Modelling Active Non-Markovian Oscillations

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Modelling noisy oscillatory Active non-Markovian systems is one of the current challenges in physics and biology. Among these systems, we are interested in describing the spontaneous oscillations of the hair bundle of the inner ear cells of the bullfrog. The hair bundle is an organelle formed by a cohesive tuft of cylindrical stereocillia that protrude from the apical surface of the namesake hair cells. This receptor cells transduces a mechanical stimulus, such as a sound wave, into a neural signal and thus facilitates hearing and other sensory processes in vertebrates. The oscillatory motion of a hair bundle is powered by an active non-Markovian process, which is essential for the organelle's sensory function, and results in the violation of the fluctuation-dissipation theorem.

Because the microscopic mechanisms governing these type of processes are difficult to model, we propose an effective description based on a stochastic system displaying periodic oscillations. For this purpose, we consider the motion of a Brownian particle in the presence of a harmonic potential whose center switches stochastically between two distinct points. Accordingly, the dynamics of the particle consists of an alternate relaxation towards the two centers of the potential. The resulting oscillatory trajectories are governed by the probability distribution of the waiting time between two consecutive switches: this mechanism makes the evolution of the system non-Markovian. Thanks to the linearity of the model, we derive analytical predictions for its most relevant dynamical and thermodynamic properties for any choice of the waiting-time distribution.

This minimal model describes accurately bistable-like oscillatory motion of hair bundles in bullfrog sacculus. We check this by using the analytical expression of the power spectrum of the model to fit that of the oscillations of the tip of the bullfrog's hair bundle, measured experimentally by A. J. Hudspeth's laboratory (Rockefeller University, NY). In order to characterize the thermodynamic properties of these trajectories, we substitute the inferred parameters of the model into the exact expression of its average stationary power, enabling us to estimate the power required to sustain such active oscillations. In agreement with the active nature of the process and with previous estimates of the entropy production, we find that the predicted average dissipated power per cycle is compatible with the consumption of ~ 10ATP molecules to fuel a single oscillation.

[1] G. Tucci, É. Roldán, A. Gambassi, R. Belousov, F. Berger, R. G. Alonso, and A. J. Hudspeth, *Modelling Active Non-Markovian Oscillators*, arXiv:2201.12171.