

INFLATION & REHEATING

→ INFLATION

(MOTIVATIONS, SIMPLEST REALIZATION,
PRIMORDIAL PERTURBATIONS)

→ PRODUCTION DURING REHEATING (WHILE THE INFLATON IS DECAYING)

→ PRODUCTION DURING INFLATION (SIGNATURES IN CMB & IN GRAVITATIONAL WAVES)

• EXPANSION IN STANDARD COSMOLOGY

DISTANCES $\propto a(t)$ SCALE FACTOR

$H \equiv \frac{\dot{a}}{a}$ HUBBLE RATE

UNIVERSE
DOMINATED BY

RADIATION
MATTER

HOMEWORK

$a \propto t^{1/2}$, $H = \frac{\dot{a}}{a} = \frac{1}{2t}$

$a \propto t^{2/3}$, $H = \frac{2}{3t}$

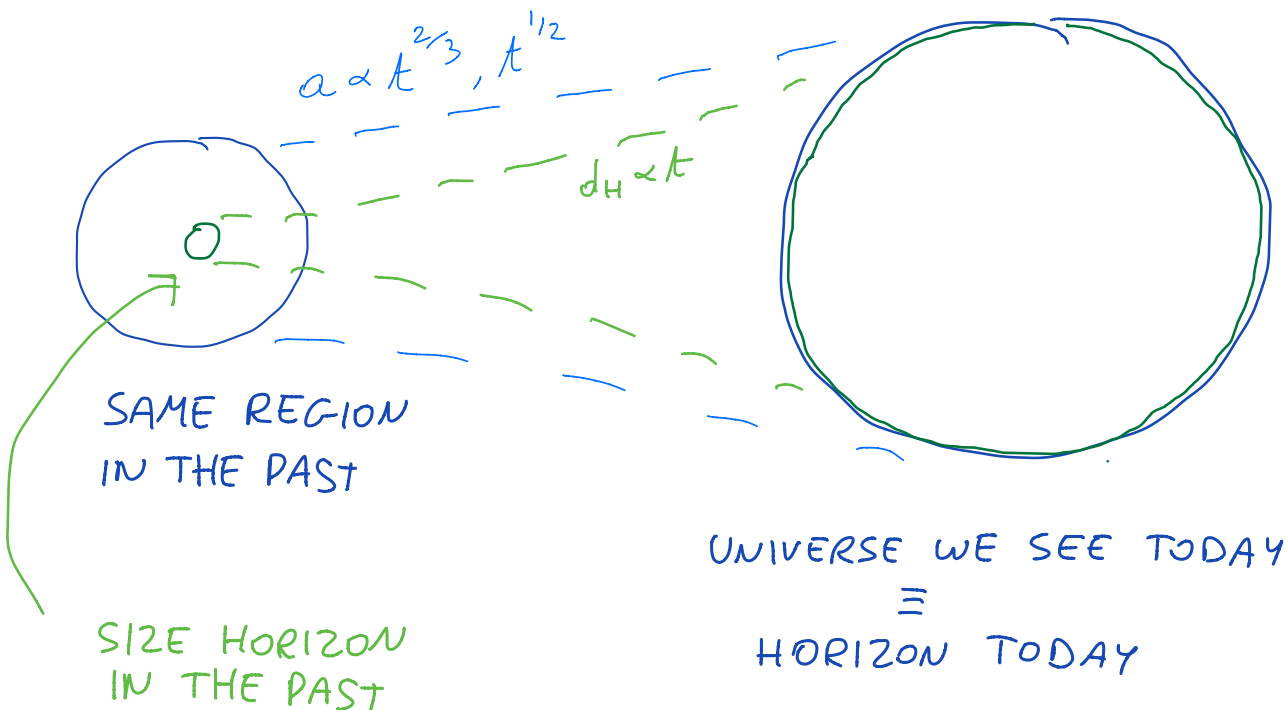
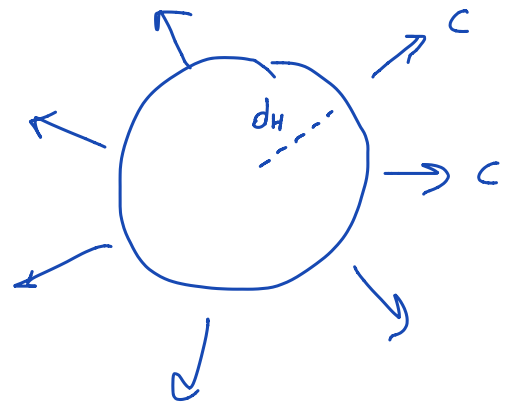
• HORIZON PROBLEM

REGION IN CAUSAL CONNECTION

$d_H \sim ct \sim \frac{1}{H}$ ($c=1$)

HORIZON GROWS FASTER

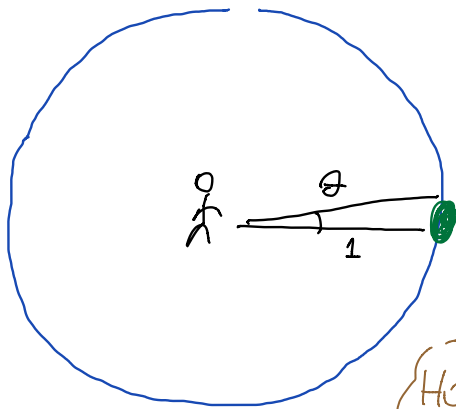
THAN SCALE FACTOR



THE SKY WE SEE TODAY IS COMPOSED OF MANY REGIONS THAT WERE CAUSALLY DISCONNECTED IN THE PAST

EXAMPLE : CMB EMITTED WHEN $a_{\text{CMB}} \sim 10^{-3} a_0$

↑ SUFFIX 0
MEANS
TODAY



PORTION OF MY SKY OCCUPIED BY A
REGION THAT WAS CAUSALLY CONNECTED
WHEN CMB WAS EMITTED

$$\theta \approx 2^\circ$$

HOMEWORK

HOWEVER, CMB RADIATION SAME
TEMPERATURE FROM ALL DIRECTIONS

PROBLEM BECAUSE $\frac{d_H}{a}$ INCREASES

$$\frac{d_H}{a} \propto \frac{t}{a} \propto \frac{1}{aH} = \frac{1}{a \frac{\dot{a}}{a}} = \frac{1}{\dot{a}} \leftarrow \text{DECREASES}$$

PROBLEM BECAUSE $\ddot{a} < 0$ IN A MATTER + RADIATION
UNIVERSE

● FLATNESS PROBLEM

EVOLUTION OF SCALE FACTOR GOVERNED BY

$$\frac{\dot{a}^2}{a^2} = -\frac{k}{a^2} + \frac{1}{3H_p^2} \left(\frac{\rho_M^{(0)}}{a^3} + \frac{\rho_R^{(0)}}{a^4} \right)$$



AS UNIVERSE EXPANDS, CURVATURE TERM BECOMES DOMINANT.
THIS TERM IS $< 0.5\%$ TODAY \Rightarrow MUST HAVE BEEN $\leq 10^{-18}$
WHEN UNIVERSE $\sim 1S$ OLD

PROBLEM BECAUSE $\rho_{\text{STANDARD}} < a^{-P}$ WITH $P > 2$

$\Rightarrow \frac{\dot{a}^2}{a^2} = \frac{\dot{a}^2}{a^2} \left(\frac{\dot{a}^2}{a^2} \right) < a^{2-P}$ DECREASES \equiv DECELERATION

AGAIN, PROBLEM BECAUSE MATTER & RADIATION LEAD
TO DECELERATED EXPANSION

● INFLATION

PROBLEMS SOLVED IF UNIVERSE UNDERWENT A PHASE
OF ACCELERATED EXPANSION, $\ddot{a} > 0$. THIS MUST HAVE
HAPPENED BEFORE BIG-BANG-NUCLEOSYNTHESIS (BBN),
 $t < 1S$.

GUTH, 1981

* DURING INFLATION, A SINGLE CAUSALLY CONNECTED
REGION GROWS SO MUCH AS TO COVER ALL THE SKY WE
SEE TODAY. THE SOURCE OF INFLATION DOMINATES OVER
CURVATURE TERM ($P < -2$) AND THEN DECAYS INTO \rightarrow MATTER
& RADIATION

WE CALL THIS SOURCE "INFLATON FIELD"

SIMPLEST ACCELERATION FROM VACUUM ENERGY

$$H^2 = \frac{\dot{\phi}^2}{2} \leftarrow \rho \text{ AND } \rho = \text{const.} \rightarrow a = e^{Ht}$$

PBM: IF $\rho > \rho_{\text{VACUUM}}$, INFLATION WILL NOT END

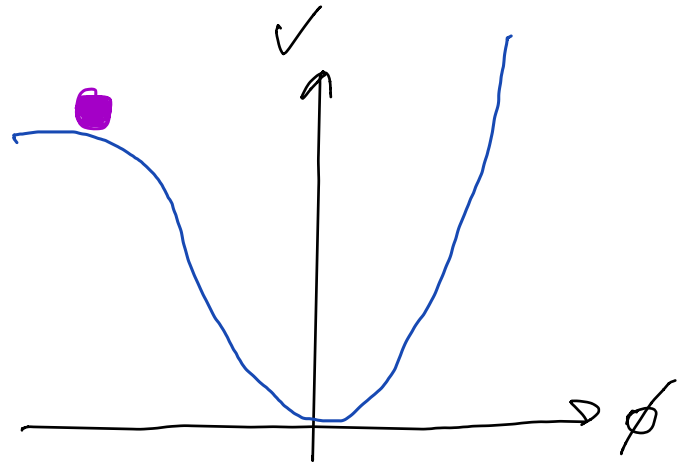
• SLOW ROLL INFLATION

LINDE 1982

ALBRECHT, STEINHARDT 1982

→ INFLATON ON A **FLAT**
PORTION OF V \Rightarrow BEHAVES \approx
VACUUM ENERGY

→ INFLATION ENDS WHEN
 ϕ REACHES NON-FLAT PART
THEN, OSCILLATIONS ABOUT
MINIMUM AND DECAY



SLOW ROLL PARAMETERS

$$\epsilon \equiv \frac{M_P^2}{2} \left(\frac{V'}{V} \right)^2 \ll 1 \quad \text{THE POTENTIAL NEEDS TO BE FLAT}$$

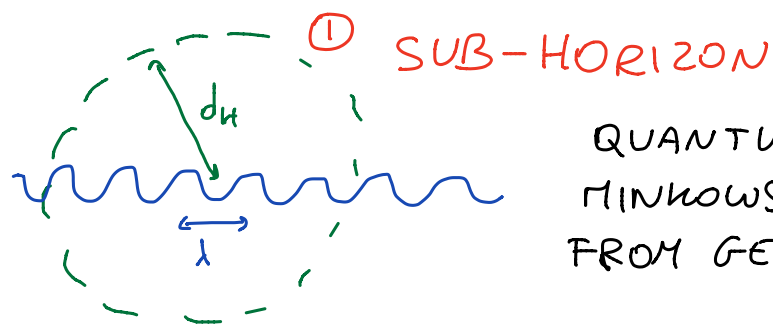
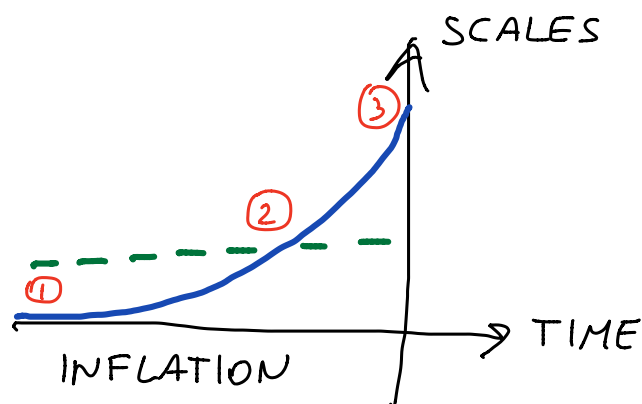
$$\eta \equiv \frac{M_P^2}{2} \frac{V''}{V} \ll 1 \quad \text{FLAT FOR LARGE SPAN OF } \phi$$

BYPRODUCT : MECHANISM FOR PRIMORDIAL PERTURBATIONS

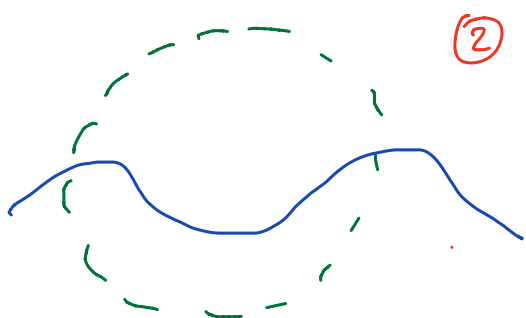
$$\delta\phi, \delta g \begin{cases} \nearrow \frac{\delta\rho}{\rho} \cong \zeta \text{ (SCALAR) DENSITY PERTURBATIONS} \\ \searrow h_+, h_x \text{ (TENSOR) GRAVITATIONAL WAVES} \end{cases}$$

CONSIDER A PERTURBATION ON OUR SKY AND TRACE IT BACK DURING INFLATION

"HUBBLE HORIZON $d_H = H^{-1} \sim \text{const.}$
WAVELENGTH $\lambda \propto a \approx e^{Ht}$



QUANTUM EVOLUTION AS IN
MINKOWSKI. NEGLIGIBLE EFFECT
FROM GEOMETRY

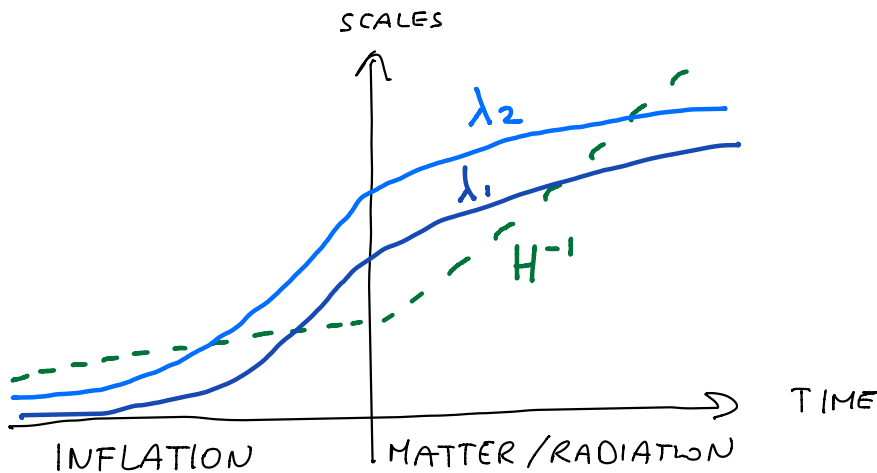


(2) HORIZON CROSSING

THIS IS WHEN THE MODE IS
IMPRINTED. PERTURBATION PROBES
CONDITION OF THE UNIVERSE AT
THIS MOMENT

③ SUPER-HORIZON

MODE SEEN AS CONSTANT IN EACH HORIZON PATCH. FROZEN (NO EVOLUTION IN TIME) DUE TO CAUSALITY



MODES OF GREATER WAVELENGTH ($\lambda_2 > \lambda_1$) CROSS THE HORIZON EARLIER. THEY PROBE EARLIER TIMES OF INFLATION.

DEPARTURE FROM dS (CONSTANT V)

$$\text{FROM } \epsilon \equiv \frac{M_P^2}{2} \left(\frac{V'}{V} \right)^2, \quad \eta \equiv M_P^2 \frac{V''}{V}$$

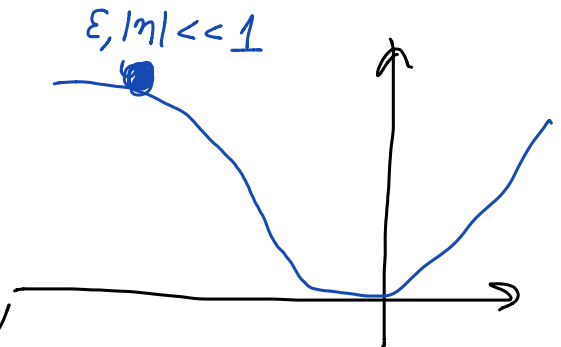
3 IMPORTANT PROPERTIES OF PRIMORDIAL PERTURBATIONS

1) NEAR SCALE INVARIANCE

MODES OF \neq SIZE HAVE NEARLY THE SAME POWER,

$$P_{\delta} \propto \lambda^{1-n_s} \quad 1-n_s \approx 6\epsilon - 2\eta$$

MODES OF \neq SIZE LEAVE THE HORIZON AT \neq TIMES, WHEN ϕ HAS MOVED TO A \neq LOCATION IN V

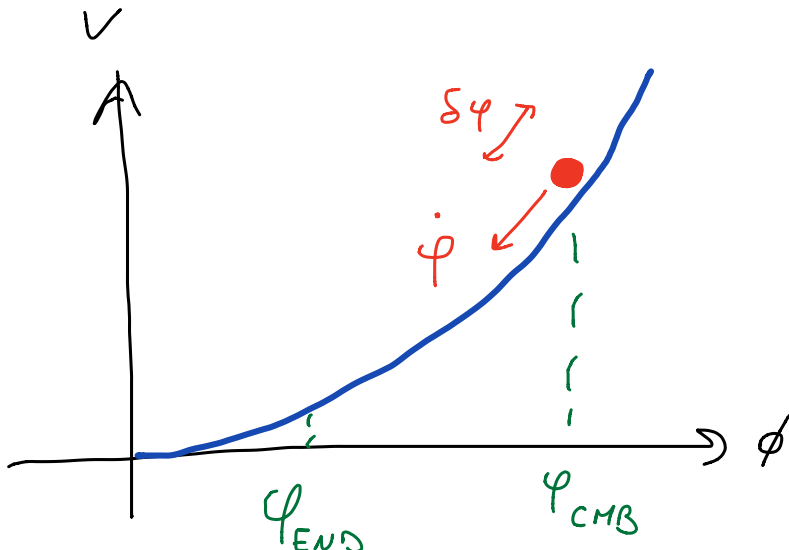


BUT, FOR SLOW ROLL, $V(\phi)$ HAS CHANGED VERY LITTLE, SO NEARLY SAME POWER AT ALL SCALES

2) $G_W \ll$ DENSITY PERTURBATIONS

INFLATION PREDICTS

$$r \equiv \frac{P_{G_W}}{P_{\delta}} = 16 \epsilon \ll 1$$



INFLATON IS A CLOCK,
MEASURING THE TIME TO
THE END OF INFLATION
SMALLER / GREATER
ENERGY DENSITY
 \equiv
EARLY / LATE CLOCK

SCALAR PERTURBATIONS $\equiv \delta\varphi \equiv$ PERTURBATIONS OF THE CLOCK

$$\mathcal{S} \sim \frac{\delta\varphi}{\epsilon} \propto \frac{\delta\varphi}{\dot{\varphi}}$$

$\dot{\varphi} \uparrow$ BACKGROUND CLOCK

THE SLOWER THE
BACKGROUND CLOCK
(\equiv SMALL $\dot{\varphi} \equiv$ SMALL ϵ)

THE GREATER THE EFFECT
OF $\delta\varphi$ ON \mathcal{S}

$$\Rightarrow P_{\mathcal{S}} \propto \frac{1}{\epsilon} \Rightarrow r = \frac{P_{\text{GW}}}{P_{\mathcal{S}}} \propto \epsilon$$

SLIDE 1

(3) HIGHLY GAUSSIAN, $\langle \mathcal{S}^3 \rangle \ll \langle \mathcal{S}^2 \rangle^{3/2}$

SMALL DEVIATIONS FROM GAUSSIANTY, PARAMETRIZED

AS $\mathcal{S} = \mathcal{S}_g + f_{NL} * \mathcal{S}_g^2$

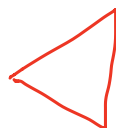
\uparrow
GAUSSIAN

\uparrow CONVOLUTION IN MOMENTUM SPACE
 \neq FUNCTIONAL DEPENDENCES GIVE
 \neq SHAPES

$$|f_{NL, \text{EQUILATERAL}}| \lesssim 10$$



$$|f_{NL, \text{EQUILATERAL}}| \lesssim 100$$



(RECALL
 $\mathcal{S} \sim 10^{-5}$)

FREE FIELDS ARE GAUSSIAN

$$\langle \phi \phi \phi \phi \rangle = \langle \overbrace{\phi \phi} \phi \phi \rangle + \langle \phi \phi \overbrace{\phi \phi} \rangle + \langle \overbrace{\phi \phi \phi \phi} \rangle$$
$$= 3 \langle \phi \phi \rangle \langle \phi \phi \rangle$$

$$\langle \phi \phi \phi \rangle = 0$$

...

\Rightarrow NON-GAUSSIANITY IS A MEASURE OF ϕ INTERACTIONS

- GRAVITATIONAL INTERACTIONS ARE WEAK
- SELF INTERACTIONS PROPORTIONAL TO DERIVATIVES OF THE POTENTIAL

$$V = V_0 + \frac{V''}{2} \delta\psi^2 + \frac{V'''}{6} \delta\psi^3 + \frac{V^{(4)}}{24} \delta\psi^4 + \dots$$

SMALL DUE TO SLOW ROLL

- \therefore LARGE NON-GAUSSIANITY POSSIBLE FROM INTERACTIONS WITH OTHER FIELDS, AS WE WILL SEE LATER

IN THESE LECTURES.