MARCO PELOSO LECTURE 2

INFLATION - ICTP 2022

SOME MORE DETAILS (STILL A SIMPLIFIED COMPUTATION)

TEST MASSLESS FIELD IN dS

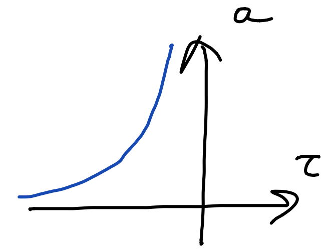
$$ds^2 = dt^2 - a^2(t)\, d\vec{x}^2 = a^2(\tau)\left(d\tau^2 - d\vec{x}^2\right)$$

PHYSICAL TIME

$$a = e^{Ht}$$

CONFORMAL TIME

$$a = -\frac{1}{H^2\tau^2}$$



$$S = \frac{1}{2}\int d^4x\, \sqrt{-g}\, g^{\mu\nu}\partial_\mu\varphi\,\partial_\nu\varphi = \frac{1}{2}\int d^4x \left[a^2(\partial_\tau\varphi)^2 - a^2(\partial_{\vec{x}}\varphi)^2\right]$$

$a\varphi = \chi$ CANONICALLY NORMALIZED

$$\chi = \int \frac{d^3k}{(2\pi)^3}\, e^{i\vec{k}\cdot\vec{x}}\, \chi_k$$

FOURIER TRANSFORM (MODES χ_k IN MOMENTUM SPACE)

$$\chi_k'' + \left(k^2 - \frac{a''}{a}\right)\chi_k = 0$$

$2a^2H^2$

NOTICE THE TRANSITION FROM STANDARD HARMONIC OSCILLATOR DEEPLY INSIDE THE HORIZON ($\gg aH$) TO SOLUTION CONTROLLED BY GEOMETRY EXPANSION

TWO SOLUTIONS; WE TAKE THE ONE REDUCING TO $\frac{e^{-ik\tau}}{\sqrt{2k}}$ DEEP INSIDE HORIZON ("STANDARD" VACUUM MODE)

$$\Rightarrow \quad \chi(t) = \frac{1-\frac{i}{k\tau}}{\sqrt{2k}} e^{-ik\tau}$$

- STANDARD MODE INSIDE HORIZON $(-k\tau \gg 1)$
- WELL OUTSIDE $\quad \chi \simeq -\frac{i}{\sqrt{2}\,k^{3/2}\,\tau} \Rightarrow \varphi = \frac{\chi}{a} \simeq \frac{iH}{\sqrt{2}\,k^{3/2}}$

"FROZEN"

$$\int d^3x \langle \varphi^2(x) \rangle = \underset{\text{GO TO MOMENTUM SPACE}}{\ldots\ldots} = \int \frac{dk}{k} \underbrace{\frac{k^3}{2\pi^2} |\varphi_k|^2}_{\text{POWER SPECTRUM}}$$

$$P_\varphi = \frac{k^3}{2\pi^2} \left| \frac{iH}{\sqrt{2}\,k^{3/2}} \right|^2 = \left(\frac{H}{2\pi} \right)^2$$

INFLATION PHENOMENOLOGY

- H SLIGHTLY DECREASING DURING INFLATION, SO H IN ABOVE RELATION SHOULD BE EVALUATED WHEN MODE CROSSES HORIZON
- IN REALITY, MORE COMPLICATED THAN THIS, SINCE $\delta g_{\mu\nu}$ MUST BE INCLUDED $\quad \delta g_{\mu\nu} \longleftrightarrow \delta\varphi$ FROM EINSTEIN EQUATIONS)

- ONE INDEED FINDS

SPECTRAL TILT $\qquad n_s \simeq 1 - 6\varepsilon + 2\eta$

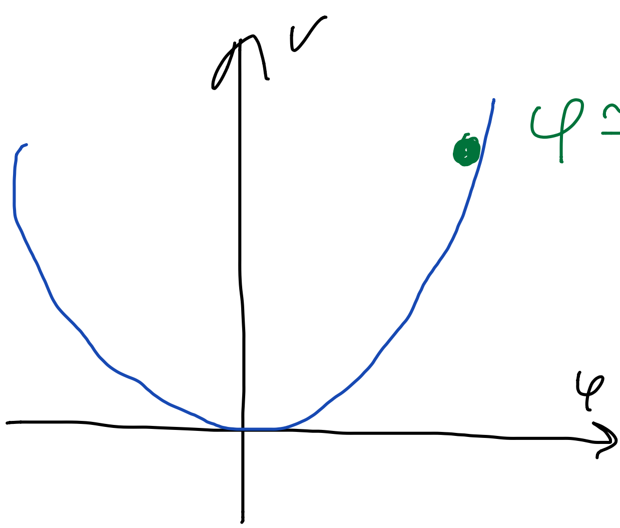
TENSOR TO SCALAR RATIO $\qquad r \simeq 16\varepsilon$

- THE SLOW ROLL PARAMETERS $\varepsilon \equiv \frac{M_p^2}{2}\frac{V'^2}{V^2}$, $\eta \equiv M_p^2 \frac{V''}{V}$ VARY DURING INFLATION, AND NEED TO BE EVALUATED AT HORIZON CROSSING

- WE USE # OF E-FOLDS $N \equiv \ln \frac{a_{END}}{a}$ AS A MEASURE OF TIME. NOTICE N DECREASES DURING INFLATION, AND $N=0$ AT THE END.

- EXAMPLE, FOR $V = \frac{m^2}{2}\varphi^2$, WE HAVE

HOMEWORK



$$\varphi \simeq \sqrt{4N}\, M_P$$

AND

$$n_s \simeq 1 - \frac{2}{N}, \quad r \simeq \frac{8}{N}$$

WHICH VALUE OF N SHOULD WE USE TO COMPARE WITH CM RESULTS?

REHEATING AFTER INFLATION

DURATION OF INFLATION MEASURED IN e-FOLDS $a = a_{END}\, e^{-N}$. AS WE WILL SEE, N IS MODEL-DEPENDENT. IN MANY MODELS $N \sim 60$.

$\Rightarrow$ DILUTION DURING INFLATION $\frac{1}{\text{VOLUME}} = \frac{1}{a^3} \sim e^{-180}$

$\Rightarrow$ ALL MATTER & RADIATION IN THE PRESENT UNIVERSE PRODUCED DURING REHEATING

REHEATING $\equiv$ ALL PROCESSES FROM THE END OF INFLATION TO THE ESTABLISHMENT OF A DOMINANT THERMAL BATH

WE REQUIRE

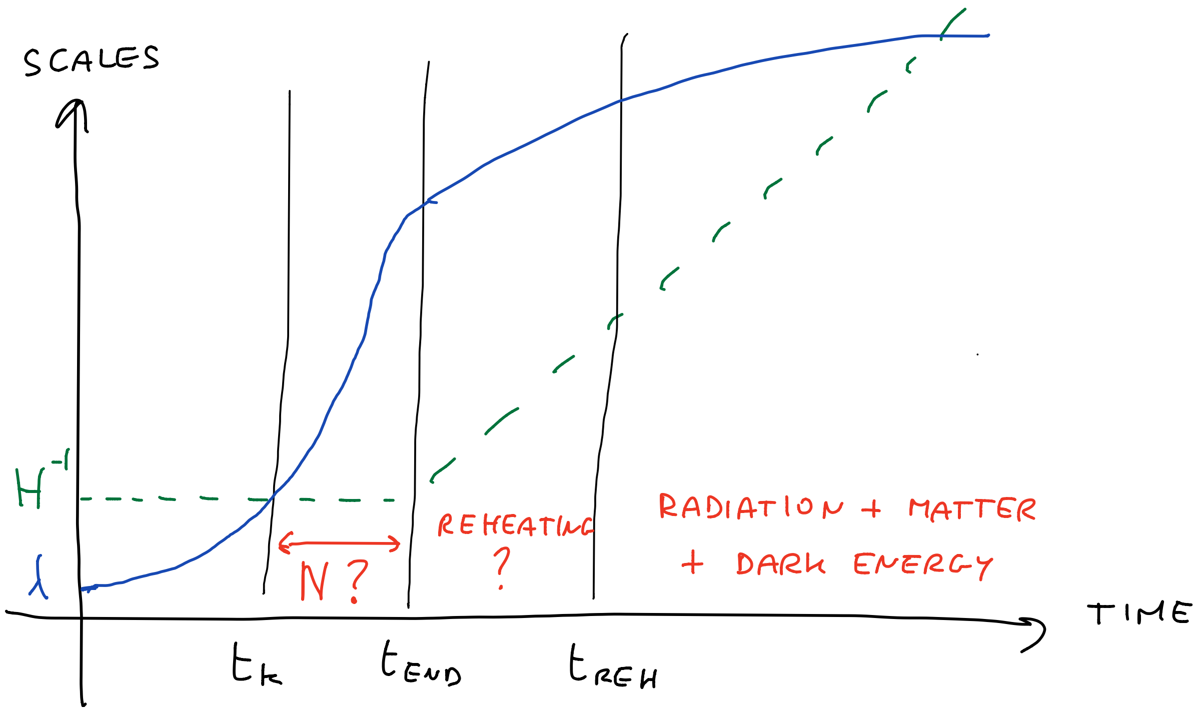
- FORMATION OF A THERMAL BATH, WITH $T \gtrsim 2\,\text{MeV}$ (REQUIRED FOR BBN)

- PRODUCTION OF DARK MATTER AND BARYON ASYMMETRY

MOST UNKNOWN COSMOLOGICAL PERIOD !

UNCERTAINTY AT REHEATING → UNCERTAINTY IN INFLATIONARY PHENOMENOLOGY

PLANCK PIVOT SCALE $k = 0.05\ \mathrm{Mpc}^{-1}$. AT WHICH E-FOLD N WAS IT PRODUCED ?



UNKNOWN INFLATIONARY PARAMETERS

- $\rho_k \simeq V_k$ (INFLATON POTENTIAL) ENERGY DENSITY AT PRODUCTION (≡ HORIZON EXIT)
- ρ_{END} ENERGY DENSITY END INFLATION

UNKNOWN REHEATING PARAMETERS

- ρ_{REH} ENERGY DENSITY WHEN REHEATING COMPLETED
- ω EQUATION OF STATE DURING REHEATING

* HORIZON CROSSING DURING INFLATION $\frac{\lambda}{2\pi} = \frac{a}{k} \sim H^{-1}$

$\rightarrow a_k H_k = k$

* CORRESPONDING E-FOLDS $a_k = e^{-N} a_{END}$

ONE CAN SHOW THAT

HOMEWORK

$$N \simeq 55.6 + \underbrace{2 \ln \frac{V_k^{1/4}}{10^{16}\,\mathrm{GeV}} + \ln \frac{10^{16}\,\mathrm{GeV}}{\rho_{END}^{1/4}}}_{\text{DEPENDS ON MODEL OF INFLATION}} + \underbrace{\frac{1-3w}{12(1+w)} \ln \frac{\rho_{REH}}{\rho_{END}}}_{\text{DEPENDS ON REHEATING}}$$

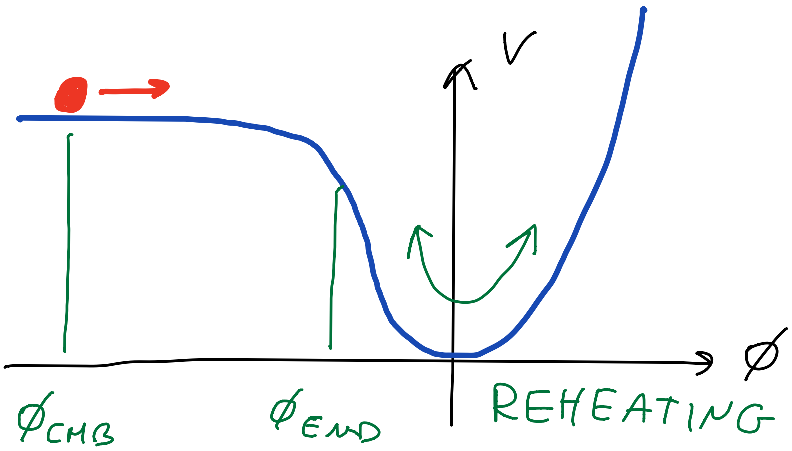
THE LAST TERM ENCODES THE UNCERTAINTY ON N PURELY DUE TO OUR IGNORANCE OF REHEATING

HOW LARGE CAN THIS TERM BE ?

WE EVALUATE THIS TERM FOR

(i) ISTANTANEOUS REHEATING AFTER INFLATION $\rightarrow \Delta N = 0$

(ii) SLOWEST POSSIBLE DECAY $T_{REH} \sim \mathrm{MeV}$



FOR A SLOW DECAY, ϕ PERFORMS MANY OSCILLATIONS ABOUT THE MINIMUM OF THE POTENTIAL BEFORE DECAYING. WHAT IS ω?

TAYLOR EXPAND $V(\varphi)$ ABOUT MINIMUM. SHIFT φ SO THAT MINIMUM AT $\varphi=0$; $V=0$ AT MINIMUM

$\Rightarrow$ $V = \frac{1}{2} m^2 \varphi^2 +$ HIGHER ORDER   ASSUME $\boxed{V = \frac{1}{2} m^2 \varphi^2}$

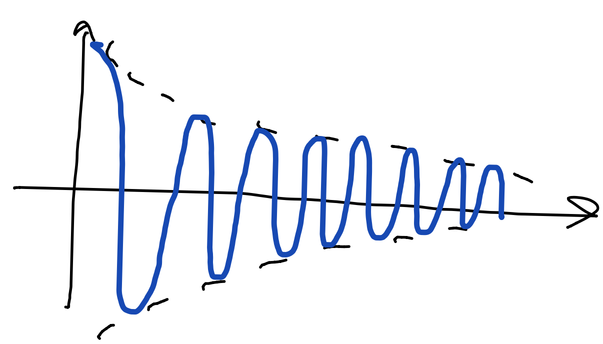
THE EQUATIONS ARE

$$\boxed{\begin{cases} \ddot{\varphi} + 3H\dot{\varphi} + m^2 \varphi = 0 \\ H^2 = \dfrac{1}{3M_p^2}\left(\frac{1}{2}\dot{\varphi}^2 + \frac{1}{2} m^2 \varphi^2\right) \end{cases}}$$

$\leftarrow$ OSCILLATOR WITH FRICTION $(a = -\gamma v)$

$\leftarrow$ FRICTION PROPORTIONAL TO ENERGY DENSITY OF THE OSCILLATOR

ANSATZ : OSCILLATIONS WITH ADIABATICALLY VARYING AMPLITUDE



TO FIND THE EVOLUTION, WE STUDY THE RELATION BETWEEN THE AVERAGE KINETIC ENERGY AND THE AVERAGE POTENTIAL ENERGY, NEGLECTING THE VARIATION OF THE AMPLITUDE DURING ONE OSCILLATION

VIRIAL THEOREM: $\langle E_k \rangle = \langle V \rangle$

$$\Rightarrow \langle \text{PRESSURE} \rangle = \left\langle \frac{1}{2}\dot{\varphi}^2 - V \right\rangle = 0$$

COHERENT INFLATON OSCILLATIONS $\equiv$ MATTER $\omega = 0$

$$\Rightarrow \phi = \frac{\Phi_0}{mt}\sin(mt) + O\left(\frac{1}{t^2}\right) \Rightarrow \rho_\phi \propto \left(\frac{\Phi_0}{mt}\right)^2 \propto \frac{1}{a^3} \quad \checkmark$$

RECALL $a \sim t^{2/3}$ IN MATTER DOMINATION

BACK TO $\Delta N = \frac{1-3\omega}{12(1+\omega)} \ln \frac{\rho_{REH}}{\rho_{END}}$

$$\rho_{END}^{1/4} \simeq \rho_h^{1/4} \simeq 3 \cdot 10^{16}\,\text{GeV}^{1/4}\, r^{1/4}$$

SLOW ROLL SEEN EARLIER

TAKING $\rho_{END}^{1/4} \simeq 10^{16}$ GeV & $\rho_{REH} \sim T_{REH}^4 \sim \text{MeV}^4$

& $w=0$ $\rightarrow$ $\Delta N = -15$

AS COMPARED TO $\Delta N = 0$ FOR ISTANTANEOUS REHEATING.

VERY LARGE UNCERTAINTY, THAT IMPACTS OUR PREDICTIONS FOR ANY GIVEN INFLATIONARY POTENTIAL

SLIDE 2

PERTURBATIVE REHEATING

- START BY ASSUMING INSTANTANEOUS INFLATON DECAY & INSTANTANEOUS THERMALIZATION

WE WILL DISCUSS TWO APPLICATIONS

↗ REHEATING TEMPERATURE?

↘ GRAVITINO PRODUCTION?

• REHEATING TEMPERATURE Γ_ϕ INFLATON DECAY RATE

IN THE TIME $t \sim H^{-1}$, THE NUMBER OF PROCESSES $\int \Gamma dt \sim \frac{\Gamma}{H}$ OCCURS $\equiv$ DECAY WHEN $H(t) = \Gamma_\phi$

ASSUMING INSTANTANEOUS THERMALIZATION OF DECAY PRODUCTS

$$H^2 = \frac{1}{3M_p^2} \rho_{THERMAL} = \frac{1}{3M_p^2} \frac{\pi^2}{30} g_* T_{RH}^4 \simeq \frac{T_{RH}^4}{M_p^2} \equiv \Gamma_\phi^2$$

RELATIVISTIC DEGREES OF FREEDOM

$$T_{RH} \simeq \sqrt{\Gamma_\phi M_p}$$