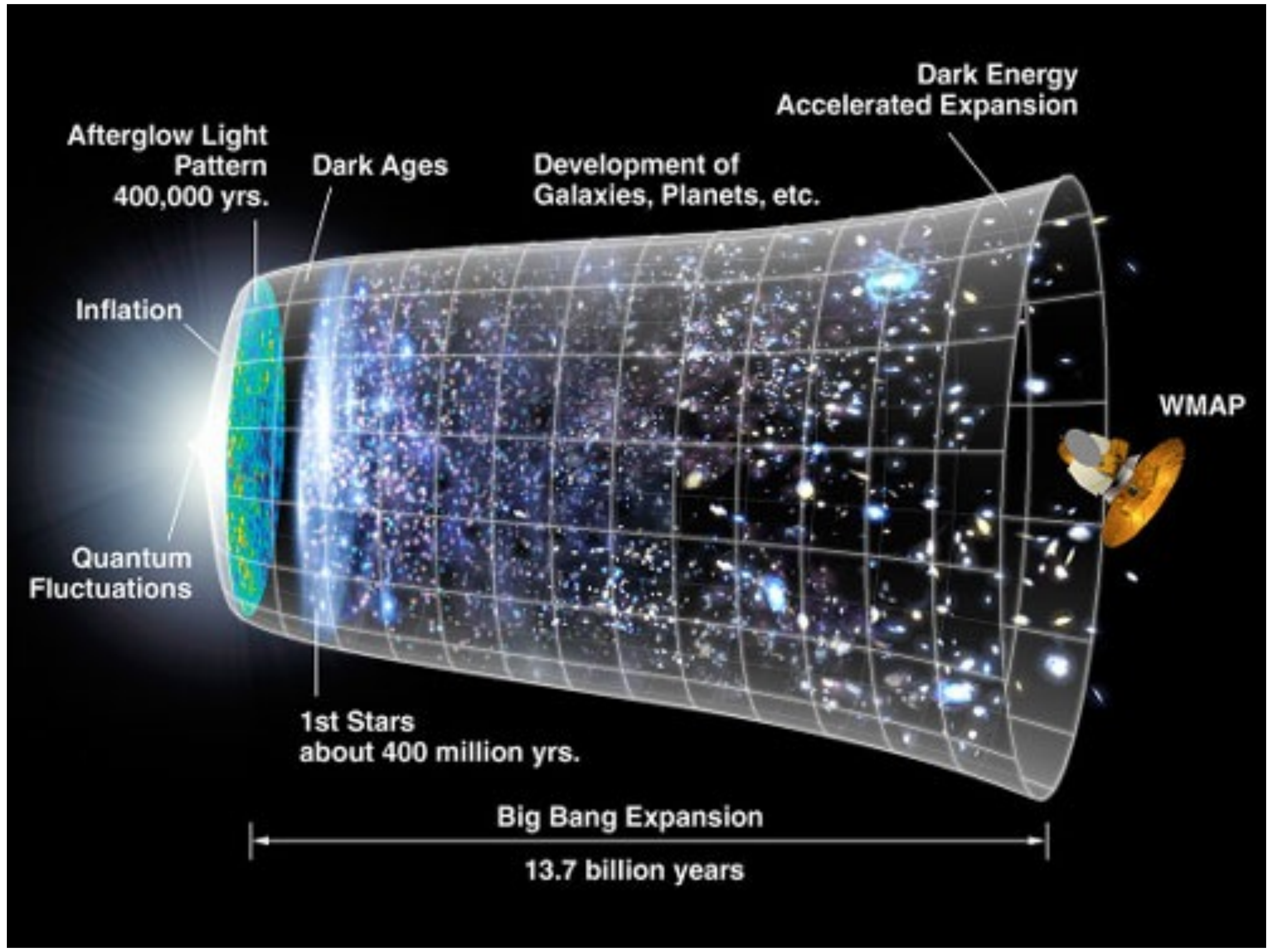
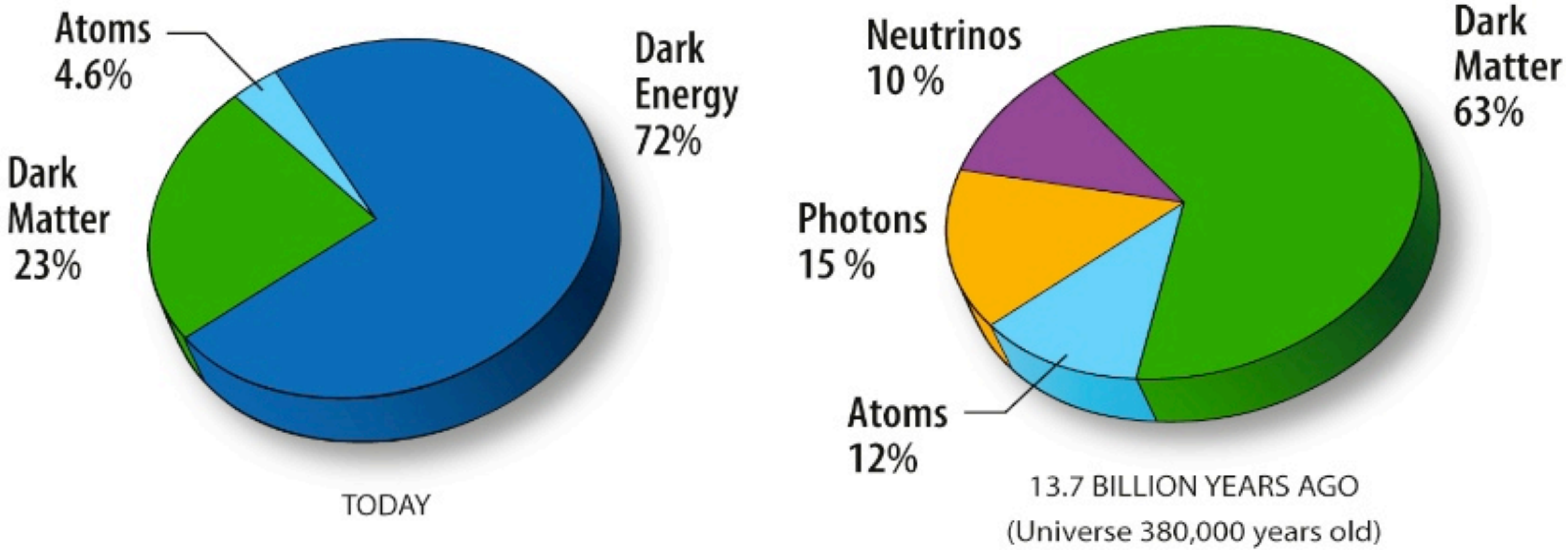
Large-Scale Structure

1. Expanding Universe and Standard Model of Cosmology





Aseem Paranjape

ICTP Summer School on Cosmology, July 2022

Literature

- **J. V. Narlikar,** *Introduction to Cosmology*
- J. A. Peacock, *Cosmological Physics*
- S. Weinberg, *Cosmology*
- P. J. E. Peebles, *Principles of Physical Cosmology*
- P. J. E. Peebles, *Large Scale Structure of the Universe*
- T. Padmanabhan, *Structure Formation in the Universe*
- E. Kolb & M. Turner, *Early Universe*
- **S. Dodelson,** *Modern Cosmology*
- A. Liddle & D. Lyth, *Cosmological Inflation & Large Scale Structure*
- R. Durrer, *Cosmic Microwave Background*
- J. Lesgourgues, *Neutrino Cosmology*
- **H. Mo, F. van den Bosch & S. White,** *Galaxy Formation & Evolution*

Distances in the Universe

A.U. → *Light year* / *Parsec (pc)* → *kiloparsec (kpc)* → *Megaparsec (Mpc)*

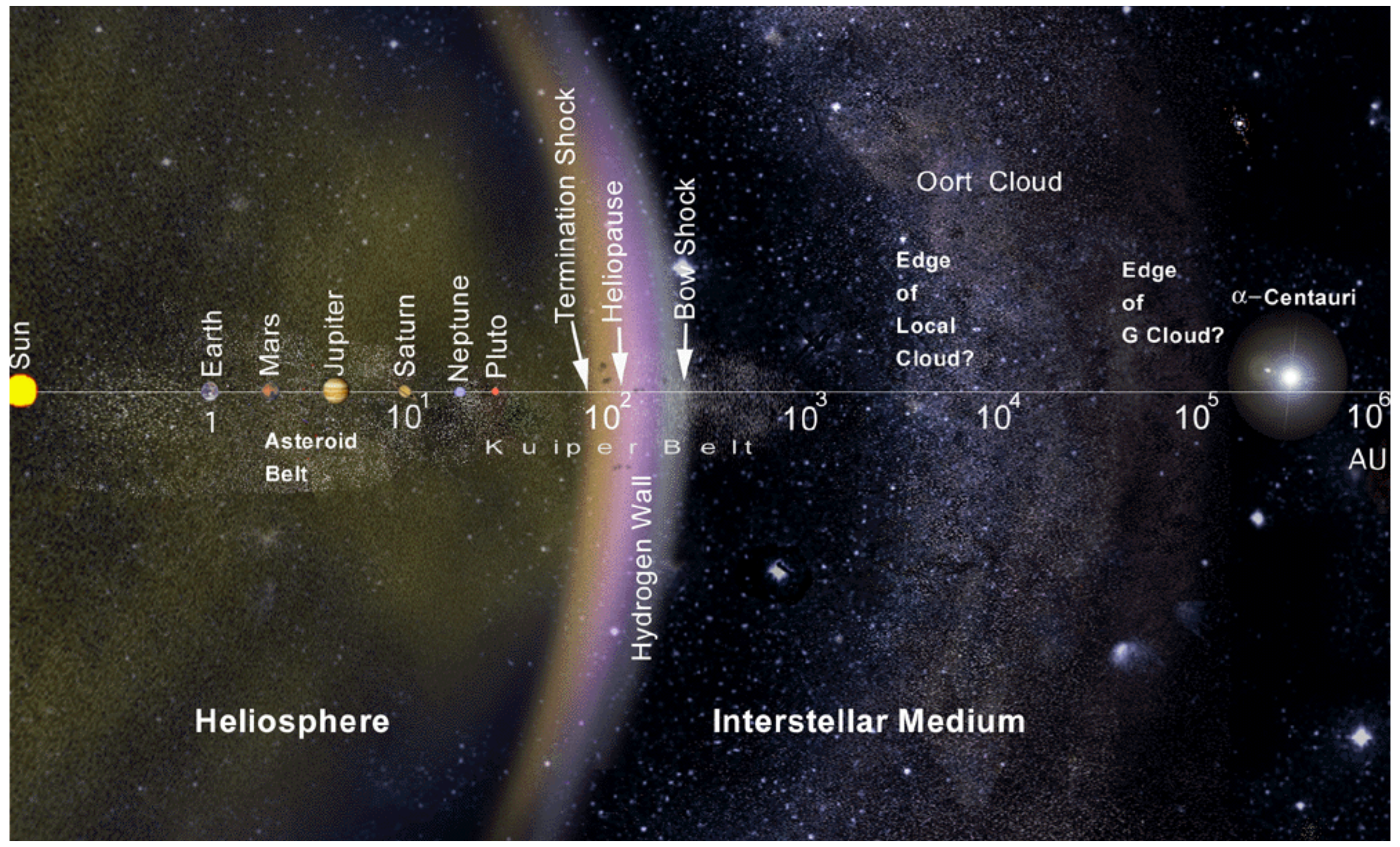
1 AU = 1.5×10^8 km

1 ly $\approx 10^{13}$ km

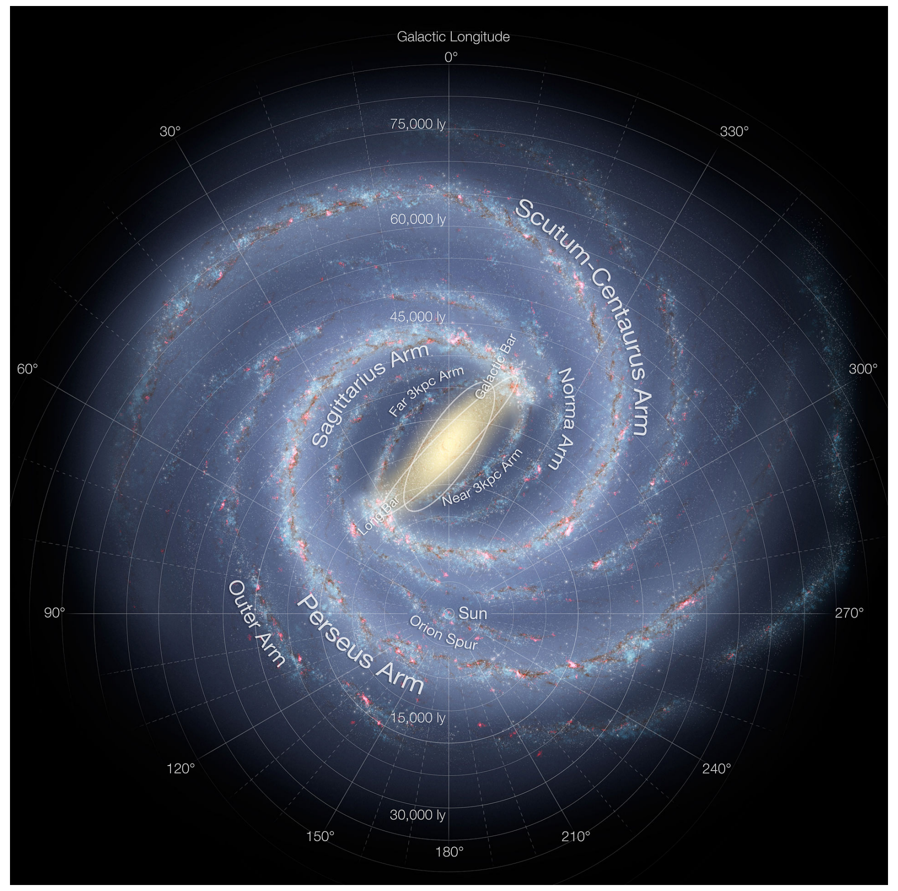
1 pc $\approx$ 3.25 ly

1 kpc = 10^3 pc

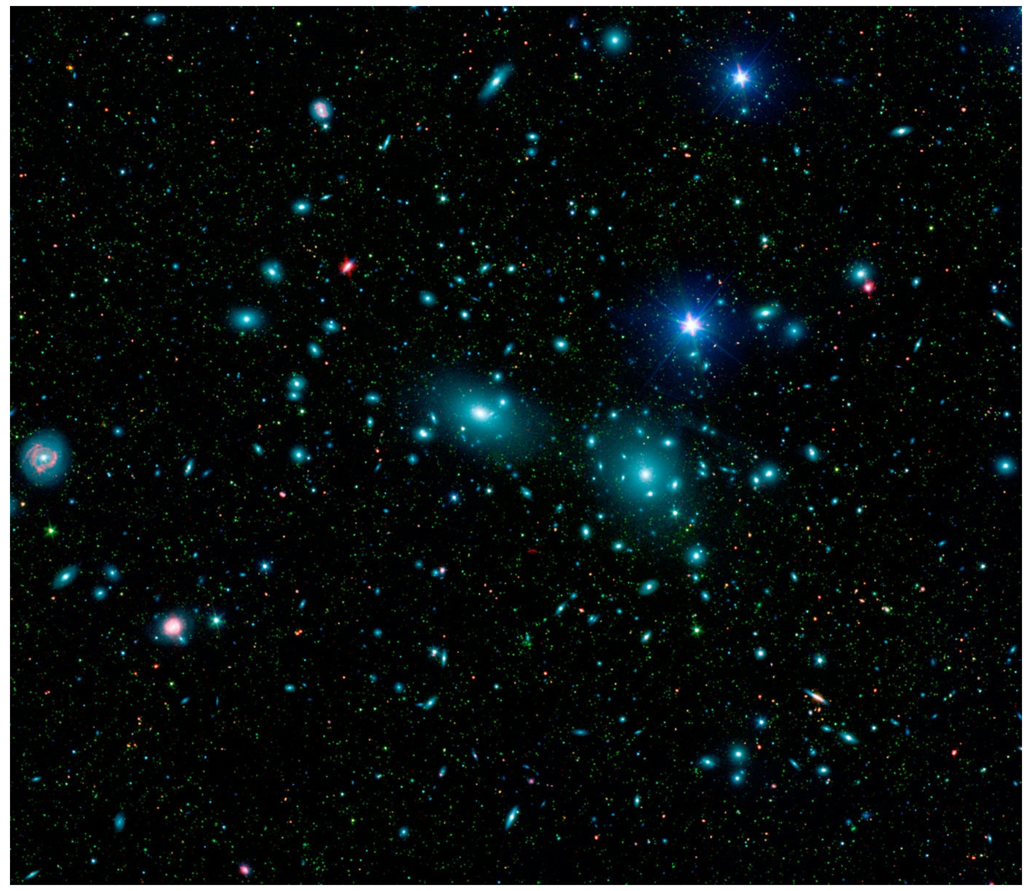
1 Mpc = 10^3 kpc



5 pc



50 kpc

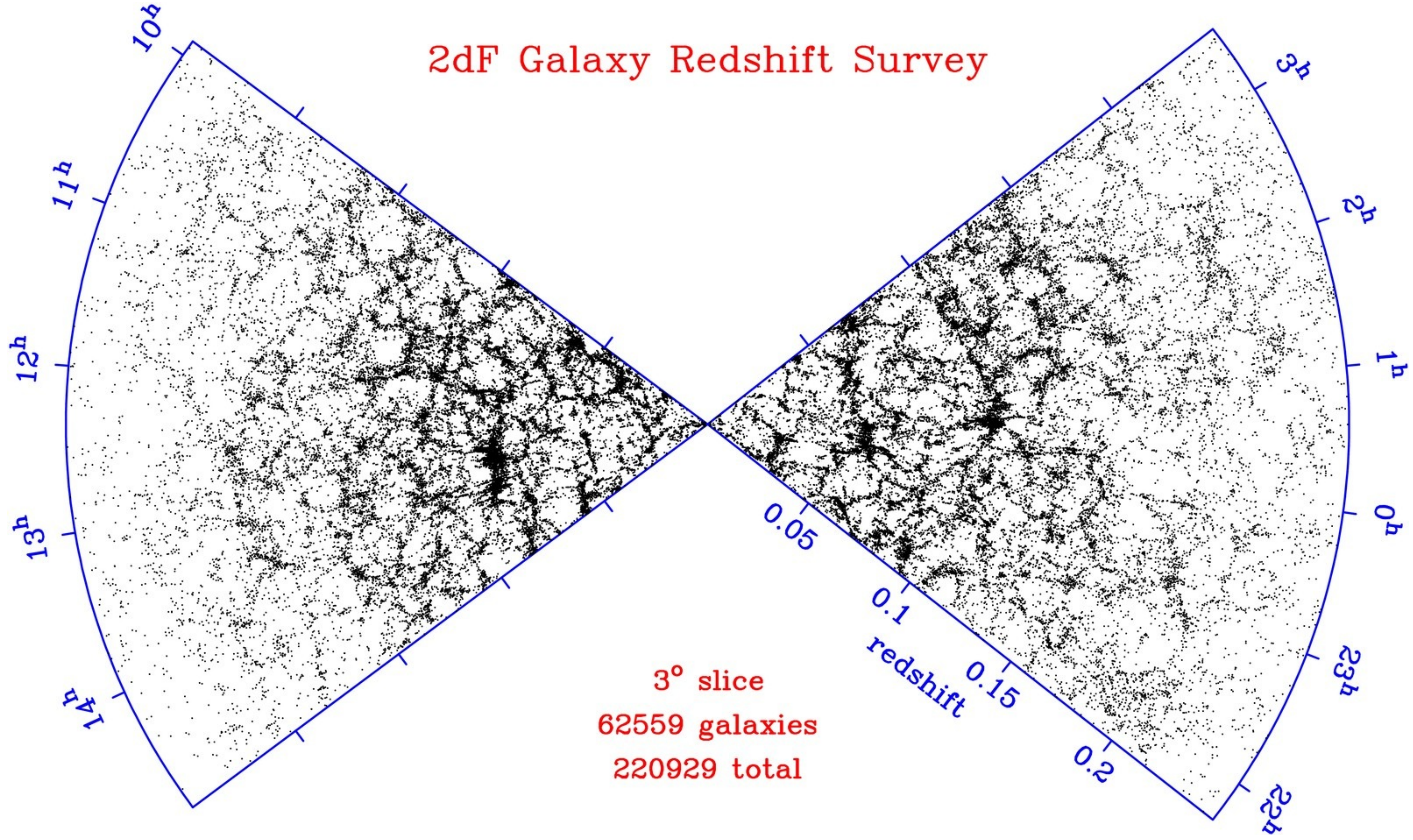


10 Mpc

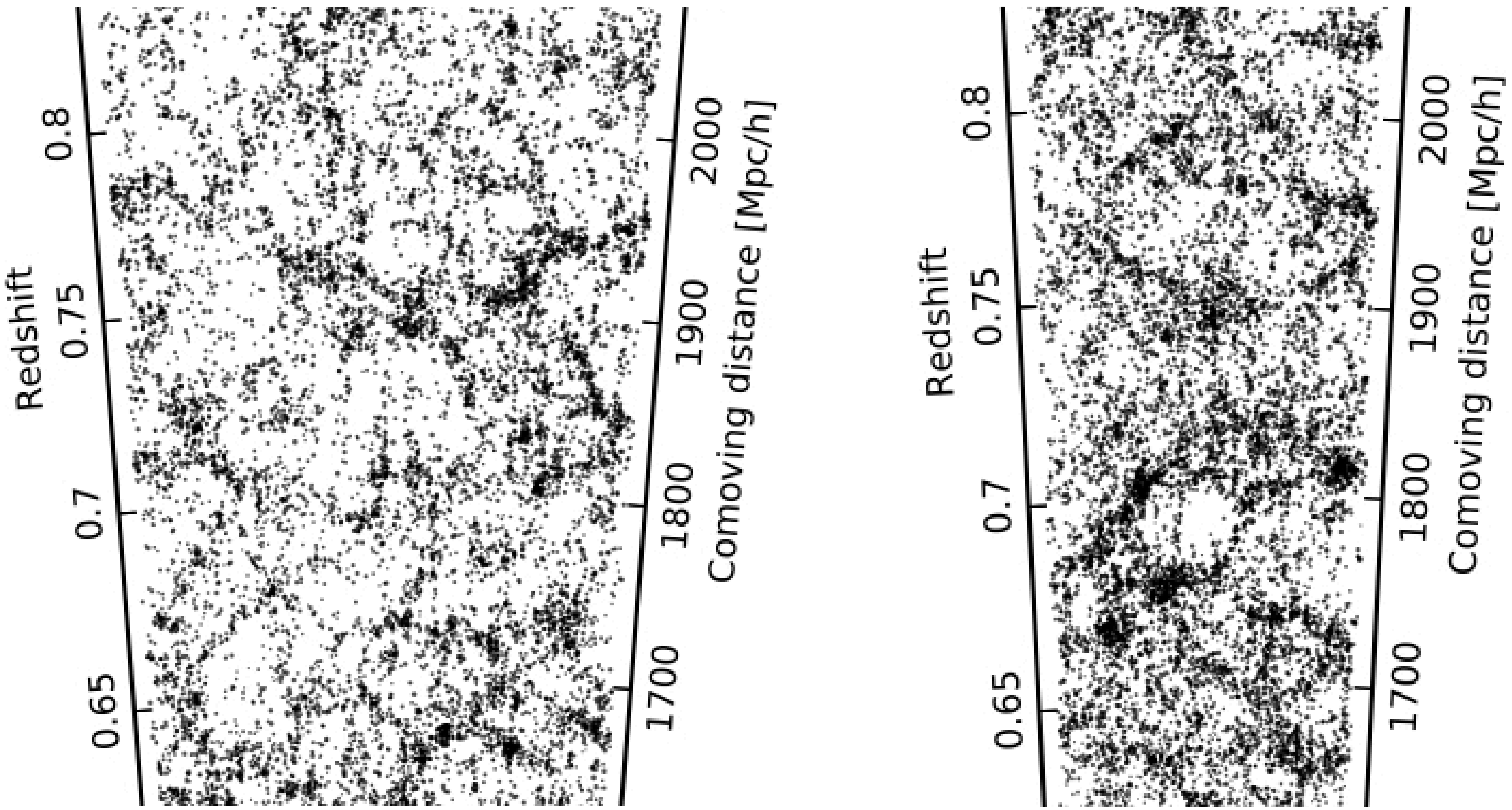
(Images: Wikimedia Commons / NASA)

The low-redshift (``nearby") Universe

(Image: Colless+01)



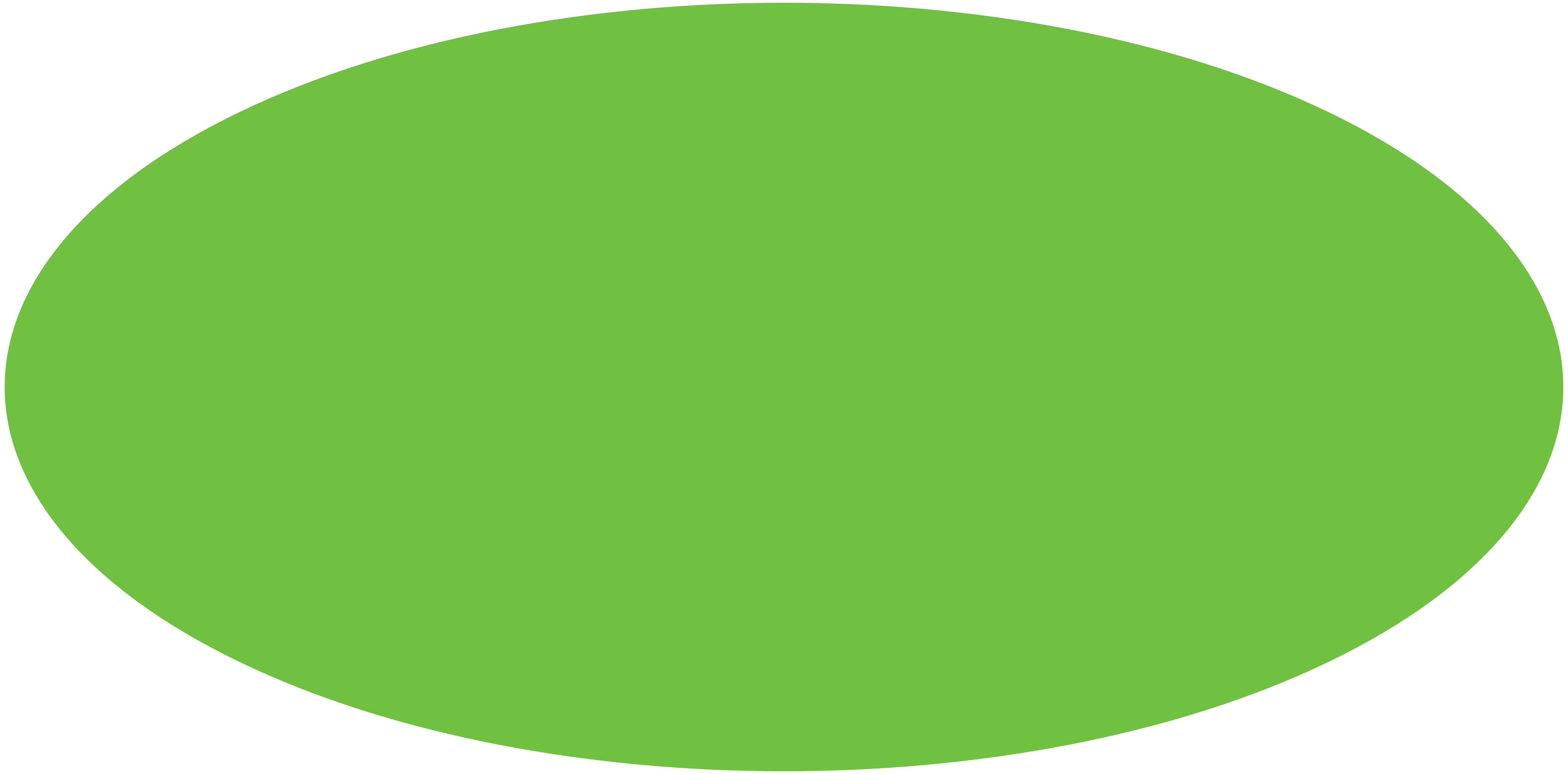
The Cosmic Web



(Image: Guzzo+13)

Cosmic Microwave Background

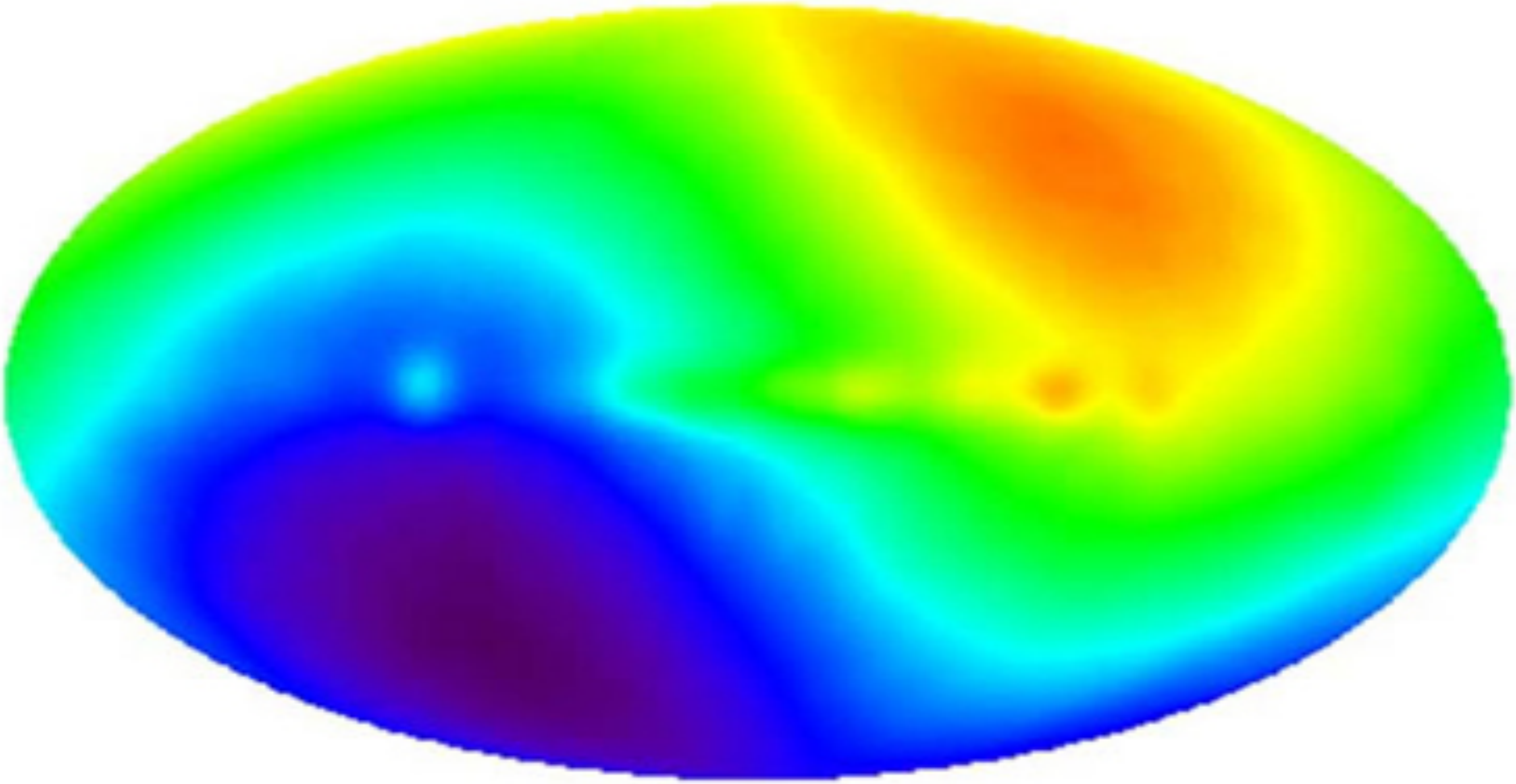
Uniform-temperature radiation bath…



$T = 2.725\,\mathrm{K}$

Cosmic Microwave Background

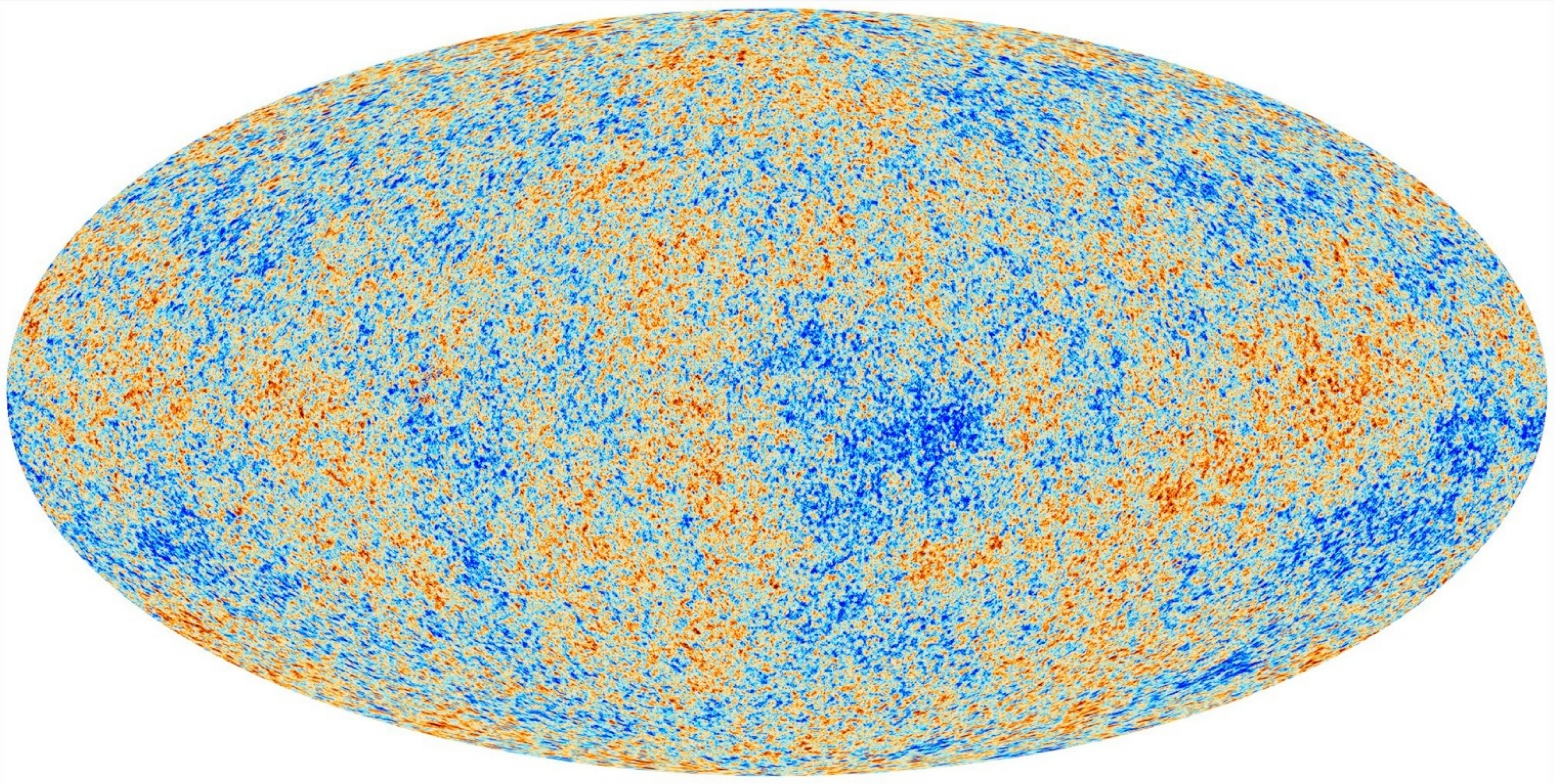
… with imprints of local motion…



$\Delta T/T \approx 10^{-3} \quad (v \sim 600\,\mathrm{km\,s^{-1}})$

Cosmic Microwave Background

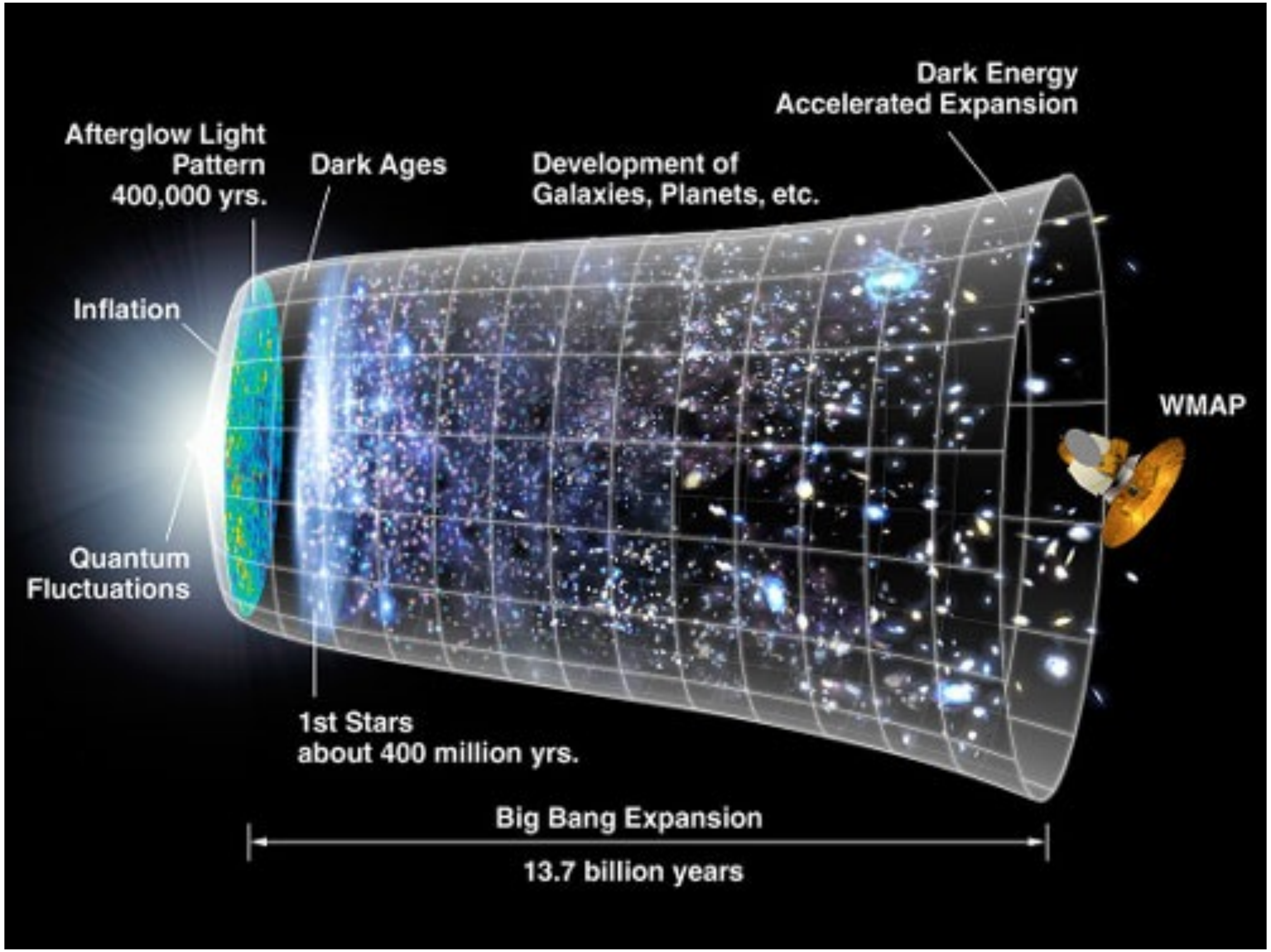
… and tiny primordial anisotropies



$\Delta T / T \approx 10^{-5}$

How did this come to be?…

Hot Big-Bang Cosmology



Relativistic Cosmology

Isotropy + homogeneity ⇒

$$\mathrm{d}s^2 = -\mathrm{d}t^2 + a^2(t)\left[\frac{\mathrm{d}r^2}{1-kr^2} + r^2\left(\mathrm{d}\theta^2 + \sin^2\theta\,\mathrm{d}\phi^2\right)\right]$$

Fundamental observers see homogeneous and isotropic ("maximally symmetric") 3-d spatial slices.

(r,θ,ϕ) → coordinates 'comoving' with fundamental observers

t → time measured on synchronised clocks carried by fundamental observers

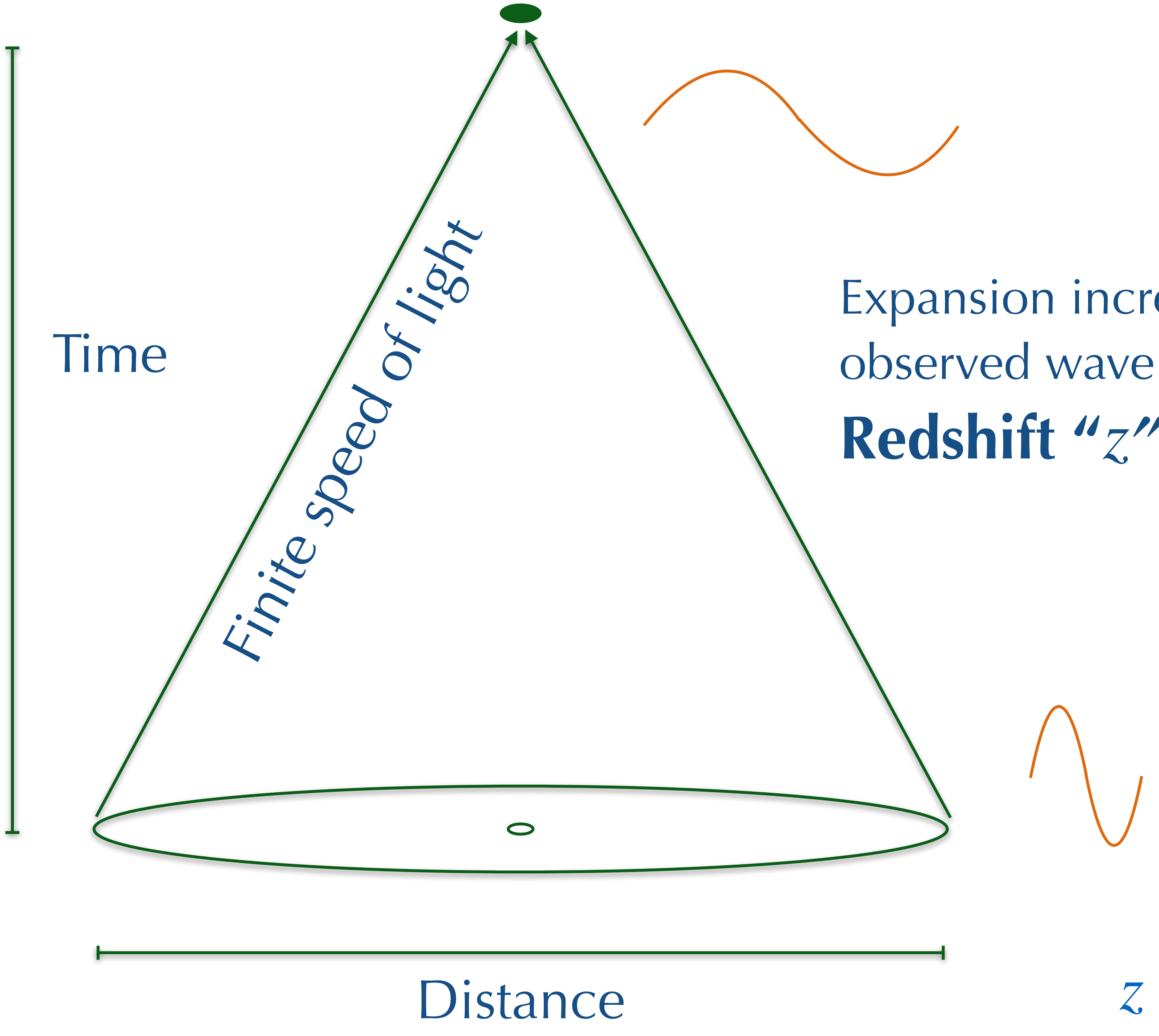
k → constant curvature of maximally symmetric slice

$a(t)$ → scale factor connecting lengths on different slices (i.e. at different times)

Corresponding matter/energy content must have "perfect fluid" form:
$T_{\mu\nu}$ → diag($-\rho, p, p, p$) where $\rho = \rho(t)$ = energy density and $p = p(t)$ = pressure

Finally, apply $G_{\mu\nu} = 8\pi G\, T_{\mu\nu}$.

Distance, time and shift of wavelength



Expansion increases observed wavelength λ:
Redshift "z"

Cosmological distance

Redshift

Time

$z \equiv \Delta\lambda/\lambda = 1/a - 1$ ($\approx v/c$ at low z)

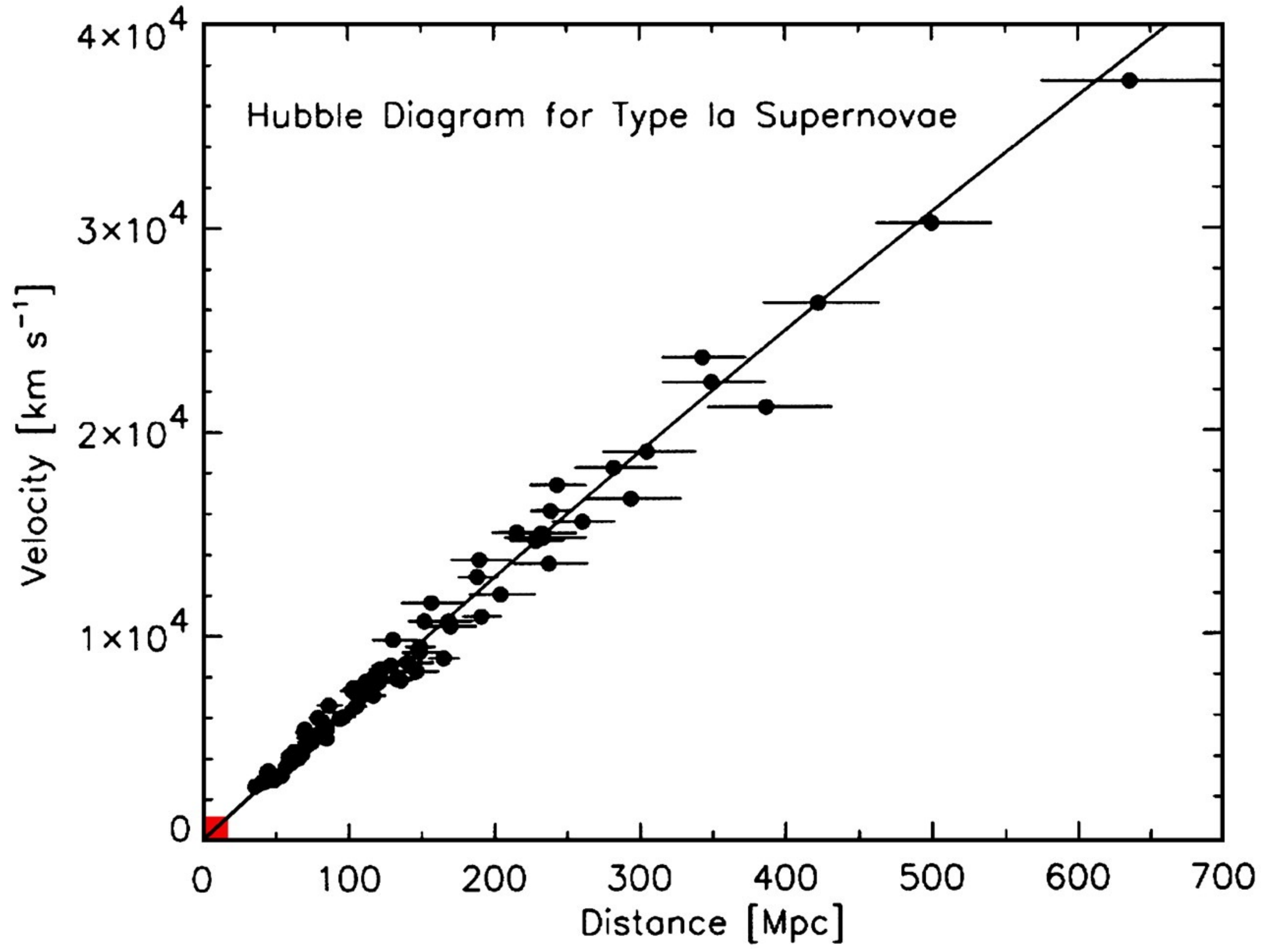
Smooth Expanding Universe

$$H^2(t) \equiv \left(\frac{1}{a}\frac{da}{dt}\right)^2 = \frac{8\pi G}{3}\bar{\rho} - \frac{k}{a^2}\,; \qquad \frac{1}{a}\frac{d^2a}{dt^2} = -\frac{4\pi G}{3}(\bar{\rho} + 3\bar{p})$$

$$\bar{\rho} \equiv \sum_i \bar{\rho}_i = \bar{\rho}_{\rm cdm} + \bar{\rho}_{\rm bary} + \bar{\rho}_{\rm rad} + \bar{\rho}_{\rm DE} \quad ; \quad \bar{p} \equiv \sum_i \bar{p}_i$$

$$\boxed{\Omega_i \equiv \frac{8\pi G}{3H^2}\bar{\rho}_i = \frac{\bar{\rho}_i}{\rho_{\rm crit}}\,; \qquad \Omega_{\rm k} \equiv -\frac{k}{a^2H^2} \qquad \sum_i \Omega_i = 1}$$

Edwin Hubble



Kirshner (2003)

Albert Einstein

Alexander Friedmann

Georges Lemaître

Foundations of modern cosmology laid during ~ 1919 - 1930

Smooth Expanding Universe

Evolution of ``energy densities":

- $\bar{\rho}_{\rm matter} \propto a^{-3}$ (think mass conservation) [matter dominated $a(t) \propto t^{2/3}$]
- $\bar{\rho}_{\rm radiation} \propto a^{-4}$ (think number conservation + momentum redshift) [radiation dominated $a(t) \propto t^{1/2}$]
- curvature $\propto a^{-2}$ [curvature dominated $a(t) \propto t$ (for $k < 0$)]
- $\bar{\rho}_{\rm DE}$ = constant ($\propto \Lambda$) in standard model, evolving in alternate cosmologies [Λ dominated $a(t) \propto e^{Ht}$]

Closed/open/flat models:

Assume $\Omega_{\rm R} = 0 = \Omega_{\rm DE} \implies \Omega_{\rm m} + \Omega_{\rm k} = 1$

- $\Omega_{\rm m} > 1,\ \Omega_{\rm k} < 0$: universe expands, turns around, collapses **(closed)**
- $\Omega_{\rm m} < 1,\ \Omega_{\rm k} > 0$: universe always expands **(open)**
- $\Omega_{\rm m} = 1,\ \Omega_{\rm k} = 0$: turn-around after infinite time **(flat)**

Distances in Cosmology

- Expansion means Euclidean notions need to be revised.
- Simplest way is to use Euclidean-like definitions and work through the maths that follows from ds^2 = 0 and photon-counting.

Define $\chi_{\rm src}(z) \equiv \int_{t_{\rm src}(z)}^{t_0} \frac{c\,\mathrm{d}t}{a(t)} = \int_0^z \frac{c\,\mathrm{d}z'}{H(z')}$

Then "comoving distance to source" is $r_{\rm src}(z) = \frac{1}{\sqrt{|k|}}\, s_k\big(\sqrt{|k|}\,\chi_{\rm src}(z)\big)$

where $k = -\Omega_{k0} H_0^2/c^2$ and $s_k(x) = \begin{cases} \sin(x), & k > 0 \\ x, & k = 0 \\ \sinh(x), & k < 0 \end{cases}$

and we have

Luminosity distance

$$d_{\rm L}(z) \equiv \sqrt{\frac{\mathcal{L}}{4\pi\mathcal{F}}} = (1+z)\, r_{\rm src}(z)$$

$\mathcal{L}$: intrinsic luminosity

$\mathcal{F}$: observed flux

Angular diameter distance

$$d_{\rm A}(z) \equiv \frac{\ell}{\Delta\theta} = (1+z)^{-1}\, r_{\rm src}(z)$$

ℓ: intrinsic physical size

$\Delta\theta$: observed angular size

Distances in Cosmology

- ***Hubble's law:***
  At $z \ll 1$, all distances reduce to Hubble's law
  $$\chi_{\rm src} = c\,z/H_0$$
  where $H_0 \equiv H(t = t_0)$.

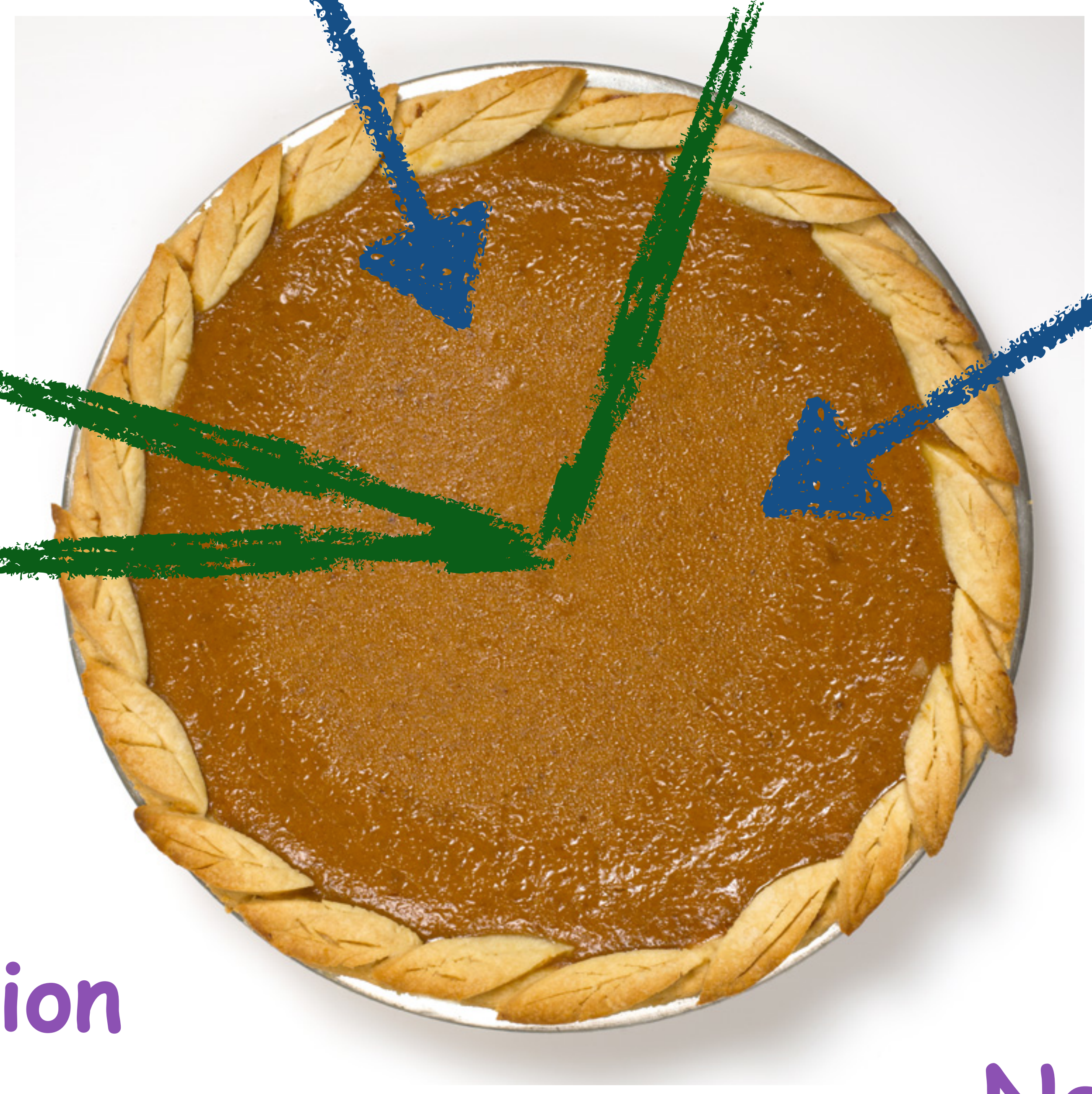
- ***Cosmography:***
  Beyond linear order in z, we have
  $$\chi_{\rm src} = \frac{c}{H_0}\left[z - \frac{z^2}{2}(1+q_0) + \mathcal{O}(z^3)\right]$$
  where $q(t) \equiv -\,a\,\ddot{a}/\dot{a}^2$ and $q_0 \equiv q(t = t_0)$ (deceleration parameter).

Standard Model of Cosmology

Matter
(cold, dark)
Ω_c

Energy
(dark, constant)
Ω_Λ

Baryons
Ω_b



Garnish with radiation
Ω_r

Neutrinos to taste
Ω_ν

Standard Model of Cosmology

- Radiation
- Baryons
- Cold Dark Matter
- Cosmological constant
- Spatial Curvature ≈ 0

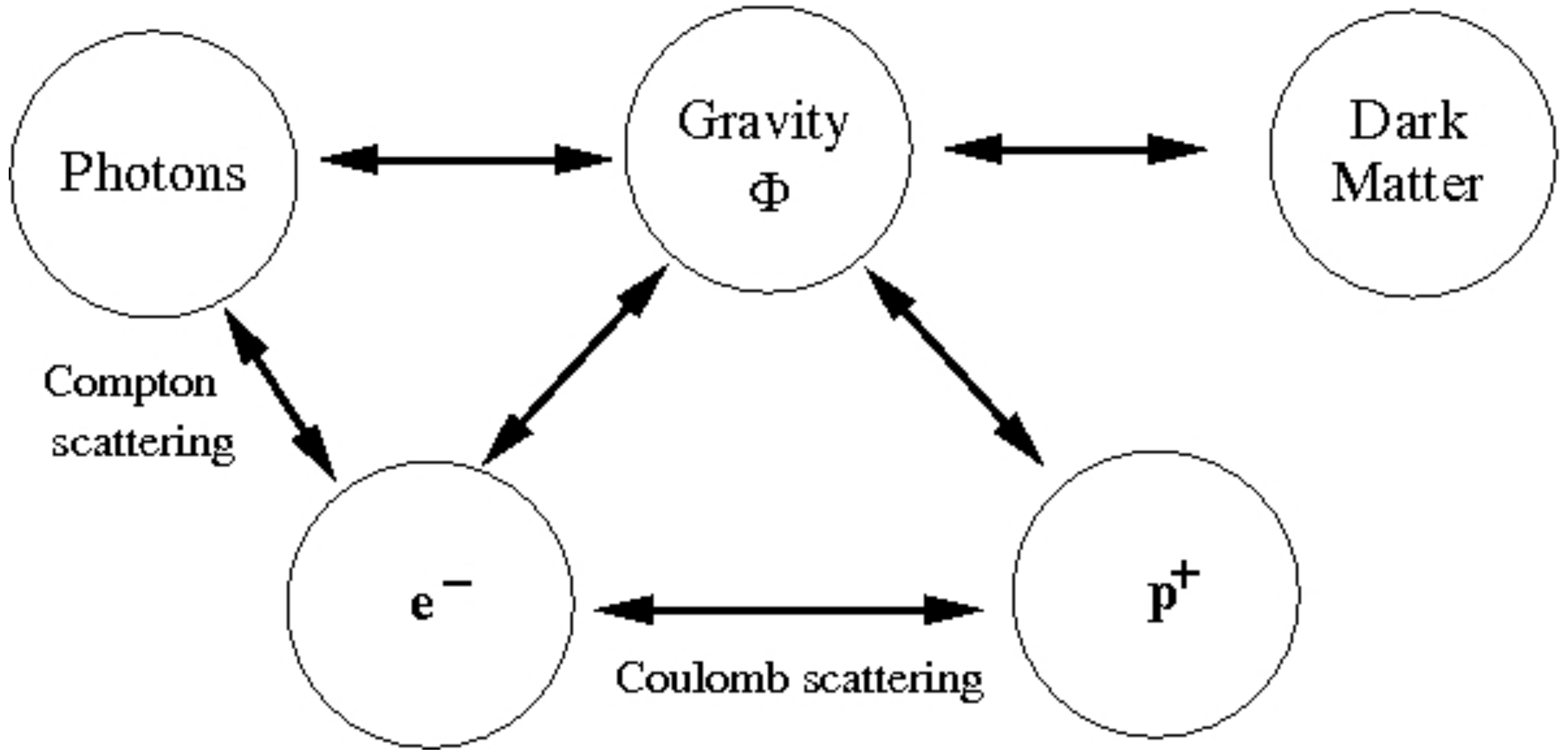
$$E(z)^2 \equiv H(z)^2/H_0^2 = \Omega_{R0}(1+z)^4 + \Omega_{m0}(1+z)^3 + \Omega_{\Lambda 0} + \Omega_{k0}(1+z)^2$$

$$[\ \Omega_{m0} = \Omega_{b0} + \Omega_{cdm0}\ ]$$

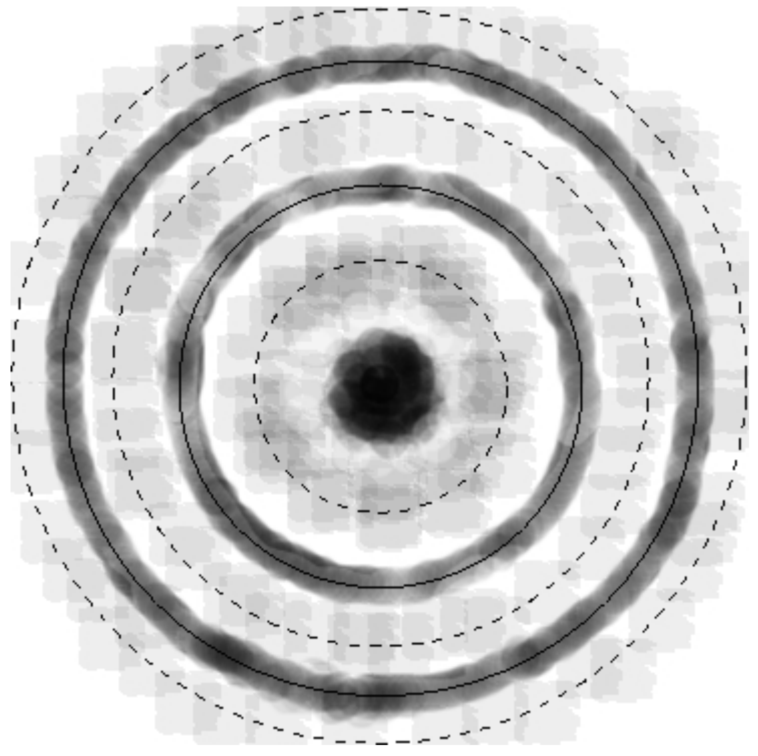
$H_0 = 100h\,\mathrm{km\,s^{-1}Mpc^{-1}}$ *[recall $H \equiv \mathrm{d}\ln a/\mathrm{dt}$ and $H_0 = H(t = t_0)$]*

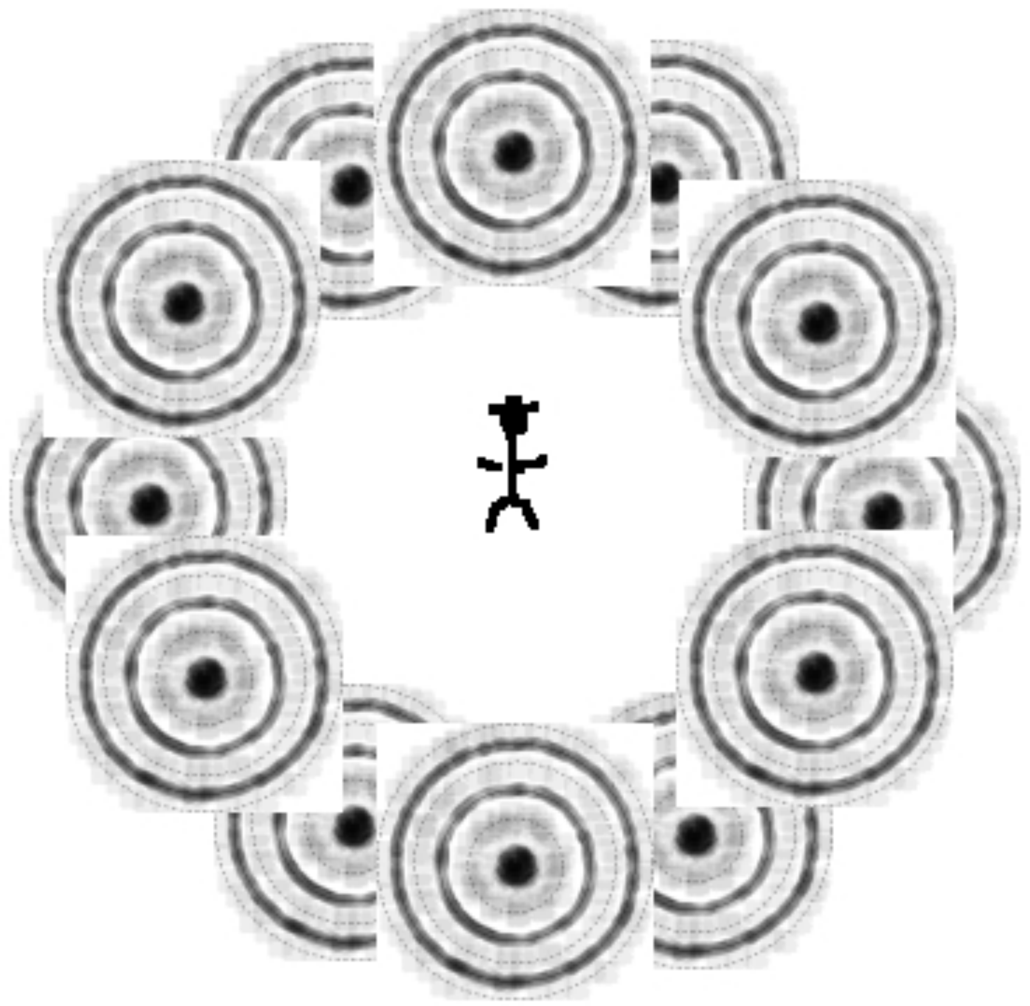
$h \approx 0.7$ [depending on probe]

Thermal history & Photon-baryon plasma



Adapted from Dodelson, *Modern Cosmology*.





- Expanding universe cools.
  [photon distribution: black body with $T \propto a^{-1}$ ~ always]
- Interactions between species remain in equilibrium until $\Gamma \gtrsim H$ after which reaction decouples.
- Species in equilibrium can also ``disappear” due to annihilations upon becoming non-relativistic.
  [can make photon T depart from a^{-1} due to entropy transfer]

- Photon-baryon fluid supports sound waves.
- These propagate until $t \sim 400{,}000$ yrs.
- Then, neutral hydrogen forms ($\Gamma \sim n_e \sigma_T v < H$), freeing photons (***recombination / photon decoupling***).
- Baryons captured by local potential wells.

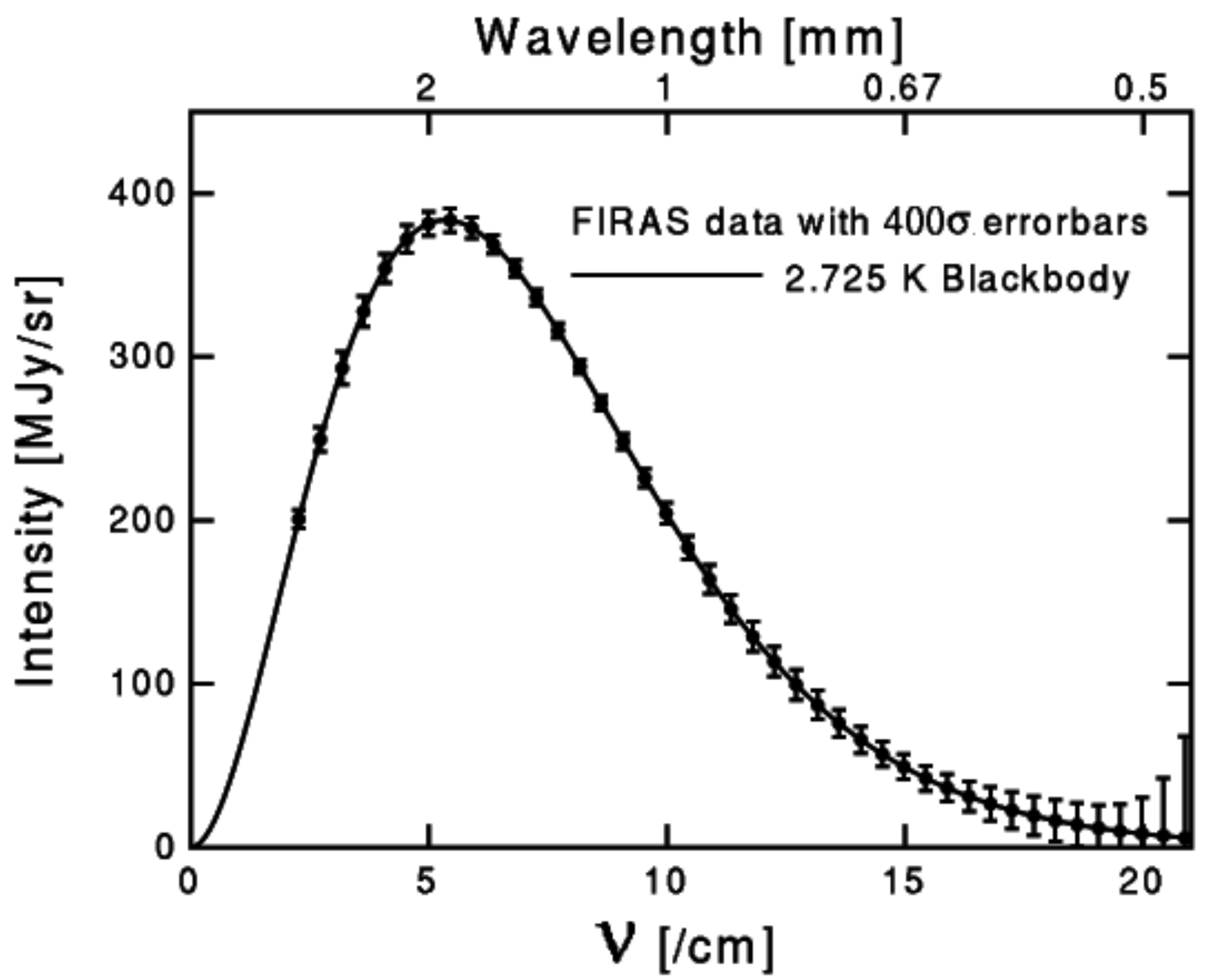
- Freed photons free-stream to us *[maintaining black-body spectrum with $T \propto a^{-1}$]* forming the Cosmic Microwave Background.
- Baryons eventually form galaxies, with remnants of acoustic correlations imprinted in number density (Baryon Acoustic Oscillations).

Radiation

$\rho_\gamma = (4\sigma/c)T^4$ where σ = Stefan-Boltzmann constant

CMB temperature today measured to be *[Fixsen 2009]*

$T_0 = 2.72548 \pm 0.00057$ K



So $\rho_{\gamma 0} = (4\sigma/c)T_0^4$

$= (4.175 \pm 0.004) \times 10^{-13}$ erg cm^{-3}

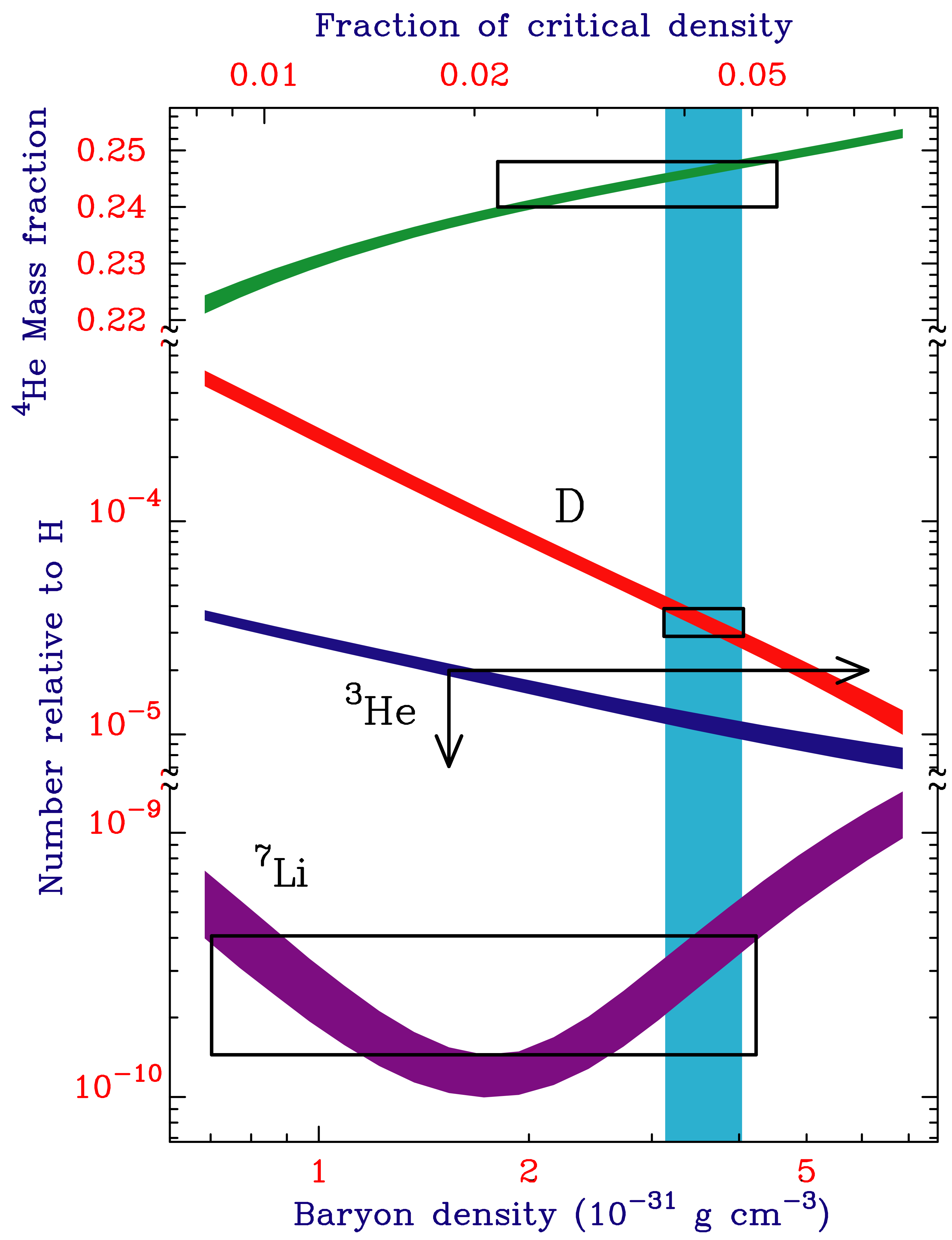
and $\rho_{\rm crit0} = 1.688 \times 10^{-8}\, h^2$erg cm^{-3}

$\therefore\ \Omega_{\gamma 0} h^2 = (2.473 \pm 0.002) \times 10^{-5}$

Including 3 relativistic neutrinos gives

$\Omega_{\rm R0} h^2 = (4.158 \pm 0.003) \times 10^{-5}$

Baryons



[Plot by Wayne Hu]

Burles, Nollett, Turner (1999)

Constraint from BBN (95% C.L.)

$\rho_{b0} = (3.76 \pm 0.38) \times 10^{-31}$ g cm^{-3}

$$\Omega_{b0}h^2 = 0.020 \pm 0.002$$

Constraint from CMB anisotropies (Planck-2018, 95% C.L.)

$$\Omega_{b0}h^2 = 0.0223 \pm 0.0003$$

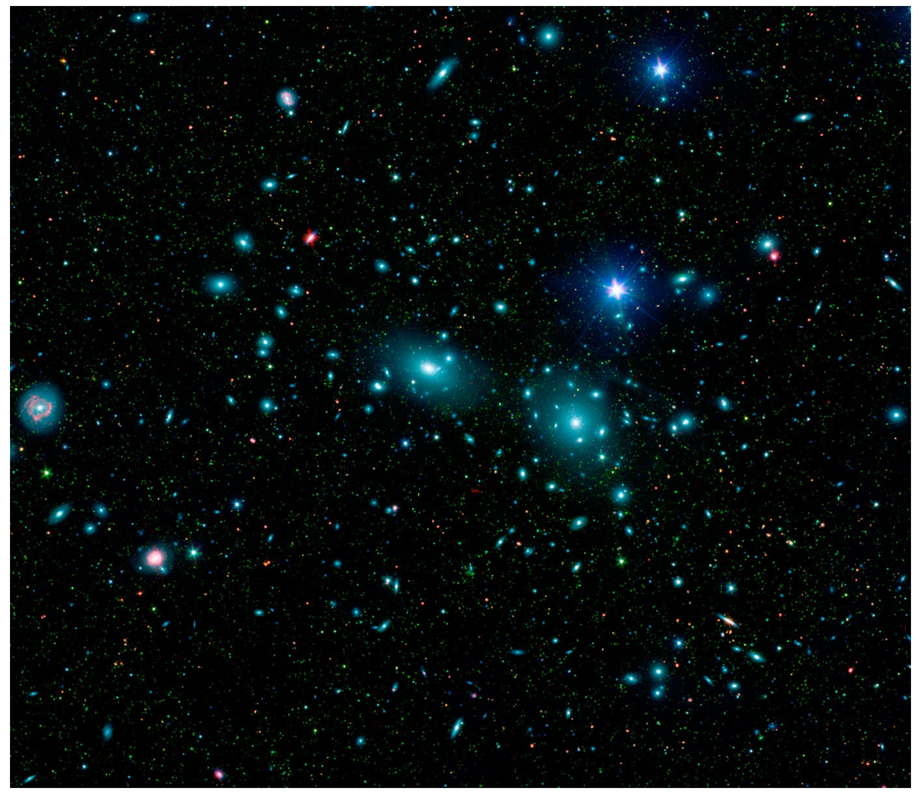
Cold Dark Matter

Non-baryonic component. Non-relativistic for most of cosmic history. Dominant interaction with ordinary matter and radiation is through gravity (*else CMB anisotropies couldn't produce today's LSS*).

Most robust evidence of collisionless nature!

Virial theorem

[Zwicky 1933;1937]



Spitzer

$\langle V^2 \rangle = GM/R_{vir}$

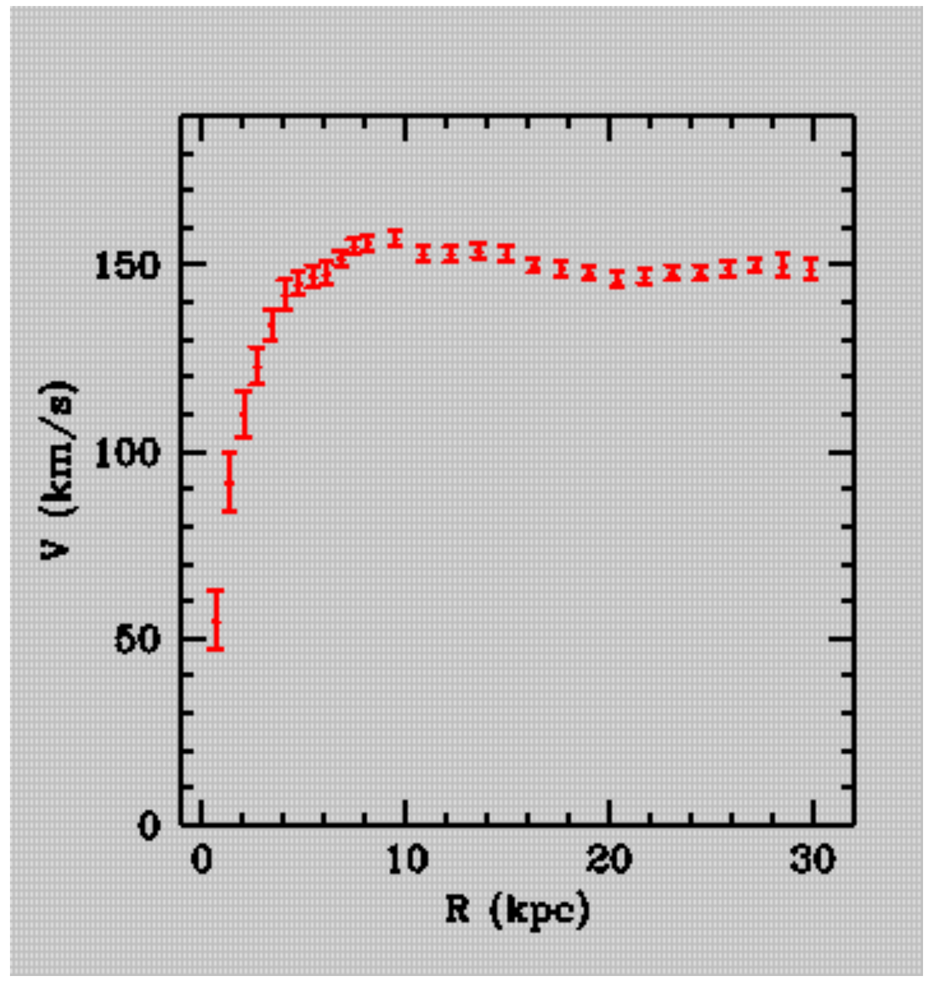
$\langle V^2 \rangle \rightarrow$ redshifts; $R_{vir} \rightarrow$ size

$M_{vir} \sim 10\, M_{gas} \sim 50\, M_{stars}$

→ Missing mass

Galaxy rotation curves

[Rubin+ 1978;1980]

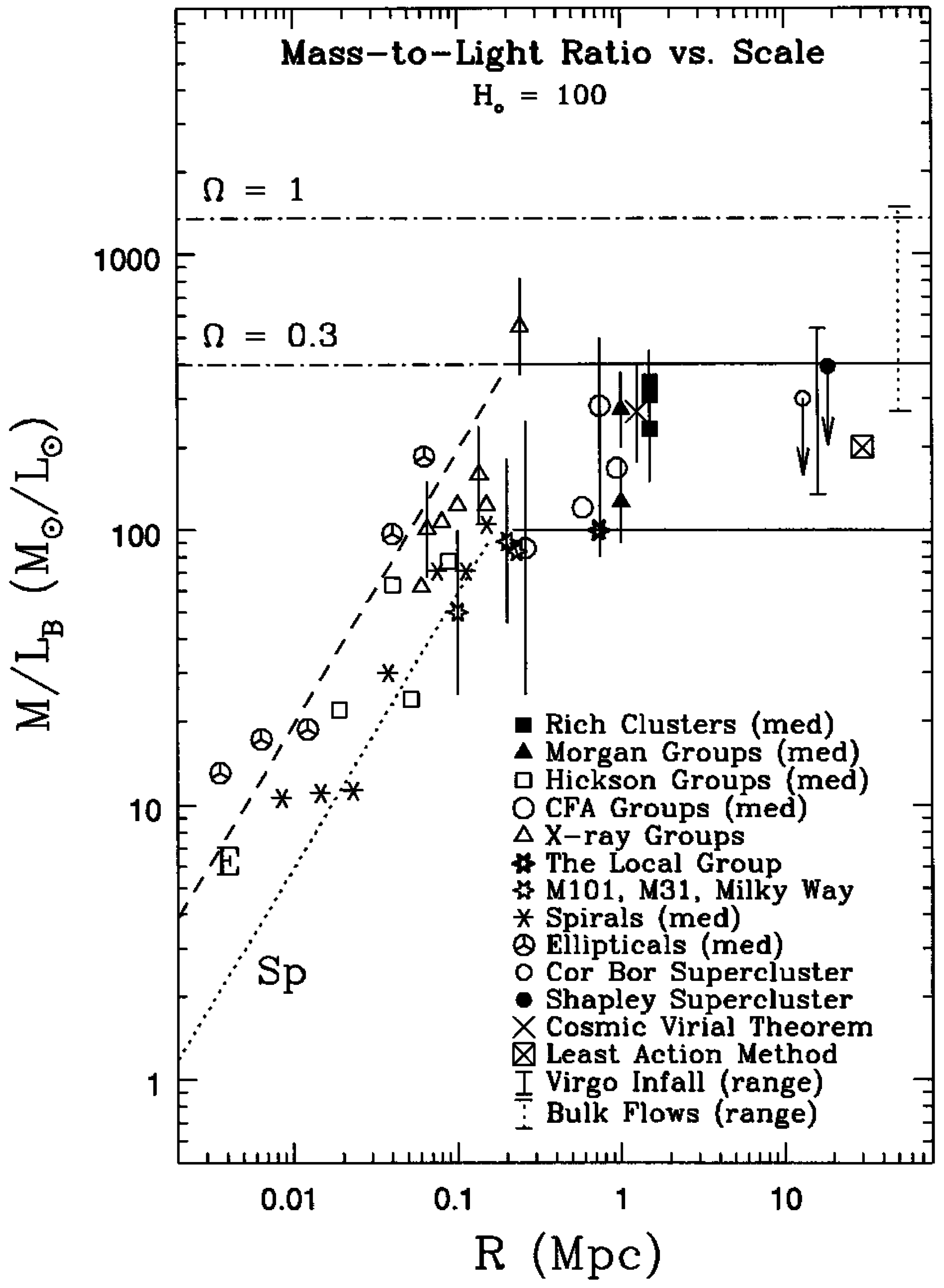


[Begeman+ 1989]

Expect $V \propto R^{-1/2}$ far from disk.
Need extra $\rho \propto R^{-2}$ to fit data.

Scale-dependent M/L

[Bahcall+ 1995;2000]



Ratio flattens at large scales.
Explained by $\Omega_{m0} \approx 0.2$-0.3

Bullet(-like) Clusters

[Clowe+ 2004]

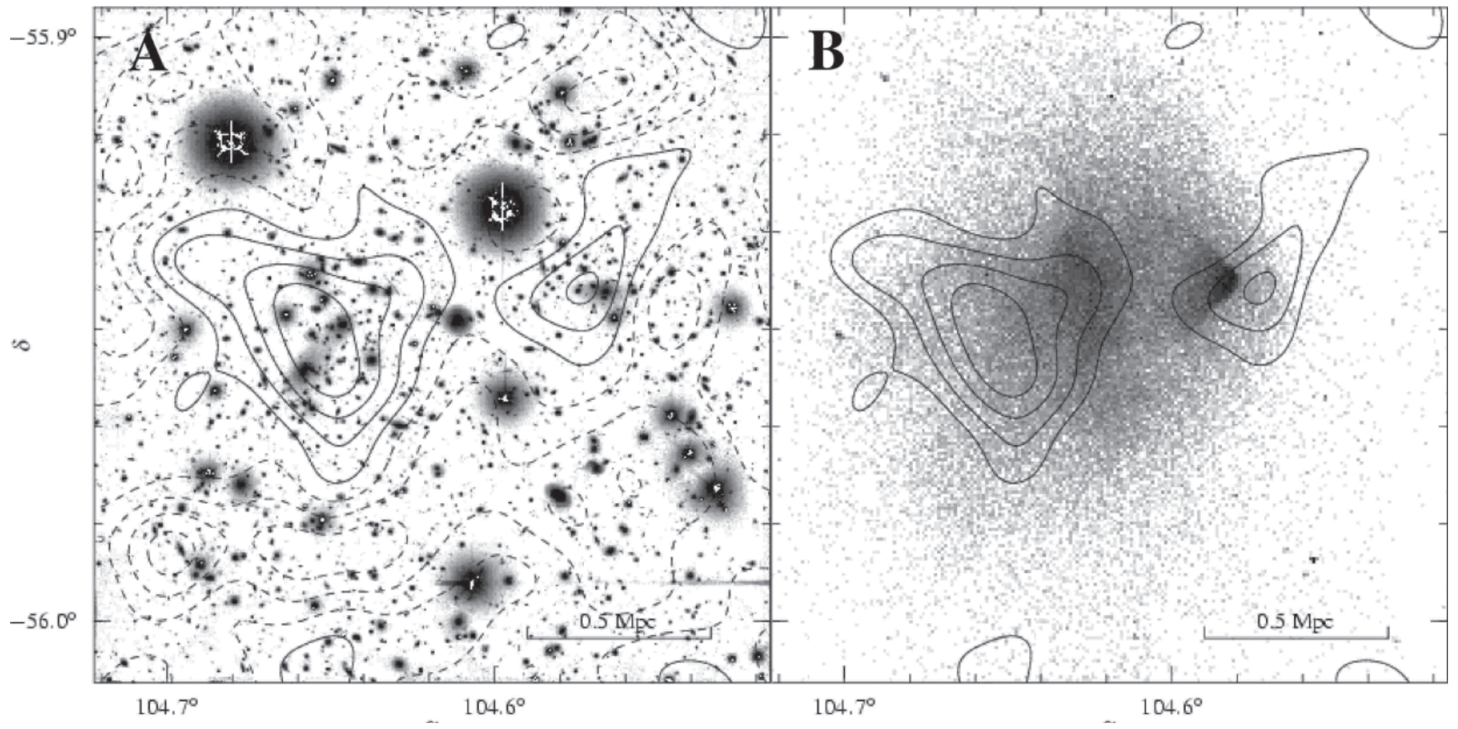


FIG. 1.—(*a*) Gray-scale *I*-band VLT image used to measure the galaxy shapes for the background galaxies. Overlain in black contours is the weak-lensing mass reconstruction, with solid contours for positive mass, dashed contours for negative mass, and the dash-dotted contour for the zero-mass level, which is set such that the mean mass at the edge of the image is zero. Each contour represents a change in the surface mass density of $2.8 \times 10^8\ M_\odot\ \mathrm{kpc}^{-2}$. (*b*) Gray-scale *Chandra* X-ray image from Markevitch et al. (2002), with the same weak-lensing contours as in (*a*). (*c*) Gray-scale luminosity distribution of galaxies with the same *B−I* colors as

Gravitational lensing mass-reconstruction is offset from gas, coincident with stellar light. Consistent with DM being collisionless.

Consistent with latest CMB determination

$$\Omega_{m0} = 0.316 \pm 0.009$$

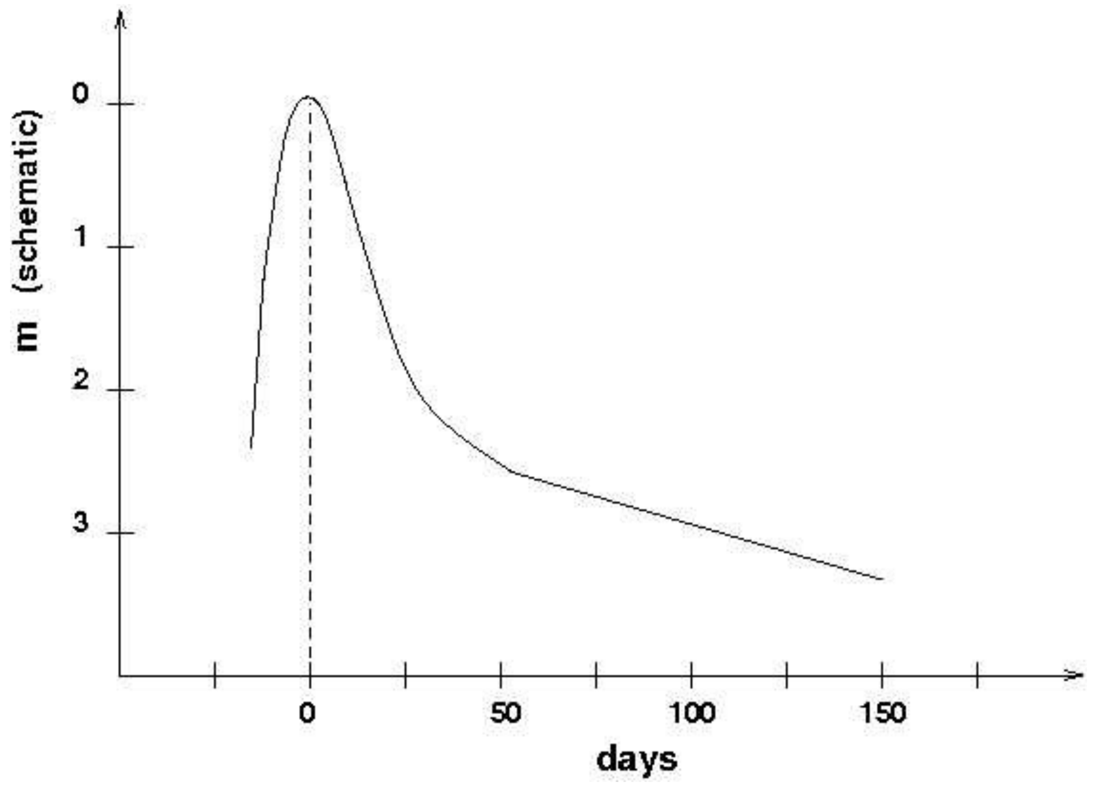
Cosmological Constant Λ

$$\boxed{\frac{d^2a}{dt^2} = -\frac{4\pi G}{3}\left(\bar{\rho} + 3\bar{p}\right)}$$

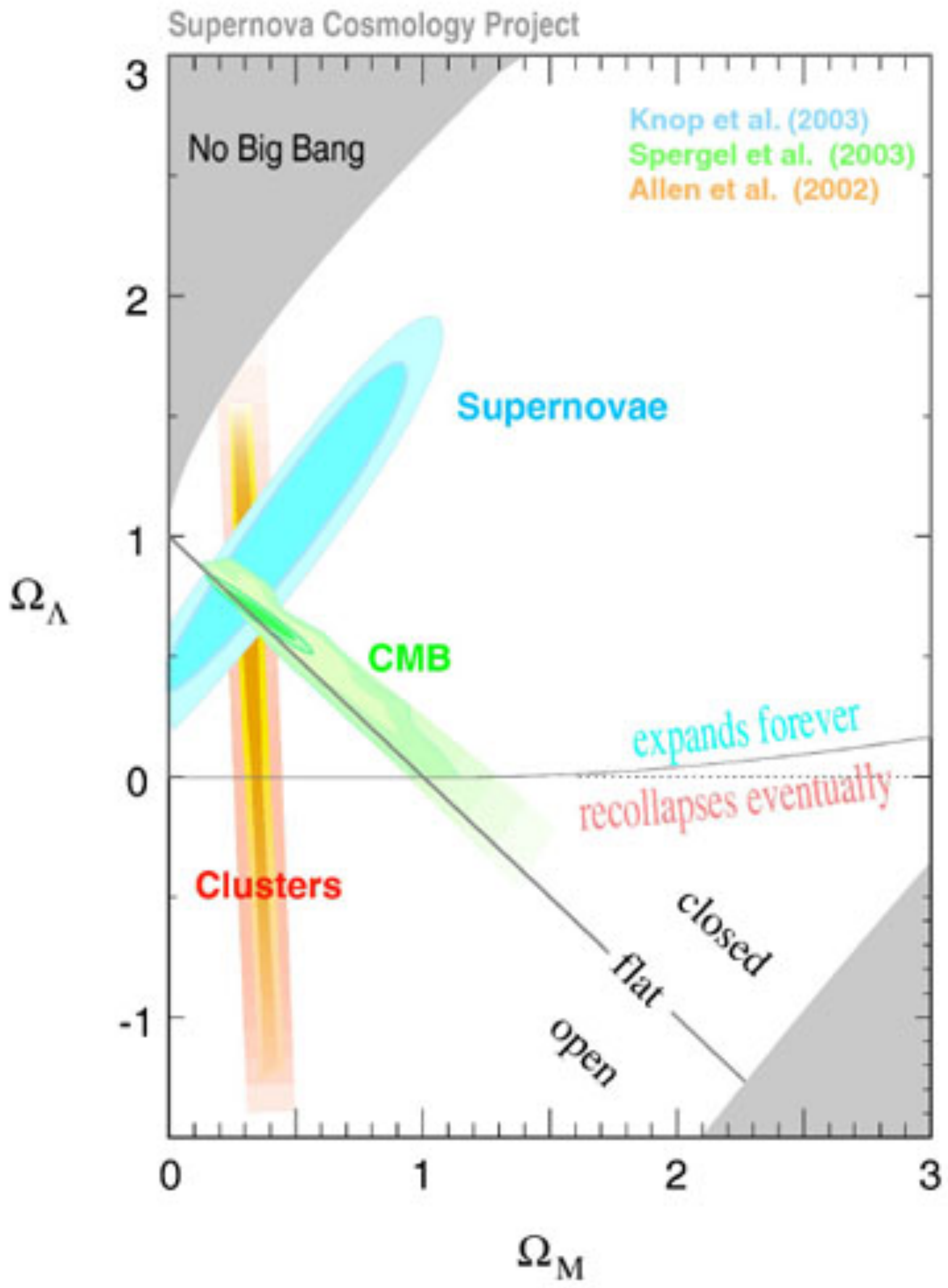
First introduced (and discarded) by Einstein, followed by a tortured history of epicycles.

Currently, cleanest evidence provided by SNe Ia cosmography which gives $q_0 \approx -0.55$ so that $d^2a/dt^2 > 0$, which requires $w = \bar{p}/\bar{\rho} < -1/3$ for at least one component.

Cosmological constant ($w = -1$) is the simplest choice consistent with data.



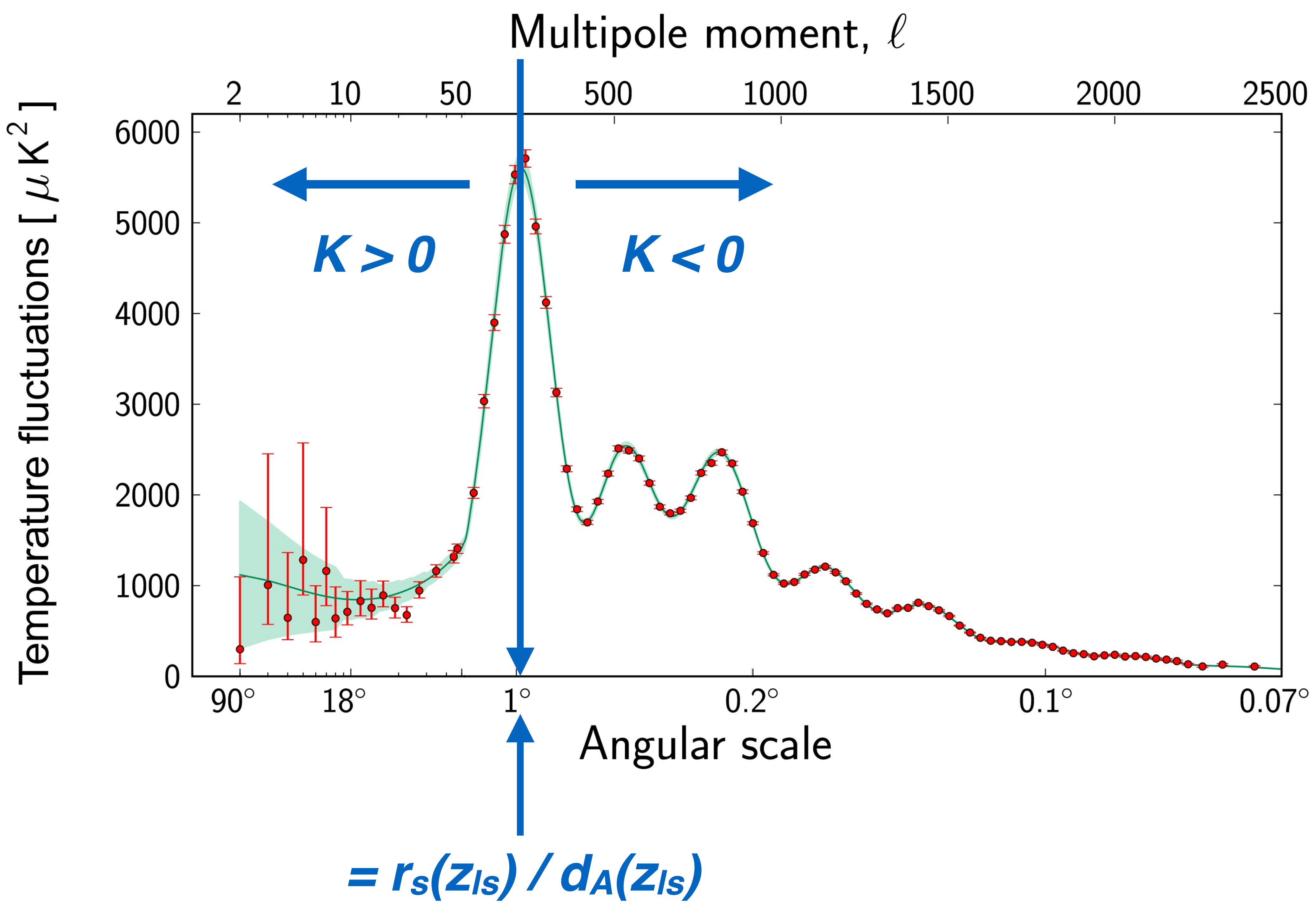
Empirical correlations between peak luminosity & light curve shape ⇒ ***standardisable*** candles



Spatial Curvature

Presence of curvature changes distances (recall expressions for $d_A(z)$ and $d_L(z)$ in terms of $S_k[\chi(z)]$).

Longest lever arm provided by angular diameter distance to last scattering surface, best accessible via 1st peak of CMB power spectrum.

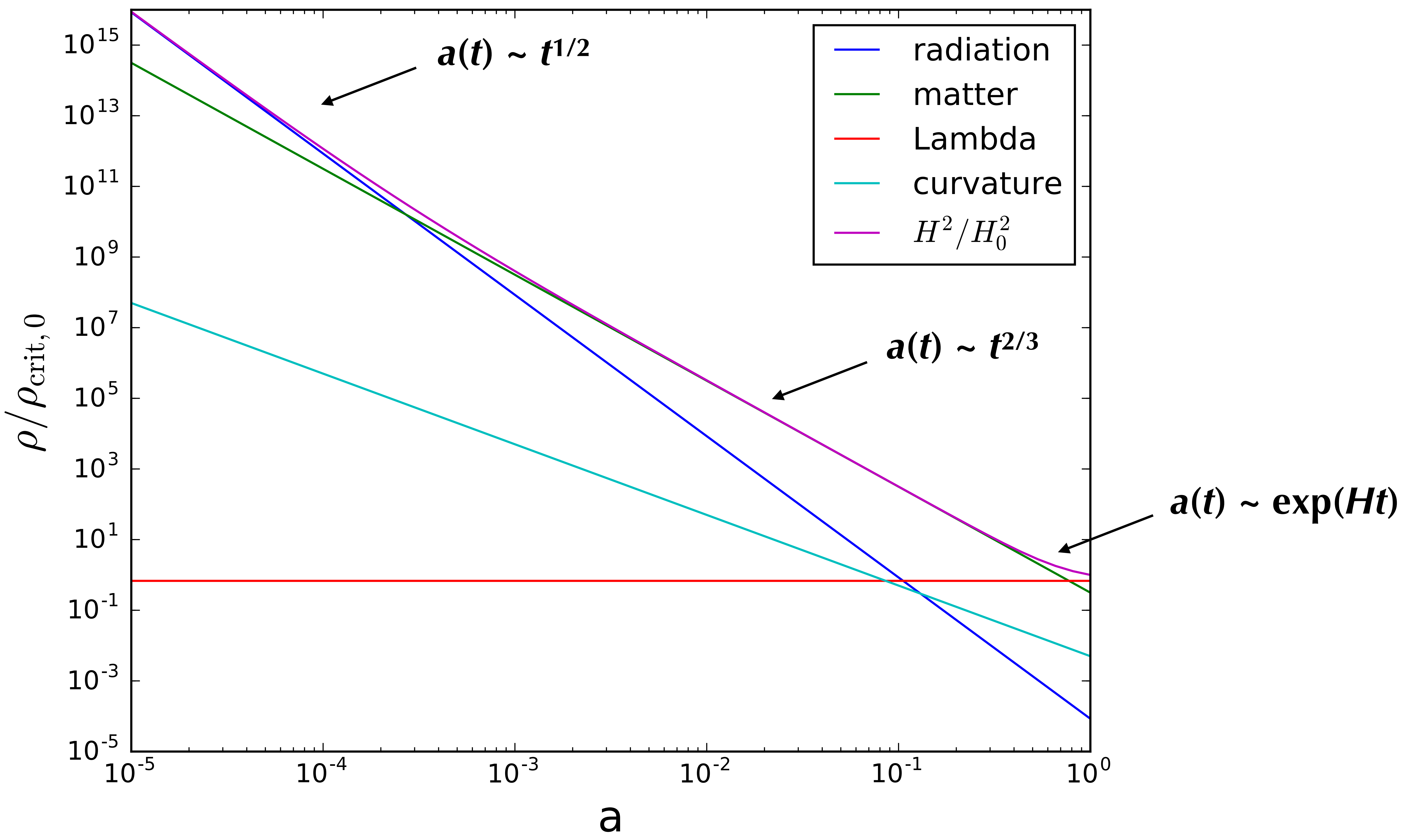


Can be strengthened by including BAO measurements [r_s at multiple z].

Current constraint, assuming ΛCDM (95% C.L.)

$$\Omega_{k0} = 0.0007 \pm 0.0037$$

Expansion History



Some human history…

Models for the evolving Universe

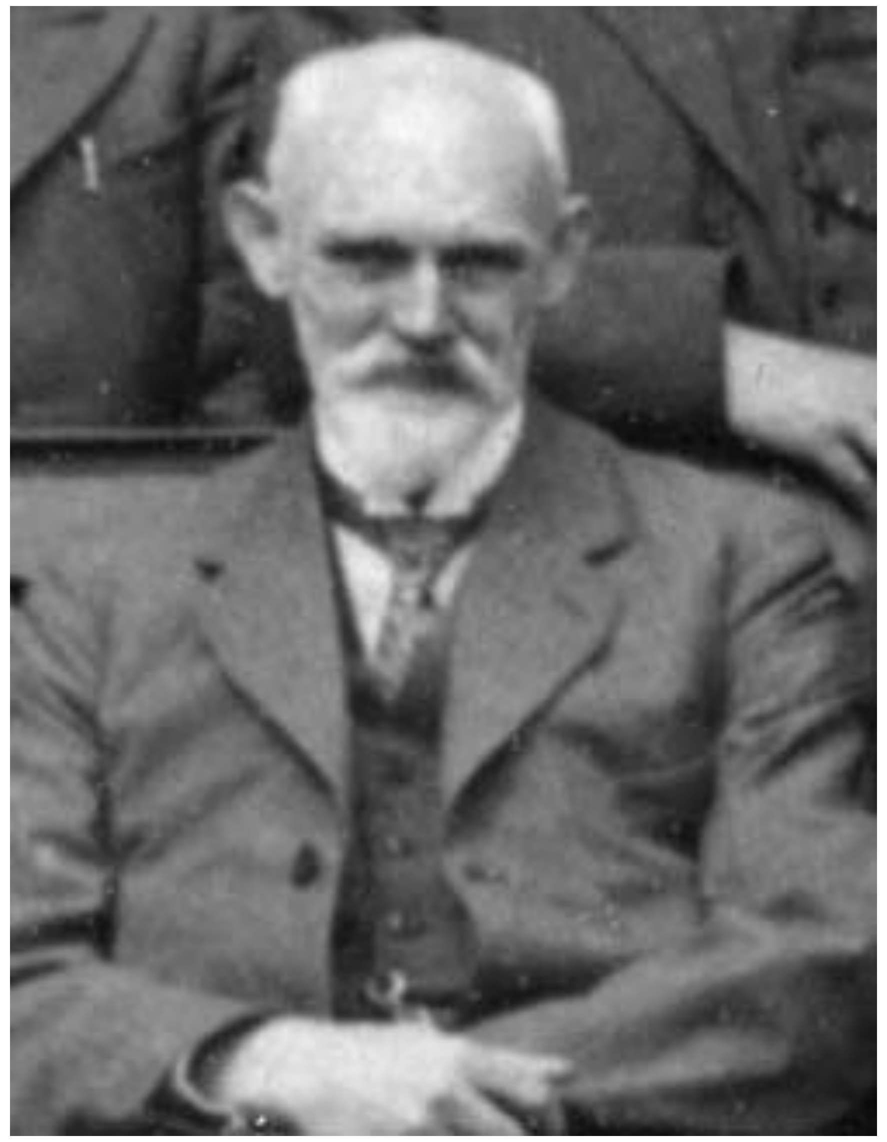
The cosmological constant Λ

1917

Matter without motion

Albert Einstein

Motion without matter



Willem deSitter

Models for the evolving Universe

Dynamical universes with matter

Alexander Friedmann

1922 Published ``closed universe” model, with and without Λ. Einstein claimed Friedmann’s algebra was wrong, but later retracted (1923).

1924 Published ``open universe” model. Same year as Hubble’s observations of Cepheids. Died 1925, aged 37.

Models for the evolving Universe

Dynamical universes with matter

Georges Lemaître

1924 - 1936

Lemaître's theoretical contributions

- In 1927 paper, constructed a solution of Friedmann's equation with nonzero Λ. Derived the linear distance-redshift relation. Explained observed redshifts in terms of an expanding Universe rather than as Doppler shifts.

- 1931: Constructed yet another solution, the coasting universe, with Λ tuned to give a long, nearly static phase, hence solving the age problem.

- Extrapolated this solution to early times, suggested notion of a beginning: seed of Big Bang cosmology. His views remained unfashionable well into the 1950's.

- 1934: Suggested notion of Λ as arising from vacuum fluctuations. Later rigorously calculated by Zel'dovich (1968) in QFT framework (but very sensitive to high-energy cutoff).