Spectral density of individual trajectories of an active Brownian particle

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As witnessed by numerous applications, the power spectral density (PSD) embodies a wealth of informations about the time evolution of a stochastic process. In a series of recent works (including - inter alia - [1,2]) it has been pointed out that the commonly used notion of PSD has certain intrinsic limitations which make its determination/interpretation rather difficult. In particular, in many experimental situations it is not possible to achieve statistically adequate sampling required in order to perform ensemble averages. Moreover, it is not possible to monitor the trajectory for infinitely long time, as demanded by the standard definition. The notion of single-trajectory spectral density (STSD) shows how to go beyond the aforementioned conceptual and practical limitations. In the first part of the talk I will summarize some of the achievements obtained within the single-trajectory analysis of stochastic processes via power spectra; the discussion encompasses the standard diffusion [1] and the fractional Brownian motion [2]. In the second part of the talk I will analyze the spectral content of the so-called active Brownian particle (ABP), which is one of the simplest models of a particle undergoing active motion; e.g., a chemically-active Janus colloid. It is known that the ABP behaves as a standard Brownian particle at large time scaled but nonetheless its spectral content is not known; this gap has been filled only recently in [3]. Firstly, I evaluate the standardly-defined spectral density, i.e. the STSD averaged over a statistical ensemble of trajectories in the limit of an infinitely long observation time T. Then, I will present results for the finite-T behavior for the power spectral density and for the coefficient of variation of the STSD distribution. The cross correlation between spatial components of the STSD provides an additional spectral fingerprint which is computed. Finally I will address the effects of translational diffusion on the functional forms of spectral densities. The exact expressions that are obtained unveil many distinctive features of active Brownian motion compared to its passive counterpart, which allow to distinguish between these two classes based solely on the spectral content of individual trajectories.

[1] D. Krapf, E. Marinari, R. Metzler, G. Oshanin, X. Xu, and A. Squarcini, *Power spectral density of a single Brownian trajectory: what one can and cannot learn from it*, New J. Phys. 20, 023029 (2018).

[2] D. Krapf, N. Lukat, E. Marinari, R. Metzler, G. Oshanin, C. Selhuber-Unkel, A. Squarcini, L. Stadler, M. Weiss, and X. Xu, *Spectral content of a single non-Brownian trajectory*, Phys. Rev. X 9, 011019 (2019).

[3] A. Squarcini, A. Solon, and G. Oshanin, *Power spectral density of trajectories of an active Brownian particle*, New J. Phys. 24, 013018 (2022).