Detection of Pairwise Hotspots on the CMB through Deep Neural Network

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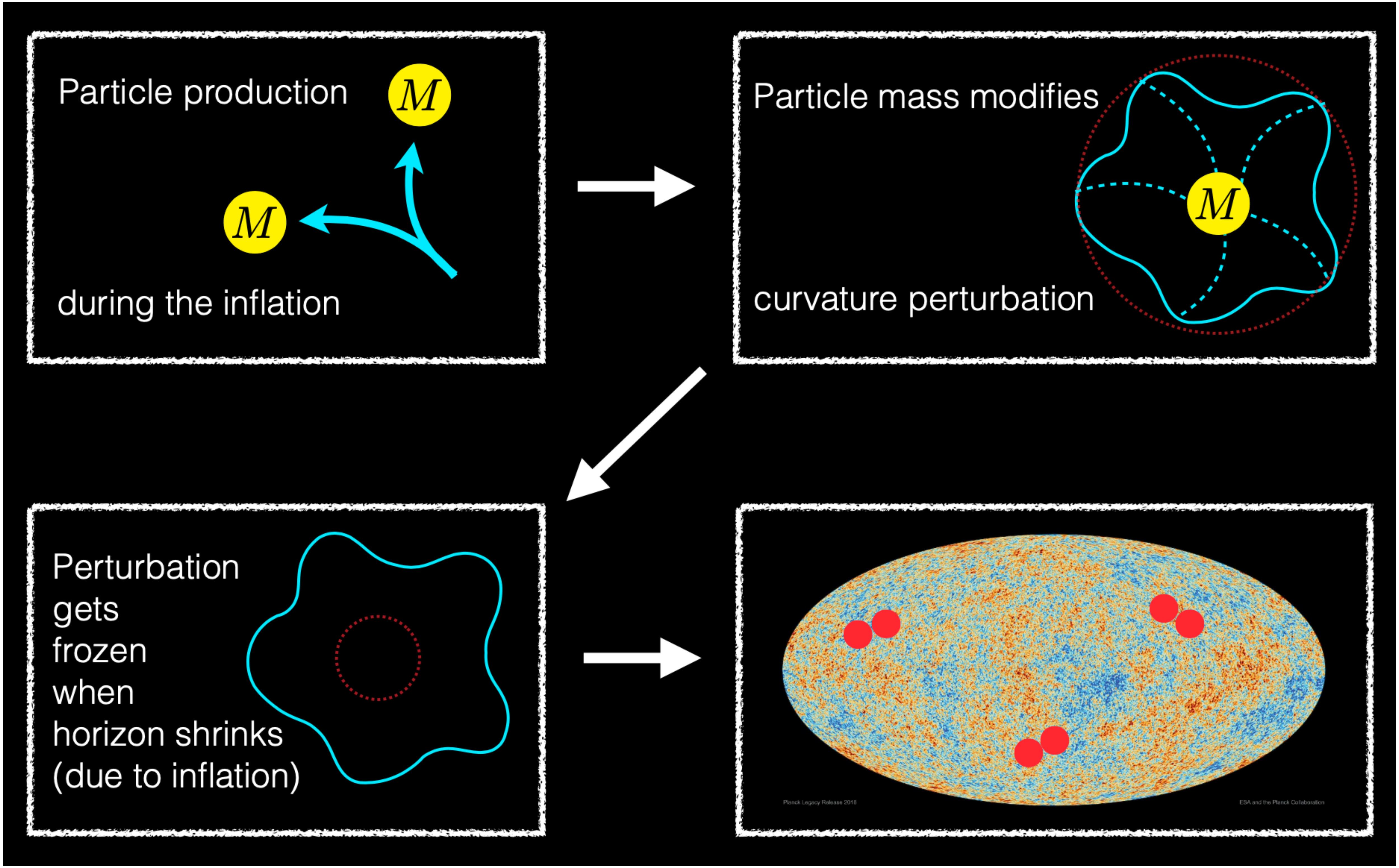
arXiv:2107.09061 and in prep

7/5/2022 ICTP

Introduction

- Cosmic inflation: seeds fluctuations in CMB and LSS
  - Inflation being (almost) scale invariant
  - Possible signals of non-Gaussianity ($\geq$ 3-pt functions)
- Probe heavy particle production during inflation ($m \gg H_{\text{inf}}$)
  - Localized signals appear in position space
  - Signature detection can be done in both position space and power spectrum
- Machine learning can be used as a tool to capture the signal on CMB

Schematics


Particle production
M
M
during the inflation
Particle mass modifies
M
curvature perturbation
Perturbation
gets
frozen
when
horizon shrinks
(due to inflation)

Toy Model

Direct coupling to Inflaton

$$V(\phi, \sigma) = V_{\text{inf}}(\phi) + \frac{1}{2}\underbrace{\boxed{\left(M_0^2 + (g\phi - M)^2\right)}}_{M^2_{\text{eff}}}\sigma^2 \quad \text{with } M \gg M_0$$

- Ultra-heavy particle (σ) is directly coupled to inflaton field (ϕ)
- Effective mass, M_{eff}, is minimized as $g\phi \approx M$ during slow roll
- Kinetic energy of inflaton can be used to produce particles
    - How to produce σ ?

Particle Production

EOM of σ

$$\sigma'' - \frac{2}{\eta}\sigma' + \left(k^2 + \frac{M_{eff}^2(\eta)}{H_*^2\eta^2}\right)\sigma = 0$$

$u = \sigma/\eta$

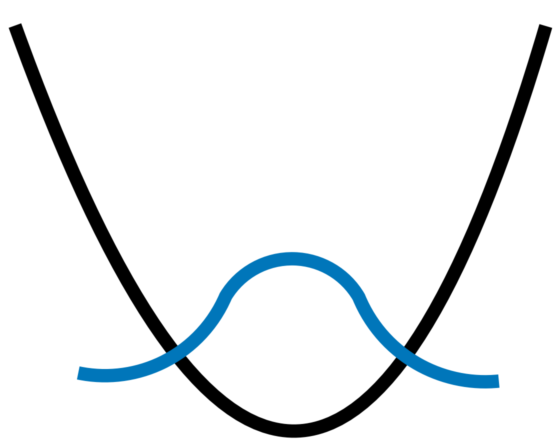
$$u'' + \left(k^2 + \frac{M_{eff}^2(\eta)/H_*^2 - 2}{\eta^2}\right)u \equiv u'' + \omega^2(\eta)\,u = 0$$

- Simple harmonic oscillator with time dependent frequency
- Compute violation of adiabaticity (ω'/ω^2)
    - Maximum at $\eta \approx \eta_*$ $(M_{eff} \approx M_0)$

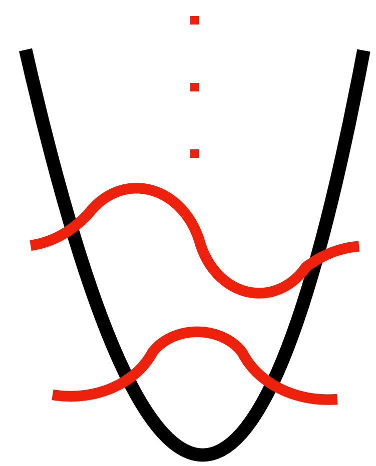
Particle Production

Bogoliubov Transformation

$\eta = -\infty$



$\eta = \eta_*$



- Changes of potential $\rightarrow$ Old ground state $\neq$ New ground state
- Detailed particle production can be expressed using **Bogoliubov Transformation**

$$
\begin{aligned}
\hat{u}(\eta, \mathbf{x}) &= \int \frac{d^3\mathbf{k}}{(2\pi)^{3/2}} \left[\hat{a}_{\mathbf{k}}\, \mathcal{I}_k(\eta)\, e^{i\mathbf{k}\cdot\mathbf{x}} + \hat{a}^\dagger_{\mathbf{k}}\, \mathcal{I}^*_k(\eta)\, e^{-i\mathbf{k}\cdot\mathbf{x}} \right] \\
&= \int \frac{d^3\mathbf{k}}{(2\pi)^{3/2}} \left[\hat{b}_{\mathbf{k}}\, \mathcal{F}_k(\eta)\, e^{i\mathbf{k}\cdot\mathbf{x}} + \hat{b}^\dagger_{\mathbf{k}}\, \mathcal{F}^*_k(\eta)\, e^{-i\mathbf{k}\cdot\mathbf{x}} \right]
\end{aligned}
$$

$$
\hat{b}_k = \alpha_k\, \hat{a}_k + \beta^*_k\, \hat{a}^\dagger_k, \qquad \hat{b}^\dagger_k = \beta_k\, \hat{a}_k + \alpha^*_k\, \hat{a}^\dagger_k
$$

Primordial Curvature Perturbation

Profile of produced heavy particles

Maldacena (1508.01082)

Fialkov et. al. (0911.2100)

- Produced heavy particles backreacts on spacetime

$$S_\sigma = \int dt \sqrt{-g_{00}}\, M_{eff} \supset \int d\eta\, \partial_\eta \zeta \frac{M_{eff}}{H}$$

- Give rise to non-trivial one-point function

$$\langle \zeta_k \rangle = -i \int_{\eta_*}^{0} d\eta\, \langle 0 \,|\, \zeta_k(\eta_0)\, \partial_\eta \zeta_k(\eta) \,|\, 0 \rangle \frac{M_{eff}}{H} + c.c.$$

- Resulting curvature profile from the spot center

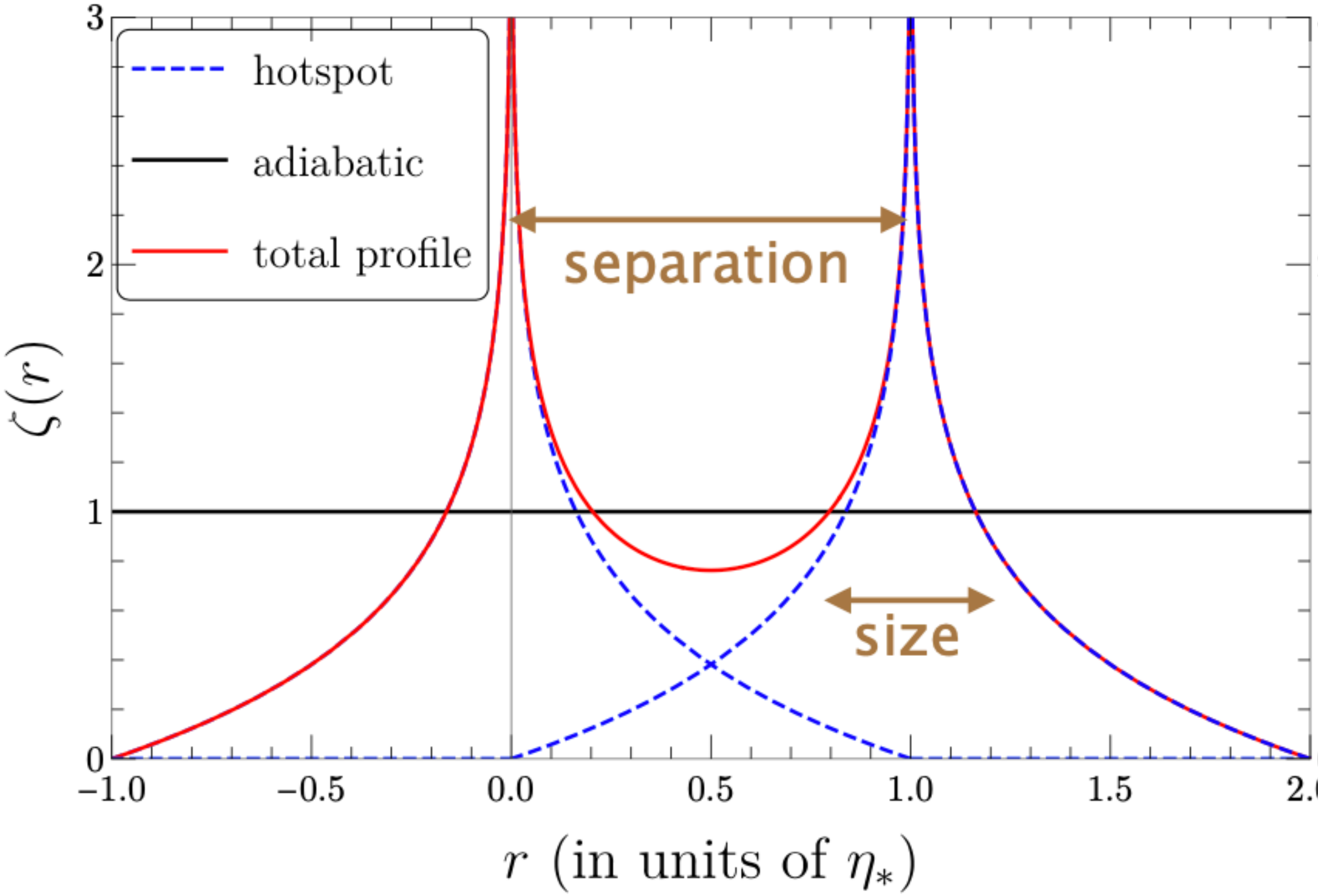
$$\langle \zeta \rangle = \boxed{\left[\frac{g}{2} \log \left(\frac{|\eta_*|}{r} \right) \right]} \boxed{\frac{H}{2\pi \sqrt{2\epsilon} M_{pl}}}$$

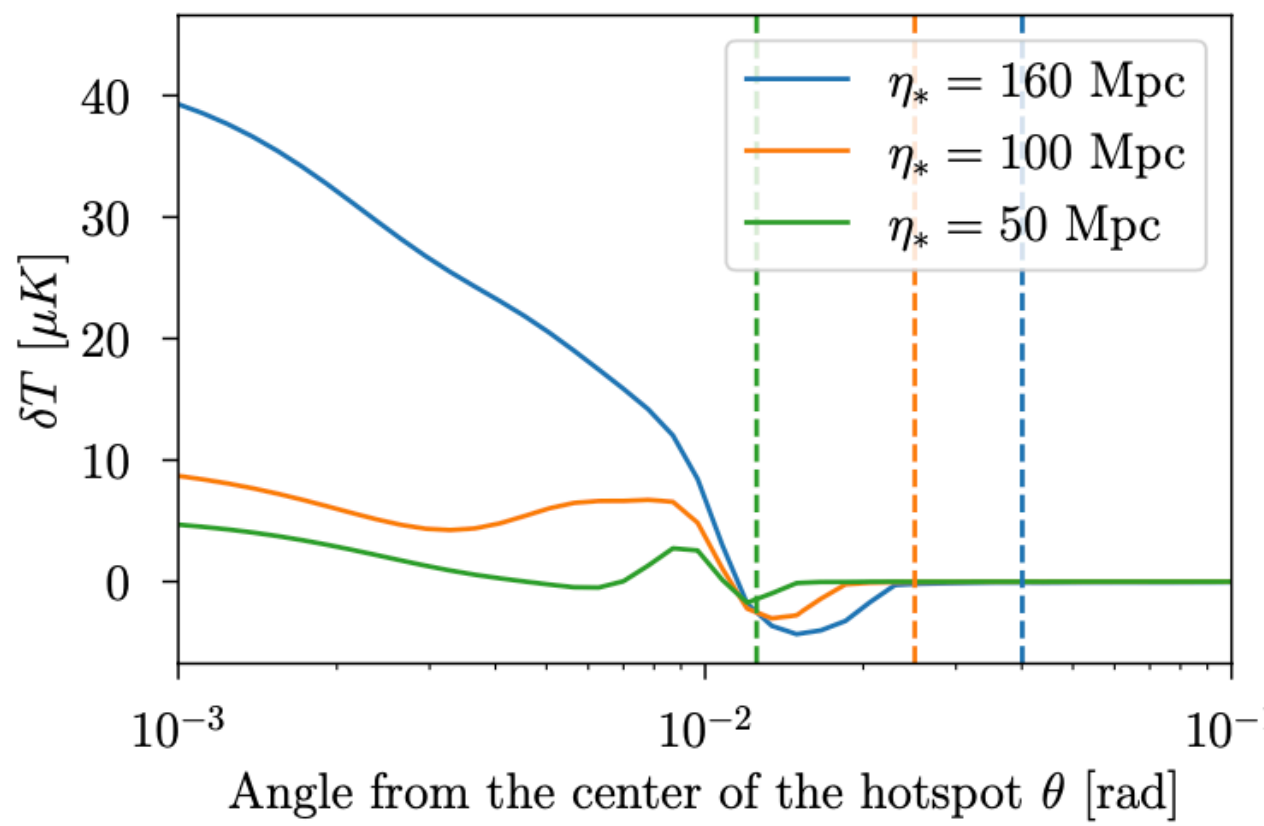
controls
visibility over the primordial fluctuation

Size of typical inflationary fluctuation
$\sim 10^{-5}$

Primordial Curvature Perturbation

Profile of produced heavy particles





- Heavy particles come in pair
- Initial exponential profile gets modified from SW and ISW effects

Primordial Curvature Perturbation

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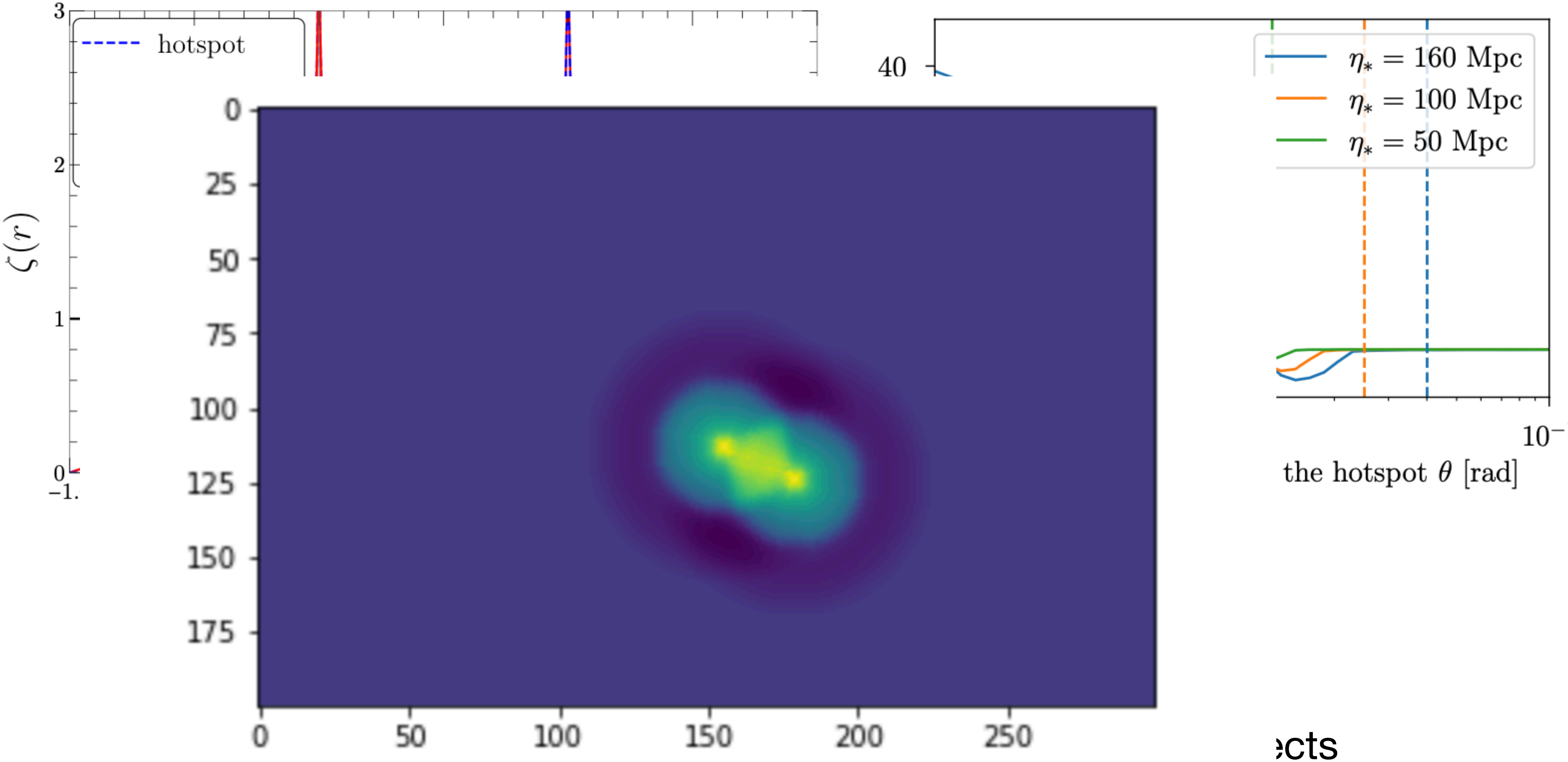


ects

Position Space search

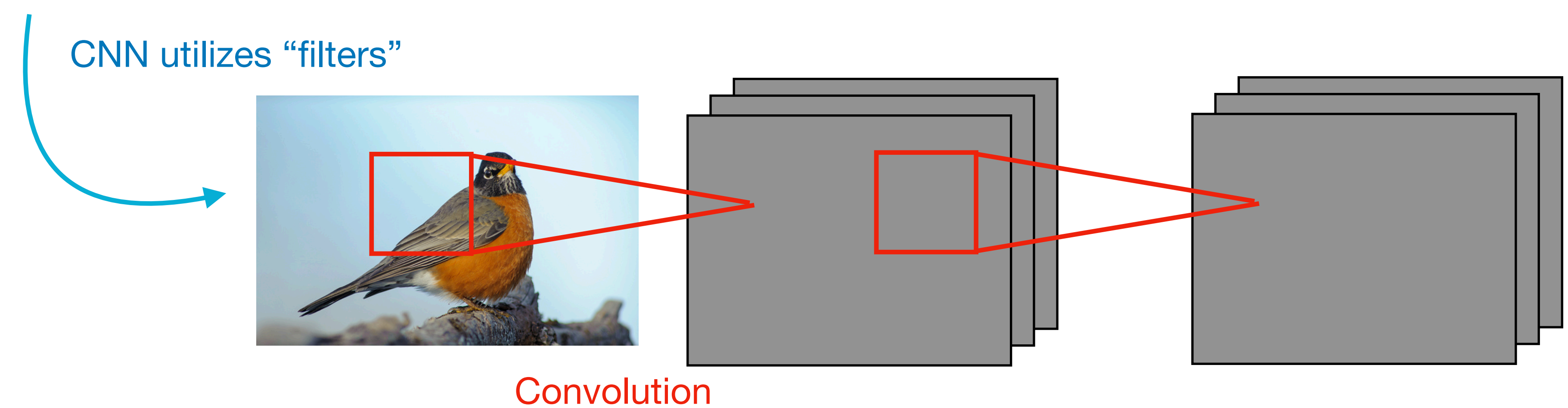
Momentum Space search (P(k))

Detection Method

Convolutional Neural Network (CNN)

Image recognition

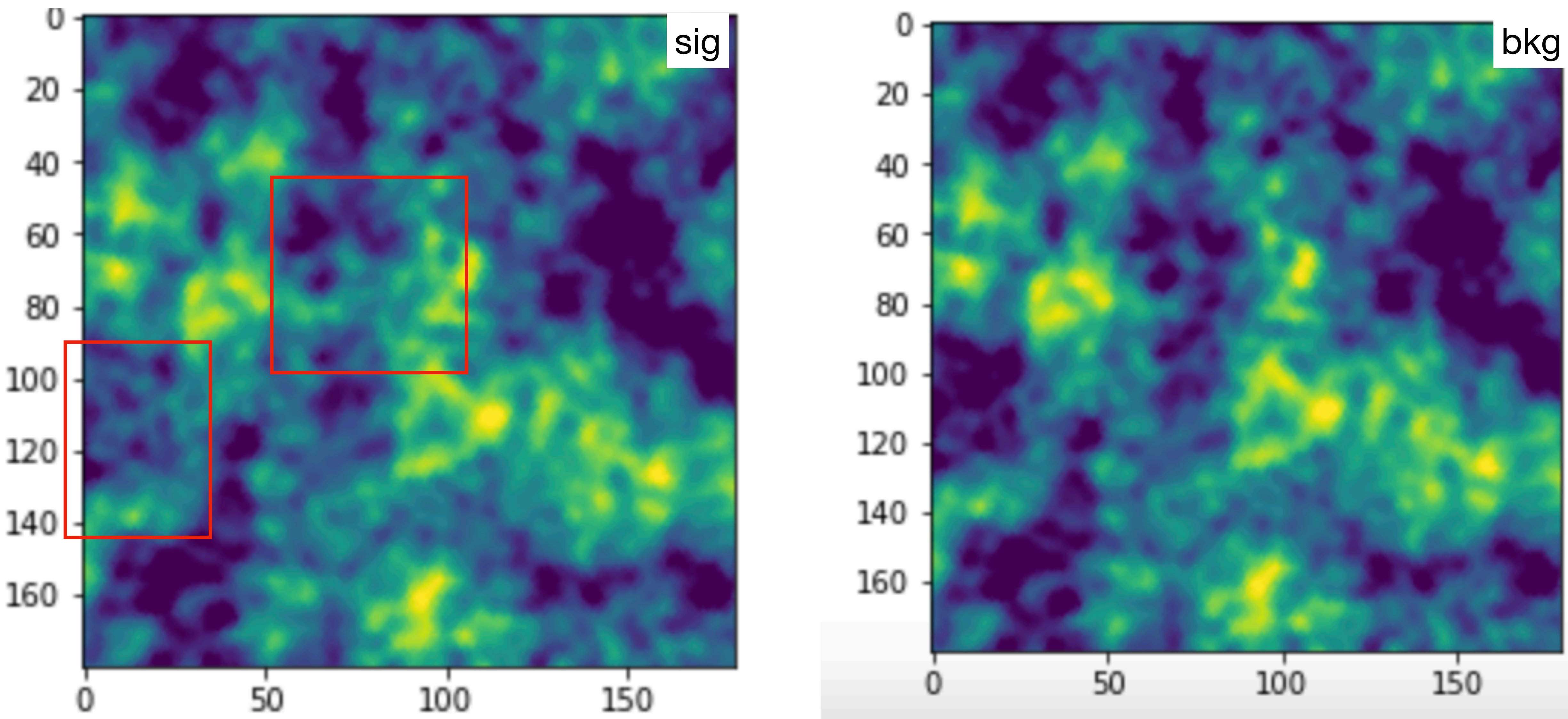
- Human recognition based on features
- Need to adjust to be programmable
- Use $n \times n$ matrix “filter”



- Classification problem
- Given the small patch of CMB background with/without Pairwise Hot spot
- The network is trained with simulated CMB maps using Healpix and CLASS

Sample Data

Benchmark $\eta_* = 160$ Mpc, g = 4

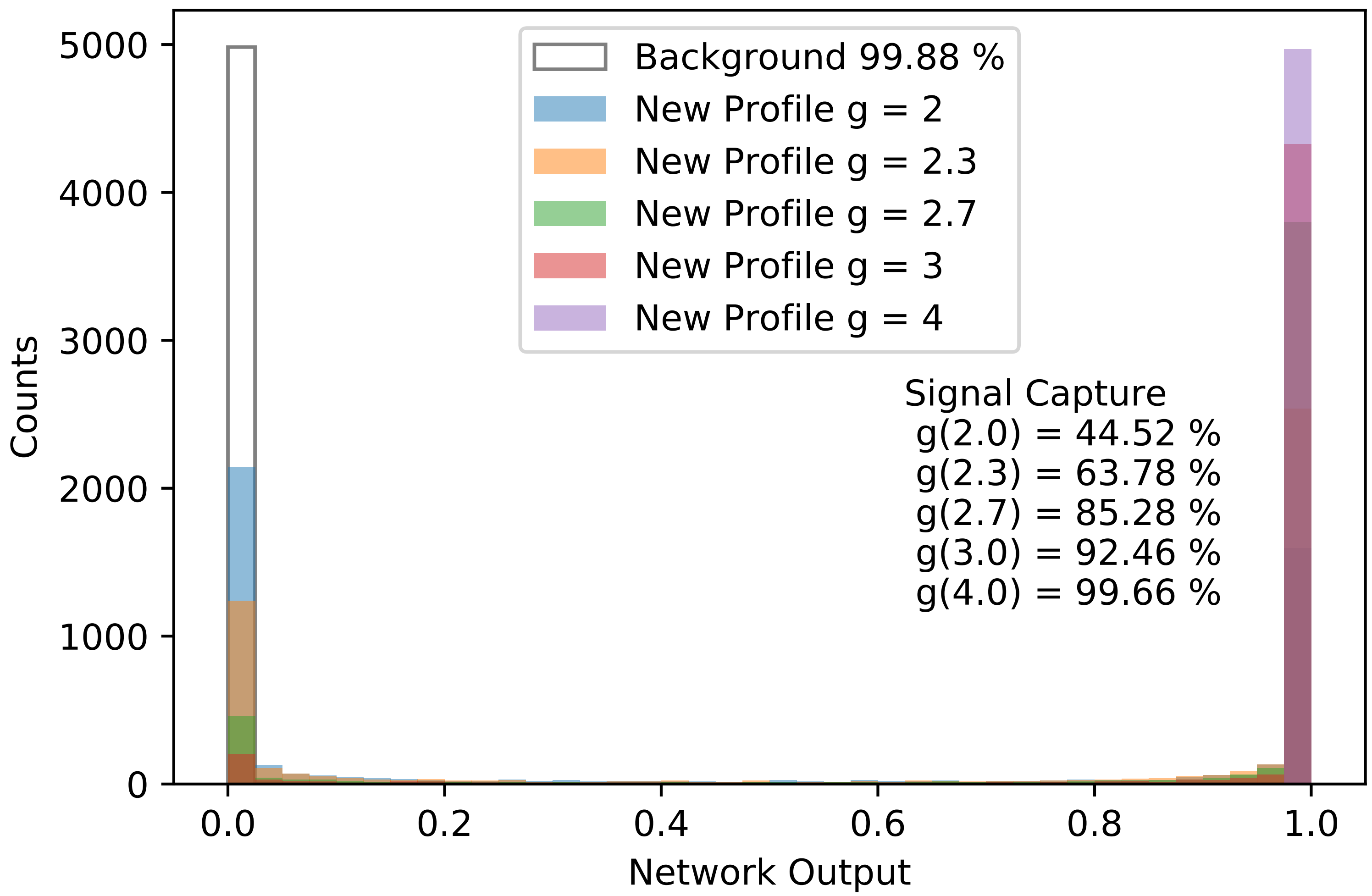


- the T enhancement depends linearly on $\phi - \sigma$ coupling, g

- Signal start to hide well enough

Training Result

Benchmark $\eta_* = 160$ Mpc



- Network training was done one time at large coupling
- Use if trained network is useful for lower g values

Conclusion

- Production of heavy particles with inflaton-dependent mass generate pairwise spots on the CMB map
- Can use “position space” studies to dig out the signal
- More to explore:
    - pairwise imprints in Large Scale Structure?
    - CMB lensing, cosmic shear, etc?

Thank you

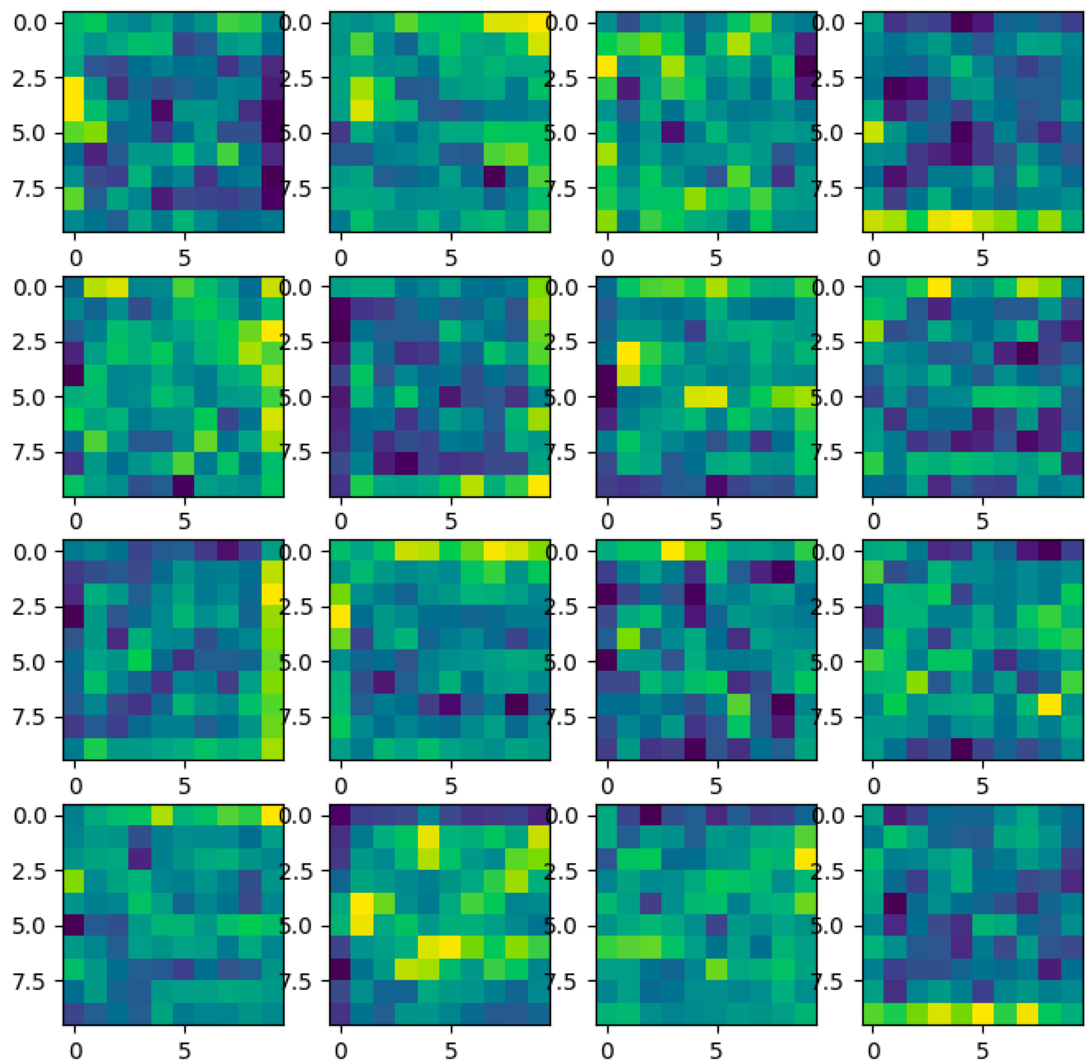
Backup slides

Number density estimate

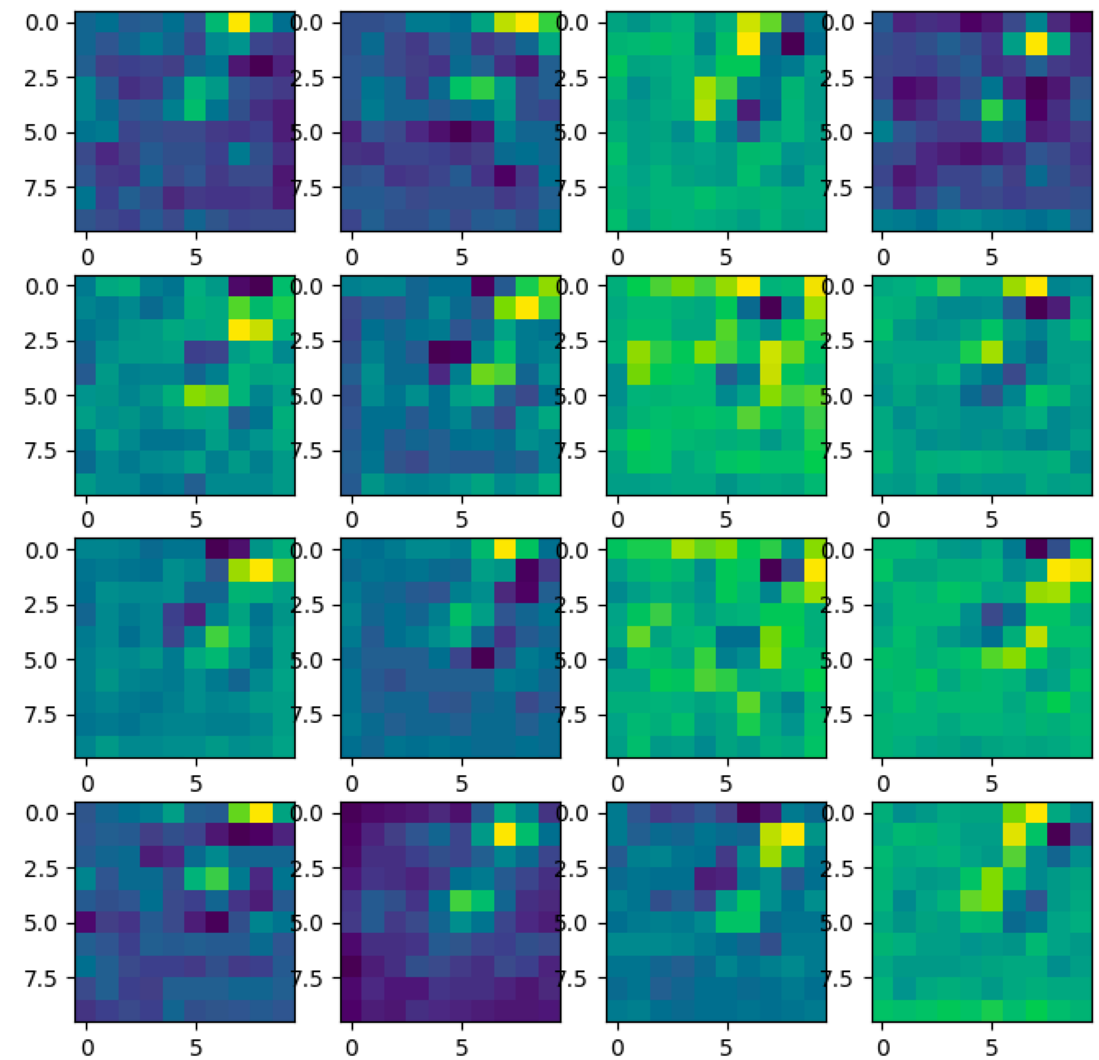
$$
\begin{aligned}
{}_{\rm univ}\langle 0|\hat{N}_k|0\rangle_{\rm univ} &= {}_{\rm univ}\langle 0|\hat{b}_k^\dagger \hat{b}_k|0\rangle_{\rm univ} \\
&= {}_{\rm univ}\langle 0|(\beta_k\,\hat{a}_k + \alpha_k^*\,\hat{a}_k^\dagger)(\alpha_k\,\hat{a}_k + \beta_k^*\,\hat{a}_k^\dagger)|0\rangle_{\rm univ} \\
&= |\beta_k|^2\delta(0)\,.
\end{aligned}
$$

$$
n \equiv \int d^3\mathbf{k}\, n_k = \int d^3\mathbf{k}\,|\beta_k|^2
$$

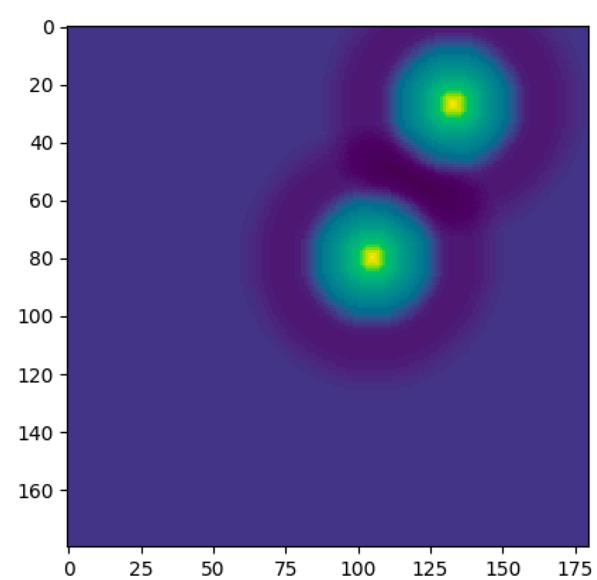
4th Convolutions (final conv)



Background

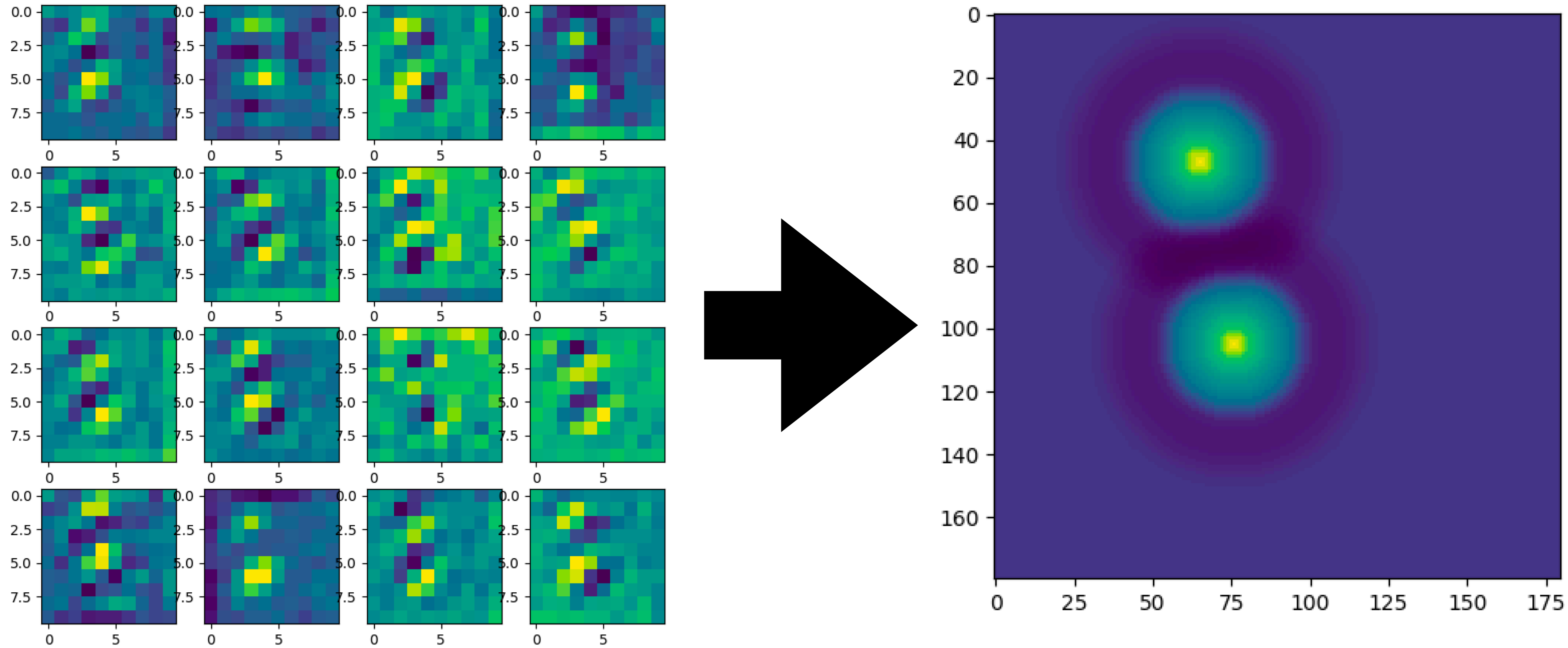


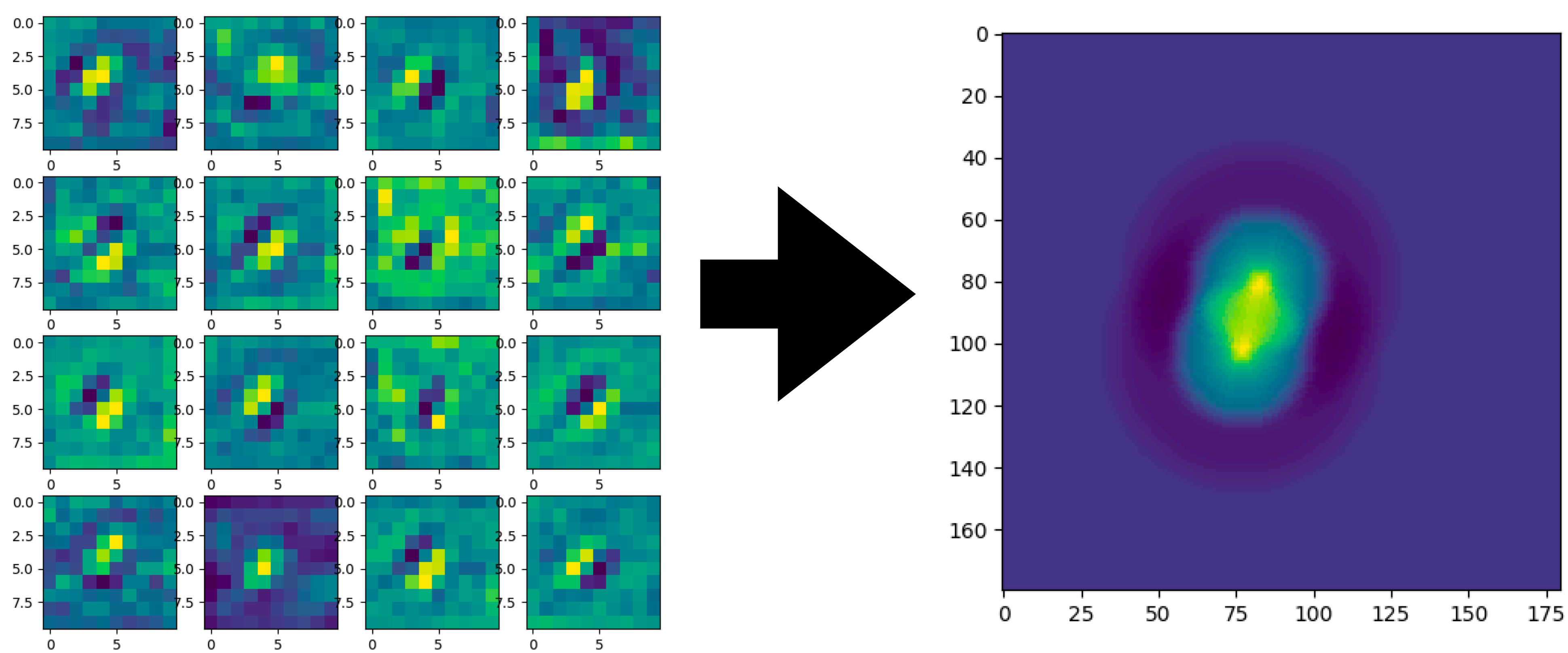
Signal



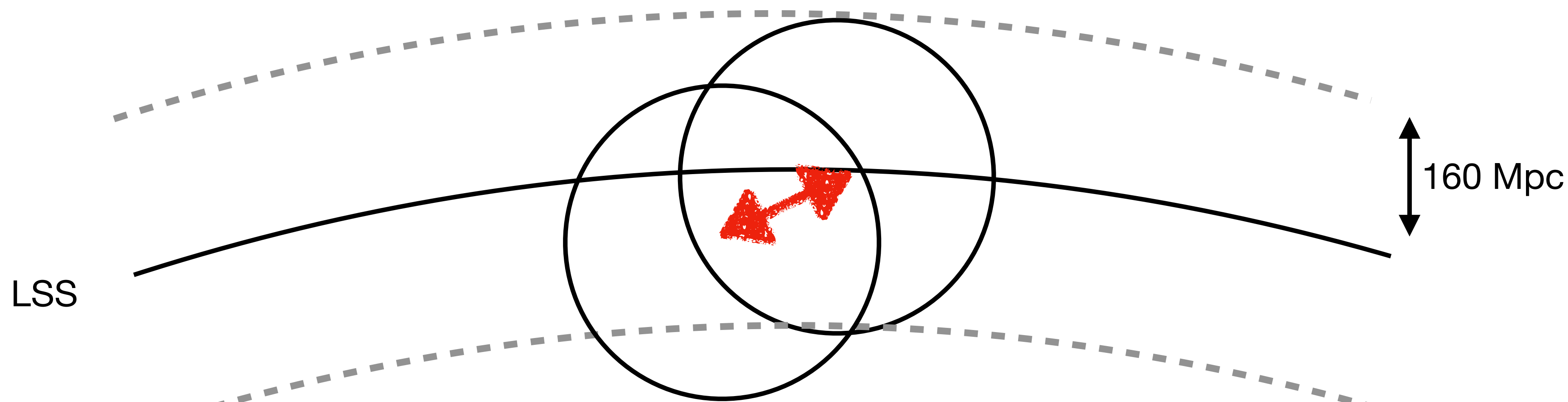
True Location

Other hot spot examples



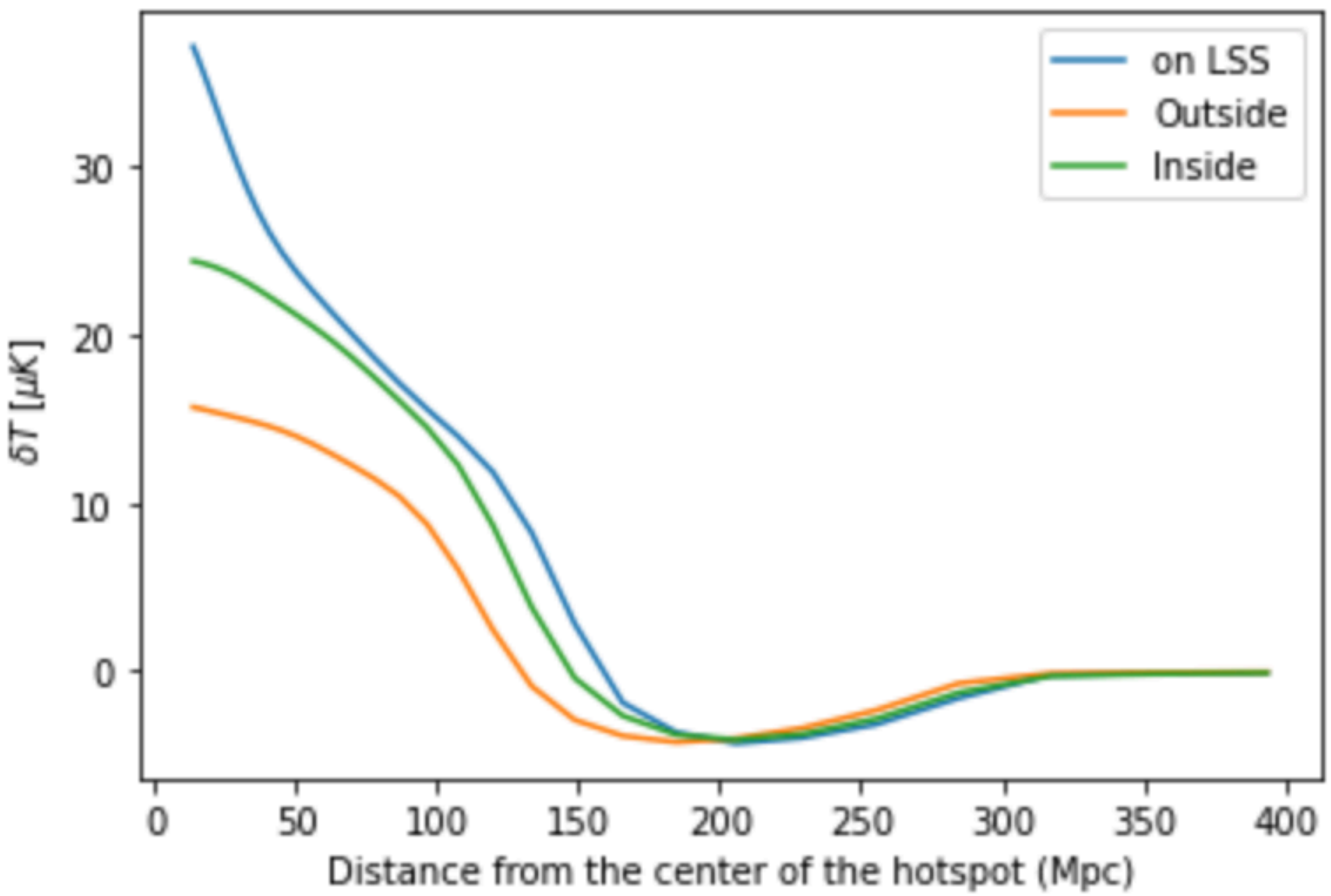


New PHS profile (Need to discuss)



- Projected on Last Scattering Surface
- $\theta_* = \sqrt{4\pi}\,\eta_*/\eta_0$: Maximum angular span of a profile
- $\eta_* = 160$ Mpc : Maximum separation
  - $\theta_{separation} = \eta_*/\eta_0$: Maximum angular separation $\theta_{separation} < \theta_*$
  - Expected to have PHS with large overlap

Radial Profile



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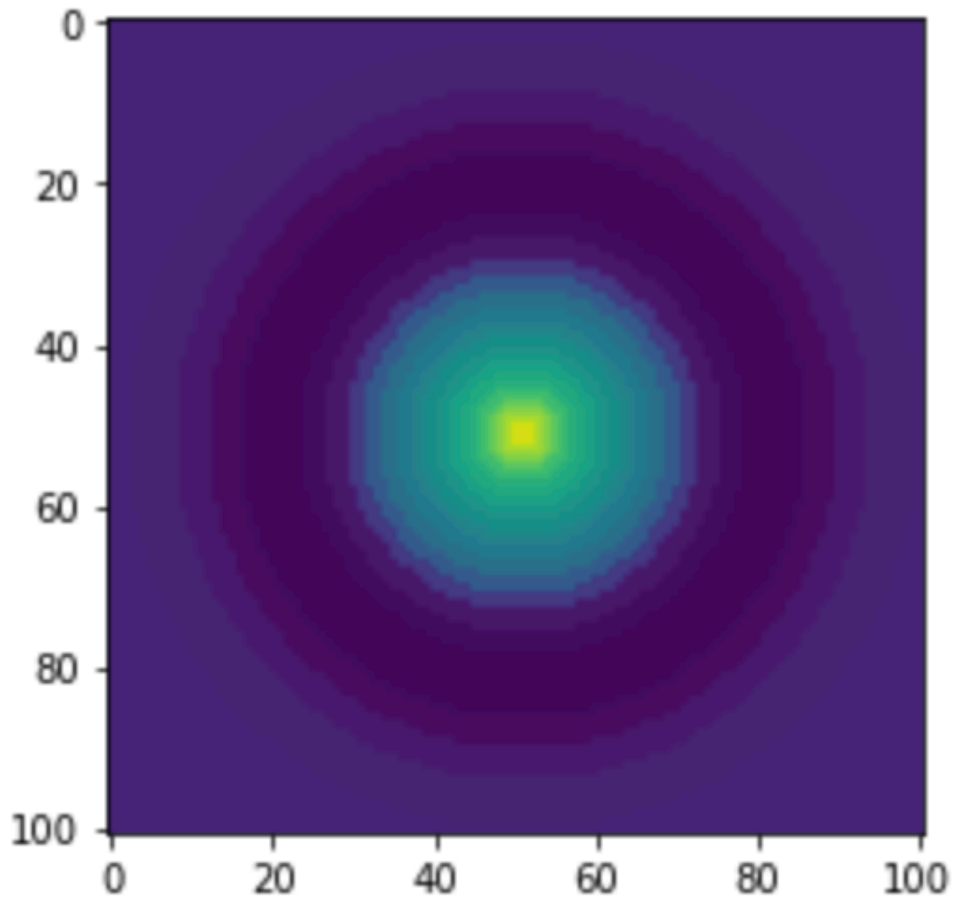

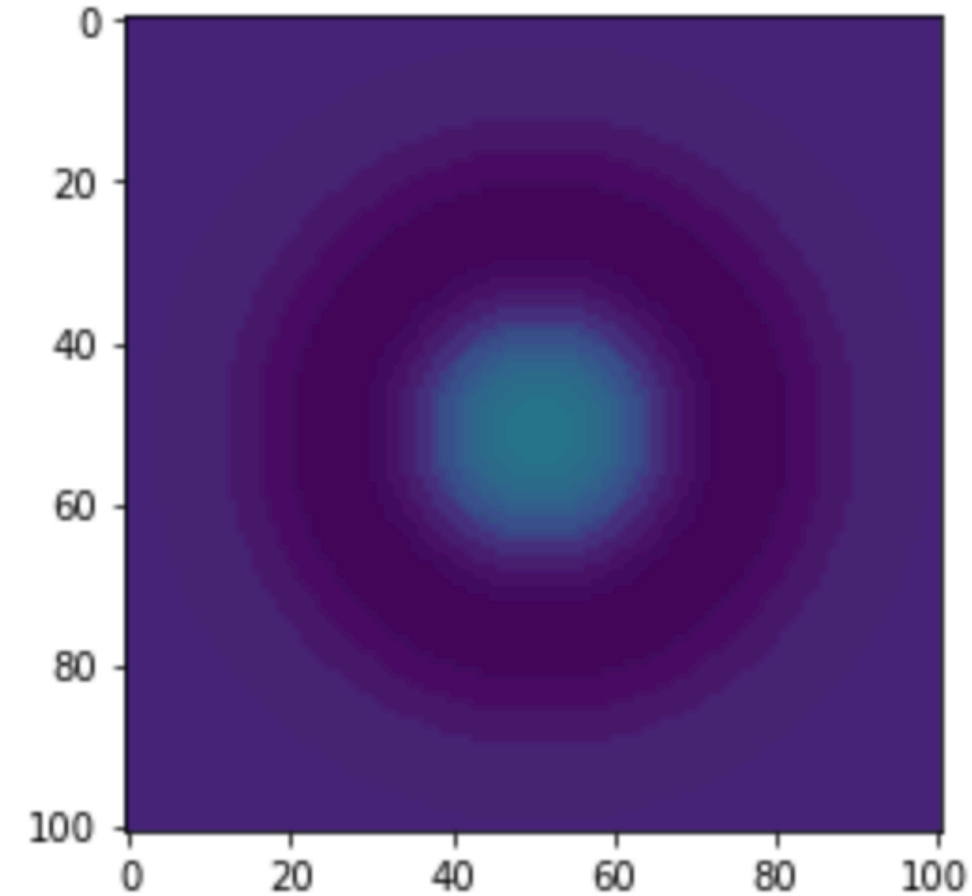

- Depending on distance from LSS, profile changes
  - Smaller radius + no apparent central peak

Evolution and Signatures of Primordial Magnetic Fields

Salome Mtchedlidze

Goettingen University & Ilia State University
(Germany & Georgia)

Collaborators:

P. Domínguez-Fernández (U. Bologna)
X. Du (Carnegie Observatories)
A. Brandenburg (NORDITA)
S. O'Sullivan (Dublin U.)
W. Schmidt (U. Hamburg)
M. Brüggen (U. Hamburg)

Supervisors:

Tina Kahniashvili (CMU, Ilia State U.)
Jens Niemeyer (Goettingen U.)

Trieste Summer School

05/07/2022

Image credit: quantamagazine.org

Primordial Magnetic Fields (PMFs)?

What are PMFs?

What we can learn from studying PMFs?

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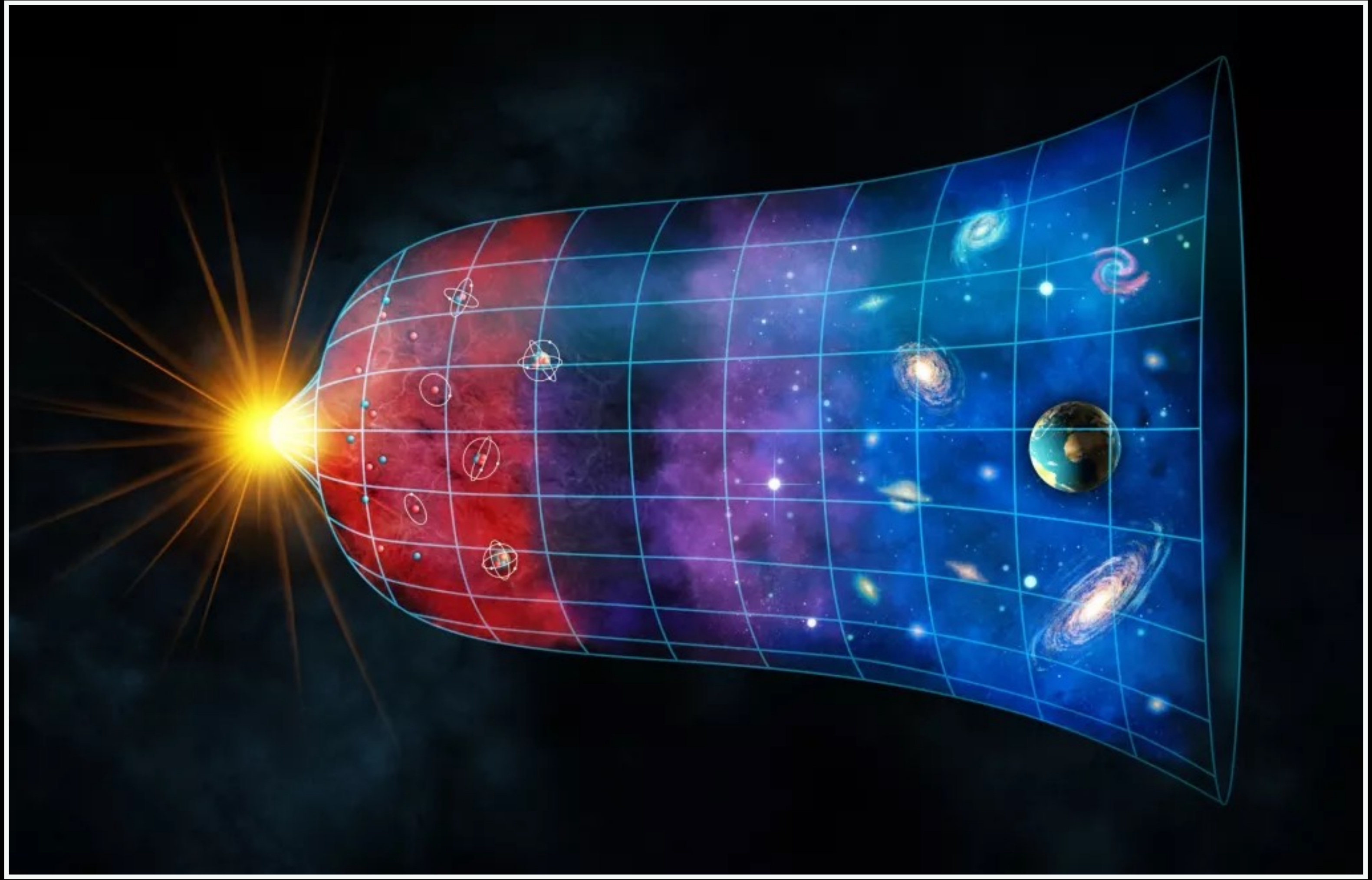

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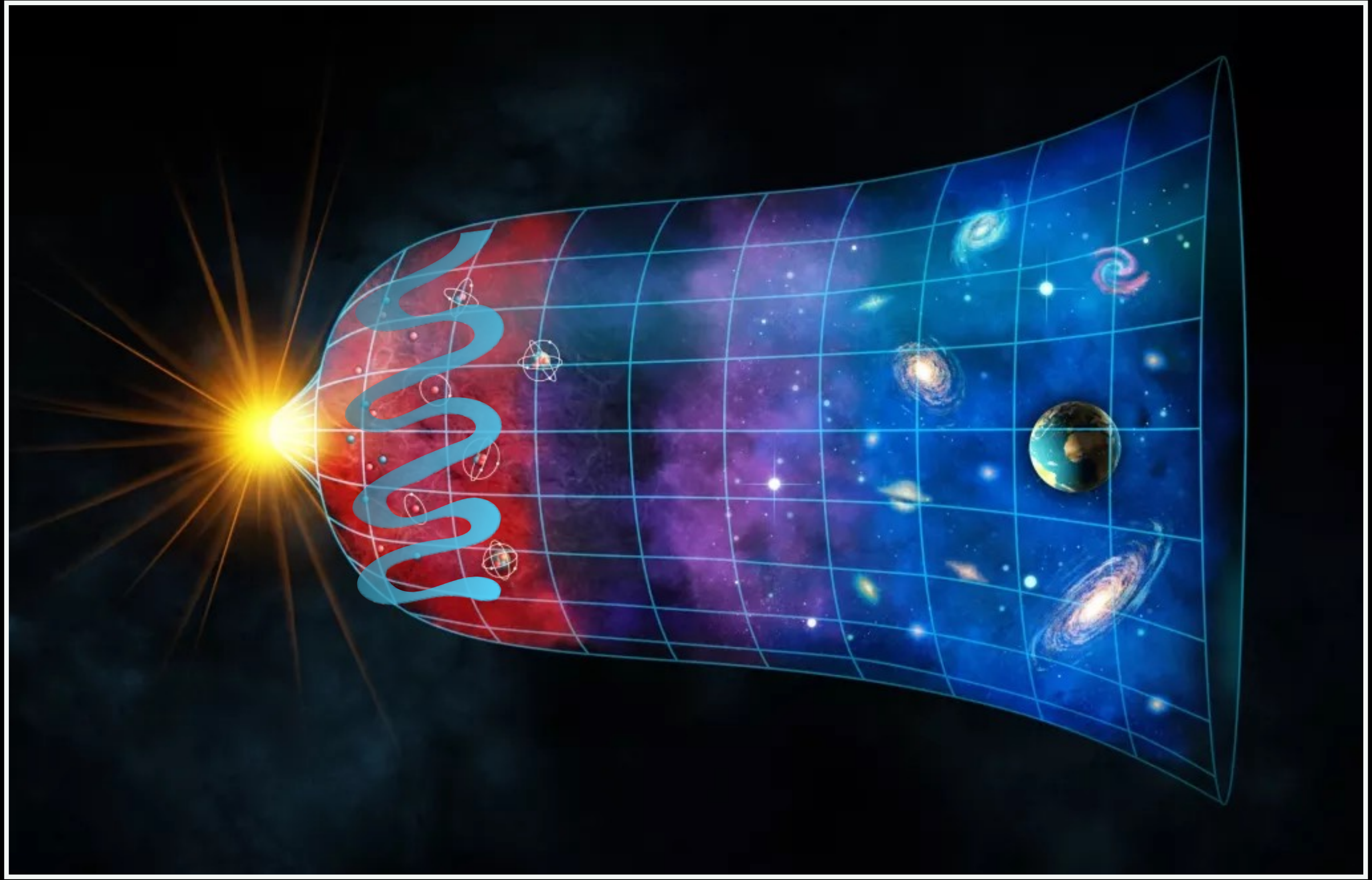

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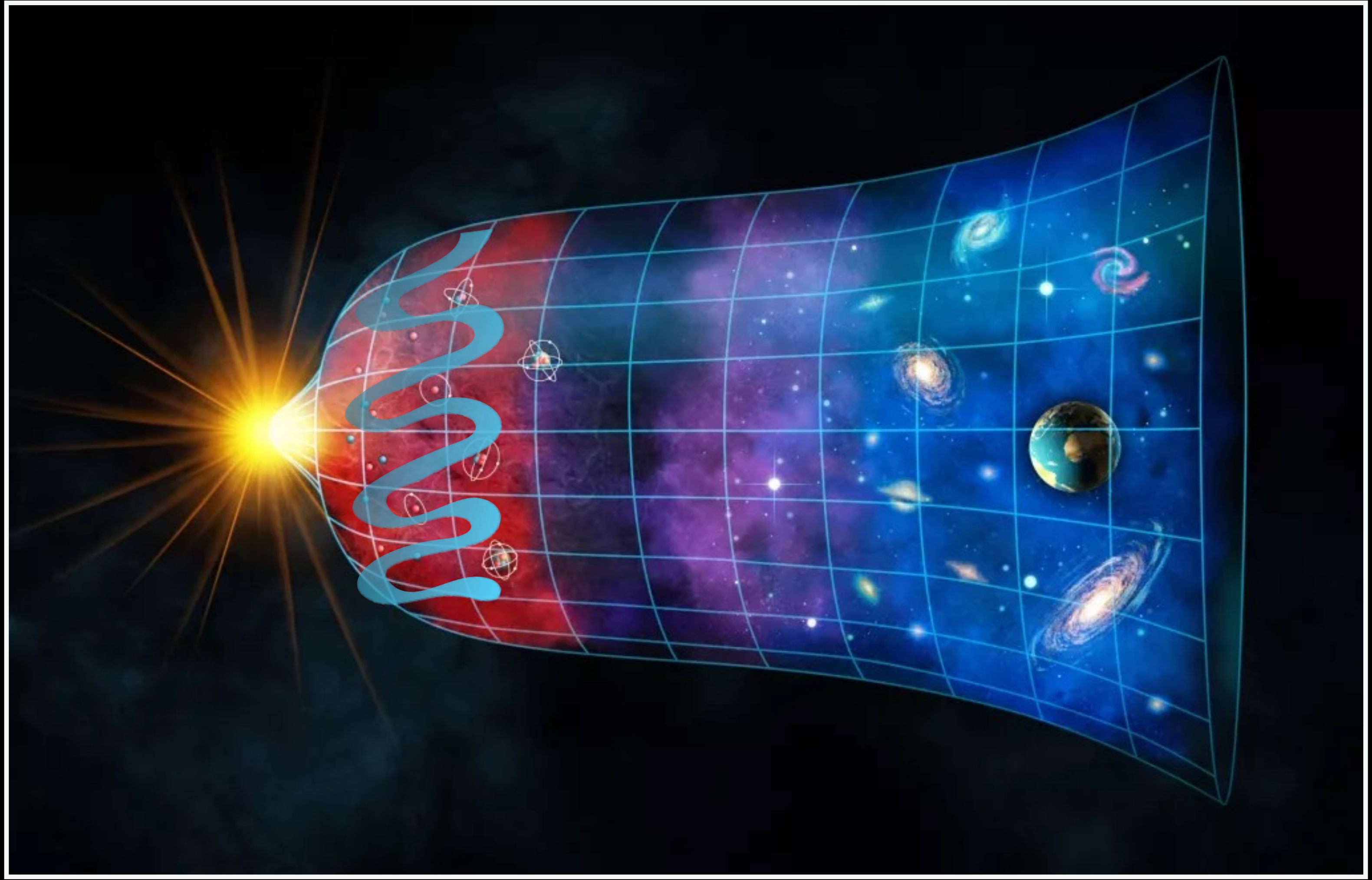

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PMFs: source of GWs, affecting CMB, BBN, reionisation history of the Universe . . .

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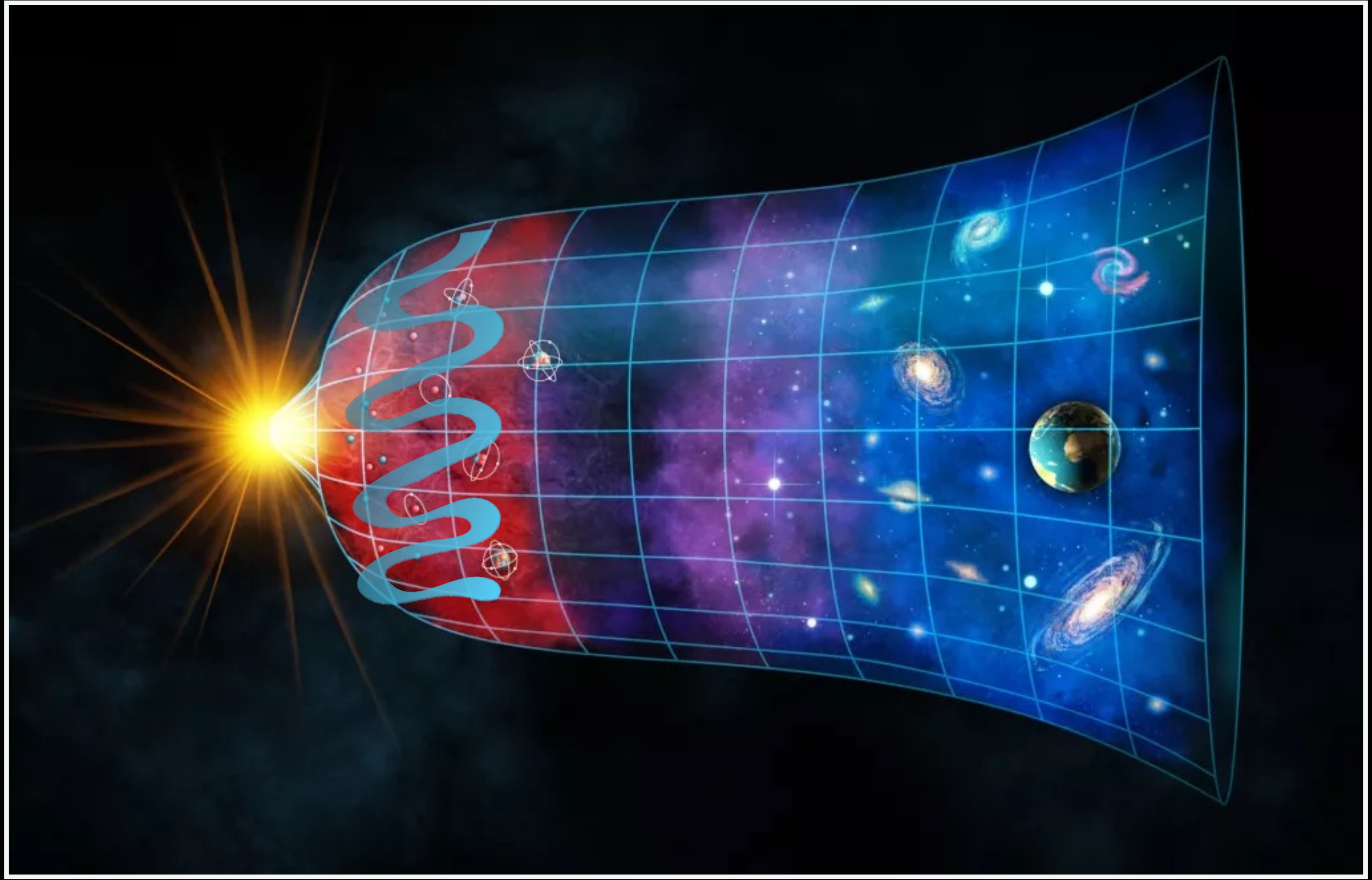

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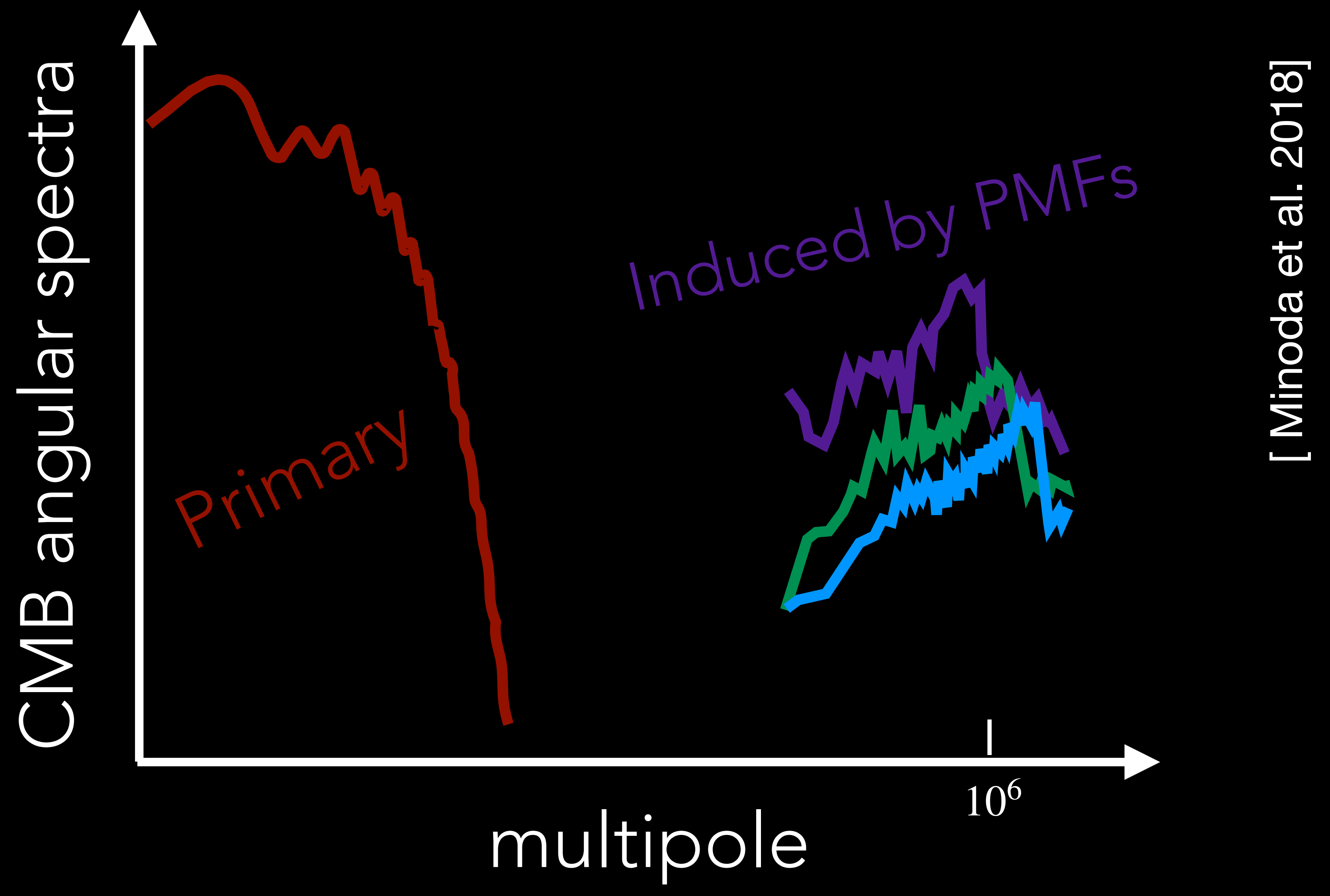

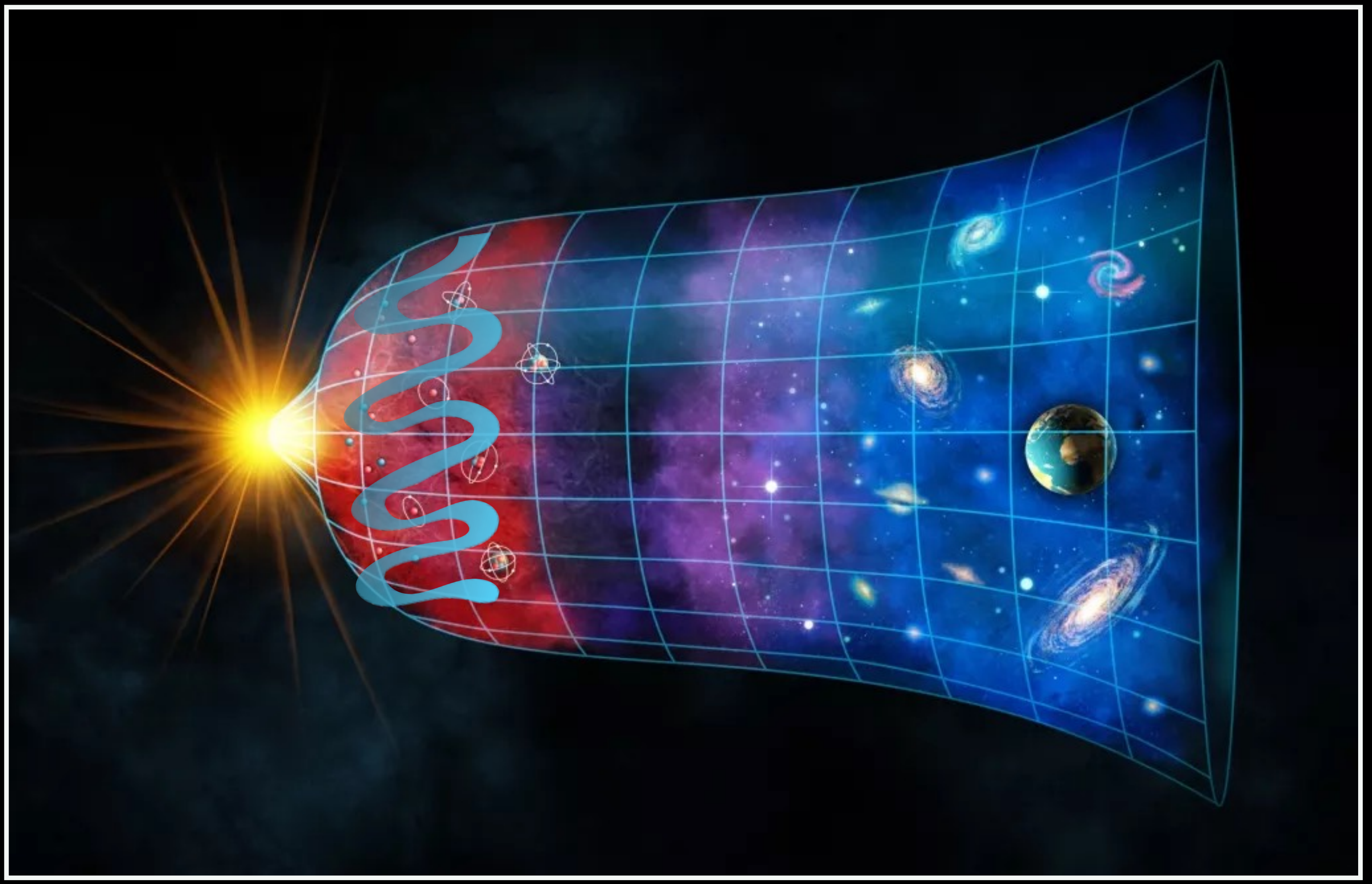

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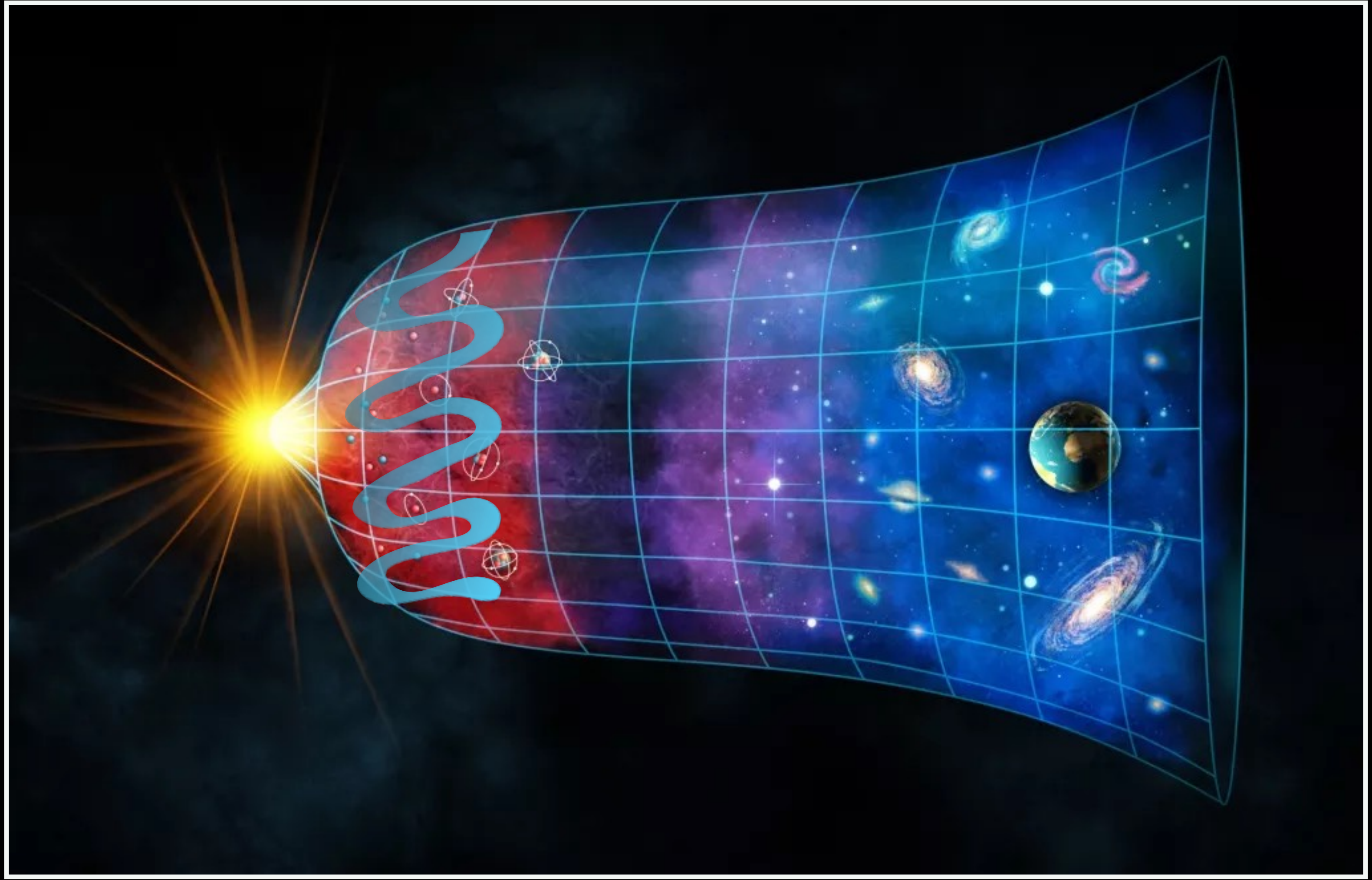

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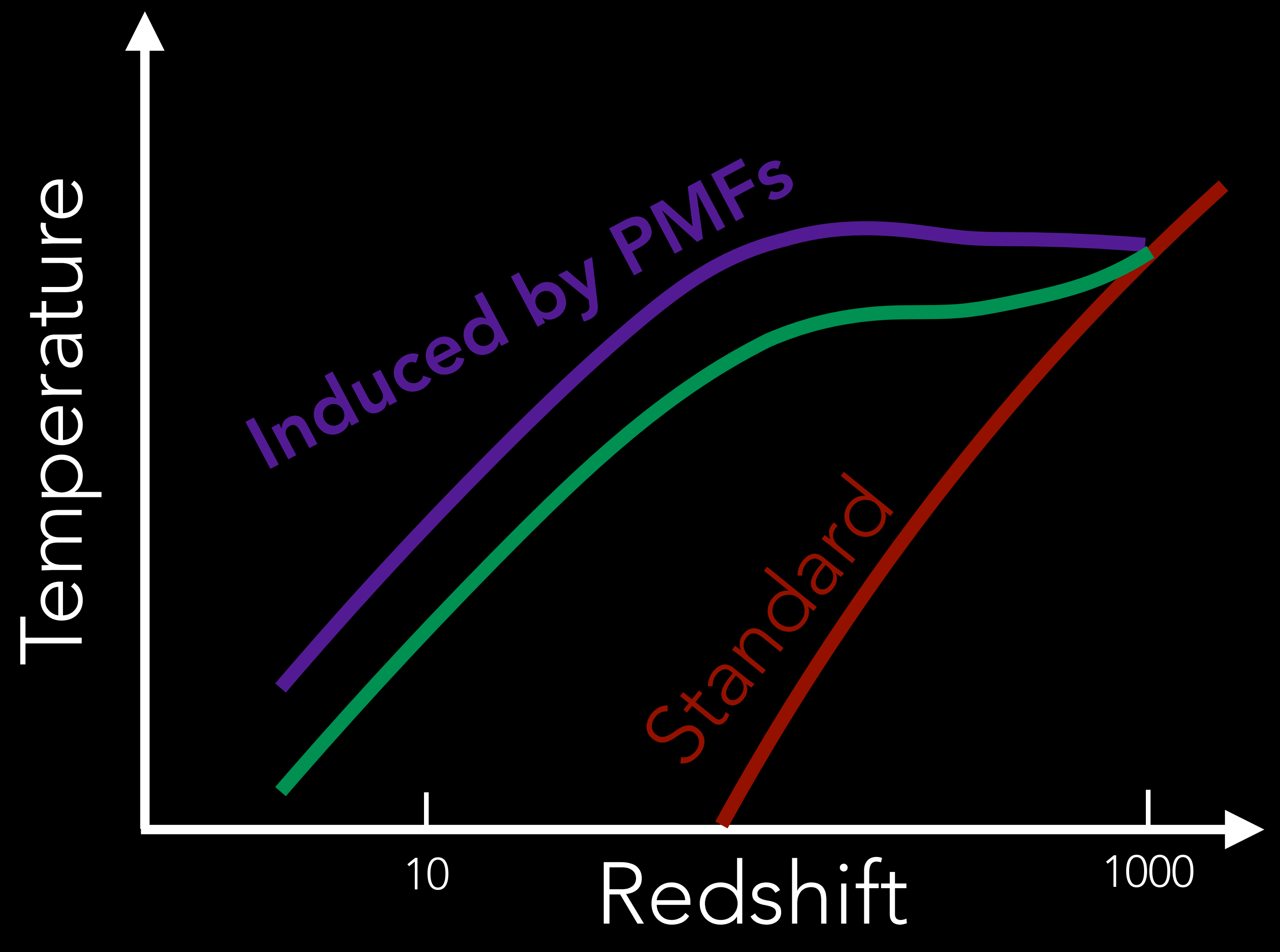
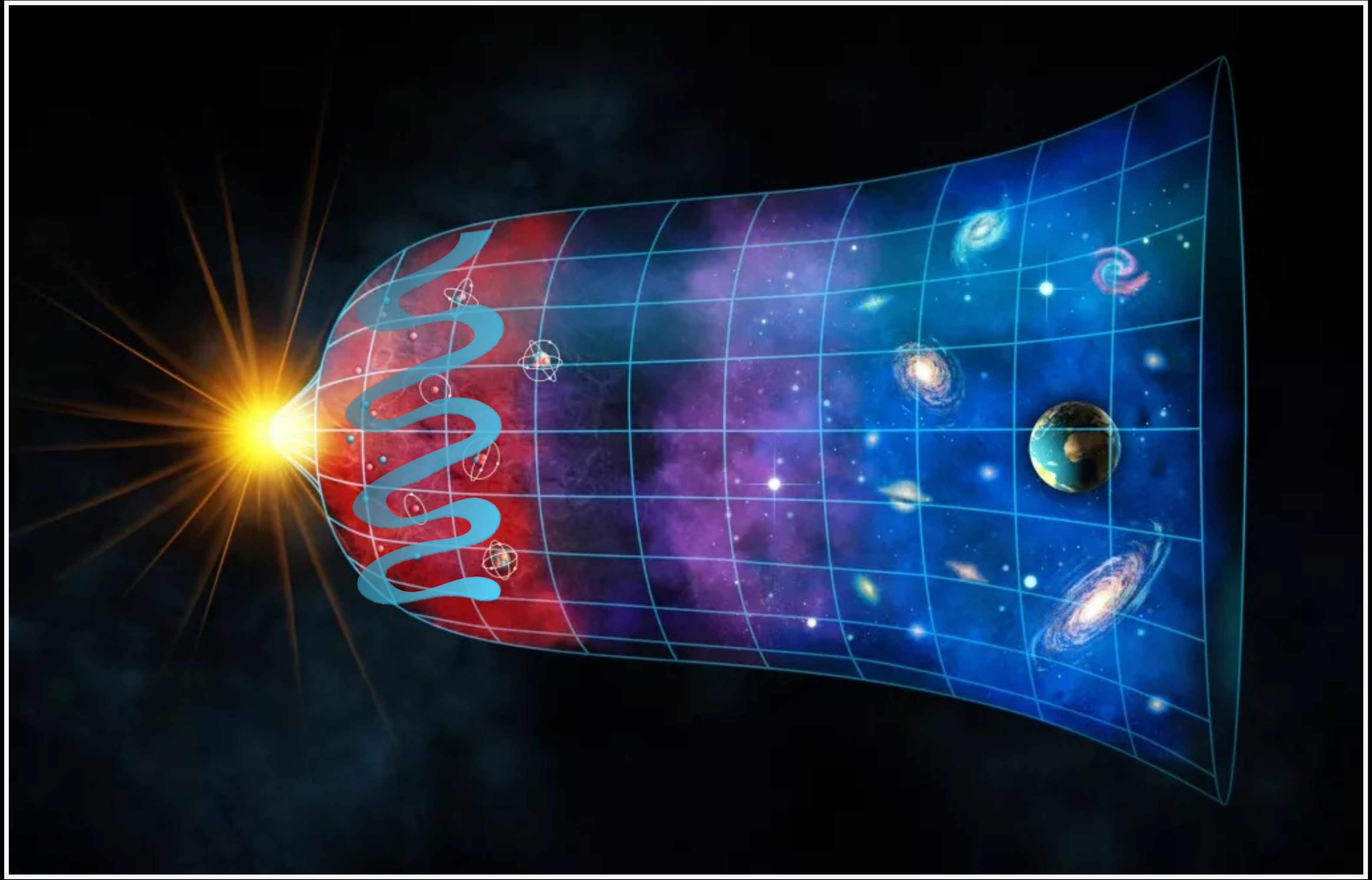


[Sethi & Subramanian 2005]



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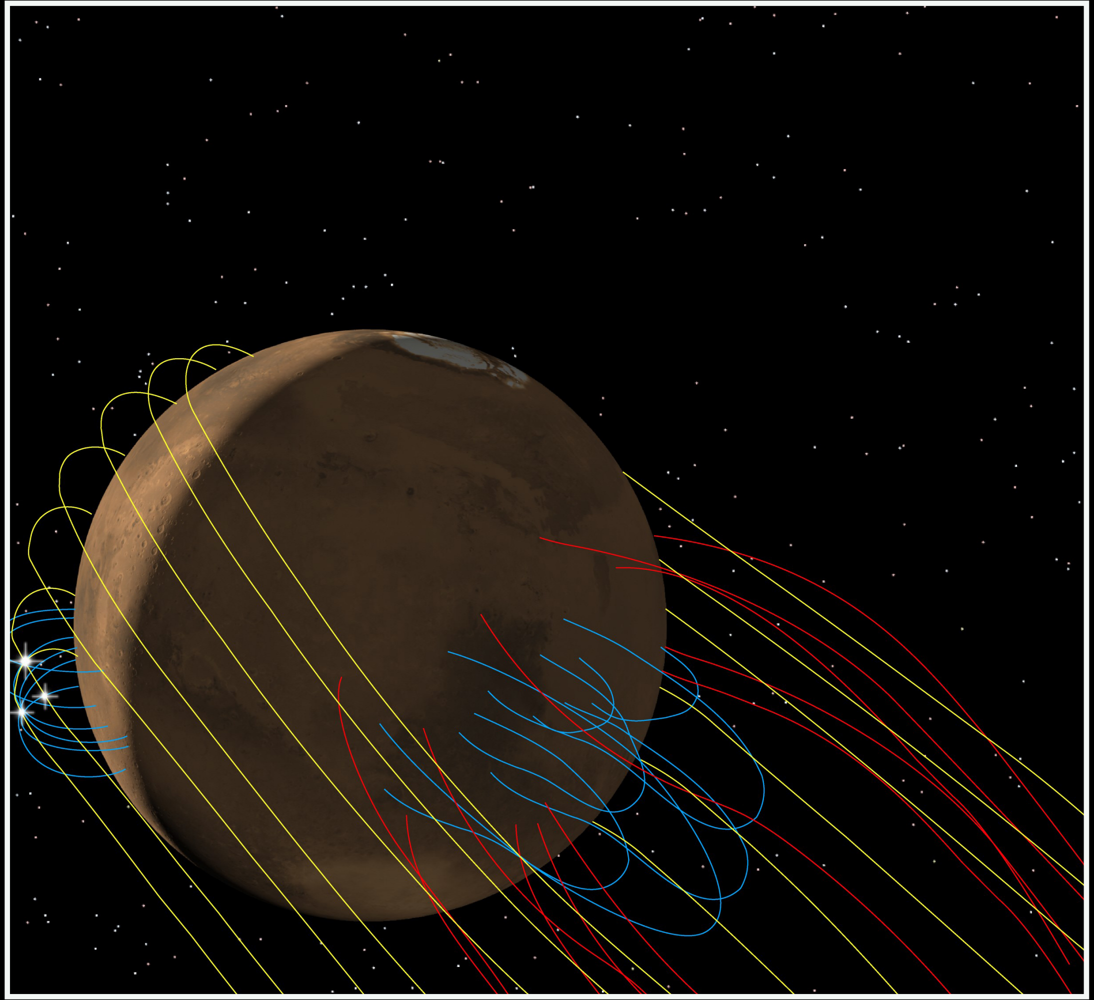
Observed Large Scale Magnetization

PLANETS



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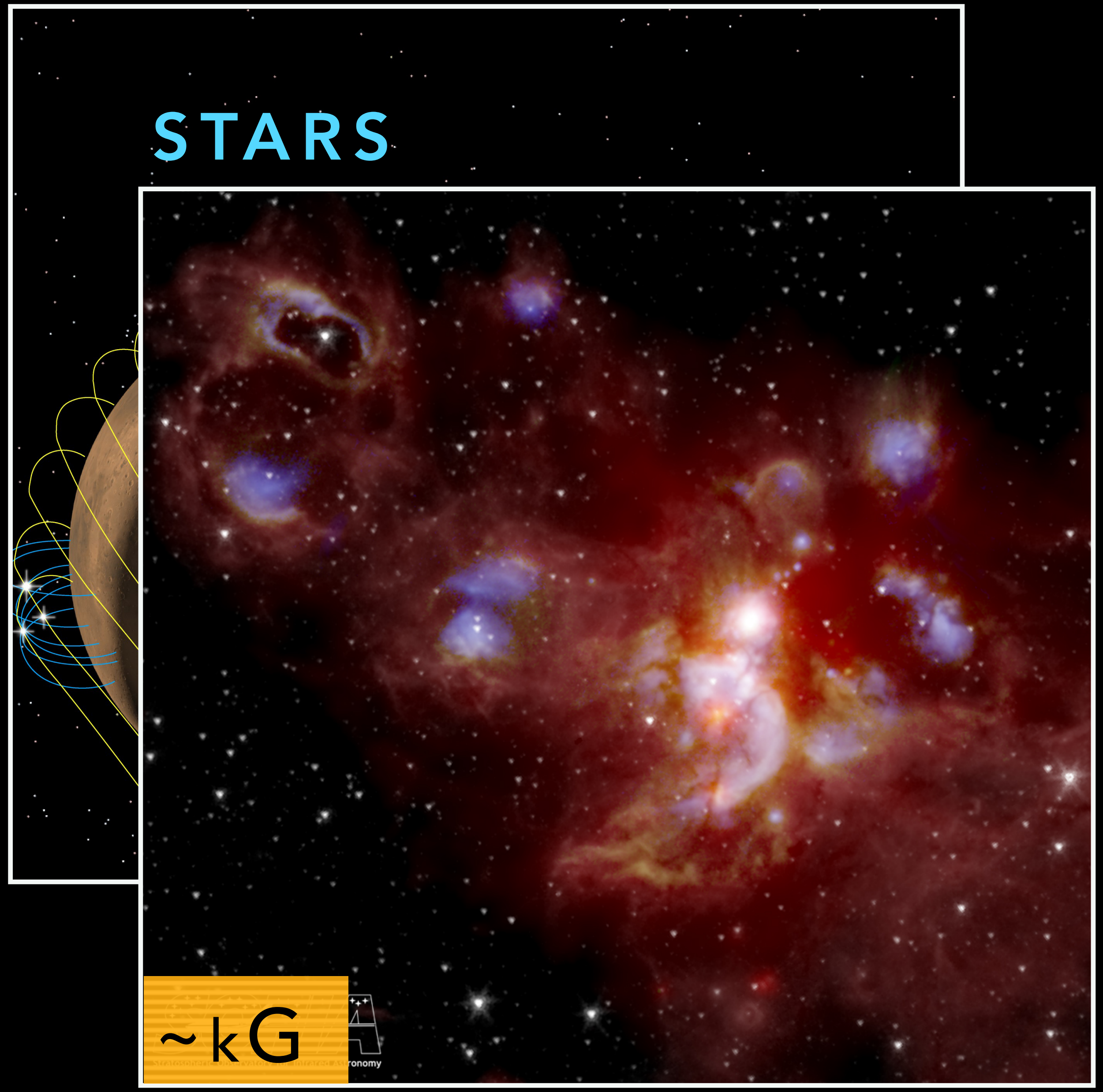

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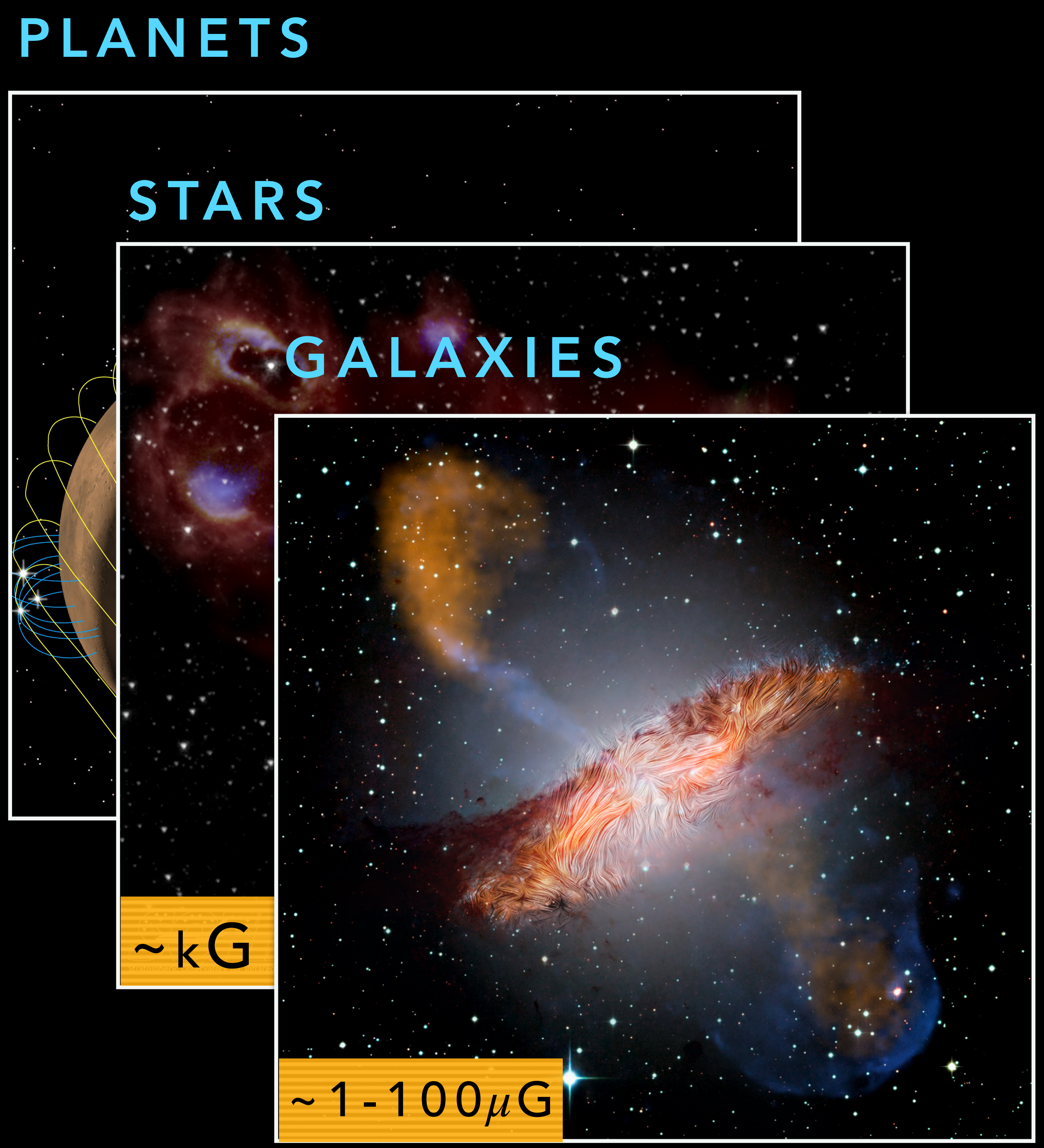

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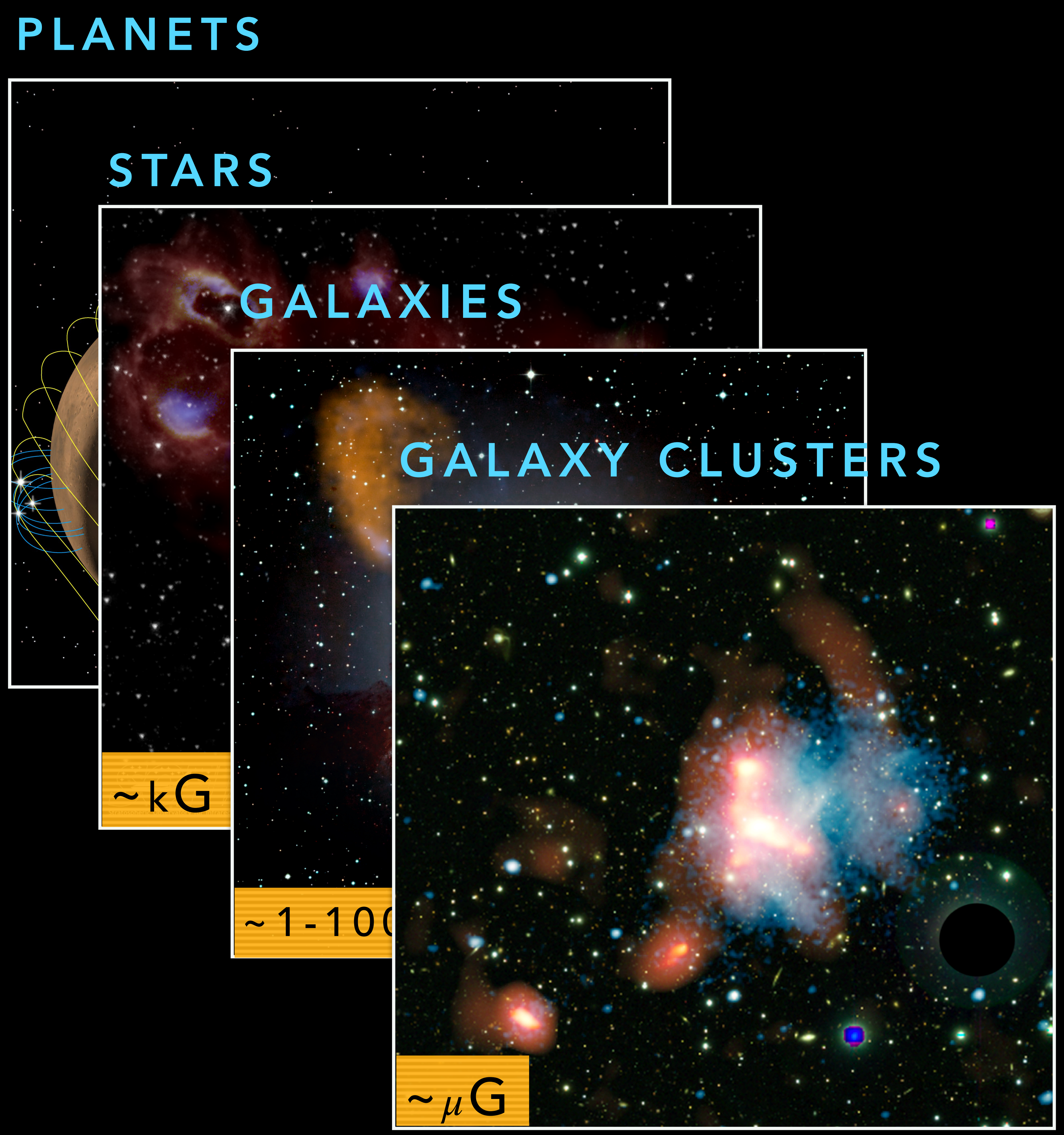

A vast range of astrophysical and cosmological scales: magnetised

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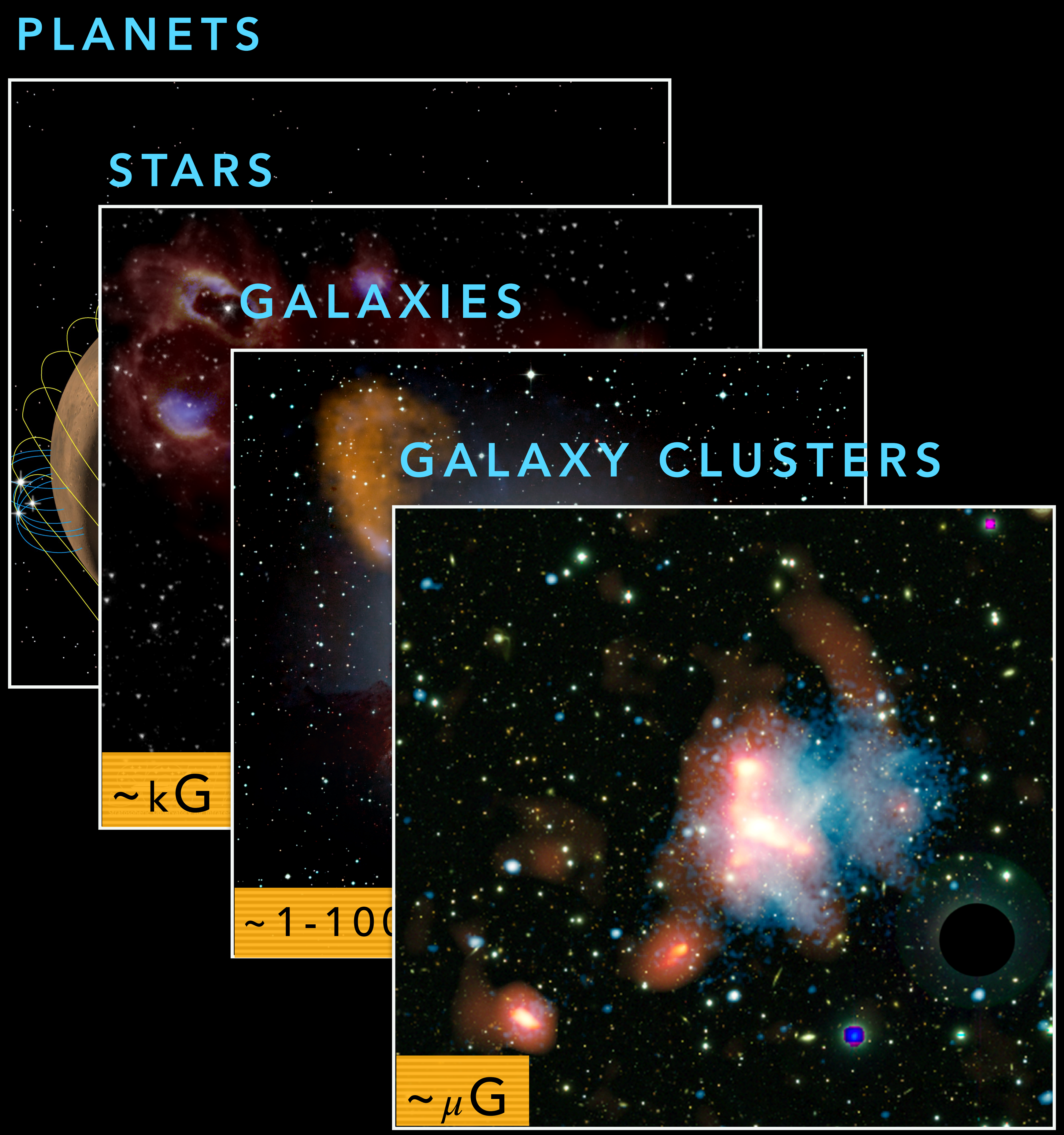

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Magnetic fields are correlated on ~kpc scales

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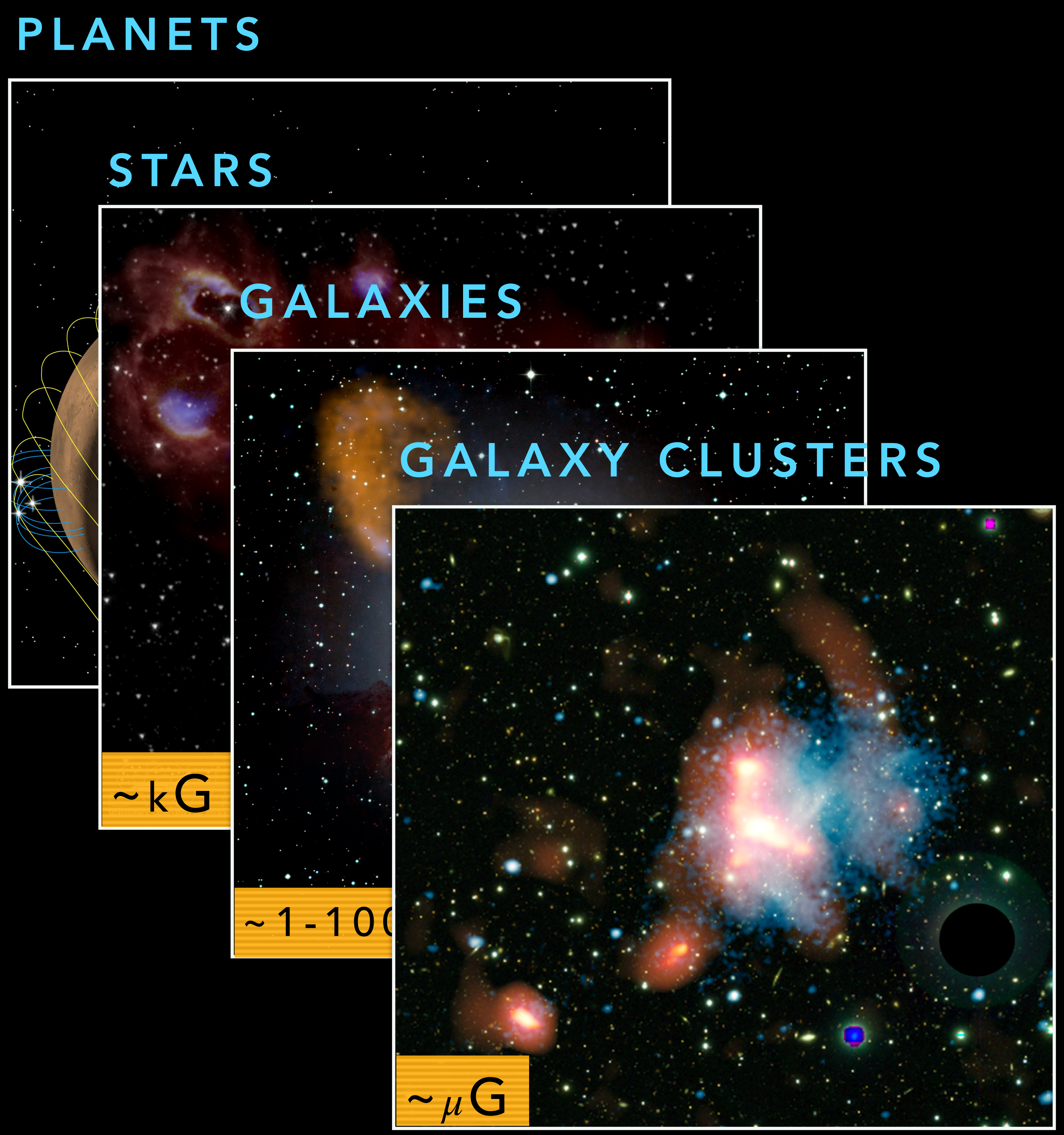

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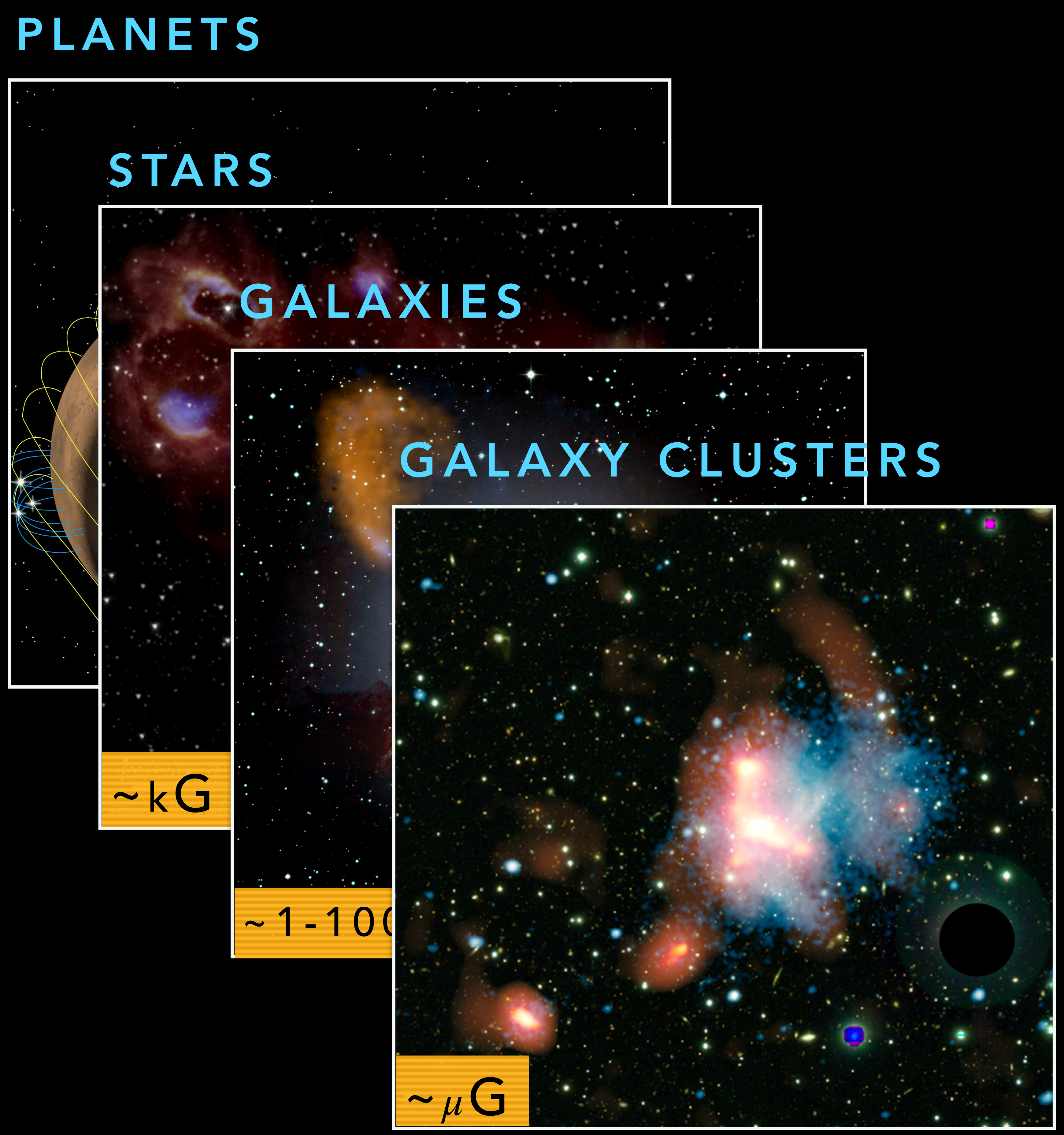
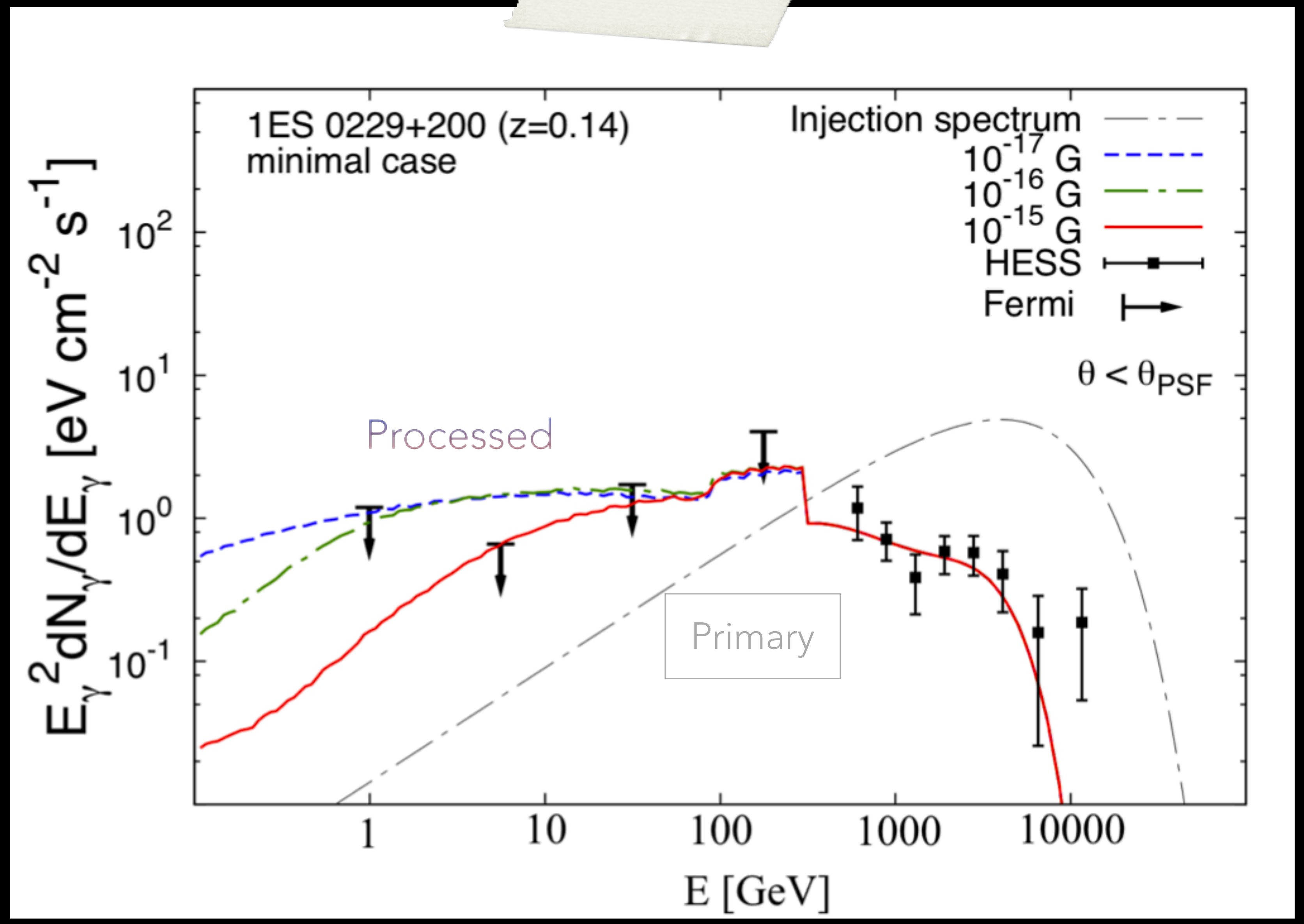
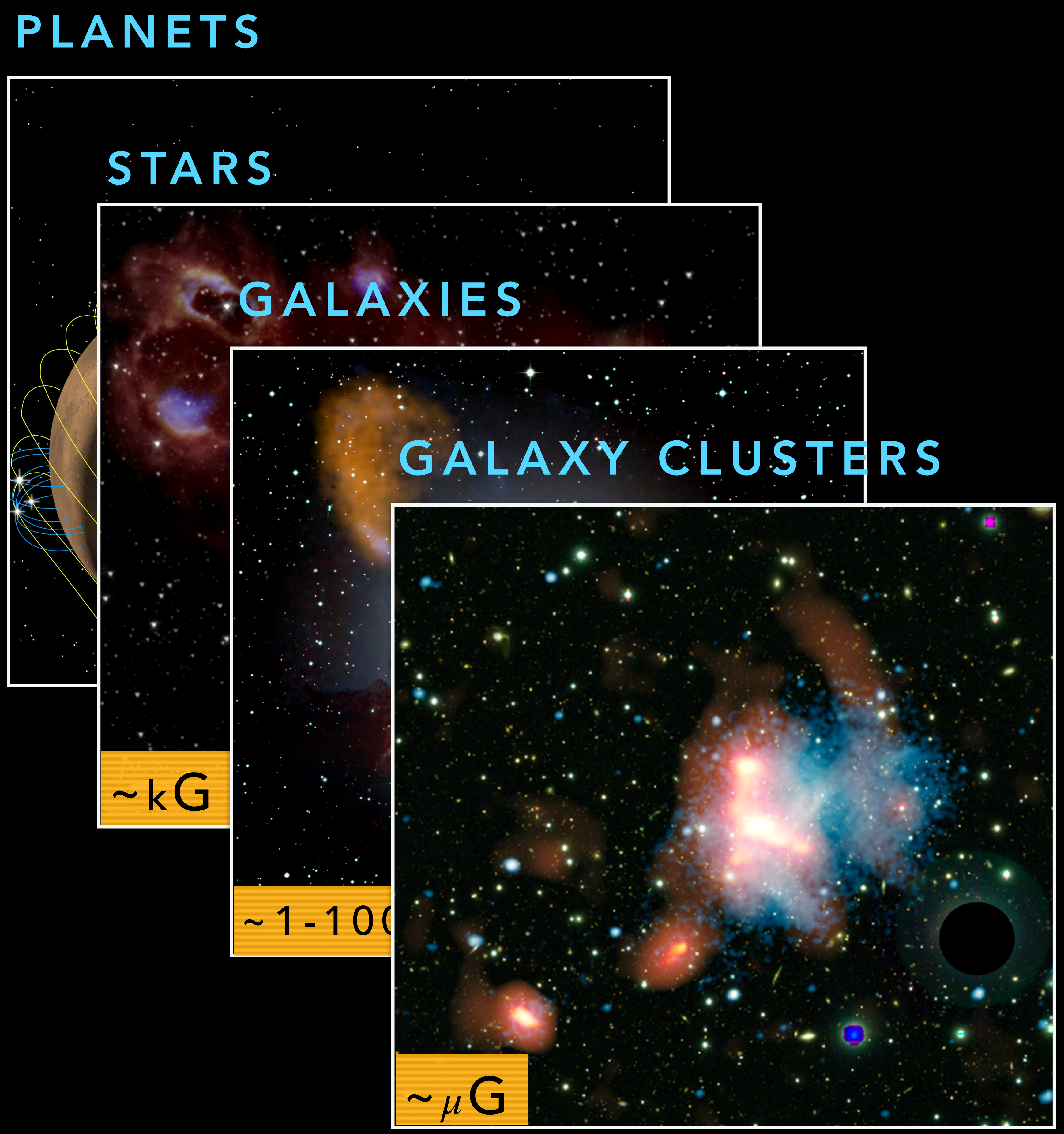


[Taylor et al. 2011]



Magnetic Fields in Voids?



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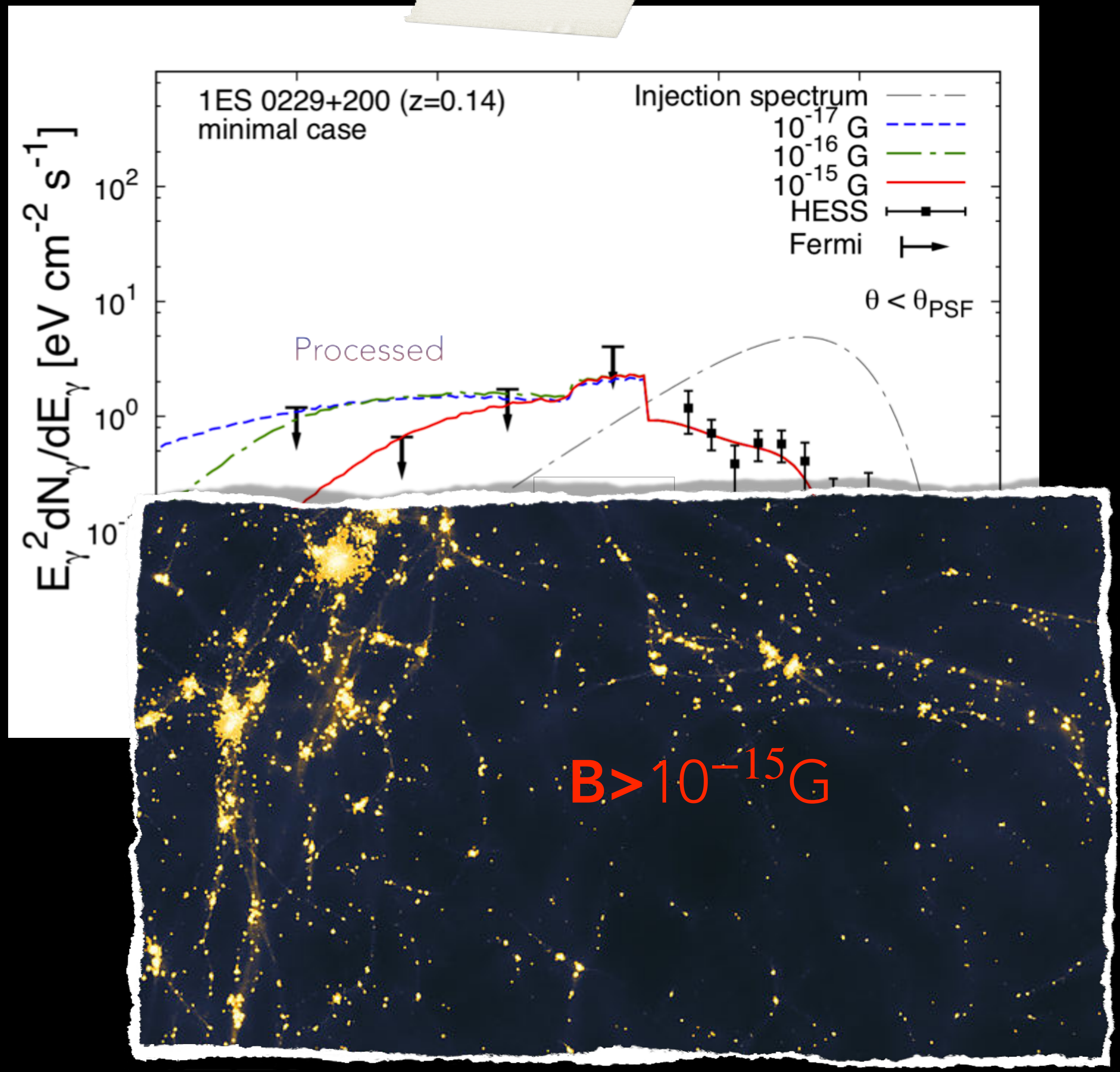
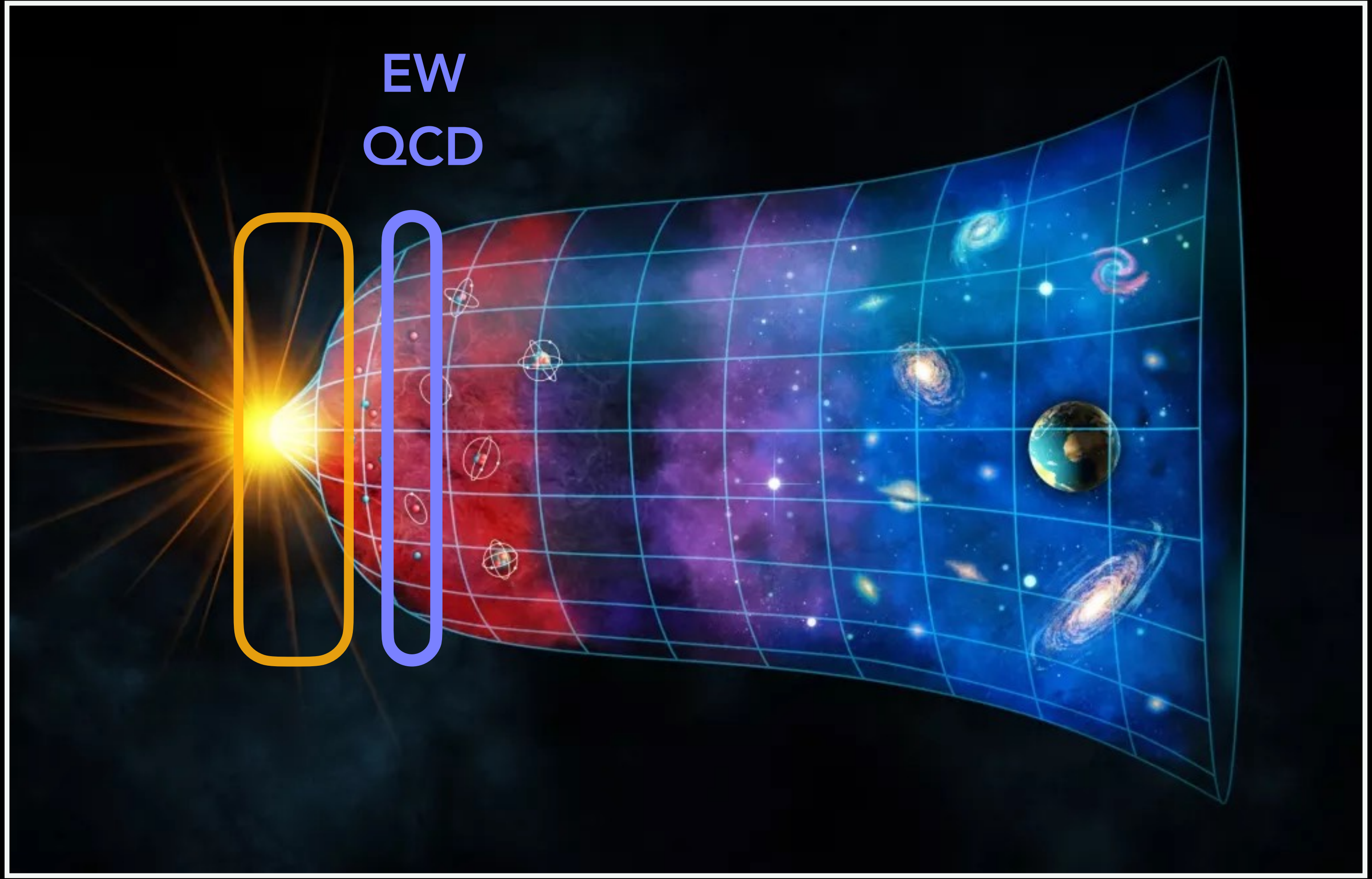
COSMIC VOIDS



Images' credits: NASA, SOFIA, Bonafede et al. 2018, Andrew Pontzen and Fabio Governato

PMFs: generation scenarios



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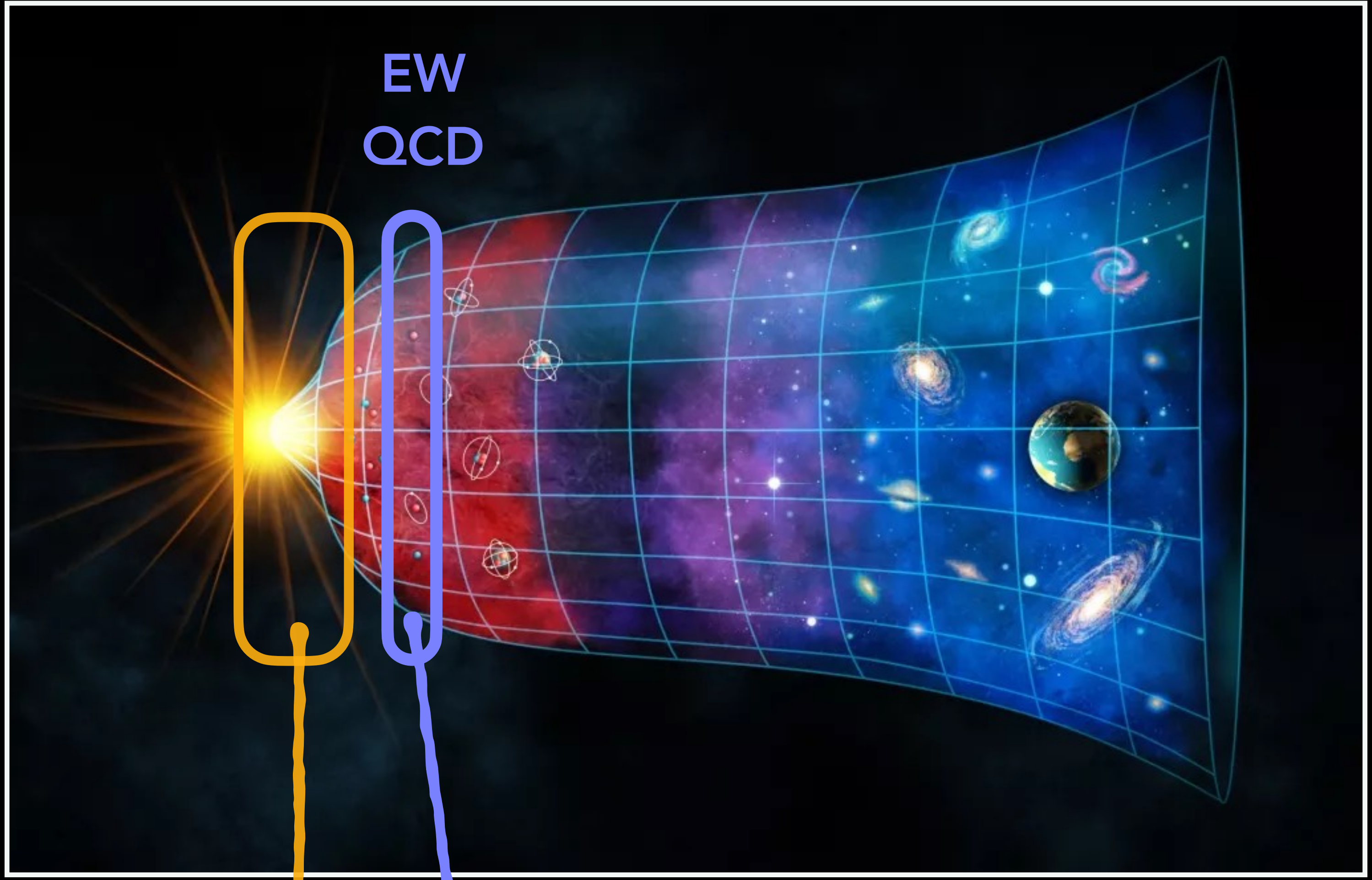


Inflationary

λ unbound

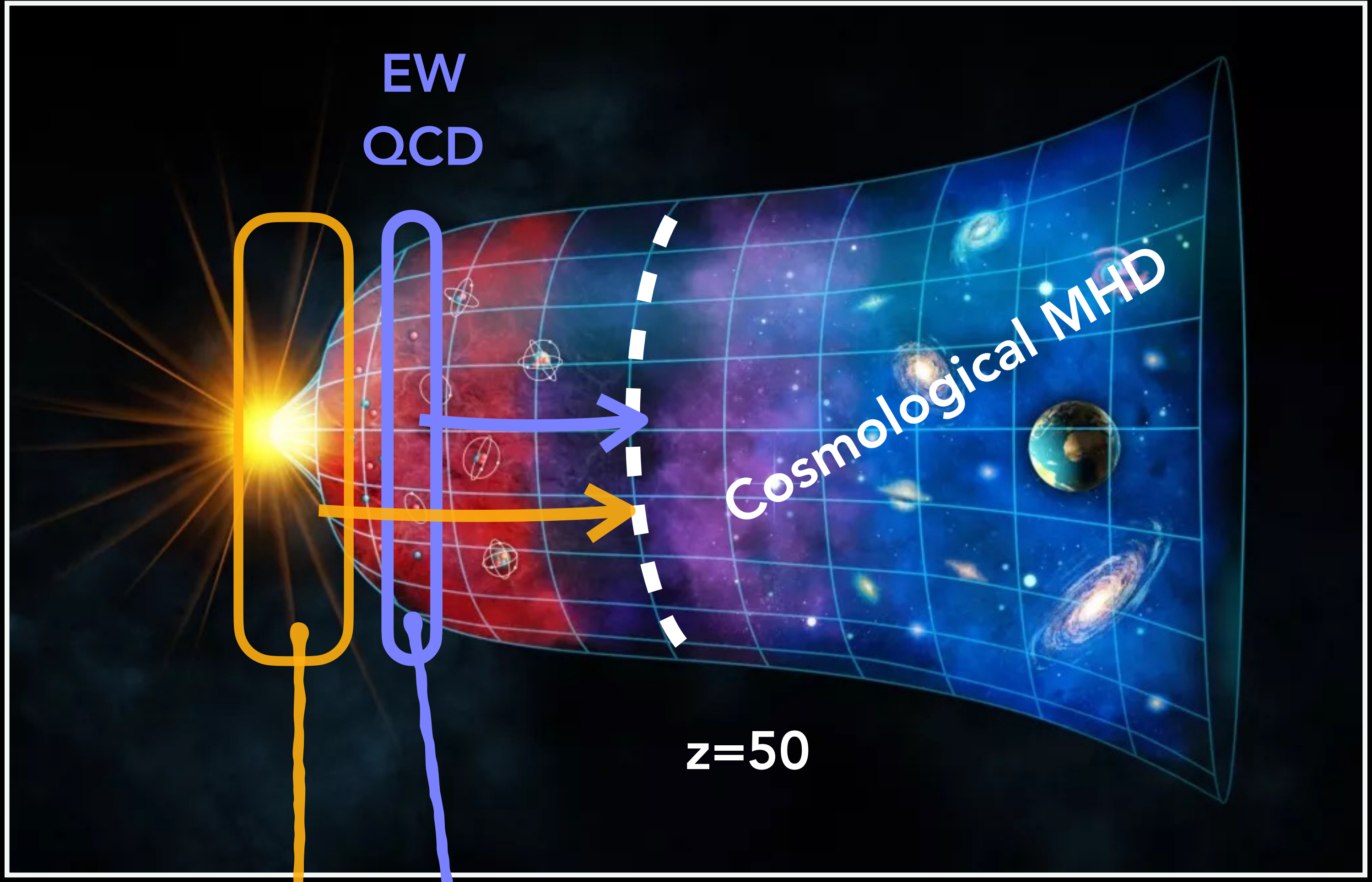
[Turner & Widrow 1988]

Phase-transitional

$\lambda < \lambda_{\text{Hubble}}$

[Hogan 1983]

How do we study PMFs?



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Cosmological MHD

ΛCDM cosmology

z_{in}=50

ENZO Bryan et al 2014

Resolution: 133 ckpc/h

Box size: 67.7 cMpc/h

No cooling and feedback

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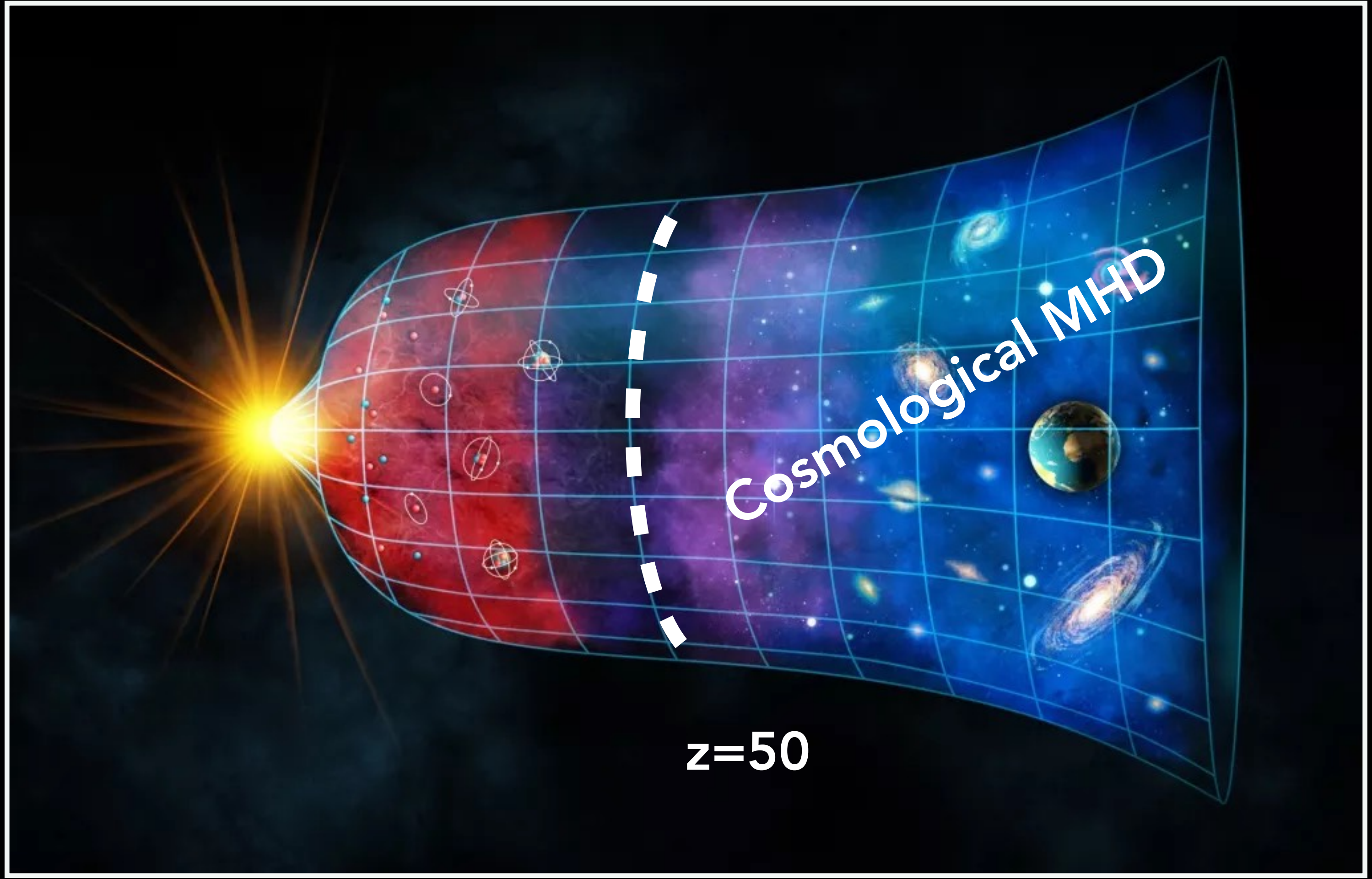

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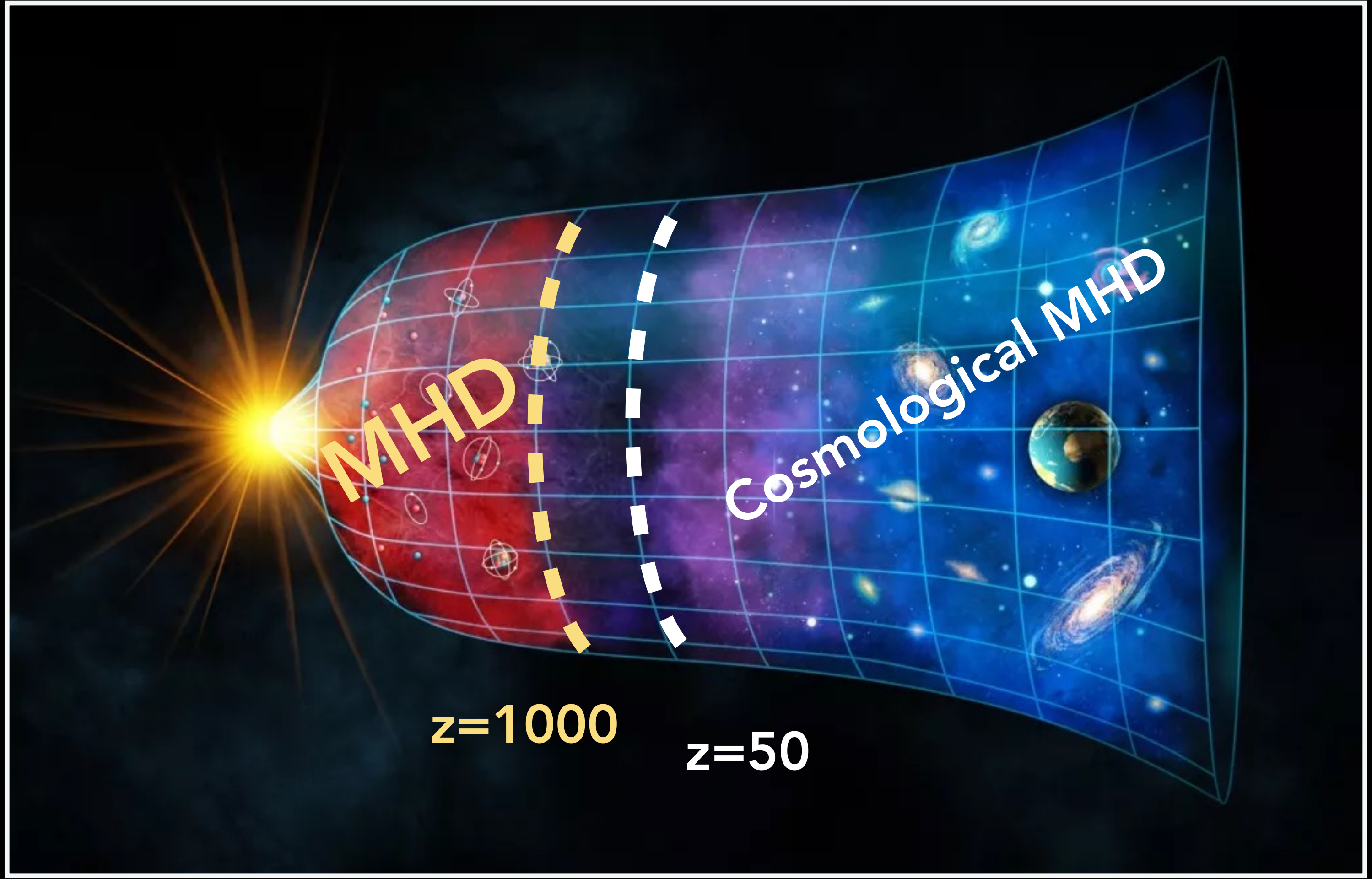

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PENCIL

Pencil collab. 2021

Novel approach:
different PMFs
results of MHD simulations
(early Universe)

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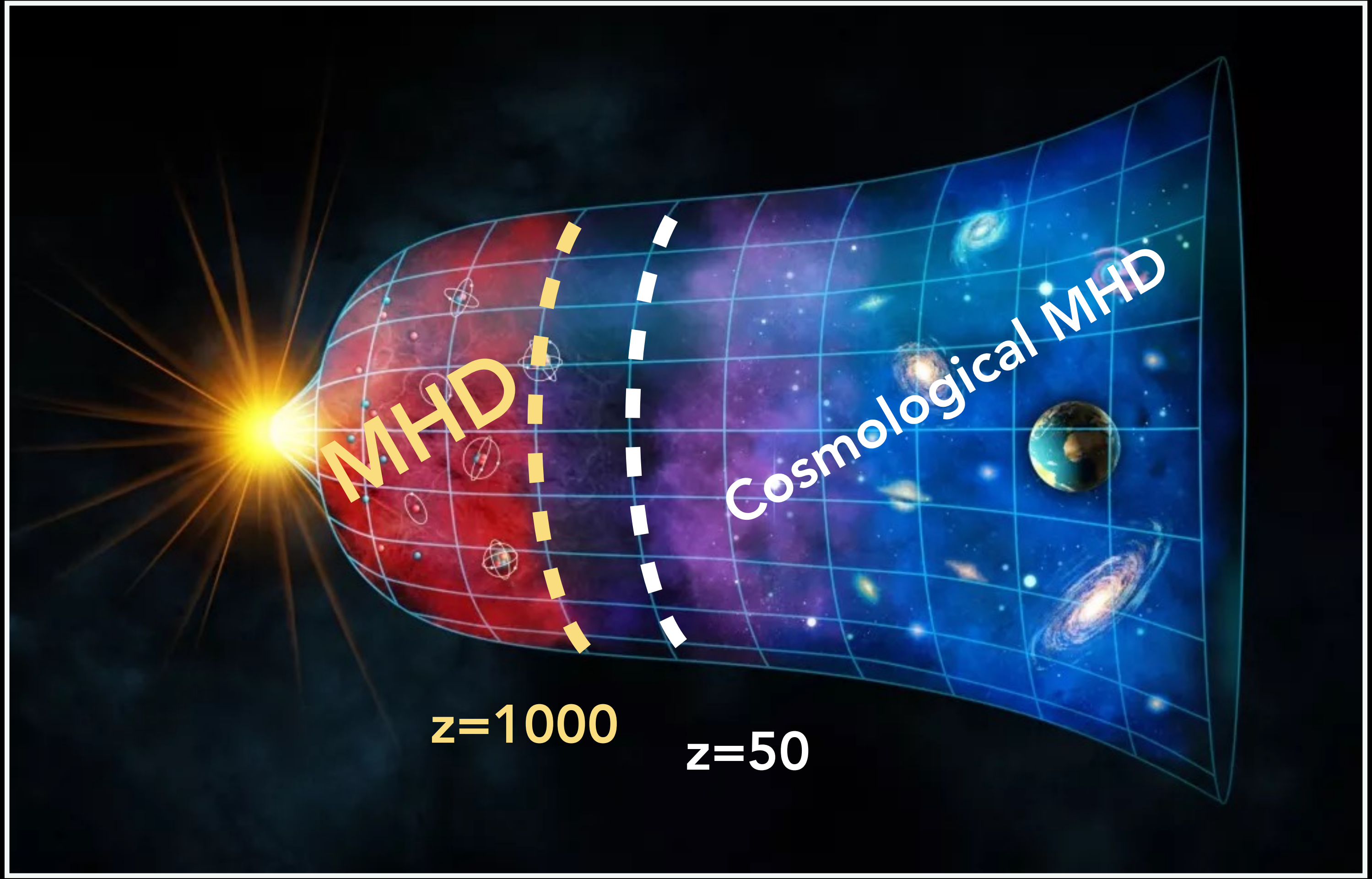

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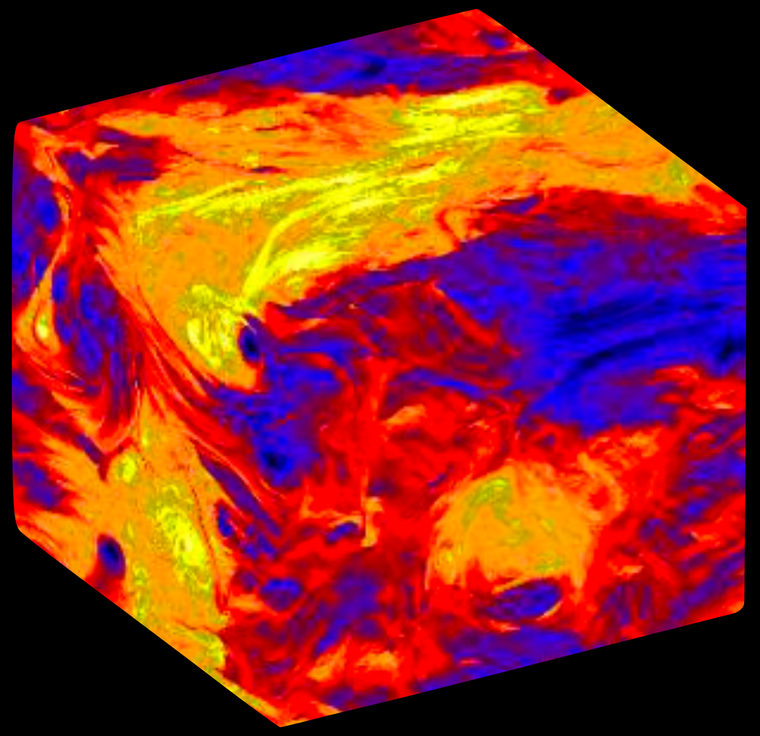

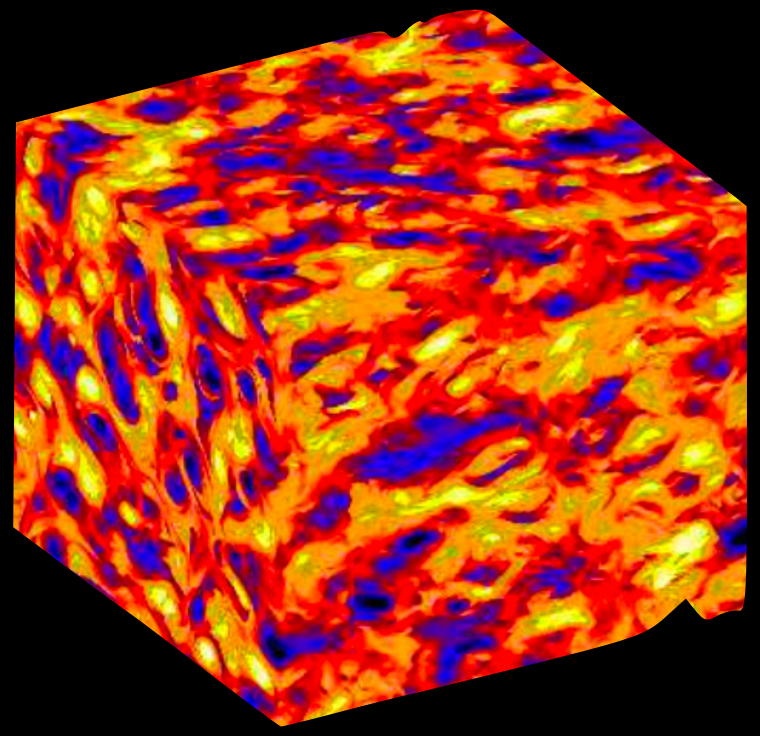

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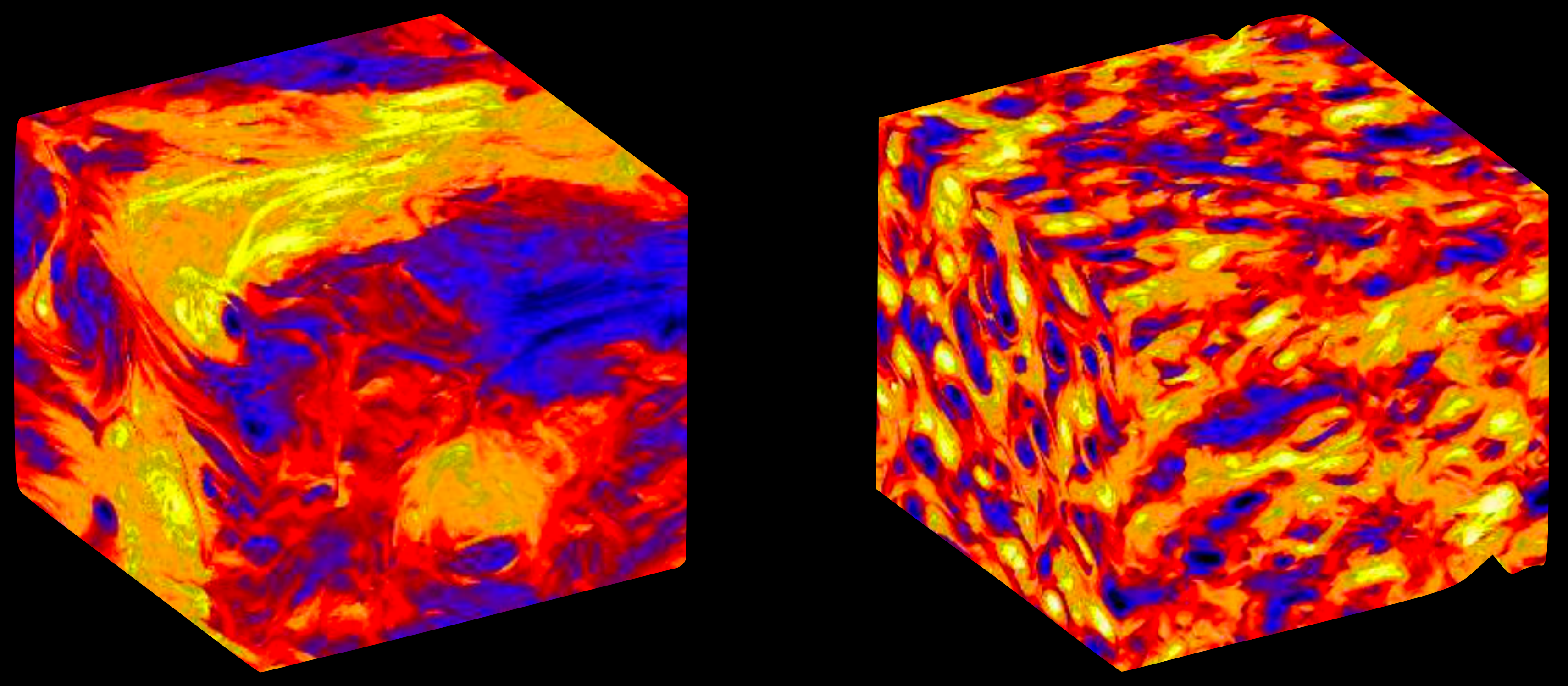

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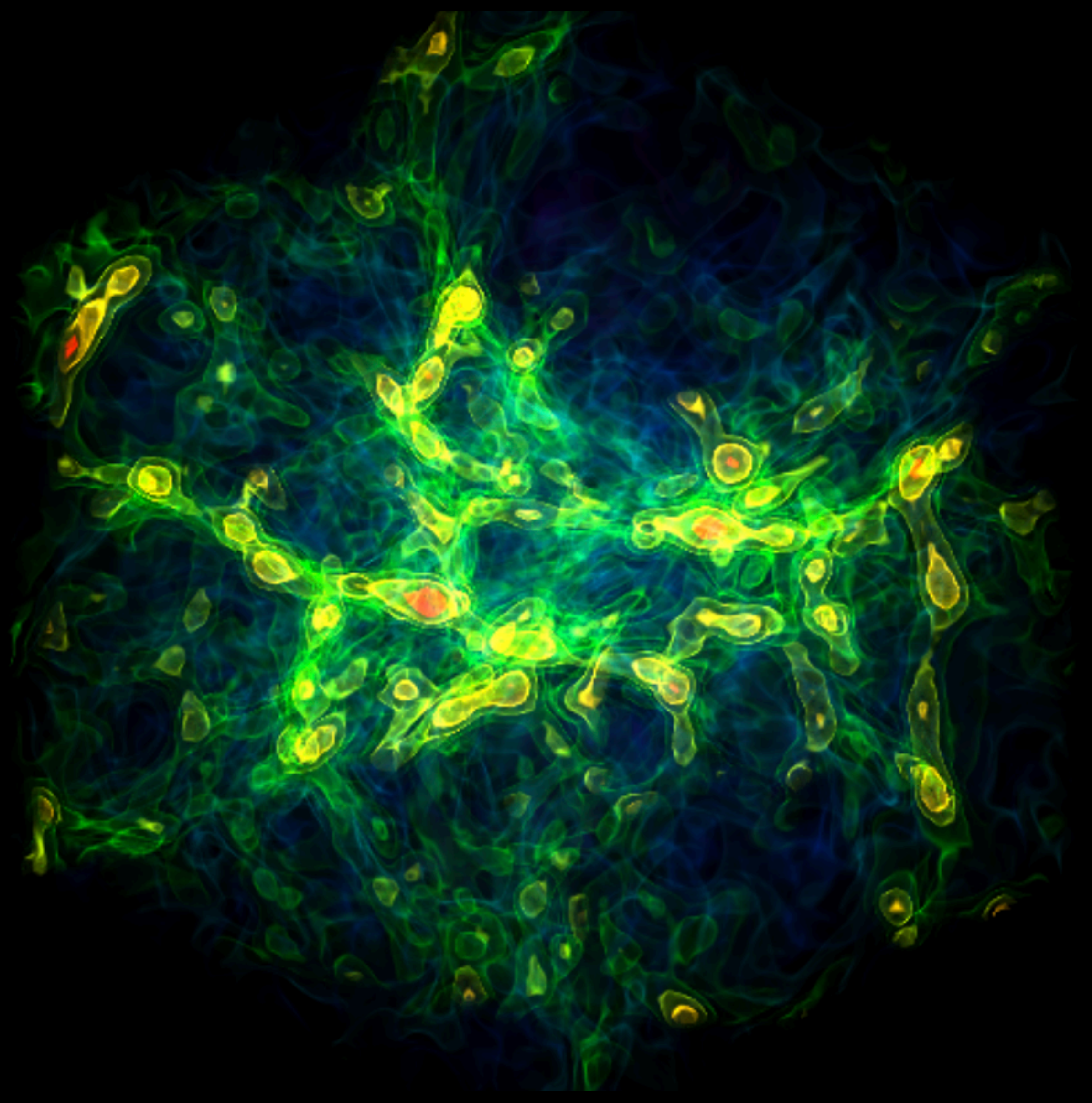
(early Universe)

How do we study PMFs?



Cosmological MHD

ΛCDM cosmology

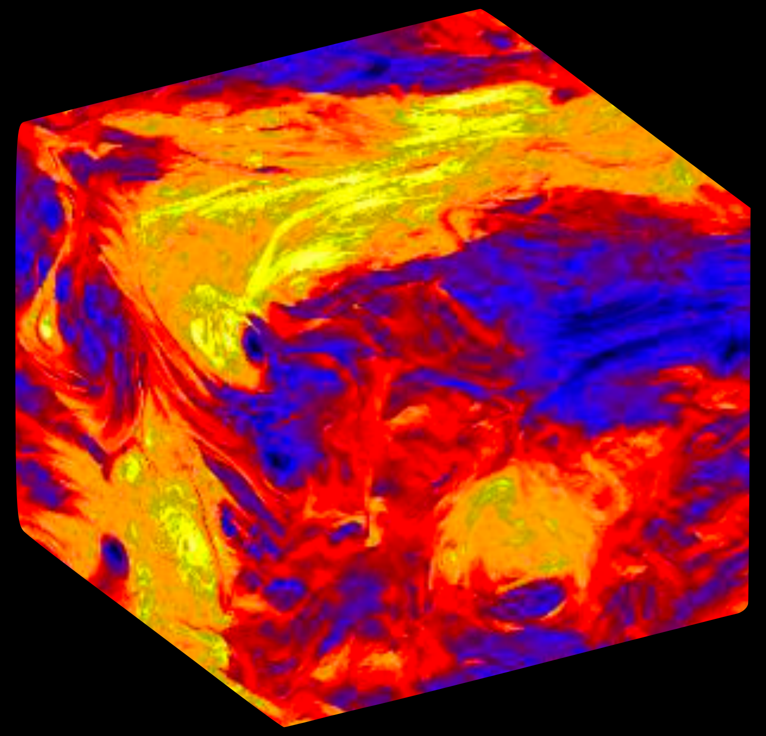
$z_{in}=50$

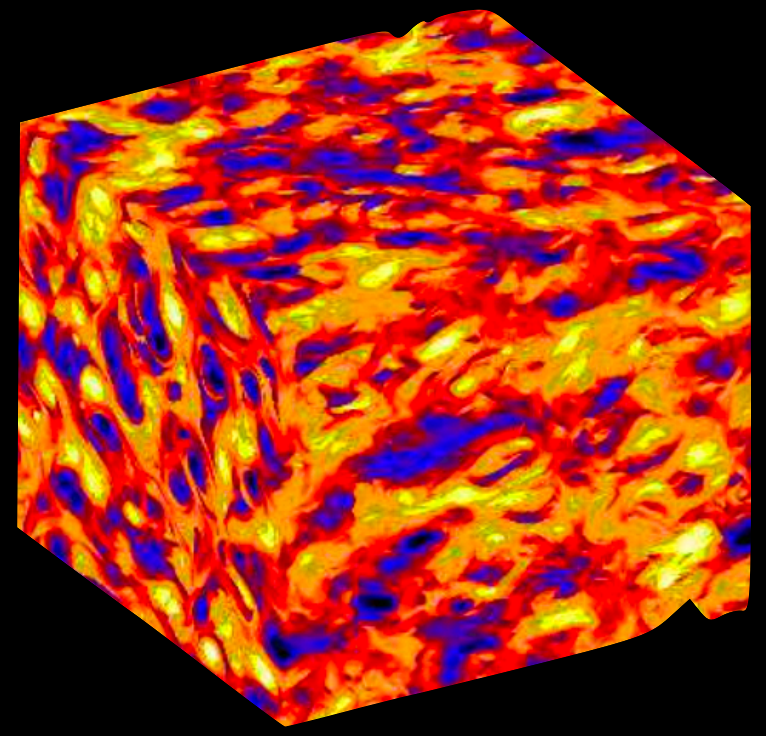
ENZO Bryan et al 2014

Resolution: 133 ckpc/h

Box size: 67.7 cMpc/h

No cooling and feedback





MHD

PENCIL

Pencil collab. 2021

Novel approach:
different PMFs
results of MHD simulations
(early Universe)

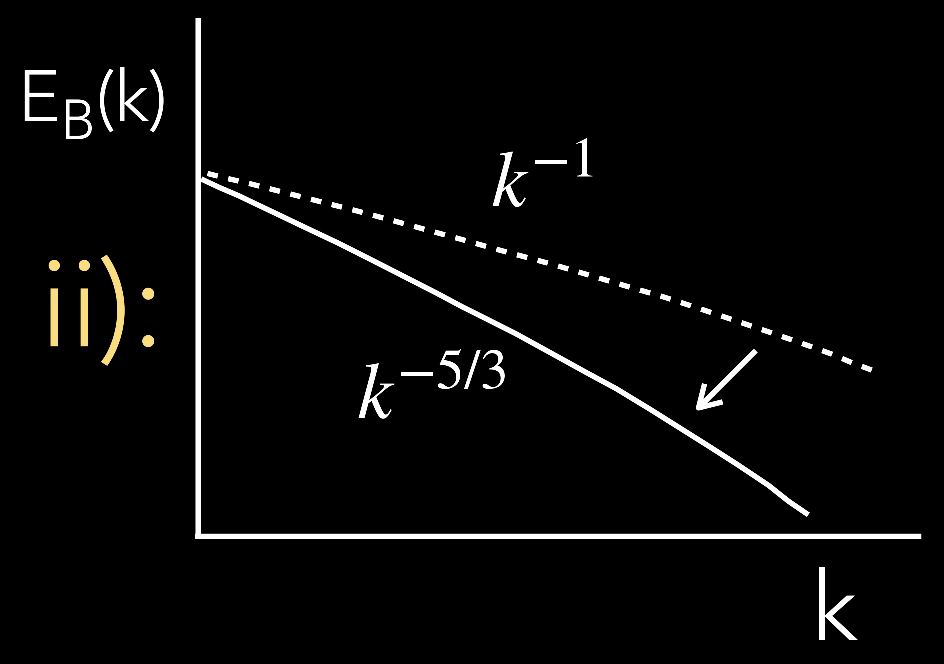
Cosmological MHD: initial conditions

Inflationary:

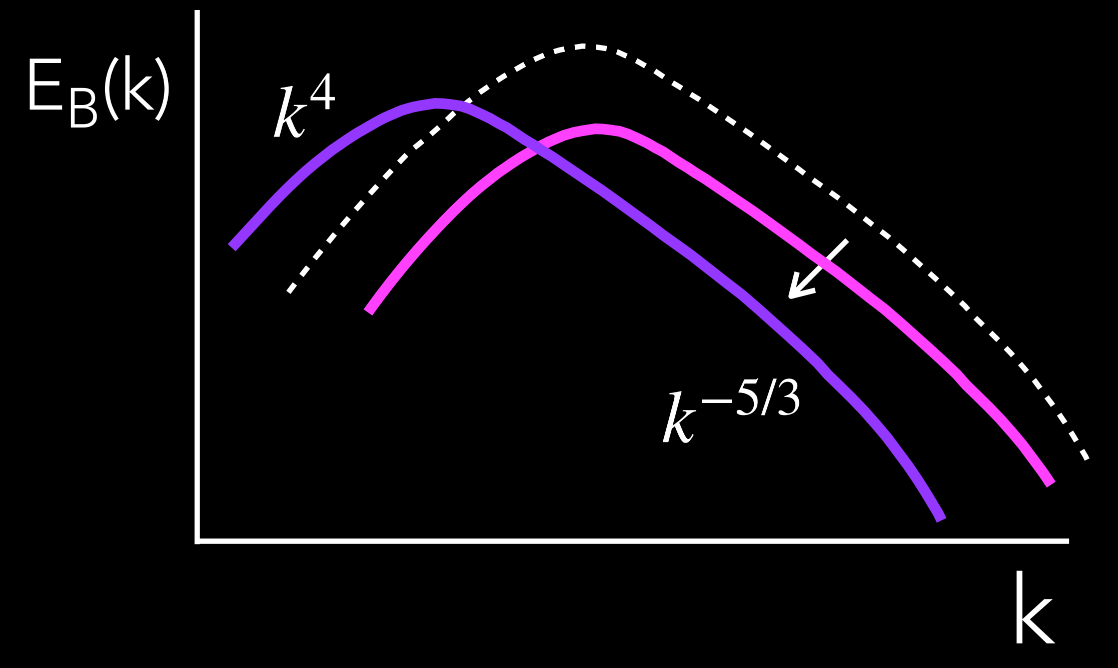
i): $B = \text{const}$

$E_B(k) = \delta(k=0)$

ii):

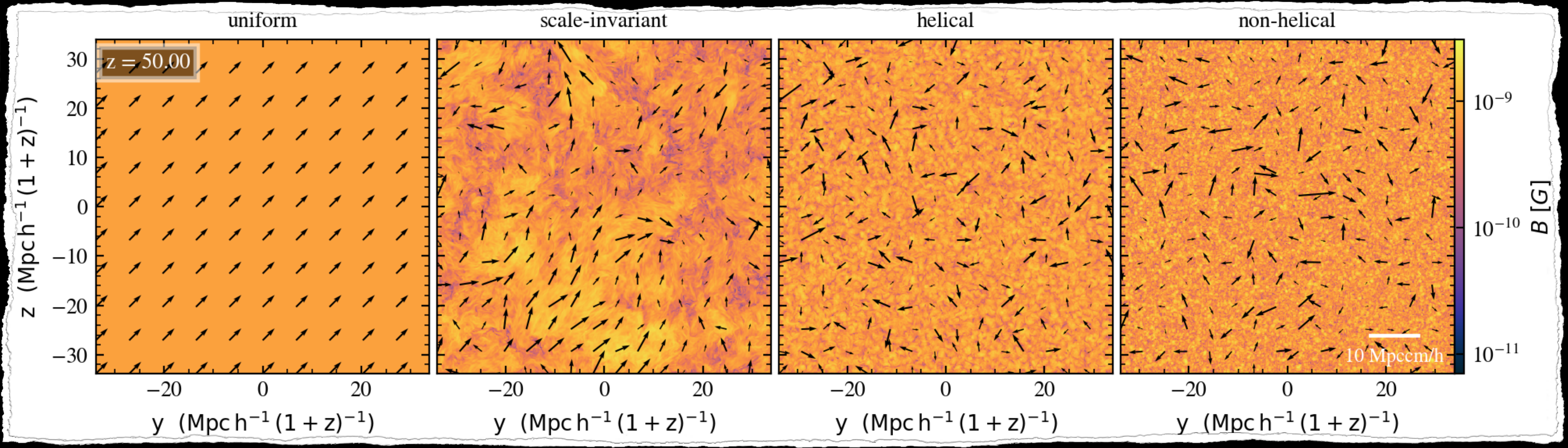


Phase-transitional:



iii): $H \neq 0$

iv): $H = 0$



Cosmological MHD: initial conditions

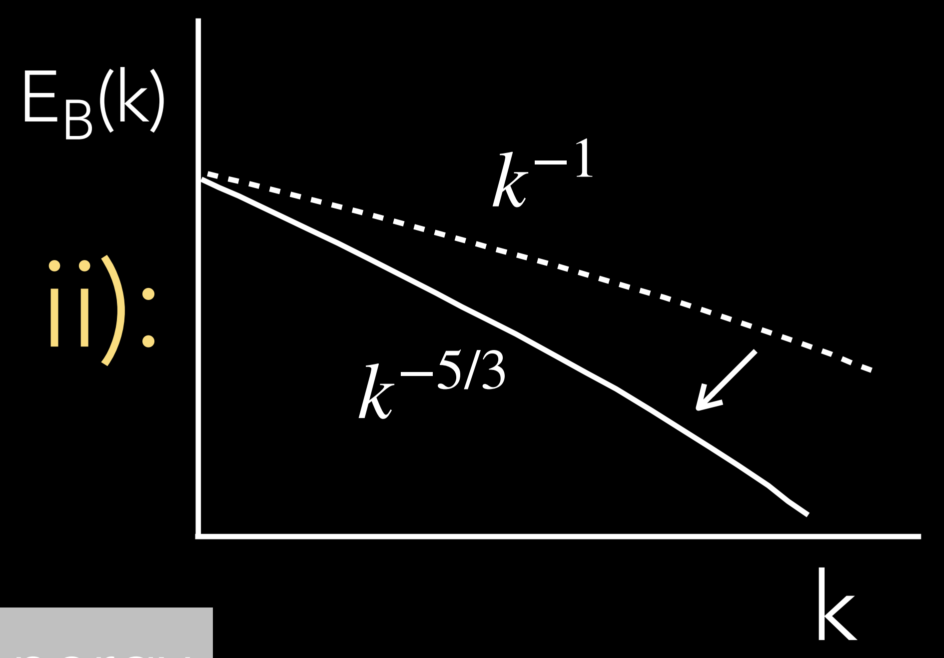
Inflationary:

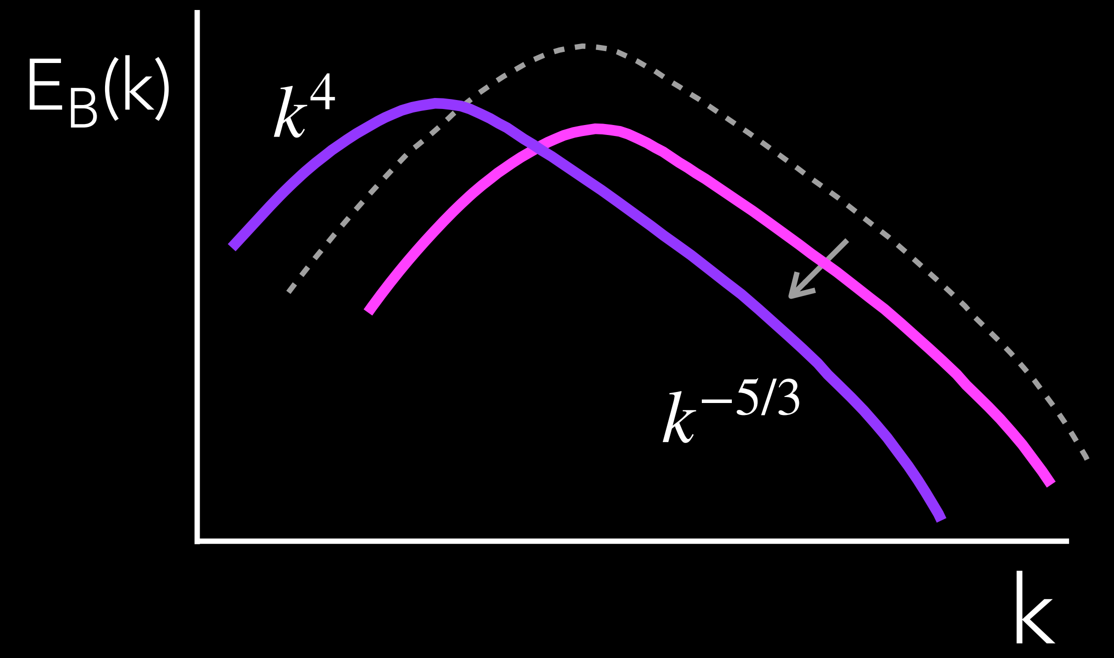
Phase-transitional:

i): $B = \text{const}$

$E_B(k) = \delta(\text{k=0})$

ii):

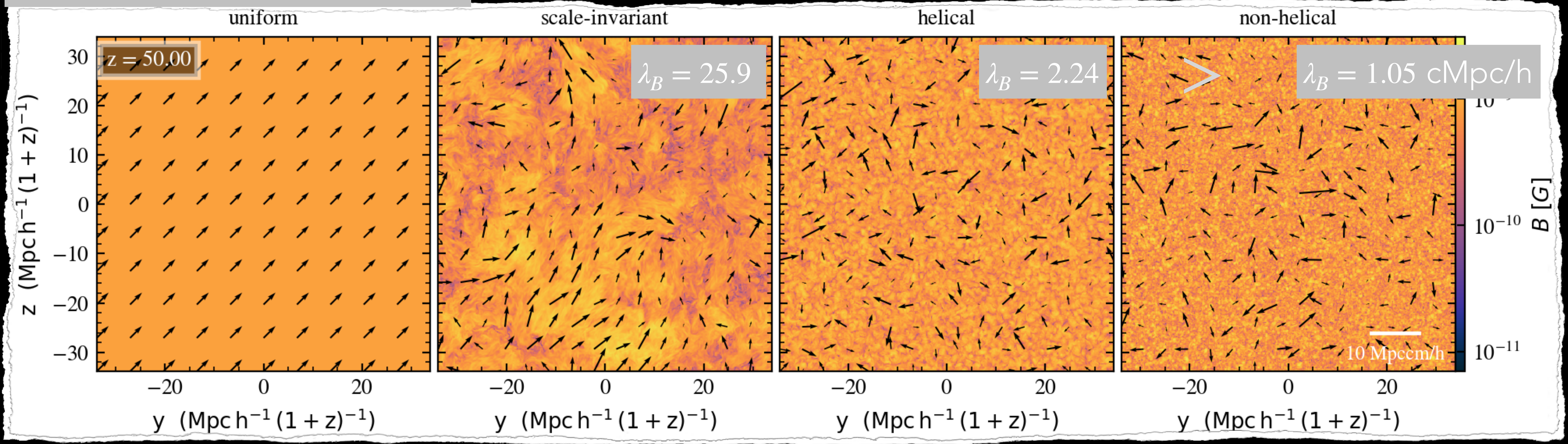




iii): **H ≠ 0**

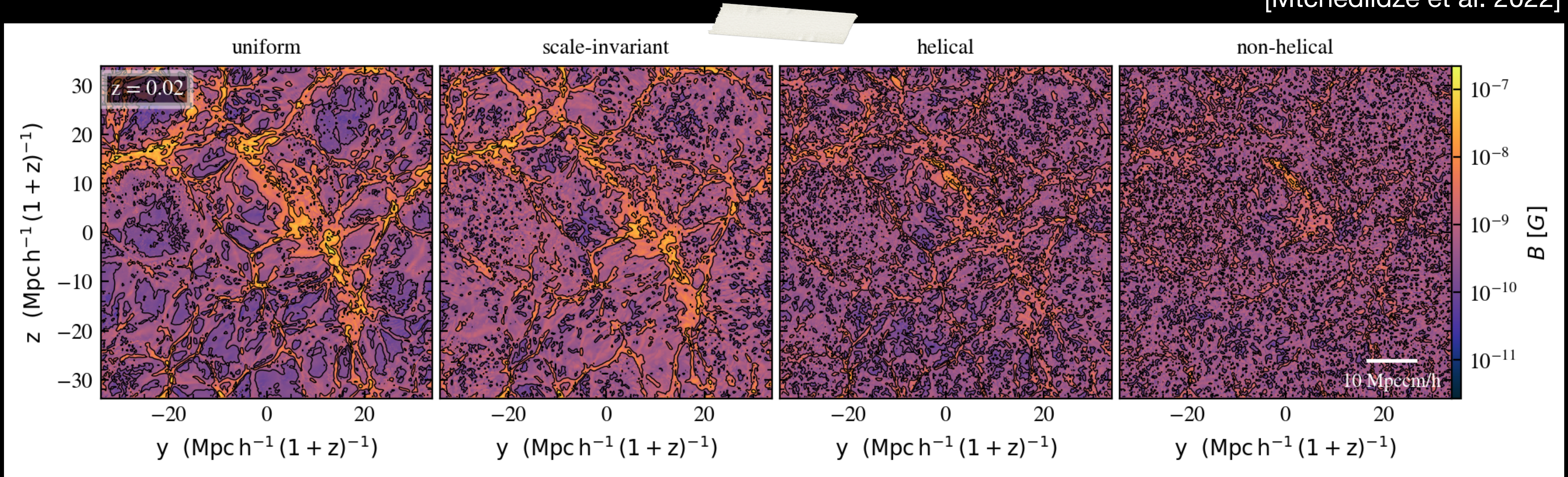
iv): **H = 0**

The same total magnetic energy



Final picture

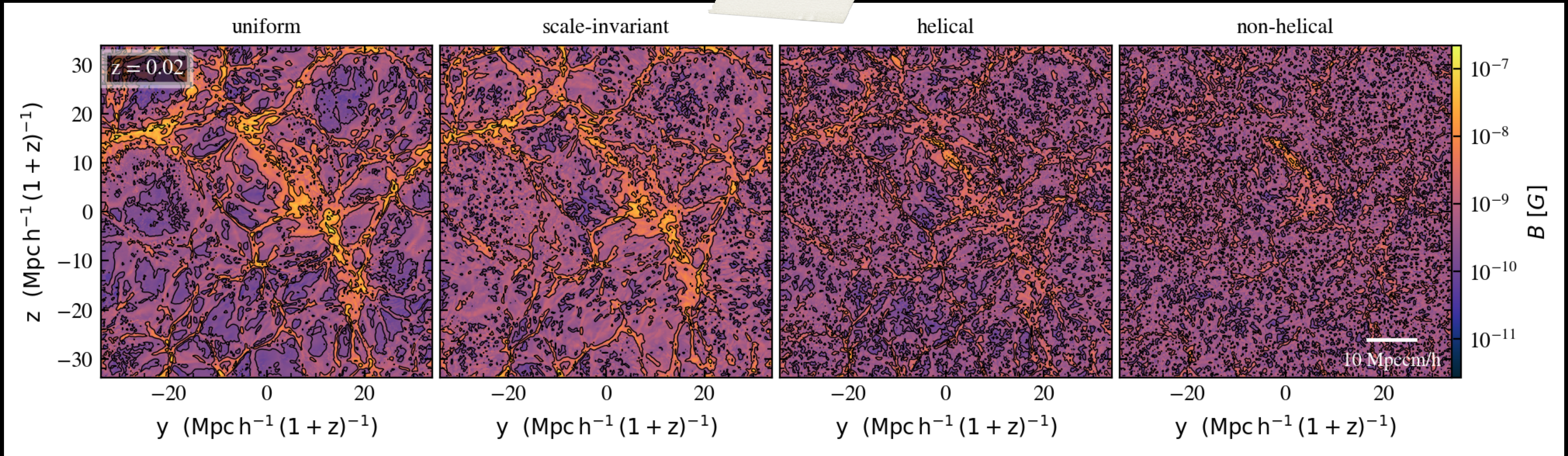
[Mtchedlidze et al. 2022]



The magnetised cosmic web shows dependence on the initial topology of PMF

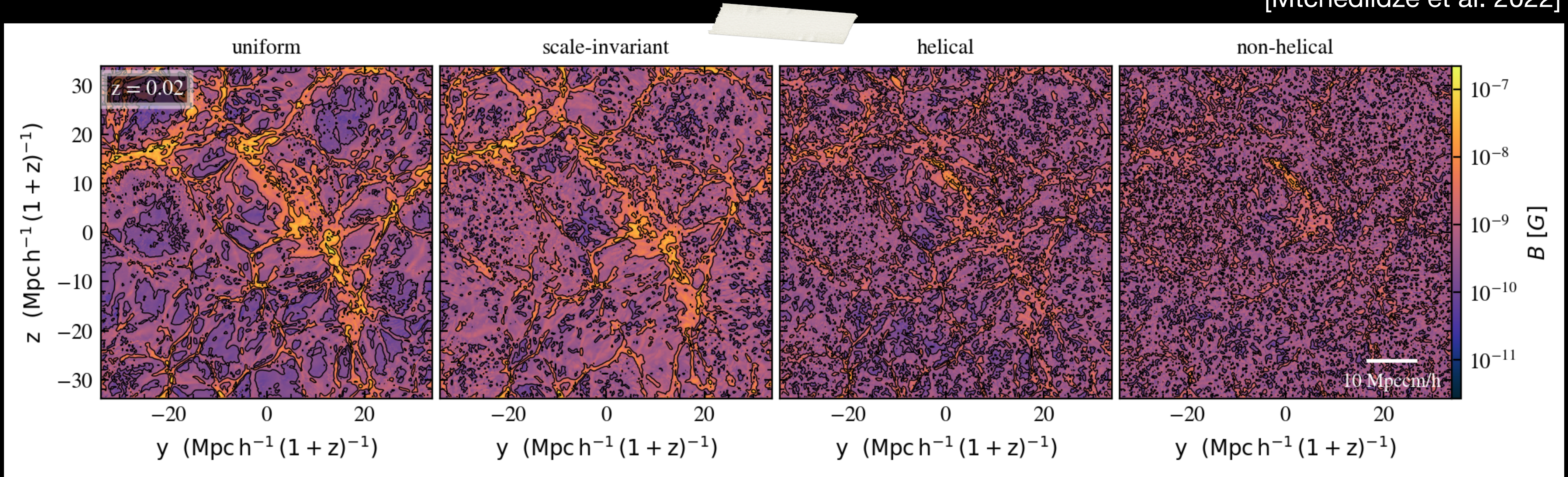
Final picture

[Mtchedlidze et al. 2022]



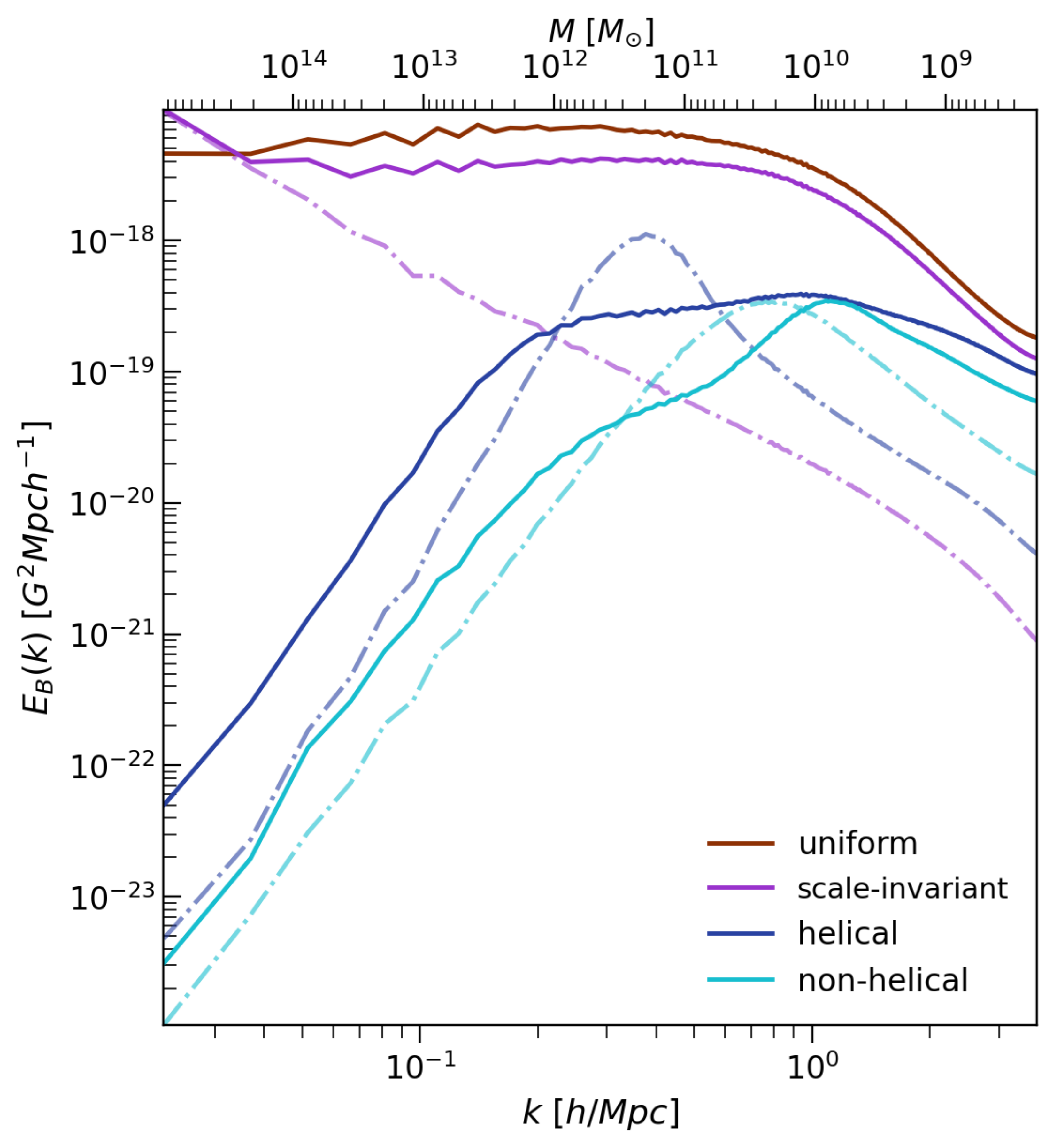
Final picture

[Mtchedlidze et al. 2022]

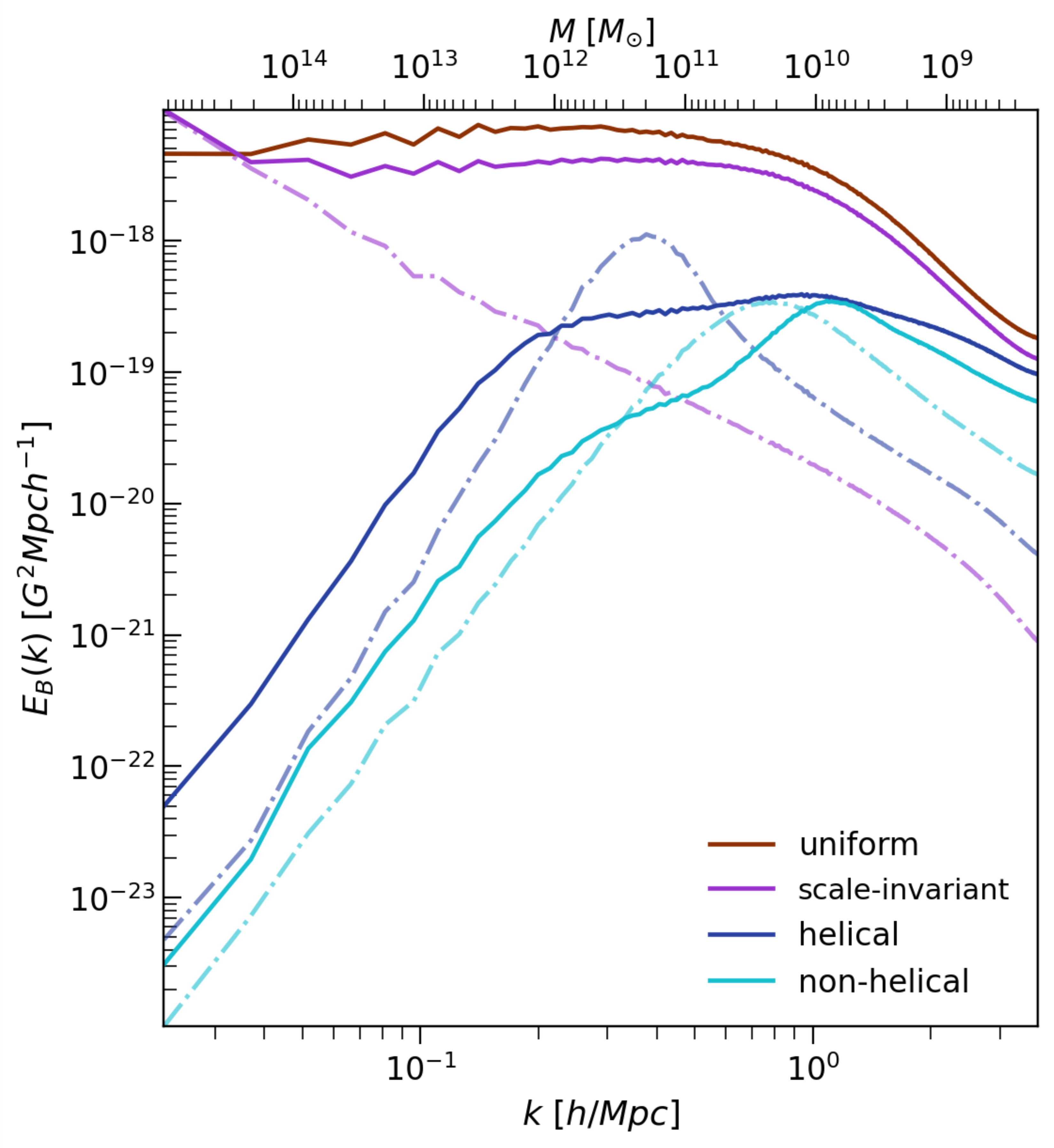


Amplification in filamentary structures is more efficient for an initially large-scale fields

Final picture



Final picture



Correlation length:

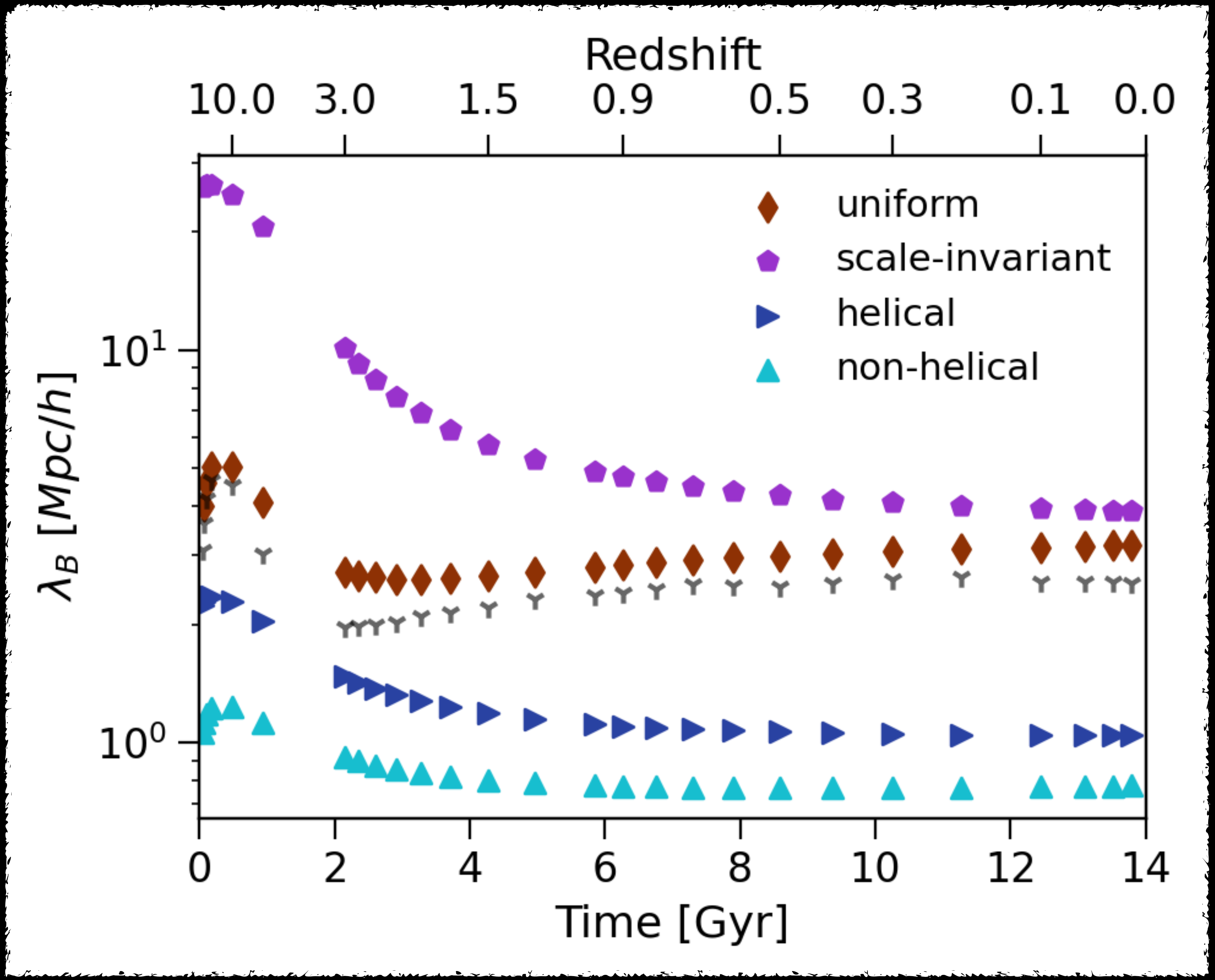
$$\lambda_B = \frac{\int_0^\infty dk\, k^{-1} E_B(k,t)}{\int_0^\infty dk\, E_B(k,t)}$$

Final picture

Correlation length:

$$\lambda_B = \frac{\int_0^\infty dk\, k^{-1} E_B(k,t)}{\int_0^\infty dk\, E_B(k,t)}$$

Final picture



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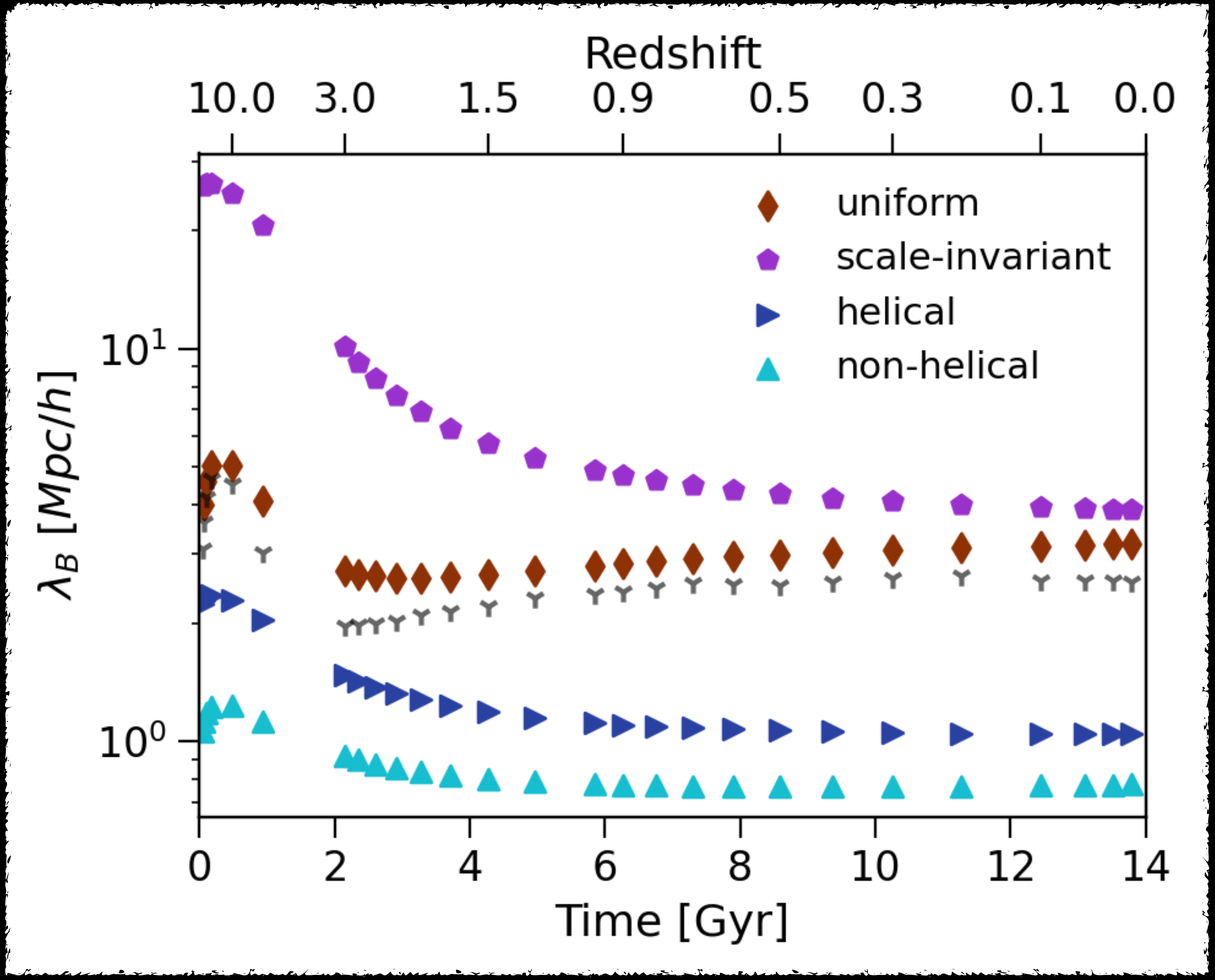
Final picture



3-4 cMpc/h

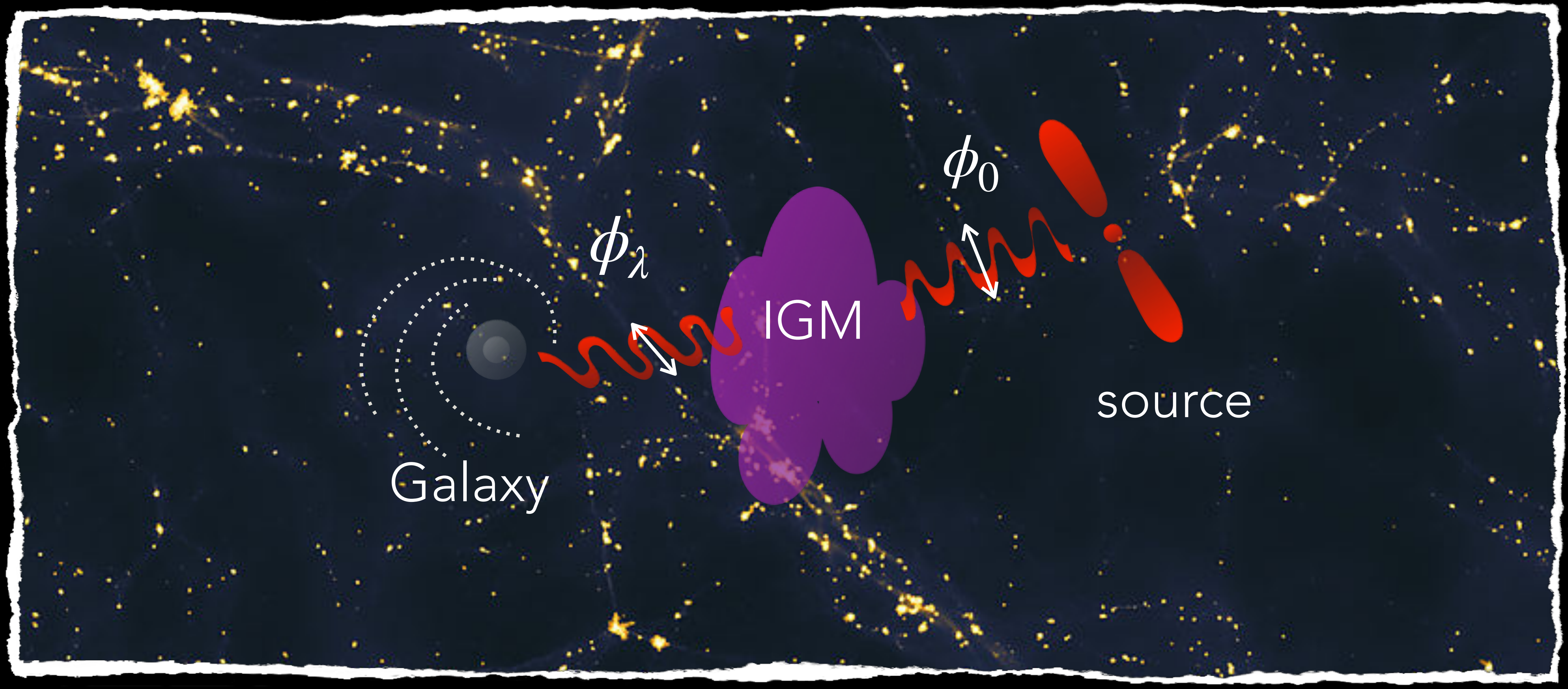
0.8-1 cMpc/h

Correlation length:

$$\lambda_B = \frac{\int_0^\infty dk\, k^{-1} E_B(k,t)}{\int_0^\infty dk\, E_B(k,t)}$$

Smallest final correlation lengths from phase-transitional models

Faraday Rotation Measure (RM)



$$\Delta\phi = \phi_\lambda - \phi_0 = \mathrm{RM}\ \lambda^2$$

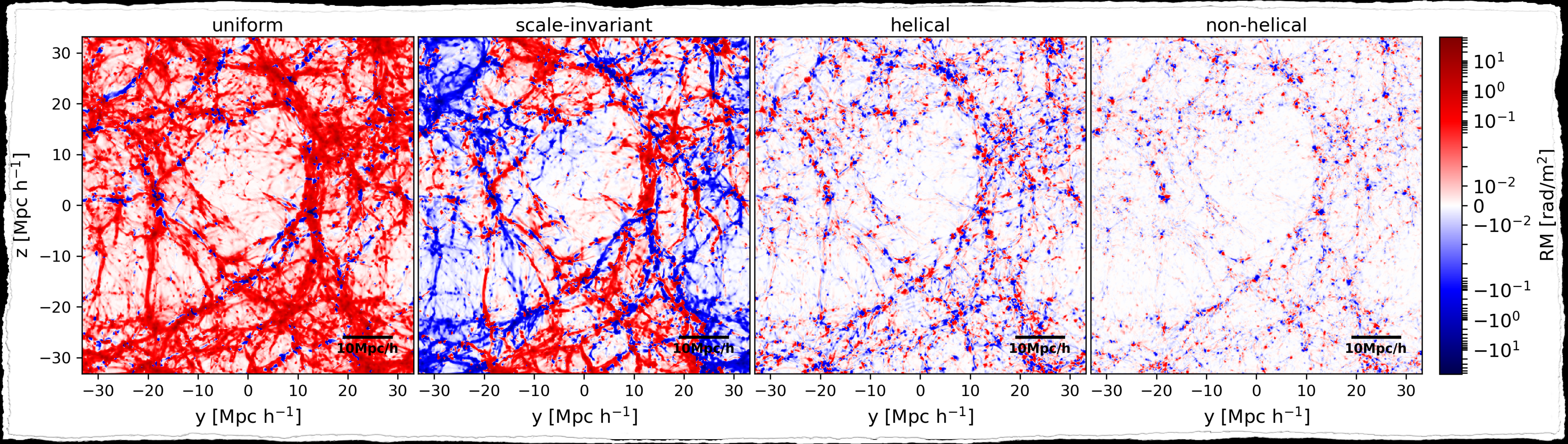
Observed polarisation

Initial polarisation

$$\mathrm{RM} \propto \int_0^L n_e B_l dl$$

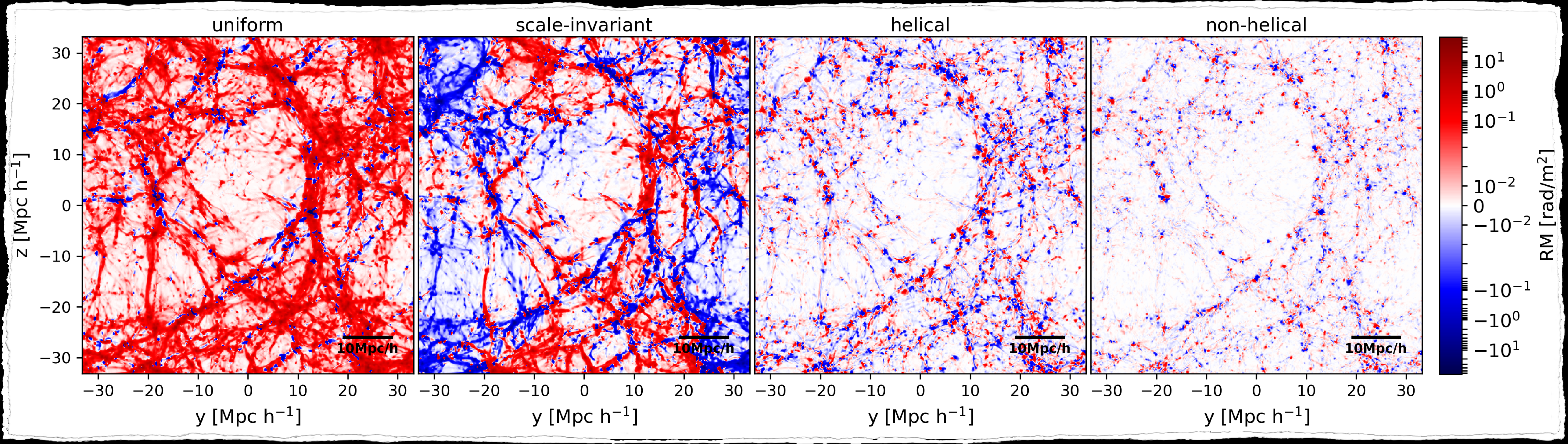
RM traces the LOS magnetic field strength

Simulated RM

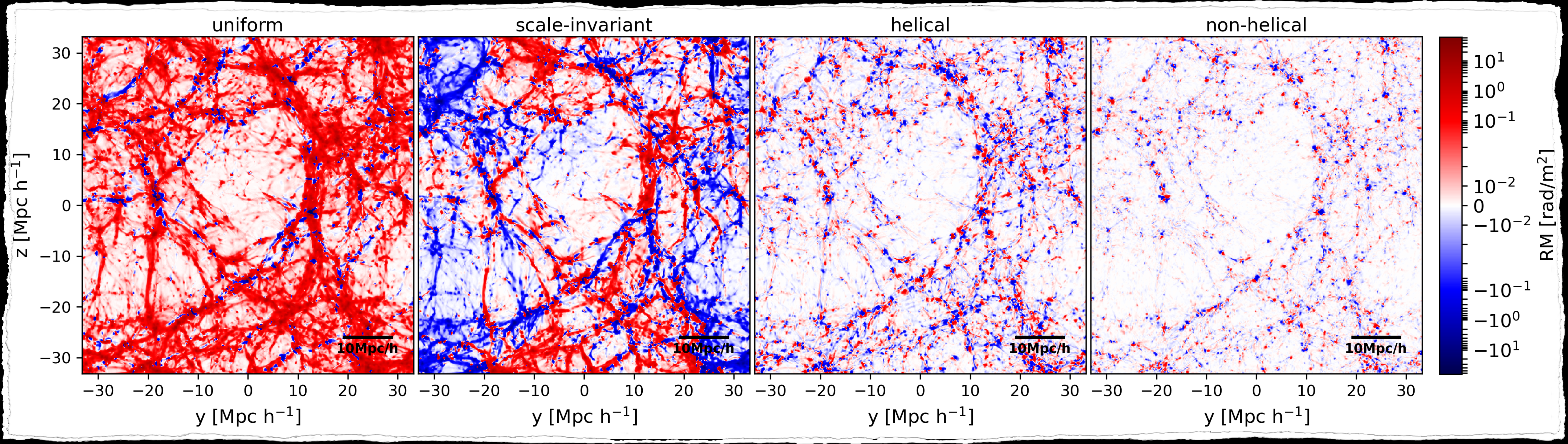


Due to RM being integrated quantity small-scale stochastic fields give rise to more incoherent structures in RM distribution

Simulated RM



Simulated RM

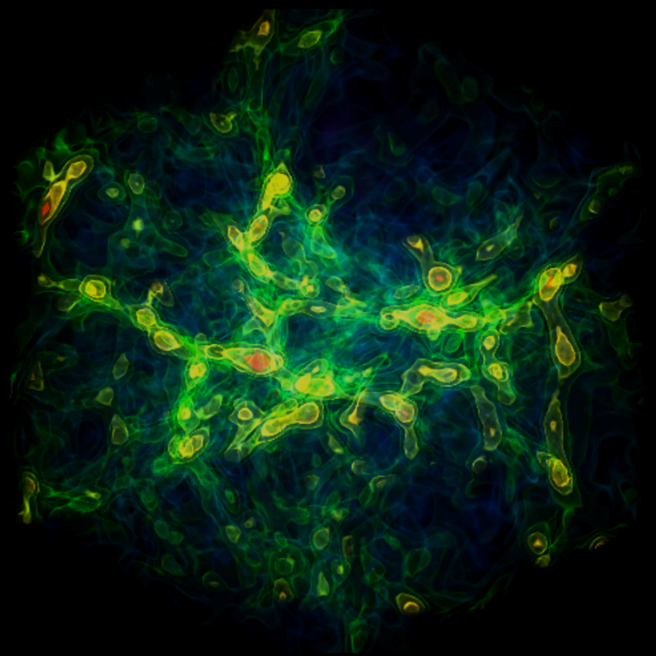


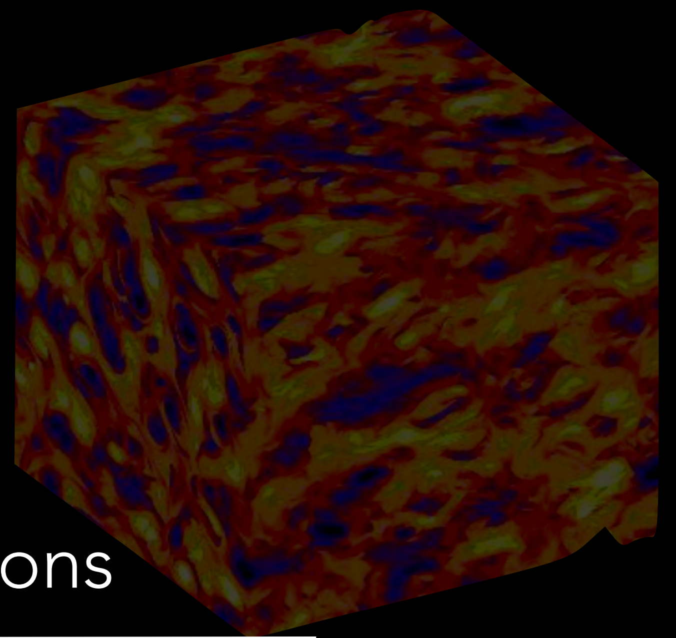
Largest and correlated signal from inflation-generated seed fields

Wrapping up and conclusions

We studied the evolution of inflation- and phase-transition generated PMFs

Topology dependent

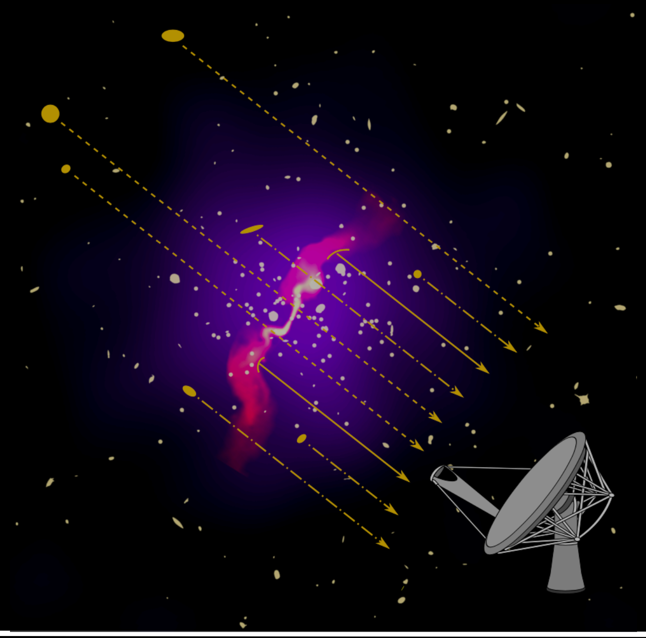




Realistic initial conditions

- Large scale fields lead to larger magnetisation level in filamentary structures (*inflation-generated PMFs*)
- Small scale, stochastic fields can heat IGM more effectively (*phase-transition generated PMFs*)
- RM analysis favours inflationary seeds also phase-transition seed fields cannot be excluded

Can future observations with SKA detect the signatures of PMFs on filamentary scales?



Thanks for listening, questions?

salome.mtchedlidze@stud.uni-goettingen.de

The projects are funded in part by Shota Rustaveli National Science Foundation of Georgia (partially supported by grants PHDF-19-4101, FR-18-1462), DAAD and Volkswagen foundation.

Helical magnetic fields lead to baryogenesis

Ashu Kushwaha

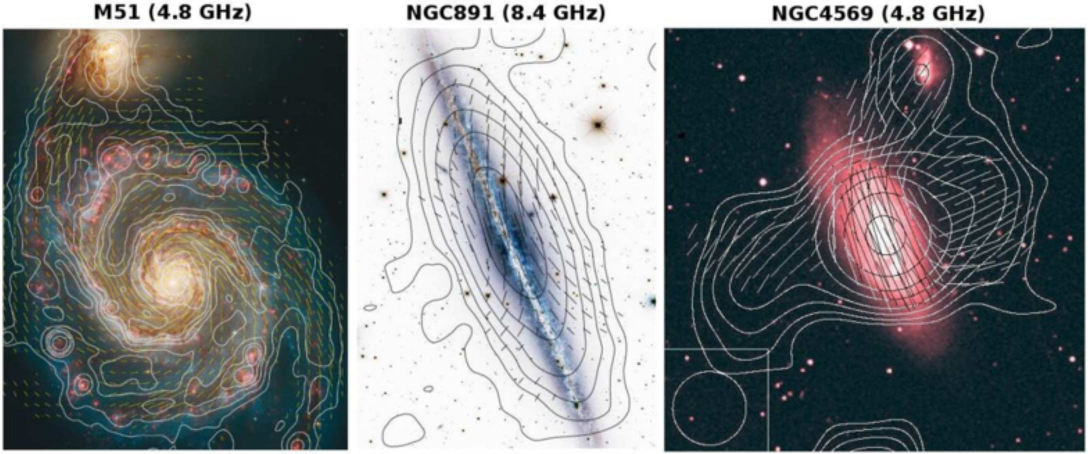
Department of Physics, IIT Bombay, India

Based on the works with **S. Shankaranarayanan**,
PRD 102,103528 (2020) and PRD 104, 063502 (2021)

ICTP cosmology summer school 2022 (contributed talk)

5th July, 2022

Observations of magnetic fields in the universe

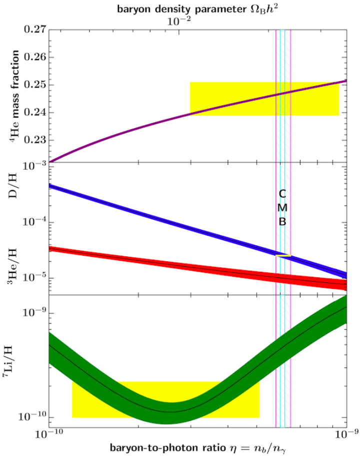


Max Planck Institute for Radio Astronomy

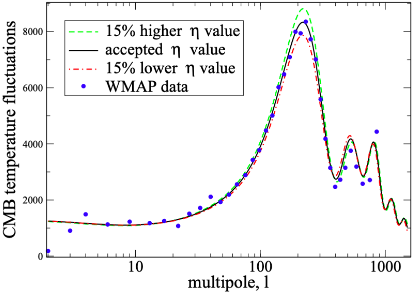
Micro-Gauss strength magnetic field over $10kpc$ coherence length scale is present in galaxies.

Origin of cosmological magnetic fields is still an unresolved problem.

Matter-Antimatter asymmetry

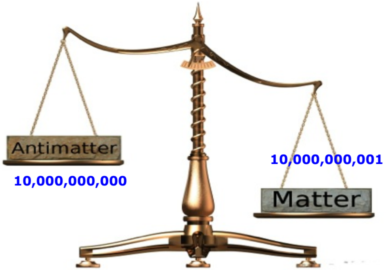


Particle Data Group (2019)



J. Cline (2006)

$$\eta_B \equiv \frac{n_B - n_{\bar{B}}}{n_\gamma} \simeq 6.1 \times 10^{-10}$$



Sakharov's conditions

Sakharov (1967)

Sakharov proposed three necessary conditions for creating the baryon asymmetry

- Baryon number violation
- Charge (C) and charge parity (CP) violation
- Departure from thermal equilibrium

Broken symmetries in the presence of magnetic field

Davidson, PLB (1996)

Davidson pointed out **an interesting relation between the primordial magnetic field and Sakharov's conditions**

- There should be some out of thermal equilibrium dynamics because in equilibrium, the photon distribution is thermal, and there are no particle currents to sustain a "long-range" field
- Since $\overrightarrow{B}$ is odd under C and CP, the presence of magnetic field will lead to CP violation.
- Since the magnetic field is a vector quantity, it chooses a particular direction hence breaks the isotropy (rotational invariance).

Presence of magnetic fields satisfy only two of Sakharov's conditions

Davidson's conditions are necessary but not sufficient. There is a key missing ingredient.

Missing ingredient: Helical magnetic fields

- For helical magnetic fields both polarization modes propagate differently and leading to non-zero **Chern-Simons number density.**
- Baryon number density $n_B = n_b - n_{\bar{b}} = a(\eta)\langle 0|J_A^0|0\rangle = \frac{e^2}{4\pi^2}a(\eta)n_{CS}$, where Chern Simon number density is

$$n_{CS} = \frac{1}{a^4}\int_{\mu}^{\Lambda}\frac{dk}{k}\frac{k^4}{2\pi^2}\left(|A_+|^2 - |A_-|^2\right) \tag{1}$$

- For **non-helical fields** $|A_+| = |A_-|$ which implies $n_{CS} = 0$.
- For **helical fields**, $n_{CS} \neq 0$ implies an imbalance between baryons and anti-baryons $\longrightarrow$ **Baryon number violation**

Hence the requirement of helical magnetic fields

to have non-zero n_{CS} is missing in Davidson's conditions.

The model

- Non-minimal coupling to the Riemann tensor generates sufficient primordial helical magnetic fields at **all observable scales.**
- **Necessary condition :** Conformal invariance breaking + parity violation

$$S = \overbrace{-\frac{M_{\rm P}^2}{2}\int d^4x\sqrt{-g}\,R}^{\text{Einstein-Hilbert term}} + \overbrace{\int d^4x\sqrt{-g}\left[\frac{1}{2}\partial_\mu\phi\partial^\mu\phi - V(\phi)\right]}^{\text{Scalar field}}$$
$$- \frac{1}{4}\int d^4x\,\sqrt{-g}\,F_{\mu\nu}F^{\mu\nu} \underbrace{- \frac{1}{M^2}\int d^4x\,\sqrt{-g}\,R_{\mu\nu}{}^{\alpha\beta}F_{\alpha\beta}\,\tilde{F}^{\mu\nu}}_{\text{Conformal breaking}} \qquad (2)$$

where M is the energy scale, which sets the scale for the breaking of conformal invariance.

- Strength of the helical magnetic fields at two different scales:

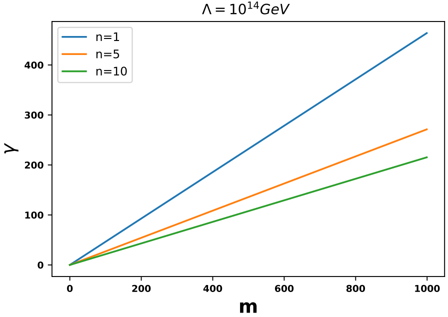
$$B|_{50\ \rm MPc} \sim 10^{-18}\ G\ (z \sim 20)\ ;\quad B|_{1\ \rm MPc} \sim 10^{-14}\ G\ (z \sim 1000)$$

Baryon asymmetry parameter

- Baryon asymmetry parameter

$$\eta_B = \frac{n_B}{s} \approx 10^{-2}\left(\frac{M}{M_P}\right)^3\left(\frac{\Lambda}{T_{\mathrm{RH}}}\right)^3 \tag{3}$$

- Using the parametrization : $M = m \times 10^{14}\,GeV,\ T_{RH} = \gamma \times 10^{12}\,GeV.$



Conclusion

- Since the curvature is large in the early Universe, the coupling term will introduce non-trivial corrections to the electromagnetic action.
- We have explicitly shown that Davidson's conditions are necessary but not sufficient. The key missing ingredient is the requirement of helical magnetic fields.
- The BAU parameter predicted by our model is independent of any specific inflation model and reheating dynamics; however, it depends on the scale at which inflation ends and reheating temperature.

Thank you

Searching for spin-2 ULDM with GW interferometers*

Juan Manuel Armaleo
(jarmaleo@df.uba.ar)

ICTP Summer School on Cosmology

Trieste, Italy. July 2022

* JCAP 04 (2021) 053, J.M. Armaleo, D.L. Nacir & F.R. Urban

Overview

Task: If Ultra-light Dark Matter (**ULDM**) has spin-2, it interacts with Gravitational Waves Interferometers (**GWIs**) in a way that, owing to its quasi-monochromaticity and persistence, closely resembles Continuous GWs (CWs).

The Spin-2 model

In a FLRW background

$$\ddot{M}_{ij} + 3H\dot{M}_{ij} - \triangle M_{ij} + m^2 M_{ij} = 0$$

On the astrophysical scales ($m \gg H$), the solution is

$$M_{ij} = \frac{\sqrt{2\rho_{\mathrm{DM}}}}{m} \cos\left(mt + \Upsilon\right)\varepsilon_{ij}$$

where ρ_{DM} is the observed local DM energy density ($\rho_{\mathrm{DM}} = 0.3$ GeV/cm^3), Υ is a random phase and the five polarisations of the spin-2 field are encoded in the ε_{ij} tensor, which has *unit norm*, *zero trace* and is *symmetric*.

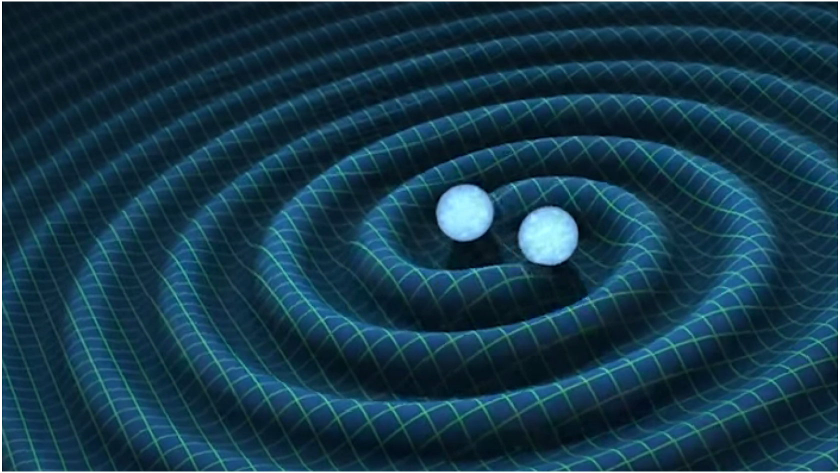
The Spin-2 model

$$M_{ij} = \frac{\sqrt{2\rho_{\mathrm{DM}}}}{m}\cos\left(mt + \Upsilon\right)\varepsilon_{ij}$$

- The solution assumes a **single frequency** $2\pi f = m$ and a **coherent polarisation structure** ($\lambda_{\mathrm{dB}}^{DM} \sim 4\mathrm{kpc}\left(\frac{10^{-3}}{V}\right)\left(\frac{10^{-24}\mathrm{eV}}{m}\right) \gg L$, where L : physical size of the GWIs).
- The coherence of the oscillation frequency, guaranteed up to a **coherence time** given by $t_{\mathrm{coh}} := 4\pi/mv^2 \sim 10^6\mathrm{yr}\left(\frac{10^{-3}}{V}\right)^2\left(\frac{10^{-24}\mathrm{eV}}{m}\right)$

GWIs

- GWs detections $\longrightarrow$ product of cataclysmic transient events (e.g. BBH merger), i.e. **strong** events ($h \sim 10^{-21}$).



GWIs

- GWs detections $\longrightarrow$ product of cataclysmic transient events (e.g. BBH merger), i.e. **strong** events ($h \sim 10^{-21}$).
- There are other events (e.g. rapidly spinning NS, ultra-compact Galactic binaries, superradiance) $\longrightarrow$ **weak** signal ($h \sim 10^{-25}$) but coherent over a longer time $\Longrightarrow$ continuous GWs (**CWs**).

The signal

Spin-2 ULDM couples to standard matter as

$$S_{\mathrm{int}}[g, M_{ij}, \Psi] := -\frac{\alpha}{2M_{\mathrm{P}}} \int \mathrm{d}^4x \sqrt{-g} M_{ij} T_{\Psi}^{ij}$$

α: strength of the interaction.

The signal

Spin-2 ULDM couples to standard matter as

$$S_{\text{int}}[g, M_{ij}, \Psi] := -\frac{\alpha}{2M_{\mathrm{P}}} \int \mathrm{d}^4x \sqrt{-g} M_{ij} T_{\Psi}^{ij}$$

α: strength of the interaction. The interaction can be absorbed into a *redefinition of the metric*

$$\tilde{g}_{ij} \to g_{ij} + \frac{\alpha M_{ij}}{M_{\mathrm{P}}}$$

$\implies$ the effect of spin-2 ULDM on the detector can be equivalently described by the **gravitational effect** of an **oscillating metric perturbation** h_{ij} given by

$$h_{ij}(t) = \frac{\alpha}{M_{\mathrm{P}}} M_{ij}(t) = \frac{\alpha\sqrt{2\rho_{\mathrm{DM}}}}{mM_{\mathrm{P}}} \cos(mt + \Upsilon)\varepsilon_{ij}$$

The signal

The **signal** is the combination of the variation of the metric perturbation and the **response function** (the differential change in the length of the detector arms) of the detector

$$D^{ij} = \frac{(n^i n^j - m^i m^j)}{2}$$

n and **m** arms of the detector.

$$\boxed{h(t) := D^{ij} h_{ij}(t) := h_s \sin(mt) + h_c \cos(mt)}$$

h_s and h_c sine and cosine amplitudes.
We define the **effective theoretical strain amplitude** h for the spin-2 ULDM-CW signal as

$$h := \langle h_s^2 + h_c^2 \rangle^{1/2} = \frac{\alpha \sqrt{\rho_{\mathrm{DM}}}}{\sqrt{5} m M_{\mathrm{P}}}$$

where $\langle \cdot \rangle$ taken over polarisation angles and random phase Υ.

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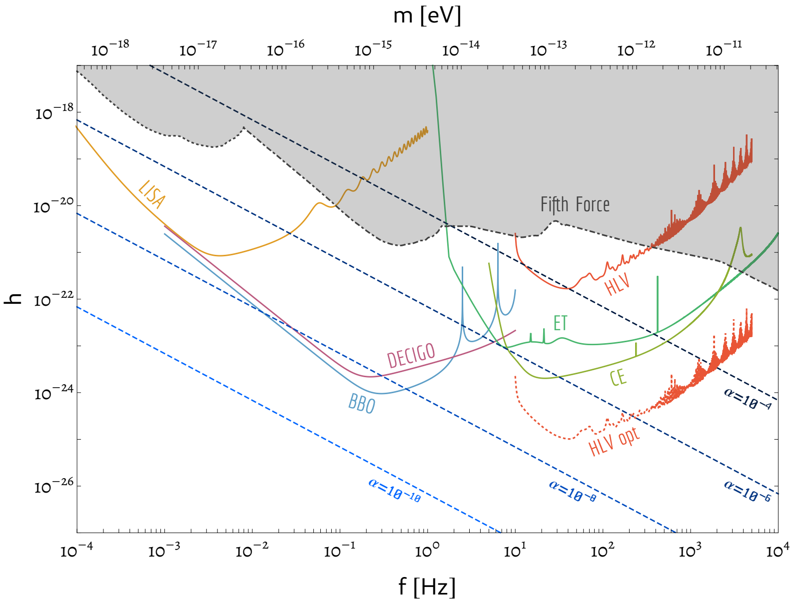
where $\langle \cdot \rangle$ taken over polarisation angles and random phase Υ.
$\longrightarrow$ we **compare** the *expected theoretical signal* h with the *design sensitivities* of a number of current and planned GWIs.

Results



m [eV]
10^{-18}
10^{-17}
10^{-16}
10^{-15}
10^{-14}
10^{-13}
10^{-12}
10^{-11}
h
10^{-18}
10^{-20}
10^{-22}
10^{-24}
10^{-26}
LISA
Fifth Force
HLV
ET
DECIGO
BBO
CE
HLV opt
$\alpha=10^{-4}$
$\alpha=10^{-6}$
$\alpha=10^{-8}$
$\alpha=10^{-10}$
10^{-4}
10^{-3}
10^{-2}
10^{-1}
10^{0}
10^{1}
10^{2}
10^{3}
10^{4}
f [Hz]

Thank you! 😄

Extra

A massive spin-2 field $M_{\mu\nu}$ described by the Fierz-Pauli lagrangian density

$$\mathcal{L} := \frac{1}{2} M_{\mu\nu} \mathcal{E}^{\mu\nu\rho\sigma} M_{\rho\sigma} - \frac{1}{4} m^2 \left(M_{\mu\nu} M^{\mu\nu} - M^2 \right)$$

Lichnerowicz operator

$$\mathcal{E}^{\mu\nu}{}_{\rho\sigma} := \delta^{\mu}_{\rho} \delta^{\nu}_{\sigma} \Box - g^{\mu\nu} g_{\rho\sigma} \Box + g^{\mu\nu} \nabla_{\rho} \nabla_{\sigma} + g_{\rho\sigma} \nabla^{\mu} \nabla^{\nu} - \delta^{\mu}_{\sigma} \nabla^{\nu} \nabla_{\rho} - \delta^{\mu}_{\rho} \nabla^{\nu} \nabla_{\sigma}$$

Extra

Searches for CWs with Earth-bound GWIs resort to **semi-coherent** methods $\longrightarrow$ not computationally feasible to analyse the data from the entire observation campaign in a fully coherent way.
The whole data set is broken into shorter time chunks such that $T_{\text{obs}} = NT_{\text{chunk}}$ ($T_{\text{chunk}} < t_{\text{coh}}$).

Bispectrum Covariance

Lina Julieth Castiblanco Tolosa

Based on : M.Biagetti, L.C, J.Noreña, E. Sefusatti (2021) arXiv:2111.05887

PONTIFICIA
UNIVERSIDAD
CATÓLICA DE
VALPARAÍSO

Primordial non-Gaussianity

The hot cosmological stage was preceded by the epoch of exponential expansion: Inflation

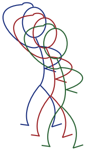
Inflation provides a mechanism to generate primordial perturbations which are the seed to structure formation.

Single field inflation?

Initial conditions: Gaussian, adiabatic and almost scale invariant.

$$\langle \zeta(\boldsymbol{k})\zeta(\boldsymbol{k}')\rangle = (2\pi)^3\delta_D(\boldsymbol{k}+\boldsymbol{k}')P_\zeta(k)$$

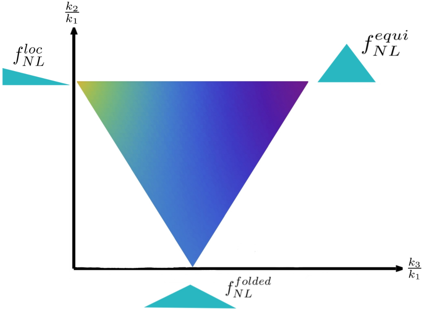
Multi-field inflation? Exotic Mechanism?



Predict large non-Gaussianity.

The **Bispectrum** is a good observable to detect primordial non-Gaussianity

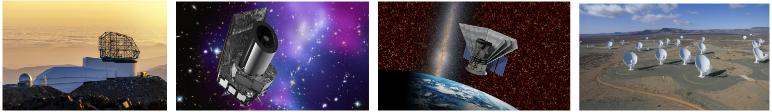
$$\langle \delta_g(\boldsymbol{k}_1)\delta_g(\boldsymbol{k}_2)\delta_g(\boldsymbol{k}_3)\rangle = (2\pi)^3\delta_D(\boldsymbol{k}_1+\boldsymbol{k}_2+\boldsymbol{k}_3)B(\boldsymbol{k}_1,\boldsymbol{k}_2,\boldsymbol{k}_3)$$



We are interested in f_{NL}^{loc}

$f_{NL}^{loc} = -0.9 \pm 5.1$ at 68% CL $\qquad k_1 \ll k_2 \sim k_3$

Planck Collaboration, Planck 2018 results.



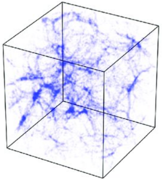
Bispectrum covariance

$$C_{ij}^{X} \equiv \left\langle \left(\hat{X}_i - \left\langle \hat{X}_i \right\rangle\right) \left(\hat{X}_j - \left\langle \hat{X}_j \right\rangle\right) \right\rangle$$

To measure the covariance in simulations it is necessary to use a large number of realizations.

We need an accurate model for the theoretical covariance of the galaxy bispectrum.

$$C = \begin{pmatrix} \mathbf{C}^{P} & \mathbf{C}^{PB} \\ \mathbf{C}^{BP} & \mathbf{C}^{B} \end{pmatrix}$$



$k_f = \frac{2\pi}{L} \rightarrow$ Fundamental mode
$k \rightarrow$ Center of the shell
$\Delta k \rightarrow$ Width of the bin

Power spectrum estimator

$$\hat{P}(k) = \frac{k_f^3}{N_k} \sum_{\boldsymbol{q} \in k} \delta(\boldsymbol{q})\delta(-\boldsymbol{q})$$

Bispectrum estimator

$$\hat{B}\left(k_1, k_2, k_3\right) = \frac{k_f^3}{N_t\left(k_1, k_2, k_3\right)} \sum_{\boldsymbol{q_1} \in k_1} \sum_{\boldsymbol{q_2} \in k_2} \sum_{\boldsymbol{q_3} \in k_3} \delta_k\left(\boldsymbol{q}_{123}\right) \delta(\boldsymbol{q}_1)\delta(\boldsymbol{q}_2)\delta(\boldsymbol{q}_3)$$

R.Scoccimarro, M.Zaldarriaga, and L.Hui (1999) E.Sefusatti, M.Crocce, Martin, S.Pueblas, R.Scoccimarro

Bispectrum covariance

$$C_{ij}^{X} \equiv \left\langle \left(\hat{X}_i - \left\langle \hat{X}_i \right\rangle \right) \left(\hat{X}_j - \left\langle \hat{X}_j \right\rangle \right) \right\rangle$$

To measure the covariance in simulations it is necessary to use a large number of realizations.

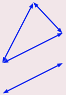
We need an accurate model for the theoretical covariance of the galaxy bispectrum.

$$C = \begin{pmatrix} \mathbf{C}^{P} & \mathbf{C}^{PB} \\ \mathbf{C}^{BP} & \mathbf{C}^{B} \end{pmatrix}$$

Power spectrum covariance

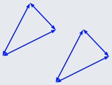
$$C_{ij}^{P} \simeq 2 \frac{\delta_{ij}^{K}}{N_{k_i}} P(k_i)^2$$

Cross-covariance P-B



$$C_{ij}^{PB} \simeq \frac{2}{N_{k_i}} P(k_i)\, B_j \left(\delta^{K}_{k_i, k_1^j} + 2\text{cyc} \right)$$

Model for the bispectrum covariance



$$C_{ij}^{B} \simeq \frac{\delta_{ij}\, s}{k_f^3 N_{tr}^{i}} P(k_1^i) P(k_2^i) P(k_3^i) + \frac{2 B(k_1^i, k_2^i, k_3^i) B(k_1^j, k_2^j, k_3^j)}{N_{tr}^{i} N_{tr}^{j}} \sum_{\mathbf{q}\text{'s} \in \{k\}\text{'s}} \delta_{\mathbf{q}_1^i + \mathbf{q}_2^i + \mathbf{q}_3^i}\, \delta_{\mathbf{q}_1^j + \mathbf{q}_2^j + \mathbf{q}_3^j}\, \delta_{\mathbf{q}_1^i + \mathbf{q}_1^j}$$

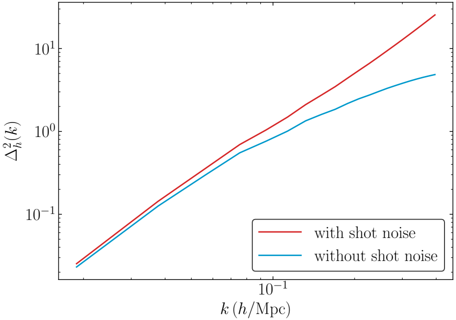
Orders of magnitude for C^B

Scales are comparable

$$\frac{C^{B,(BB)}}{C^{B,(PPP)}} \sim \left(\frac{\Delta k}{k}\right)^2 \Delta_h^2(k)$$

Squeezed configuration

$$\frac{C^{B,(BB)}}{C^{B,(PPP)}} \sim \left(\frac{\Delta k}{k_L}\right)^2 \Delta_h^2(k_L)\left(\frac{k_s^2}{k_L^2}+4\right)$$

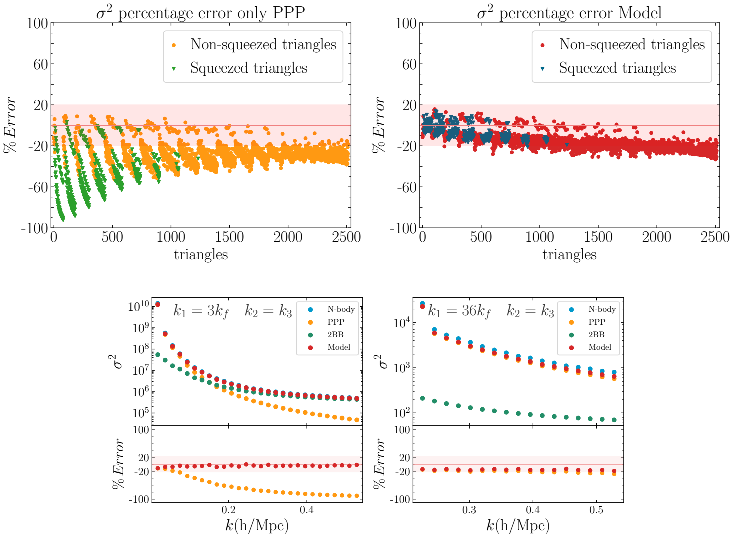


$$\Delta_h^2(k) = 4\pi k^3 P(k) \qquad \Delta k = 0.009 h/Mpc$$

Comparison with N-body simulations

https://quijote-simulations.readthedocs.io/en/latest/

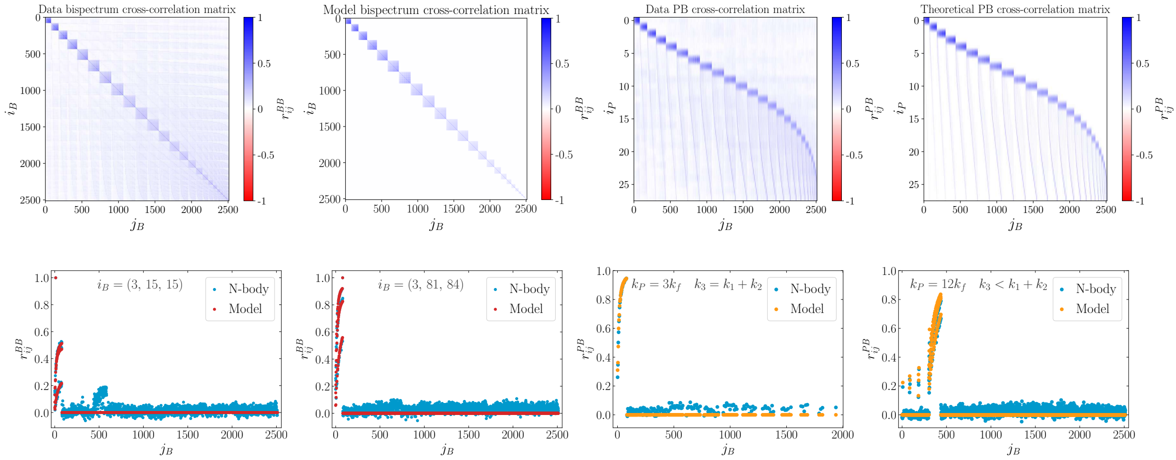
Bispectrum variance



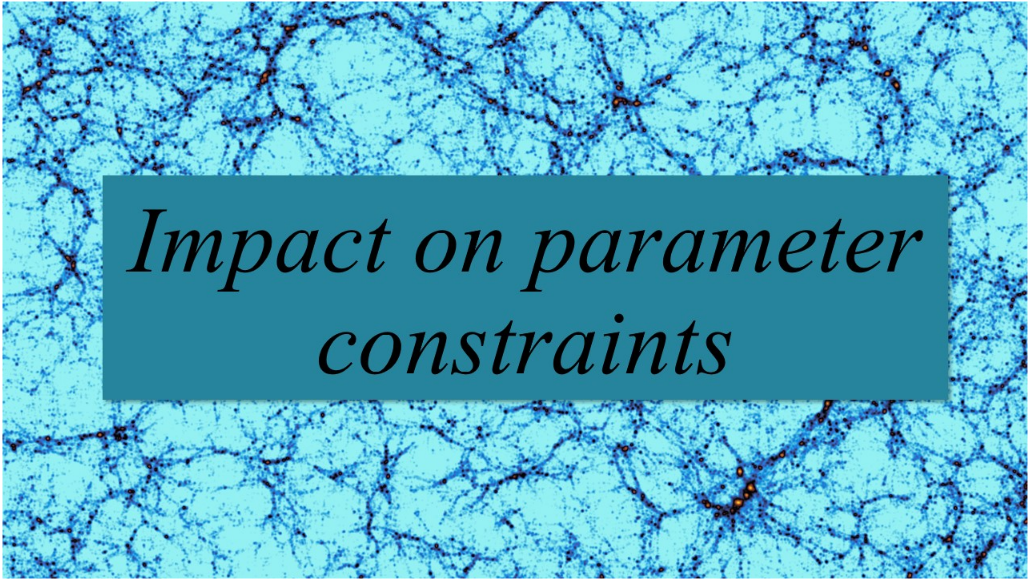
M.Biagetti, L.C, J.Noreña, E. Sefusatti (2021)

Correlation Matrix

$$r_{ij} = \frac{C_{ij}}{\sqrt{C_{ii}C_{jj}}}$$



M.Biagetti, L.C, J.Noreña, E. Sefusatti (2021)

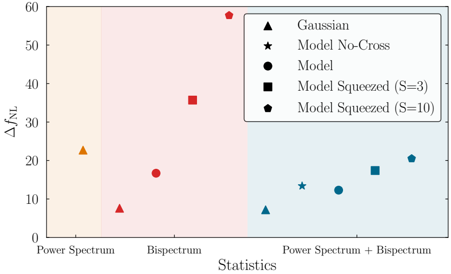

Impact on parameter constraints

Fisher analysis for PNG

$$F_{f_{NL}} = \frac{\partial \mathbf{D}}{\partial f_{NL}} \cdot \mathbf{C}^{-1} \cdot \frac{\partial \mathbf{D}^T}{\partial f_{NL}}$$

$$\frac{\partial \mathbf{D}}{\partial f_{NL}} = \frac{\mathbf{D}(\mathsf{NG250L}) - \mathbf{D}(\mathsf{NGm250L})}{2\bar{f}_{\mathrm{NL}}}$$

$$\Delta f_{NL}^{loc} = \frac{1}{\sqrt{F_{f_{NL}^{loc}}}}$$



	P	B	P+B
Gaussian	22.8	7.6	7.2
Model No-Cross	22.8	16.7	13.45
Model	22.8	16.7	12.5
Model Squeezed (S=3)	22.8	35.7	17.4
Model Squeezed (S=10)	22.8	57.7	20.5

https://mbiagetti.gitlab.io/cosmos/nbody/eos M.Biagetti, L.C, J.Noreña, E. Sefusatti (2021)


Thank you

Primordial black holes and gravitational waves from dissipative effects during inflation

Based on work in progress with G. Ballesteros, M.A.G. García, M. Pierre, J. Rey

Alejandro Pérez Rodríguez

Universidad Autónoma de Madrid, IFT UAM-CSIC

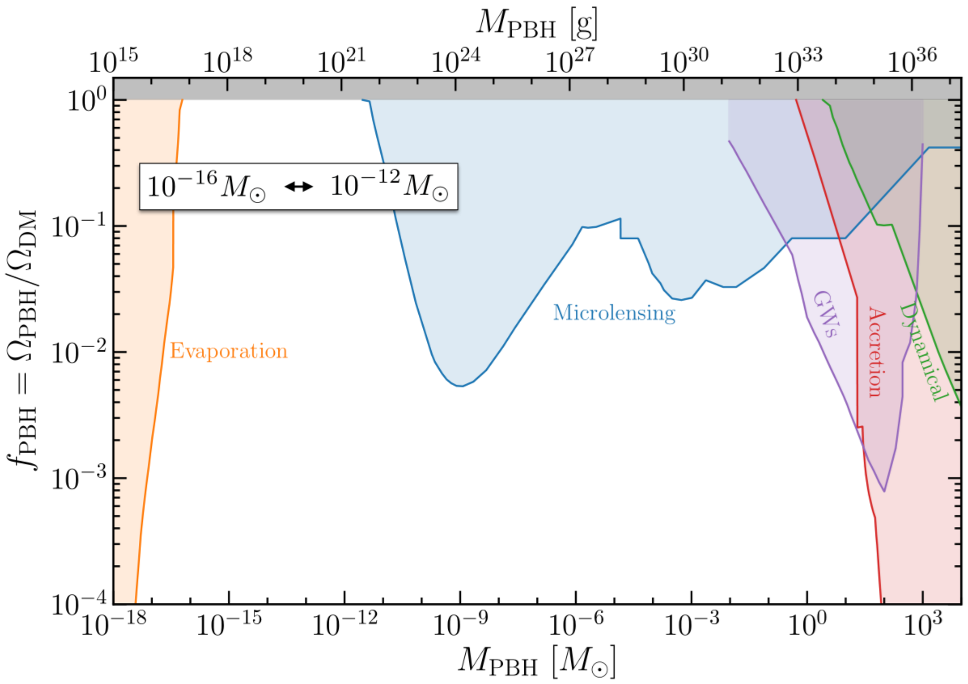
-ICTP summer school on Cosmology, july 2022-

BASIC CONCEPTS. Primordial black holes and GWs

- $\mathcal{P}_{\mathcal{R}}$ peaked at scale k enhances $\begin{cases} \Omega_{PBH}(M(k)) & \text{(Press-Schechter)} \\ \Omega_{GW}(f(k)) & \text{(Second-order scalar perturbations)} \end{cases}$

- Assuming **gaussianity**, certain value for δ_c, radiation domination… $\rightarrow$ $\boxed{\text{Peak} \sim 10^{-2} \rightarrow f_{PBH} \sim 1}$

- Corresponding GW background potentially detectable by **LISA**



Green, Kavanagh (2020, modified)

BASIC CONCEPTS. Dissipative effects during inflation

- Coupling between inflaton and radiation
- **Background**: extra friction $\ddot{\phi} + (3H + \Gamma) + V_{,\phi} = 0$
- **Perturbations**:
  - Introduce radiation perturbations
  - **Fluctuation – dissipation** → stochastic terms

$$\boxed{\begin{aligned} \delta\ddot{\phi} + [...] &= f_\phi(t)\xi(t) \\ \delta\dot{\rho}_r + [...] &= f_{\rho_r}(t)\xi(t) \\ \dot{\varphi} + [...] &= 0 \end{aligned}}$$

- C.f. **warm inflation**. *Arya (2019); Bastero-Gil, Subías Díaz-Blanco (2021), …*

SOLVING SDEs. Numerical approaches

Main idea: solve for the **thermally averaged** power spectrum $\mathcal{P}_{\mathcal{R}} = \frac{k^3}{2\pi^2}\langle|\mathcal{R}|^2\rangle$

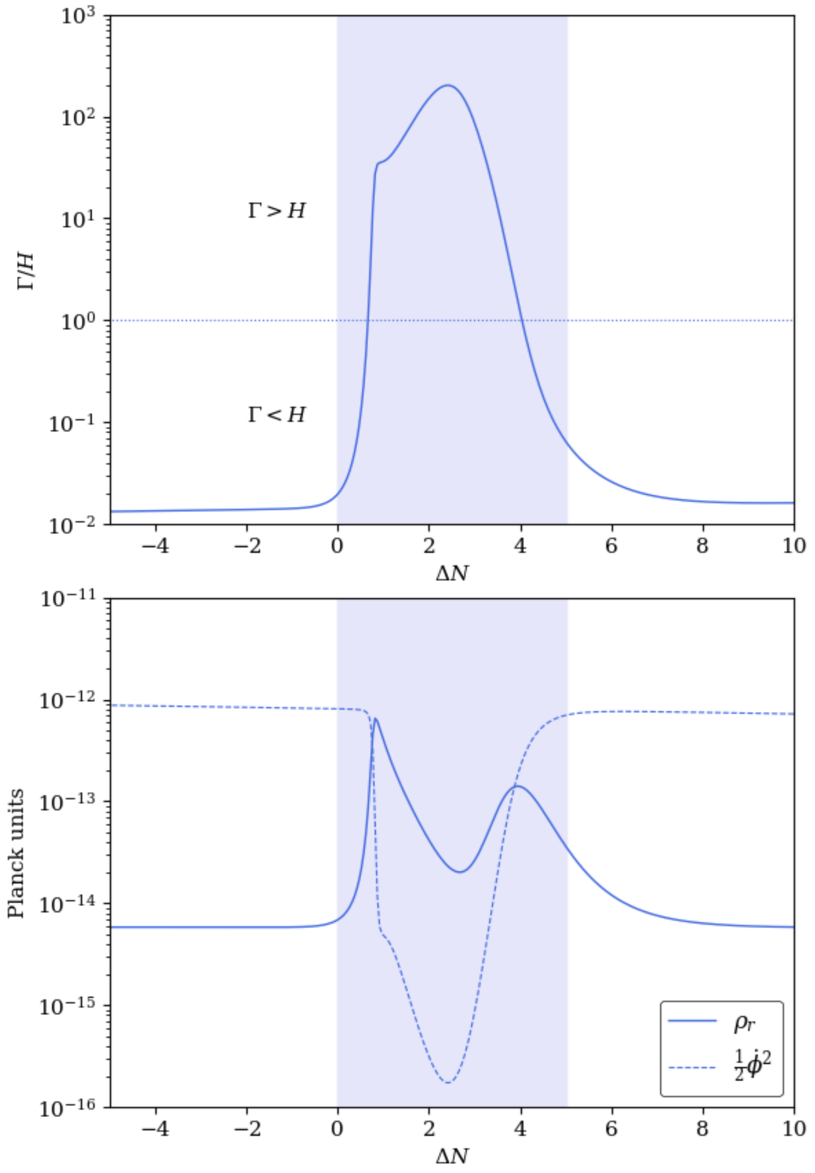
Fokker-Planck

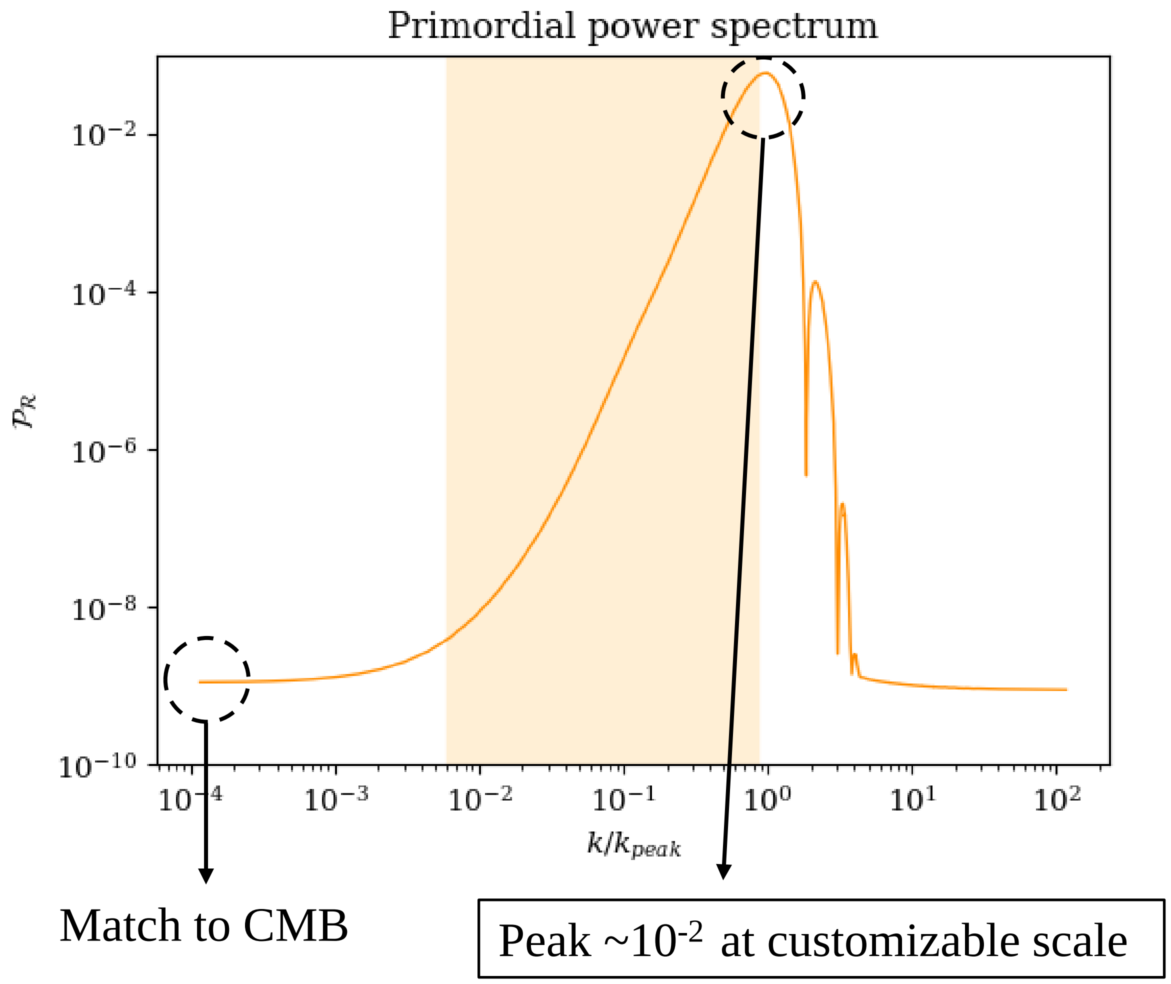
- SDEs → ODEs for the **correlations**
- Solve for
  $\langle|\delta\phi|^2\rangle, \langle|\delta\dot{\phi}|^2\rangle, \langle|\delta\rho_r|^2\rangle, \langle|\varphi|^2\rangle,$
  $\langle\delta\phi^*\delta\dot{\phi}\rangle, ..., \langle\delta\rho_r^*\varphi\rangle$
- Recast into $\langle|\mathcal{R}|^2\rangle$

Montecarlo

- Randomize source ξ for each time
- Compute particular realization of $|\mathcal{R}|^2$
- Iterate and take average $\langle|\mathcal{R}|^2\rangle$

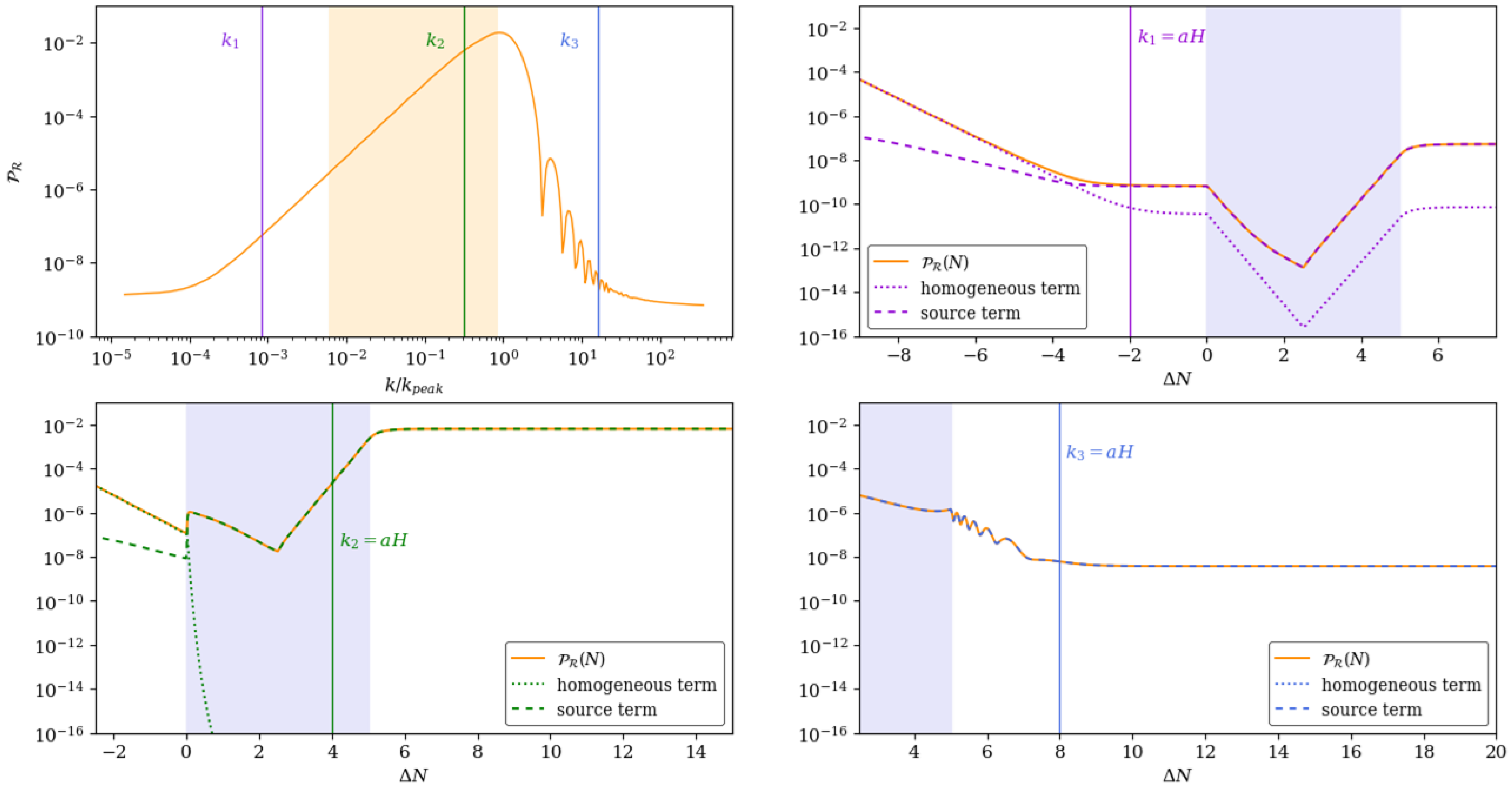
A SPECIFIC MODEL. Background and power spectrum





A SPECIFIC MODEL. Perturbation dynamics

Analytical **approximation** $\longrightarrow$ $\mathcal{P}_{\mathcal{R}}(N) = \mathcal{P}_{\mathcal{R}}^{(h)}(N) + \mathcal{P}_{\mathcal{R}}^{(\mathrm{source})}(N)$



C.f. *Hall et al. (2003), López Nacir et al. (2012)*

CONCLUSIONS AND PROSPECTS

- Dissipative effects in inflation→ interesting **perturbation** physics:
  - **Stochastic dynamics** of the perturbations
  - **Thermal enhancement**
- Enhancement of **certain** modes → **peak** in the power spectrum
- This could explain **PBHs dark matter** and **SGW LISA** signals

Self-similar solutions for Fuzzy Dark Matter

Published in Phys. Rev. D 105, 123528 (2022)
arXiv:2203.05995

Raquel Galazo García
Supervisors: Philippe Brax & Patrick Valageas

Institut de Physique Théorique (IPhT)

ICTP Summer School on Cosmology, July 5, 2022

Introduction

Dark matter

ΛCold Dark Matter Model(ΛCDM) to describe the Universe.

- **DM particles not detected.**
- **CDM has tensions at small scales.**

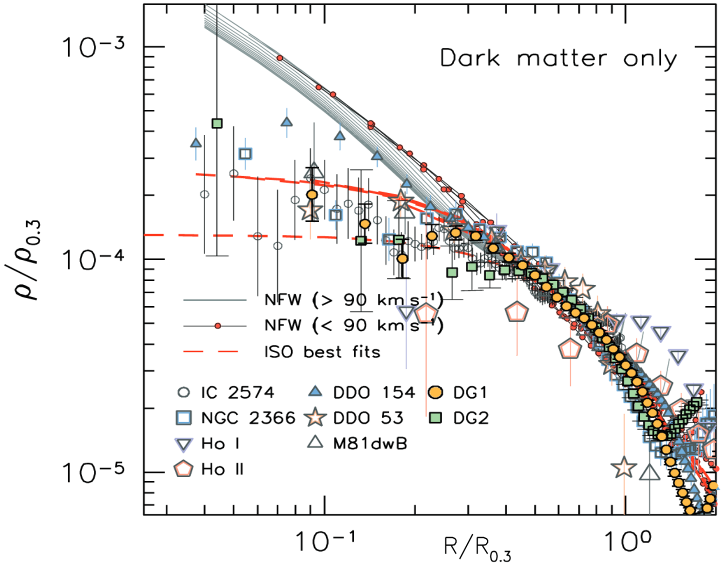
Alternative scenarios → Scalar field dark matter (SFDM).

- **Solitons (eq. configuration) → Solve CDM tensions.**

Hydrostatic equilibrium

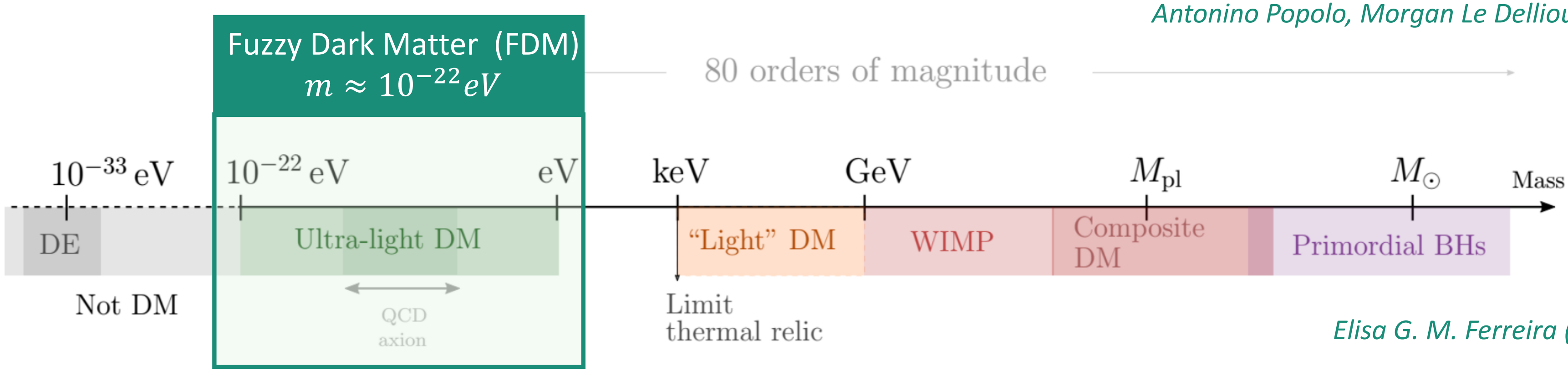
$$\nabla\left(\Phi_{\mathrm{N}}+\Phi_{\mathrm{Q}}\right)=0$$

Core-cusp problem



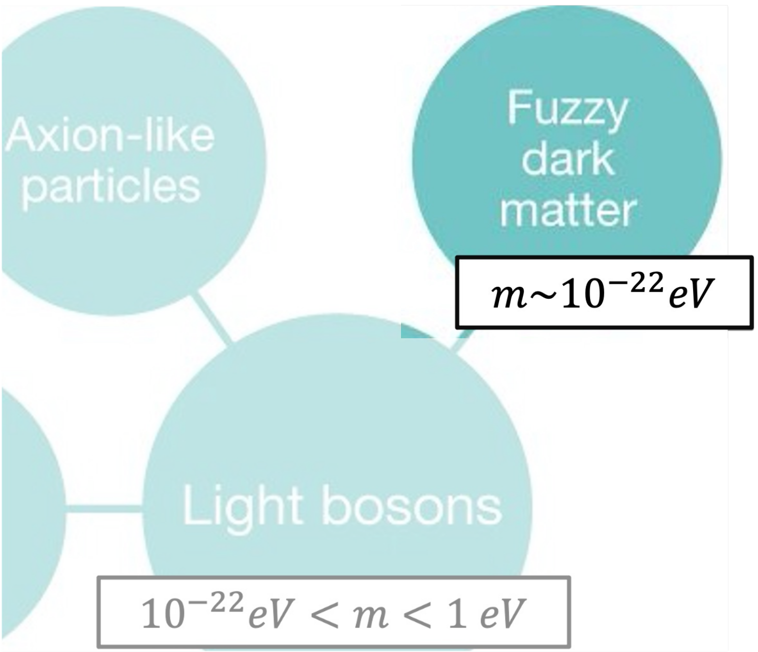
Density profiles observations and simulations

Antonino Popolo, Morgan Le Delliou (2017)



Elisa G. M. Ferreira (2020)

Dynamics of FDM: Fluid picture



Action:

$$S=\int d^4x\sqrt{-g}\left[-\frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi-\frac{m^2}{2}\phi^2\right]$$

$$\boxed{\begin{aligned}\frac{\partial\rho}{\partial t}+\nabla\cdot(\rho\vec{v})&=0,\\ \frac{\partial\vec{v}}{\partial t}+(\vec{v}\cdot\nabla)\vec{v}&=-\nabla\left(\Phi_{\mathrm{N}}+\Phi_{\mathrm{Q}}\right)\\ \nabla^2\Phi_{\mathrm{N}}&=4\pi\rho.\end{aligned}}$$

Continuity, Euler and Poisson

Quantum pressure

$$\boxed{\Phi_{\mathrm{Q}}=-\frac{\epsilon^2}{2}\frac{\nabla^2\sqrt{\rho}}{\sqrt{\rho}}.}$$

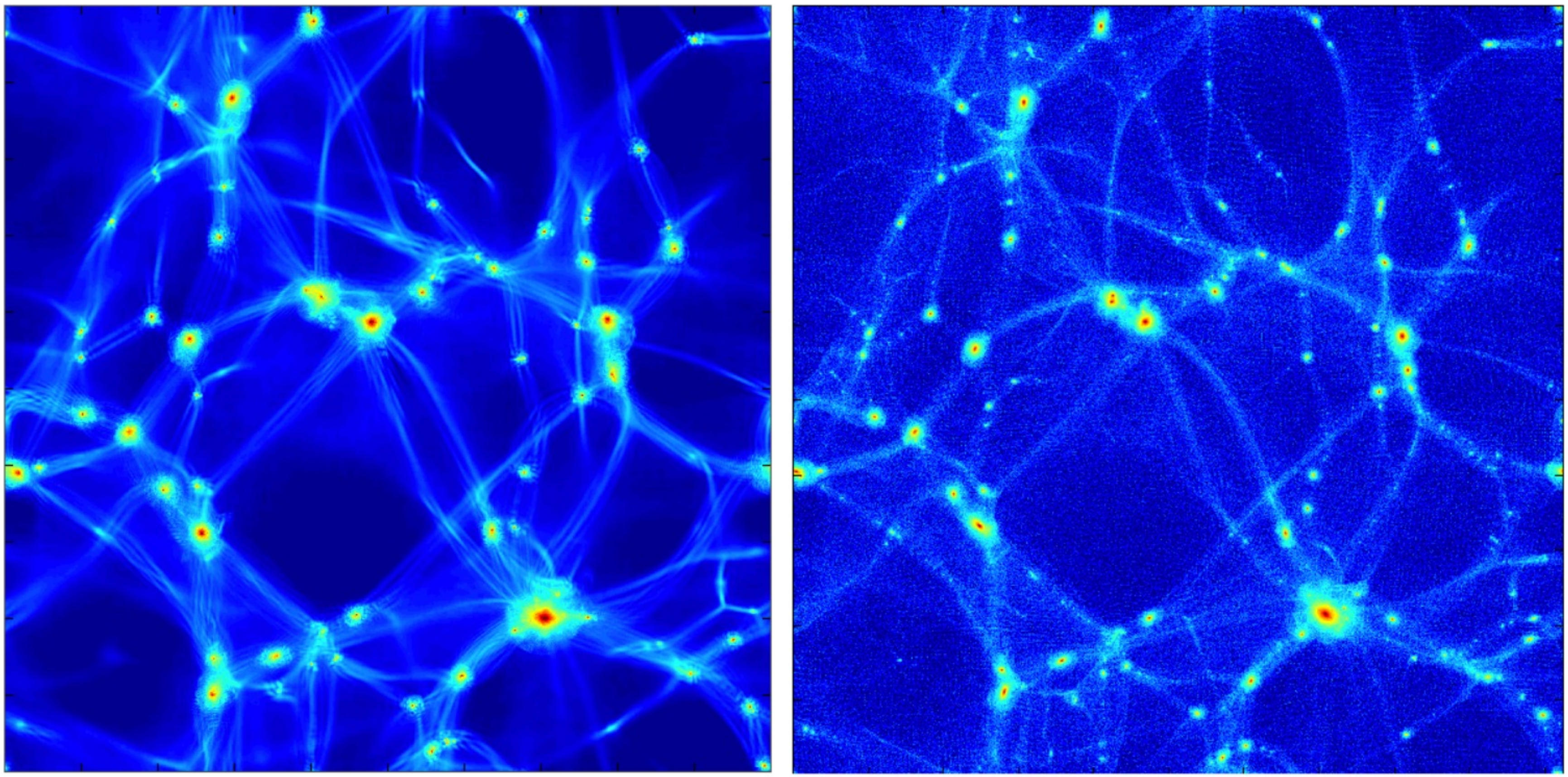
$$\epsilon=\frac{T}{mL^2}\sim\frac{\lambda_{DB}}{L}$$

$\lambda_{DB}\sim0.5\ kpc$

FDM large scale distribution

Recover the success of CDM large scale distribution of filaments and voids.

a **Fuzzy Dark Matter (FDM)** b **Cold Dark Matter (CDM)**



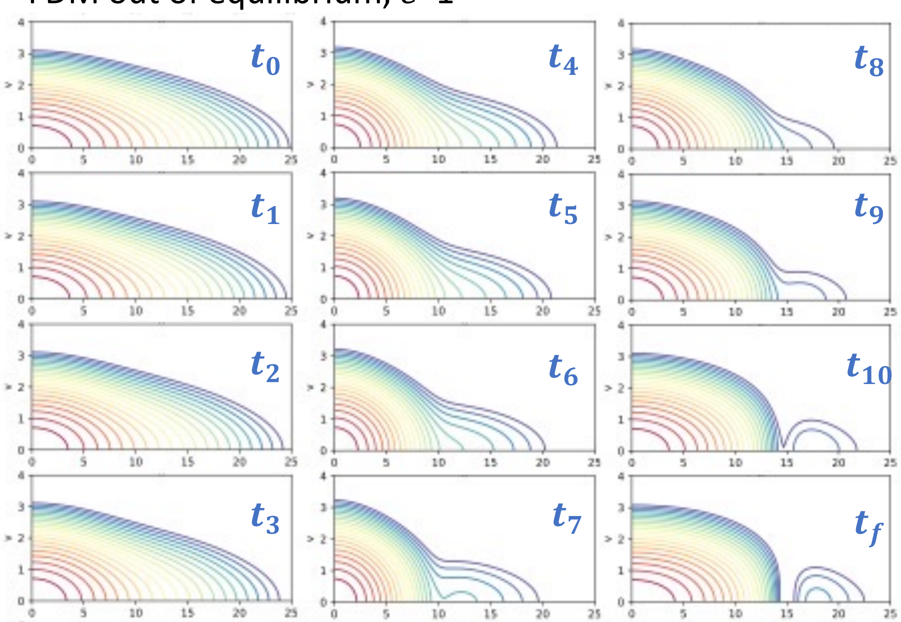
Schive, Chiueh, and Broadhurst (2014)

Motivation of this work

1. **Go beyond the static solitons** by investigating **dynamical self-similar solutions.**
2. Understand physical processes: **gravitational cooling.**
3. Understand **comparison with self similar solutions for CDM.**

Dynamics: 3D Numerical simulations

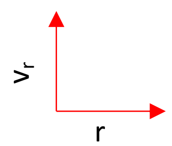
FDM out of equilibrium, ε=1



RGG

Husimi Phase-space distribution

Axis:



Gravitational Cooling

E.Seidel, W.M.Suen(1994)
F.S Guzmán, L.A. Ureña-López (2004)

Self-similar solutions for FDM

Cosmological Self-similar solutions

FLUID PICTURE

$$\rho = t^{-2} f\left(\frac{r}{\sqrt{t}}\right), \quad v = t^{-1/2} g\left(\frac{r}{\sqrt{t}}\right), \quad \Phi_{\mathrm{N}} = t^{-1} h\left(\frac{r}{\sqrt{t}}\right)$$

Continuity, Euler and Poisson

PERTURBATIONS AROUND THE EXPANDING COSMOLOGICAL BACKGROUND

$$\rho = \bar{\rho}(1+\delta), \quad \vec{v} = \bar{\vec{v}} + \vec{u}, \quad \Phi_{\mathrm{N}} = \bar{\Phi}_{\mathrm{N}} + \varphi_{\mathrm{N}},$$

- Comoving spatial coordinates $\vec{x} = \vec{r}/a$.
- Spherical symmetry.

Einstein de-Sitter Universe: matter era & the scale factor : $a \propto t^{2/3}$

Self-similar form

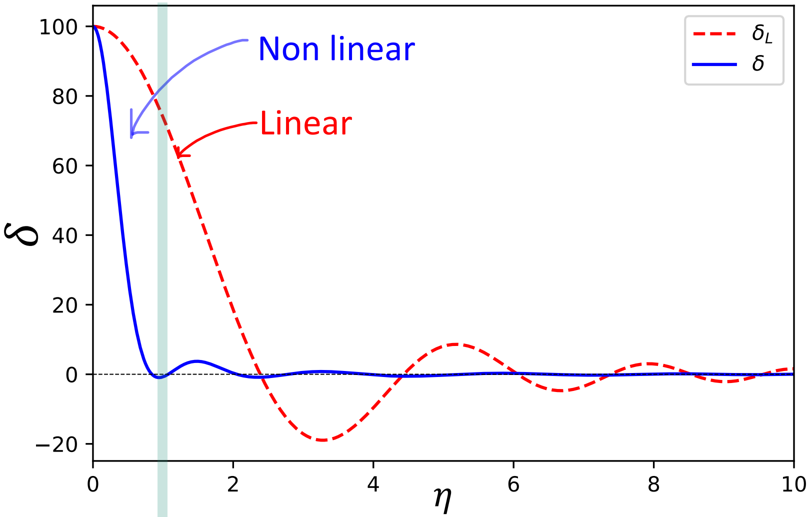
SCALING VARIABLE

$$\eta = \frac{t^{1/6} x}{\epsilon^{1/2}} = \frac{r}{\sqrt{\epsilon t}}.$$

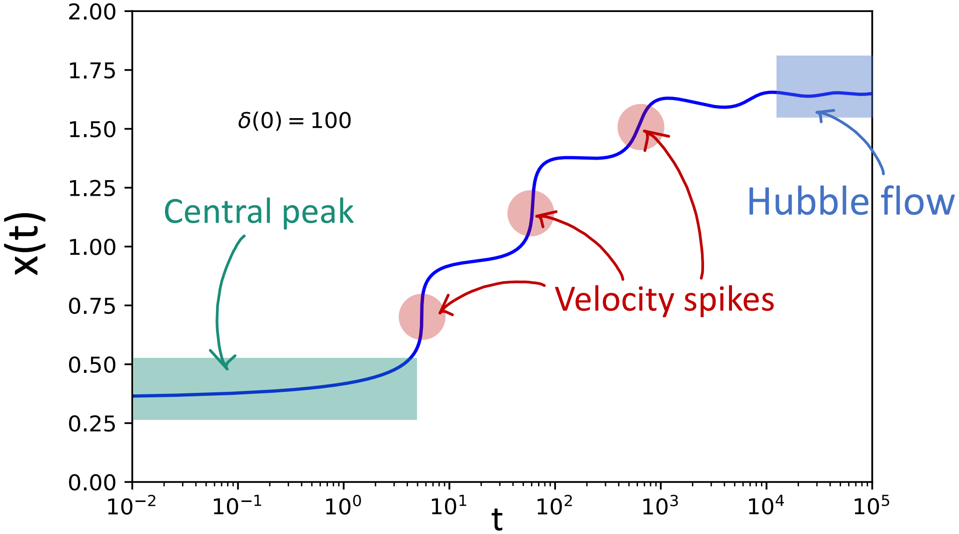
Non-linear regime: Overdensity, $\delta(0) = 100$

Gravitational cooling effect!

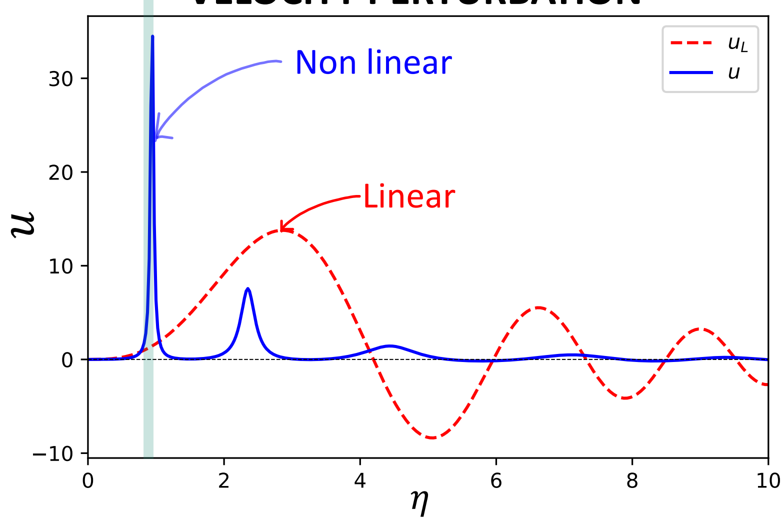
DENSITY PERTURBATION



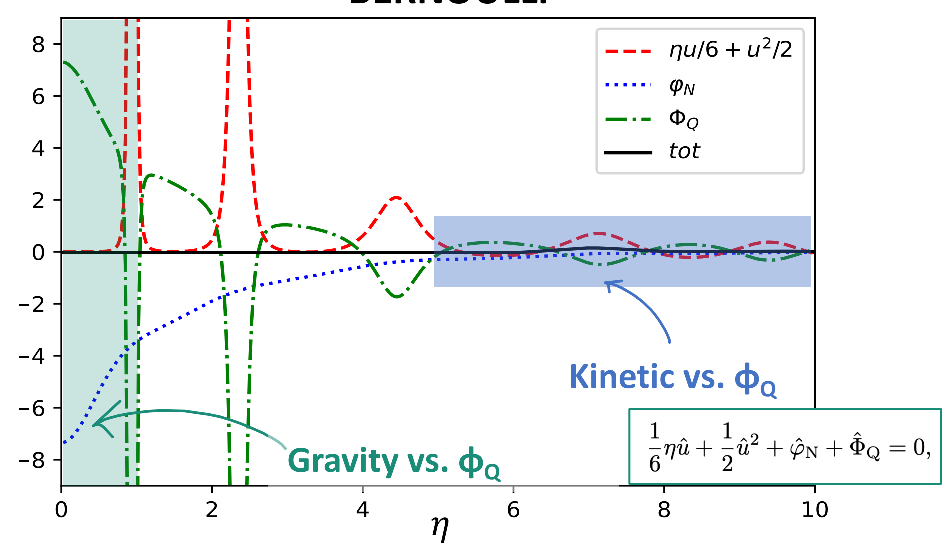
TRAJECTORY



VELOCITY PERTURBATION



BERNOULLI

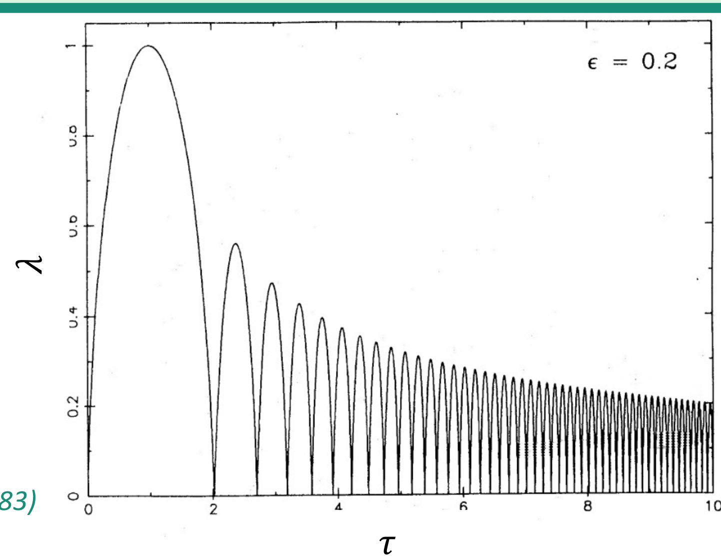


Comparison with CDM self-similar solutions & Conclusions

Comparison: CDM vs FDM

CDM

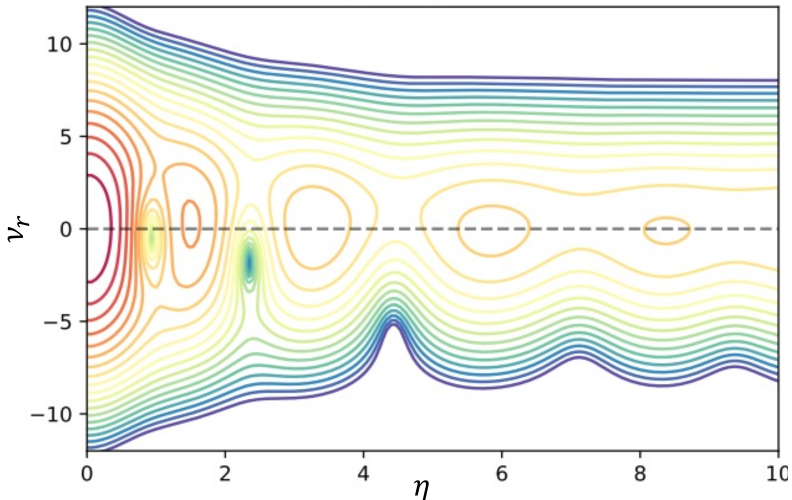
- **Transition from the linear regime to the non-linear regime.**
- **Gravitational collapse**→ Virial equilibrium in the inner nonlinear core.
- **The size grows** with time, in **physical and comoving** coordinates.

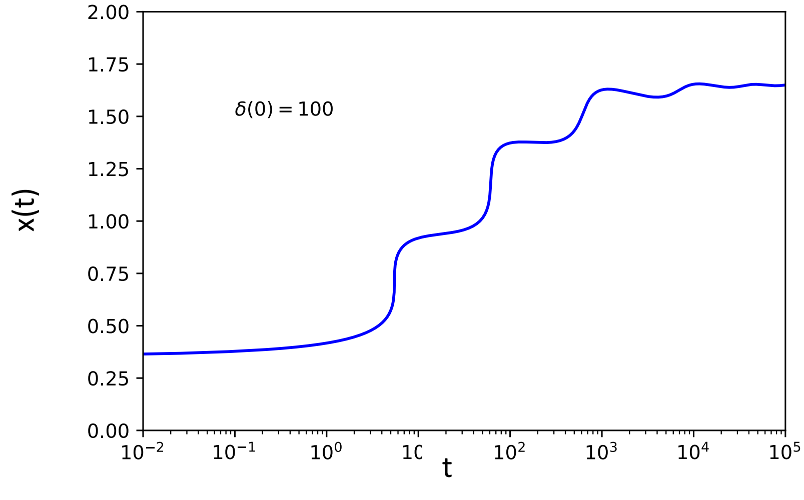


Bertschinger, E.(1985) Fillmore, J. A. and Goldreich, P. (1983)

FDM

- **No transition from the linear to the non-linear regime.**
- **Gravitational cooling**.
- **The size grows in physical** coordinates but **shrinks in comoving** coordinates.





Upcoming work

- **New numerical studies** at **galactic scales** to compare **different scalar field dark matter models (SFDM).**

- Extend self-similar solutions.

- **SFDM cosmological simulations** to study how **the formation of large scale structure is modified compared to the CDM scenario.**

Self-similar solutions for Fuzzy Dark Matter

Published in Phys. Rev. D 105, 123528 (2022)
arXiv:2203.05995

Thank you for your attention

Raquel Galazo García

Institut de Physique Théorique (IPhT)

ICTP Summer School on Cosmology, July 5, 2022

Back up: Introduction

Dynamics of Fuzzy Dark Matter: Hydrodynamical picture

$m \sim 10^{-22}\ eV$ De Broglie wavelength $\sim 0.5kpc$

Action:

$$S = \int d^4x\sqrt{-g}\left[-\frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - \frac{m^2}{2}\phi^2\right]$$

Non-Relativistic regime:

$$\phi = \frac{1}{\sqrt{2m}}\left(\psi \exp^{-imt} + \psi^* \exp^{imt}\right)$$

$|\ddot{\psi}| \ll m|\dot{\psi}|$ Factor-out the fast time oscillation of $\boldsymbol{\phi}$

SCHRÖDINGER-POISSON SYSTEM (SP)

$$i\frac{\partial\psi}{\partial t} = -\frac{1}{2m}\nabla^2\psi + m\Phi_\mathrm{N}\,\psi,$$

$$\nabla^2\Phi_\mathrm{N} = 4\pi\mathcal{G}_\mathrm{N}\rho, \quad \rho = m\psi\psi^*$$

HYDRODYNAMICAL PICTURE

$$\boxed{\psi = \sqrt{\rho}\,e^{iS/\epsilon}, \quad \vec{v} = \nabla S,}$$

$$\frac{\partial\rho}{\partial t} + \nabla\cdot(\rho\vec{v}) = 0,$$

$$\frac{\partial\vec{v}}{\partial t} + (\vec{v}\cdot\nabla)\vec{v} = -\nabla\left(\Phi_\mathrm{N} + \Phi_\mathrm{Q}\right)$$

$$\nabla^2\Phi_\mathrm{N} = 4\pi\rho.$$

SP system scaling law

$$\{t, \vec{r}, \Phi_\mathrm{N}, \psi, \rho\} \to \left\{\lambda^{-2}t, \lambda^{-1}\vec{r}, \lambda^2\Phi_\mathrm{N}, \lambda^2\psi, \lambda^4\rho\right\}.$$

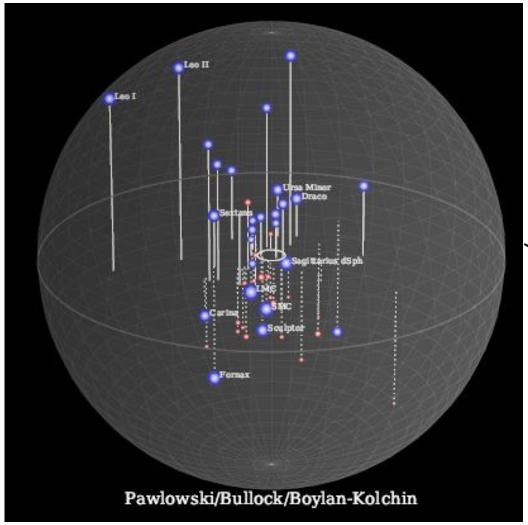
Quantum pressure

$$\boxed{\Phi_\mathrm{Q} = -\frac{\epsilon^2}{2}\frac{\nabla^2\sqrt{\rho}}{\sqrt{\rho}}.}$$

$$\epsilon = \frac{T}{mL^2} \sim \frac{\lambda_{DB}}{L}$$

FDM Motivation 1) Explanation to CDM small-scales tensions

Missing satellite problem



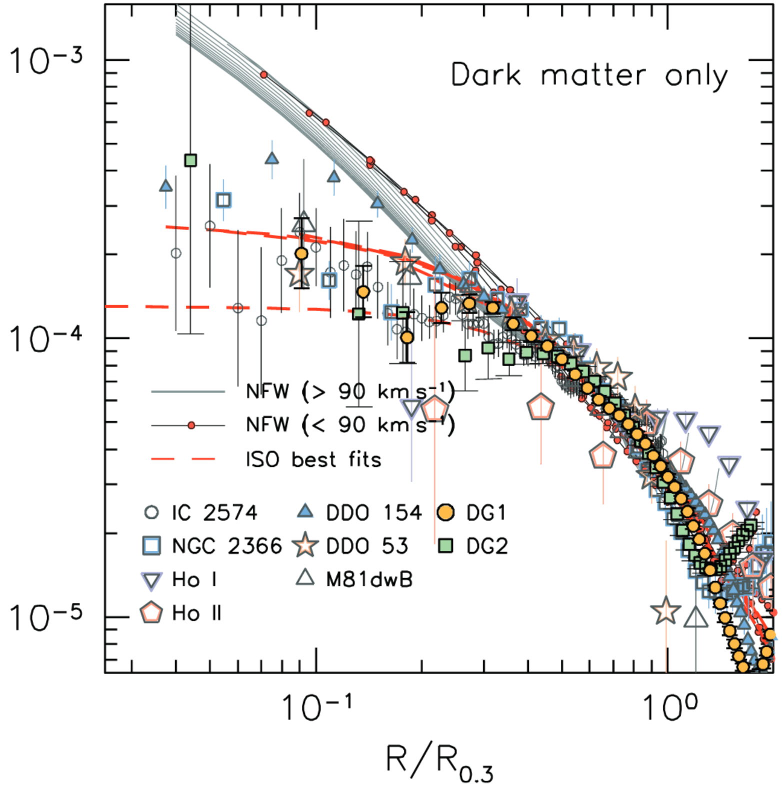
Predicted ΛCDM substructure

Simulation by V. Robles and T. Kelley and collaborators.

Known Milky Way satellites

James S. Bullock, M. Boylan-Kolchin, M. Pawlowski

Core/cusp problem



Density profiles observations and simulations

Antonino Popolo, Morgan Le Delliou (2017)

FDM Motivation 2) Alternative to CDM N-Body simulations

CDM: A classical collisionless fluid is governed by the **Vlasov-Poisson equations**. $f = f(\boldsymbol{x}, \boldsymbol{p}, t)$

$$\frac{\mathrm{d}f}{\mathrm{d}t} = \frac{\partial f}{\partial t} + \frac{\boldsymbol{p}}{ma^2} \cdot \boldsymbol{\nabla}_x f - m(\boldsymbol{\nabla}_x \phi) \cdot \boldsymbol{\nabla}_p f = 0, \qquad \nabla_x^2 \phi = 4\pi G \bar{\rho}(t)\, a^2 \delta(\boldsymbol{x}, t),$$

$$\rho = \bar{\rho}(t)[1 + \delta(\boldsymbol{x}, t)], \qquad \rho = \int f(\boldsymbol{x}, \boldsymbol{p}, t)\, \mathrm{d}^3 p.$$

- **Wigner** quasi-probability distribution: **link the Schrödinger** wave function ψ to a function f in phase space.

$$f_{\mathrm{W}}(\vec{r}, \vec{v}) = \int \frac{d\vec{r}\,'}{(2\pi)^3}\, e^{i\vec{v}\cdot\vec{r}\,'} \psi\left(\vec{r} - \frac{\epsilon}{2}\vec{r}\,'\right) \psi^*\left(\vec{r} + \frac{\epsilon}{2}\vec{r}\,'\right),$$

- **Husimi** representation: **smoothing of the Wigner** distribution by a Gaussian filter of width σx and σp in x and p space.

$$f_{\mathrm{H}}(\vec{r}, \vec{v}) = \int \frac{d\vec{r}\,' d\vec{v}\,'}{(2\pi\epsilon)^3 \sigma_r^3 \sigma_v^3}\, e^{-(\vec{r}-\vec{r}\,')^2/(2\epsilon\sigma_r^2) - (\vec{v}-\vec{v}\,')^2/(2\epsilon\sigma_v^2)} \times f_{\mathrm{W}}(\vec{r}\,', \vec{v}\,'),$$

- Husimi is a positive-semidefinite function → No fast oscillations of Wigner

FDM comoving Vlasov equation

$$\boxed{\frac{\partial f_{\mathrm{W}}}{\partial t} + \frac{\vec{p}}{a^2} \cdot \frac{\partial f_{\mathrm{W}}}{\partial \vec{x}} - \vec{\nabla}_x \varphi_{\mathrm{N}} \cdot \frac{\partial f_{\mathrm{W}}}{\partial \vec{p}} + \mathcal{O}(\epsilon) = 0.}$$

Kaiser (1993)
C. Uhlemann, M. Kopp, & T. Haugg (2014)

FDM Motivation 2) Alternative to CDM N-Body simulations

FDM comoving Vlasov equation

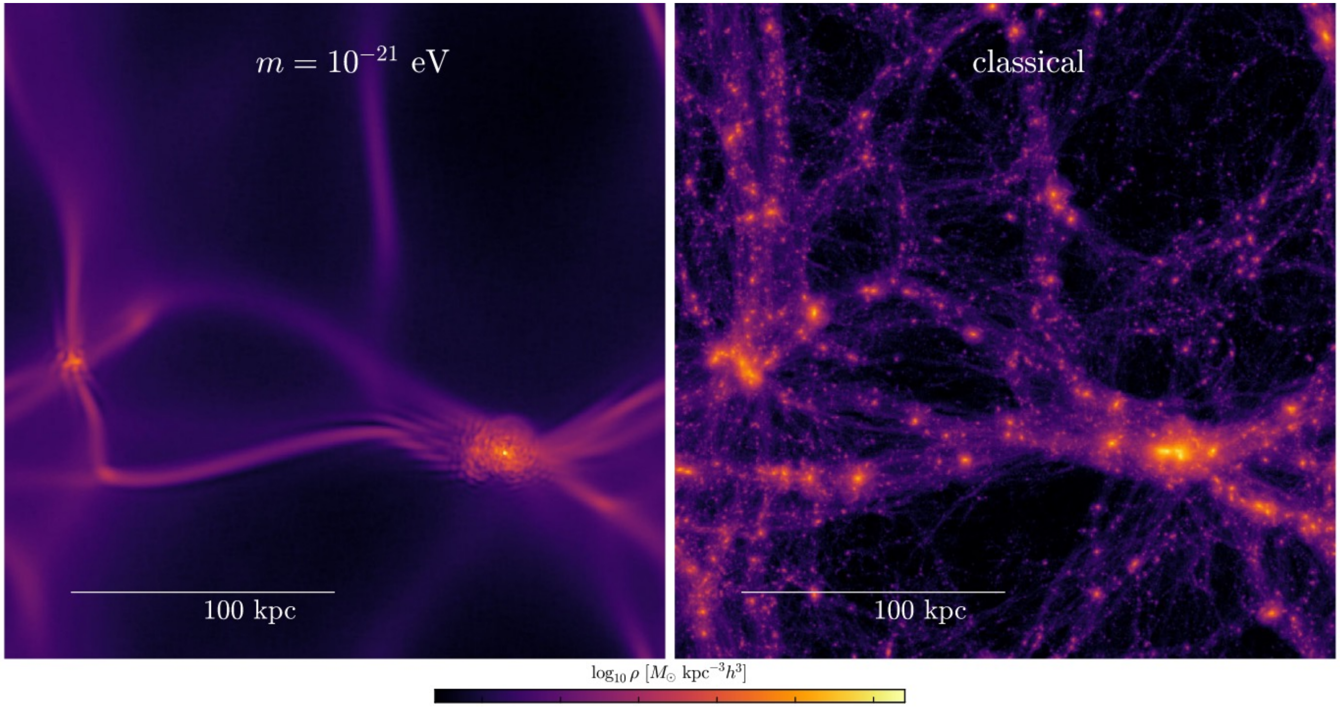
$$\boxed{\frac{\partial f_{\mathrm{W}}}{\partial t} + \frac{\vec{p}}{a^2} \cdot \frac{\partial f_{\mathrm{W}}}{\partial \vec{x}} - \vec{\nabla}_x \varphi_{\mathrm{N}} \cdot \frac{\partial f_{\mathrm{W}}}{\partial \vec{p}} + \mathcal{O}(\epsilon) = 0.}$$

CDM comoving Vlasov equation

$$\frac{\mathrm{d}f}{\mathrm{d}t} = \frac{\partial f}{\partial t} + \frac{\boldsymbol{p}}{ma^2} \cdot \boldsymbol{\nabla}_x f - m(\boldsymbol{\nabla}_x \phi) \cdot \boldsymbol{\nabla}_p f = 0,$$

Kaiser (1993)

Link the Schrödinger wave function ψ to a function f in phase space.



Cosmological simulation at z=3, evolved either as CDM (VP eq) or as FDM (SP)

Philip Mocz and Lachlan Lancaste,Anastasia Fialkov and Fernando Becerra, Pierre-Henri Chavanis (2018).

Back up: Self-similar solutions for FDM

Cosmological Self-similar solutions

FLUID PICTURE

$$\boxed{\rho = t^{-2} f\left(\frac{r}{\sqrt{t}}\right), \quad v = t^{-1/2} g\left(\frac{r}{\sqrt{t}}\right), \quad \Phi_{\mathrm{N}} = t^{-1} h\left(\frac{r}{\sqrt{t}}\right)}$$

Continuity, Euler and Poisson

COSMOLOGICAL BACKGROUND

Einstein de-Sitter Universe: matter era & the scale factor : $a \propto t^{2/3}$

$$\bar{\rho} = \frac{1}{6\pi t^2}, \quad \bar{v} = \frac{2r}{3t}, \quad \bar{\Phi}_{\mathrm{N}} = \frac{r^2}{9t^2}.$$

Self-similar form ✓

Comoving spatial coordinates $\vec{x} = \vec{r}/a$

PERTURBATIONS AROUND THE EXPANDING BACKGROUND

$$\rho = \bar{\rho}(1+\delta), \quad \vec{v} = \vec{\bar{v}} + \vec{u}, \quad \Phi_{\mathrm{N}} = \bar{\Phi}_{\mathrm{N}} + \varphi_{\mathrm{N}},$$

Comoving Self-similar solutions: Scaling variable

$\rho = \bar{\rho}(1+\delta), \quad \vec{v} = \bar{\vec{v}} + \vec{u}, \quad \Phi_{\rm N} = \bar{\Phi}_{\rm N} + \varphi_{\rm N},$ → Substituting into the Euler, Poisson and continuity eq.

COMOVING FLUID EQUATIONS

$$\frac{\partial \delta}{\partial t} + \frac{1}{a}\nabla_x \cdot [(1+\delta)\vec{u}] = 0,$$
$$\frac{\partial \vec{u}}{\partial t} + \frac{1}{a}(\vec{u}\cdot\nabla_x)\vec{u} + H\vec{u} = -\frac{1}{a}\nabla_x(\varphi_{\rm N} + \Phi_{\rm Q}),$$
$$\nabla_x^2 \varphi_{\rm N} = \frac{2}{3}\frac{\delta}{a},$$

Quantum Pressure

$$\Phi_{\rm Q} = -\frac{\epsilon^2}{2a^2}\frac{\nabla_x^2\sqrt{\rho}}{\sqrt{\rho}}.$$

Spherical self-similar solutions will be of the form:

$$\boxed{\begin{aligned} &\delta(x,t) = \hat{\delta}(\eta), \quad u(x,t) = \epsilon^{1/2} t^{-1/2}\,\hat{u}(\eta), \\ &\varphi_{\rm N}(x,t) = \epsilon t^{-1}\,\hat{\varphi}_{\rm N}(\eta), \quad \Phi_{\rm Q}(x,t) = \epsilon t^{-1}\,\hat{\Phi}_{\rm Q}(\eta), \\ &\delta M(x,t) = \epsilon^{3/2} t^{-1/2}\,\delta\hat{M}(\eta), \end{aligned}}$$

Where the mass perturbation inside the radius r :

$$\delta M(r) = 4\pi\int_0^r dr\, r^2 \delta\rho(r) = \frac{2}{3}\int_0^x dx\, x^2 \delta(x),$$

Self-similar ansatz

$$v = t^{-1/2} g\left(\frac{r}{\sqrt{t}}\right)$$
$$\rho = t^{-2} f\left(\frac{r}{\sqrt{t}}\right) \quad \Phi_{\rm N} = t^{-1} h\left(\frac{r}{\sqrt{t}}\right)$$

SCALING VARIABLE

$$\boxed{\eta = \frac{t^{1/6}x}{\epsilon^{1/2}} = \frac{r}{\sqrt{\epsilon t}}.}$$

Scaling variable

Spherical self-similar solutions:

$$\delta(x,t) = \hat{\delta}(\eta), \quad u(x,t) = \epsilon^{1/2} t^{-1/2}\, \hat{u}(\eta),$$
$$\varphi_{\rm N}(x,t) = \epsilon t^{-1}\, \hat{\varphi}_{\rm N}(\eta), \quad \Phi_{\rm Q}(x,t) = \epsilon t^{-1}\, \hat{\Phi}_{\rm Q}(\eta),$$
$$\delta M(x,t) = \epsilon^{3/2} t^{-1/2}\, \delta\hat{M}(\eta),$$

SCALING VARIABLE

$$\eta = \frac{t^{1/6} x}{\epsilon^{1/2}} = \frac{r}{\sqrt{\epsilon t}}.$$

$$M = \bar{M} + \delta M = \epsilon^{3/2} t^{-1/2} \left[\frac{2}{9}\eta^3 + \delta\hat{M}(\eta) \right].$$

- The size **grows as $\sim\sqrt{t}$ in physical units** but more slowly than the scale factor, → shrink **as $t^{-1/6}$** in comoving units.
- The associated **mass decreases as $M \sim 1/\sqrt{t}$**. (≠CDM: grow both in comoving size and in mass.)

Back up: Linear regime

Linear regime: Fourier space

FOURIER SPACE

$$\boxed{\ddot{\delta}_L + \frac{4}{3t}\dot{\delta}_L - \frac{2}{3t^2}\delta_L + \frac{\epsilon^2 k^4}{4t^{8/3}}\delta_L = 0.}$$

FDM Growing and decaying modes: $D_\pm(k,t)$

$$D_+(k,t) = t^{-1/6} J_{-5/2}\left(\frac{3}{2}\epsilon k^2 t^{-1/3}\right),$$
$$D_-(k,t) = t^{-1/6} J_{5/2}\left(\frac{3}{2}\epsilon k^2 t^{-1/3}\right).$$

CDM Growing and decaying modes: $D_\pm(k,t)$

- Semi-classical limit, $\epsilon \to 0$, or on large scales $k \to 0$ →

$$D_+(k,t) \propto t^{2/3} \propto a$$
$$D_-(k,t) \propto t^{-1}$$

- At late times → CDM behaviour → damping of Φ_Q term $t^{-8/3}$
- For $\epsilon \neq 0$, Φ_Q → Acoustic waves: $D_+(k,t) \sim \cos(3\epsilon k^2 t^{-1/3}/2)$ $D_-(k,t) \sim \sin(3\epsilon k^2 t^{-1/3}/2)$

To recover the **BKGD** density on large scales, we keep the decaying mode:

$$\delta_L(x,t) = 1 + \frac{\eta^4}{45} - \frac{8\eta^2}{9\pi}\, {}_2F_3\left(-\frac{1}{2}, 2; \frac{3}{2}, \frac{5}{4}, \frac{7}{4}; -\frac{\eta^4}{144}\right)$$

Linear regime: Real space

REAL SPACE

$$\boxed{\delta_L^{(4)} + \frac{4}{\eta}\delta_L^{(3)} + \frac{\eta^2}{9}\delta_L'' + \frac{\eta}{3}\delta_L' - \frac{8}{3}\delta_L = 0,}$$

FDM 4 independent linear modes

$$\delta_{L1} = 45 + \eta^4, \quad \cancel{\delta_{L2} = \frac{1}{\eta}\,{}_2F_3\left(-\frac{5}{4}, \frac{5}{4}; \frac{1}{4}, \frac{1}{2}, \frac{3}{4}; -\frac{\eta^4}{144}\right)},$$

$$\cancel{\delta_{L3} = \eta\,{}_2F_3\left(-\frac{3}{4}, \frac{7}{4}; \frac{3}{4}, \frac{5}{4}, \frac{3}{2}; -\frac{\eta^4}{144}\right)},$$

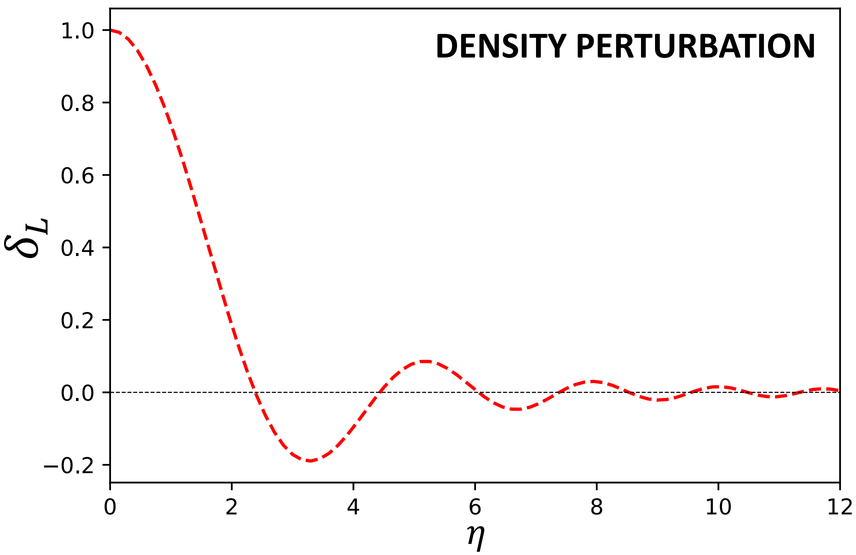
$$\delta_{L4} = \eta^2\,{}_2F_3\left(-\frac{1}{2}, 2; \frac{5}{4}, \frac{3}{2}, \frac{7}{4}; -\frac{\eta^4}{144}\right).$$

- Smooth solution at $\eta = 0$
- Satisfy the boundary conditions at infinity with $\delta(0) = 1$

$$\boxed{\delta_L = -\frac{8}{9\pi}\left(\delta_{L4} - \frac{\pi}{40}\delta_{L1}\right)}$$

Recover Fourier solution

Linear regime self-similar solution



SCALING VARIABLE

$$\boxed{\eta = \frac{t^{1/6}x}{\epsilon^{1/2}} = \frac{r}{\sqrt{\epsilon t}}.}$$

$$\delta(x,t) = \hat{\delta}(\eta),$$

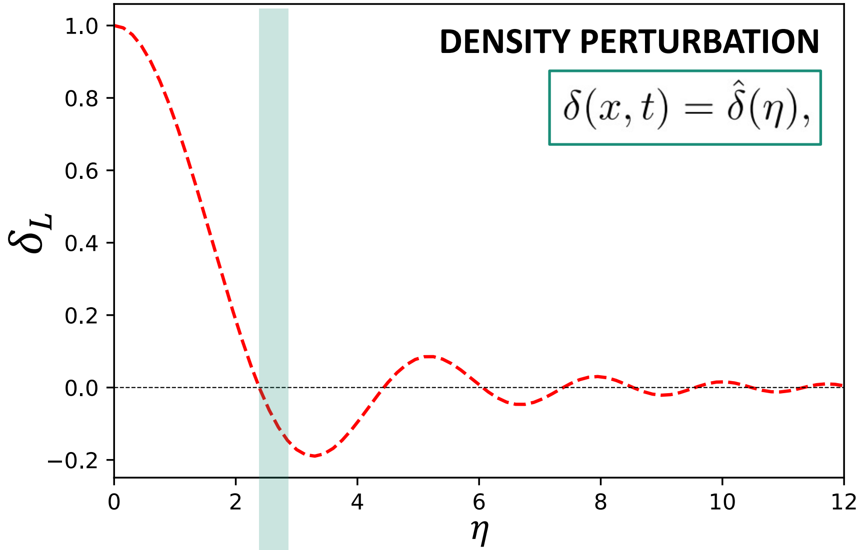
$$\boxed{\delta_L(x,t) = 1 + \frac{\eta^4}{45} - \frac{8\eta^2}{9\pi}\,{}_2F_3\left(-\frac{1}{2}, 2; \frac{3}{2}, \frac{5}{4}, \frac{7}{4}; -\frac{\eta^4}{144}\right)}$$

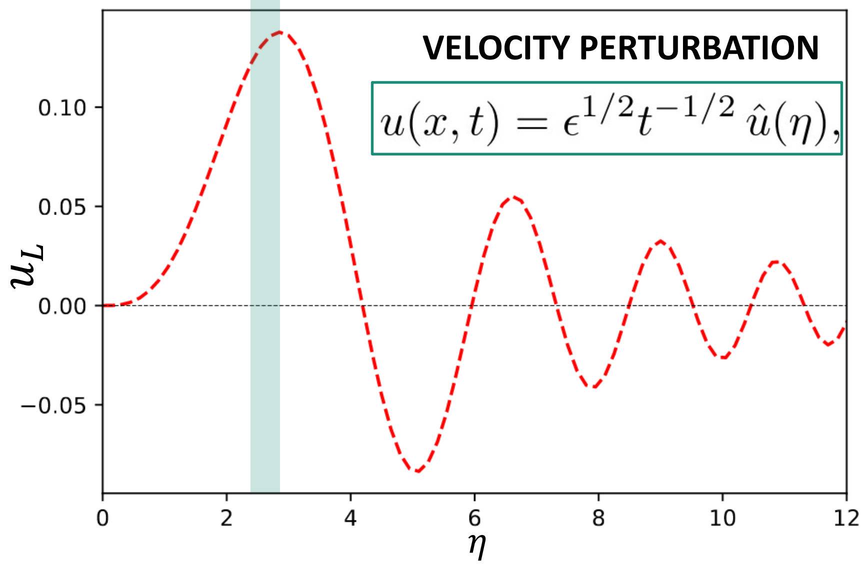
- **Central peak much higher** than the next peaks.

- **Amplitude of δ_L does not grow with time and remains constant. (≠ CDM)**
  - It is stable and **does not reach the nonlinear regime at late times**.
  - **To reach the nonlinear regime → start with a large nonlinear perturbation.**

- As time grows, self-similar solution **grows in physical** coordinates but **shrinks in comoving** coordinates.

I) Small perturbations: Linear regime, $\delta(0) = 1$





- **FDM: δ_L constant amplitude. ($\neq$ CDM)**
- **FDM: δ_L grows in physical** coordinates but **shrinks in comoving** coordinates. **($\neq$ CDM)**
- **FDM** : Fields with **oscillations ,** Φ_Q **. ($\neq$ CDM)**

Back up: Non-linear regime

Closed equation over δM

Euler equation in terms of η :

$$\frac{1}{6}(\hat{u} + \eta \hat{u}') + \hat{u}\hat{u}' + \hat{\varphi}'_{\mathrm{N}} + \hat{\Phi}'_{\mathrm{Q}} = 0,$$

The fields in terms of η :

$$\hat{\Phi}_{\mathrm{Q}} = -\frac{1}{2\eta^2\sqrt{1+\hat{\delta}}}\frac{d}{d\eta}\left(\eta^2 \frac{d}{d\eta}\sqrt{1+\hat{\delta}}\right) \qquad \hat{\delta} = \frac{3}{2\eta^2}\delta\hat{M}'$$

Poisson equation in terms of η :

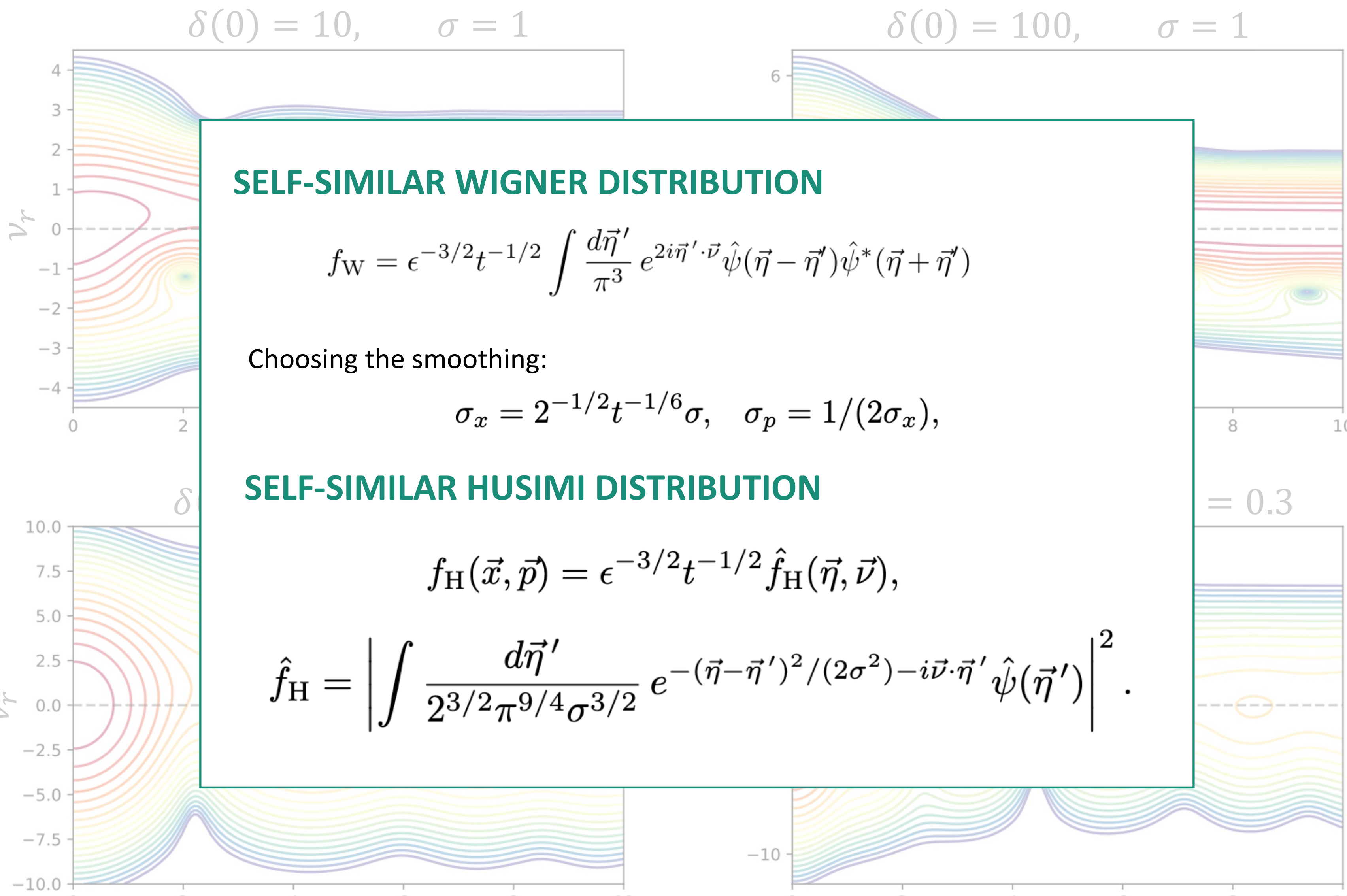
$$\frac{1}{\eta^2}\frac{d}{d\eta}\left(\eta^2 \frac{d\hat{\varphi}_{\mathrm{N}}}{d\eta}\right) = \frac{2}{3}\hat{\delta},$$

Integrating the continuity equation: $\boxed{\hat{u} = \frac{3\delta\hat{M} - \eta\delta\hat{M}'}{4\eta^2 + 6\delta\hat{M}'}.}$

CLOSED NON LINEAR EQUATION OVER δM

$$\boxed{\begin{aligned}&9(2\eta^3 + 3\eta\delta\hat{M}')^2\delta\hat{M}^{(4)} - (144\eta^5 + 216\eta^3\delta\hat{M}' + 108\eta^4\delta\hat{M}'' \\ &+162\eta^2\delta\hat{M}'\delta\hat{M}'')\delta\hat{M}^{(3)} + (4\eta^8 + 288\eta^4 + 36\eta^5\delta\hat{M} \\ &-216\eta^2\delta\hat{M}' + 324\eta^3\delta\hat{M}'' + 81\eta^2\delta\hat{M}^2 + 81\eta^2\delta\hat{M}''^2)\delta\hat{M}'' \\ &-3(4\eta^7 + 96\eta^3 + 180\eta^4\delta\hat{M} + 243\eta^2\delta\hat{M}\delta\hat{M}' - 3\eta^3\delta\hat{M}'^2 \\ &+108\delta\hat{M}\delta\hat{M}'^2)\delta\hat{M}' - 12\eta^3(7\eta^3 - 9\delta\hat{M})\delta\hat{M} = 0.\end{aligned}}$$

Self-similar Husimi phase-space distribution $\hat{f}_H\,(\eta; v_r; v_t = 0)$



SELF-SIMILAR WIGNER DISTRIBUTION

$$f_{\rm W} = \epsilon^{-3/2} t^{-1/2} \int \frac{d\vec{\eta}\,'}{\pi^3}\, e^{2i\vec{\eta}\,'\cdot\vec{\nu}}\, \hat{\psi}(\vec{\eta}-\vec{\eta}\,')\hat{\psi}^*(\vec{\eta}+\vec{\eta}\,')$$

Choosing the smoothing:

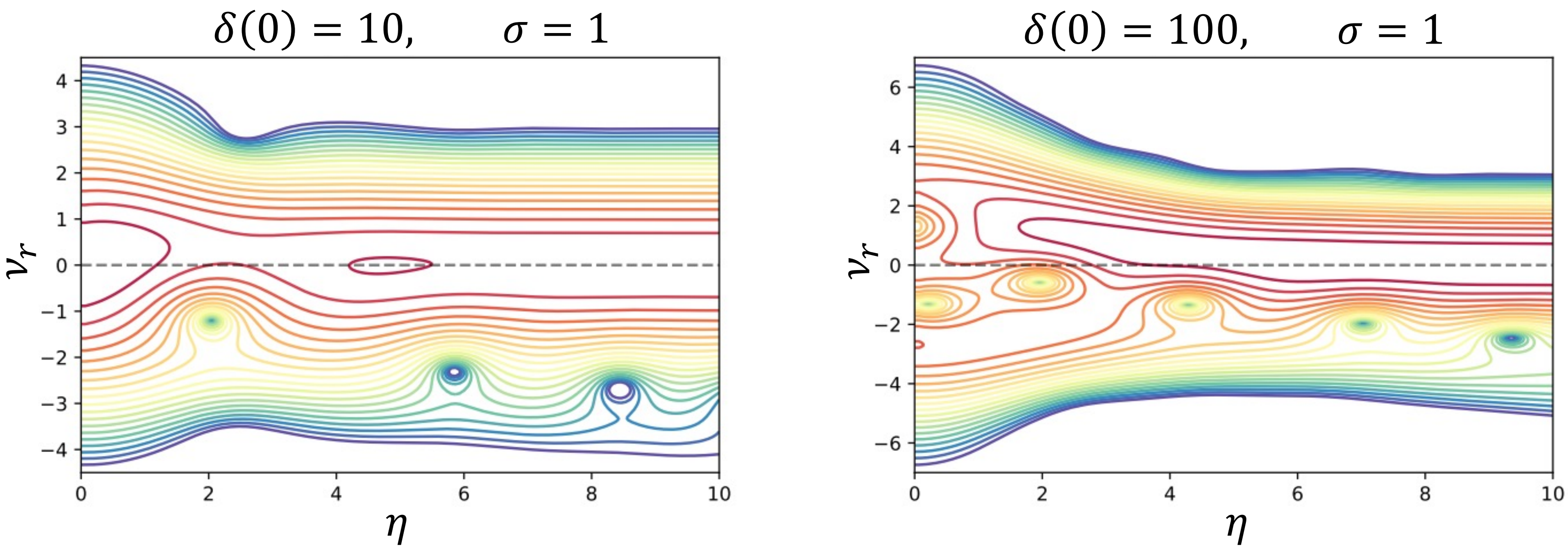
$$\sigma_x = 2^{-1/2} t^{-1/6} \sigma, \quad \sigma_p = 1/(2\sigma_x),$$

SELF-SIMILAR HUSIMI DISTRIBUTION

$$f_{\rm H}(\vec{x},\vec{p}) = \epsilon^{-3/2} t^{-1/2} \hat{f}_{\rm H}(\vec{\eta},\vec{\nu}),$$

$$\hat{f}_{\rm H} = \left| \int \frac{d\vec{\eta}\,'}{2^{3/2}\pi^{9/4}\sigma^{3/2}}\, e^{-(\vec{\eta}-\vec{\eta}\,')^2/(2\sigma^2) - i\vec{\nu}\cdot\vec{\eta}\,'}\, \hat{\psi}(\vec{\eta}\,') \right|^2 .$$

Radial Husimi phase-space distribution $\hat{f}_H\,(\eta; v_r; v_t = 0), \sigma = 1$

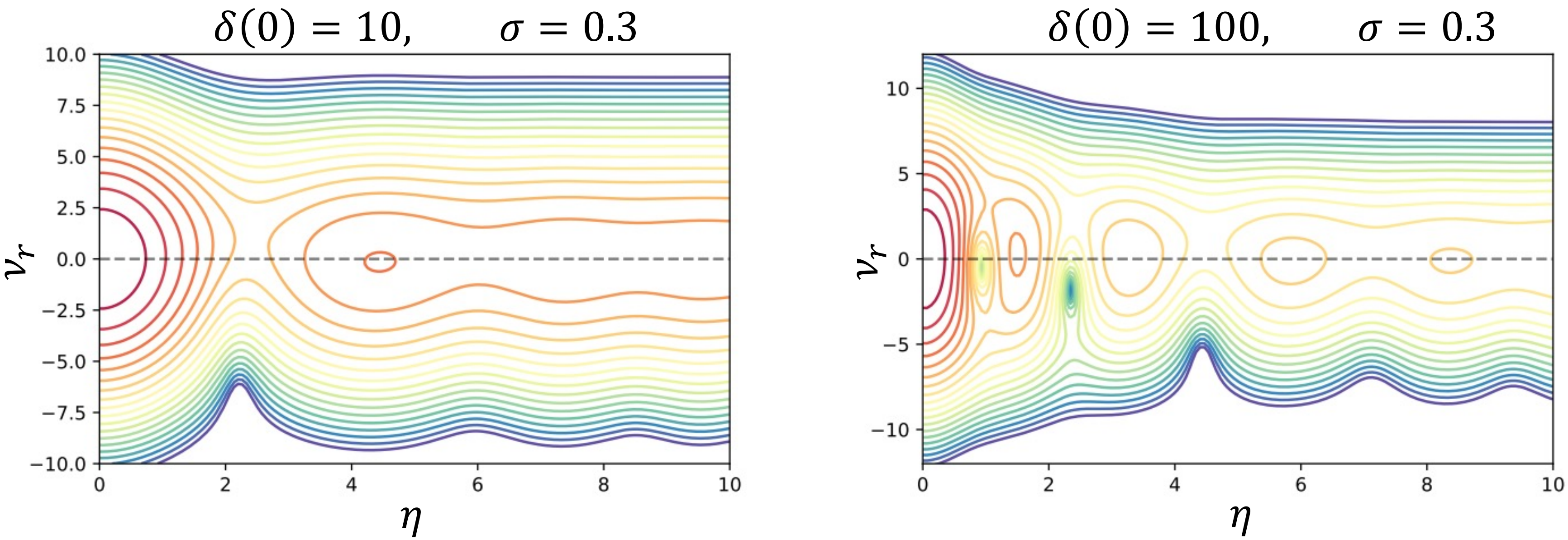


- As the spatial coarsening $\sigma_x \propto \sigma$ increases → the velocity coarsening, $\sigma_p \propto 1/\sigma$ decreases : **Heisenberg uncertainty principle.**

- **Velocity asymmetries but spatial profile is smoothed out.**
- At large distance, the profile → **cosmological BKGD.**

- **The coarsening $\sigma = 1$ is no longer sufficient to separate the first few peaks.**
- Artificial interferences between these peaks and to a Husimi distribution that is difficult to interpret and far from the semiclassical expectations

Radial Husimi phase-space distribution $\hat{f}_H\,(\eta; v_r; v_t = 0), \sigma = 0.3$



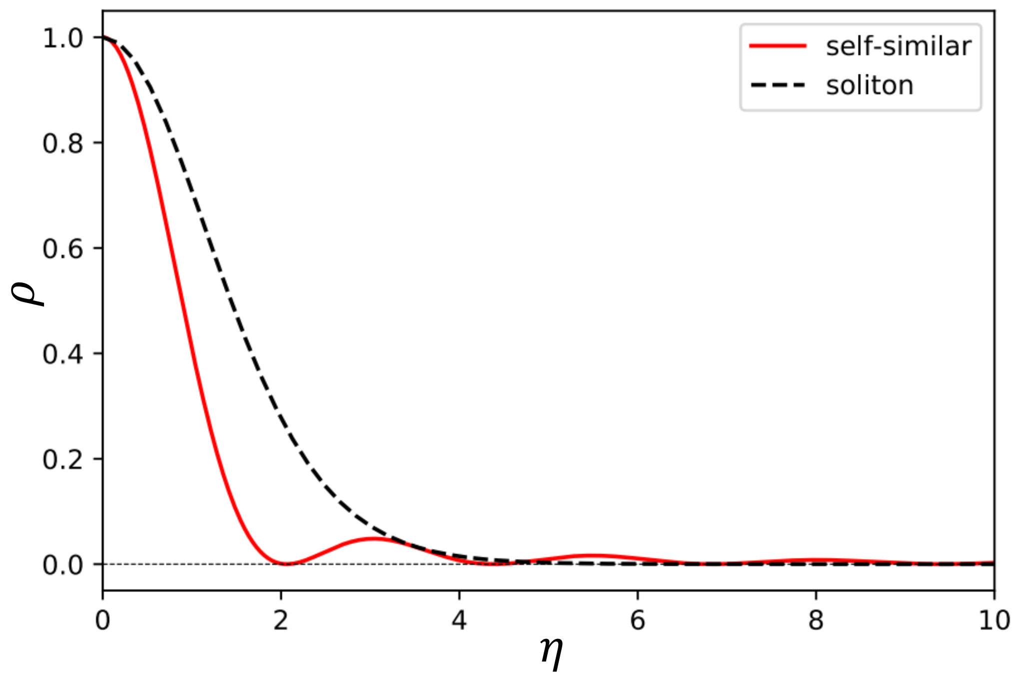
- **Well-defined peaks increasing** with δ(0) and preserves **signs of the density fluctuations.**
- **At large distance→ the cosmological BKGD.**

- More **faithful representation**: Sequence of scalar-field clumps
- **COST: This erases most of the information about the velocity field.**
- Different choices of σ→ different pictures →Difficult to relate to the underlying dynamics

- **The hydrodynamical mapping clearer picture of the dynamics.**

Back up: High-density asymptotic limit

High-density asymptotic limit



- The BKGD density becomes negligible as compared with the central density.
- The inner profile converge to a limiting shape that obeys the scaling law:

$$\{\eta, \psi, \rho, M\} \to \{\lambda^{-1}\eta, \lambda^2\psi, \lambda^4\rho, \lambda M\}$$

- The central peak of the **self-similar solution is narrower than the soliton peak**.
- **The shape of the central peak of the self-similar profile does not converge to the soliton equilibrium**. → **kinetic effects** (dominate near the boundary of the central peak).

Soliton

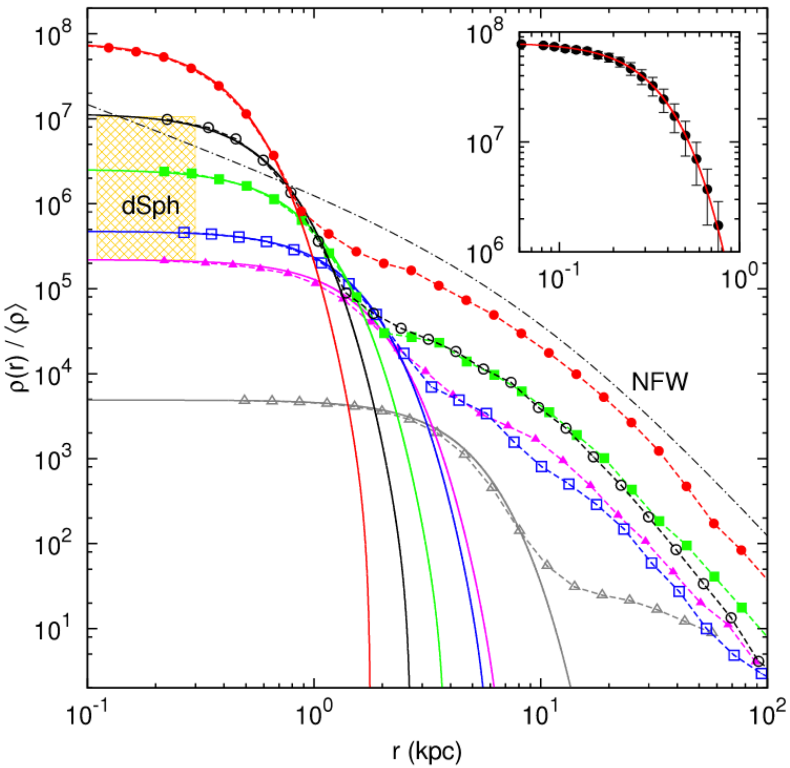
Hydrostatic equilibrium

$$\Phi_N + \Phi_Q = \alpha,$$

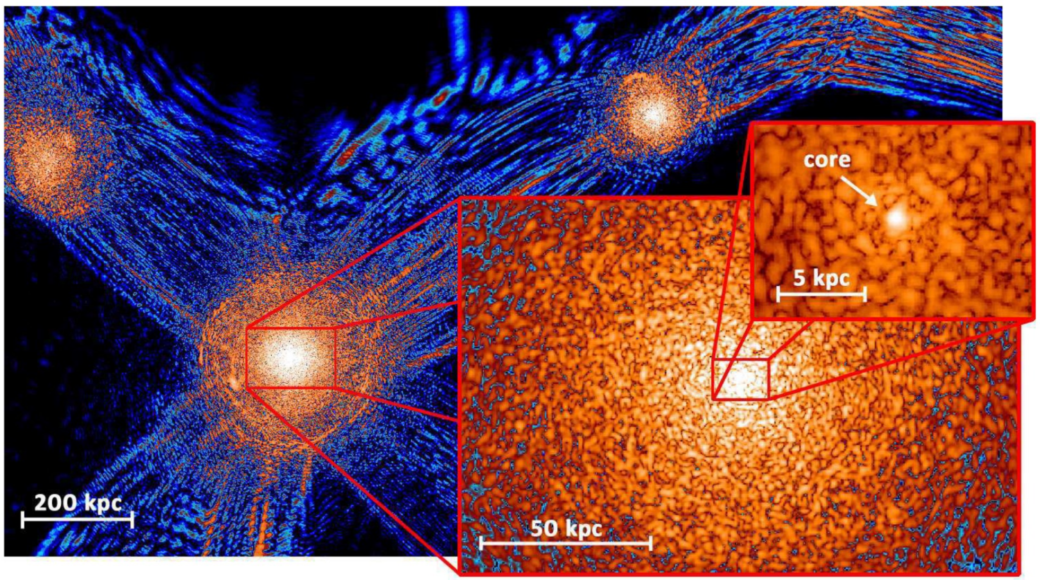
Soliton profile

$$\epsilon^2 \nabla^2 \psi_{sol} = 2(\Phi_N - \alpha)\psi_{sol}.$$

$$\nabla^2 \Phi_N = 4\pi \psi_{sol}^2$$



Radial density profiles of haloes formed in the ψDM model



A slice of density field of ψDM simulation on various scales at z=0.1

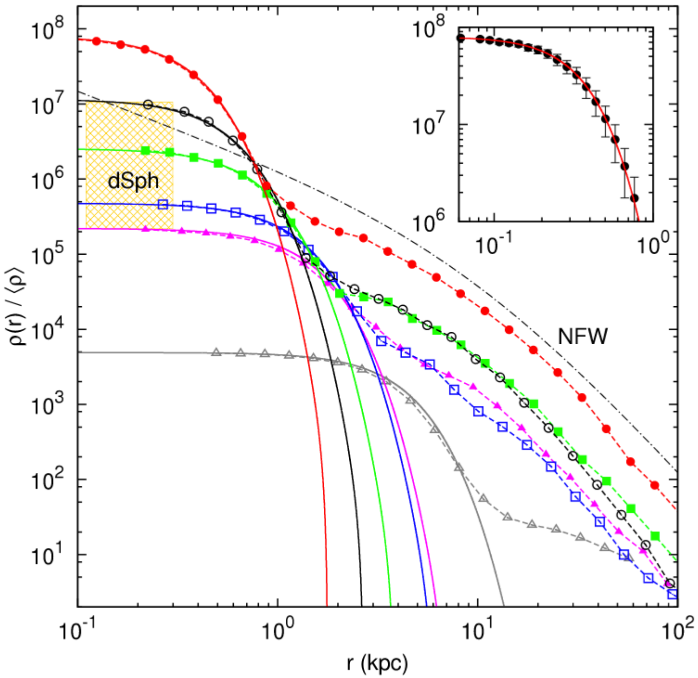
Schive, Chiueh, and Broadhurst (2014)

FDM at small scales

SOLITON: Static equilibrium configuration.

Hydrostatic equilibrium

$$\nabla\left(\Phi_{\rm N}+\Phi_{\rm Q}\right)=0$$

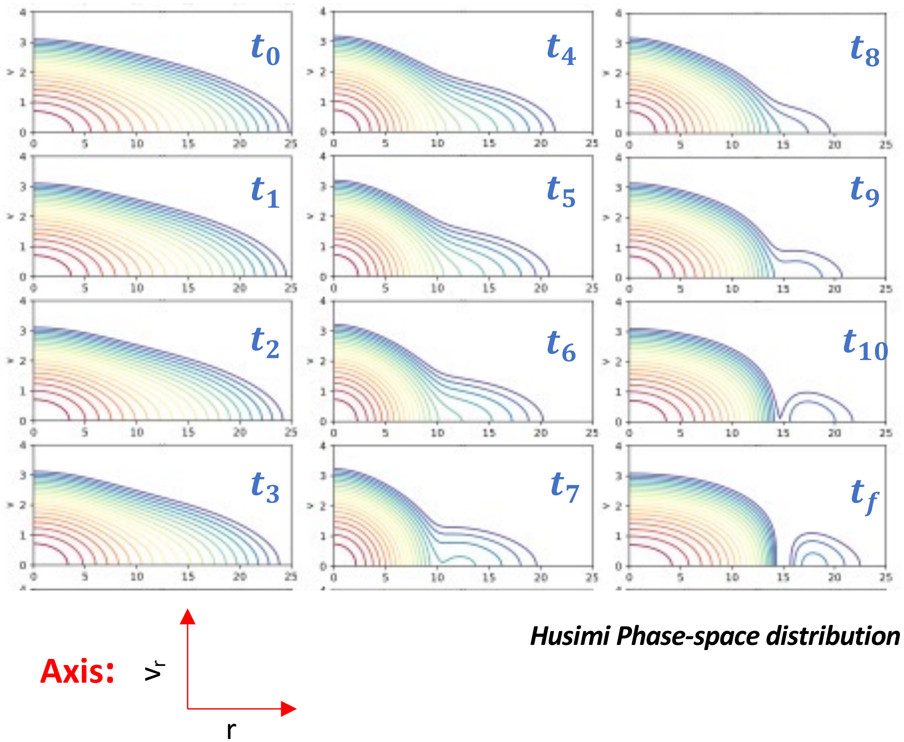


Radial density profiles of haloes formed in the ψDM model

Schive, Chiueh, and Broadhurst (2014)

Dynamics: 3D Numerical simulations

FDM out of equilibrium, ε=1



Husimi Phase-space distribution

Gravitational Cooling

E.Seidel, W.M.Suen(1994)
F.S Guzmán, L.A. Ureña-López (2004)

Rare Events are Non-Perturbative

Sina Hooshangi
IPM, Tehran

Based on:
arxiv: 2112.04520
(with M.H. Namjoo, M. Noorbala)

Rare Events

- Large rare fluctuations that can be bigger than a threshold, Collapse to form a BH.
- We need Non-perturbative methods to explore the tail.
- I will focus on Probability Distribution Function(PDF) for curvature perturbation. To calculate correctly the mass fraction you need the PDF for compaction function.
  (De Luca and Riotto, arxiv:2201.09008
  Biagetti, De Luca, Franciolini, Kehagias, Riotto, arxiv: 2105.07810
  Musco, De Luca, Franciolini, Riotto, arxiv:2011.03014
  Escriva arxiv:2111.12693)

Non-Perturbative Formalisms

- Wave-Function of Universe (Exp(- \zeta ^ 3/2))
  (Celoria, Creminelli, Tambalo, Yingcharoenrat, 2021, arxiv:2103.09244)

- Stochastic Inflation (Exponential Tail)
  (Ezquiaga, Garcia-Bellido, Vennin, 2020, arxiv:1912.05399)

- delta-N Formalism

Heavy-Tailed Distribution

Mathematical definition:

$$\bar{F}(\zeta) = \int_{\zeta}^{+\infty} \rho_\zeta(\zeta')d\zeta' \xrightarrow{\text{for} \quad \lambda > 0} \limsup_{\zeta \to +\infty} e^{\lambda\zeta} \bar{F}(\zeta) = +\infty$$

Practical definition:

$$\mathcal{D} = -\frac{\mathrm{d} \log \rho_\zeta}{\mathrm{d}\zeta} \longrightarrow \lim_{\zeta \to +\infty} \mathcal{D} = 0$$

Some Examples of Heavy-tailed PDFs

$$p(x) = \frac{1}{\sqrt{2\pi\sigma x}} \exp\left(-\frac{\ln^2 x}{2\sigma^2}\right)$$

$$p(x) = \frac{1}{2\pi} \frac{\exp\left(\frac{1}{2x}\right)}{x^{3/2}}$$

$$p(x) \sim \exp(-kx^p) \quad 0 < p < 1 \,, \; k > 0$$

$$p(x) \sim \frac{1}{x^n} \quad n > 1$$

Our Model (Power Law Tail)

$$V(\phi) = V_0\left[1 + \frac{1}{3}\alpha\left(\phi - \bar{\phi}\right)^3\right]$$

$$\big\downarrow$$

$$\mathcal{N}\left(\phi\right)$$

$$\zeta = \mathcal{N}(\bar{\phi} + \delta\phi) - \mathcal{N}(\bar{\phi}) \longrightarrow \text{Power Law}$$

$$\zeta^{(1)} \approx \mathcal{N}'(\bar{\phi})\delta\phi \longrightarrow \text{Gaussian}$$

$$\zeta^{(2)} \approx \mathcal{N}'(\bar{\phi})\delta\phi + \frac{1}{2}\mathcal{N}''(\bar{\phi})\delta\phi^2 \longrightarrow \text{Exponential}$$

$$\rho_\zeta = \left|\frac{d\delta\phi}{d\zeta}\right|\rho_{\delta\phi}$$

The PDF

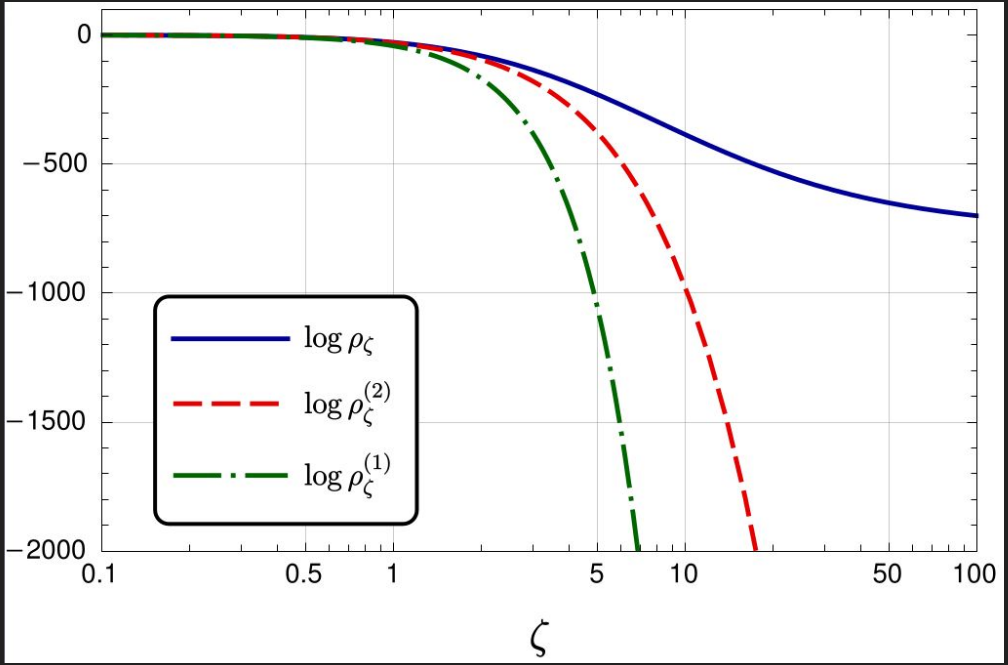
$$\bar{\phi} = 0\ \mathbf{M_P}$$

$$V_0 = 10^{-10}\ \mathrm{M_P^4}$$

$$\bar{\pi} = -10^{-4}\mathrm{M_P}$$

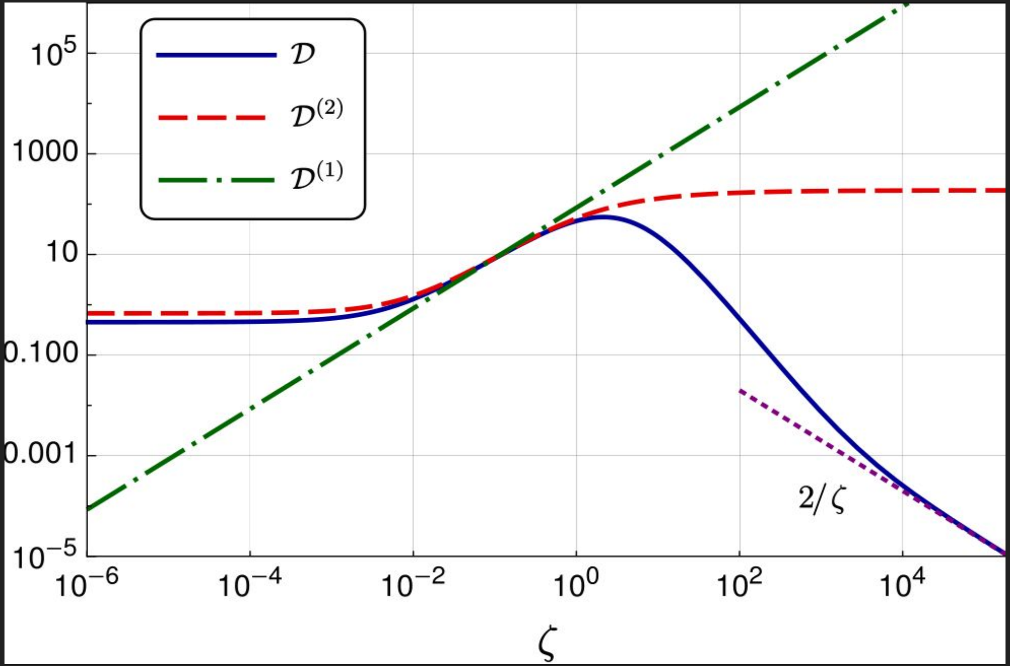
$$\alpha = 10^4\ \mathrm{M_P}$$

$$\phi_e = -3.5 \times 10^{-4}\mathrm{M_P}$$



The Behavior of the Tail

$$\mathcal{D} = -\frac{\mathrm{d}\log \rho_\zeta}{\mathrm{d}\zeta}$$



$$\beta = \int_{\zeta_c=1} \rho_\zeta \mathrm{d}\zeta$$

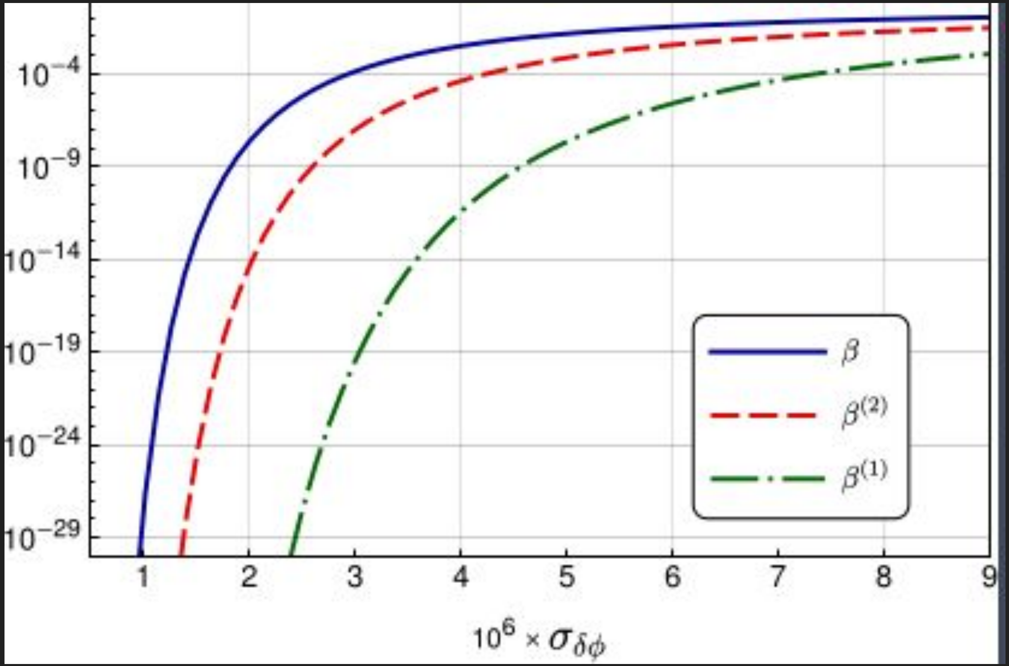
$\bar{\phi} = \mathbf{0\ M_P}$

$V_0 = 10^{-10}\ \mathrm{M_P^4}$

$\bar{\pi} = -10^{-4}\mathrm{M_P}$

$\alpha = 10^4\ \mathrm{M_P}$

$\phi_e = -2.5 \times 10^{-5}\mathrm{M_P}$



Thanks!

ICTP Summer School on Cosmology 2022
04 -15 July

Gravitational Reheating

based on Ref. arxiv: 2201.02348

Md Riajul Haque
Department of Physics,
IIT Guwahati, Assam, India

- How the present state of our universe has been created?
- Is it possible to explain our current universe from purely gravitational production during reheating?

Difficulty in probing the early Universe

Our knowledge about the cosmic history of the Universe

Cosmic microwave background (CMB)

Provides evidence for an early inflationary phase with
- ❖ Energy scale $\sim 10^{16}$ GeV
- ❖ Duration $\Delta t_{inf} \geq 10^{36}$ Sec

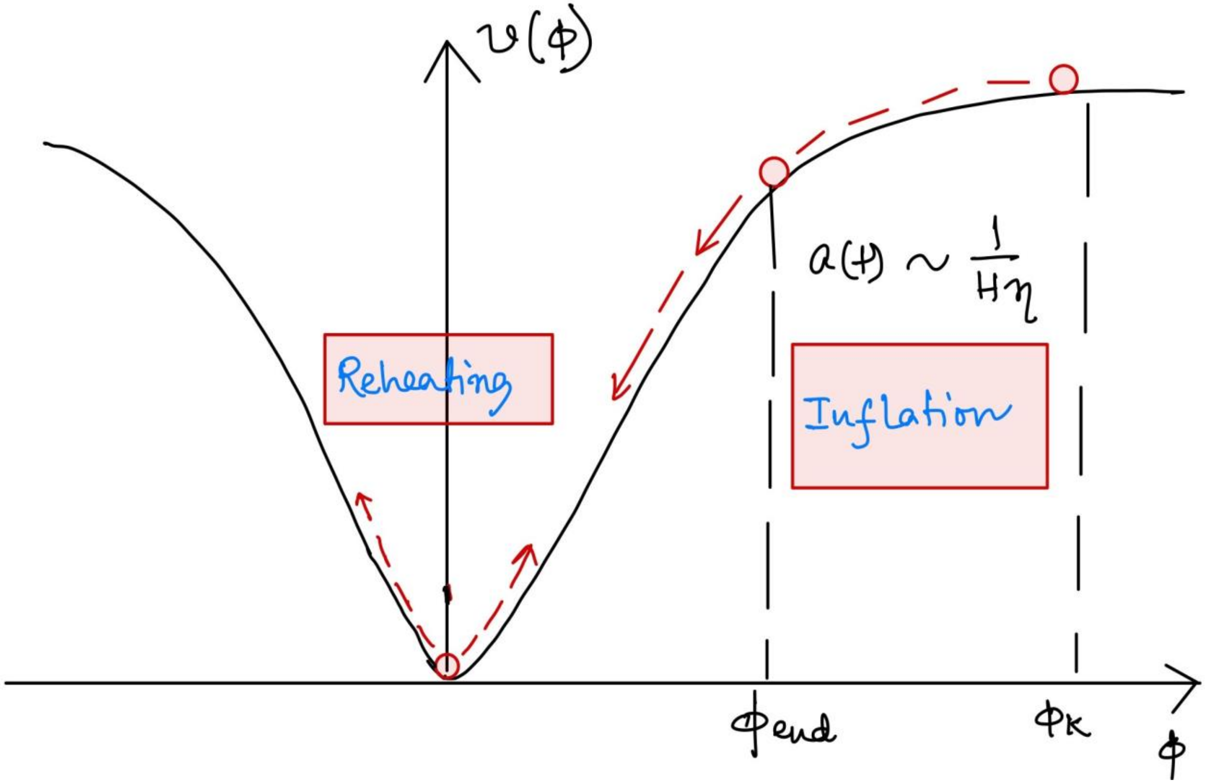
Big-Bang Nucleosynthesis (BBN)

Predicts quantities such as light-element abundances
- ❖ Energy scale $E_{BBN} \sim 1$ MeV
- ❖ Time scale $t_{BBN} \sim 1$ Sec

- ❖ There is a massive gap in terms of energy (and time) scale between the periods of inflation and BBN, which is poorly understood from both theory and observation

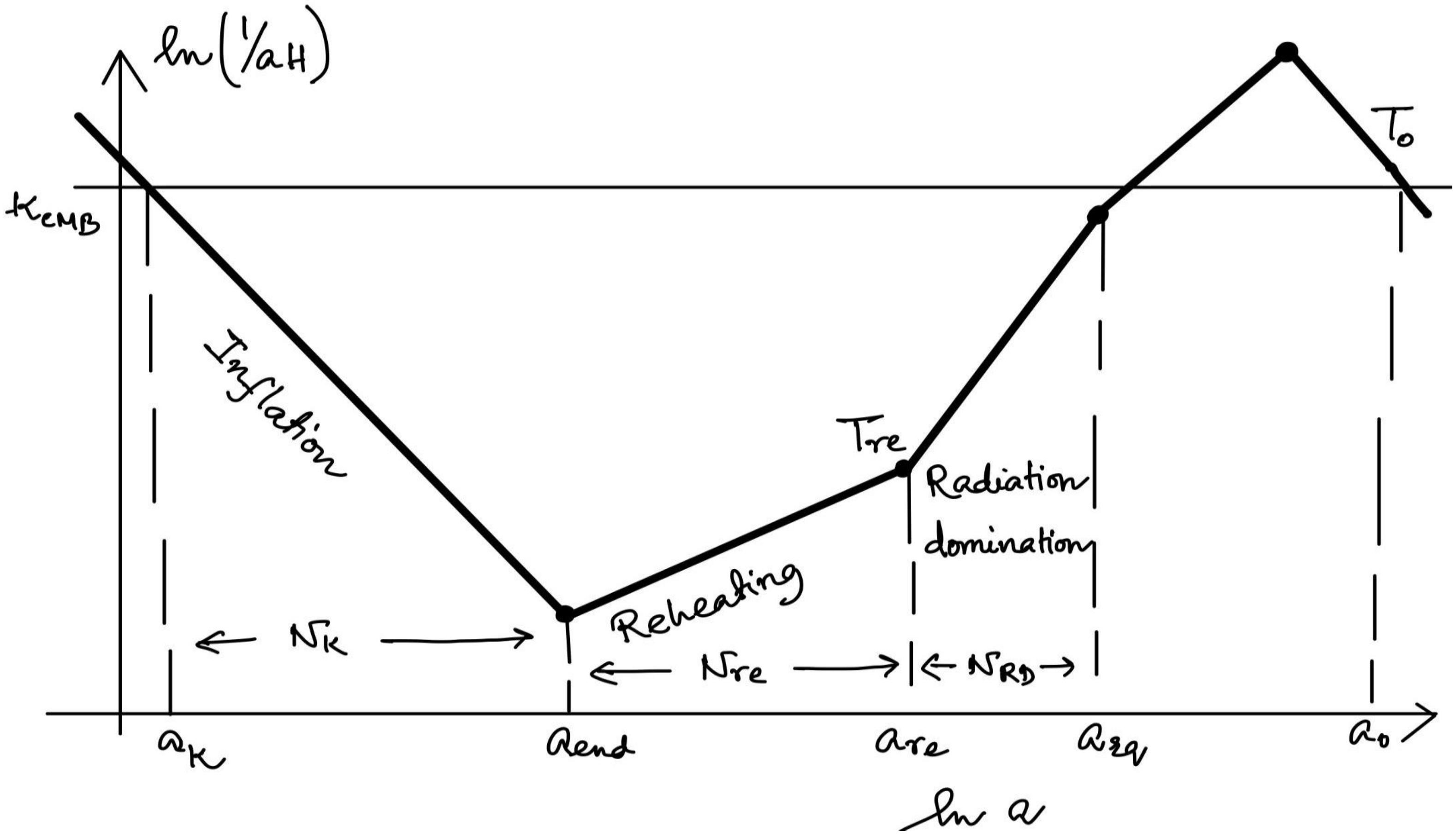
Why do we need reheating phase?

- The end point of inflation
  - The universe is cold, dark, and dominated by the homogeneous inflaton field.
- How does the Universe transition to a the hot, thermalized, radiation-dominated state after inflation, which is required for nucleosynthesis.
- Reheating!



- Natural consequence after inflation: filled the empty space with matters (**generate entropy)**

Schematic diagram of the evolution of the Comoving Hubble radius



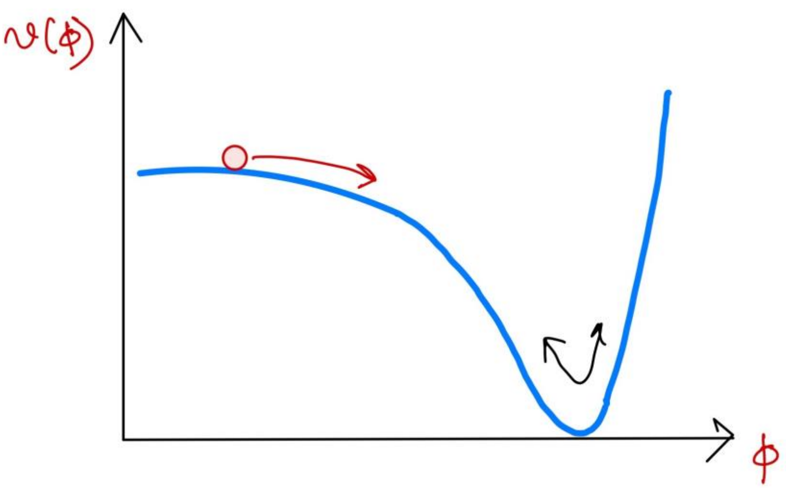
❖ We need to understand how the modified expansion history influences the prediction for cosmological observables.

Inflationary parameters: Initial conditions for reheating

- Slow roll parameters: $\epsilon_v = \frac{M_p^2}{2}\left(\frac{V'}{V}\right)^2 \qquad \eta_v = M_p^2\left(\frac{V''}{V}\right)$

- e-folding number & inflationary energy scale :

$$N_k = \log\left(\frac{a_{end}}{a_k}\right) = \int_{\phi_k}^{\phi_{end}} \frac{|d\phi|}{\sqrt{2\epsilon_v}M_p}\ , H_k = \frac{\pi M_p\sqrt{r_k A_s}}{\sqrt{2}}$$



- CMB observable :

$$n_s = 1-6\epsilon_k(\phi_k)+2\eta_k(\phi_k)\ ,\ r = 16\epsilon_k(\phi_k)$$

- End of inflation : Initial condition for reheating

$$\epsilon(\phi_{end}) = \frac{1}{2M_p^2}\left(\frac{V'(\phi_{end})}{V(\phi_{end})}\right)^2 = 1$$

Reheating phenomenology

- Usual approach: Through parametric resonance (Preheating)/ Perturbative decay → $g\phi S^2,\ g_1\phi^2 S^2,\ h\phi\bar{f}f, \ldots\ldots$

- Gravitational decay → $\sim \frac{1}{M_P} h_{\mu\nu} T_i^{\mu\nu}, i = S, f, X, \phi$

The gravitational decay channel was always ignored due to this Planck mass suppression. It was always thought that only gravitational production could not be sufficient to reheat the universe successfully.

Gravitational reheating set up

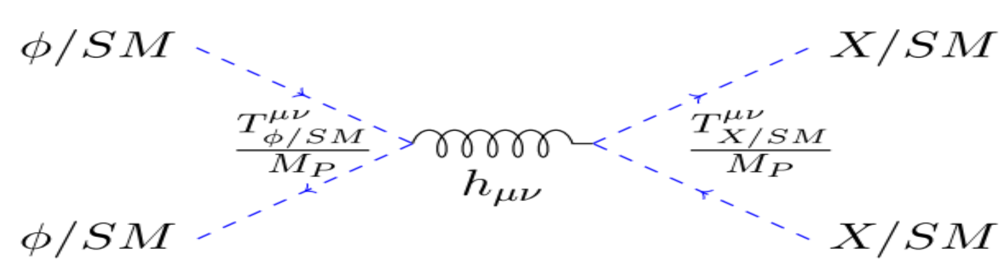
- Inflaton gravitationally decaying into Radiation (massless) + Dark matter (massive)

$$
\begin{aligned}
&\dot{\rho}_\phi + 3H(1+\omega_\phi)\rho_\phi + \Gamma^T_\phi \rho_\phi (1+\omega_\phi) = 0\,, \\
&\dot{\rho}_R + 4H\rho_R - \Gamma^{Rad}_{\phi\phi\to RR}\,\rho_\phi(1+\omega_\phi) = 0\,, \\
&\dot{n}_Y + 3H n_Y - \frac{\Gamma^{DM}_{\phi\phi\to YY}}{m_\phi}\,\rho_\phi(1+\omega_\phi) = 0
\end{aligned}
$$

$$
\begin{aligned}
\Gamma_{\phi\phi\to SS} &= \frac{\rho_\phi\, m_\phi}{1024\,\pi\, M_p^4}\left(1+\frac{m_S^2}{2\,m_\phi^2}\right)\sqrt{1-\frac{m_S^2}{m_\phi^2}}\,, \\
\Gamma_{\phi\phi\to ff} &= \frac{\rho_\phi\, m_f^2}{4096\pi\, M_p^4 m_\phi}\left(1-\frac{m_f^2}{m_\phi^2}\right)^{\frac{3}{2}}\,, \qquad (0.1)\\
\Gamma_{\phi\phi\to XX} &= \frac{\rho_\phi\, m_\phi}{32768\,\pi\, M_p^4}\left(4+4\frac{m_X^2}{m_\phi^2}+19\frac{m_X^4}{m_\phi^4}\right)\sqrt{1-\frac{m_X^2}{m_\phi^2}}.
\end{aligned}
$$

$$
\begin{aligned}
\Omega_X h^2 &= \frac{\rho_X(T_F)}{\rho_R(T_F)}\frac{T_F}{T_{now}}\Omega_R h^2, \\
&= \langle E_X\rangle \frac{X(T_F)}{R(T_F)}\frac{T_F}{T_{now}}\frac{A_F}{m_\phi}\Omega_R h^2
\end{aligned}
$$

Parameters: $H_{end},\ \omega_\phi,\ M_{DM}$ $\quad \Gamma^{Rad} = (\Gamma^S + \Gamma^f + \Gamma^X)$



M. R. Haque and D. Maity [arXiv:2201.02348
Y. Mambrini and K. A. Olive [Phys.Rev.D 103 (2021) 11, 115009]

Initial conditions and constraints

- Initial conditions : $\boxed{\rho_\phi^{in} = 3M_p^2 H_{end}^2, \rho_R = \rho_{DM} = 0}$

- Constraint conditions: Present state of our universe

1. Entropy conservation

$$\boxed{T_{re} = \left(\frac{43}{11\, g_*^{re}}\right)^{1/3} \left(\frac{a_0\, H_{end}}{k}\right) e^{-(N_k+N_{re})}\, T_0\,,}$$

with $k/a_0 = 0.05\ \mathrm{Mpc}^{-1}$, $T_0 = 2.725^0\ \mathrm{K}$

2. Present DM abundance $\boxed{\Omega_X h^2 = 0.12}$

3. Universe must be radiation dominated before $\boxed{T_{re} > T_{BBN} \sim 10MeV}$

4. Upper limit on Inflationary energy scale $\boxed{H_{end}^{max} > \pi M_p \sqrt{r\, A_s/2} \sim 5 \times 10^{13} GeV}$

Present state of the universe is completely fixed by $\boxed{H_{end},\ \omega_\phi,\ M_{DM}}$

L. Dai, M. Kamionkowski and J. Wang, PRL. 113, 041302 (2014)
J. L. Cook, etal JCAP 1504 (2015) 047

Model independent predictions

- Assuming Slow-roll inflation (with out taking any particular model)

$$\boxed{m_\phi^{end} \simeq \sqrt{(1+\omega_\phi)(4+12\omega_\phi)/(1-\omega_\phi)^2}\, H_{end}}$$

$$\boxed{N_{re} = \frac{1}{3\omega_\phi - 1} \ln\left(\frac{512\,\pi\, M_p^2\,(1+15\,\omega_\phi)}{3\,(1+\gamma)H_{end}\, m_\phi^{end}\,(1+\omega_\phi)}\right)}$$

$$\boxed{T_{re} = \left(\frac{9\,(1+\gamma)\,H_{end}^3\, m_\phi^{end}\,(1+\omega_\phi)}{512\,\beta\,\pi\,(1+15\omega_\phi)}\, e^{-4\,N_{re}}\right)^{1/4}}$$

$$\boxed{\begin{aligned} n_f^{com} &\simeq \frac{3H_{end}^3}{2048\pi}\frac{1+\omega_\phi}{1-\omega_\phi}\left(\frac{m_f}{m_\phi^{end}}\right)^2\left(1-e^{-\frac{3N_{re}}{2}(1-\omega_\phi)}\right), \\ n_S^{com} &= 8n_X^{com} = \frac{3H_{end}^3\,(1+\omega_\phi)}{512(\pi+3\pi\omega_\phi)}, \end{aligned}}$$

Gravitational Reheating prediction:

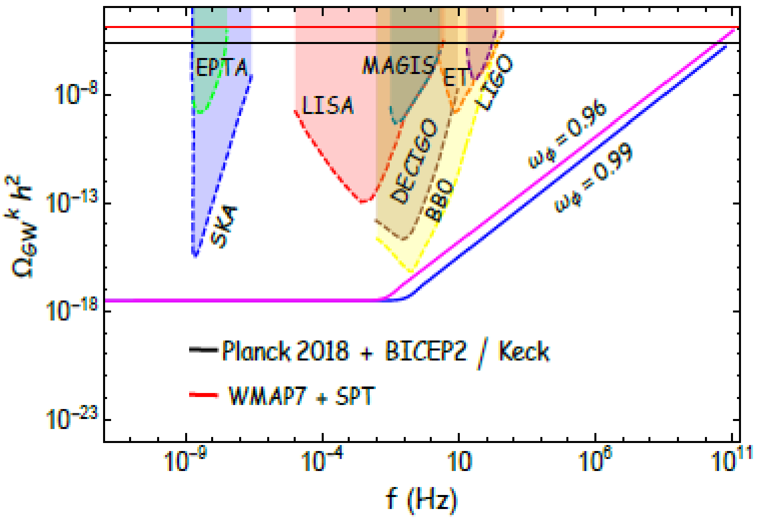
Dark matter sector

- Fermionic DM: $2\times10^5 < m_f < 3\times10^8\, GeV$
- Bosonic DM: $50 < (m_S, (1/8)m_X) < 1000\, eV$

Inflaton sector

- Inflaton equation of state $\omega_\phi = (0.6, 0.99)$
- Energy scale $H_{end} = (1\times10^9, 5\times10^{13})\, GeV$
- Inflationary e-folds $62 < N_{efold} < 63$

Predictions from primordial gravitational waves



Spectrum of the gravitational today

$$\Omega^k_{GW} h^2 \simeq \Omega_R h^2 P_T(k) \frac{4\mu^2}{\pi} \Gamma^2 \left(\frac{5+3\omega_\phi}{2+6\omega_\phi} \right) \left(\frac{k}{2\mu k_{re}} \right)^{n_{GW}}$$

$$\mu = \frac{1}{2}(1+3\omega_\phi) \quad P_T(k) = H^2_{end}/12\pi^2 M_p^2.$$

Index of the GW spctrum:

$$n_{GW} = \frac{(6\omega_\phi - 2)}{(3\omega_\phi + 1)}$$

From BBN bounds set by Plank 2018 data:

$$\boxed{\omega_\phi \to 0.98 \sim 1.0}$$

R. Haque, DM, S. T. Paul, L. Sriramkumar, Phys.Rev.D 104 (2021) 6, 063513

Constraining specific models

$$V(\phi) = \Lambda^4 \left[1 - e^{-\sqrt{\frac{2}{3\,\alpha}}\phi/M_p}\right]^{2n}$$

Predictions:

$\alpha = 1 \rightarrow 0.9681 \leq n_s \leq 0.9687$

$n \geq 4.75$

$7 \times 10^6 < m_f < 9 \times 10^7\ GeV$

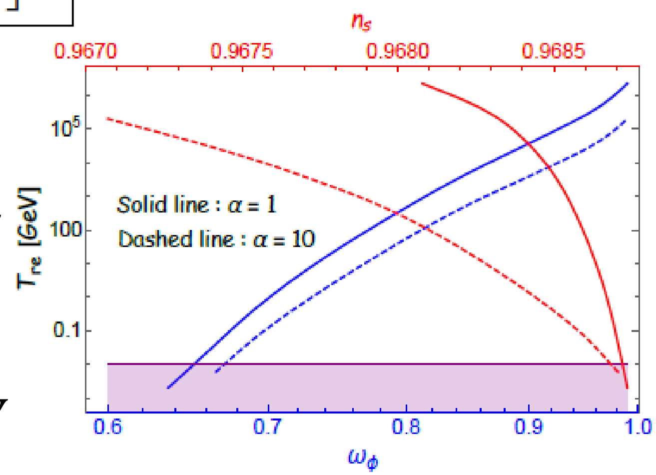
$60 < (m_S, (1/8)m_X) < 1000\ eV$

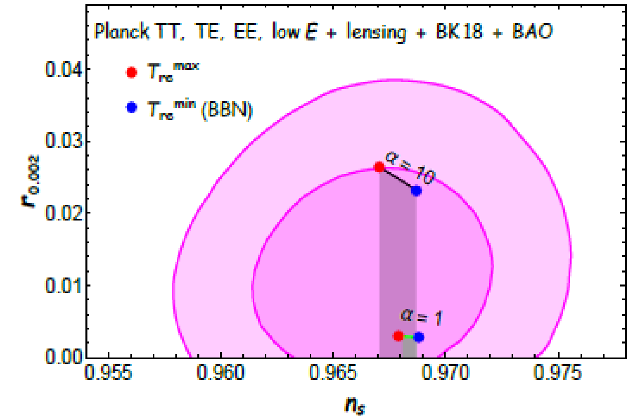
$\alpha = 10 \rightarrow 0.9611 \leq n_s \leq 0.9687$

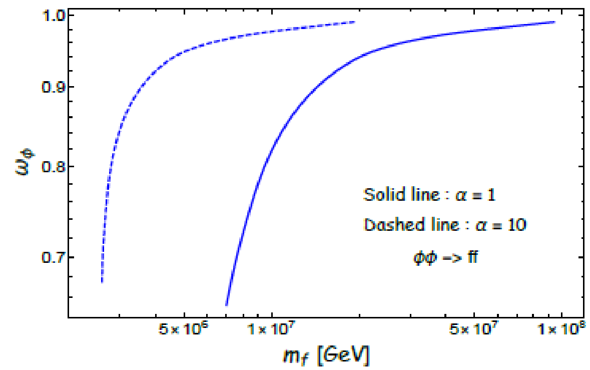
$n \geq 5.15$

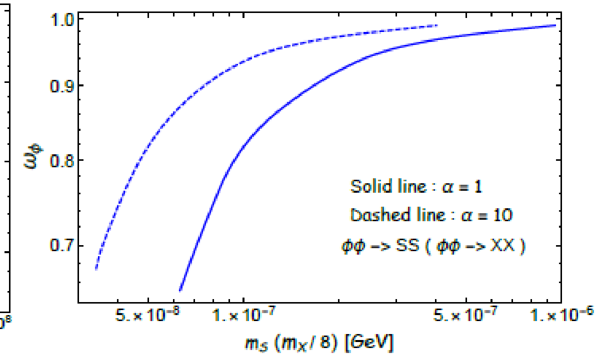
$3 \times 10^6 < m_f < 2 \times 10^7\ GeV$

$30 < (m_S, (1/8)m_X) < 400\ eV$

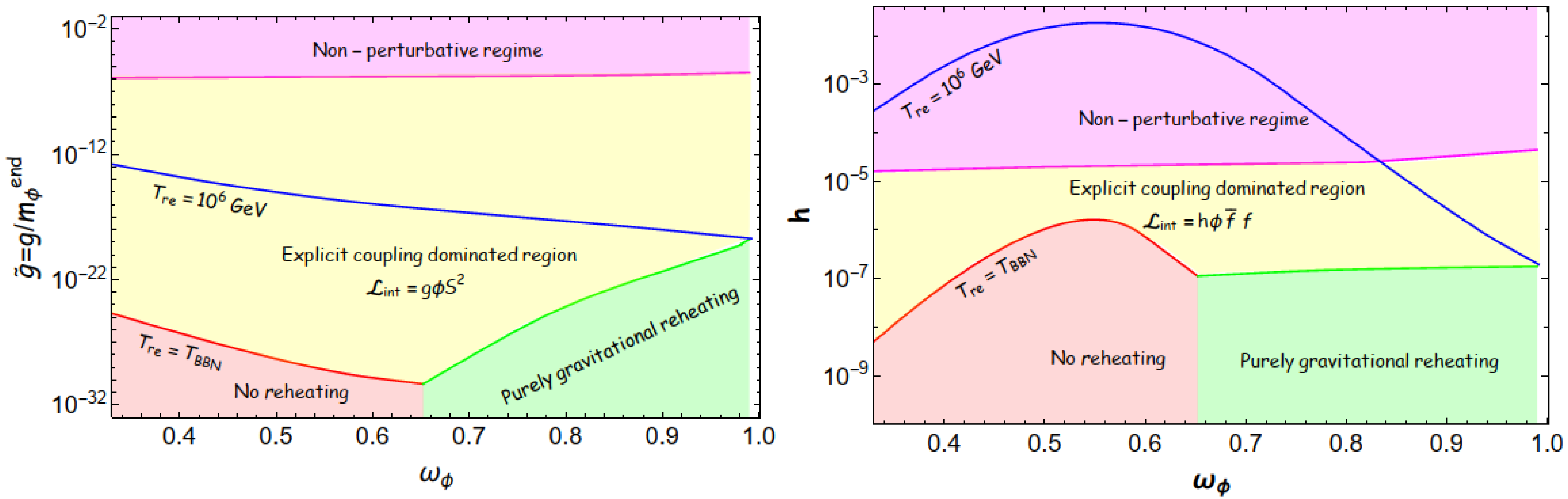








Compare the Graviational reheating scenario with the case where explicit coupling present between Inflaton and radiation sector



Main points and outcomes

- We took a model-independent approach to acquire a precise cosmological prediction. We switch off all possible unknown parameters, implying that the inflaton sector is coupled with the observable sector only through gravitational interaction.

- Predicts Very narrow range of DM mass, low reheating temperature, unique GW spectrum, Stiff reheating equation of state.

- The scenario discards a large number of possible models of dark matter and inflation that are otherwise consistent with PLANCK.

Thank You