottingham, Nottingham, NG7 2RD, United Kingdom

In the first part of the talk, we consider closed quantum many-body systems subject to stochastic resetting. This means that their unitary time evolution is interrupted by resets at randomly selected times. The study of the non-equilibrium stationary state that emerges from the combination of stochastic resetting and coherent quantum dynamics has recently raised significant interest. The connection between this non-equilibrium stationary state, an effective open dynamics and non-equilibrium signatures of quantum phase transitions is, however, not fully understood. We provide a unified understanding of these phenomena by combining techniques from quantum quenches in closed systems and semi-Markov processes. We discuss as an application the paradigmatic transverse-field quantum Ising chain. We show that signatures of its ground-state quantum phase transition are visible in the steady state of the reset dynamics as a sharp crossover.

In the second part of the talk, we consider the case where stochastic resetting is superimposed to a Markovian open quantum dynamics. We show that the ensuing dynamics is non-Markovian and has the form of a generalized Lindblad equation. Interestingly, the large-deviation statistics of quantum-jumps can be exactly derived. This is achieved by combining techniques from the thermodynamics of quantum-jump trajectories with the renewal structure of the resetting dynamics. Our findings show that stochastic resetting may be exploited as a tool to tailor the statistics of the quantum-jump trajectories and the dynamical phases of open quantum systems.

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[2] G. Perfetto, F. Carollo, I. Lesanovsky, Arxiv:2112.05078 (2021).

T14

Spectral density of individual trajectories of an active Brownian particle

A. Squarcini [1,2,3], A. Solon[4], and G. Oshanin[4]

[1]Max-Planck-Institute for Intelligent Systems, Heisenbergstr. 3, D-70569, Stuttgart, Germany
[2] IV. Institut für Theoretische Physik, Universität Stuttgart, Pfaffenwaldring 57, D-70569 Stuttgart, Germany
[3] Institut für Theoretische Physik, Universität Innsbruck, Technikerstraße 21A, A-6020 Innsbruck, Austria
[4] Sorbonne Université, CNRS, Laboratoire de Physique Théorique de la Matière Condensée (UMR CNRS 7600), 4 Place Jussieu, 75252 Paris Cedex 05, France

As witnessed by numerous applications, the power spectral density (PSD) embodies a wealth of informations about the time evolution of a stochastic process. In a series of recent works (including - inter alia - [1,2]) it has been pointed out that the commonly used notion of PSD has certain intrinsic limitations which make its determination/interpretation rather difficult. In particular, in many experimental situations it is not possible to achieve statistically adequate sampling required in order to perform ensemble averages. Moreover, it is not possible to monitor the trajectory for infinitely long time, as demanded by the standard definition. The notion of single-trajectory spectral density (STSD) shows how to go beyond the aforementioned conceptual and practical limitations. In the first part of the talk I will summarize some of the achievements obtained within the single-trajectory analysis of stochastic processes via power spectra; the discussion encompasses the standard diffusion [1] and the fractional Brownian motion [2]. In the second part of the talk I will analyze the spectral content of the so-called active Brownian particle (ABP), which is one of the simplest models of a particle undergoing active motion; e.g., a chemically-active Janus colloid. It is known that the ABP behaves as a standard Brownian particle at large time scaled but nonetheless its spectral content is not known; this gap has been filled only recently in [3]. Firstly, I evaluate the standardly-defined spectral density, i.e. the STSD averaged over a statistical ensemble of trajectories in the limit of an infinitely long observation time T. Then, I will present results for the finite-T behavior for the power spectral density and for the coefficient of variation of the STSD distribution. The cross correlation between spatial components of the STSD provides an additional spectral fingerprint which is computed. Finally I will address the effects of translational diffusion on the functional forms of spectral densities. The exact expressions that are obtained unveil many distinctive features of active Brownian motion compared to its passive counterpart, which allow to distinguish between these two classes based solely on the spectral content of individual trajectories.

[1] D. Krapf, E. Marinari, R. Metzler, G. Oshanin, X. Xu, and A. Squarcini, *Power spectral density of a single Brownian trajectory: what one can and cannot learn from it*, New J. Phys. 20, 023029 (2018).

[2] D. Krapf, N. Lukat, E. Marinari, R. Metzler, G. Oshanin, C. Selhuber-Unkel, A. Squarcini, L. Stadler, M. Weiss, and X. Xu, *Spectral content of a single non-Brownian trajectory*, Phys. Rev. X 9, 011019 (2019).

[3] A. Squarcini, A. Solon, and G. Oshanin, *Power spectral density of trajectories of an active Brownian particle*, New J. Phys. 24, 013018 (2022).

T15

Memory effects in confined colloidal motion

Arthur Straube [1] **and Felix Höfling** [2]

[1] *Modeling and Simulation of Complex Processes, Zuse Institute Berlin, Germany*
[2] *Department of Mathematics and Computer Science, Freie Universität Berlin, Germany*

Generalized Lagevin equations (GLEs) have proven as useful models to reduce the complexity of a system, by incorporating a large number of irrelevant degrees of freedom into a memory kernel. A formidable challenge is parameterizing the GLE from observations of such a complex system, that is estimating the memory kernel from simulation or experimental data. We have recently developed a general method to estimate memory functions from the mean-square displacement (MSD) as sole input [1], combining representations in the time and frequency domains. Further, while the mainstream of studies utilizing the GLE formalism tacitly implies the presence of inertia, colloidal systems belong to a wide class of strongly overdamped systems. In this contribution, I will show how our method [1] can be adjusted for an overdamped system and provide an analytically tractable example for a colloidal particle confined to a periodic energy landscape. The gained insights can be important for understanding from the perspective of non-Markovian dynamics active colloidal propulsion over periodic geometry landscapes [2] as well as passive colloids in (or driven over) periodic energy landscapes [3,4].

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T16

Modelling Active Non-Markovian Oscillations

G. Tucci[1], É. Roldán[2], A. Gambassi[1], R. Belousov[2,3], F. Berger[4], R. G. Alonso[5], and A. J. Hudspeth[5]

[1] *SISSA — International School for Advanced Studies and INFN, via Bonomea 265, 34136 Trieste, Italy*

[2] *ICTP — The Abdus Salam International Centre for Theoretical Physics, Strada Costiera 11, 34151 Trieste, Italy*

[3] EMBL — European Molecular Biology Laboratory, Meyerhofstr. 1, 69117, Heidelberg, Germany

[4] Cell Biology, Neurobiology and Biophysics, Department of Biology, Faculty of Science, Utrecht University, 3584 CH, Utrecht, The Netherlands

[5] Howard Hughes Medical Institute and Laboratory of Sensory Neuroscience, The Rockefeller University, 1230 York Avenue, New York, NY 10065, USA

Modelling noisy oscillatory Active non-Markovian systems is one of the current challenges in physics and biology. Among these systems, we are interested in describing the spontaneous oscillations of the hair bundle of the inner ear cells of the bullfrog. The hair bundle is an organelle formed by a cohesive tuft of cylindrical stereocillia that protrude from the apical surface of the namesake hair cells. This receptor cells transduces a mechanical stimulus, such as a sound wave, into a neural signal and thus facilitates hearing and other sensory processes in vertebrates. The oscillatory motion of a hair bundle is powered by an active non-Markovian process, which is essential for the organelle's sensory function, and results in the violation of the fluctuation-dissipation theorem.

Because the microscopic mechanisms governing these type of processes are difficult to model, we propose an effective description based on a stochastic system displaying periodic oscillations. For this purpose, we consider the motion of a Brownian particle in the presence of a harmonic potential whose center switches stochastically between two distinct points. Accordingly, the dynamics of the particle consists of an alternate relaxation towards the two centers of the potential. The resulting oscillatory trajectories are governed by the probability distribution of the waiting time between two consecutive switches: this mechanism makes the evolution of the system non-Markovian. Thanks to the linearity of the model, we derive analytical predictions for its most relevant dynamical and thermodynamic properties for any choice of the waiting-time distribution.

This minimal model describes accurately bistable-like oscillatory motion of hair bundles in bullfrog sacculus. We check this by using the analytical expression of the power spectrum of the model to fit that of the oscillations of the tip of the bullfrog's hair bundle, measured experimentally by A. J. Hudspeth's laboratory (Rockefeller University, NY). In order to characterize the thermodynamic properties of these trajectories, we substitute the inferred parameters of the model into the exact expression of its average stationary power, enabling us to estimate the power required to sustain such active oscillations. In agreement with the active nature of the process and with previous estimates of the entropy production, we find that the predicted average dissipated power per cycle is compatible with the consumption of ~ 10ATP molecules to fuel a single oscillation.

[1] G. Tucci, É. Roldán, A. Gambassi, R. Belousov, F. Berger, R. G. Alonso, and A. J. Hudspeth, *Modelling Active Non-Markovian Oscillators*, arXiv:2201.12171.

T17

Unified non-Hermitian-Lindblad approach and its applications to quantum dissipative systems

We compare two approaches to open quantum systems far from equilibrium, namely, the non-Hermitian dynamics and the Lindblad master equation. In order to deal with more general dissipative phenomena, we propose the unified master equation that combines the characteristics of both of these approaches. This allows us to assess the differences between them as well as to clarify which observed features come from the Lindblad or the non-Hermitian part, when it comes to experiment. Using a generic two-mode single-atom laser system as a practical example, we analytically solve the dynamics of the normalized density matrix operator.