Generation of multipartite entanglement in Josephson parametric systems at microwave frequencies

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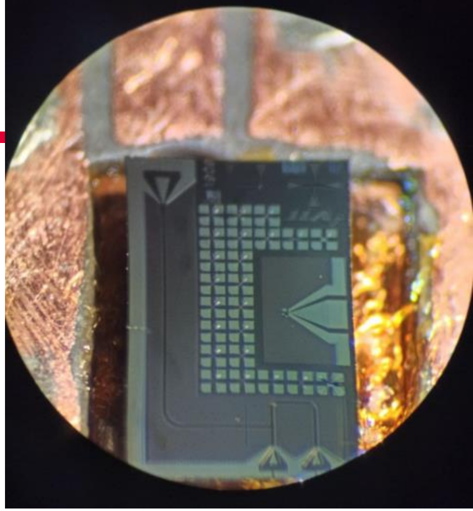
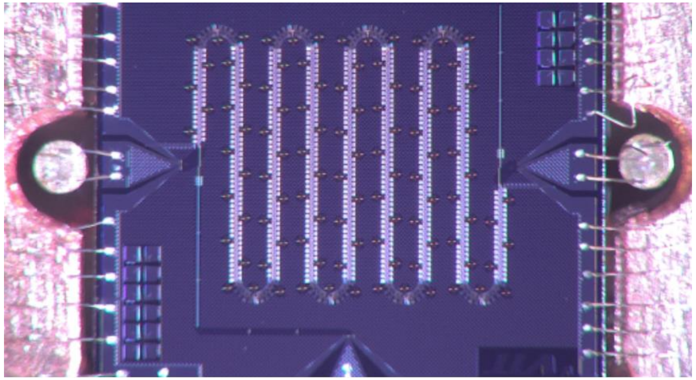
Kirill Petrovnin, …, PJH, arXiv:2203.09247

Michael Perelshtein, …, PJH, arXiv:2111.06145

Outline

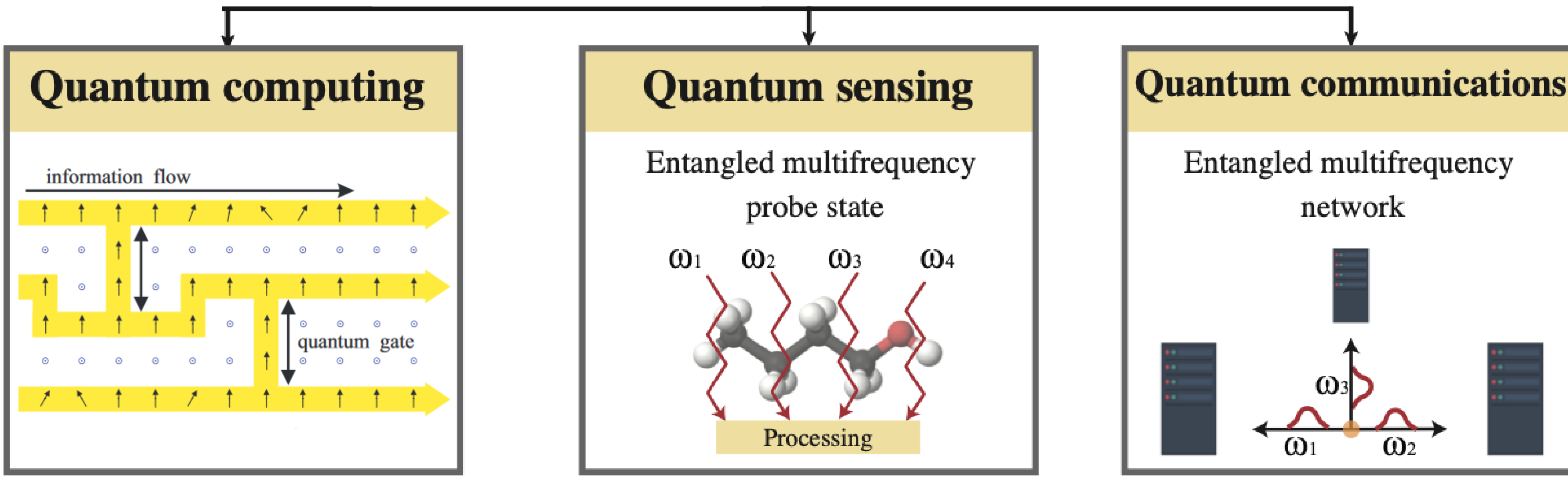


- Introduction
- Parametric system with multiple pumps
  - structure of states
- Experimental setting and basic characteristics
- Experimental results on genuine multipartite entanglement
- Broad band generation of correlations using TWPA
  - two mode squeezing
  - genuine multipartite entanglement
- Future plans
- Summary



Motivation for quantum technology

Fast robust generation of entangled states allows:



-Cluster states for one-way QC

-Quantum illumination

-Quantum key sharing

Parametric system with multiple pumps

Hamiltonian for parametrically-driven cavity, RWA (frame $\omega_\Sigma/2$)

$$\omega_\Sigma = \frac{1}{p}\sum_{d=1}^{p}\omega_d \qquad \Delta_d = \omega_d - \omega_\Sigma \qquad \Delta_r = \omega_r - \omega_\Sigma$$

$$H_{\rm sys,rwa}(t) = \hbar\Delta_r\tilde{a}^\dagger\tilde{a} + \frac{\hbar}{2}\sum_{d=1}^{p}(\alpha_d^* e^{i\Delta_d t}\tilde{a}^2 + \alpha_d e^{-i\Delta_d t}\tilde{a}^{\dagger 2}) + 6\hbar K\tilde{a}^\dagger\tilde{a}^\dagger\tilde{a}\tilde{a},$$

Linear part: analytics, interaction graphs, gain and squeezing coefficient below threshold

Nonlinear part (numerical simulations, dynamics above threshold, oscillation, transition between states)

Quantum Langevin Equation (QLE)

$$\dot{\tilde{a}}(t) = (-i\Delta_r - \frac{\kappa+\gamma}{2})\tilde{a} - i\sum_{d=1}^{p}\alpha_d e^{i\Delta_d t}\tilde{a}^\dagger + \sqrt{\kappa}\tilde{b}_{in} + \sqrt{\gamma}\tilde{c}_{in} - 12iK\tilde{a}^\dagger\tilde{a}\tilde{a}$$

input-output relationship

$$\tilde{b}_{out}(t) = \tilde{b}_{in}(t) - \sqrt{\kappa}\tilde{a}(t)$$

Simplified theoretical analysis

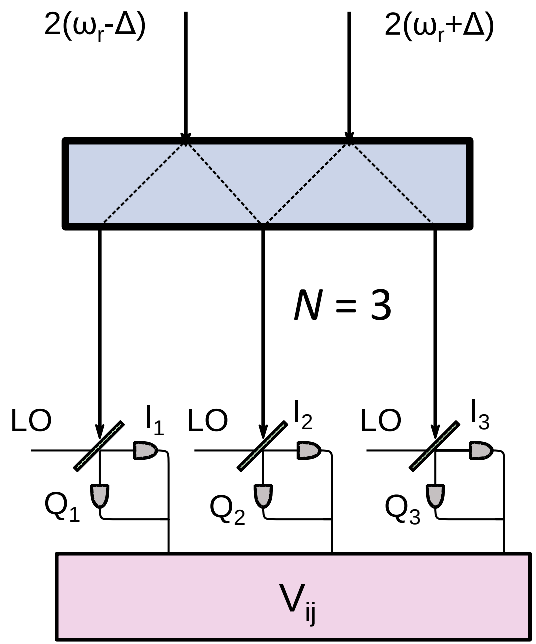
➢ Simplification of conditions: zero detuning, working point far below critical threshold ($\alpha < \alpha_{\text{crit}}$), no internal dissipation ($\gamma = 0$)

$$\dot{\tilde{a}}(t) = (-i\Delta_r - \tfrac{\kappa+\gamma}{2})\tilde{a} - i\sum_{d=1}^{p} \alpha_d e^{i\Delta_d t}\tilde{a}^\dagger + \sqrt{\kappa}\tilde{b}_{in} + \sqrt{\gamma}\tilde{c}_{in} - 12iK\tilde{a}^\dagger\tilde{a}\tilde{a}$$

$$\hat{a} = \{a_1, a_2, a_3, a_1^\dagger, a_2^\dagger, a_3^\dagger\}^{Tr}$$ - 3 mode case

$$(-i(\omega - \Delta_r) + \tfrac{\kappa}{2})\tilde{a}(\omega) + i\alpha(\int \tilde{a}^\dagger(t)e^{i\omega t}e^{-i\Delta_d t}dt + \int \tilde{a}^\dagger(t)e^{i\omega t}e^{i\Delta_d t}dt) = \sqrt{\kappa}\tilde{b}_{in}(\omega)$$

$$\hat{M}\tilde{a}(\omega) = \sqrt{\kappa}\tilde{b}_{\text{in}}(\omega)$$



Matrix analysis of modes (N = 3, Δ_r=0)

Input-output relation

$$\hat{M}\tilde{a}(\omega) = \sqrt{\kappa}\tilde{b}_{\text{in}}(\omega) \Rightarrow \tilde{a}(\omega) = \sqrt{\kappa}\hat{M}^{-1}\tilde{b}_{\text{in}}(\omega) \Rightarrow \tilde{b}_{\text{out}}(\omega) = (\hat{I} - \kappa\hat{M}^{-1})\tilde{b}_{\text{in}}(\omega).$$

$$\hat{M} = \begin{bmatrix} c_1 & 0 & 0 & 0 & i\alpha & 0 \\ 0 & c_1 & 0 & i\alpha & 0 & i\alpha \\ 0 & 0 & c_1 & 0 & i\alpha & 0 \\ 0 & -i\alpha^\dagger & 0 & c_2 & 0 & 0 \\ -i\alpha^\dagger & 0 & -i\alpha^\dagger & 0 & c_2 & 0 \\ 0 & -i\alpha^\dagger & 0 & 0 & 0 & c_2 \end{bmatrix}$$

Inverse

$$\hat{M}^{-1} = \frac{1}{c^2 - 2\alpha^2} \cdot \begin{bmatrix} c - \frac{\alpha^2}{c} & 0 & \frac{\alpha^2}{c} & 0 & -i\alpha & 0 \\ 0 & c & 0 & -i\alpha & 0 & -i\alpha \\ \frac{\alpha^2}{c} & 0 & c - \frac{\alpha^2}{c} & 0 & -i\alpha & 0 \\ 0 & i\alpha & 0 & c - \frac{\alpha^2}{c} & 0 & \frac{\alpha^2}{c} \\ i\alpha & 0 & i\alpha & 0 & c & 0 \\ 0 & i\alpha & 0 & \frac{\alpha^2}{c} & 0 & c - \frac{\alpha^2}{c} \end{bmatrix}$$

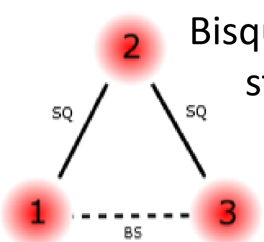
Mode basis -> Quadrature basis

$$\hat{S}^{-1} = \frac{1}{c^2 - 2\alpha^2} \cdot \begin{bmatrix} c + \frac{\alpha^2}{c} & 0 & 0 & -\alpha & \frac{\alpha^2}{c} & 0 \\ 0 & c + \frac{\alpha^2}{c} & -\alpha & 0 & 0 & \frac{\alpha^2}{c} \\ 0 & -\alpha & c & 0 & 0 & -\alpha \\ -\alpha & 0 & 0 & c & -\alpha & 0 \\ \frac{\alpha^2}{c} & 0 & 0 & -\alpha & c + \frac{\alpha^2}{c} & 0 \\ 0 & \frac{\alpha^2}{c} & -\alpha & 0 & 0 & c + \frac{\alpha^2}{c} \end{bmatrix}$$

$$x_i = \left(a_i^\dagger + a_i\right)/2$$
$$p_i = \left(a_i - a_i^\dagger\right)/2i$$

Covariance matrix
For Gaussian system

$$\hat{V}_a = \hat{S}^{-1}\hat{V}_{in}(\hat{S}^{-1})^T$$



Bisqueezed state

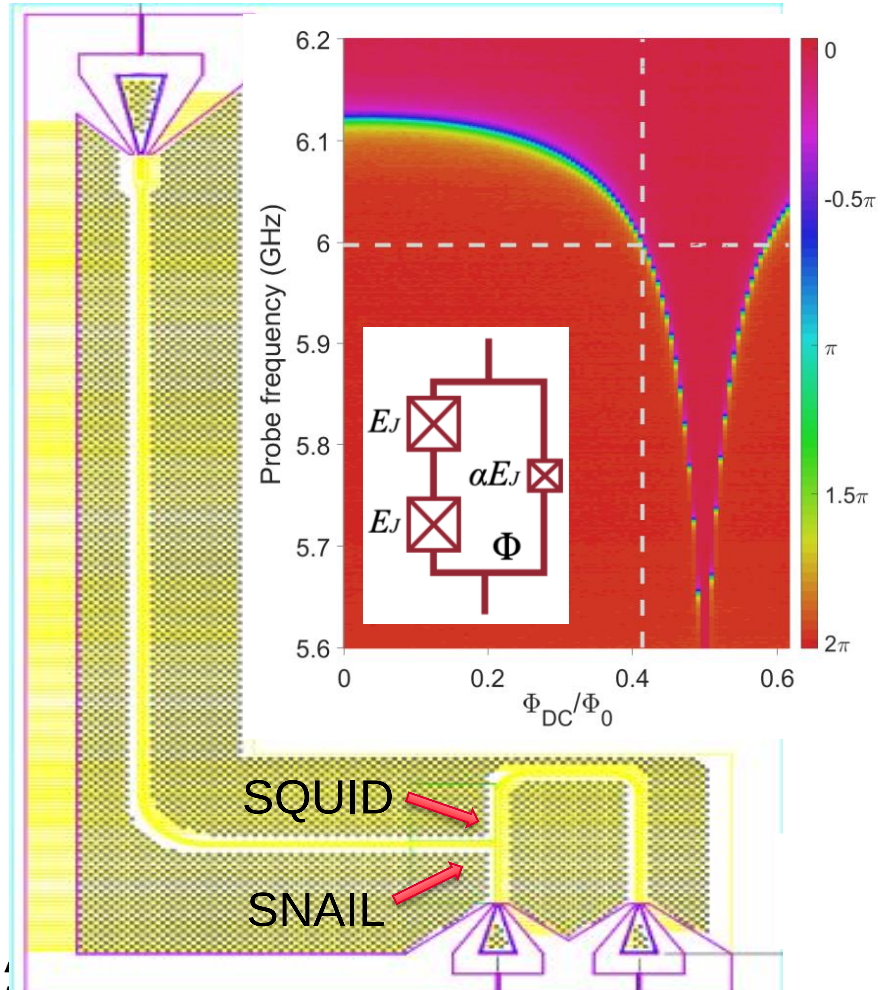
$$\hat{V}_a = \frac{\kappa}{4(c^2 - 2\alpha^2)^2} \cdot \begin{bmatrix} -\alpha^2 + \frac{2\alpha^4}{c^2} + c^2 & 0 & 0 & -2\alpha c & 3\alpha^2 - \frac{2\alpha^4}{c^2} & 0 \\ 0 & -\alpha^2 + \frac{2\alpha^4}{c^2} + c^2 & -2\alpha c & 0 & 0 & 3\alpha^2 - \frac{2\alpha^4}{c^2} \\ 0 & -2\alpha c & 2\alpha^2 + c^2 & 0 & 0 & -2\alpha c \\ -2\alpha c & 0 & 0 & 2\alpha^2 + c^2 & -2\alpha c & 0 \\ 3\alpha^2 - \frac{2\alpha^4}{c^2} & 0 & 0 & -2\alpha c & -\alpha^2 + \frac{2\alpha^4}{c^2} + c^2 & 0 \\ 0 & 3\alpha^2 - \frac{2\alpha^4}{c^2} & -2\alpha c & 0 & 0 & -\alpha^2 + \frac{2\alpha^4}{c^2} + c^2 \end{bmatrix}$$

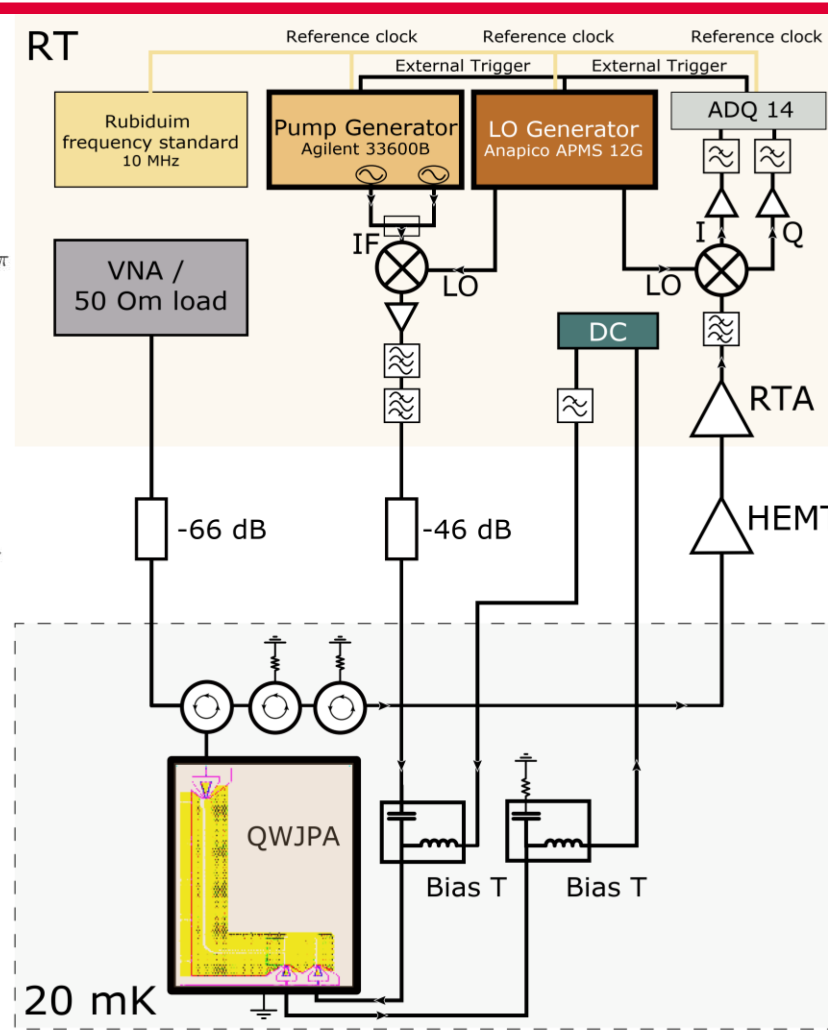
BS - beam splitter correlation

Differ from regular H-graph states: generalized H-graph state

λ/4 parametric device and measurement setting

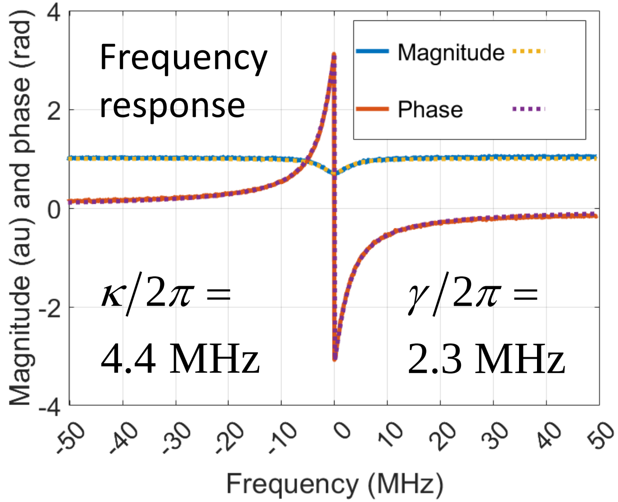
The device:

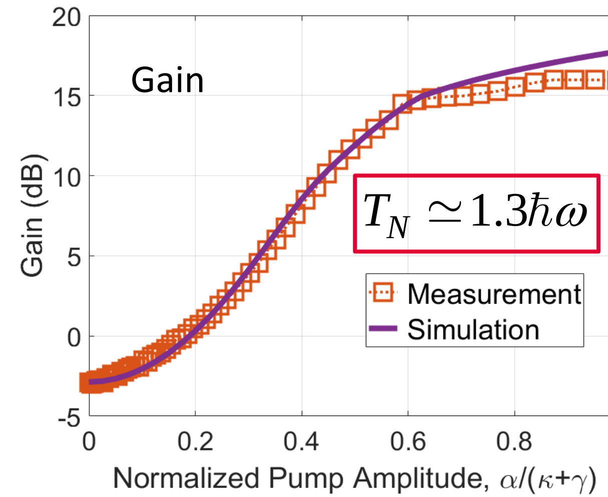


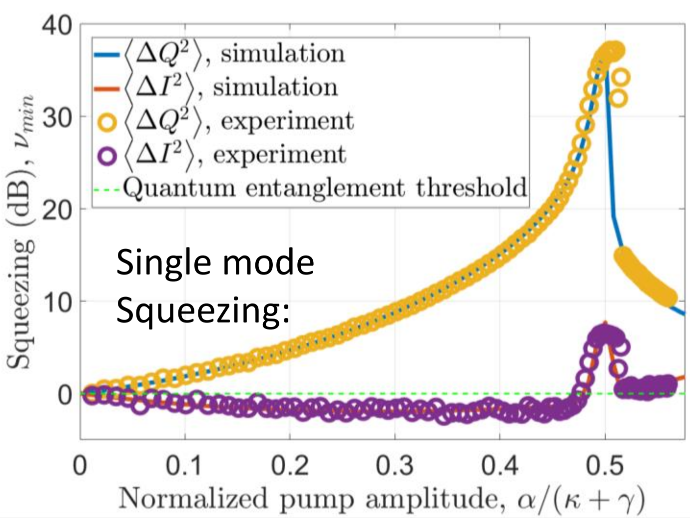


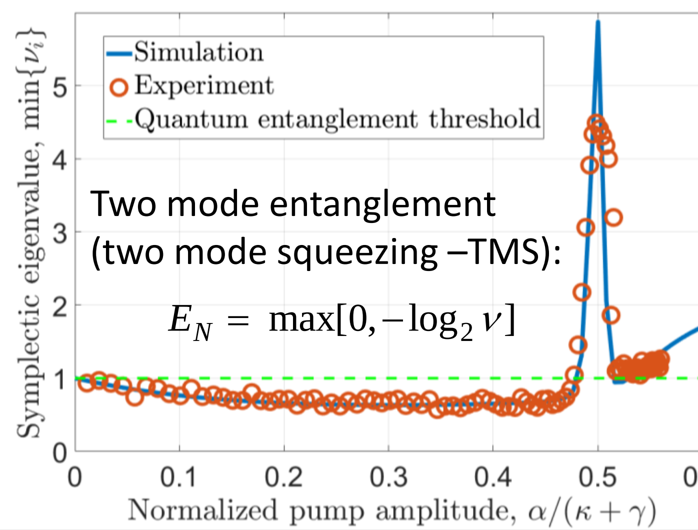
50 MS/s

Basic characteristics of parametric device









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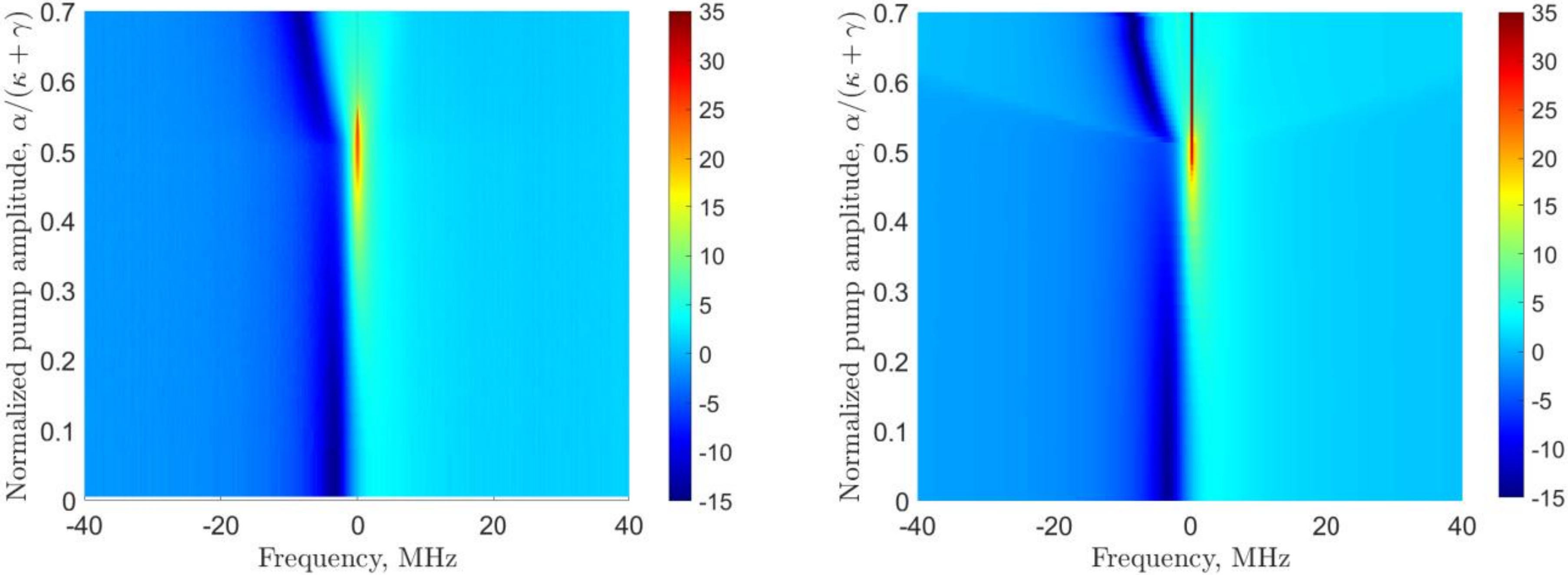
Measured and simulated gain



$$\dot{\tilde{a}}(t) = (-i\Delta_r - \frac{\kappa+\gamma}{2})\tilde{a} - i\sum_{d=1}^{p}\alpha_d e^{i\Delta_d t}\tilde{a}^\dagger + \sqrt{\kappa}\tilde{b}_{in} + \sqrt{\gamma}\tilde{c}_{in} - 12iK\tilde{a}^\dagger\tilde{a}\tilde{a}, \quad K = 0.54\omega_r$$

Kirill Petrovnin, …, PJH, arXiv:2203.09247

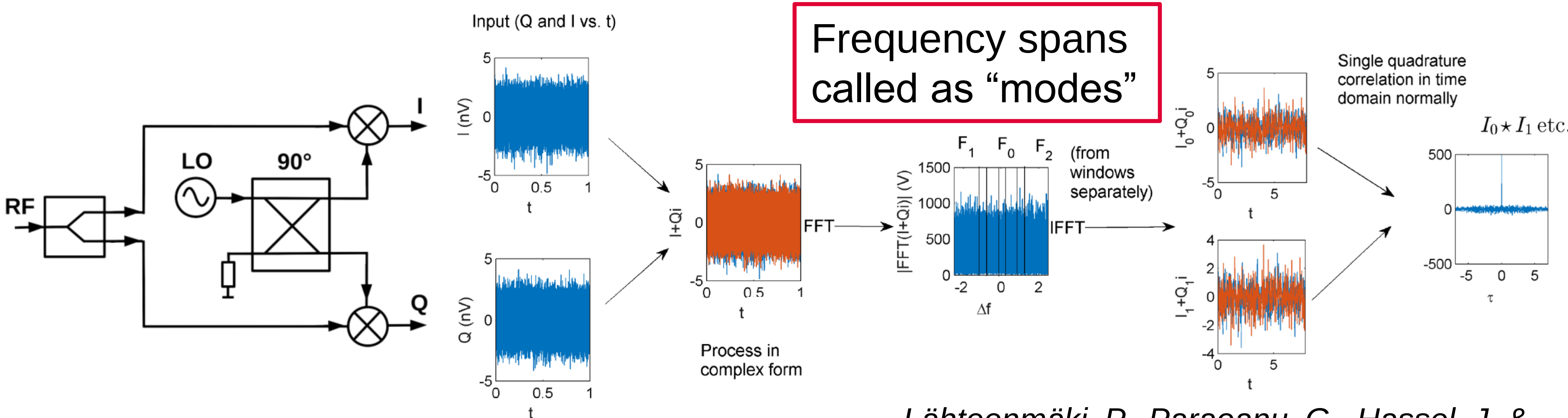
Data analysis

Gaussian states are characterized by the **covariance matrix**

$$\hat{x} = \frac{\tilde{a}_i + \tilde{a}_i^\dagger}{2} \qquad \hat{p} = \frac{\tilde{a}_i - \tilde{a}_i^\dagger}{2i}$$

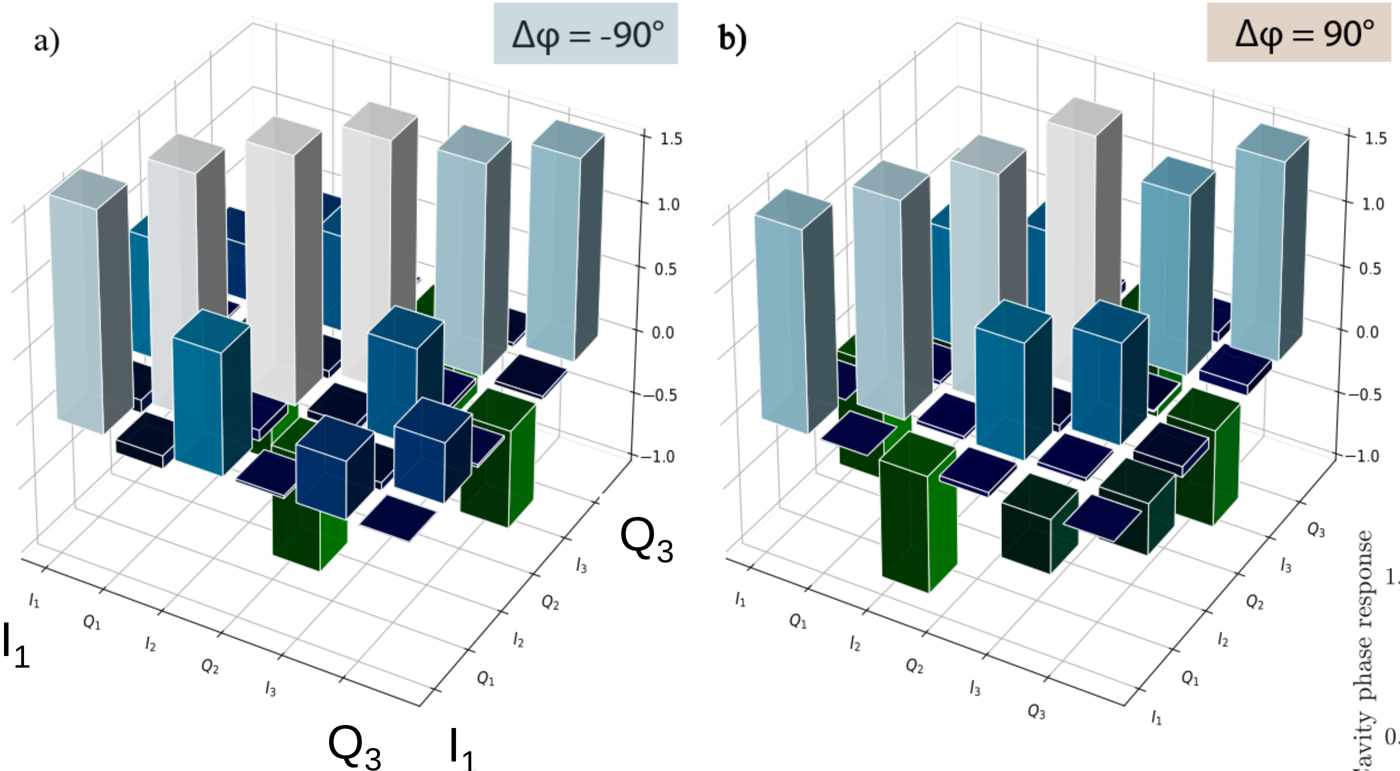
$$\hat{r} = (\hat{x}_1, \hat{p}_1, \ldots \hat{x}_N, \hat{p}_N)^T$$

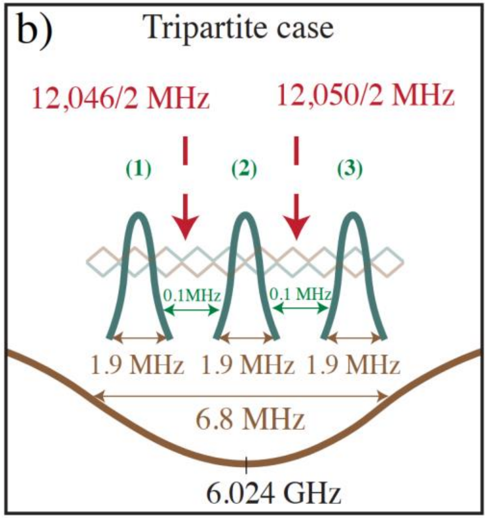
$$V_{i,j} = \frac{1}{2} \langle \Delta\hat{r}_i \Delta\hat{r}_j + \Delta\hat{r}_j \Delta\hat{r}_i \rangle - \langle \Delta\hat{r}_i \rangle \langle \Delta\hat{r}_j \rangle$$



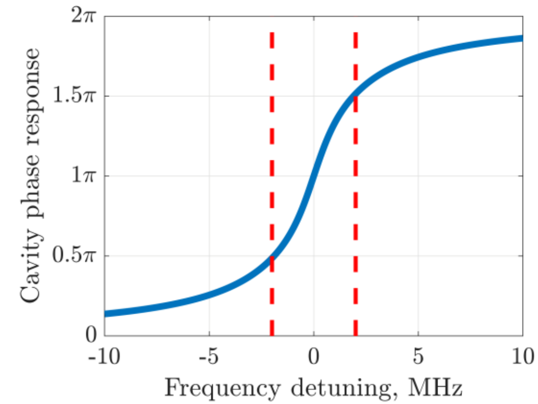
Lähteenmäki, P., Paraoanu, G., Hassel, J. & Hakonen, P. J., Nature Comm. 7 (2016).

Covariance matrix with two pumps



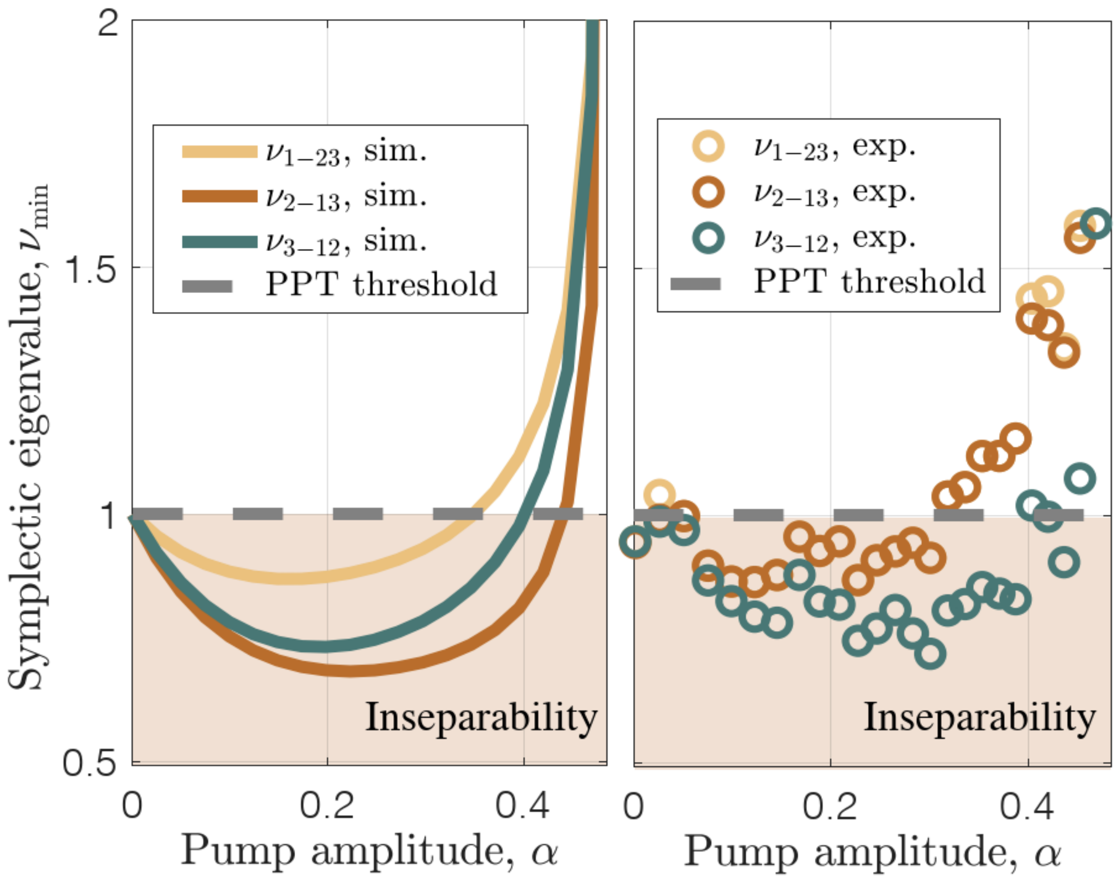


(spectrum) modes



- rotation TMS correlations (1 − 2 or 2 − 3).
- BS correlations follow the signs of TMS

PPT criterion for three partite case



Change sign of momentum of one subsection:

$$\begin{bmatrix} I_0 I_0 & I_0 Q_0 & I_0 I_1 & I_0 Q_1 \\ Q_0 I_0 & Q_0 Q_0 & Q_0 I_1 & Q_0 Q_1 \\ I_1 I_0 & I_1 Q_0 & I_1 I_1 & -I_1 Q_1 \\ Q_1 I_0 & Q_1 Q_0 & -Q_1 I_1 & Q_1 Q_1 \end{bmatrix}$$

Entanglement criterion

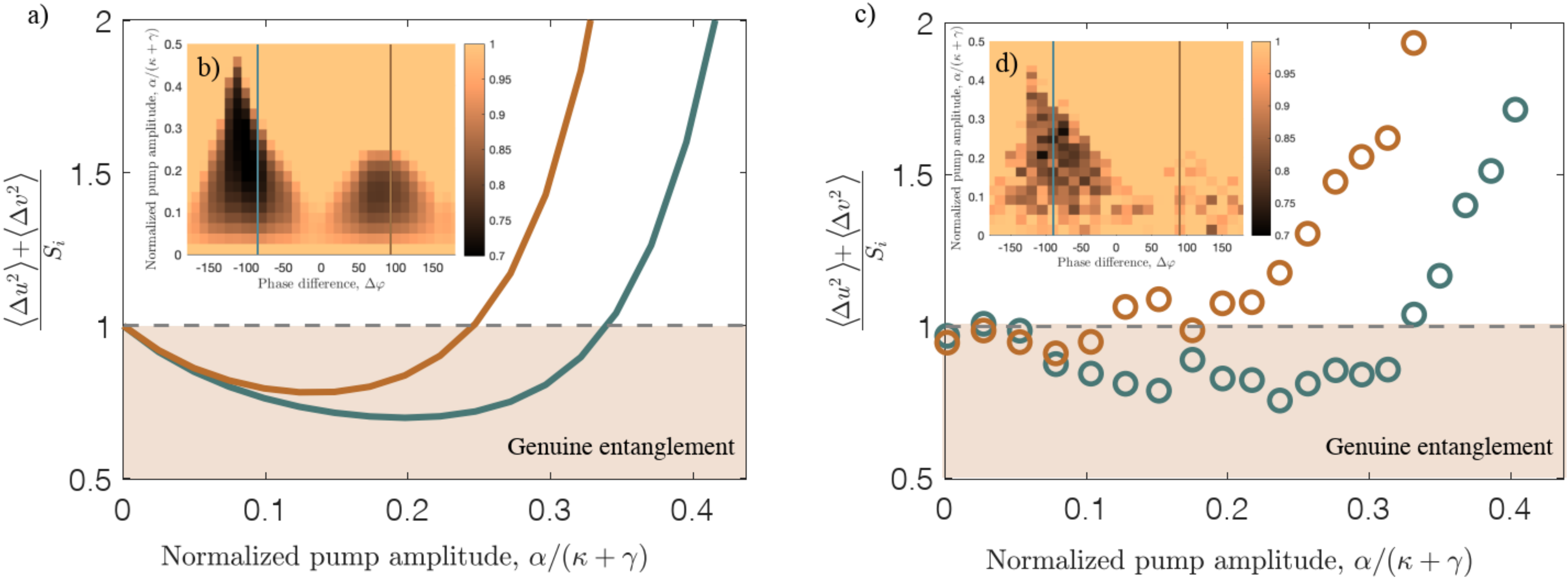
$$\tilde{\nu}_k = \mathrm{P}^{\mathrm{T}} \tilde{\mathrm{V}}_{i,j} \mathrm{P} \qquad \tilde{\nu}_i < 1$$

P is symplectic matrix

A. Peres, Phys. Rev. Lett. 77, 1413 (1996).

M. Horodecki, P. Horodecki, and R. Horodecki, Phys. Lett. A223, 1 (1996).

Genuine three partite entanglement



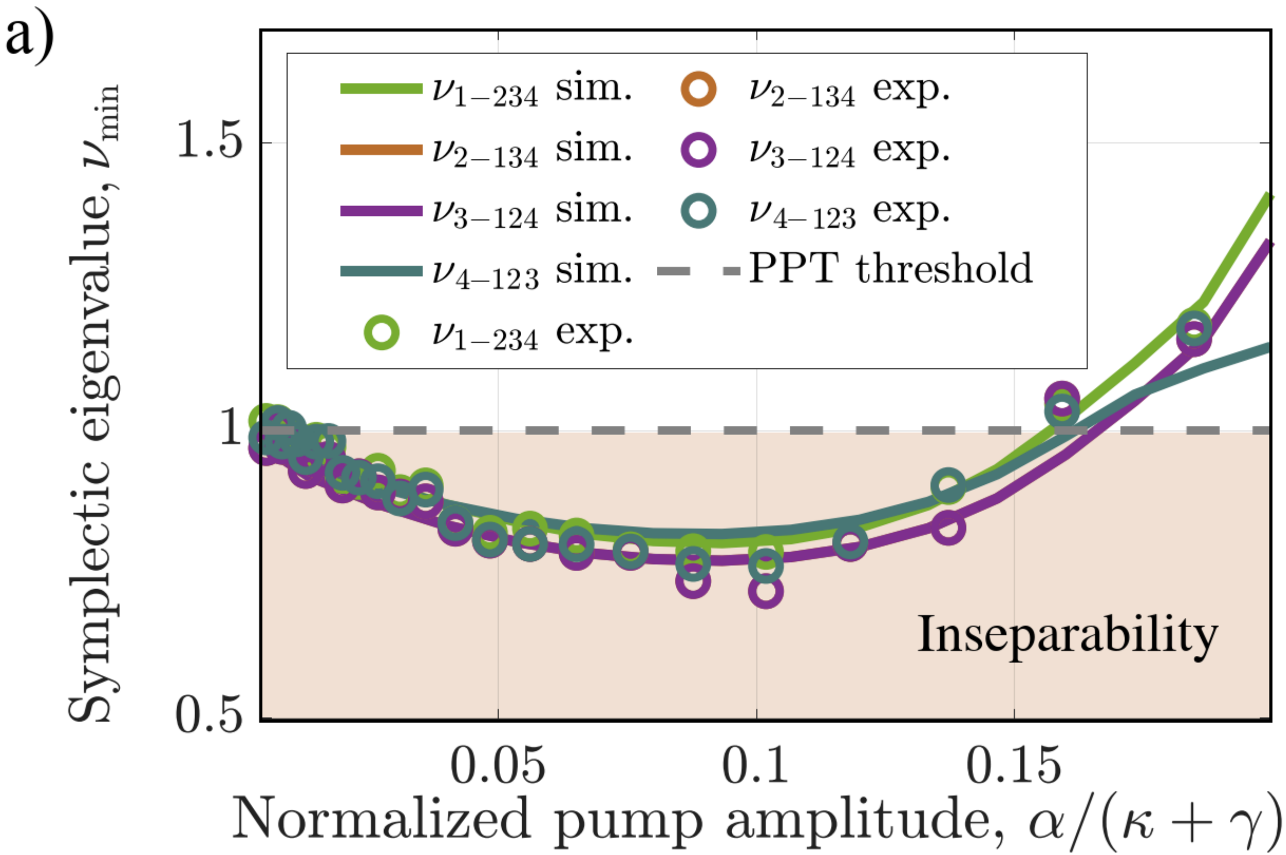
$$\frac{\langle \Delta u^2 \rangle + \langle \Delta v^2 \rangle}{S} \geq 1 \qquad u = \sum_i h_i I_i \qquad v = \sum_k h_k Q_k$$

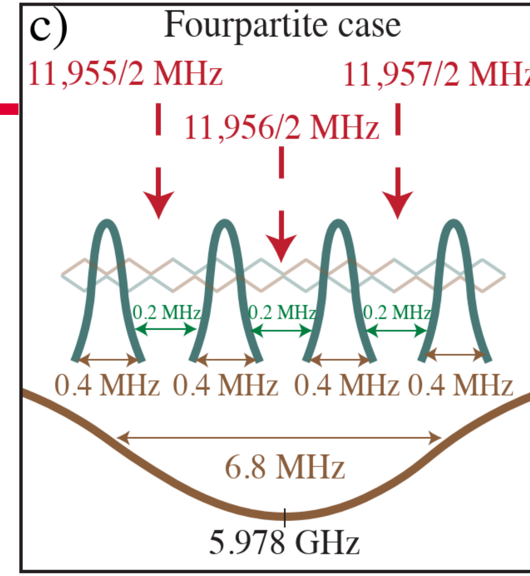
$$S = 2\min\{|h_1 g_1 + h_2 g_2| + |h_3 g_3|, |h_3 g_3 + h_2 g_2| + |h_1 g_1|, |h_1 g_1 + h_3 g_3| + |h_2 g_2|\}$$

Same result if the phase selected on the basis of measured Covar-matrix

P. Van Loock and A. Furusawa, Phys. Rev. A ***67****, 052315 (2003).*
R. Y. Teh and M. D. Reid, Phys. Rev. A ***90****, 062337 (2014).*

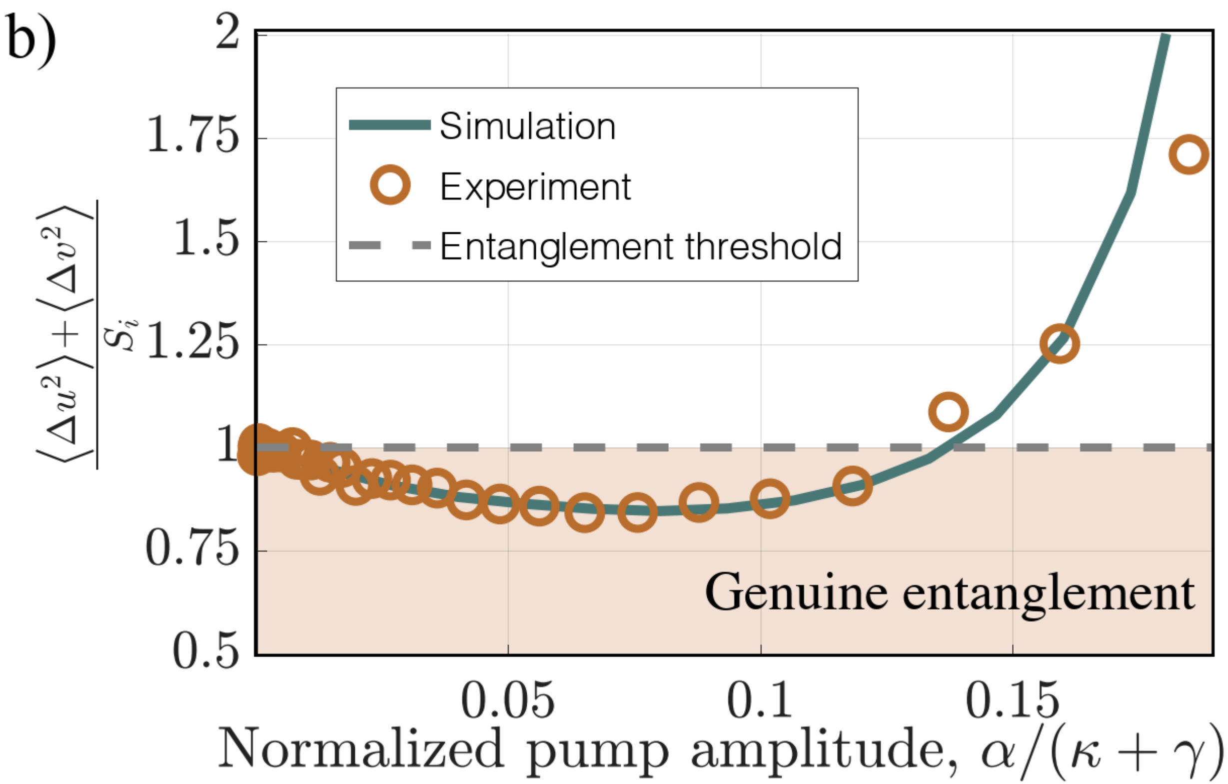
PPT criterion for four partite case



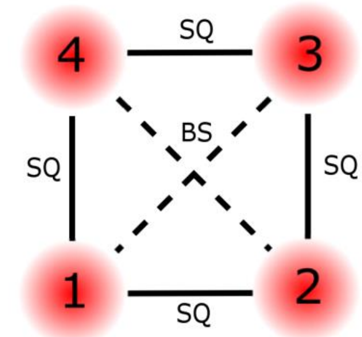


α far below critical pumping

Genuine four partite entanglement

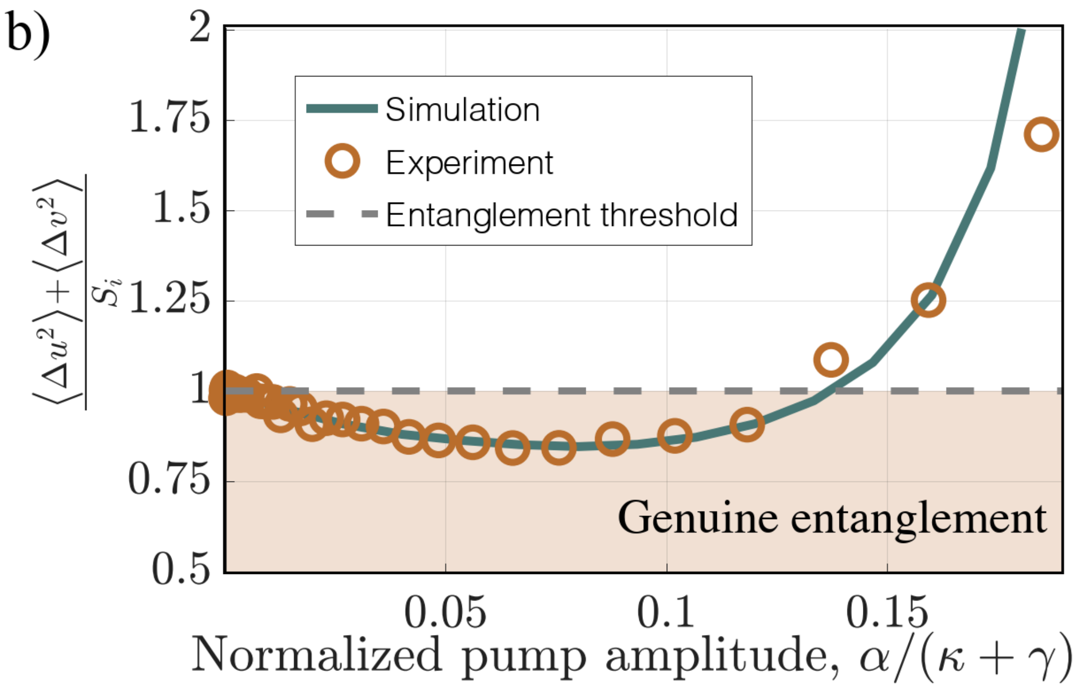


Generalized H-graph state



Expand to TWPA to reach more bandwidth

Genuine four partite entanglement



$$S \equiv \frac{\langle \Delta u^2 \rangle + \langle \Delta v^2 \rangle}{f_4(h_i, g_i)} \geq 1,$$

$$\begin{aligned} f_4(h_i, g_i) = \min\{ & |h_1 g_1 + h_2 g_2 + h_3 g_3| + |h_4 g_4|, \\ & |h_4 g_4 + h_2 g_2 + h_3 g_3| + |h_1 g_1|, \\ & |h_4 g_4 + h_1 g_1 + h_3 g_3| + |h_2 g_2|, \\ & |h_4 g_4 + h_1 g_1 + h_2 g_2| + |h_3 g_3|, \\ & |h_1 g_1 + h_2 g_2| + |h_3 g_3 + h_4 g_4|, \\ & |h_1 g_1 + h_3 g_3| + |h_2 g_2 + h_4 g_4|, \\ & |h_2 g_2 + h_3 g_3| + |h_1 g_1 + h_4 g_4|\} \end{aligned}$$

$$h_1 = g_1 = 1$$

$$h_i = h, \; g_i = g, \; \{i = 2, 3, 4\}$$

Criterion from R. Y. Teh and M. D. Reid, Phys. Rev. A ***90****, 062337 (2014).*

Control with phases

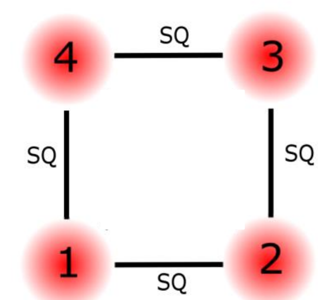
$$\mathbf{V}_\mathrm{a} = \frac{\kappa}{4(c^2-4\alpha^2)^2}\cdot$$

$$\begin{bmatrix}
-2\alpha^2+\frac{8\alpha^4}{c^2}+c^2 & 0 & 2\alpha c & 0 & \boldsymbol{6\alpha^2-\frac{8\alpha^4}{c^2}} & 0 & 2\alpha c & 0 \\
0 & -2\alpha^2+\frac{8\alpha^4}{c^2}+c^2 & 0 & -2\alpha c & 0 & \boldsymbol{6\alpha^2-\frac{8\alpha^4}{c^2}} & 0 & -2\alpha c \\
2\alpha c & 0 & -2\alpha^2+\frac{8\alpha^4}{c^2}+c^2 & 0 & 2\alpha c & 0 & \boldsymbol{6\alpha^2-\frac{8\alpha^4}{c^2}} & 0 \\
0 & -2\alpha c & 0 & -2\alpha^2+\frac{8\alpha^4}{c^2}+c^2 & 0 & -2\alpha c & 0 & \boldsymbol{6\alpha^2-\frac{8\alpha^4}{c^2}} \\
\boldsymbol{6\alpha^2-\frac{8\alpha^4}{c^2}} & 0 & 2\alpha c & 0 & -2\alpha^2+\frac{8\alpha^4}{c^2}+c^2 & 0 & 2\alpha c & 0 \\
0 & \boldsymbol{6\alpha^2-\frac{8\alpha^4}{c^2}} & 0 & -2\alpha c & 0 & -2\alpha^2+\frac{8\alpha^4}{c^2}+c^2 & 0 & -2\alpha c \\
2\alpha c & 0 & \boldsymbol{6\alpha^2-\frac{8\alpha^4}{c^2}} & 0 & 2\alpha c & 0 & -2\alpha^2+\frac{8\alpha^4}{c^2}+c^2 & 0 \\
0 & -2\alpha c & 0 & \boldsymbol{6\alpha^2-\frac{8\alpha^4}{c^2}} & 0 & -2\alpha c & 0 & -2\alpha^2+\frac{8\alpha^4}{c^2}+c^2
\end{bmatrix}.$$

Pump phases of three pumps: $\{\alpha e^{\frac{i\pi}{2}}; \alpha e^{\frac{i\pi}{2}}; \alpha e^{\frac{i\pi}{2}}\} \Rightarrow \{\alpha e^{\frac{-i\pi}{2}}; \alpha e^{\frac{i\pi}{2}}; \alpha e^{\frac{i\pi}{2}}\}$

$$\mathbf{V}_\mathrm{a} = \frac{\kappa}{4(c^2-2\alpha^2)^2}\cdot$$

$$\begin{bmatrix}
c^2+2\alpha^2 & 0 & -2\alpha c & 0 & 0 & 0 & 2\alpha c & 0 \\
0 & c^2+2\alpha^2 & 0 & 2\alpha c & 0 & 0 & 0 & -2\alpha c \\
-2\alpha c & 0 & c^2+2\alpha^2 & 0 & 2\alpha c & 0 & 0 & 0 \\
0 & 2\alpha c & 0 & c^2+2\alpha^2 & 0 & -2\alpha c & 0 & 0 \\
0 & 0 & 2\alpha c & 0 & c^2+2\alpha^2 & 0 & 2\alpha c & 0 \\
0 & 0 & 0 & -2\alpha c & 0 & c^2+2\alpha^2 & 0 & -2\alpha c \\
2\alpha c & 0 & 0 & 0 & 2\alpha c & 0 & c^2+2\alpha^2 & 0 \\
0 & -2\alpha c & 0 & 0 & 0 & -2\alpha c & 0 & c^2+2\alpha^2
\end{bmatrix}.$$



gives cluster state

Entanglement in Travelling Wave Parametric Amplifier

Potential energy:

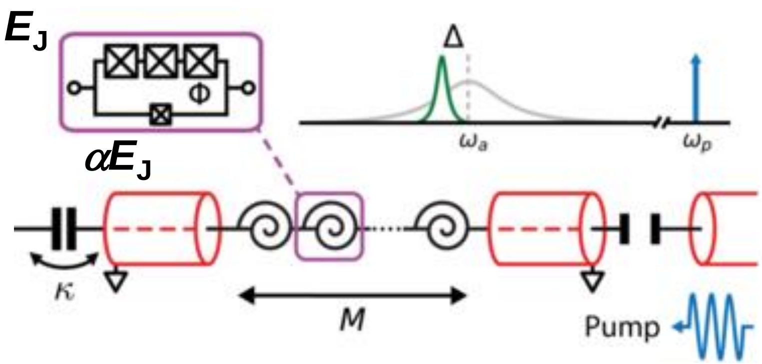
$$U(\varphi) = E_J(c_2\varphi^2 + c_3\varphi^3 + c_4\varphi^4 + \dots)$$

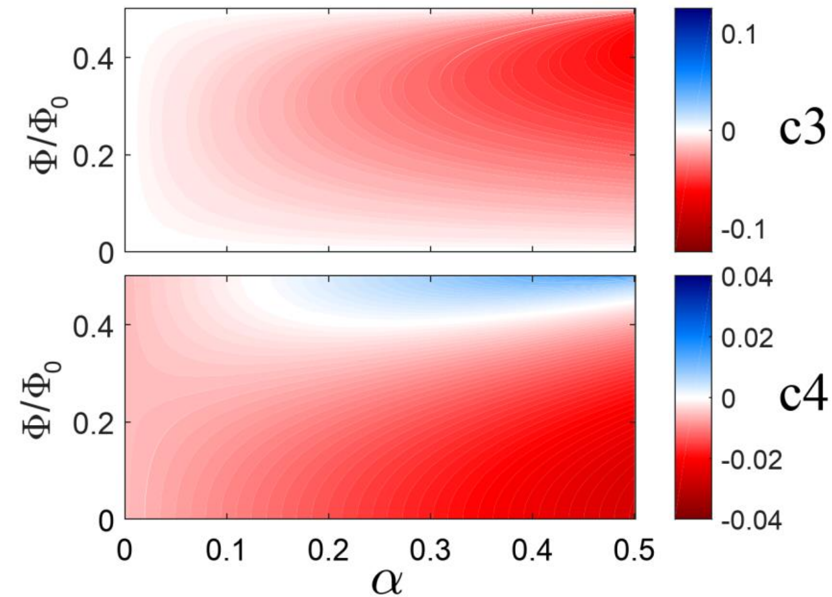
3-wave mixing

4-wave mixing

SNAIL elements technology:

=> 3-wave mixing dominates





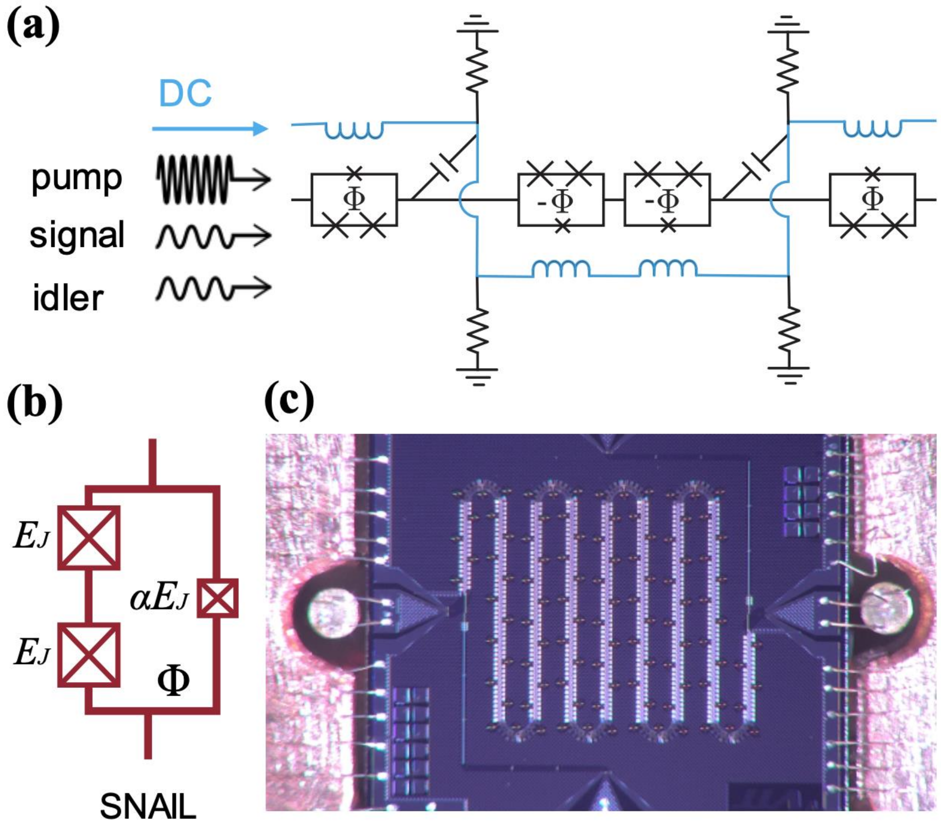
$$\tilde{c}_4 = \frac{p^4}{M^3}\left(c_4 - \frac{3c_3^2}{c_2}(1-p)\right)$$

$$p = \frac{ML_J/L}{c_2 + ML_J/L}$$

*N. E. Frattini, et al. Phys. Rev. Applied **10,** 054020* (2018)

Device

- VTT SNAIL elements technology with spiral resonators for phase matching



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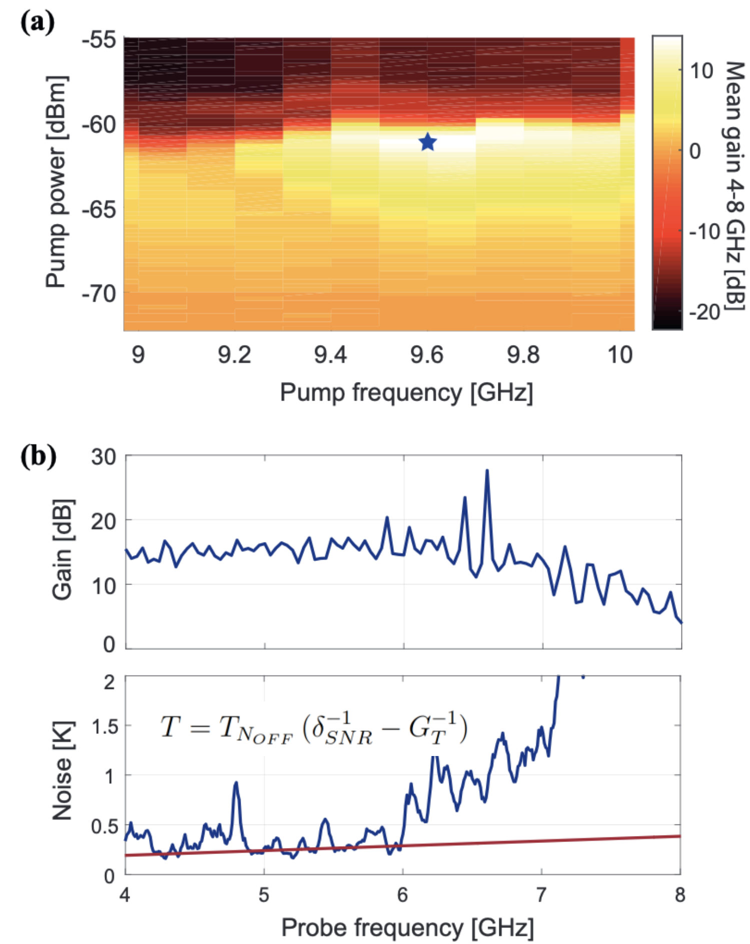
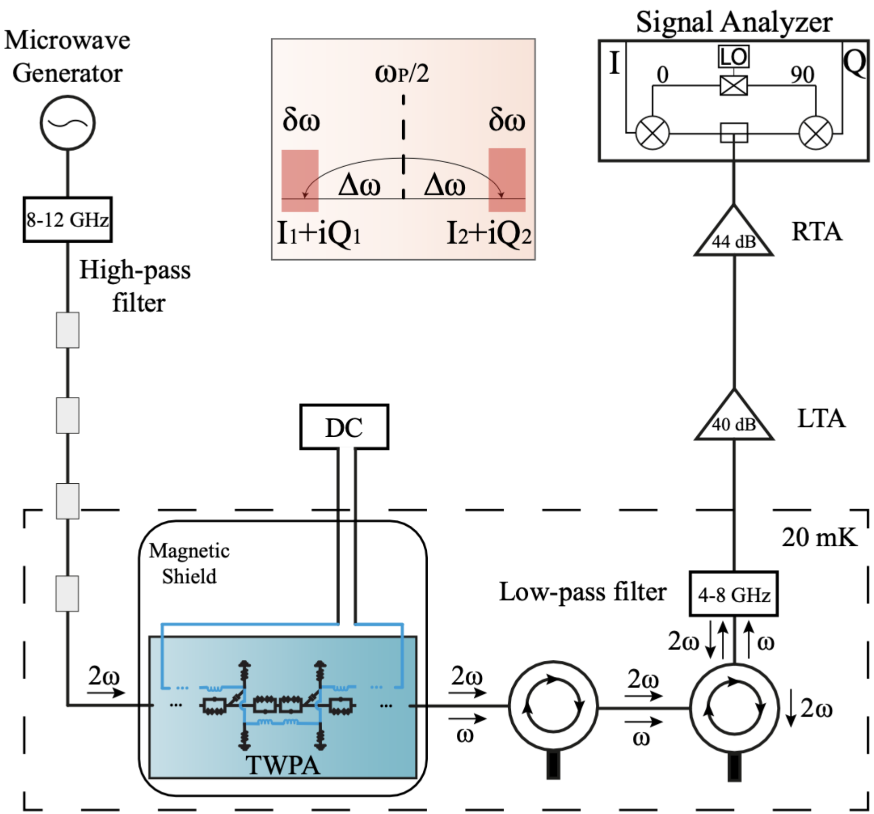
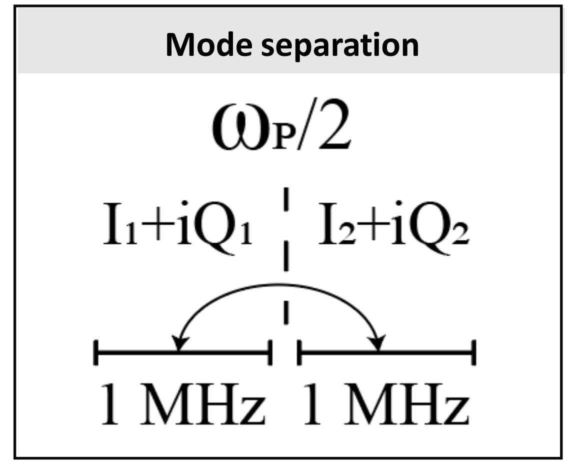


M. Perelshtein, …, PJH, arXiv:2111.06145

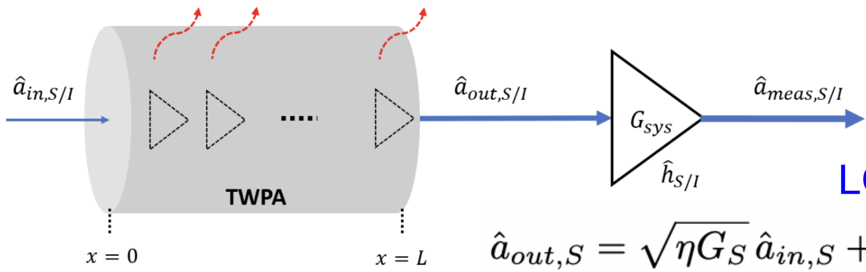
Measurement setup



Spectrum modes are symmetric with respect to the half of the pump frequency



Calibration



LOSS by η NOISE by 1-η

$$\hat{a}_{out,S} = \sqrt{\eta G_S}\,\hat{a}_{in,S} + \sqrt{\eta G_I}\,\hat{a}^{\dagger}_{in,I} + \sqrt{G_S(1-\eta)}\,\hat{\xi}_S + \sqrt{G_I(1-\eta)}\,\hat{\xi}^{\dagger}_I$$

Measured signals

$$\hat{a}^{(ON)}_{meas,S} = \sqrt{G_{sys}}\left(\sqrt{\eta G_S}\,\hat{v}_S + \sqrt{\eta G_I}\,\hat{v}^{\dagger}_I + \sqrt{G_S(1-\eta)}\,\hat{\xi}_S + \sqrt{G_I(1-\eta)}\,\hat{\xi}^{\dagger}_I\right) + \sqrt{G_{sys}-1}\hat{h}^{\dagger}_S,$$

$$\hat{a}^{(ON)}_{meas,I} = \sqrt{G_{sys}}\left(\sqrt{\eta G_I}\,\hat{v}_I + \sqrt{\eta G_S}\,\hat{v}^{\dagger}_S + \sqrt{G_I(1-\eta)}\,\hat{\xi}_I + \sqrt{G_S(1-\eta)}\,\hat{\xi}^{\dagger}_S\right) + \sqrt{G_{sys}-1}\hat{h}^{\dagger}_I.$$

Signals with pump off

$$\hat{a}^{(OFF)}_{meas,S} = \sqrt{G_{sys}\,\eta}\,\hat{v}_S + \sqrt{G_{sys}\,(1-\eta)}\,\hat{\zeta}_S + \sqrt{G_{sys}-1}\hat{h}^{\dagger}_S,$$

$$\hat{a}^{(OFF)}_{meas,I} = \sqrt{G_{sys}\,\eta}\,\hat{v}_I + \sqrt{G_{sys}\,(1-\eta)}\,\hat{\zeta}_I + \sqrt{G_{sys}-1}\hat{h}^{\dagger}_I.$$

$$\mathbf{V}^{(ON)}_{meas} - \mathbf{V}^{(OFF)}_{meas} = G_{sys}\mathbf{V}_{out} - \frac{G_{sys}\eta}{4}\mathbb{I}_4 - G_{sys}(1-\eta)\mathbf{V}_{noise}$$

$$\eta = G_T(P_p = 0),\; G_{OFF} = \eta G_{sys}$$

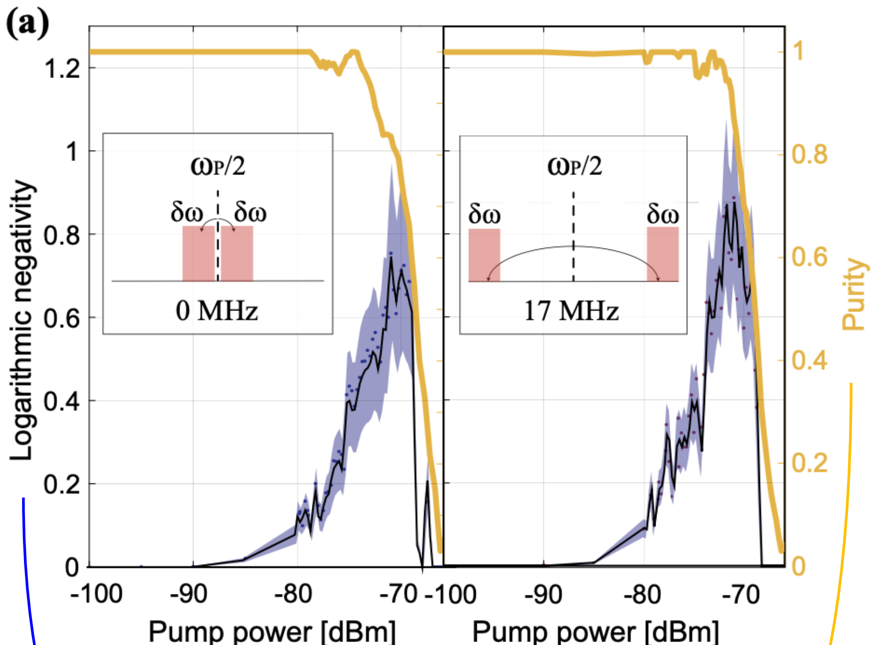
$$\mathbf{V}_{out} = \frac{\eta\left(\mathbf{V}^{(ON)}_{meas} - \mathbf{V}^{(OFF)}_{meas}\right)}{G_{OFF}} + \frac{1}{4}\mathbb{I}_4.$$

η = 0.65 dB

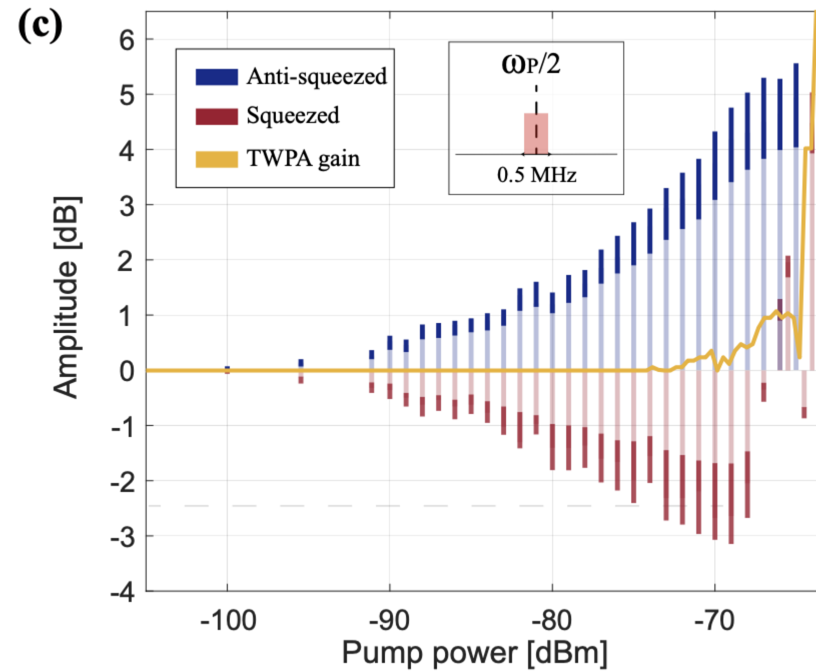
tanδ = 0.003

Results

Two-mode entanglement



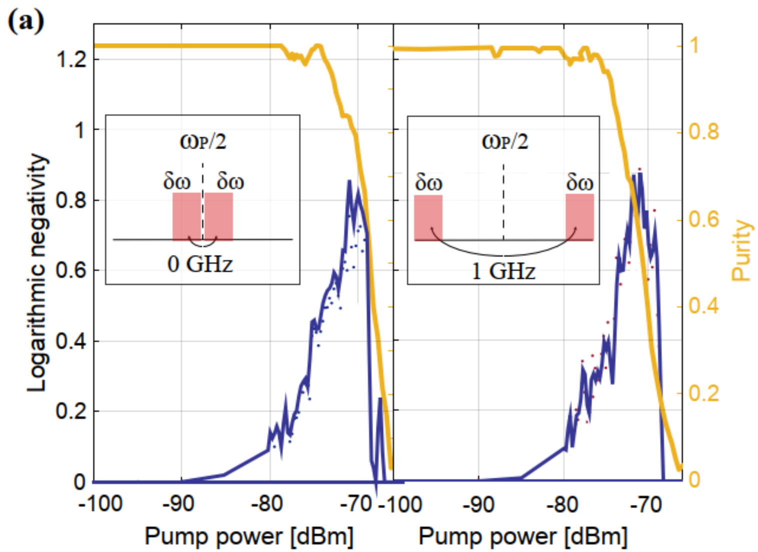
Single mode squeezing

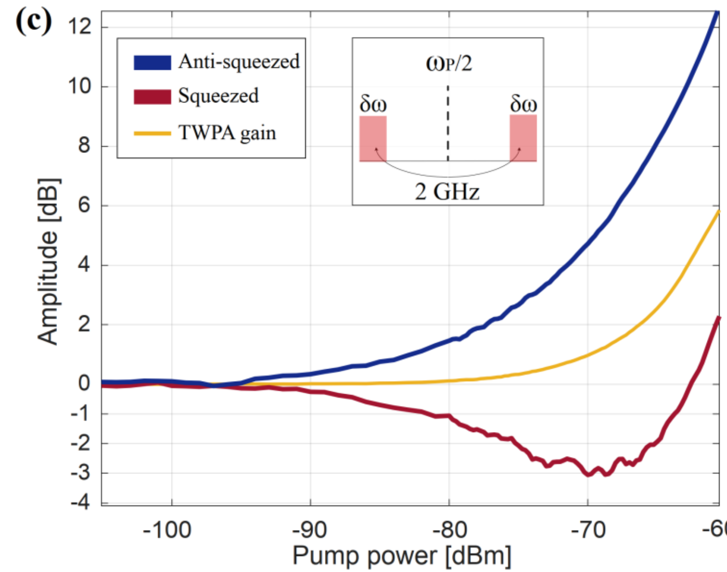


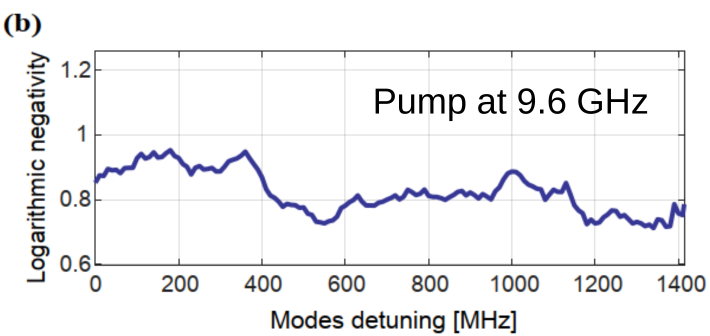
$$E_N = \max[0, -\log_2 \nu]$$

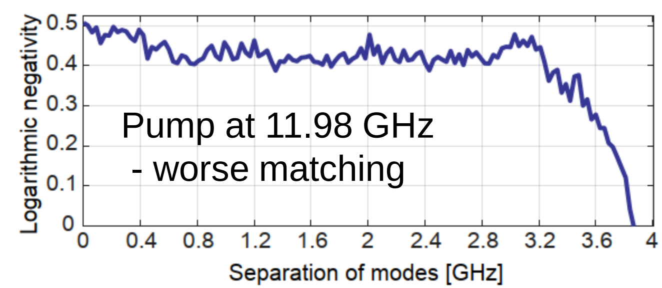
$$\mu = \frac{1}{\sqrt{|\det(V_{\text{out}})|}}$$

Entanglement in Travelling Wave Parametric Amplifier





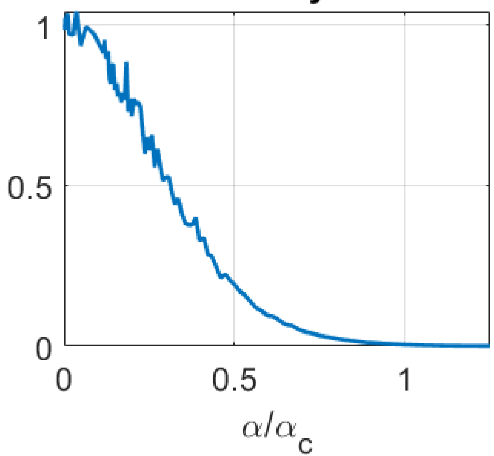


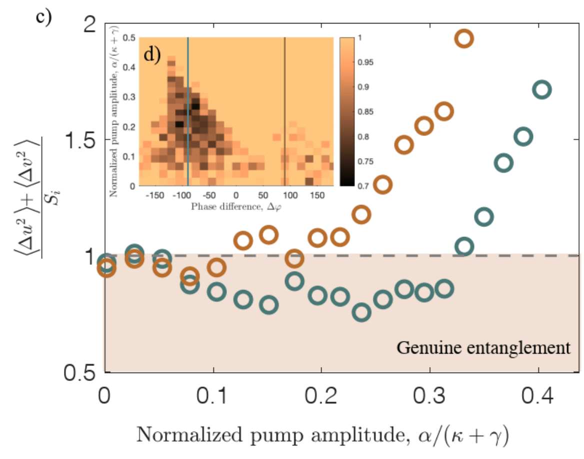


M. Perelshtein et al., arXiv:2111.06145

Multipartite entanglement in TWPA (N = 3)

Purity

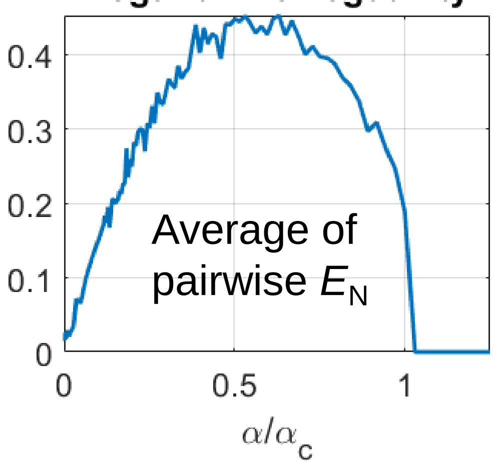




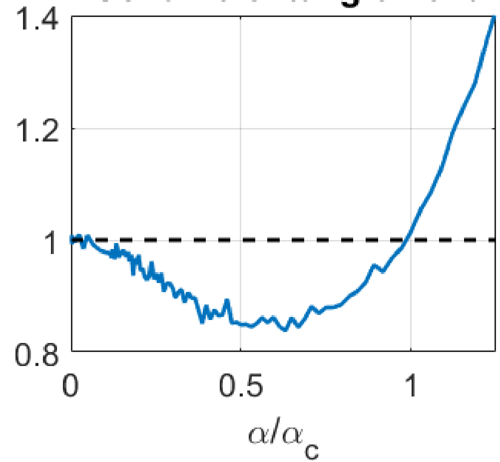
Critical pumping power small compared with the λ/4 JPA case:

GAIN 1 dB vs. 16 dB

Logarithmic negativity



Genuine entanglement



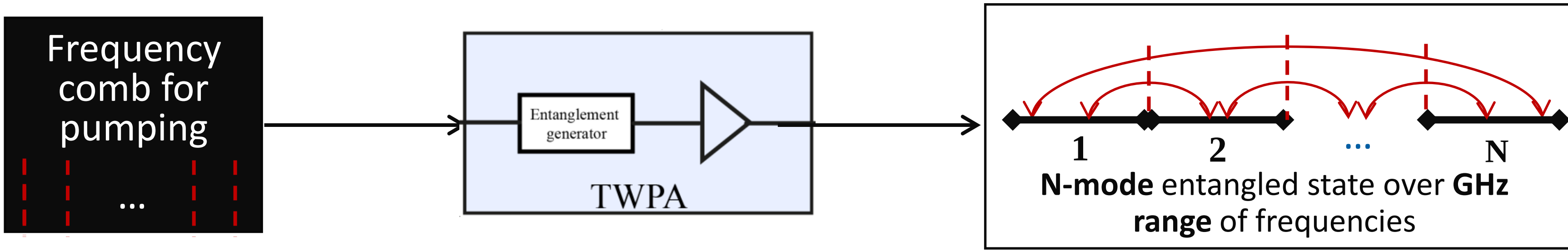
$$\frac{\langle\Delta u^2\rangle+\langle\Delta v^2\rangle}{S}\geq 1 \qquad u=\sum_i h_i I_i \qquad v=\sum_k h_k Q_k$$

$$S = 2\min\{|h_1g_1+h_2g_2|+|h_3g_3|,\ |h_3g_3+h_2g_2|+|h_1g_1|,\ |h_1g_1+h_3g_3|+|h_2g_2|\}$$

I. Lilja, et al., unpublished

Future plans

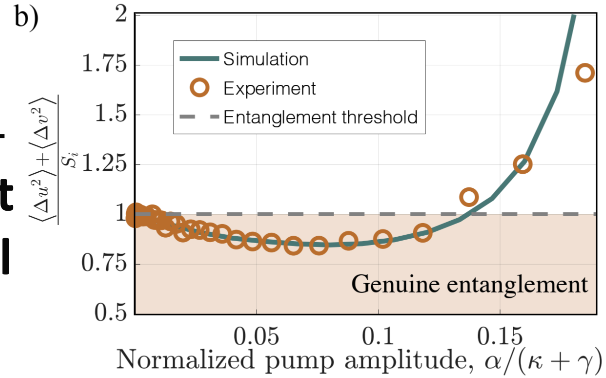
- **To understand TWPA** operation better
- **Large multimode** entangled state generation

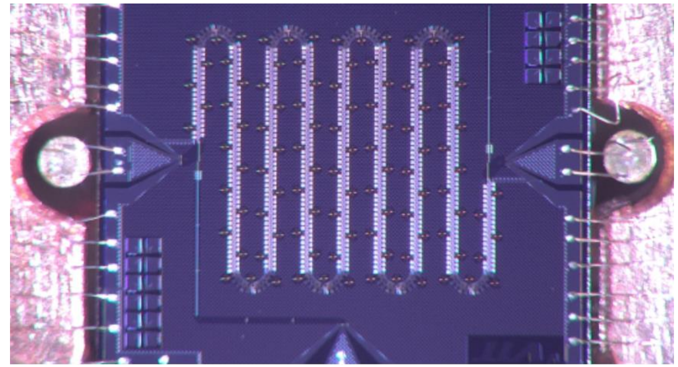


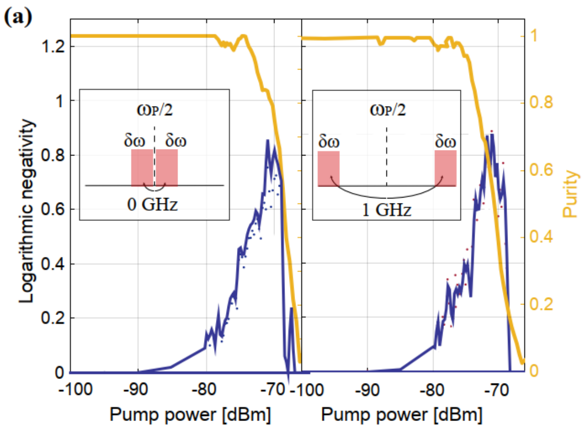
- **Quantum illumination/sensing, data transmission protocols**
- **CV quantum network**
- **Cluster state CV computing –** processing and measurement

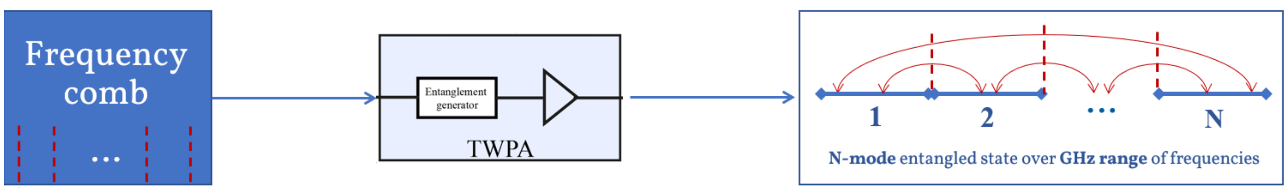
Summary

- Demonstration of genuine **four partite entanglement** at microwaves – and three partite with **phase control**
- Novel method of broadband **entanglement generation (up to 1 Gebit/s)**
- **Squeezing** in TWPA -2.4 dB (for the first time)
- Scalable N-mode states generation for **quantum sensing** and **CV quantum computing**









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