Small Scale Structure in Vector Dark Matter

Mustafa A. Amin — RICE




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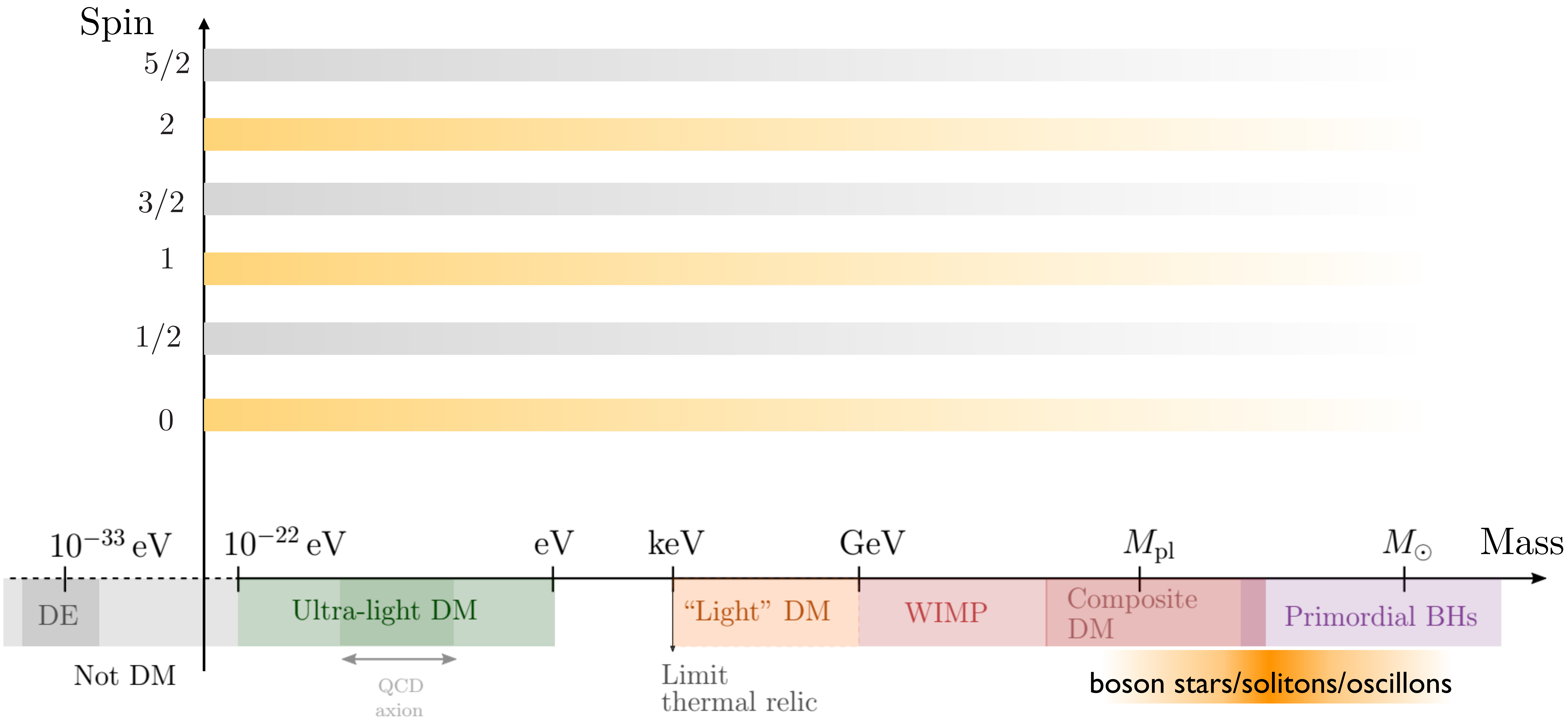
1 Polarized solitons in higher-spin wave dark matter arXiv: 2111.08700

2 Polarized vector oscillons arXiv: 2109.04892

3 Small-scale structure in vector dark matter arXiv: 2203.11935

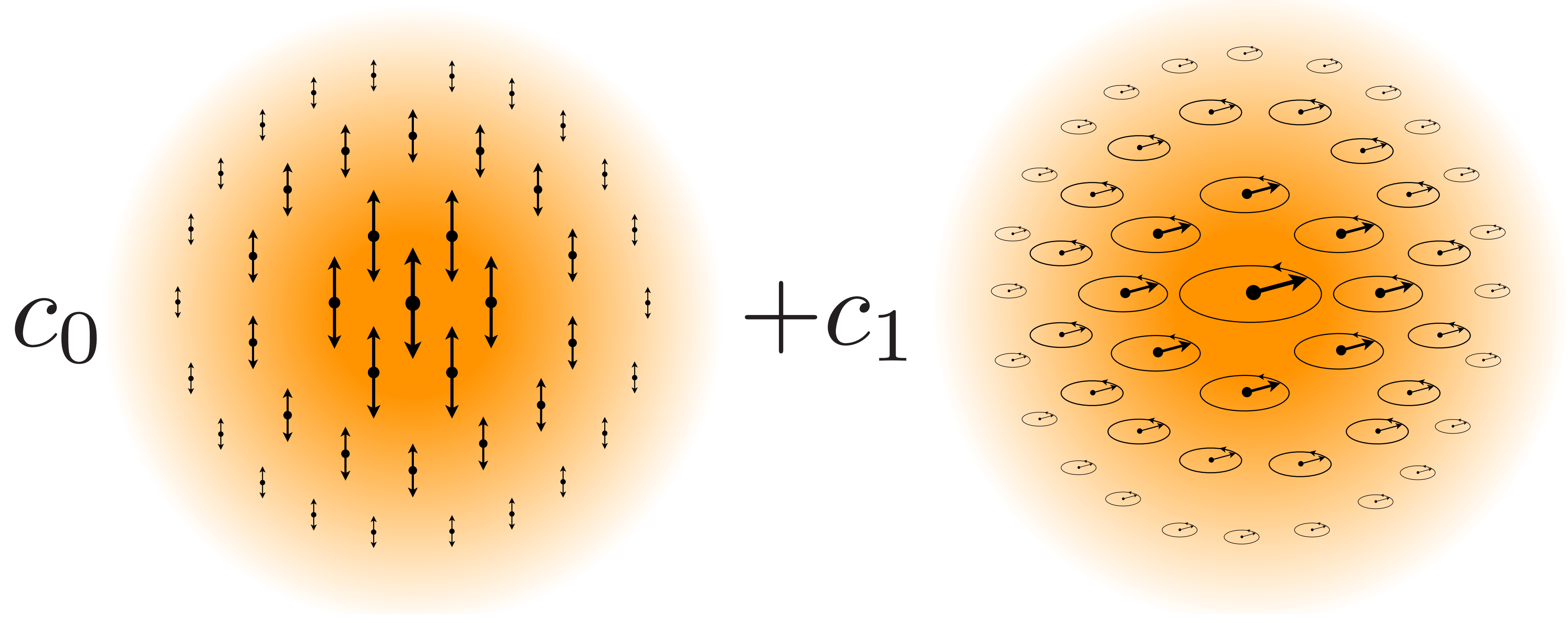
@ICTP Trieste (2022), seminar during the Cosmology Summer School

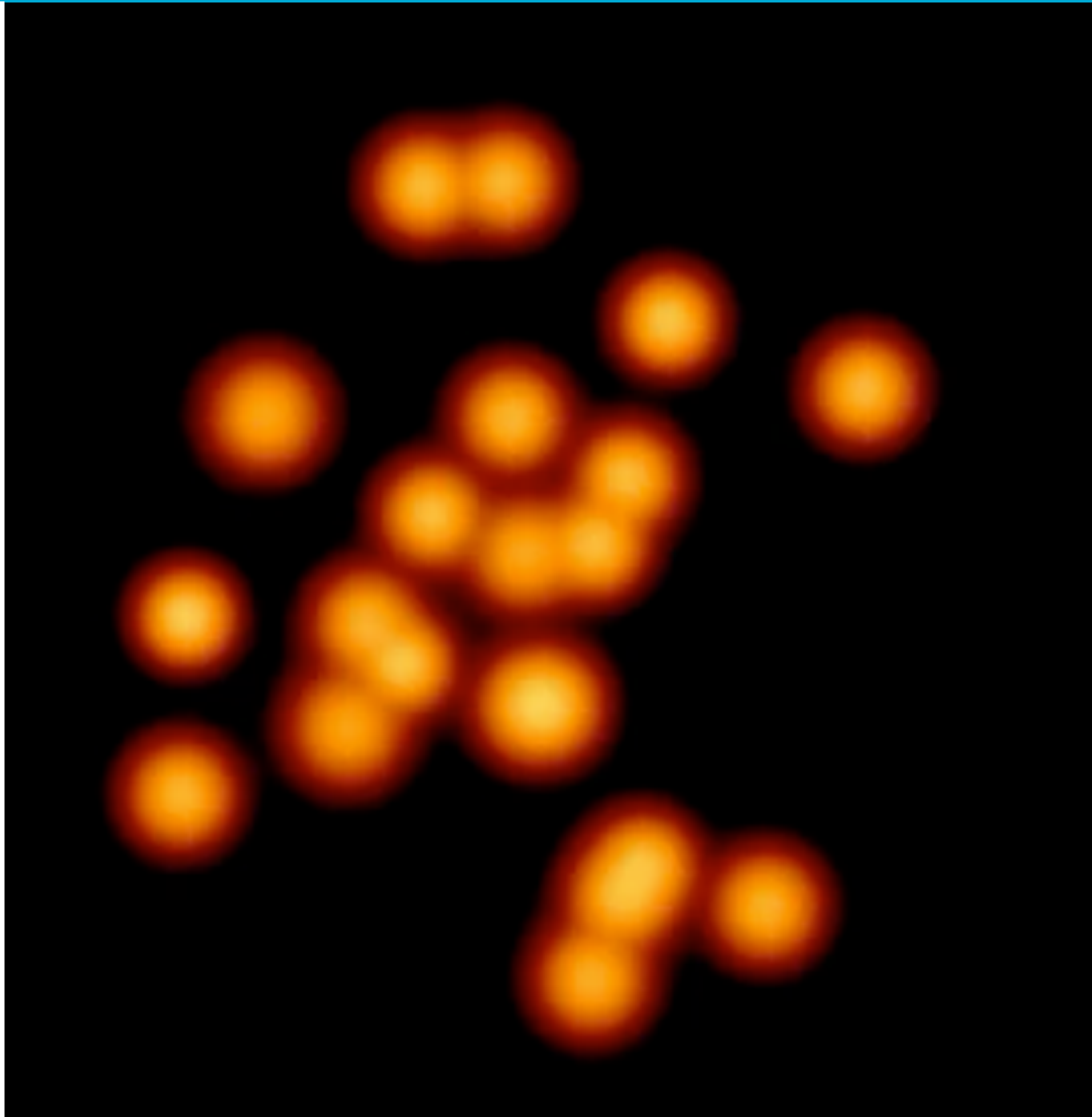
dark matter spin from astrophysical observations?



distinct phenomenology in (ultra)light vector dark matter

- new class of polarized vector solitons with macroscopic spin





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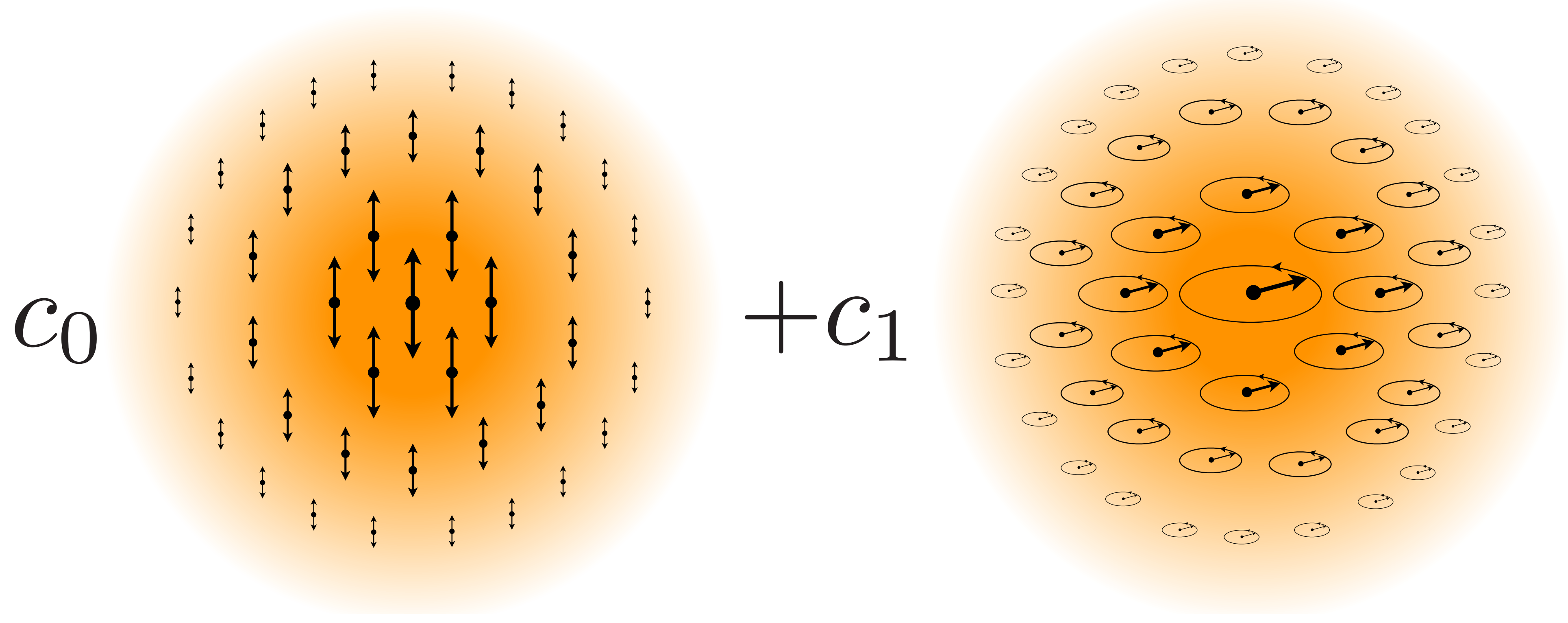
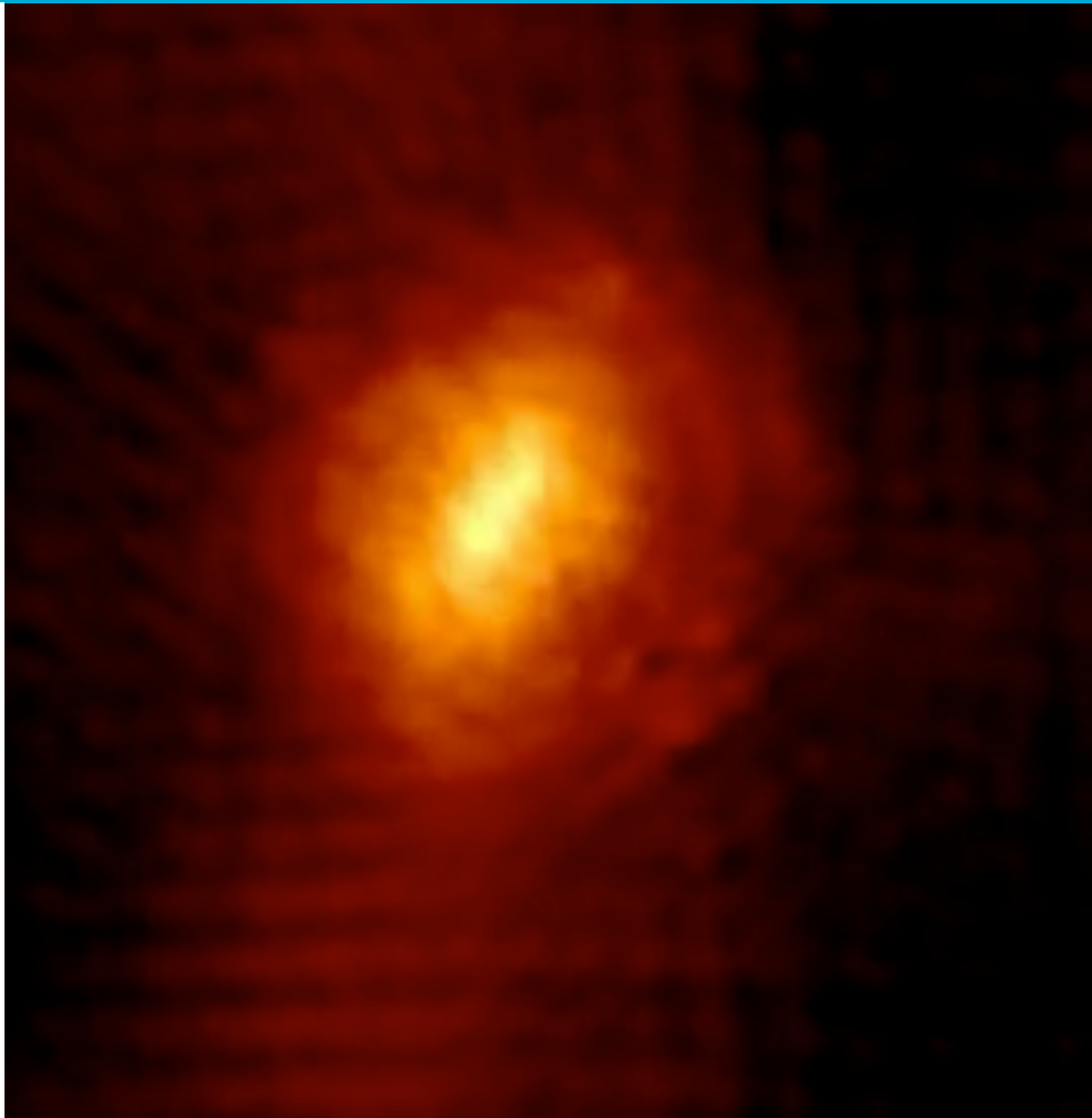


- interference patterns, and halo density profiles



distinct phenomenology in (ultra)light vector dark matter

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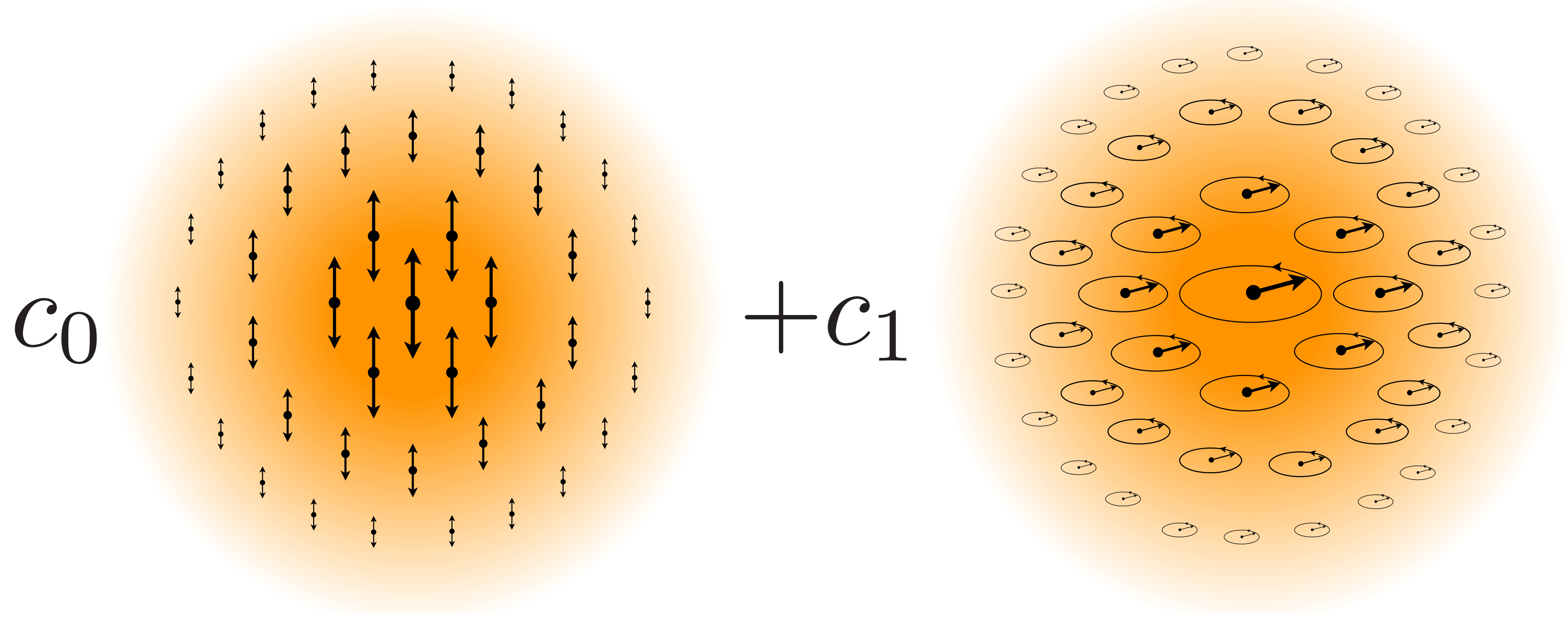
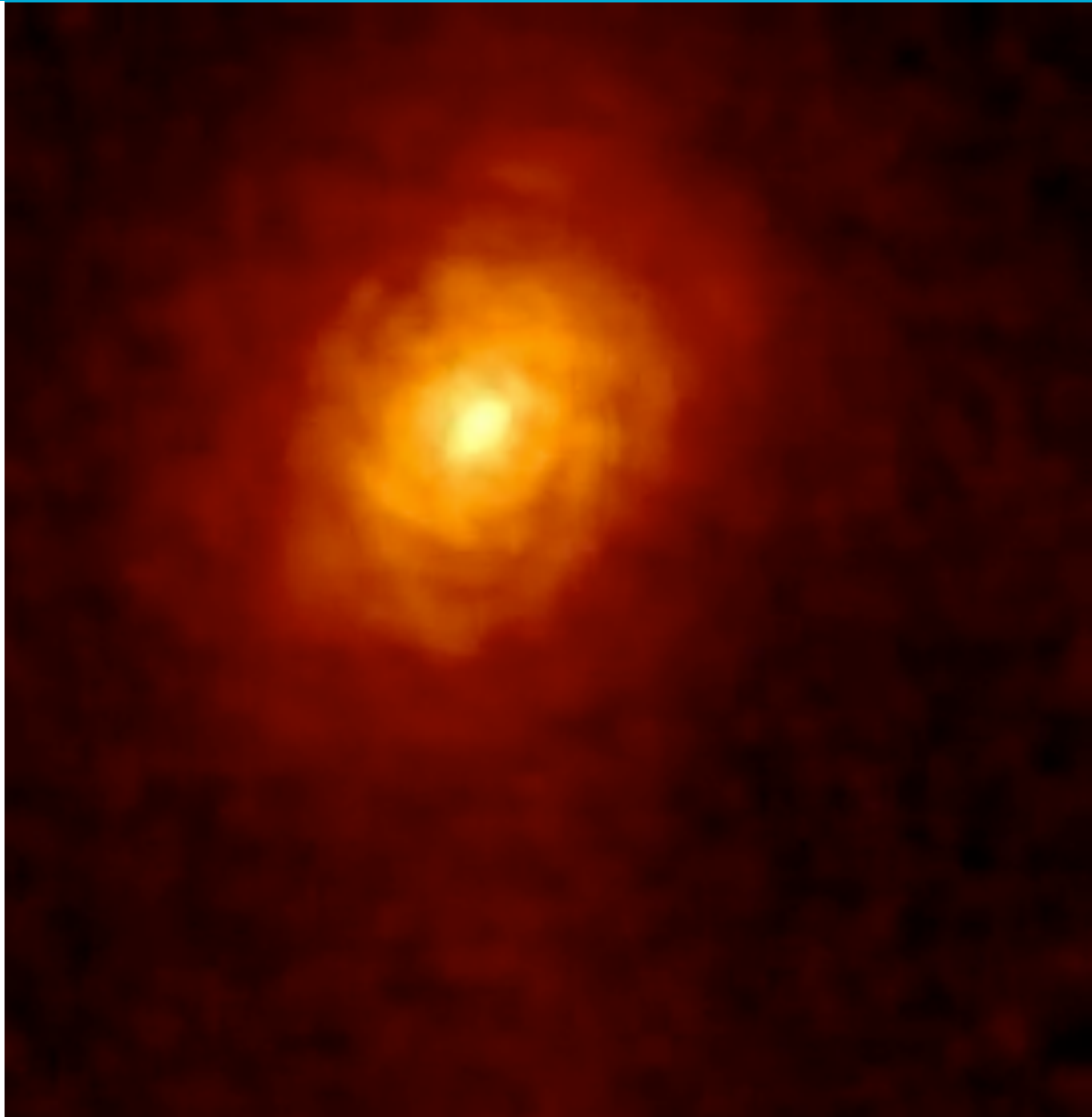


- interference patterns, and halo density profiles

- formation mechanisms



talk also includes pedagogical elements

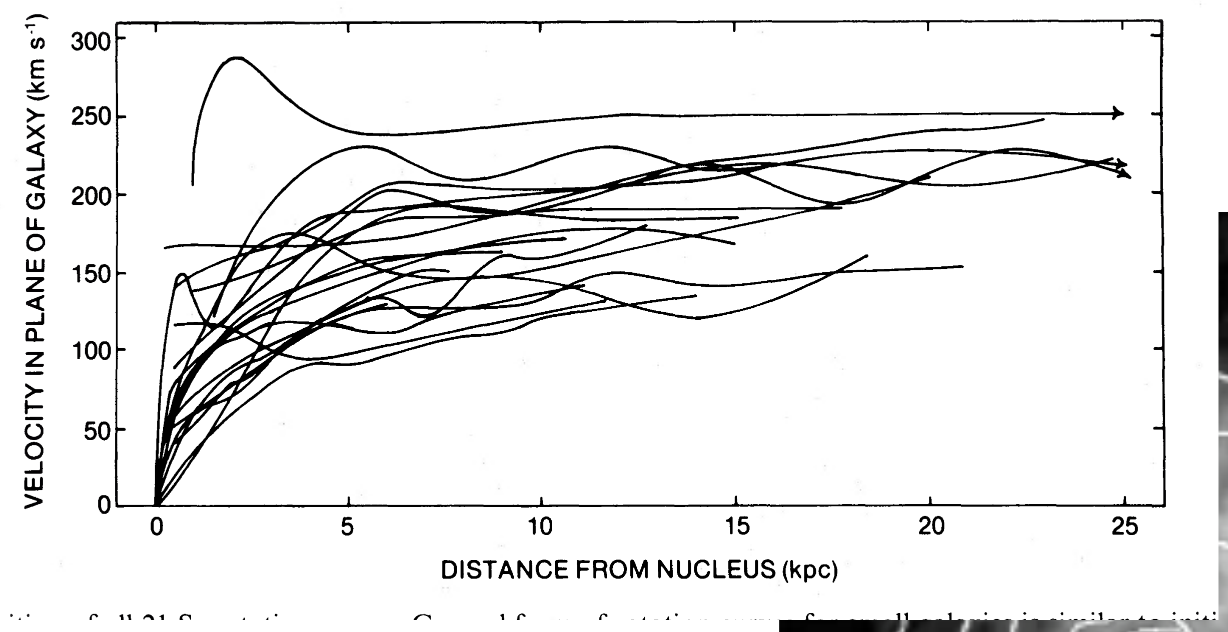
Introduction to:

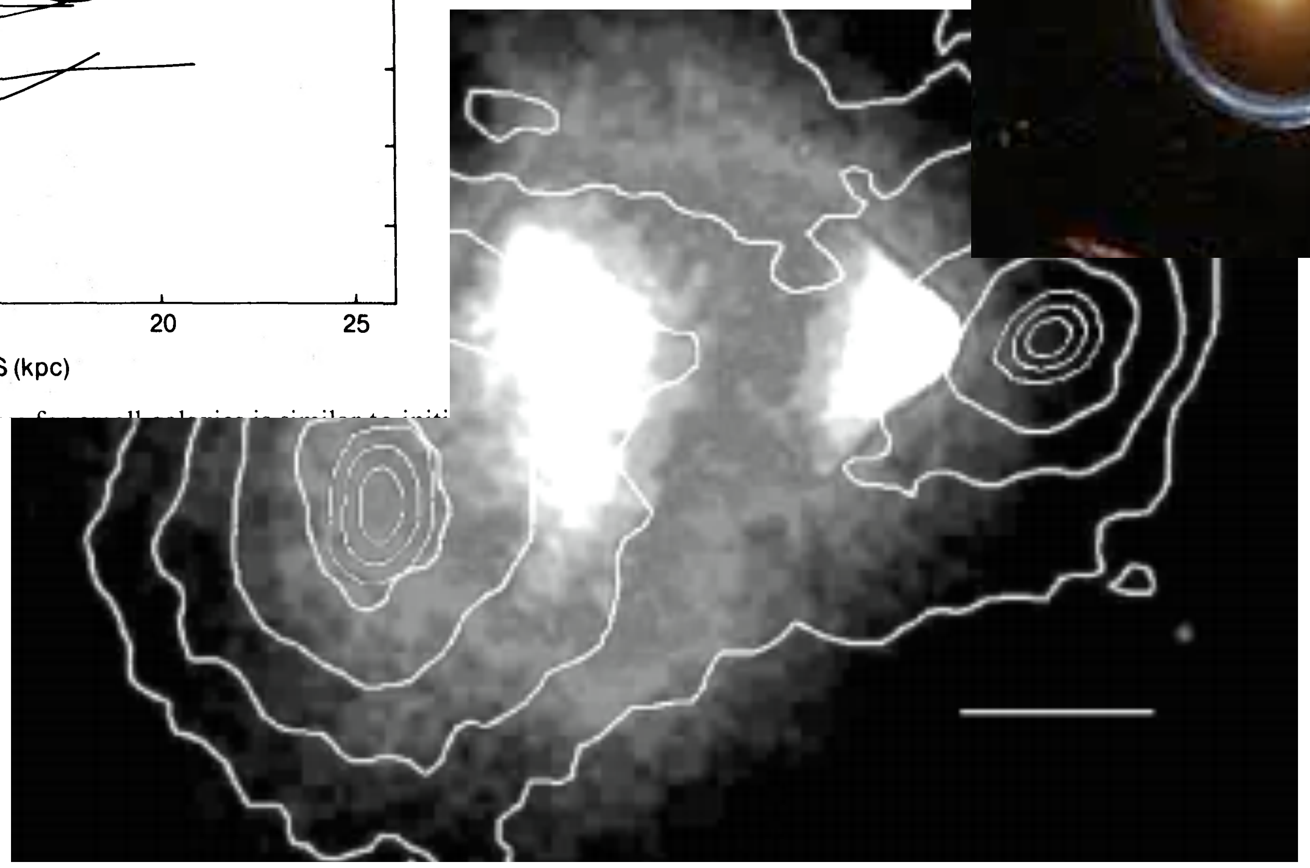
- dark matter (DM) and its properties
- field/wave-like dark matter
- non-topological solitons
- power spectrum of density fluctuations in standard CDM, scalar DM and vector DM

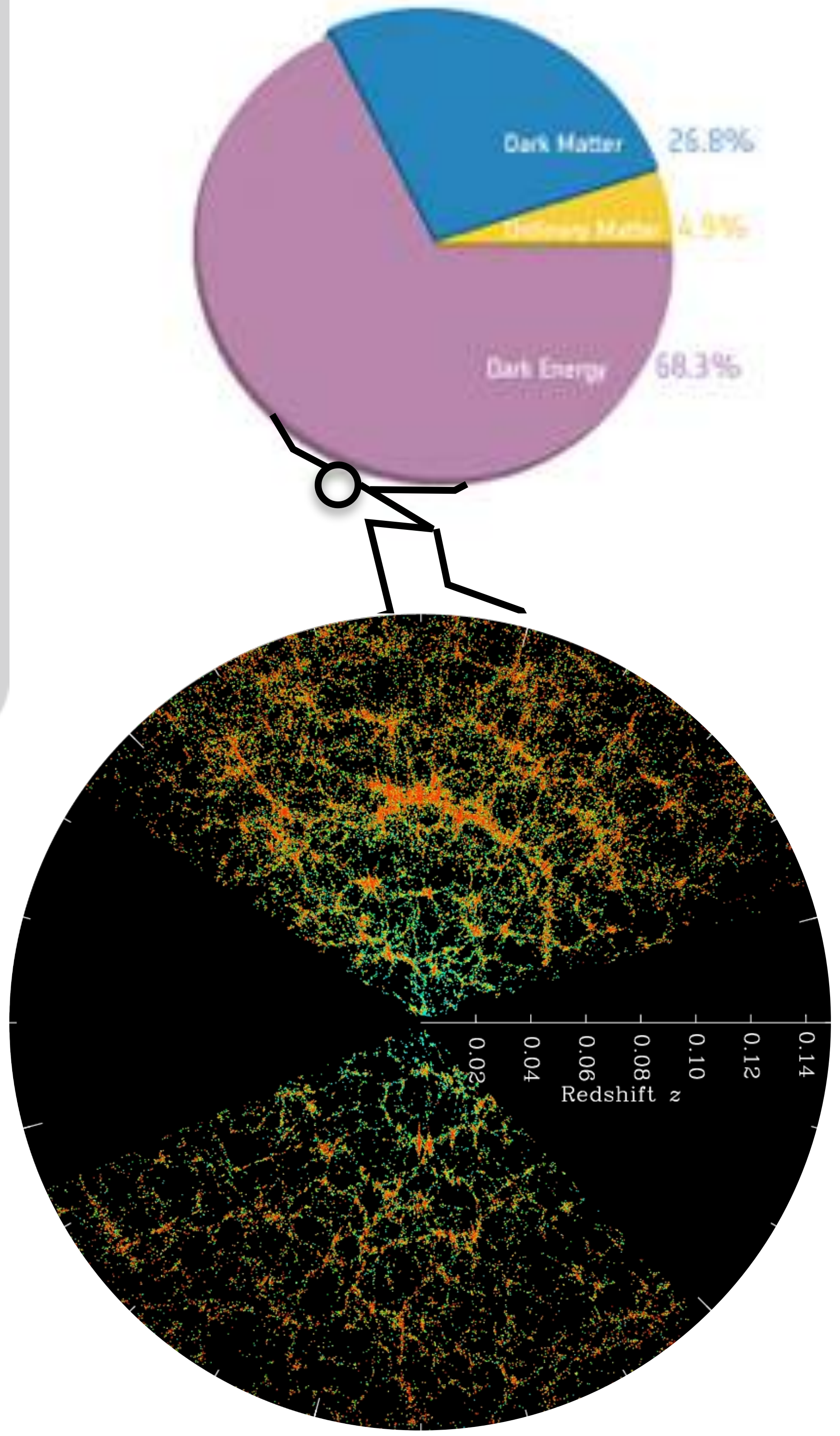
motivation & introduction

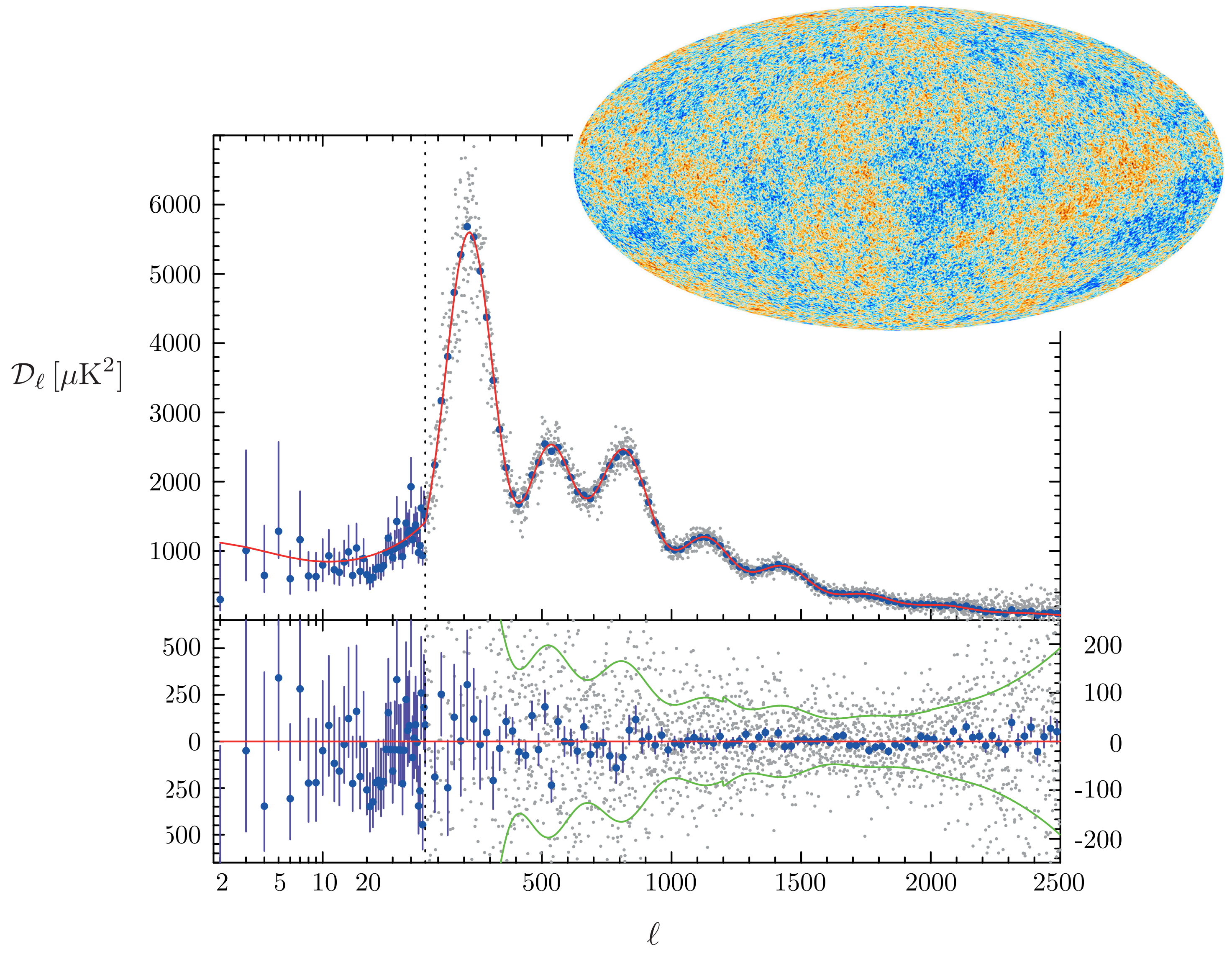
evidence for dark matter

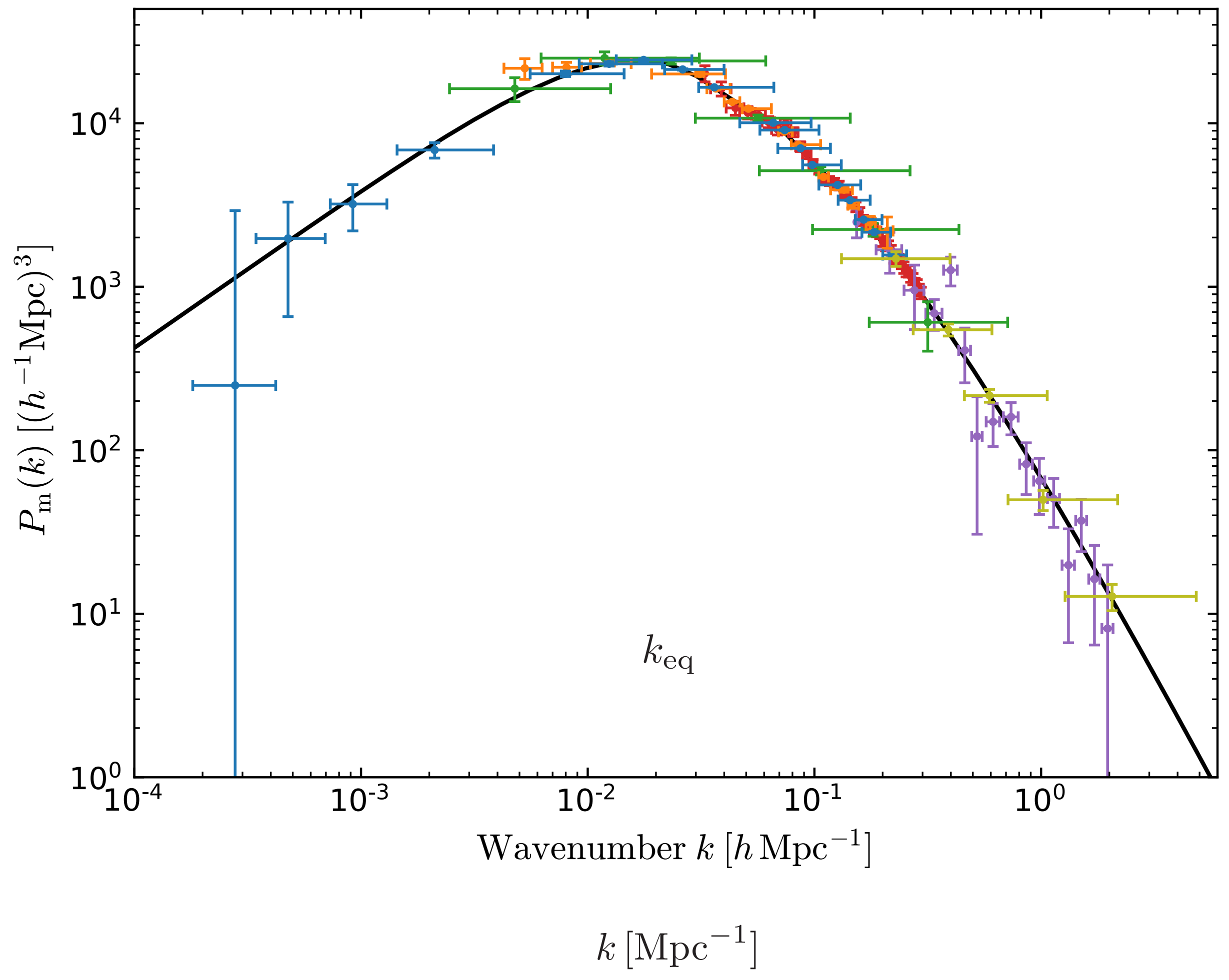
- dark matter exists
- gravitational interactions ✓











*incomplete sample

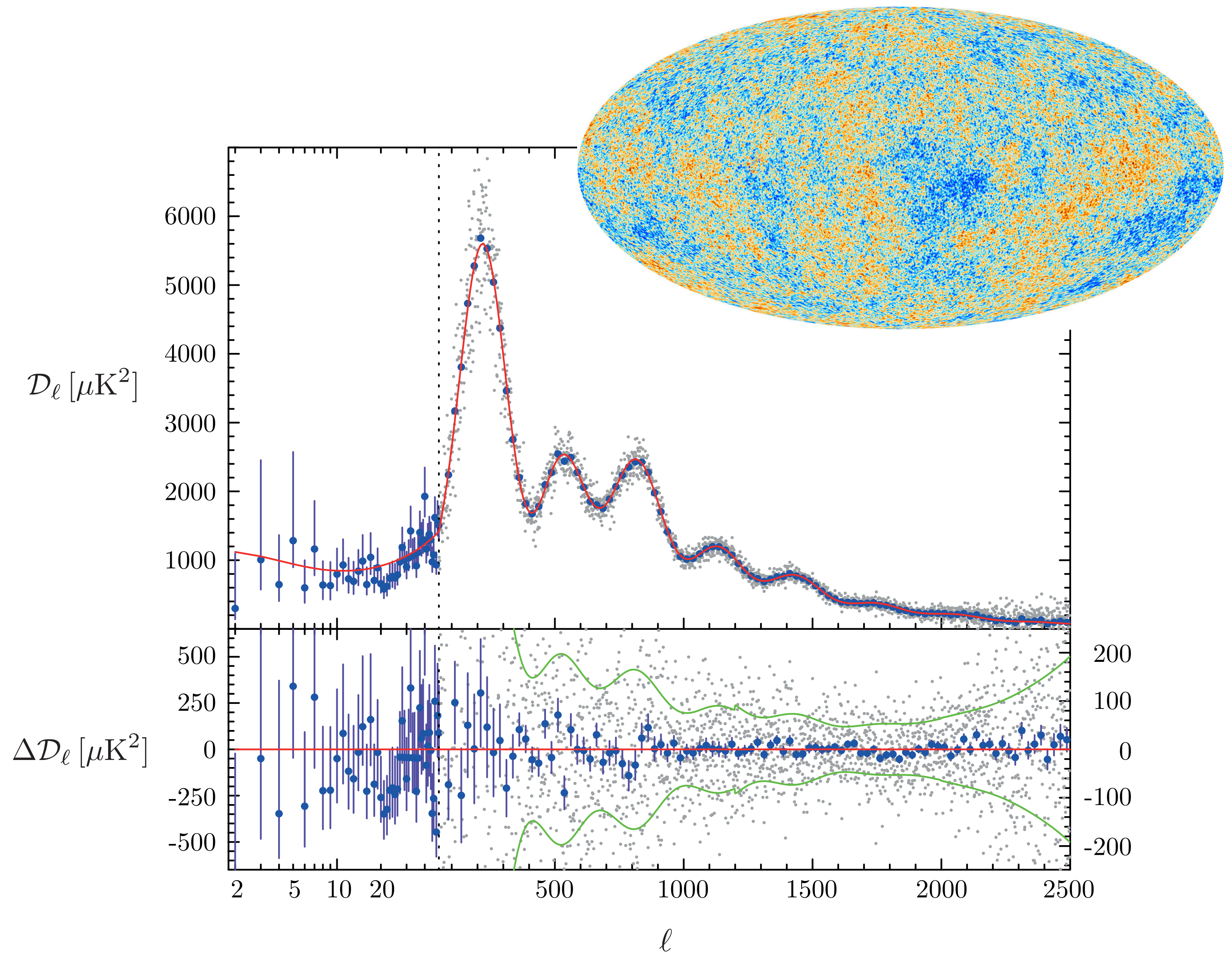
what is dark matter?

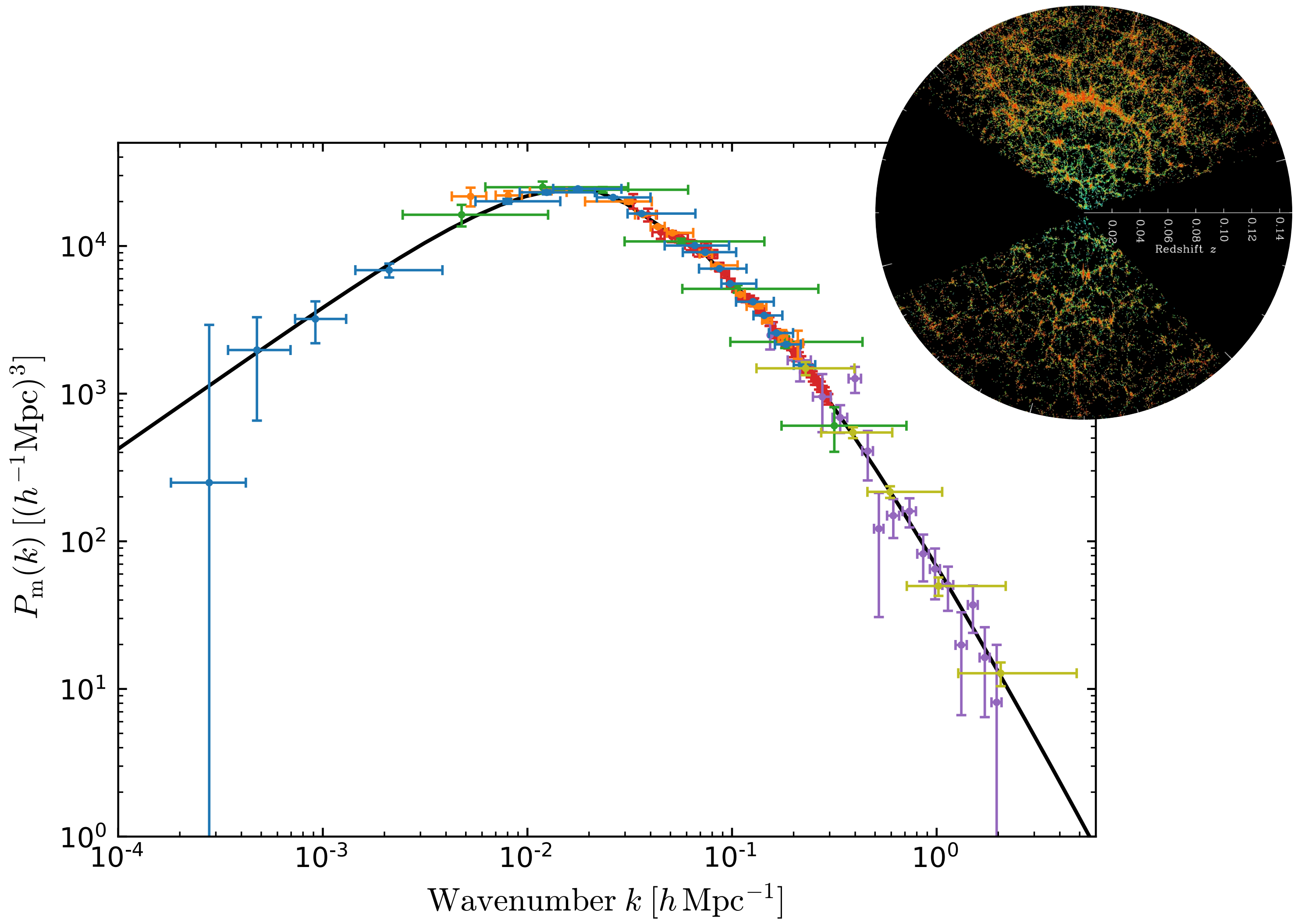
- what is it: charge, spin, mass, σ ?
- elementary / composite ?
- formation — thermal, non-thermal ?

*I recommend my colleague, Andrew Long's notes on cosmological relics

charge of dark matter?

- what is it: **charge**, spin, mass, σ ?
- elementary / composite ?
- formation — thermal, non-thermal ?





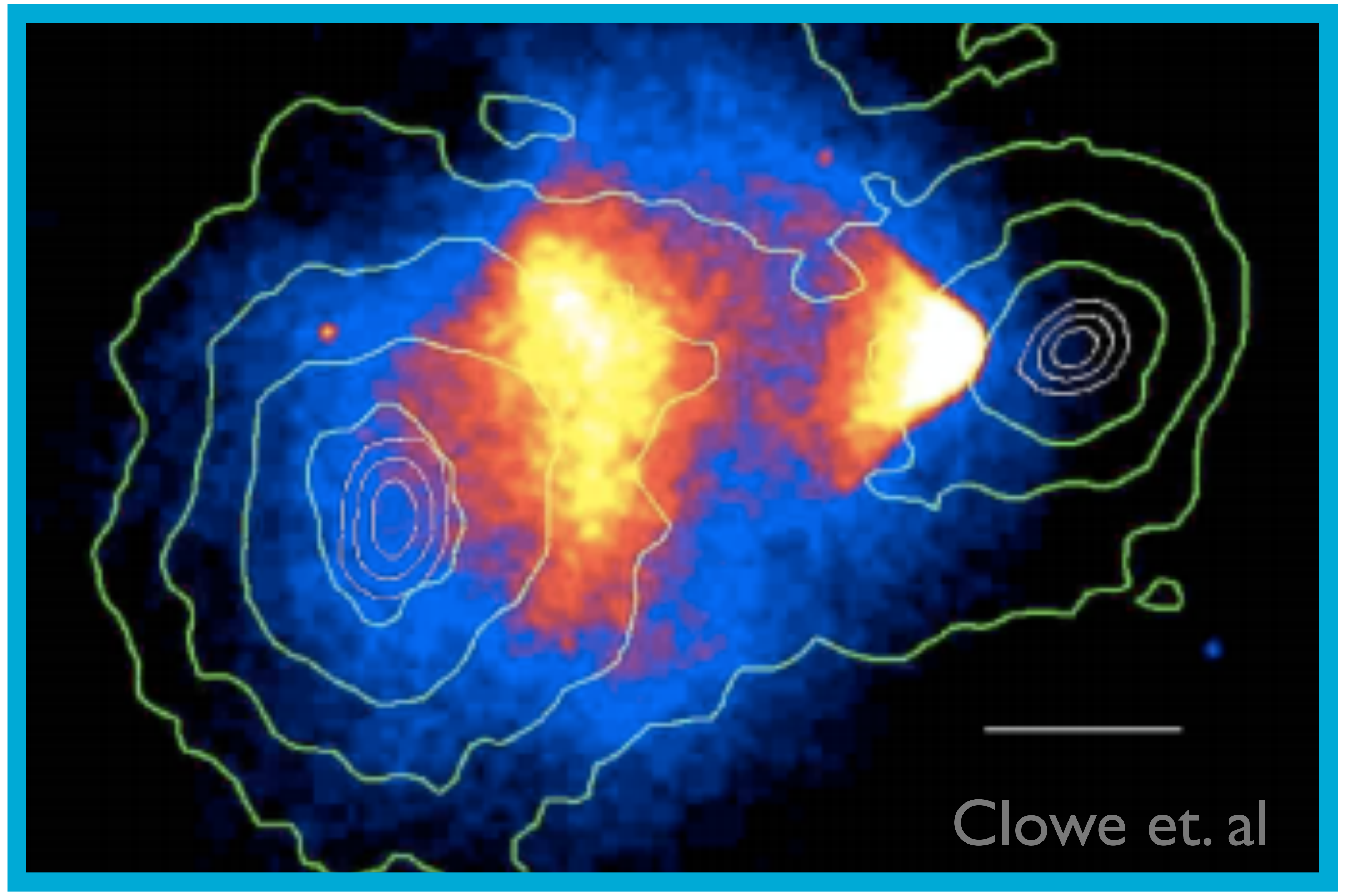
$$e_{\rm dm} \lesssim 10^{-7} \left(\frac{m}{\rm GeV}\right)^{0.5} e$$

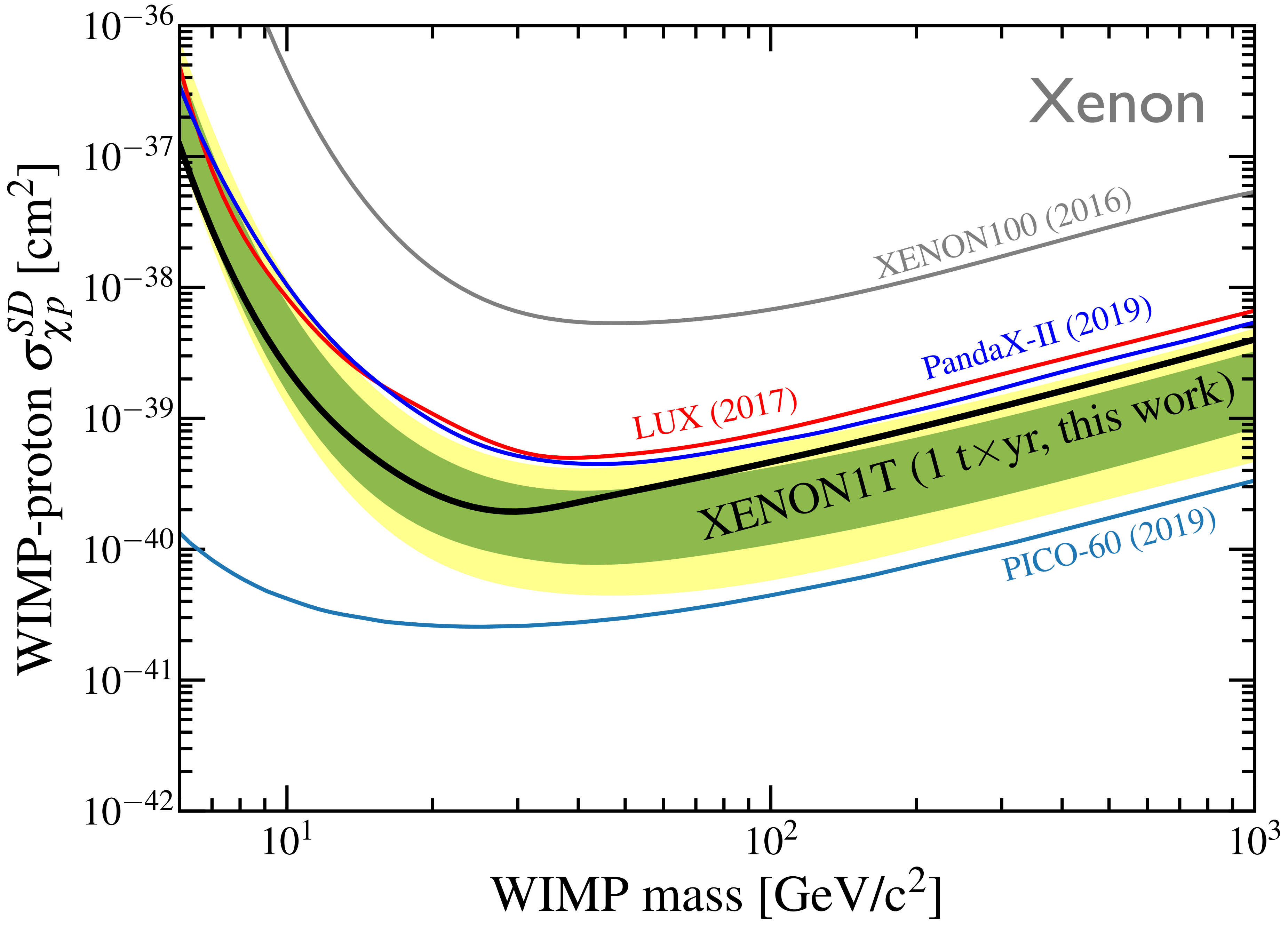
See PDG for references and details

interactions of dark matter?

- what is it: charge, spin, mass, σ ?
- elementary / composite ?
- formation — thermal, non-thermal ?

$$\frac{\sigma}{m} \lesssim 10^{-24} \frac{\text{cm}^2}{\text{GeV}}$$





See PDG for refs.

dark matter mass ?

- additional constraints possible depending on details of models and formation mechanism

$\gtrsim 10^{-21}\,\mathrm{eV}$

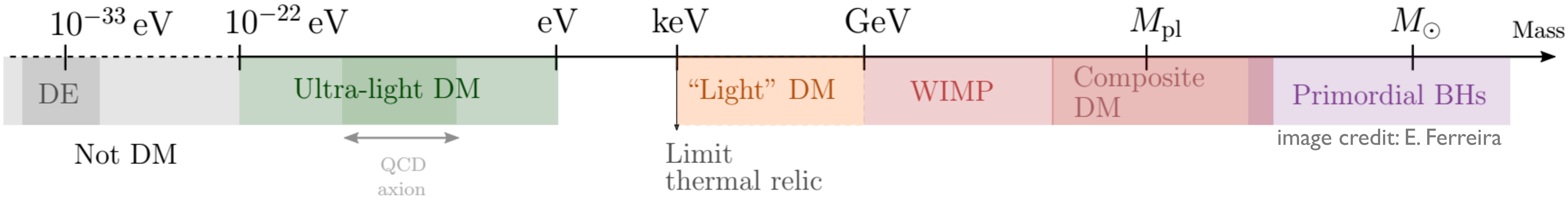
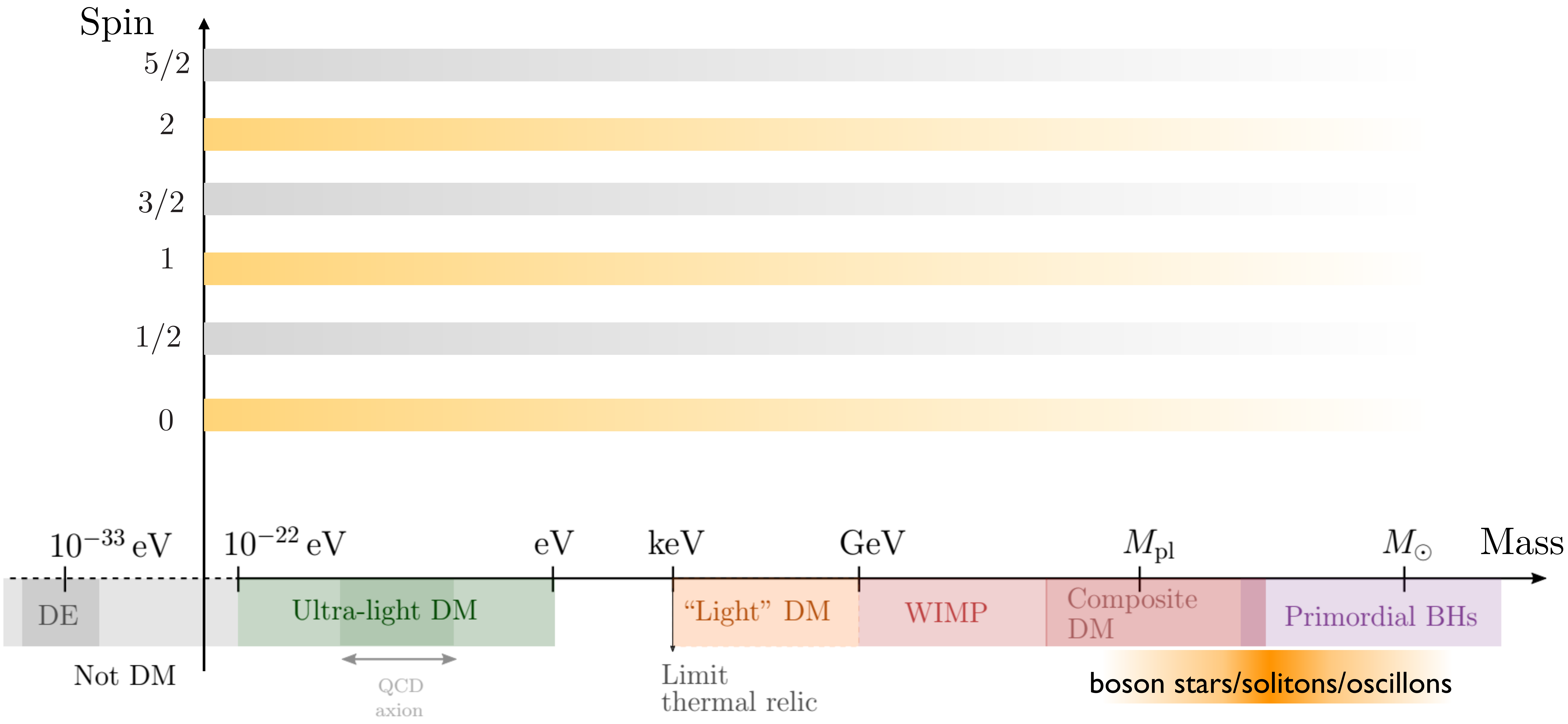


image credit: E. Ferreira

dark matter spin ?



fermionic DM cannot be too light

$$m \gtrsim 100\,\mathrm{eV}$$

$$\lambda_{\mathrm{dB}} \lesssim 10^{-6}\mathrm{meters}$$

$$n_{\mathrm{dm}} \lambda_{\mathrm{dB}}^3 \lesssim 1$$

$$n_{\mathrm{dm}} \sim \frac{0.3\,\mathrm{GeV\,cm}^{-3}}{m}$$

$$\lambda_{\mathrm{dB}} \sim \frac{\hbar}{mv} \qquad \text{with} \quad v \sim 10^{-3} c$$

bosonic dark matter can be very light

~~$n_{\rm dm}\lambda_{\rm dB}^3 \lesssim 1$~~

$$n_{\rm dm}\lambda_{\rm dB}^3 \sim 10^{23}\left(\frac{10^{-5}\,{\rm eV}}{m}\right)^4 \sim 10^{83}\left(\frac{10^{-20}\,{\rm eV}}{m}\right)^4$$

- large occupation numbers possible

light bosonic dark matter

$$\cancel{n_{\mathrm{dm}}\lambda_{\mathrm{dB}}^3 \lesssim 1}$$

$$n_{\mathrm{dm}}\lambda_{\mathrm{dB}}^3 \sim 10^{23}\left(\frac{10^{-5}\,\mathrm{eV}}{m}\right)^4 \sim 10^{83}\left(\frac{10^{-20}\,\mathrm{eV}}{m}\right)^4$$

$$\lambda_{\mathrm{dB}} \sim 10^3\mathrm{cm} \times \left(\frac{10^{-5}\,\mathrm{eV}}{m}\right)\left(\frac{10^{-3}c}{v}\right) \sim 1\mathrm{pc}\left(\frac{10^{-20}\,\mathrm{eV}}{m}\right)\left(\frac{10^{-3}c}{v}\right)$$

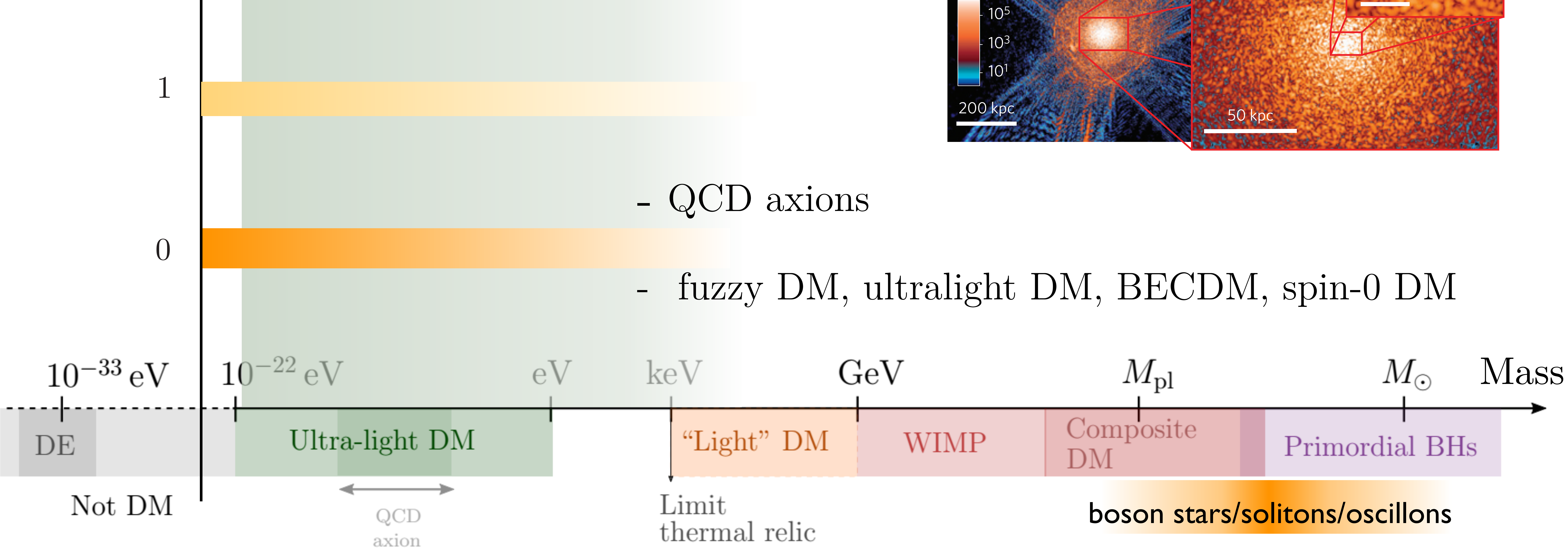
- large occupation numbers possible
- macroscopic/astrophysical deBroglie scales possible possible

light bosonic “wave” dark matter

- classical wave description sensible
- linear and nonlinear wave dynamics
- interference, solitons etc.

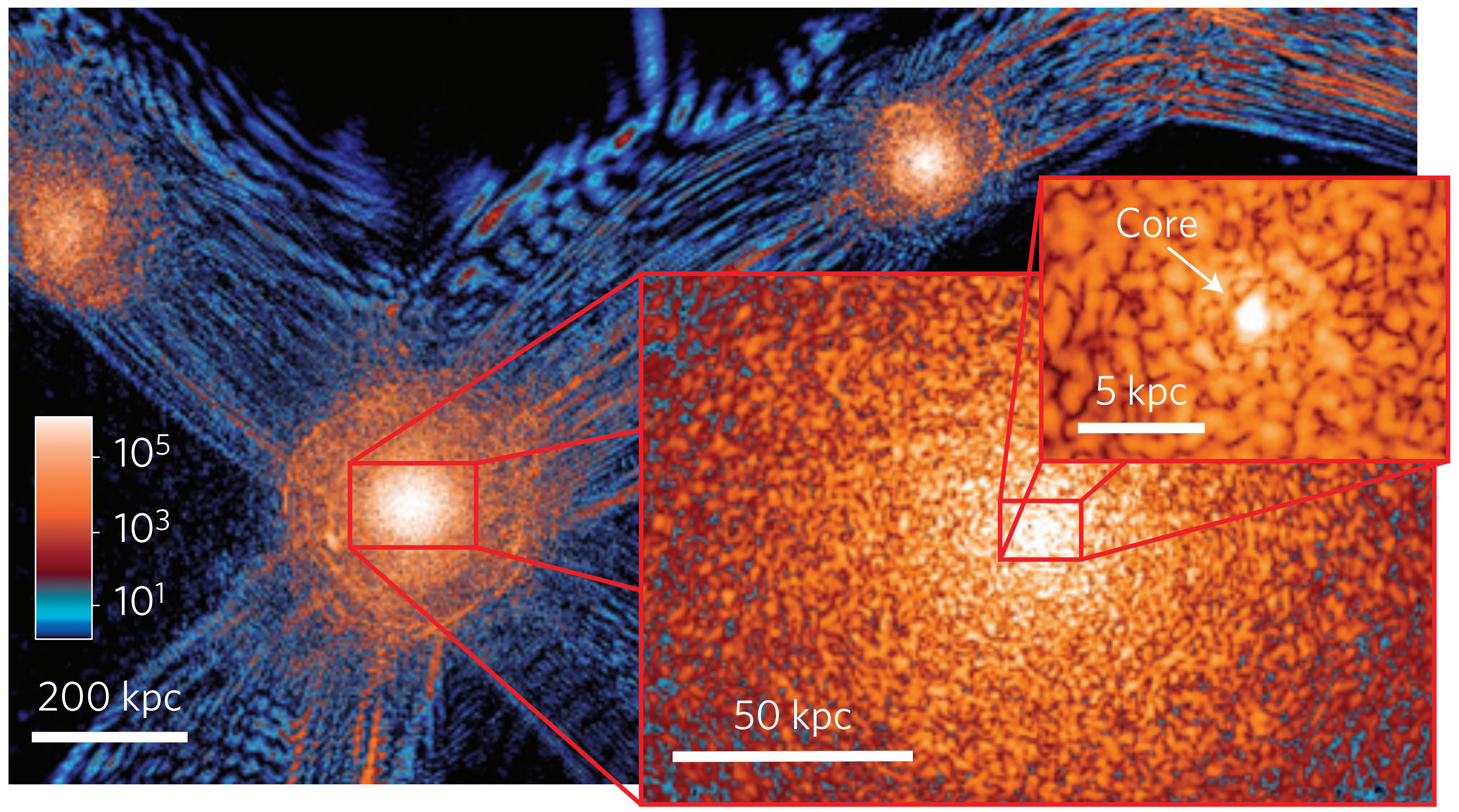
- condensed matter systems (BECs)
- similar ideas already used in early universe cosmology

scalar dark matter

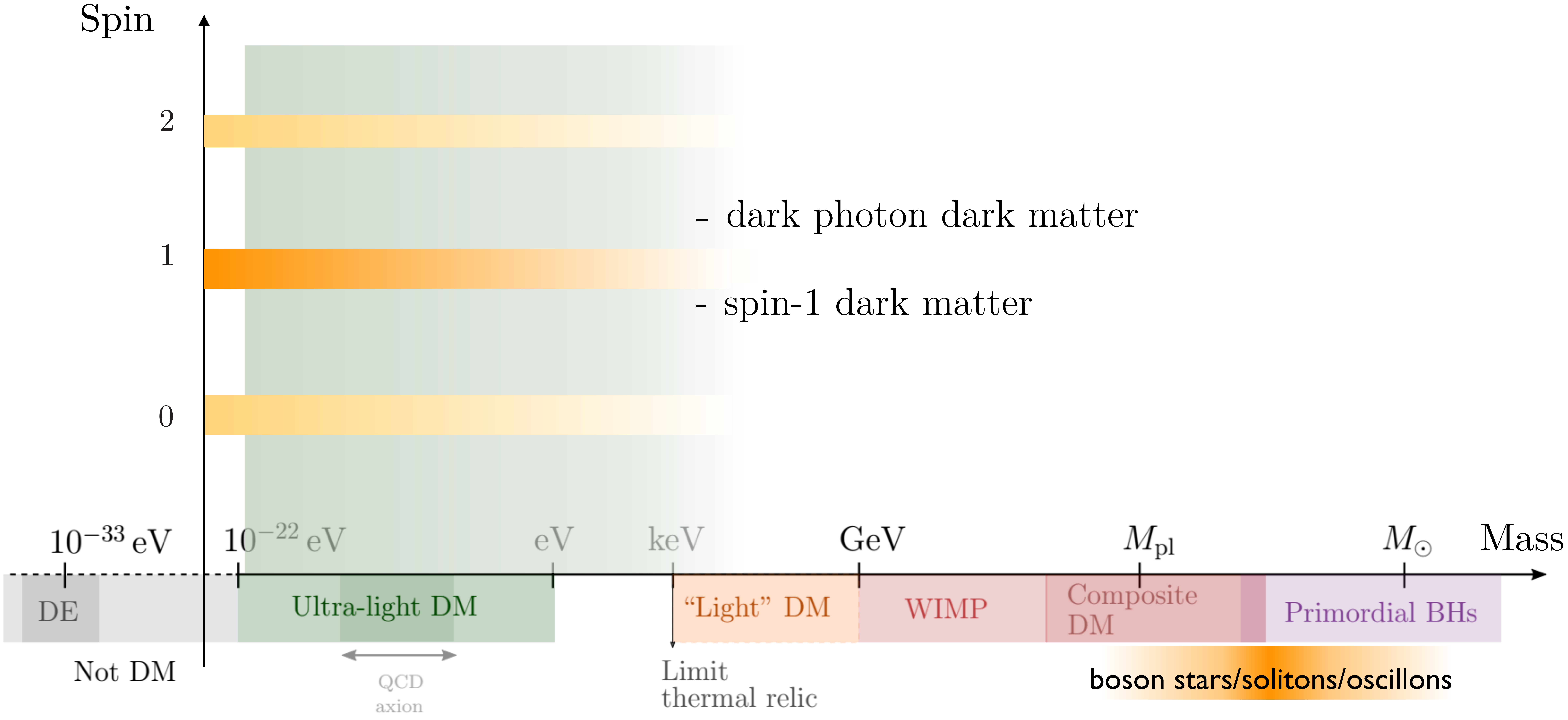


- QCD axions
- fuzzy DM, ultralight DM, BECDM, spin-0 DM

Schive et. al (2014)



vector dark matter



- dark photon dark matter
- spin-1 dark matter

model

vector dark matter

$$S = \int \mathrm{d}^4x\,\sqrt{-g}\Big[-\frac{1}{4}g^{\mu\alpha}g^{\nu\beta}\,\mathcal{G}_{\mu\nu}\mathcal{G}_{\alpha\beta} + \frac{1}{2}\frac{m^2c^2}{\hbar^2}\,g^{\mu\nu}W_\mu W_\nu + \frac{c^3}{16\pi G}R + ...\Big]$$

vector case

$$\mathcal{G}_{\mu\nu} = \partial_\mu W_\nu - \partial_\nu W_\mu$$

$$S = \int d^4x\,\sqrt{-g}\,\left[-\frac{1}{2}\partial_\mu\phi\partial^\mu\phi + \frac{1}{2}\frac{m^2c^2}{\hbar^2}\phi^2 + \frac{c^3}{16\pi G}R + \ldots\right]$$

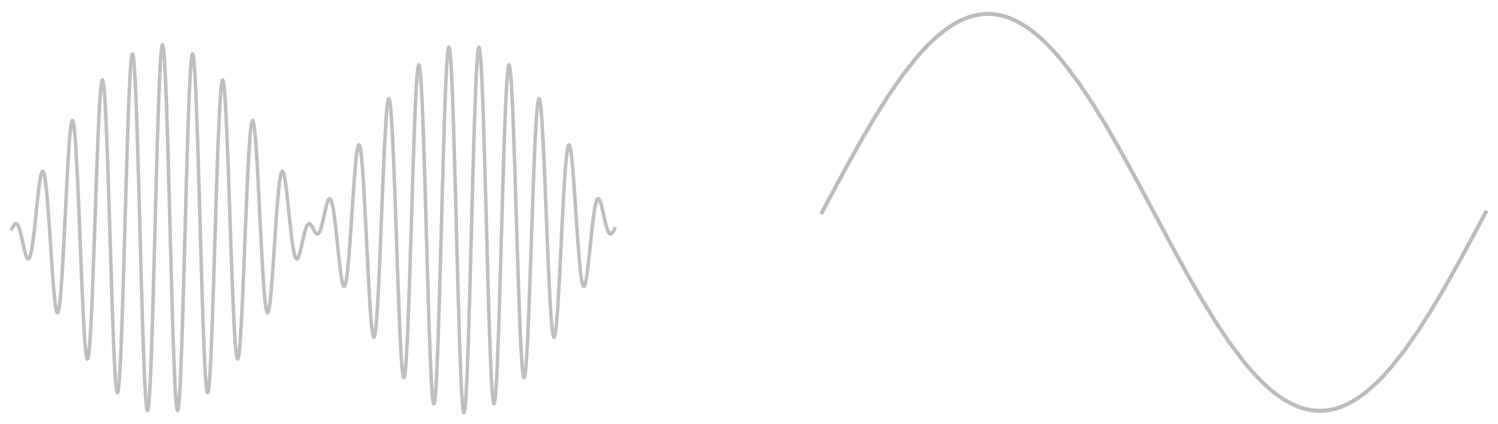
scalar case

*we wil ignore non-gravitational interactions (self or with other fields) for the moment, but include them later

non-relativistic limit = multicomponent Schrödinger-Poisson

$$S = \int \mathrm{d}^4x\,\sqrt{-g}\Big[-\frac{1}{4}g^{\mu\alpha}g^{\nu\beta}\,\mathcal{G}_{\mu\nu}\mathcal{G}_{\alpha\beta} + \frac{1}{2}\frac{m^2c^2}{\hbar^2}\,g^{\mu\nu}W_\mu W_\nu + \frac{c^3}{16\pi G}R + \dots\Big]$$

$$\mathcal{G}_{\mu\nu} = \partial_\mu W_\nu - \partial_\nu W_\mu$$



split in "fast" and "slow" parts

non-relativistic limit

$$\boldsymbol{W}(t,\boldsymbol{x}) \equiv \frac{\hbar}{\sqrt{2mc}}\Re\left[\boldsymbol{\Psi}(t,\boldsymbol{x})e^{-imc^2t/\hbar}\right]$$

$$\mathcal{S}_{nr} = \int \mathrm{d}t\mathrm{d}^3x\left[\frac{i\hbar}{2}\boldsymbol{\Psi}^\dagger\dot{\boldsymbol{\Psi}} + \mathrm{c.c.} - \frac{\hbar^2}{2m}\nabla\boldsymbol{\Psi}^\dagger\cdot\nabla\boldsymbol{\Psi} + \frac{1}{8\pi G}\Phi\nabla^2\Phi - m\,\Phi\,\boldsymbol{\Psi}^\dagger\boldsymbol{\Psi}\right]$$

Adshead & Lozanov (2021) for vectors

Jain & MA (2021) for spin-s fields

Recent work: Salehian et. al (2021) for scalar case including higher order terms

non-relativistic limit = multicomponent Schrödinger-Poisson

$[\mathbf{\Psi}]_i = \psi_i$ with $i = 1, 2, 3$ vector case

$$i\hbar \frac{\partial}{\partial t} \mathbf{\Psi} = -\frac{\hbar^2}{2m} \nabla^2 \mathbf{\Psi} + m\,\Phi\,\mathbf{\Psi}\,, \qquad \nabla^2 \Phi = 4\pi G m\, \mathbf{\Psi}^\dagger \mathbf{\Psi}$$

$[\mathbf{\Psi}]_i = \psi_i$ with $i = 1$ scalar case

generalization to spin -s field, Jain & MA (2022)

at this level this is just $2s{+}1$ scalar fields

the fluid version (Madelung eq.)

$$i\frac{\partial}{\partial t}\boldsymbol{\Psi} = -\frac{1}{2m}\nabla^2\boldsymbol{\Psi} + m\,\Phi\,\boldsymbol{\Psi}$$
$$\nabla^2\Phi = \frac{m}{2m_{\rm pl}^2}\,\mathrm{Tr}[\boldsymbol{\Psi}^\dagger\boldsymbol{\Psi}].$$

$$[\boldsymbol{\Psi}]_j = \sqrt{\rho_j/m}\,e^{iS_j}$$

$$\boldsymbol{u}_i \equiv \hbar\nabla S_i/m$$

$$\frac{\partial\rho_j}{\partial t} + \nabla\cdot(\rho_j\boldsymbol{u}_j) = 0\,,$$

$$\frac{\partial\boldsymbol{u}_j}{\partial t} + (\boldsymbol{u}_j\cdot\nabla)\boldsymbol{u}_j = \frac{1}{m}\nabla(Q_j - m\Phi)$$

$$\nabla^2\Phi = 4\pi G\rho\sum_j\rho_j$$

$$Q_j = (\hbar^2/2m)\nabla^2\sqrt{\rho_j}/\sqrt{\rho_j}.$$

conserved quantities

$[\boldsymbol{\Psi}]_i = \psi_i$ with $i = 1, 2, 3$

$$N = \int \mathrm{d}^3x \boldsymbol{\Psi}^\dagger \boldsymbol{\Psi}, \quad \text{and} \quad M = mN, \qquad \text{(particle number and rest mass)}$$

$$E = \int \mathrm{d}^3x \Big[\frac{\hbar^2}{2m} \nabla\boldsymbol{\Psi}^\dagger \cdot \nabla\boldsymbol{\Psi} - \frac{Gm^2}{2} \boldsymbol{\Psi}^\dagger\boldsymbol{\Psi} \int \frac{\mathrm{d}^3y}{4\pi|\boldsymbol{x}-\boldsymbol{y}|} \boldsymbol{\Psi}^\dagger(\boldsymbol{y})\boldsymbol{\Psi}(\boldsymbol{y}) \Big], \qquad \text{(energy)}$$

$$\boldsymbol{S} = \hbar \int \mathrm{d}^3x \, i\boldsymbol{\Psi} \times \boldsymbol{\Psi}^\dagger, \qquad \text{(spin angular momentum)}$$

$$\boldsymbol{L} = \hbar \int \mathrm{d}^3x \, \Re\left(i\, \boldsymbol{\Psi}^\dagger \nabla \boldsymbol{\Psi} \times \boldsymbol{x} \right). \qquad \text{(orbital angular momentum)}$$

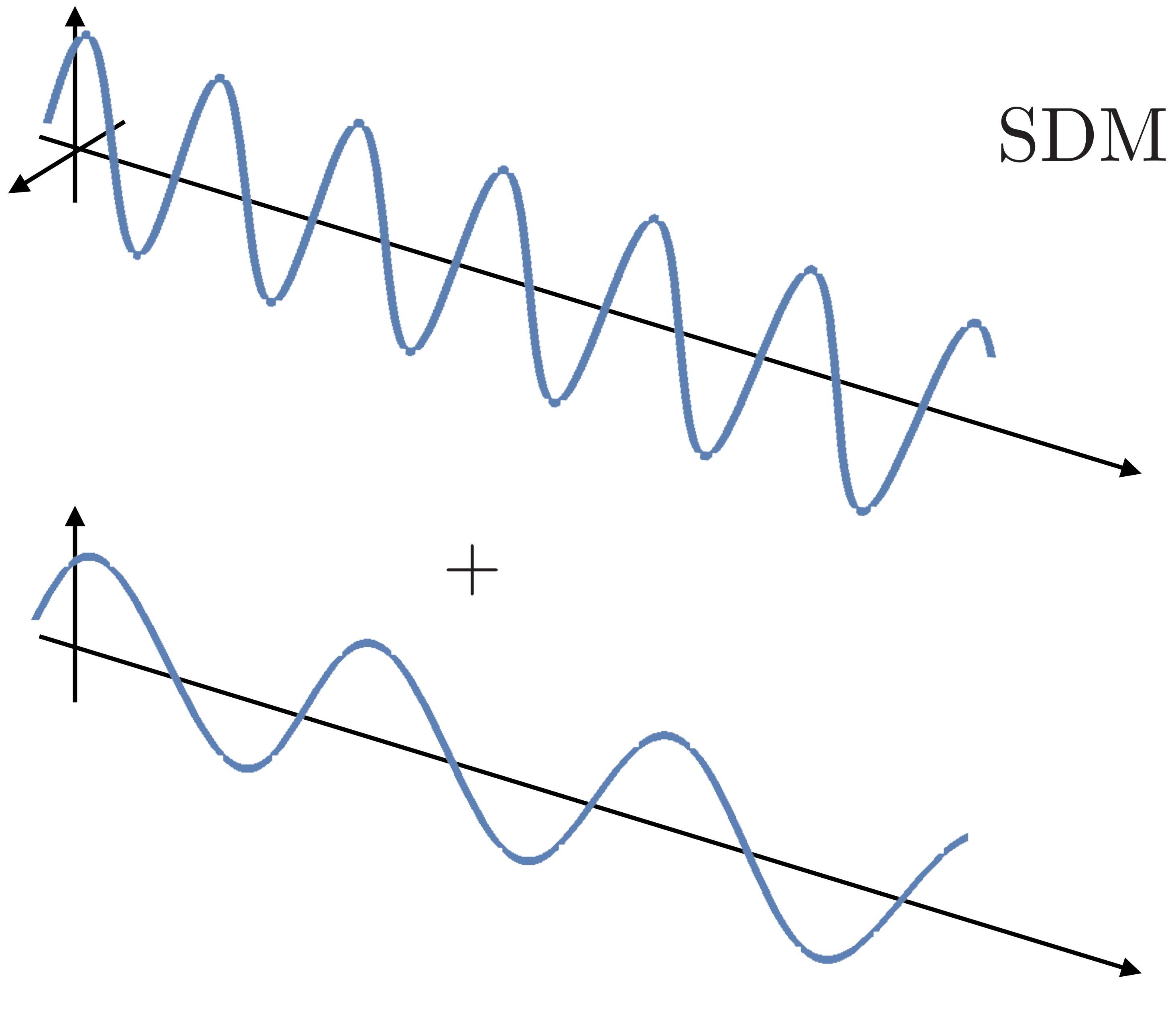
vector vs. scalar DM: two key differences

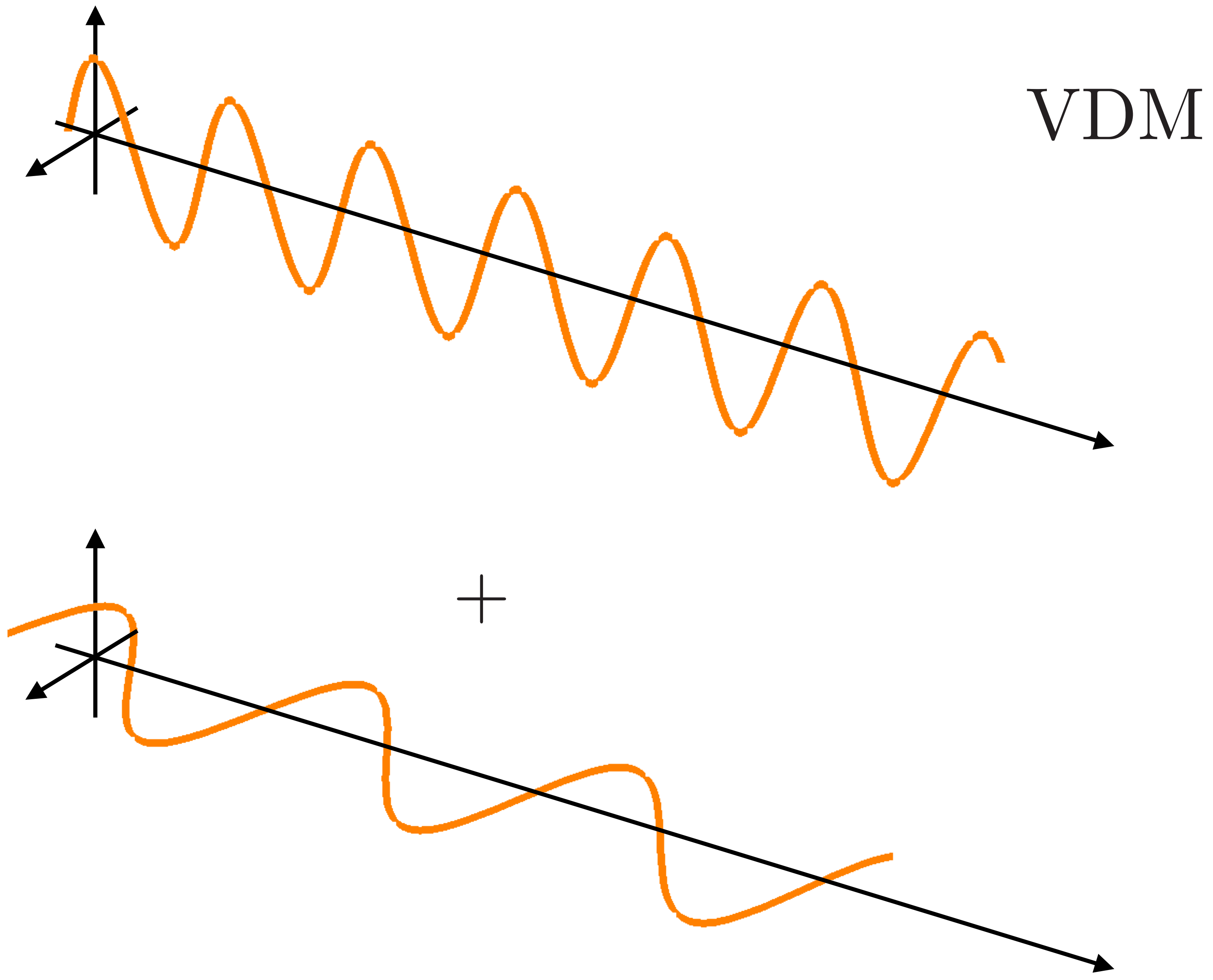
interference

polarized solitons

analytics

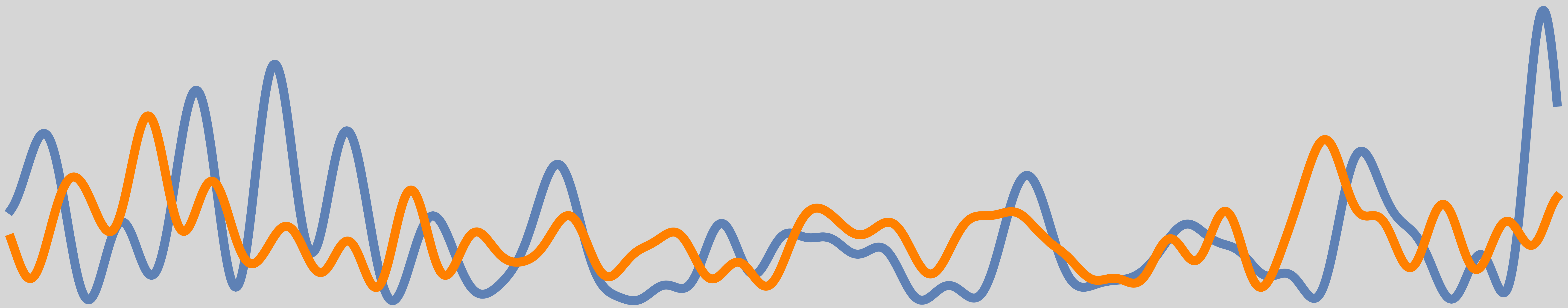
wave interference





$$|\Psi_a(\boldsymbol{x}) + \Psi_b(\boldsymbol{x})|^2 \neq |\Psi_a(\boldsymbol{x})|^2 + |\Psi_b(\boldsymbol{x})|^2$$

$$|\Psi_a(\boldsymbol{x}) + \Psi_b(\boldsymbol{x})|^2 = |\Psi_a(\boldsymbol{x})|^2 + |\Psi_b(\boldsymbol{x})|^2$$



reduced wave interference in VDM

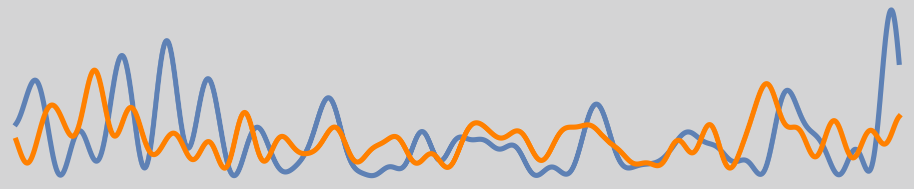
$s = 0$ for SDM and $s = 1$ for VDM

$$\boldsymbol{\Psi}_a(\boldsymbol{x}) = \boldsymbol{\epsilon}_a^{(s)} e^{i\boldsymbol{k}_a\cdot\boldsymbol{x}}$$

$$|\boldsymbol{\Psi}_a(\boldsymbol{x}) + \boldsymbol{\Psi}_b(\boldsymbol{x})|^2 = 2\left(1 + \Re\left[\boldsymbol{\epsilon}_a^{(s)\dagger}\cdot\boldsymbol{\epsilon}_a^{(s)} e^{-i(\boldsymbol{k}_a-\boldsymbol{k}_b)\cdot\boldsymbol{x}}\right]\right) = 2\left(1 + \mathrm{int}_{(s)}\right)$$

$$\sqrt{\langle \mathrm{int}^2_{(s)}\rangle} = \frac{1}{\sqrt{2(2s+1)}}$$

$$\frac{\sqrt{\langle \mathrm{int}^2_{(1)}\rangle}}{\sqrt{\langle \mathrm{int}^2_{(0)}\rangle}} = \frac{1}{\sqrt{3}}$$



MA, Jain, Karur & Mocz (2022)

soliton ?

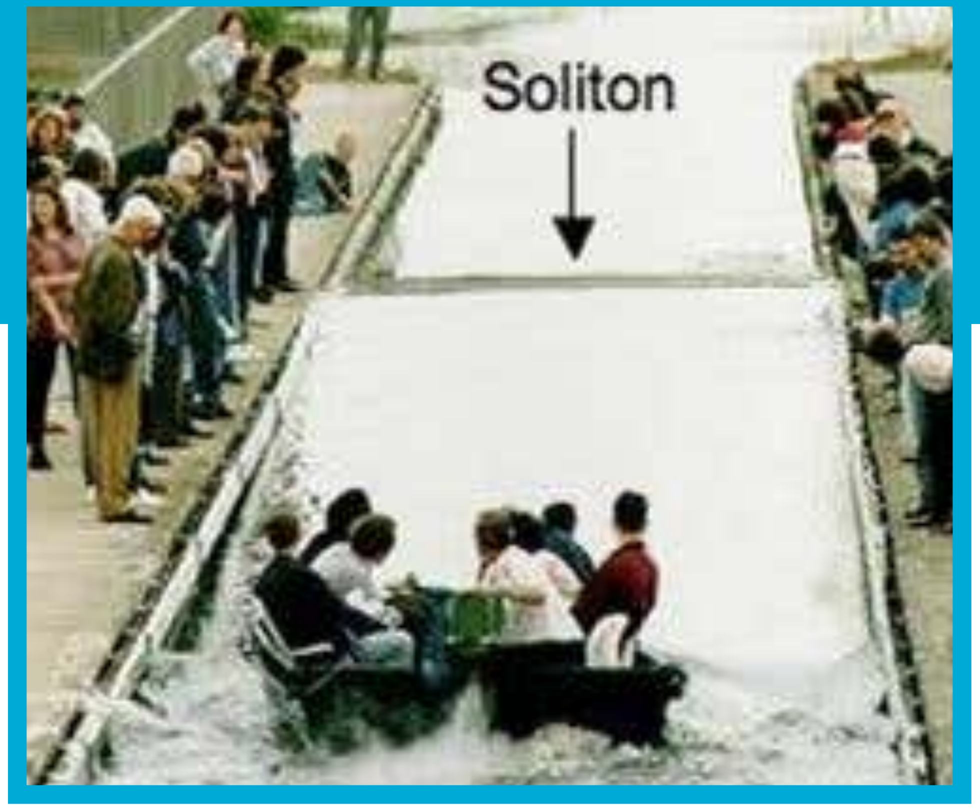


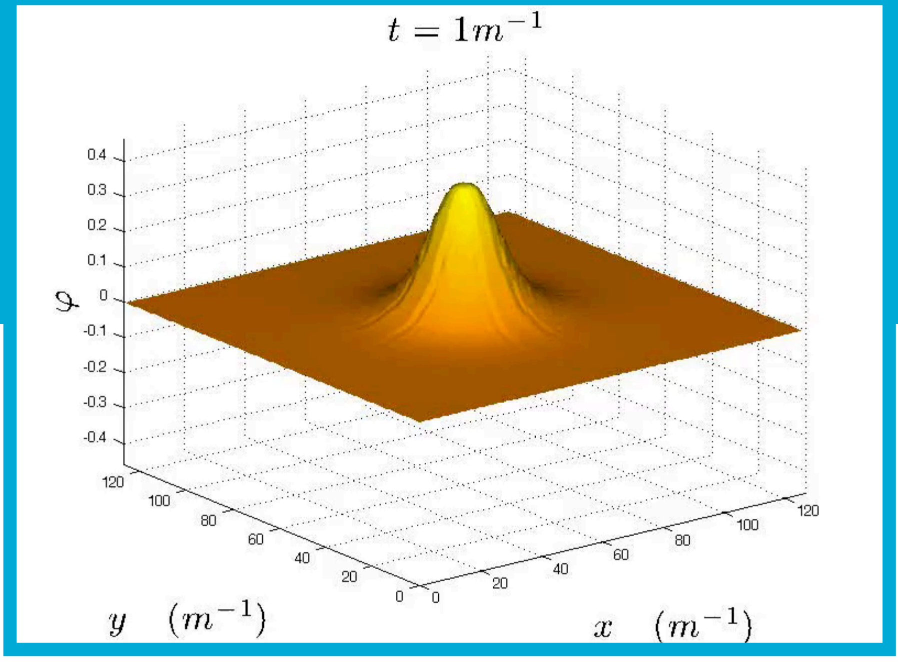
Image Credit: Heriot-Watt University

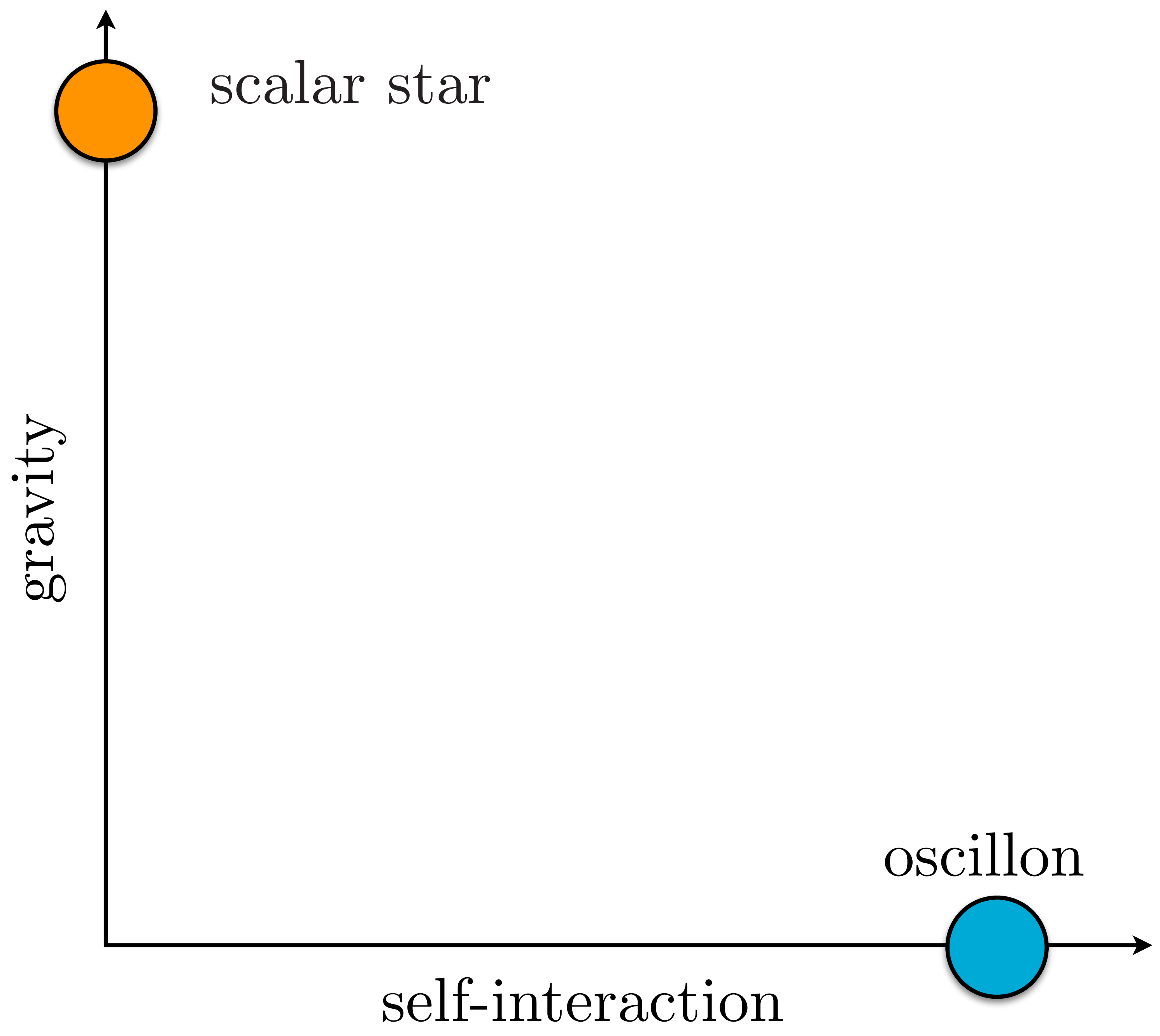
very-long lived, dynamical excitation in a field, with nonlinearities balancing dispersion

- discovered in nonlinear waves in water in canals (John Scott Russell, 1834)
- optics, hydrodynamics, BECs, high energy physics, and cosmology

non-topological solitons

spatially localized, coherently oscillating, long-lived





spatially localized

coherently oscillating (components)

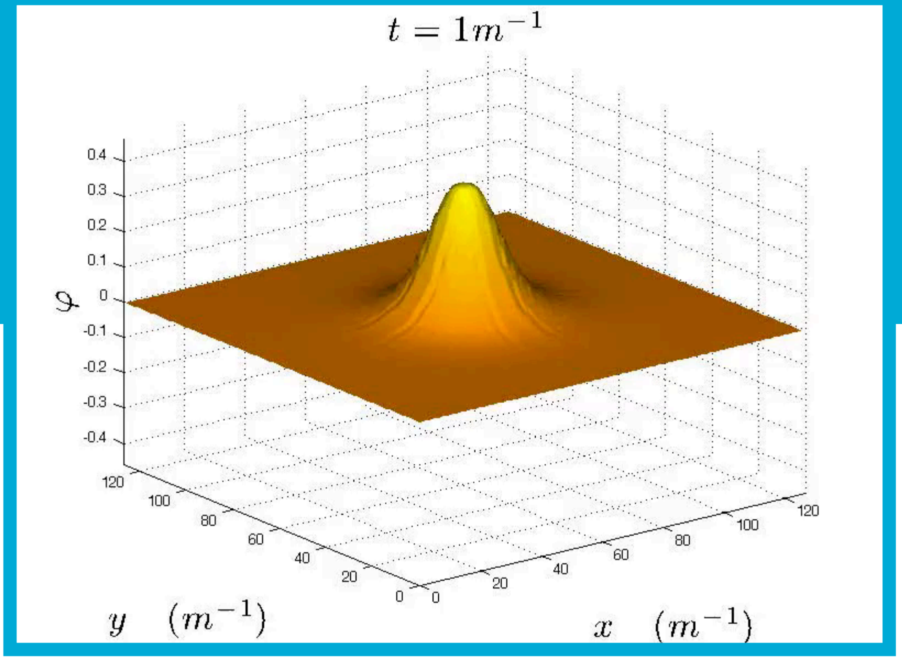
exceptionally long-lived

Makhanakov, Bolglubovsky, Kruskal & Seagur

Seidel & Sun …

Gleiser, Copeland, Muller … + many more

solitons in massive spin-0 (scalar fields)



$$\phi(t, \boldsymbol{x}) = \frac{\hbar}{\sqrt{2mc}} \Re[\Psi_{\rm sol}(\mu, \boldsymbol{x}) e^{imc^2 t/\hbar}]$$

$$\Psi_{\rm sol}(t, \boldsymbol{x}) = \psi_{\rm sol}(\mu, \boldsymbol{x}) e^{i\mu c^2 t/\hbar}$$

$$M_{\rm sol} \approx 60.7 \frac{m_{\rm pl}^2}{m} \sqrt{\frac{\mu}{m}}$$

$$R_{\rm sol} \sim \sqrt{\frac{m}{\mu}} \frac{\hbar}{mc}$$

lowest energy solution for fixed total *Mass (in non-relativistic limit)*

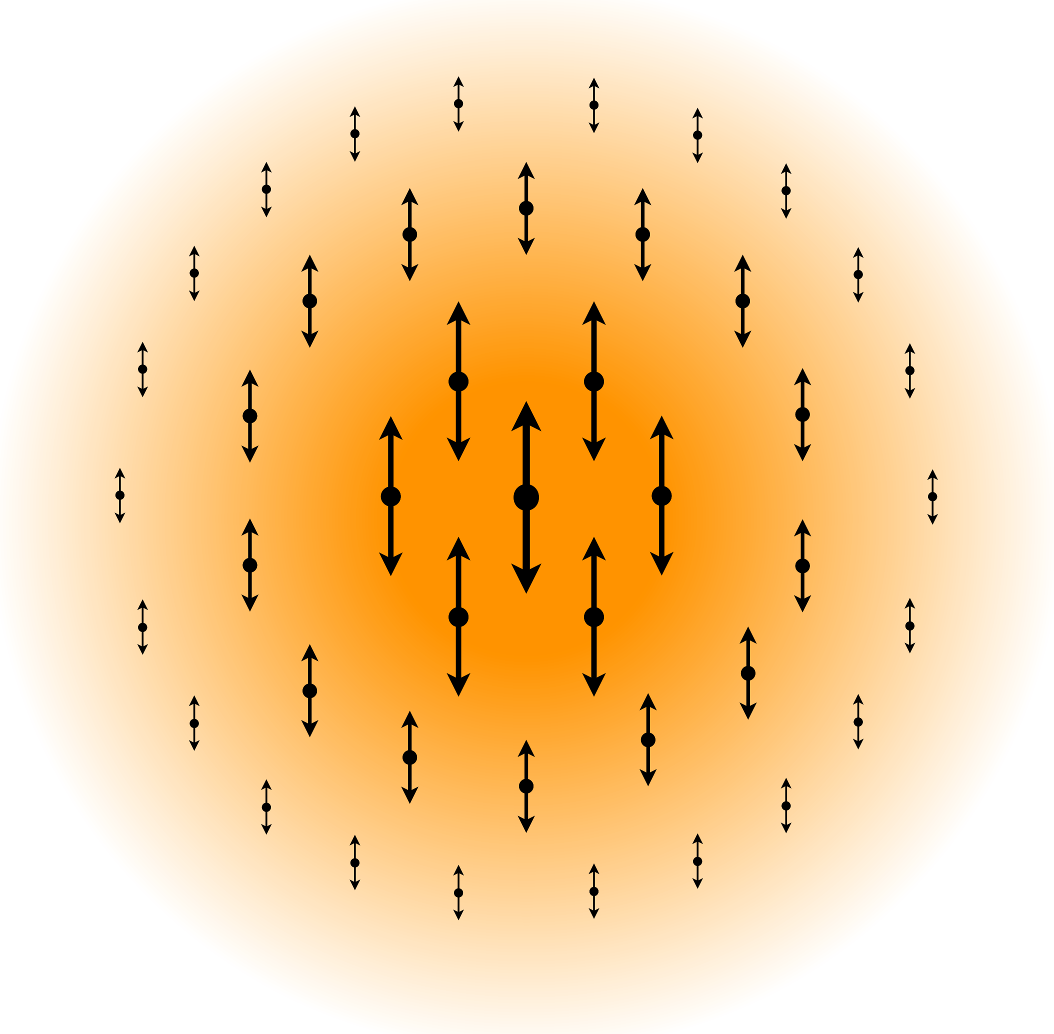
$\mu/m \ll 1$

"polarized" vector solitons

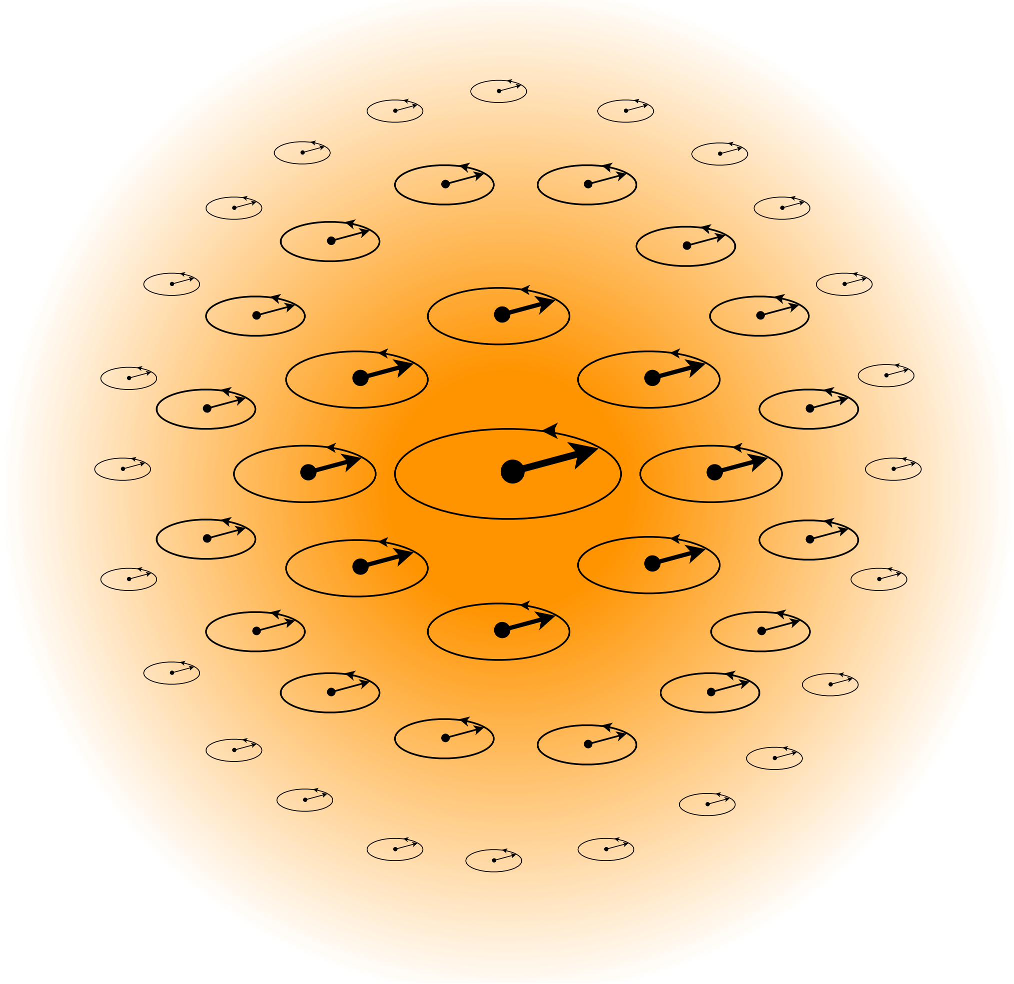
$$\boldsymbol{W}(t,\boldsymbol{x}) \equiv \frac{\hbar}{\sqrt{2mc}}\Re\left[\boldsymbol{\Psi}(t,\boldsymbol{x})e^{-imc^2t/\hbar}\right]$$

$$\boldsymbol{\Psi}_{\rm sol}(t,\boldsymbol{x}) = \psi_{\rm sol}(\mu, r)e^{i\mu c^2 t/\hbar}\boldsymbol{\epsilon}$$

$$\boldsymbol{\epsilon}^\dagger\boldsymbol{\epsilon} = 1$$



$$\boldsymbol{\epsilon}^{(0)}_{1,\hat{z}} = \begin{pmatrix}0\\0\\1\end{pmatrix}$$



$$\boldsymbol{\epsilon}^{(\pm 1)}_{1,\hat{z}} = \frac{1}{\sqrt{2}}\begin{pmatrix}1\\ \pm i\\0\end{pmatrix}$$

MA & Jain (2022)

Adshead & Lozanov (2022)

"polarized" vector solitons

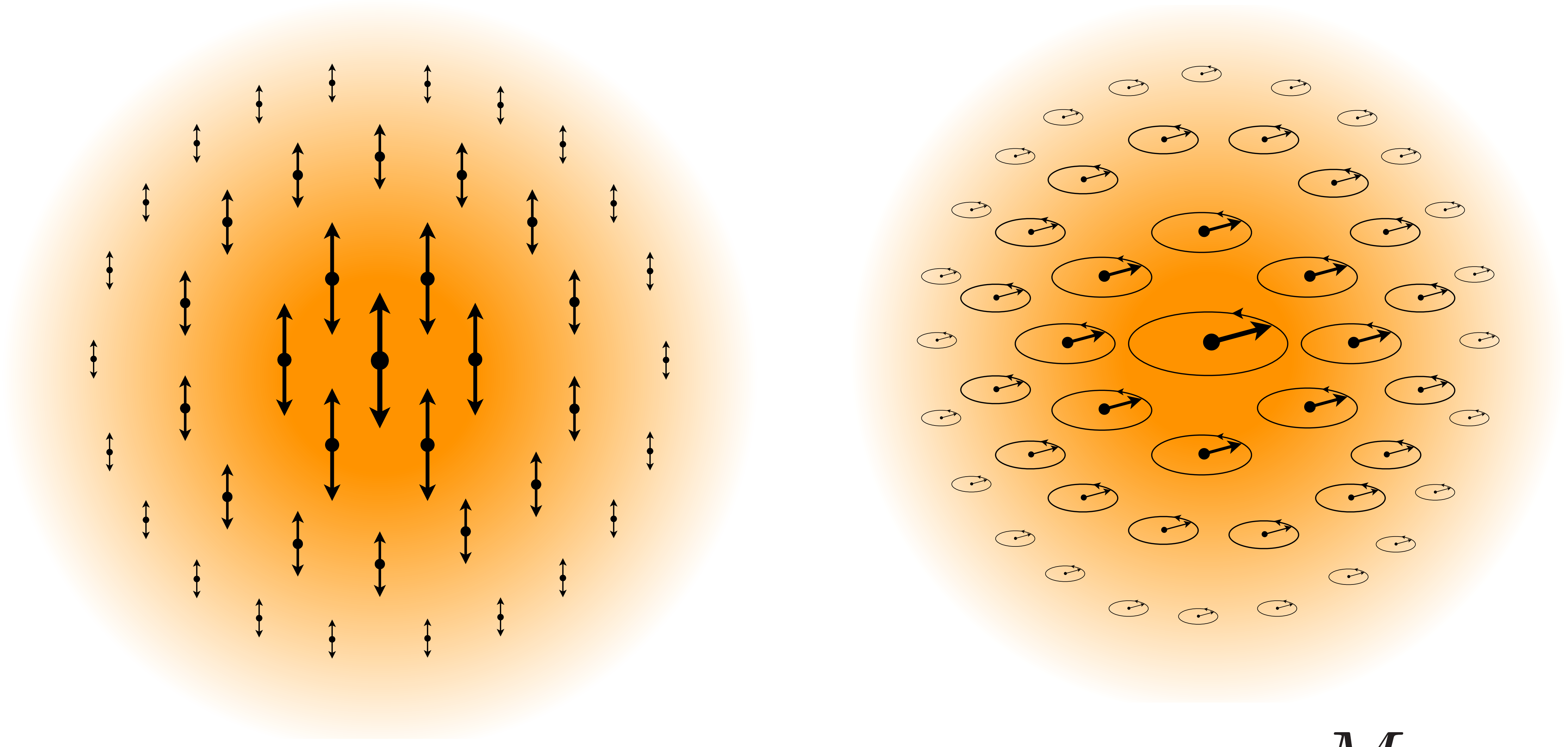
$$\boldsymbol{W}(t,\boldsymbol{x}) \equiv \frac{\hbar}{\sqrt{2mc}}\Re\left[\boldsymbol{\Psi}(t,\boldsymbol{x})e^{-imc^2t/\hbar}\right]$$

$$\boldsymbol{\Psi}_{\rm sol}(t,\boldsymbol{x}) = \psi_{\rm sol}(\mu,r)e^{i\mu c^2 t/\hbar}\boldsymbol{\epsilon} \qquad \boldsymbol{\epsilon}^\dagger\boldsymbol{\epsilon} = 1$$

$$\boldsymbol{S}_{\rm sol} \approx i(\boldsymbol{\epsilon}\times\boldsymbol{\epsilon}^\dagger)60.7\frac{m_{\rm pl}^2}{m^2}\sqrt{\frac{\mu}{m}}\hbar \quad = i(\boldsymbol{\epsilon}\times\boldsymbol{\epsilon}^\dagger)\frac{M_{\rm sol}}{m}\hbar$$

macroscopic spin $\quad \boldsymbol{S}_{\rm tot}/\hbar = \lambda N\hat{z}$

$N =$ # of particles in soliton



$\boldsymbol{S}_{\rm tot} = 0\hat{z}$ $\qquad$ $\boldsymbol{S}_{\rm tot} = \hbar\frac{M_{\rm sol}}{m}\hat{z}$

"polarized" vector solitons

$$\boldsymbol{W}(t,\boldsymbol{x}) \equiv \frac{\hbar}{\sqrt{2mc}} \Re\left[\boldsymbol{\Psi}(t,\boldsymbol{x}) e^{-imc^2t/\hbar}\right]$$

$$\boldsymbol{\Psi}_{\rm sol}(t,\boldsymbol{x}) = \psi_{\rm sol}(\mu, r) e^{i\mu c^2 t/\hbar} \boldsymbol{\epsilon} \qquad \boldsymbol{\epsilon}^\dagger \boldsymbol{\epsilon} = 1$$

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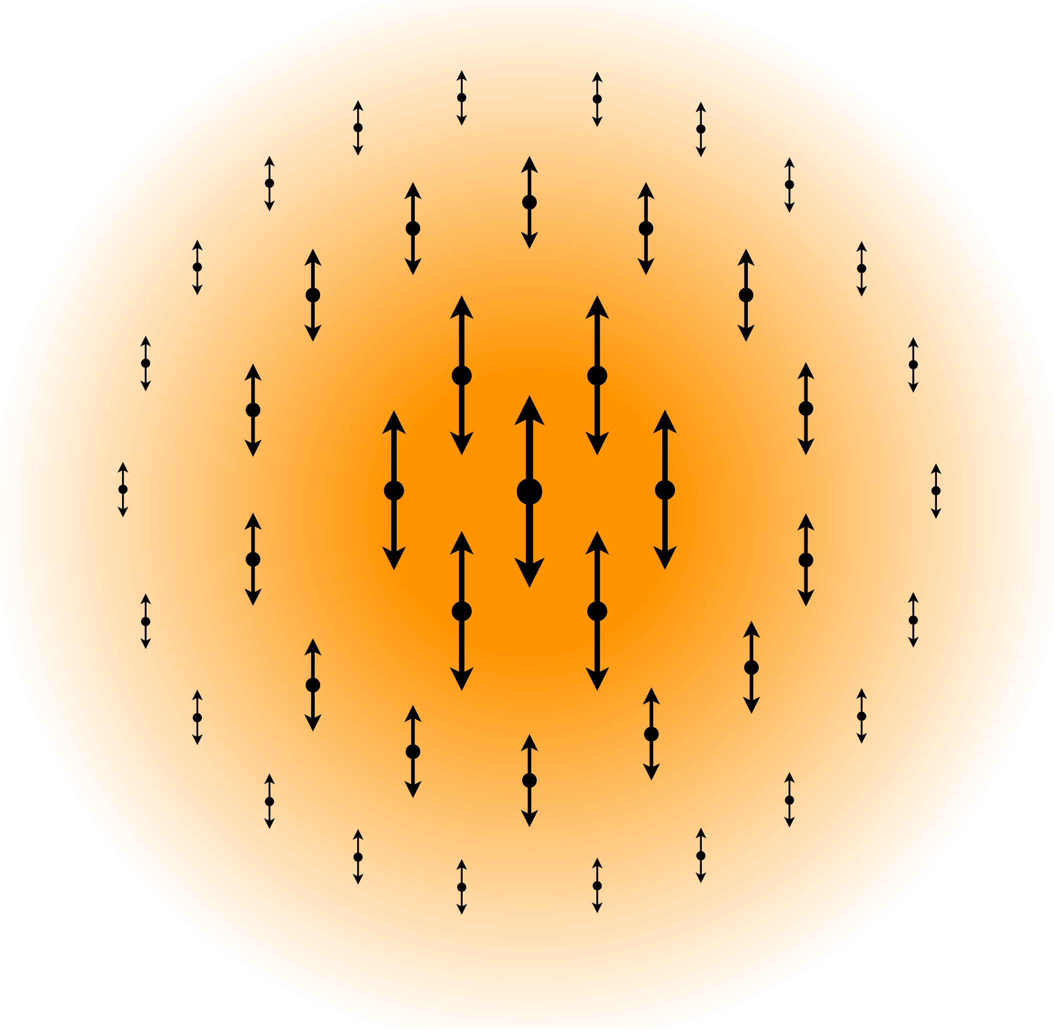
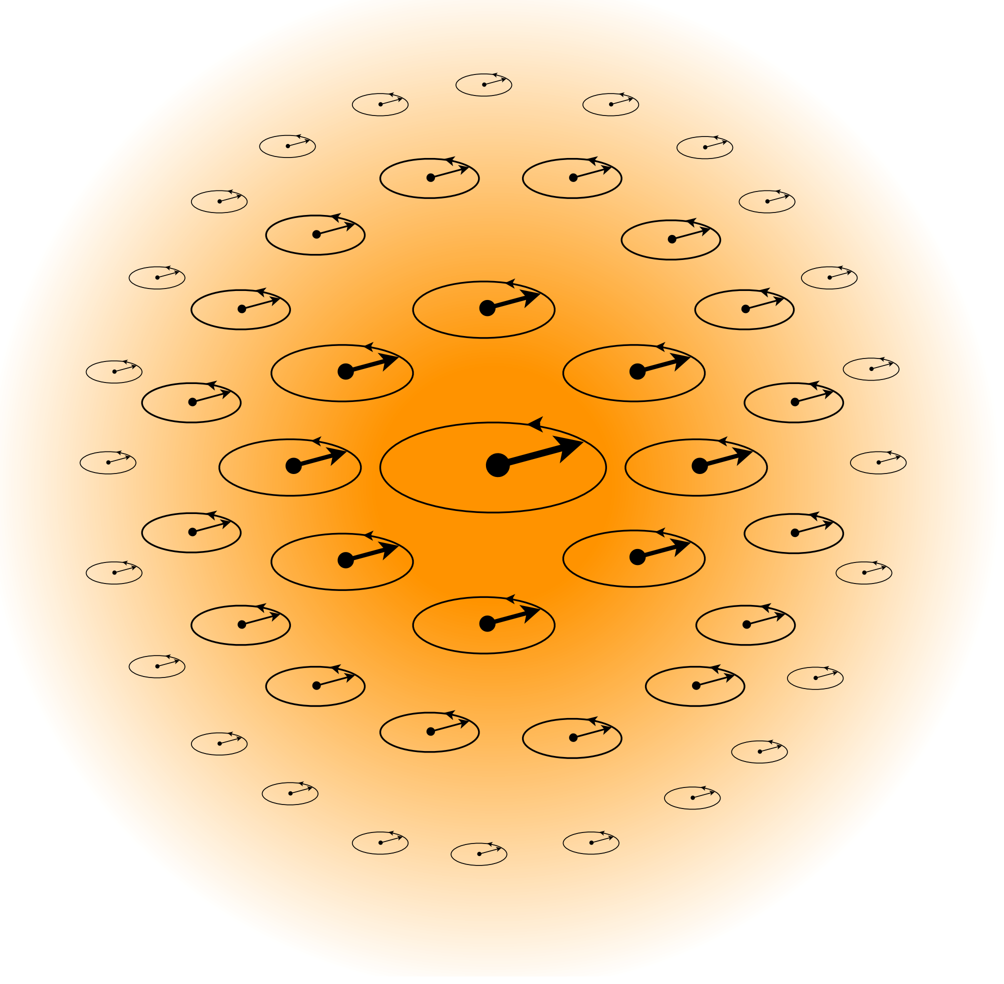


$\boldsymbol{S}_{\rm tot} = 0\hat{z}$

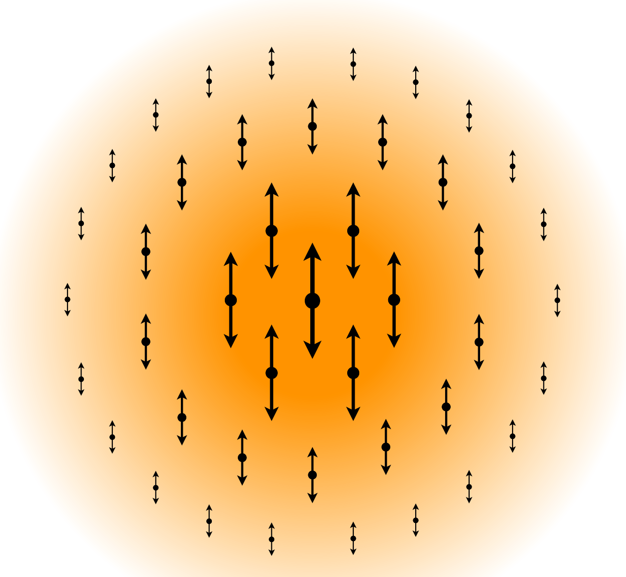
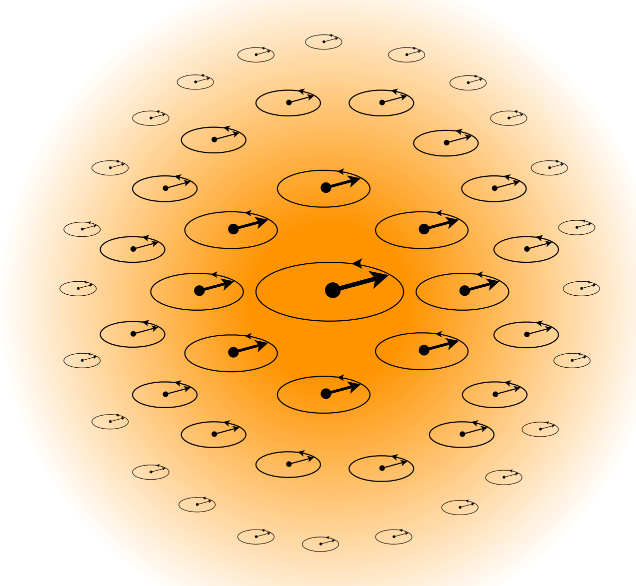
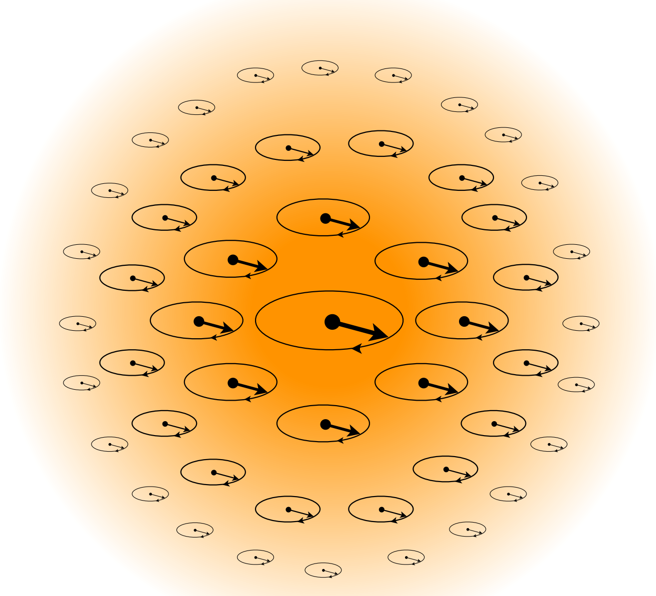


$\boldsymbol{S}_{\rm tot} = \hbar \dfrac{M_{\rm sol}}{m}\hat{z}$

macroscopic spin $\quad \boldsymbol{S}_{\rm tot}/\hbar = \lambda N \hat{z}$

$N =$ # of particles in soliton

- all lowest energy for fixed M
- bases for partially-polarized solitons

c_0  $+c_1$  $+c_{-1}$ 

$$0 \le |\boldsymbol{S}_{\rm tot}| \le \frac{M_{\rm sol}}{m}\hbar$$

Jain and MA (2021)

a different higher energy soliton: the “hedgehogs”

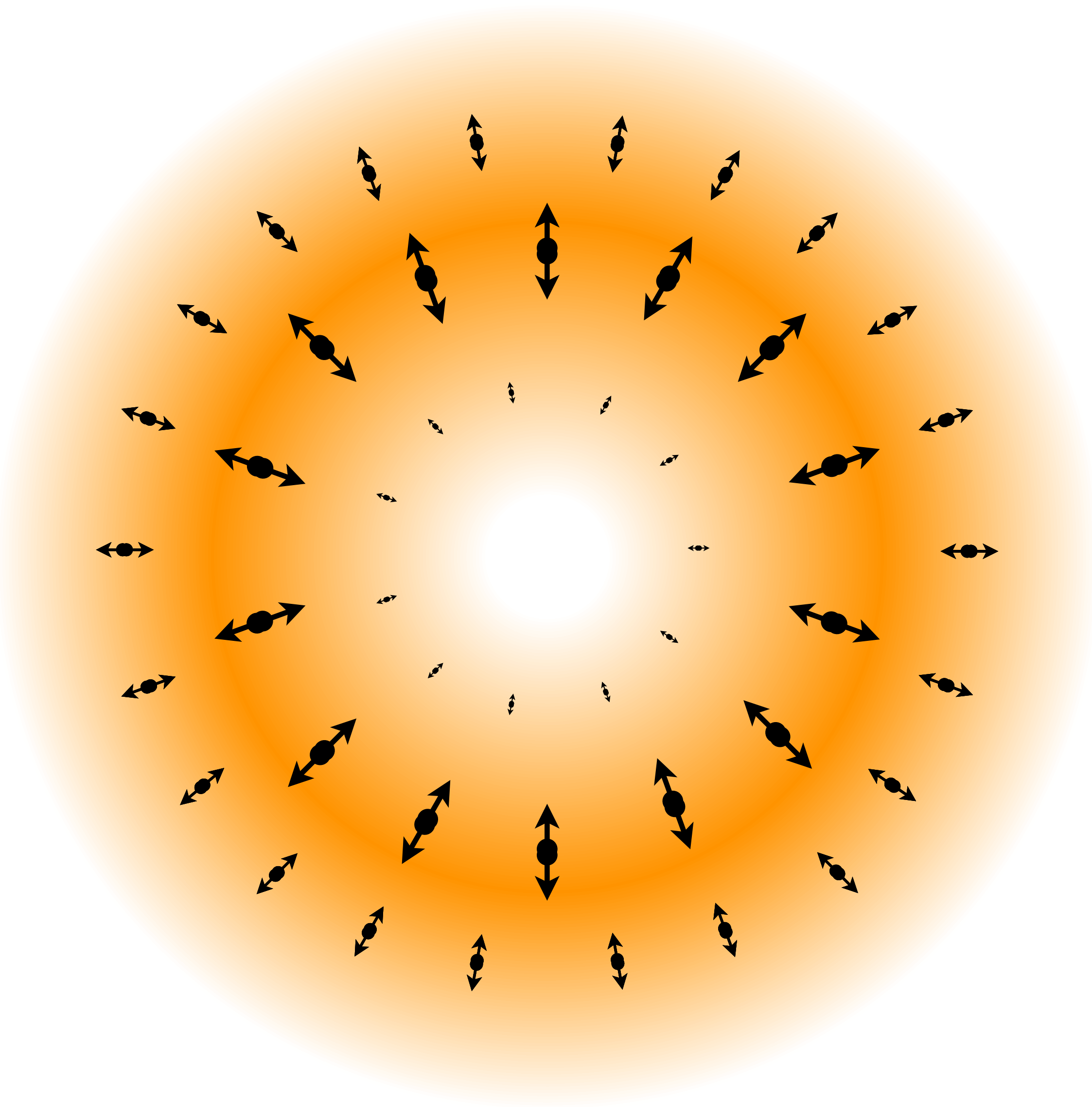
earlier literature

$$W_j(\mathbf{x}, t) = f(r)\frac{x^j}{r}\cos\omega t\,,$$

hedgehogs
not ground states

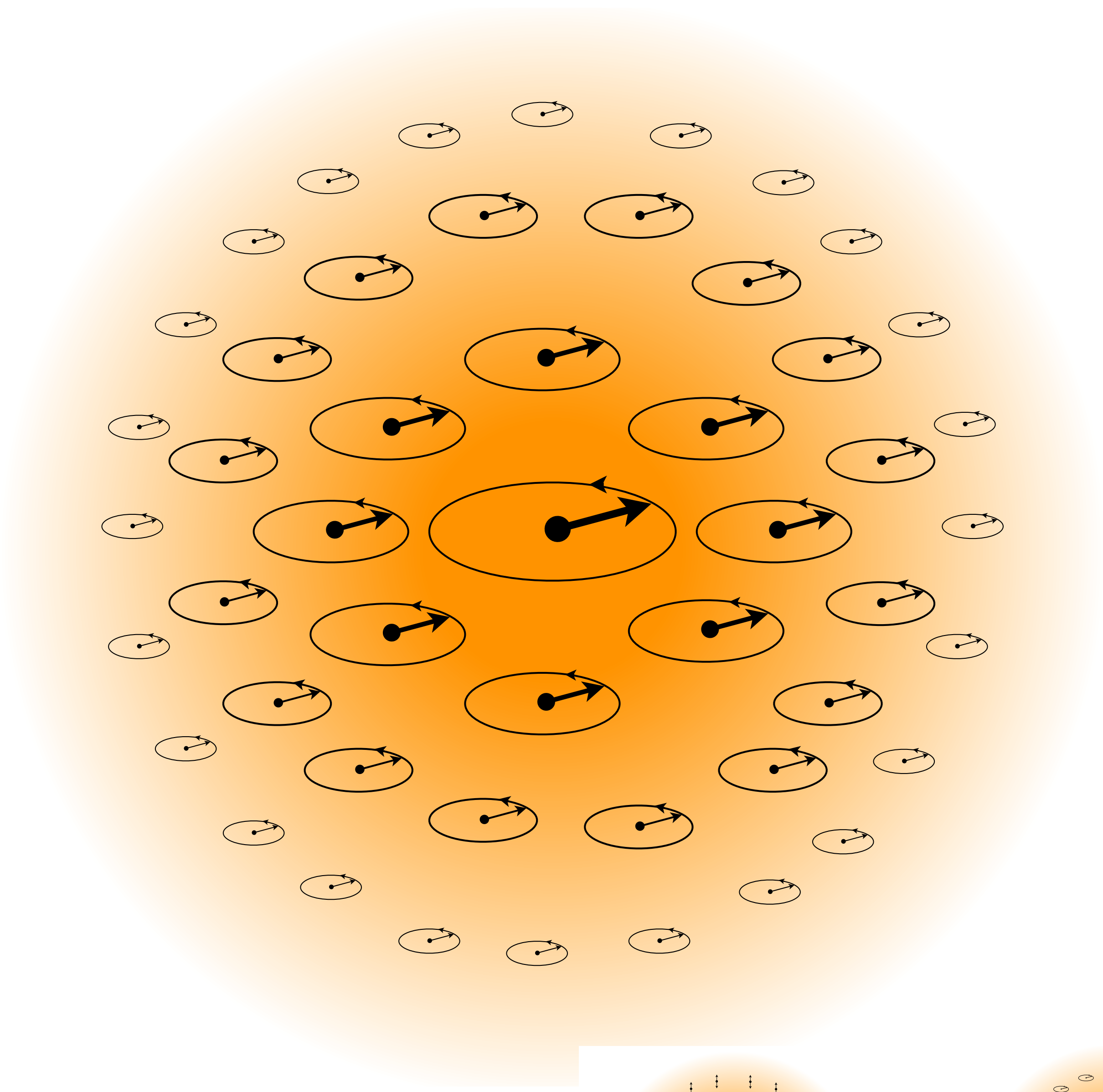
Lozanov & Adshead (2021)

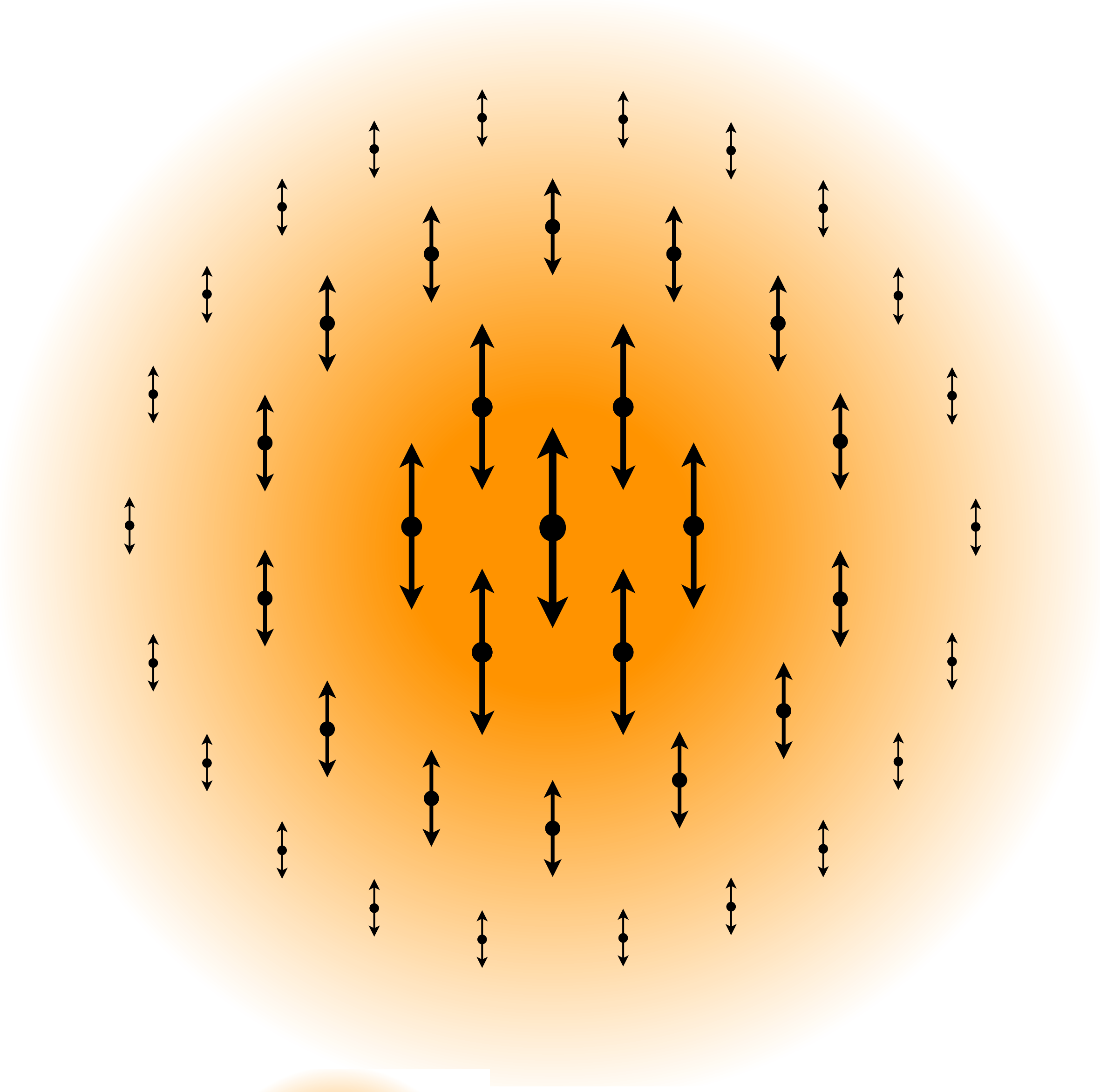
at least when non-relativistic

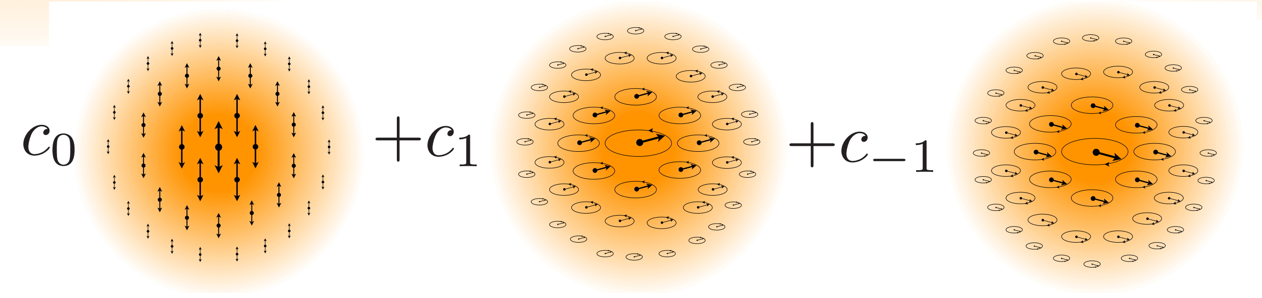


$$E^s_{\rm hh} > E$$

$$E^{s=1}_{\rm hh} \approx 0.33E \; < 0$$







a different higher energy soliton: the "hedgehogs"

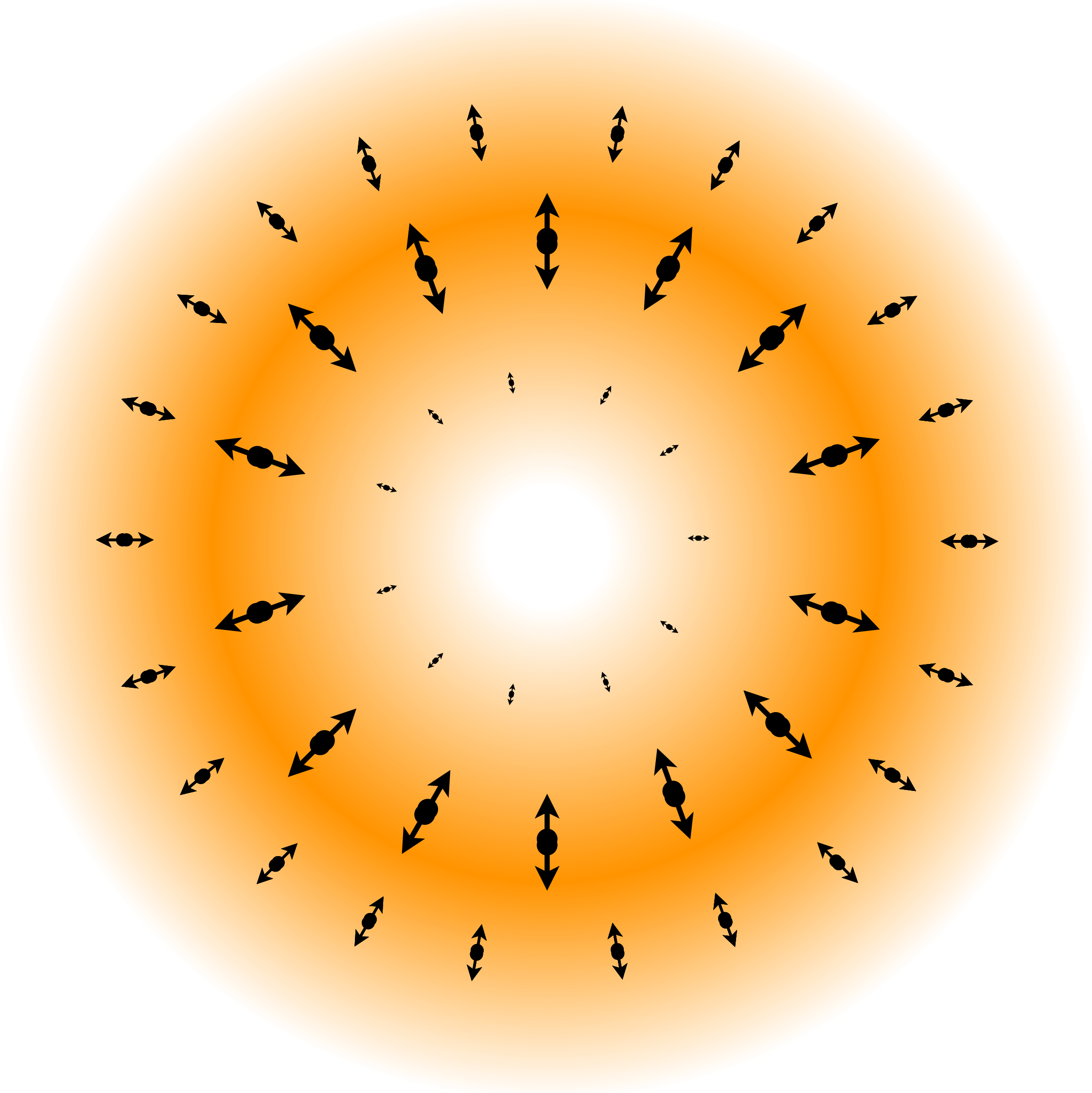
earlier literature

$$W_j(\mathbf{x},t) = f(r)\frac{x^j}{r}\cos\omega t\,,$$

hedgehogs
not ground states

Lozanov & Adshead (2021)

at least when non-relativistic



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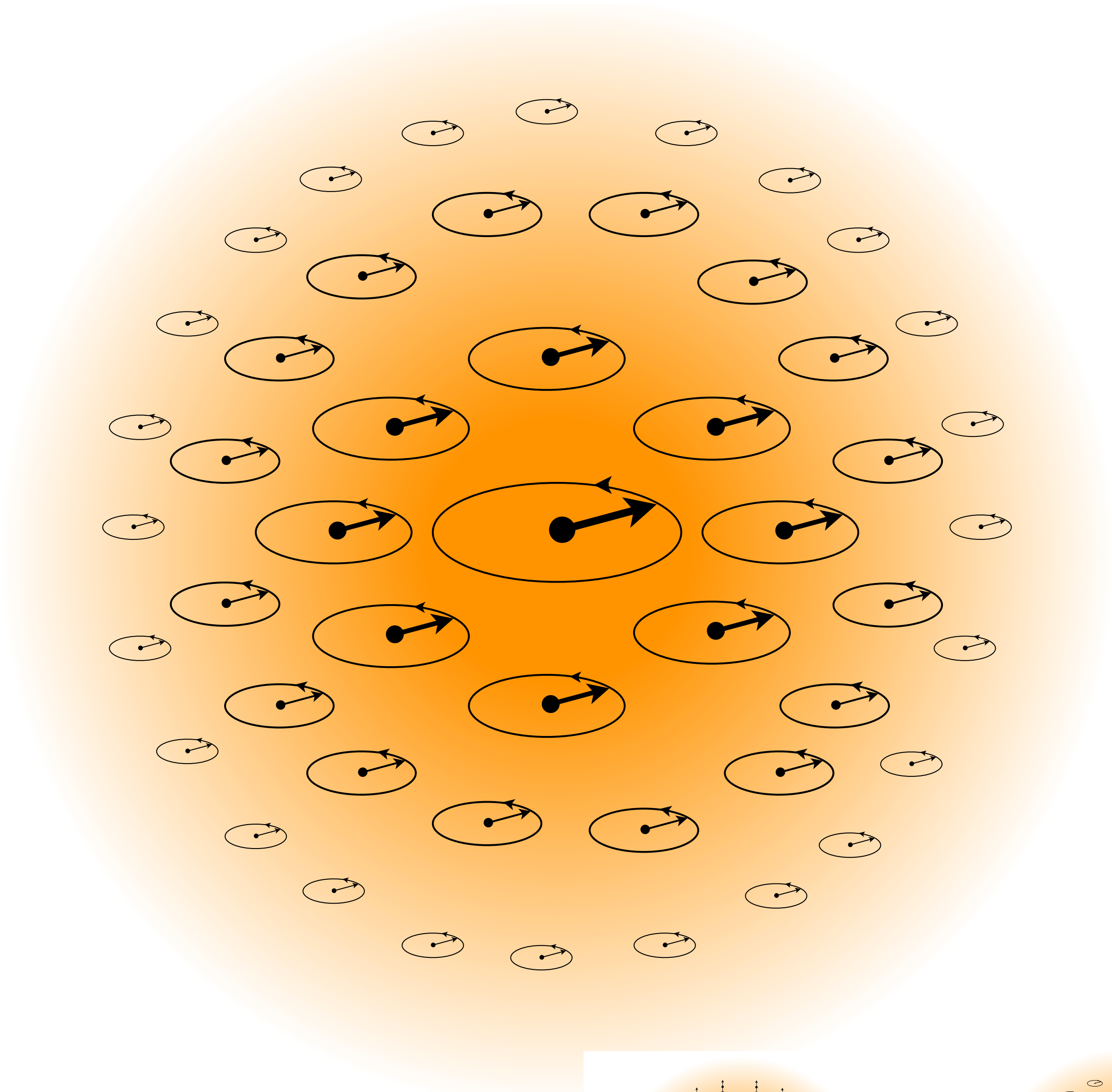
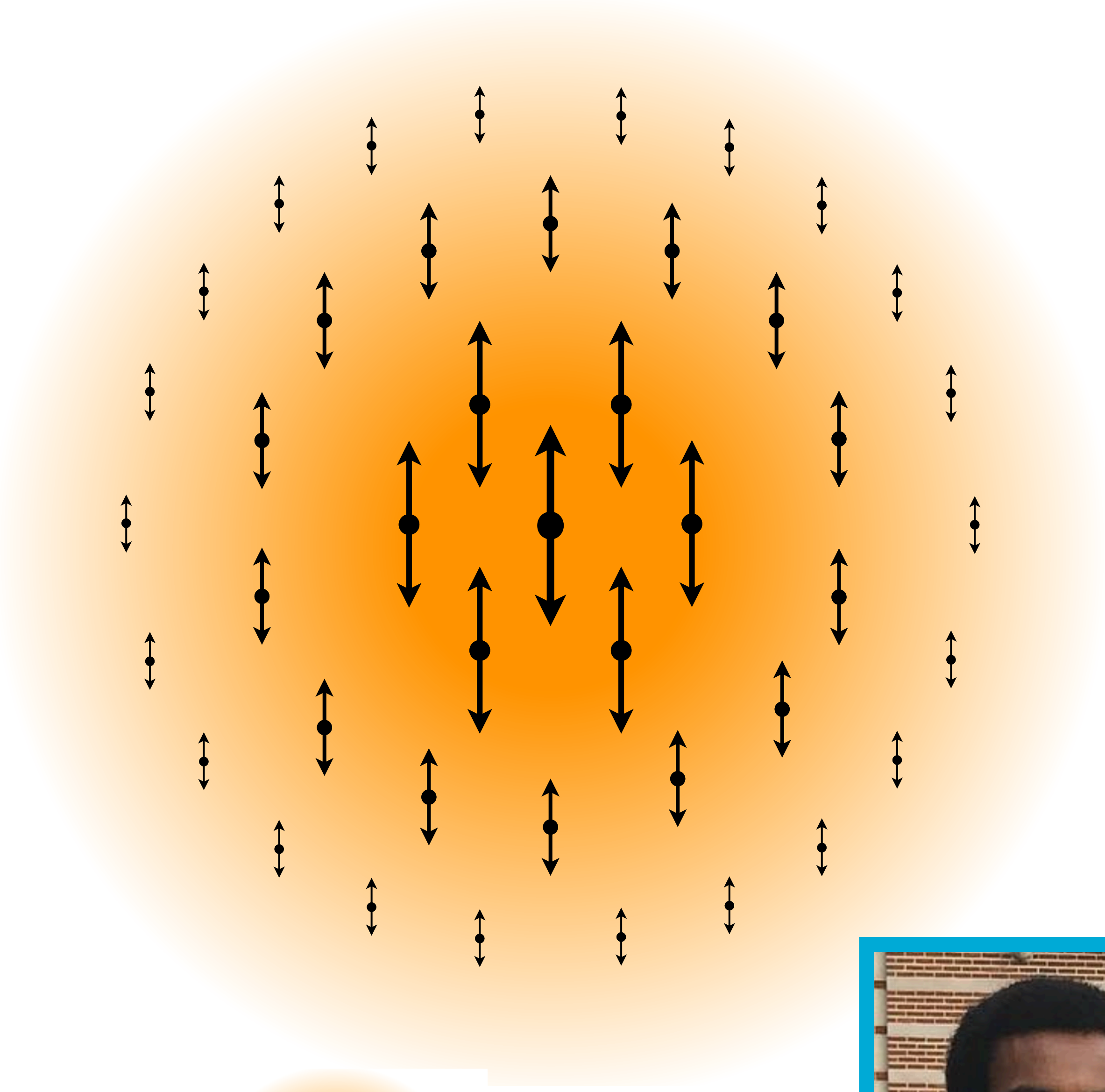
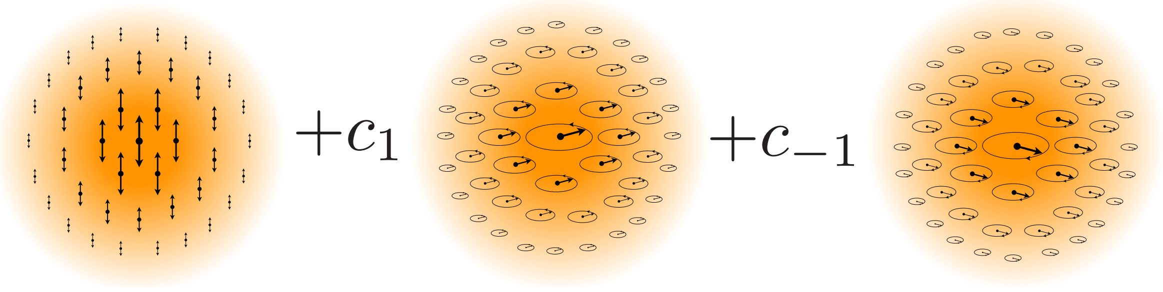




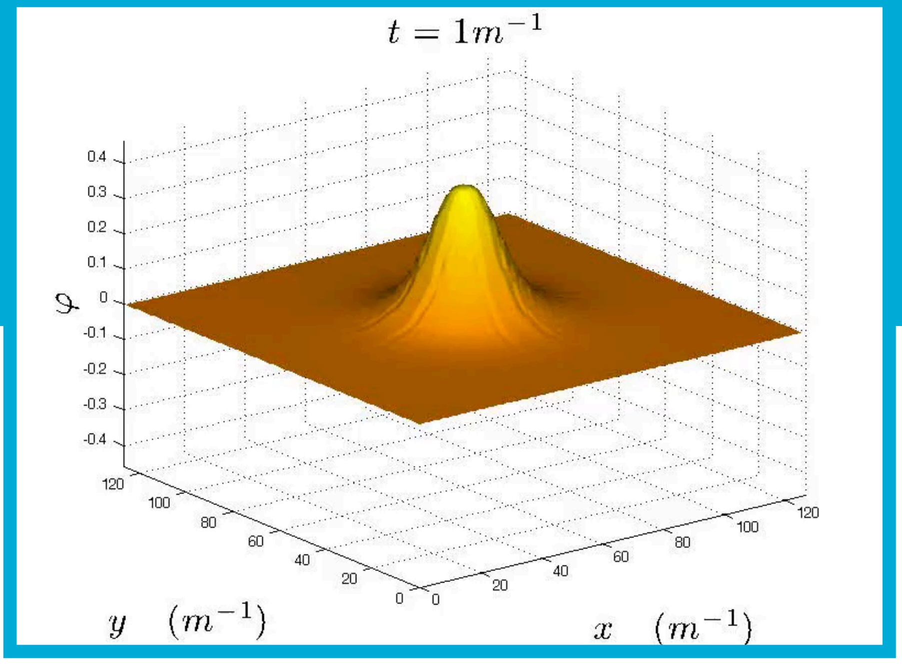


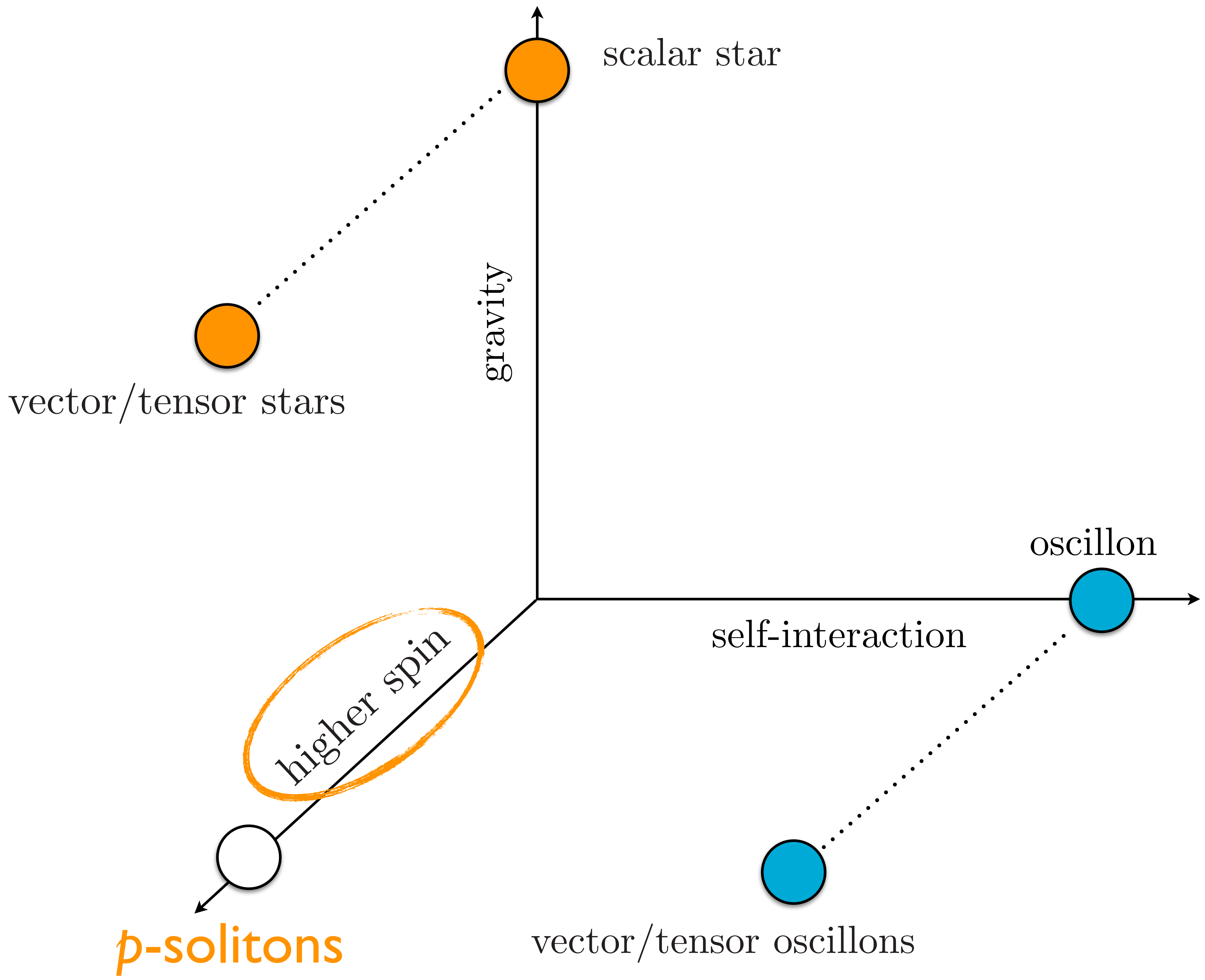
c_0 $+c_1$ $+c_{-1}$

J.Thomas (ongoing efforts "transitions")

non-topological solitons

spatially localized, coherently oscillating, long-lived





spatially localized

coherently oscillating (components)

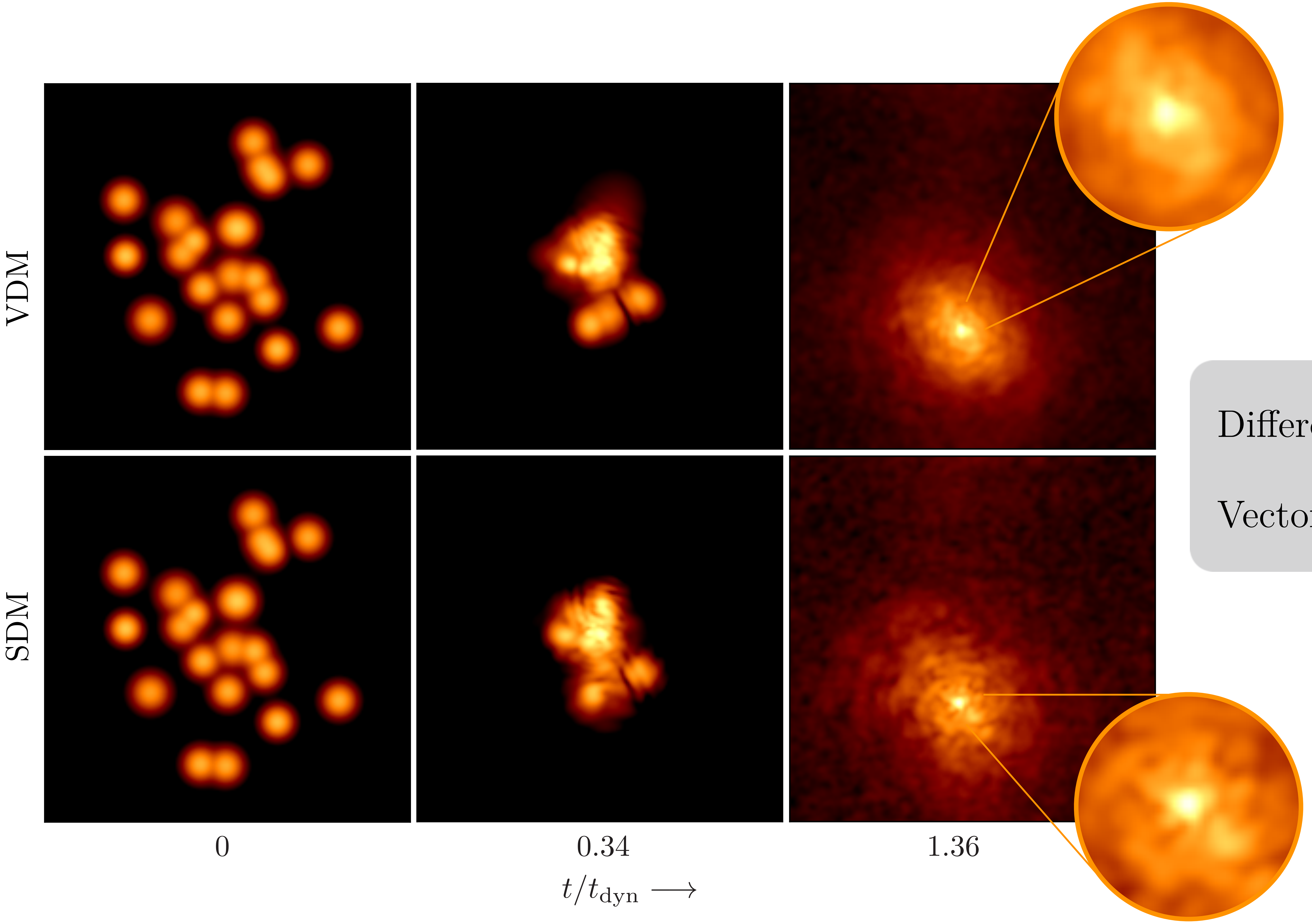
exceptionally long-lived

vector vs. scalar DM: two key differences

interference

polarized solitons

3+1 dimensional simulations



Difference between

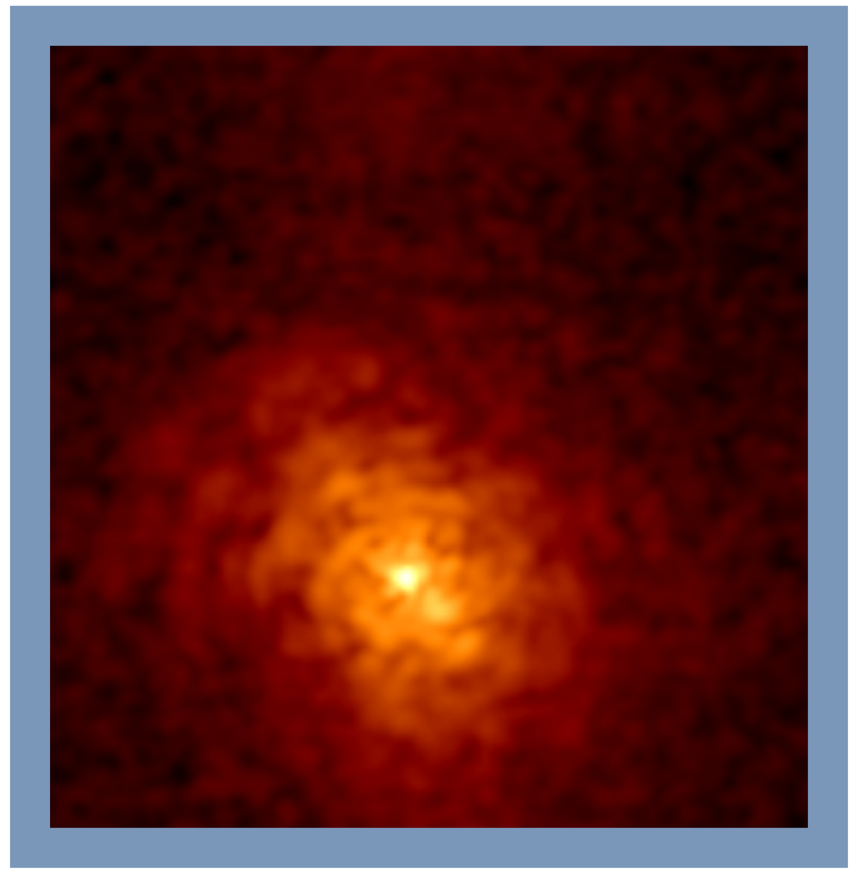
Vector & Scalar Dark Matter

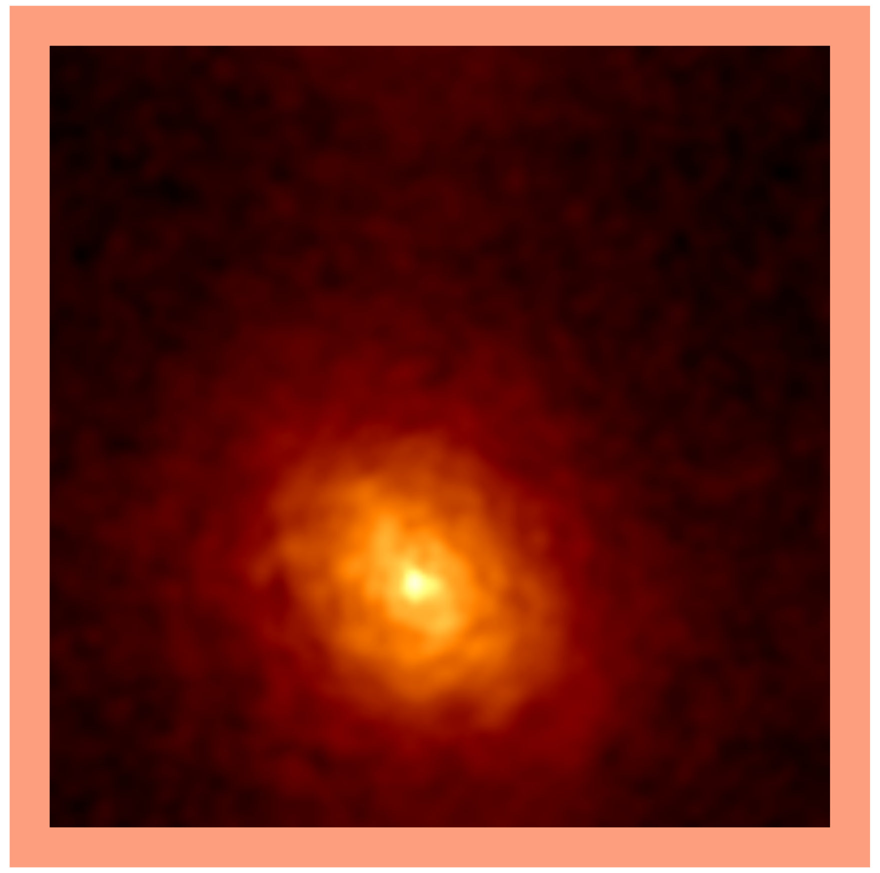
MA, Jain, Karur & Mocz (2022)

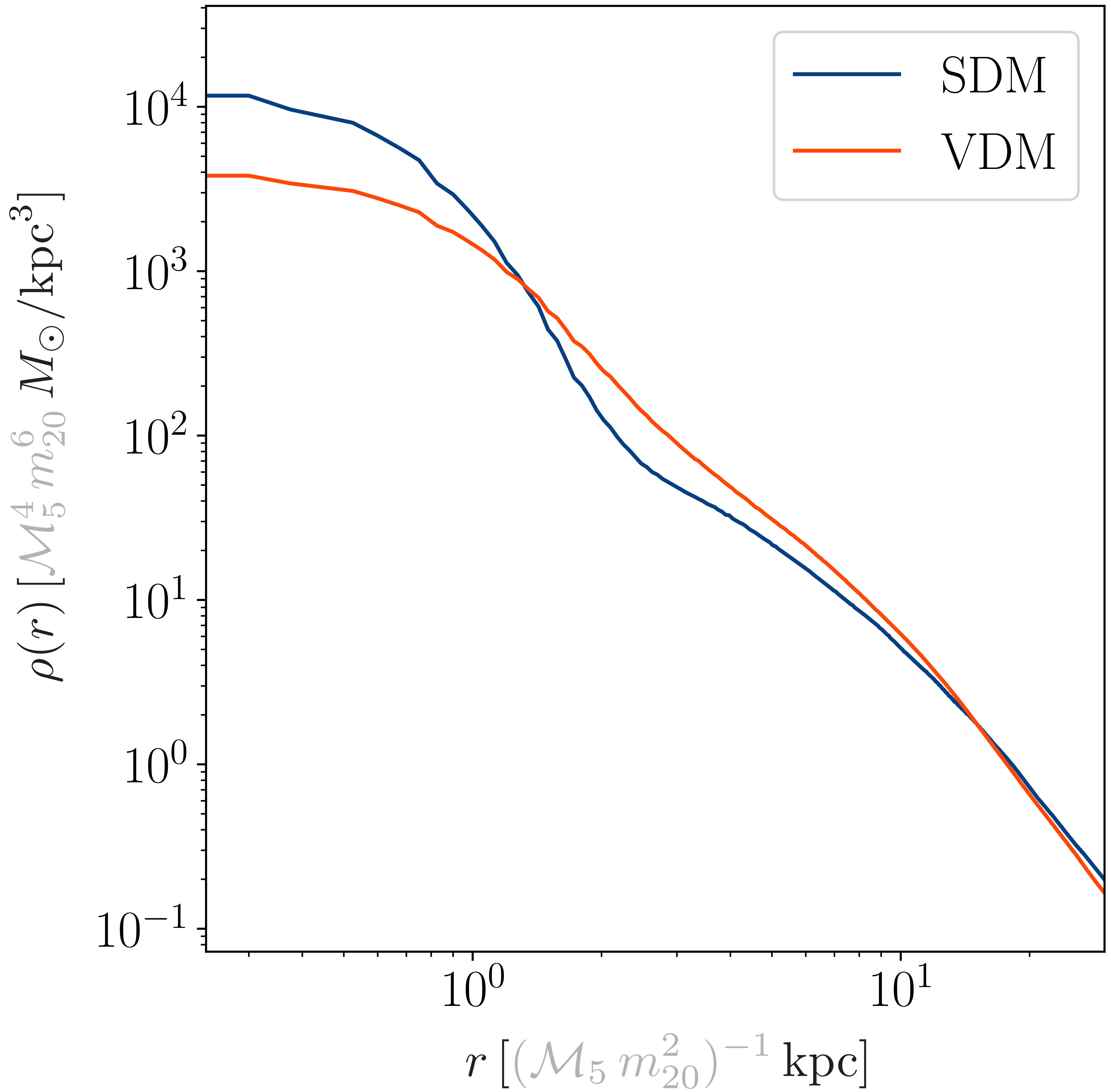
radial density profiles

scalar vs. vector dark matter

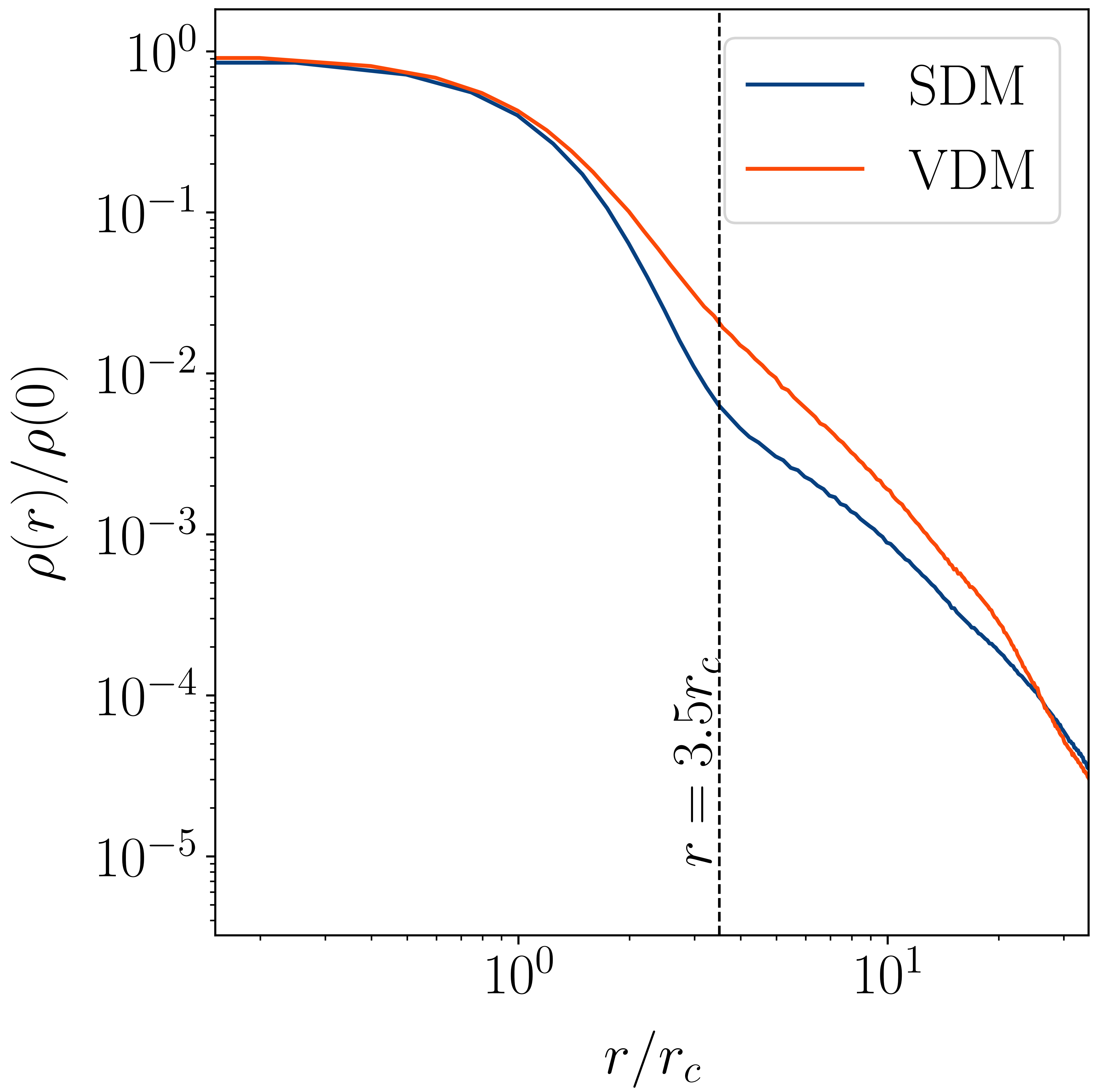
- less dense & broader core
- smoother transition to r^{-3} tail





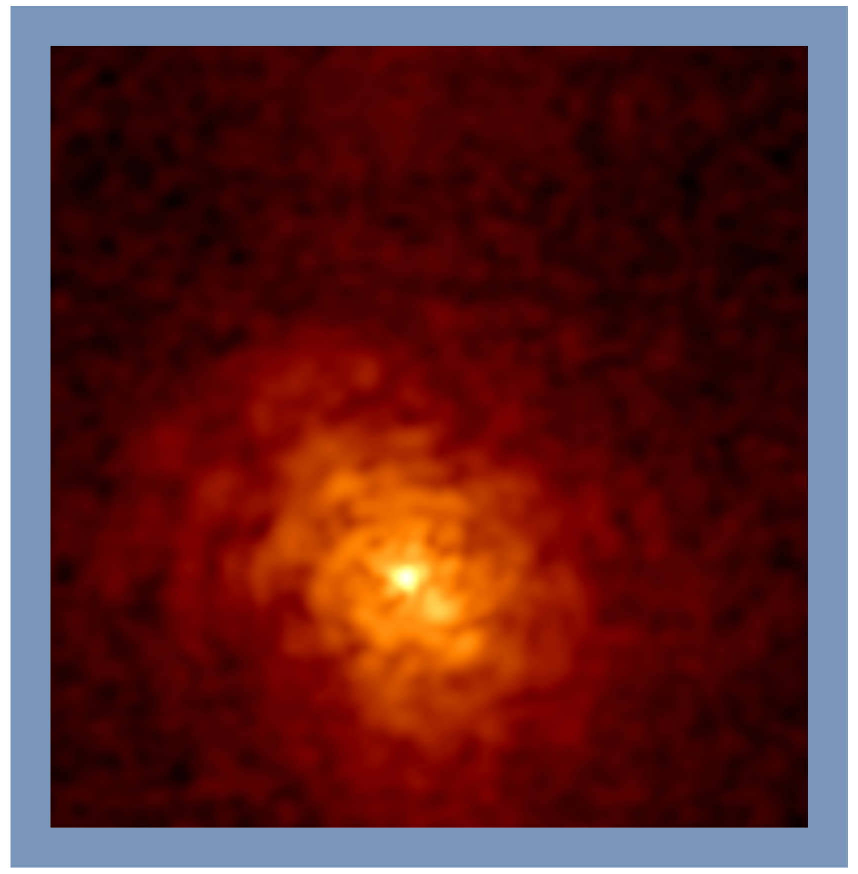


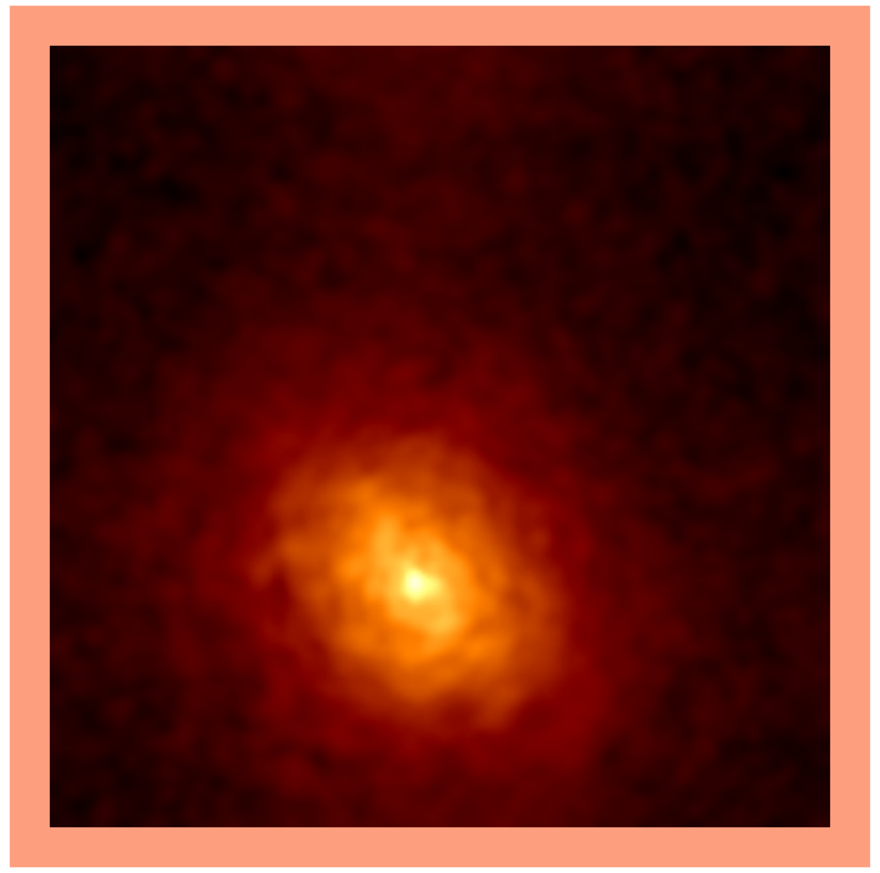
radial density profiles



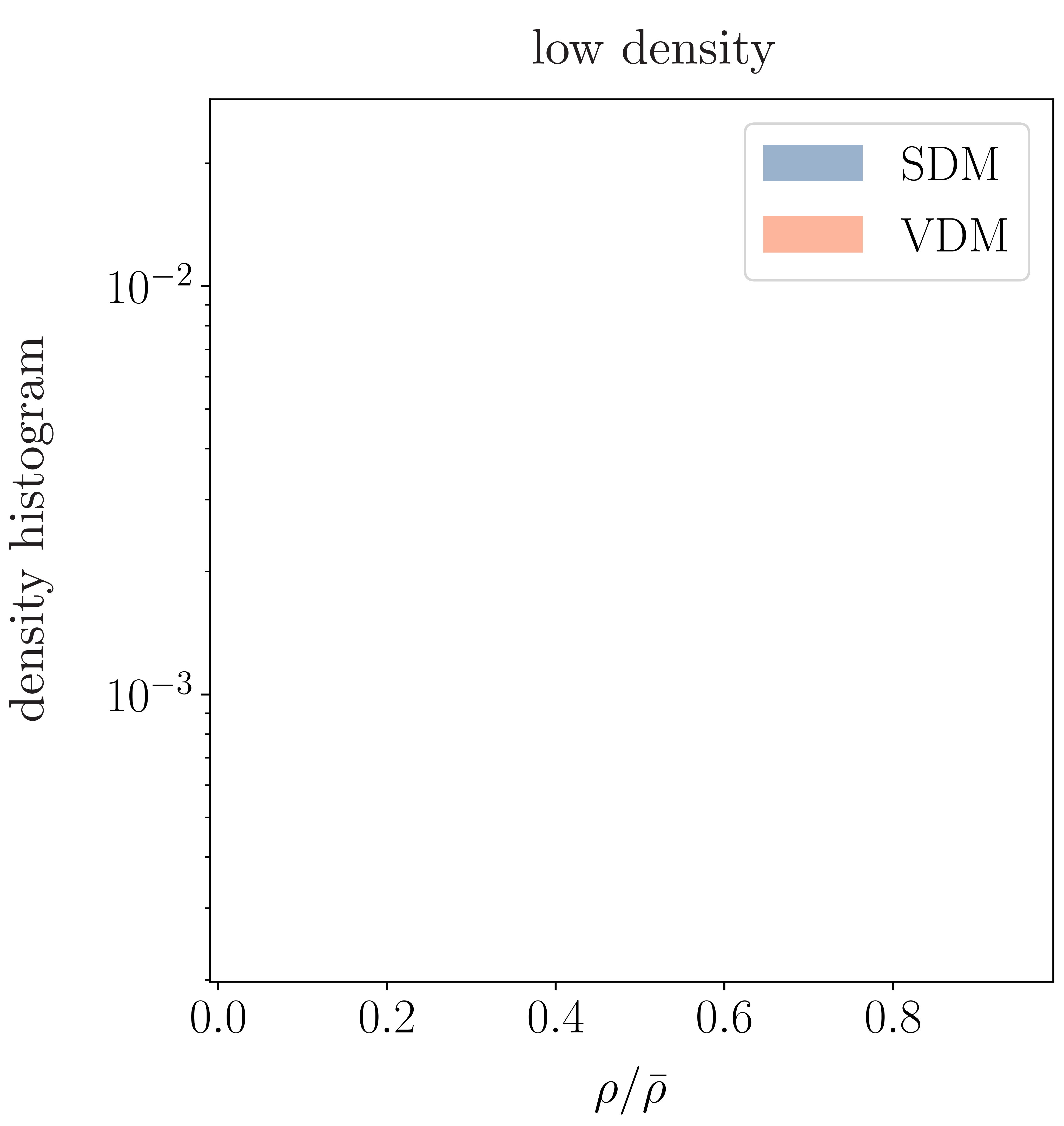
vector vs. scalar dark matter

- *shape* difference
- smoother transition to r^{-3} tail

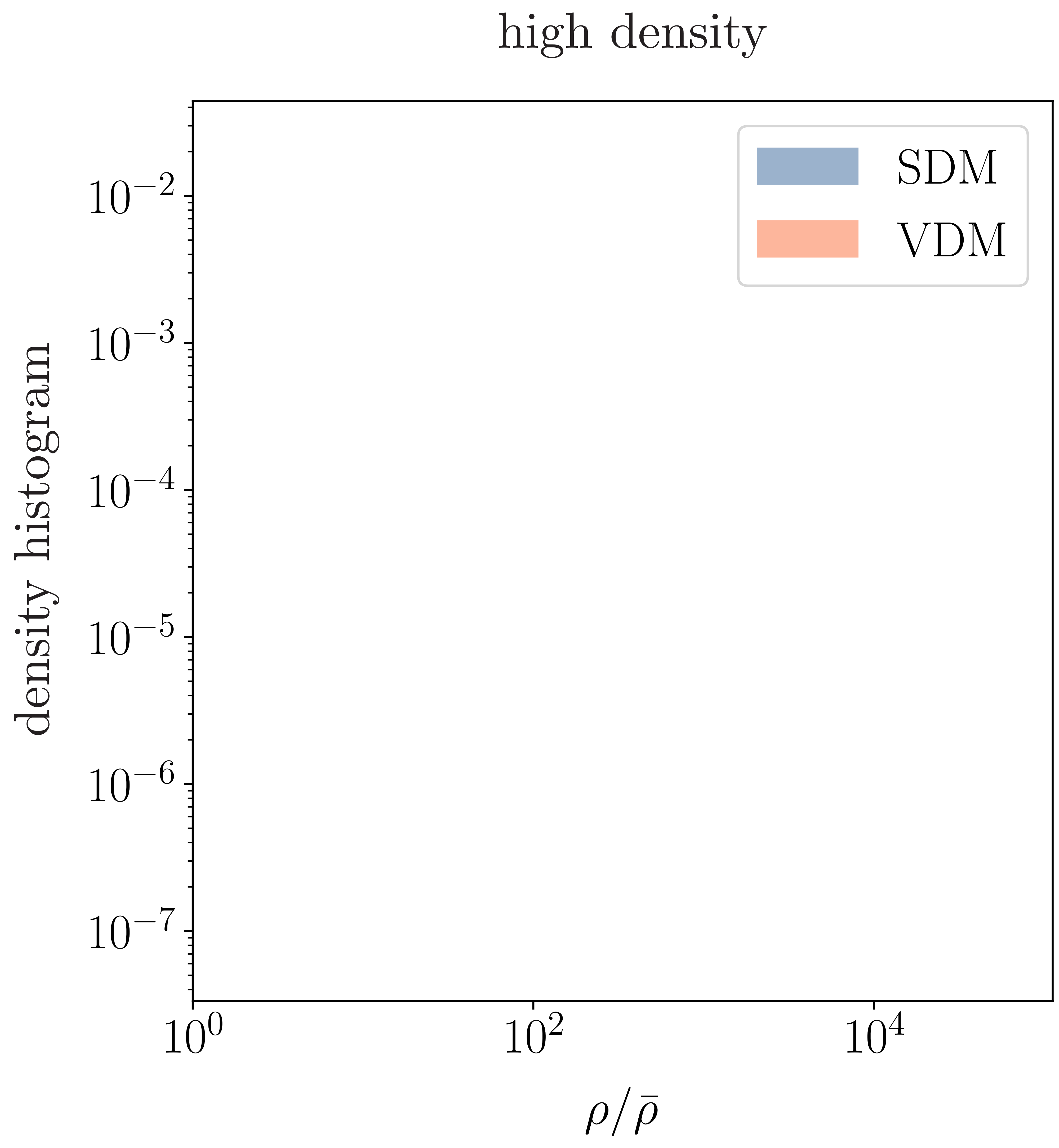




density pdf



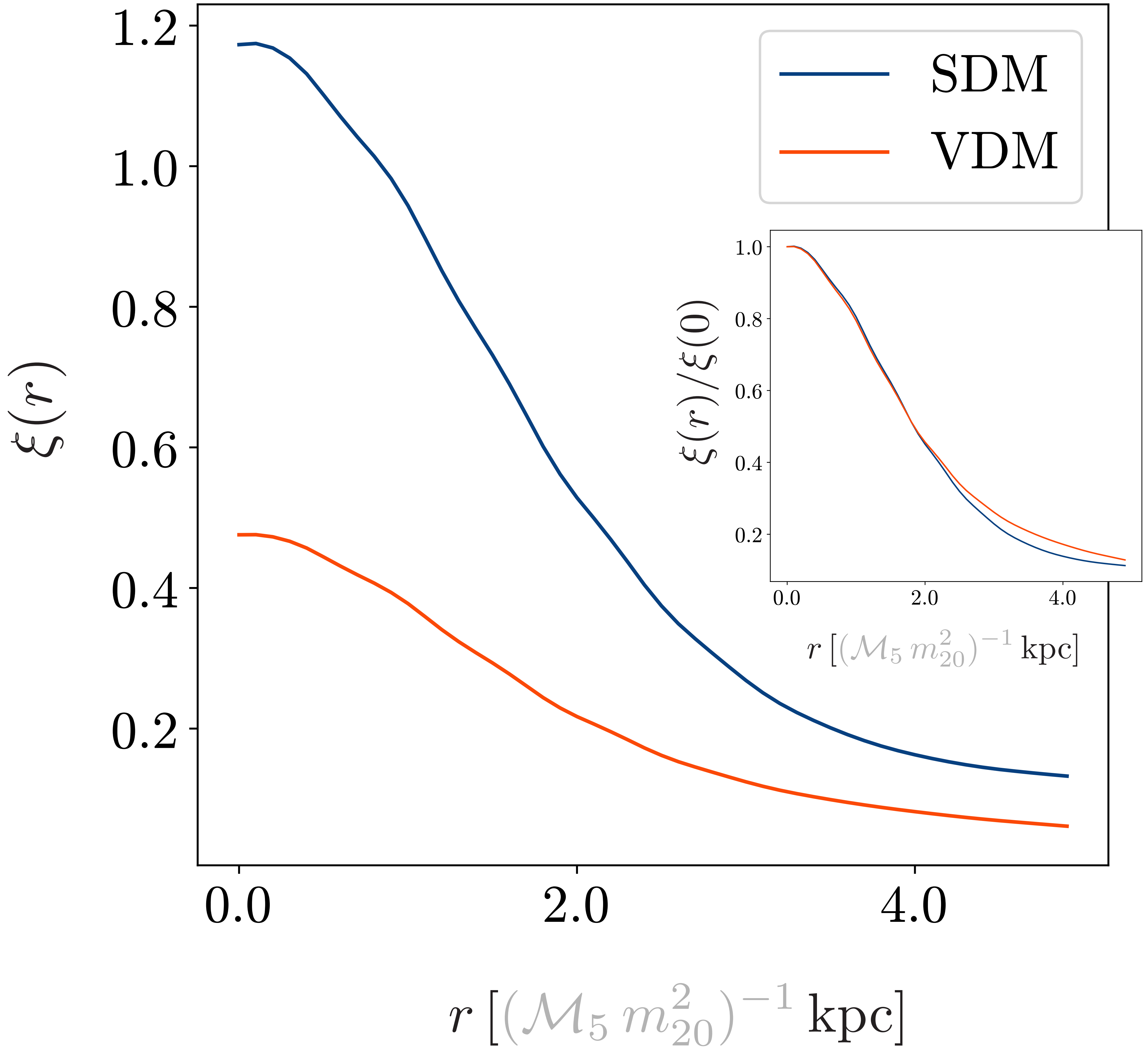
low density
SDM
VDM
density histogram
10^{-2}
10^{-3}
0.0
0.2
0.4
0.6
0.8
$\rho/\bar{\rho}$

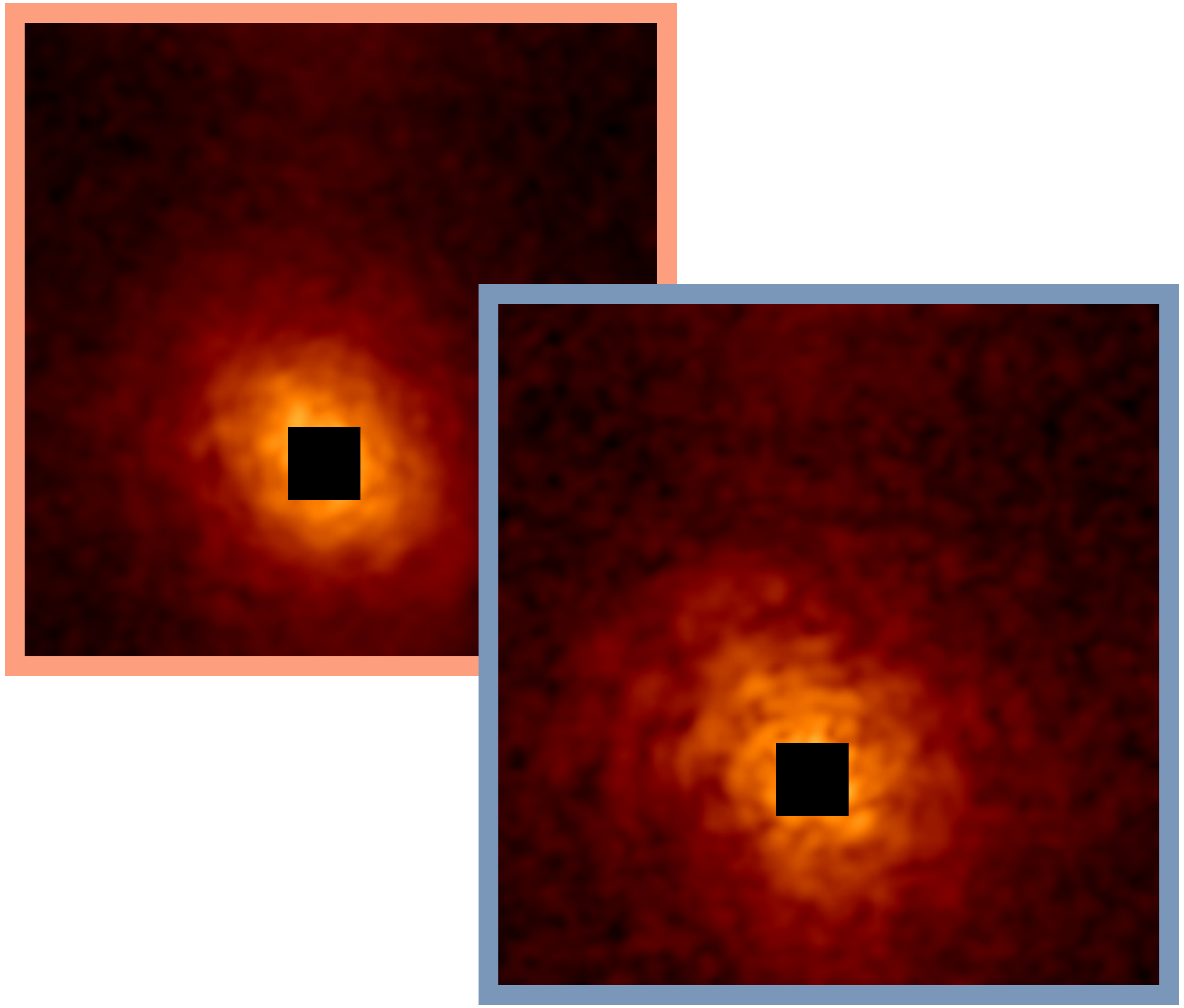


high density
SDM
VDM
density histogram
10^{-2}
10^{-3}
10^{-4}
10^{-5}
10^{-6}
10^{-7}
10^{0}
10^{2}
10^{4}
$\rho/\bar{\rho}$

2-point density correlation

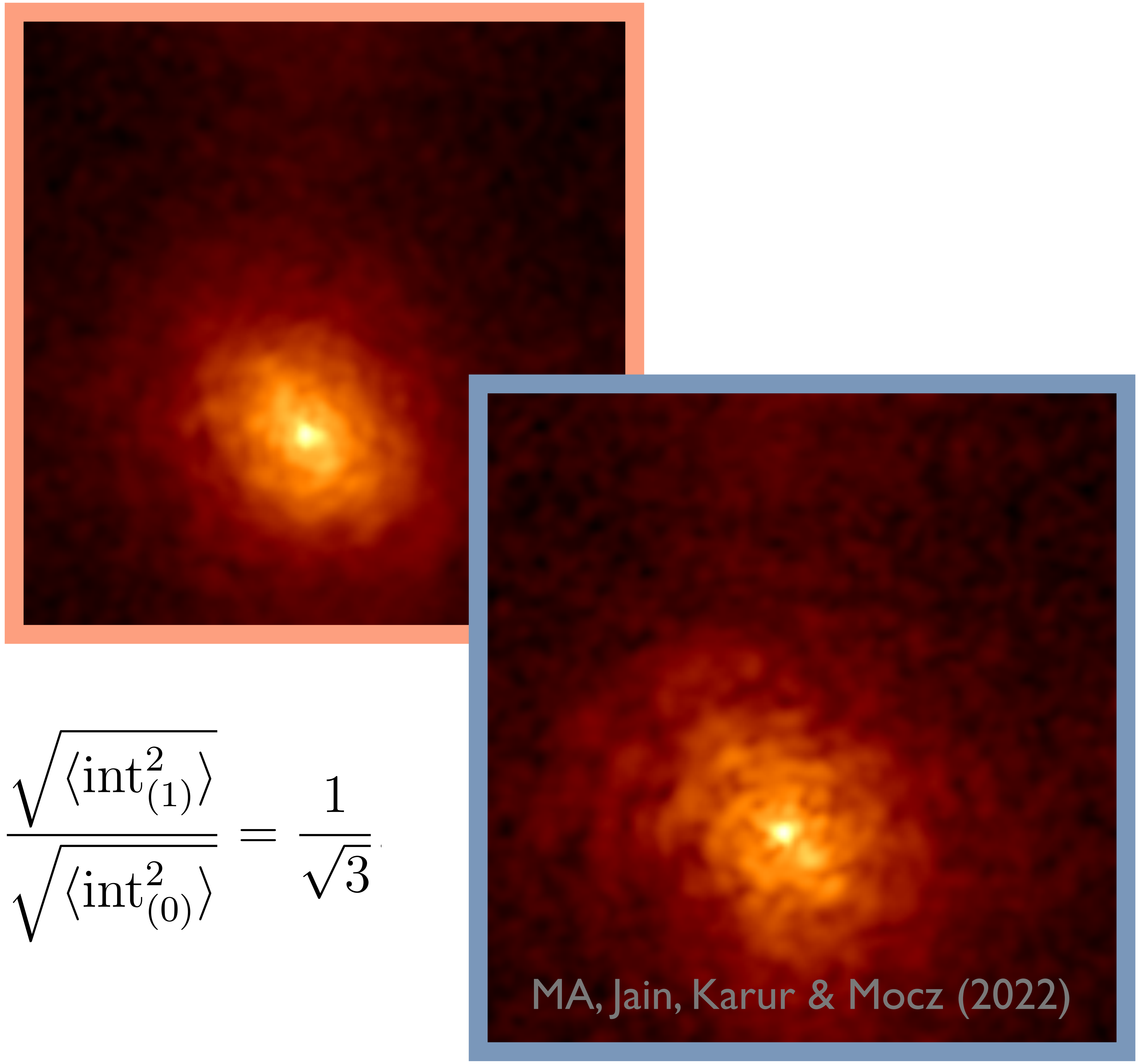
same $\lambda_{\rm dB}$ in the outer regions.





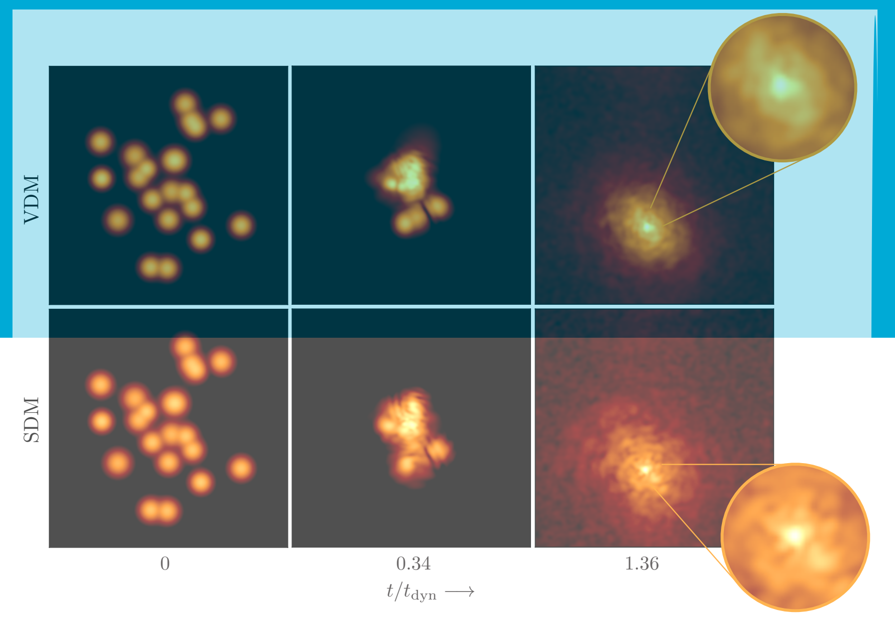
gravitational implications (examples)

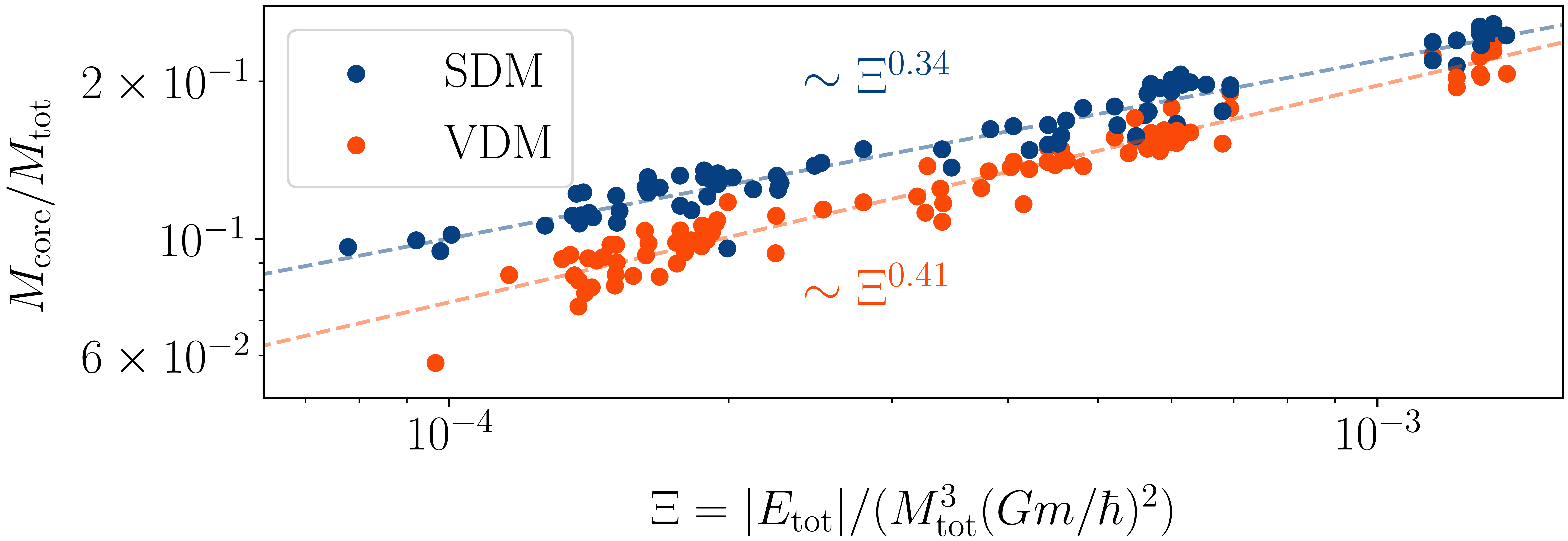
- dynamical heating of stars
  Church et. al (2021), Dalal & Kravtsov (2022)
- cusp-core, diversity
- lensing



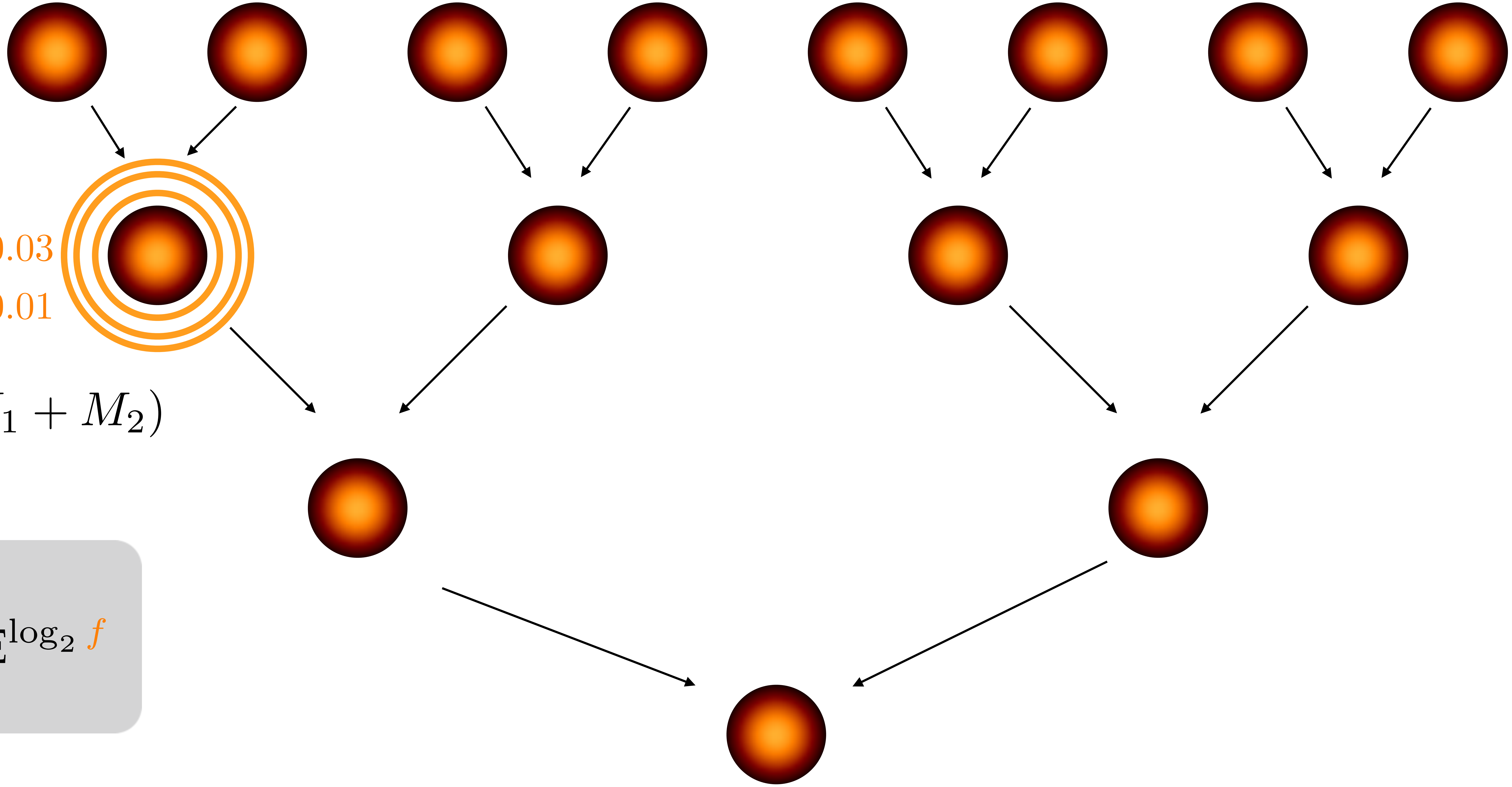
$$\frac{\sqrt{\langle \mathrm{int}^2_{(1)} \rangle}}{\sqrt{\langle \mathrm{int}^2_{(0)} \rangle}} = \frac{1}{\sqrt{3}}$$

core-halo mass relation





merger tree



$f_{\rm v} = 0.56 \pm 0.03$

$f_{\rm s} = 0.61 \pm 0.01$

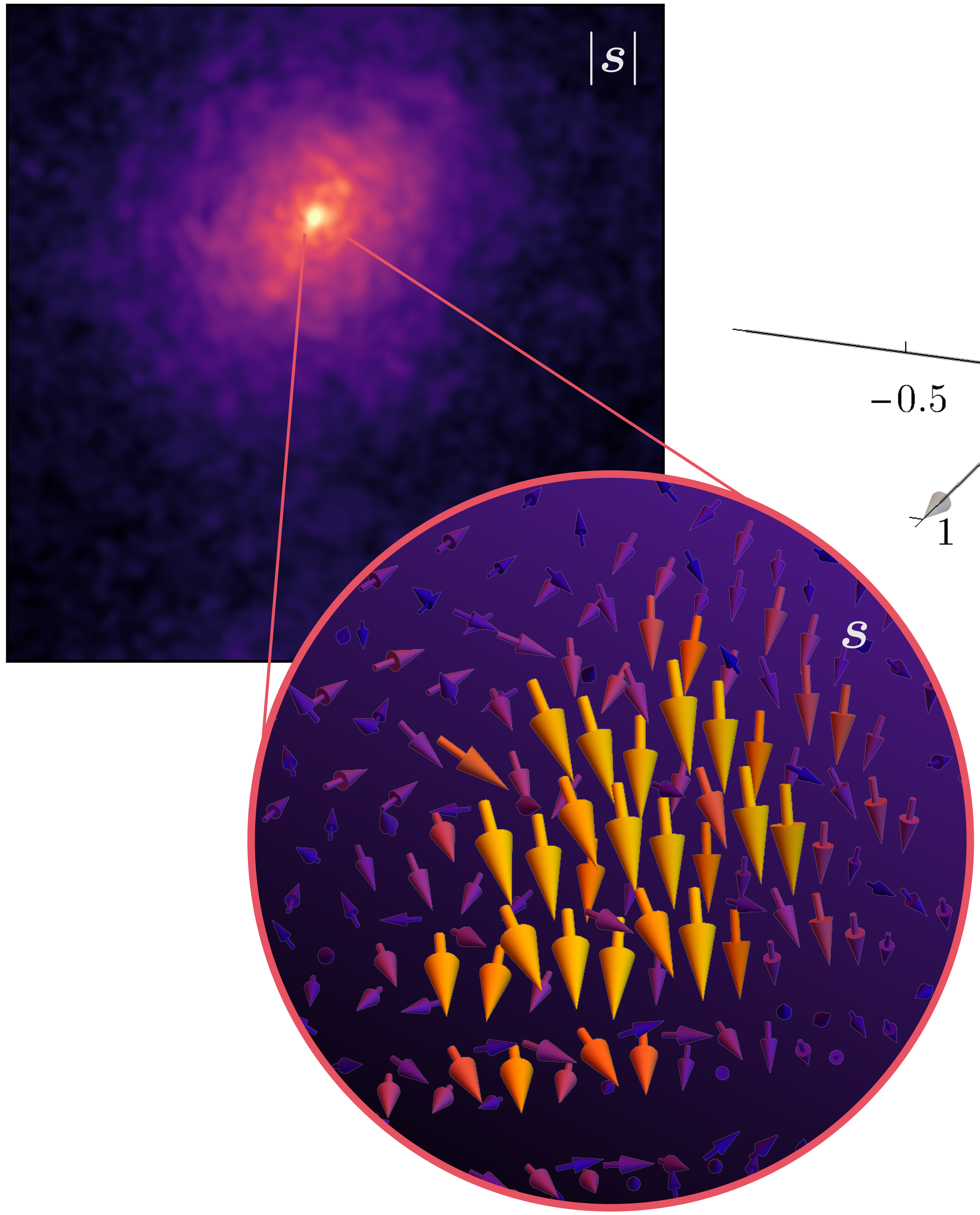
$M_{\rm f} = f(M_1 + M_2)$

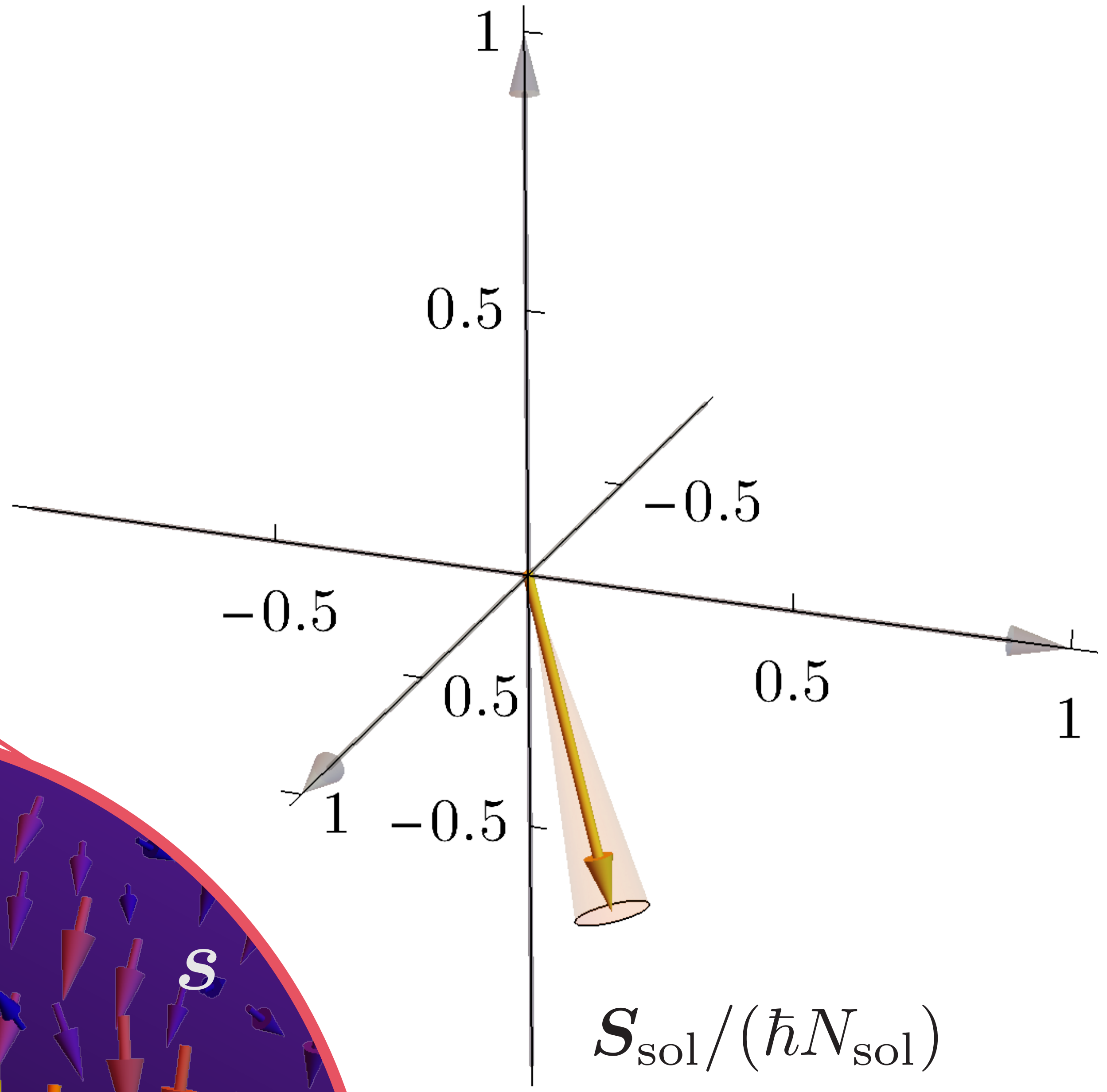
$$\frac{M_{\rm core}}{M_{\rm tot}} \propto \Xi^{\log_2 f}$$

Du et. al (2017)

intrinsic spin

spin





spin density

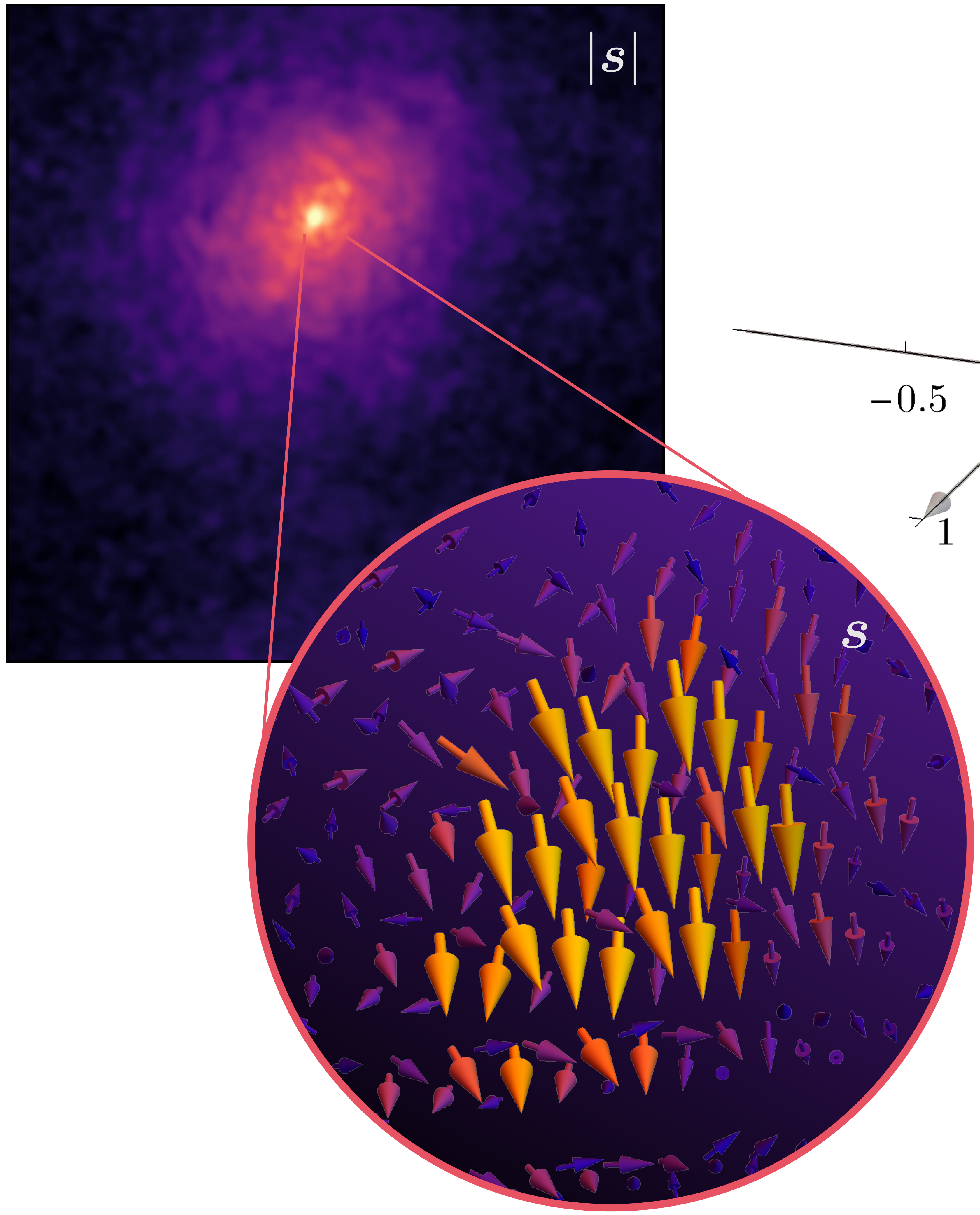
$$s = i\hbar \mathbf{\Psi} \times \mathbf{\Psi}^\dagger$$

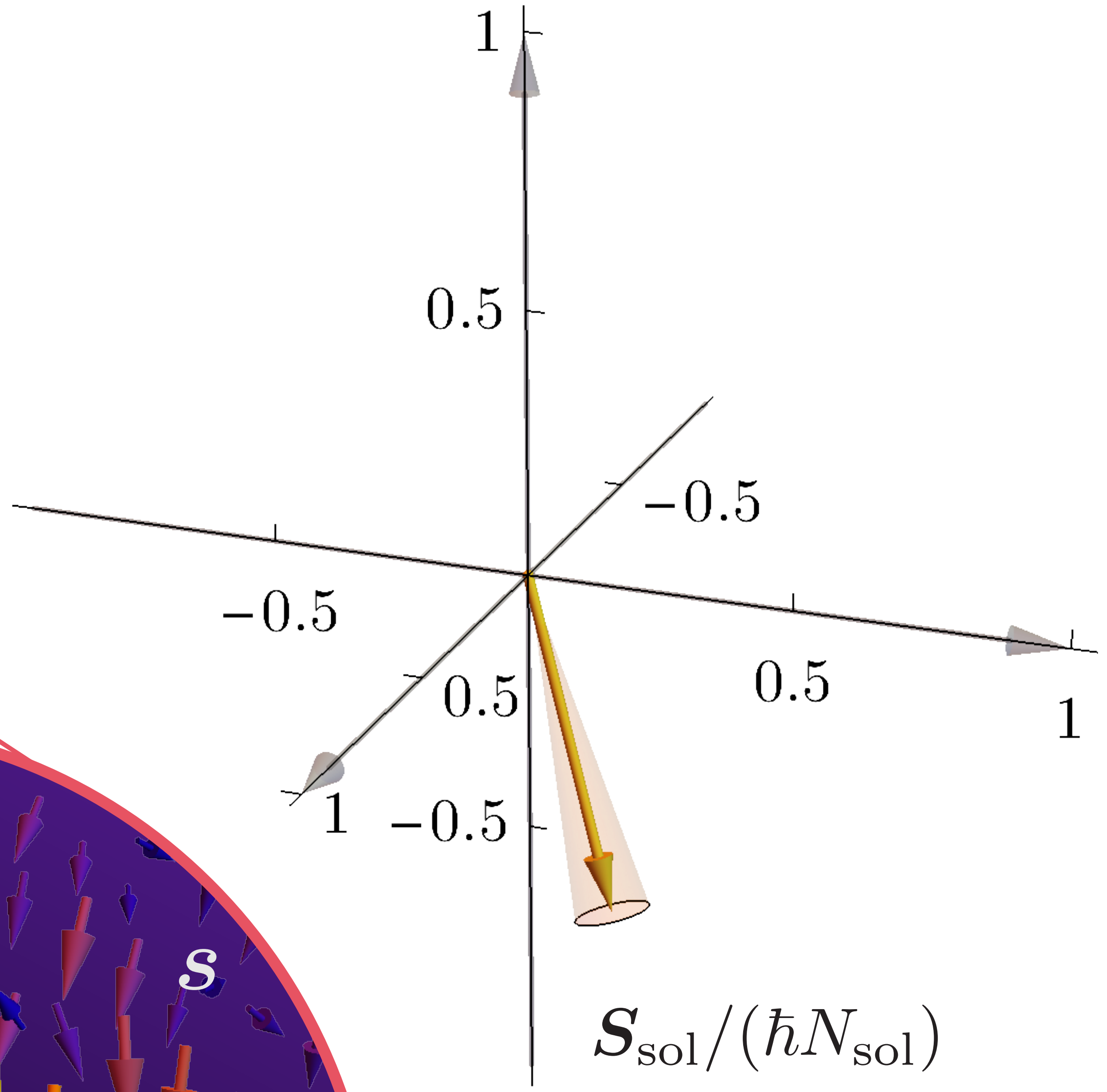
spin

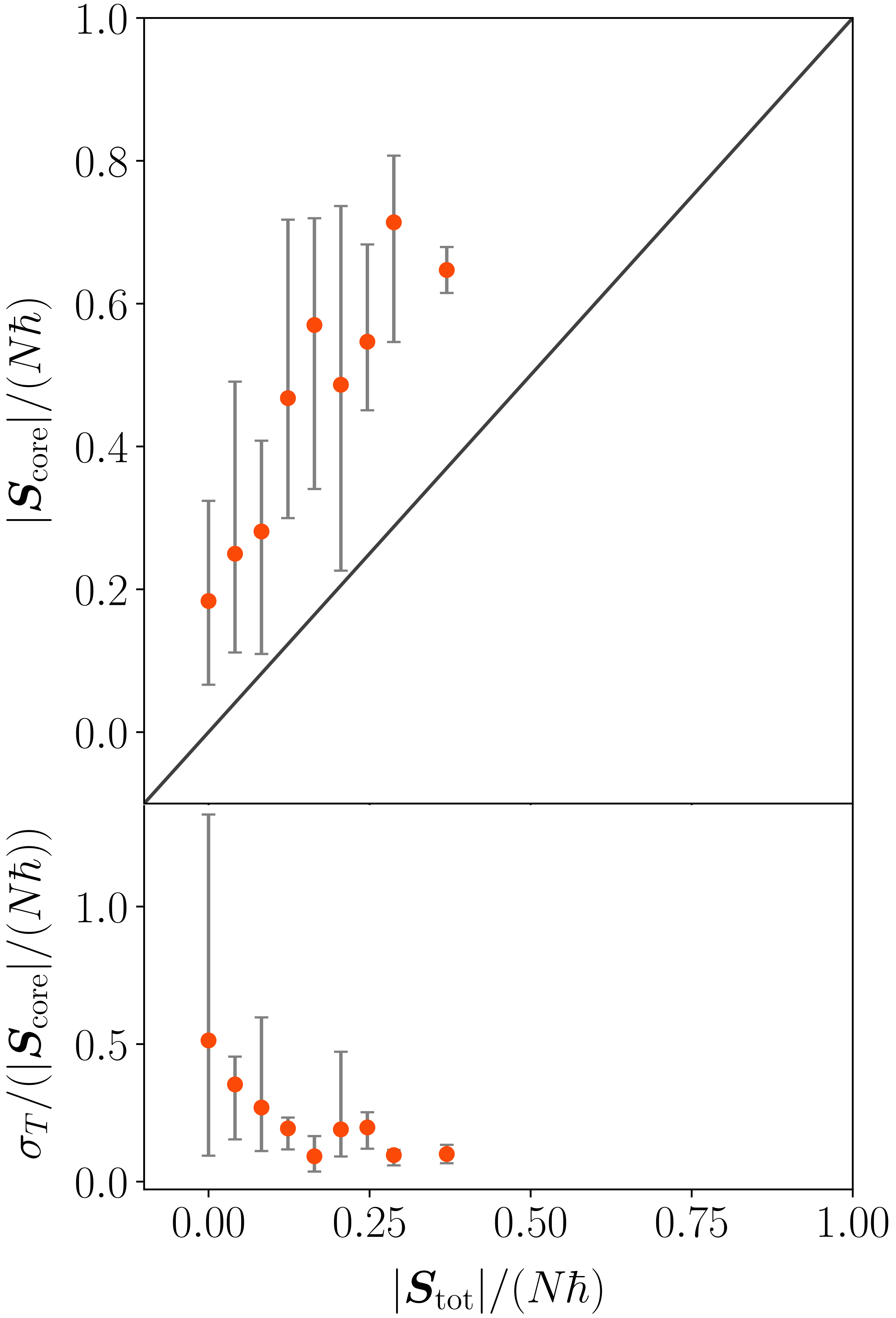
$$S = \hbar \int d^3x \, i \mathbf{\Psi} \times \mathbf{\Psi}^\dagger$$

$$s = (2mc/\hbar) \mathbf{W} \times (\dot{\mathbf{W}} - \nabla W_0)$$

generation of spin density







MA, Jain, Karur & Mocz (2022)

“polarized” vector solitons

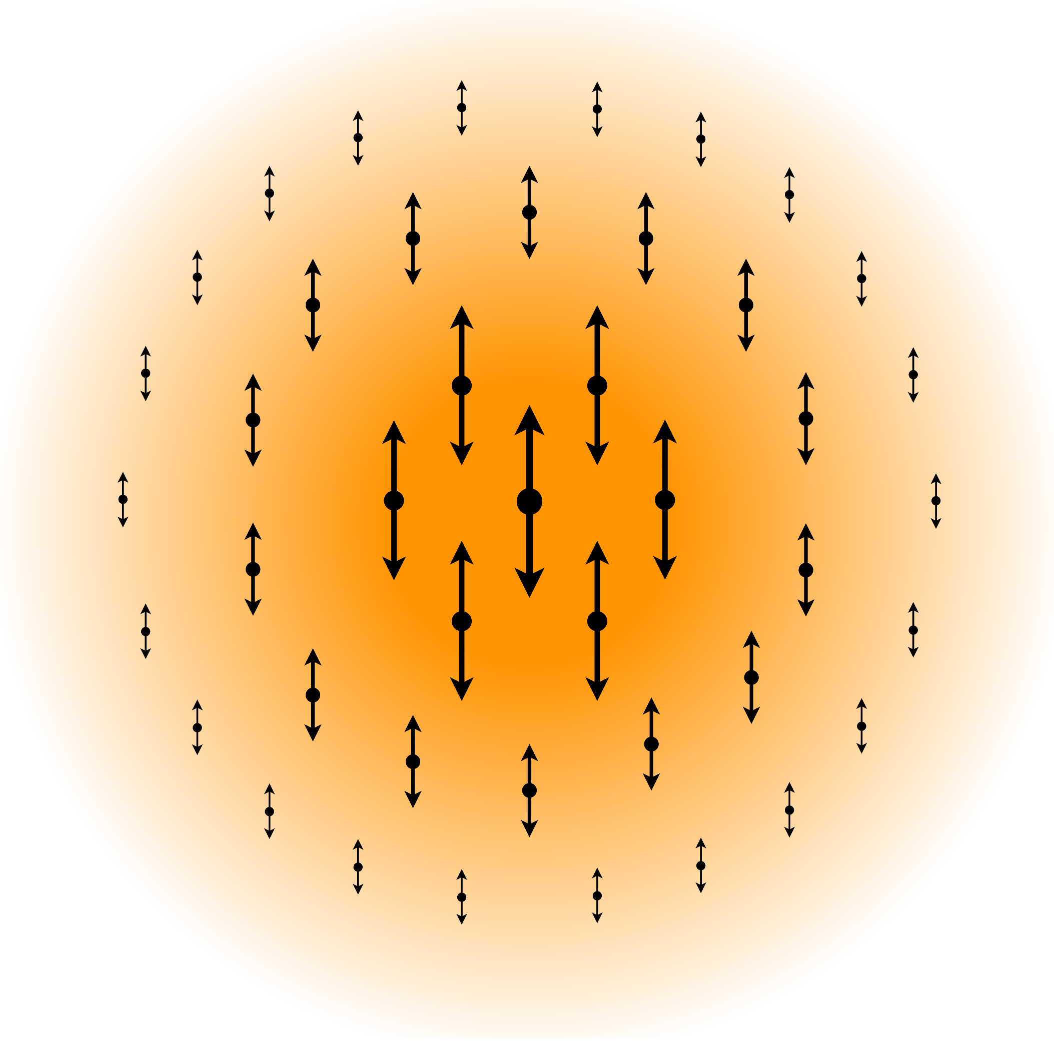
$$\boldsymbol{W}(t,\boldsymbol{x}) \equiv \frac{\hbar}{\sqrt{2mc}}\Re\left[\boldsymbol{\Psi}(t,\boldsymbol{x})e^{-imc^2t/\hbar}\right]$$

$$\boldsymbol{\Psi}_{\rm sol}(t,\boldsymbol{x}) = \psi_{\rm sol}(\mu,r)e^{i\mu c^2 t/\hbar}\boldsymbol{\epsilon} \qquad \boldsymbol{\epsilon}^\dagger\boldsymbol{\epsilon} = 1$$

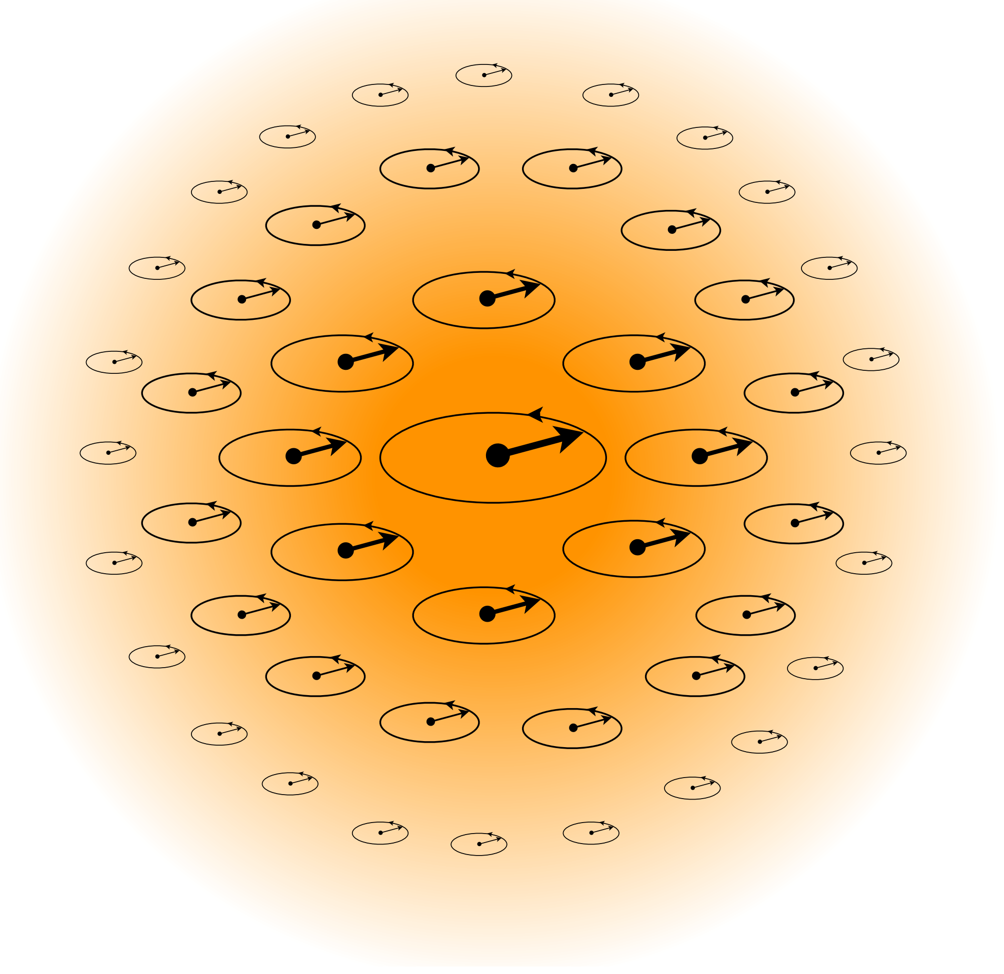
$$\boldsymbol{S}_{\rm sol} \approx i(\boldsymbol{\epsilon}\times\boldsymbol{\epsilon}^\dagger)60.7\frac{m_{\rm pl}^2}{m^2}\sqrt{\frac{\mu}{m}}\hbar \quad = i(\boldsymbol{\epsilon}\times\boldsymbol{\epsilon}^\dagger)\frac{M_{\rm sol}}{m}\hbar$$

macroscopic spin $\quad \boldsymbol{S}_{\rm tot}/\hbar = \lambda N\hat{z}$

$N =$ # of particles in soliton

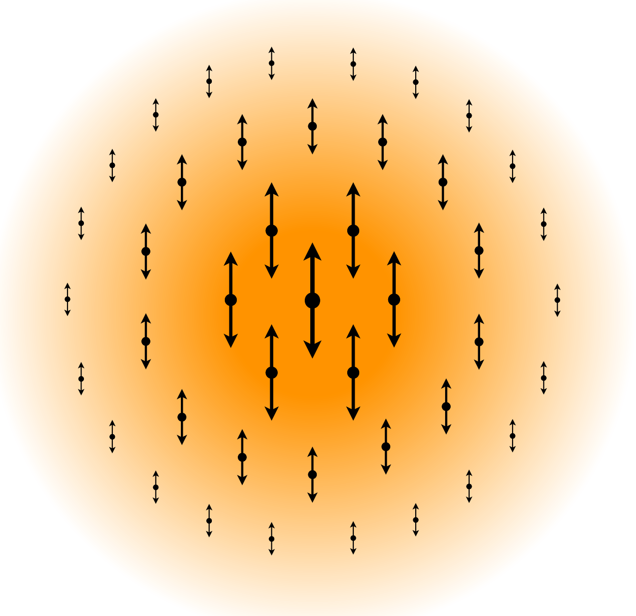
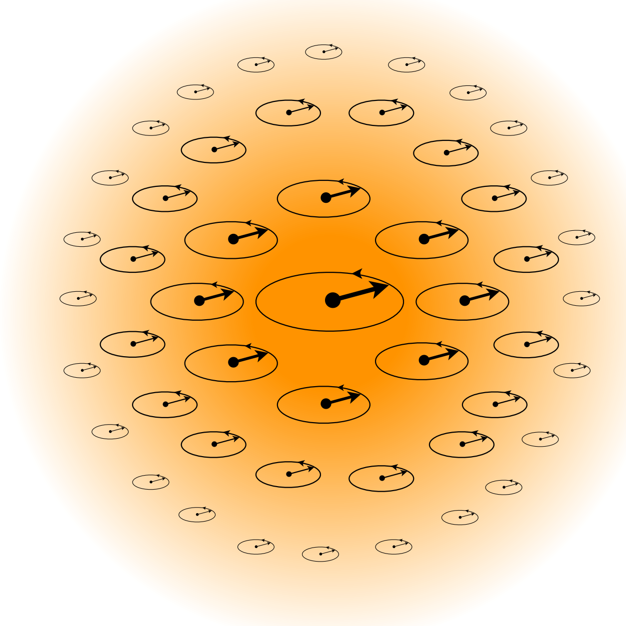
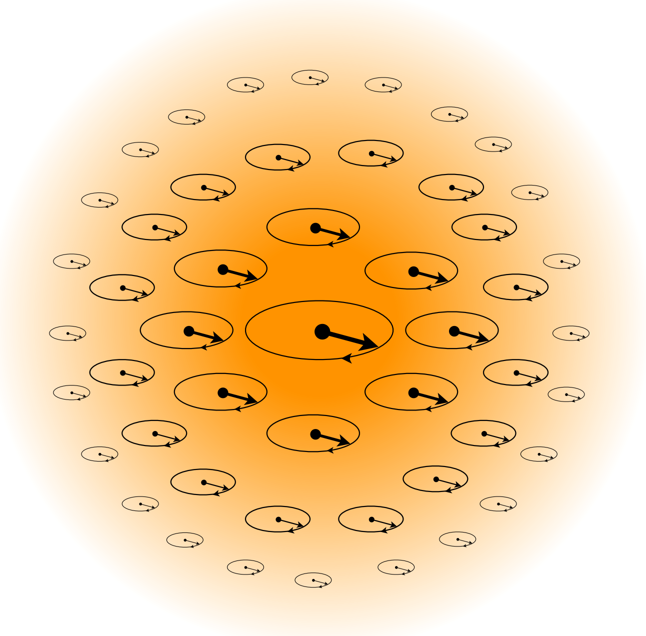


$\boldsymbol{S}_{\rm tot} = 0\hat{z}$



$\boldsymbol{S}_{\rm tot} = \hbar\frac{M_{\rm sol}}{m}\hat{z}$

- all lowest energy for fixed N
- bases for partially-polarized solitons

c_0  $+c_1$  $+c_{-1}$ 

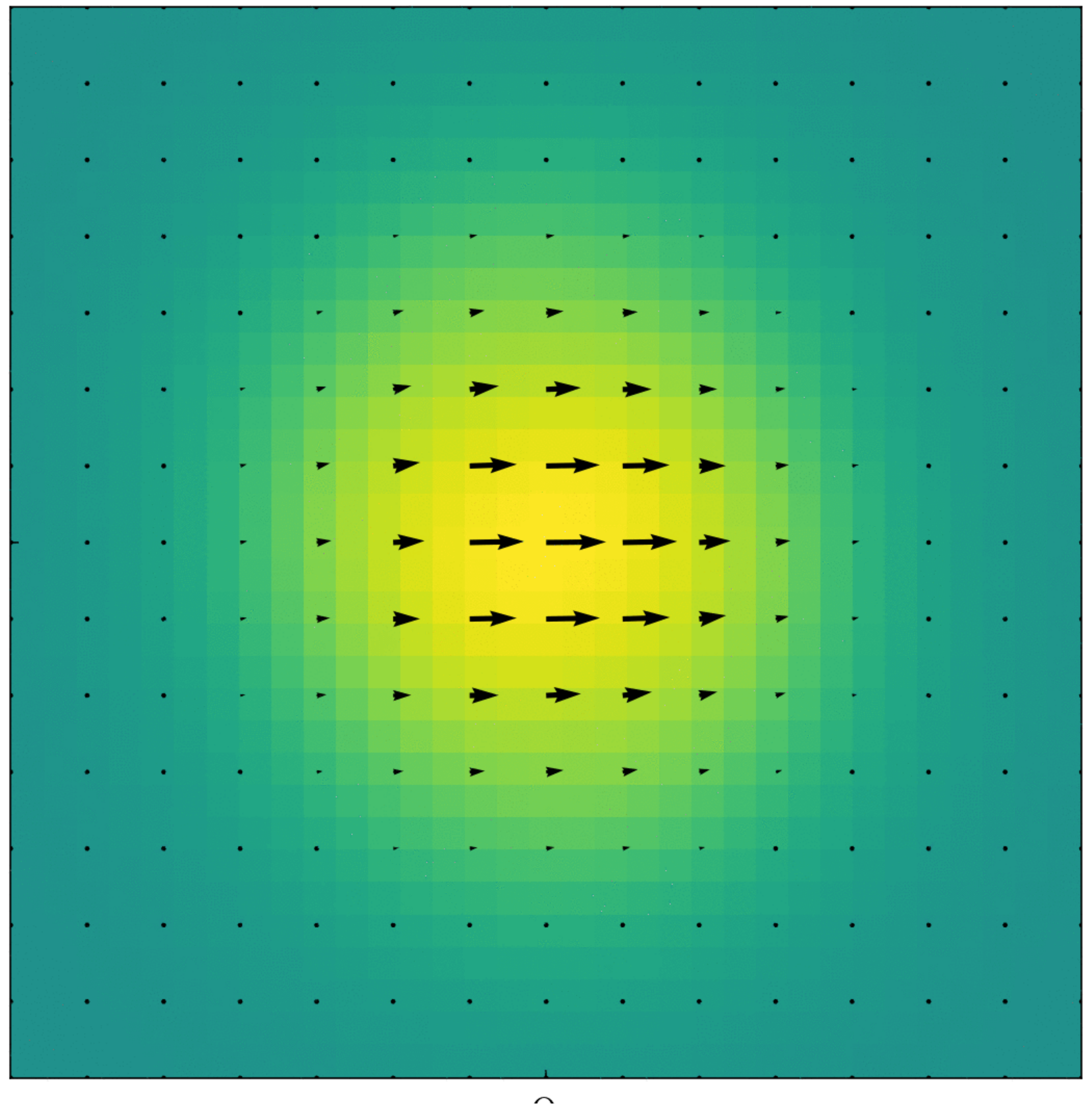
$$0 \le |\boldsymbol{S}_{\rm tot}| \le \frac{M_{\rm sol}}{m}\hbar$$

spin with self-interactions: vector oscillons

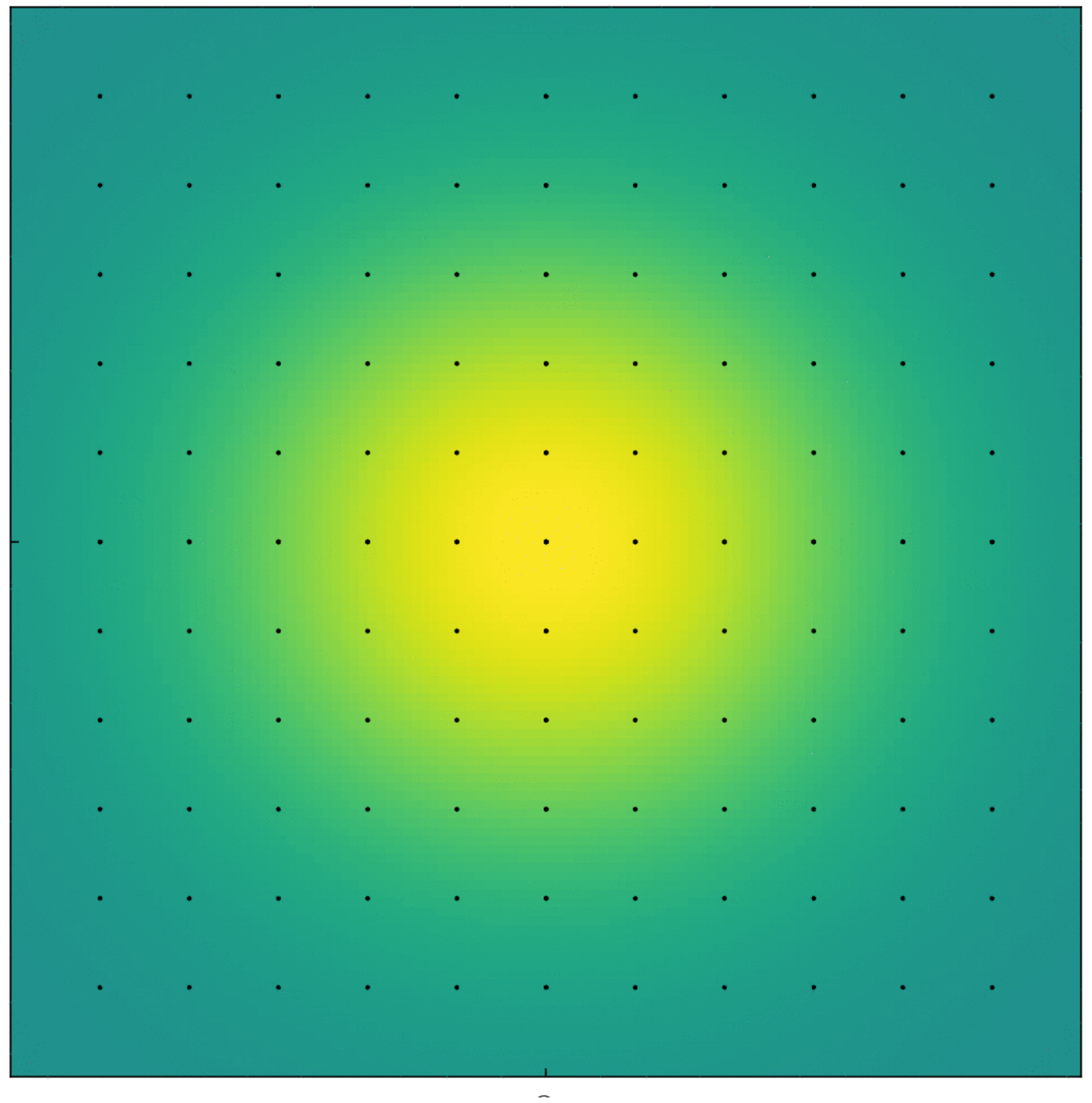
Zhang, Jain & MA (2022)

$$\mathcal{L} = -\frac{1}{4}\mathcal{G}_{\mu\nu}\mathcal{G}^{\mu\nu} - \frac{m^2}{2}W_\mu W^\mu - \frac{\lambda}{4}(W_\mu W^\mu)^2 - \frac{\gamma}{6}(W_\mu W^\mu)^3$$

$\boldsymbol{S}_{\rm tot} \neq 0$



$\boldsymbol{S}_{\rm tot} = 0$



self-interaction supported

NOT degenerate in energy

spin-spin interaction matters!

$$\mathcal{L} = \Re[i\boldsymbol{\Psi}^\dagger\dot{\boldsymbol{\Psi}}] - \frac{1}{2m}\nabla\boldsymbol{\Psi}^\dagger \cdot \nabla\boldsymbol{\Psi} - V_{\rm nl}(\boldsymbol{\Psi}^\dagger, \boldsymbol{\Psi})$$

$$V_{\rm nl}(\boldsymbol{\Psi}^\dagger, \boldsymbol{\Psi}) = \frac{3\lambda}{8m^2}(\boldsymbol{\Psi}^\dagger\boldsymbol{\Psi})^2 + \frac{5\gamma}{12m^3}(\boldsymbol{\Psi}^\dagger\boldsymbol{\Psi})^3 - \left[\frac{\lambda}{8m^2} + \frac{\gamma}{4m^3}(\boldsymbol{\Psi}^\dagger\boldsymbol{\Psi})\right](\boldsymbol{s}\cdot\boldsymbol{s}),$$

$$\boldsymbol{s} = i\hbar\boldsymbol{\Psi} \times \boldsymbol{\Psi}^\dagger$$

spin with self-interactions: vector oscillons

Zhang, Jain & MA (2022)

$$\mathcal{L} = -\frac{1}{4}\mathcal{G}_{\mu\nu}\mathcal{G}^{\mu\nu} - \frac{m^2}{2}W_\mu W^\mu - \frac{\lambda}{4}(W_\mu W^\mu)^2 - \frac{\gamma}{6}(W_\mu W^\mu)^3$$

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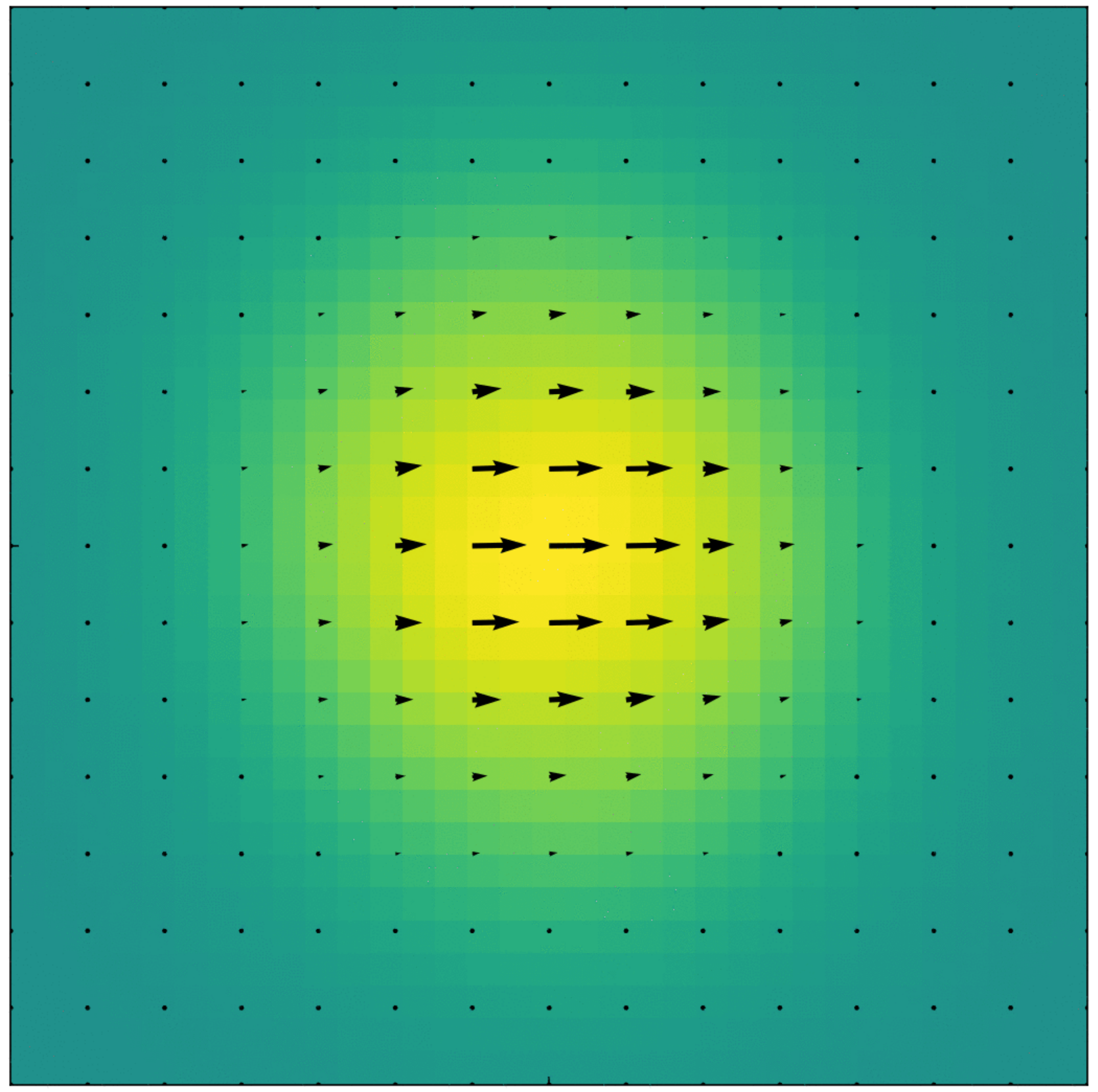
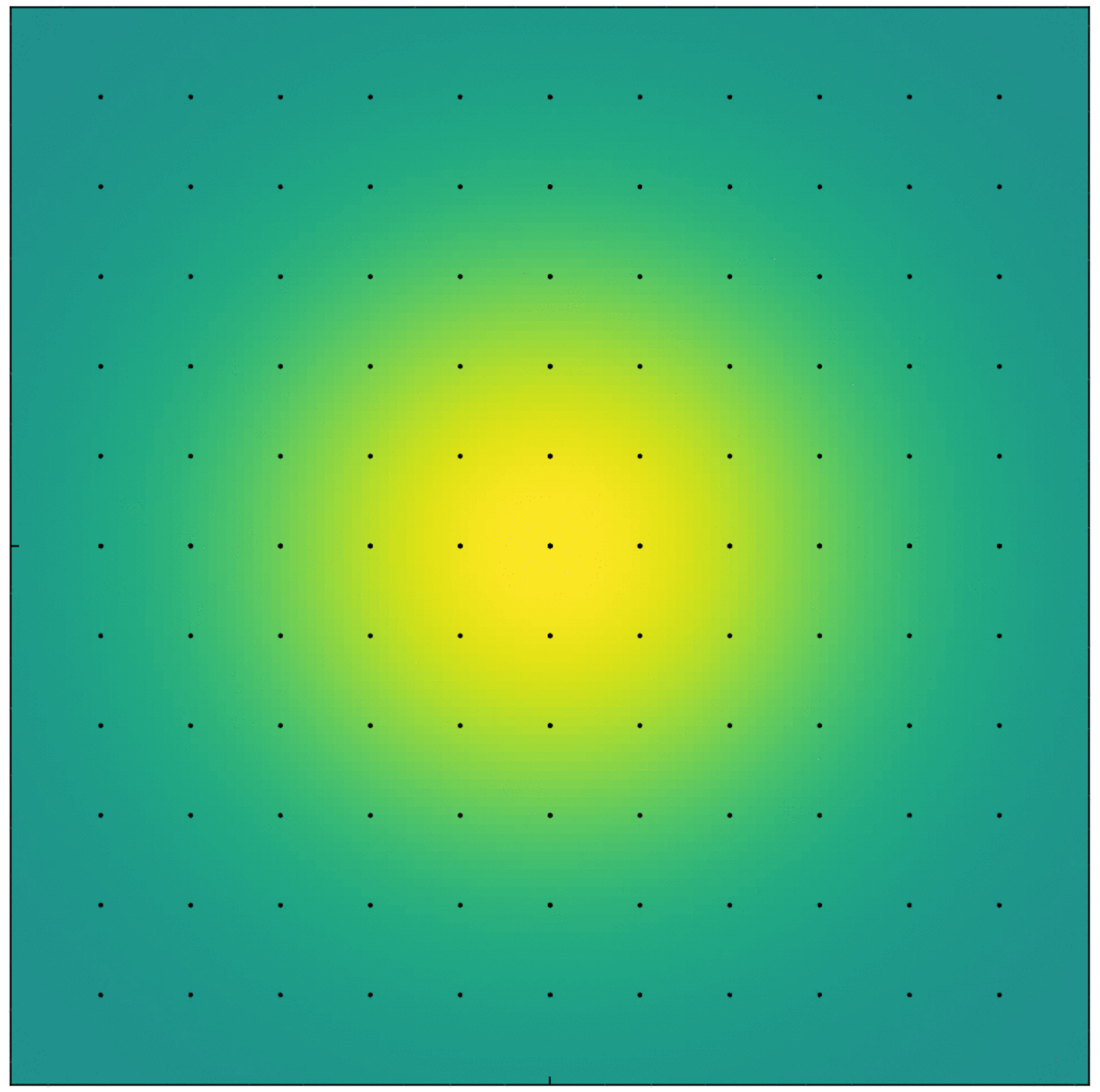


$\boldsymbol{S}_{\rm tot} = 0$



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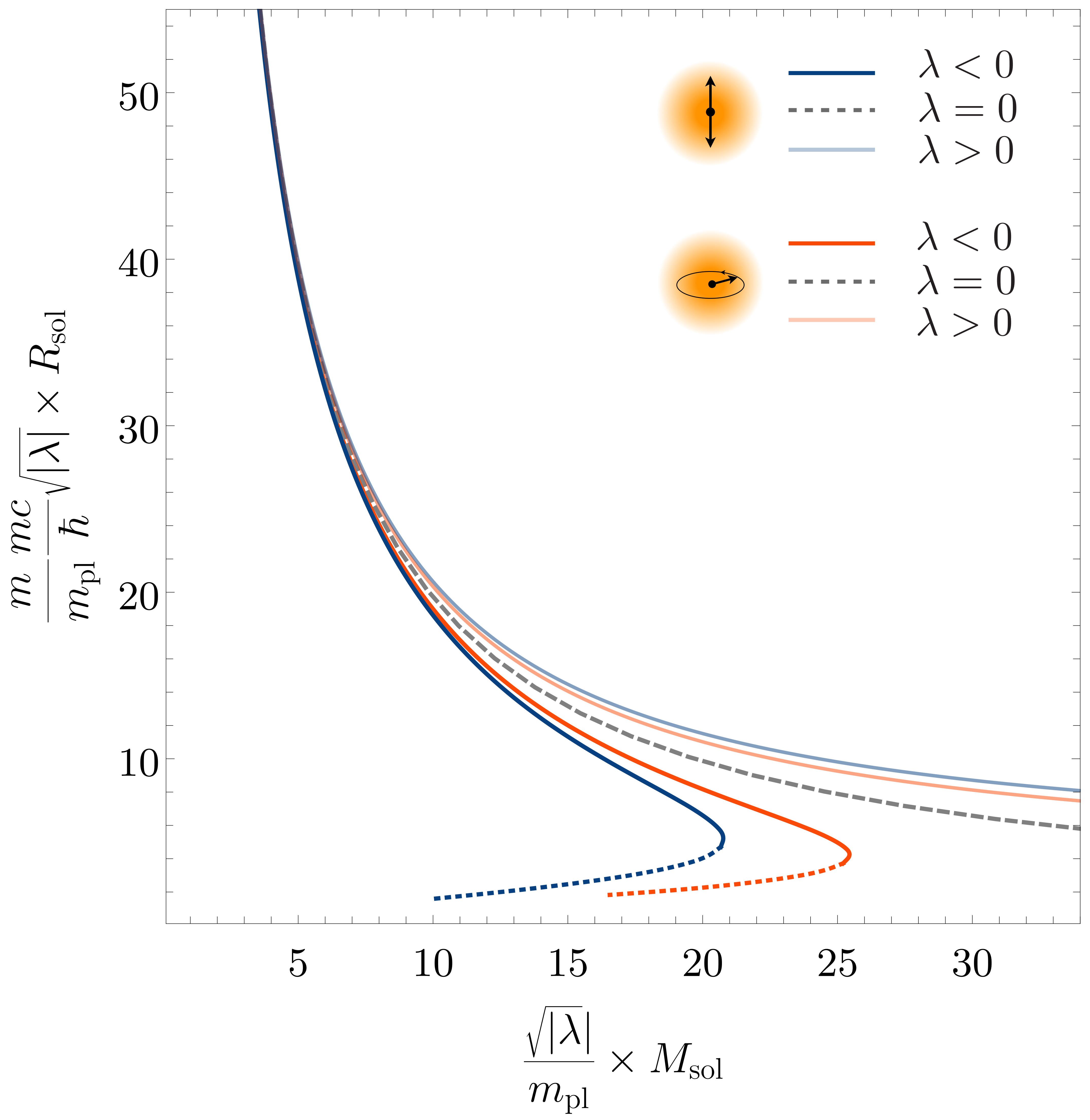
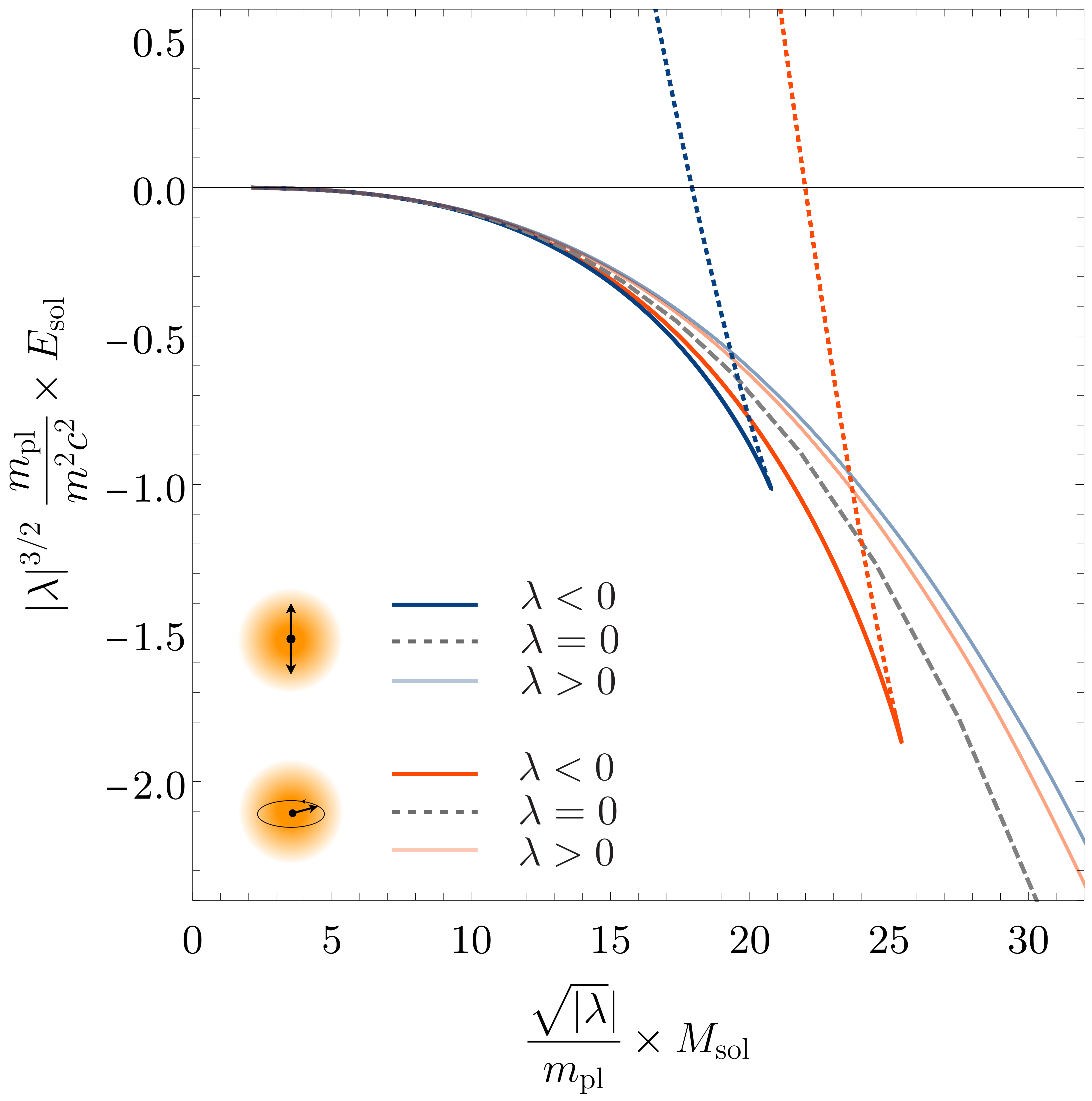
For concerns with sufficiently large amplitude fields, including superradiance, see Mou and Zhang (2022), Clough, Helfer, Witek and Berti (2022)

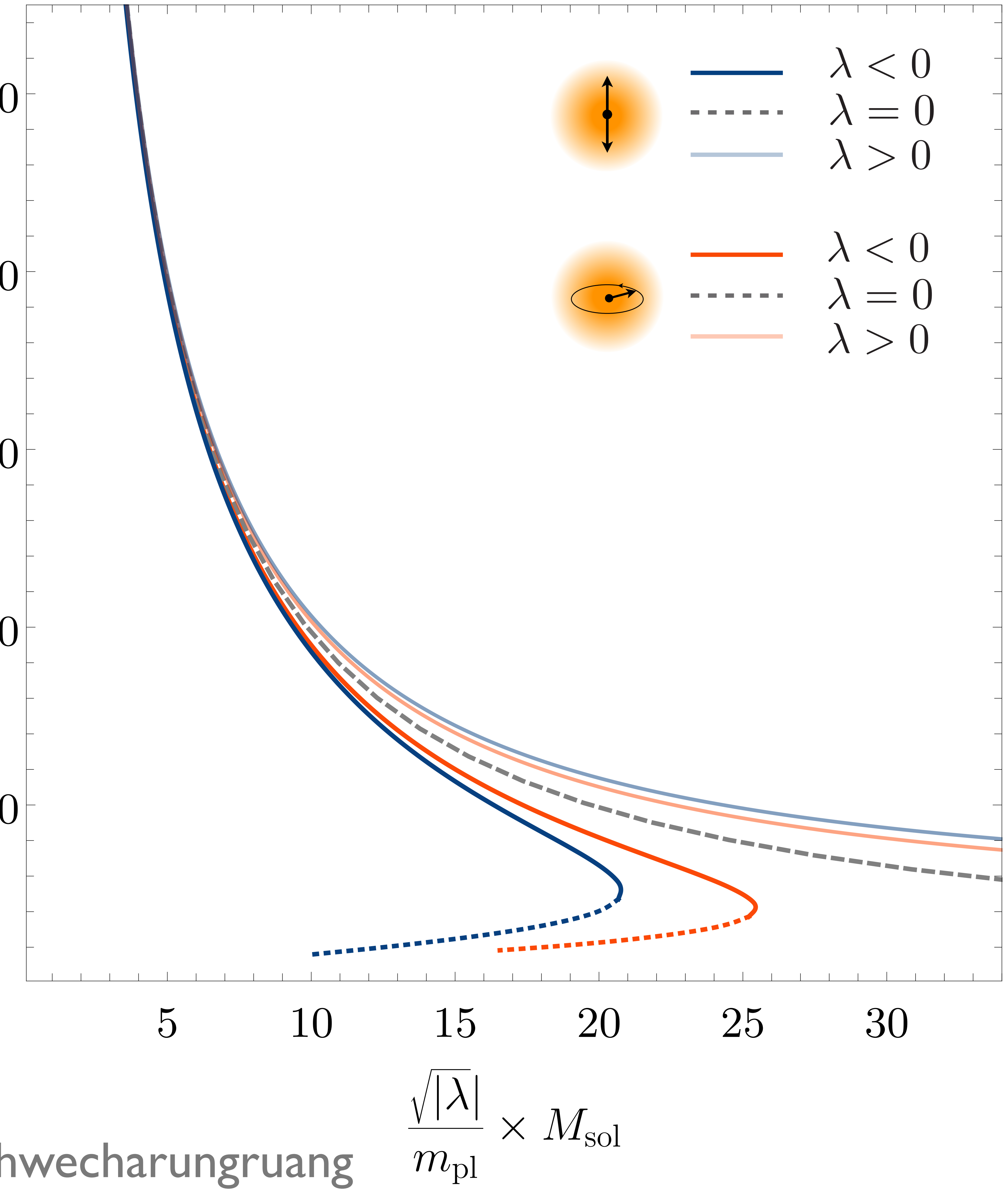
self-interaction and gravity included

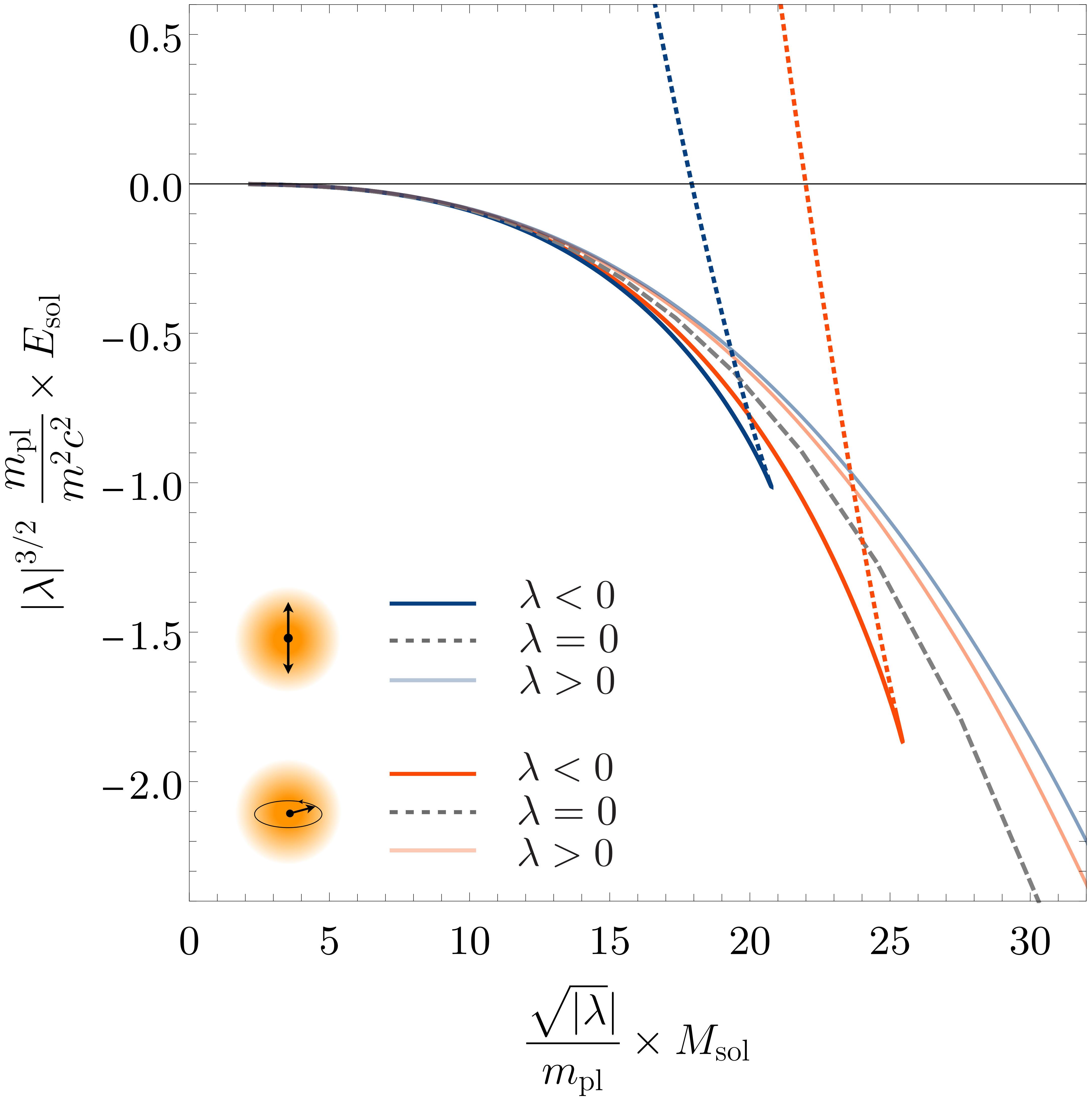




(these are non-YM cases in Jain 2022)

self-interaction and gravity included



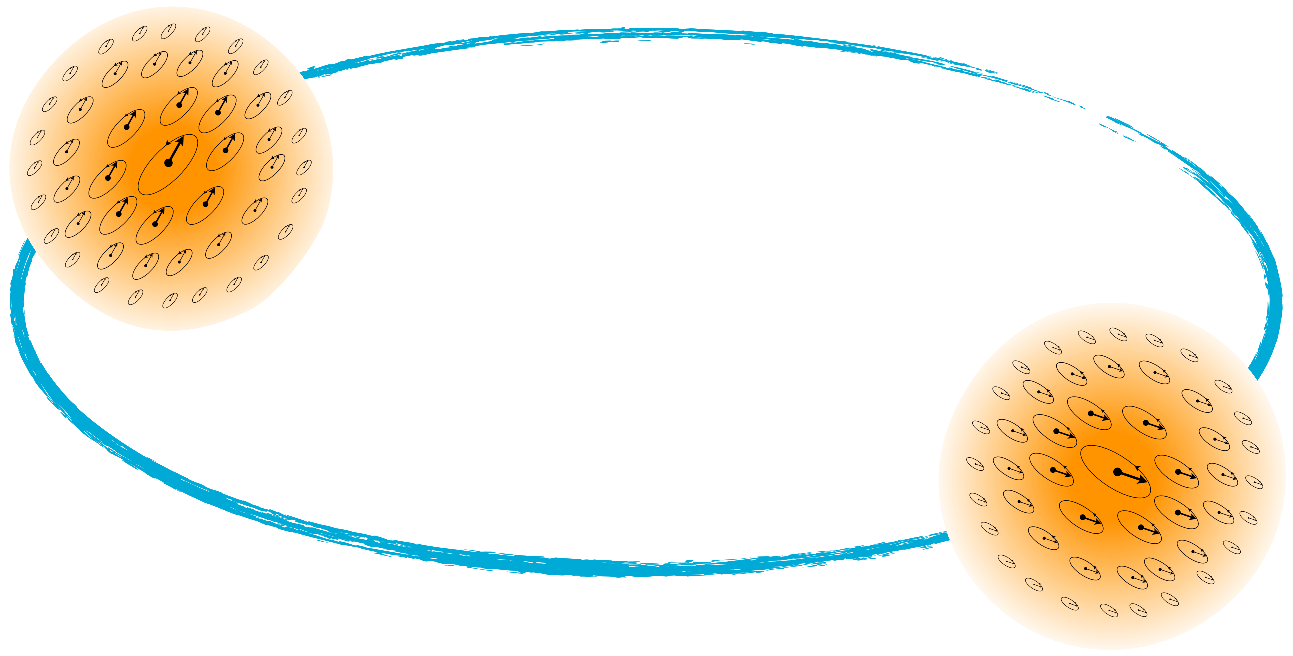


W. Wanichwecharungruang

ongoing efforts to simulate collisions with non-grav. interactions

implications of non-zero spin solitons?

post-Newtonian effects



$$V = -\frac{GM_1M_2}{r}\Bigg[1$$

$$+\frac{1}{2c^2}\left\{3(v_1^2+v_2^2)-7(\boldsymbol{v}_1\cdot\boldsymbol{v}_2)-(\boldsymbol{v}_1\cdot\hat{\boldsymbol{r}})(\boldsymbol{v}_2\cdot\hat{\boldsymbol{r}})-\frac{G(M_1+M_2)}{r}+\frac{r}{4GM_1M_2}\sum_{p=1}^{2}(M_a v_a^4)\right\}$$

$$-\frac{2}{rc}[\hat{\boldsymbol{r}}\times(\boldsymbol{v}_1-\boldsymbol{v}_2)]\cdot\sum_{a=1}^{2}\frac{\boldsymbol{S}_a}{M_a}$$

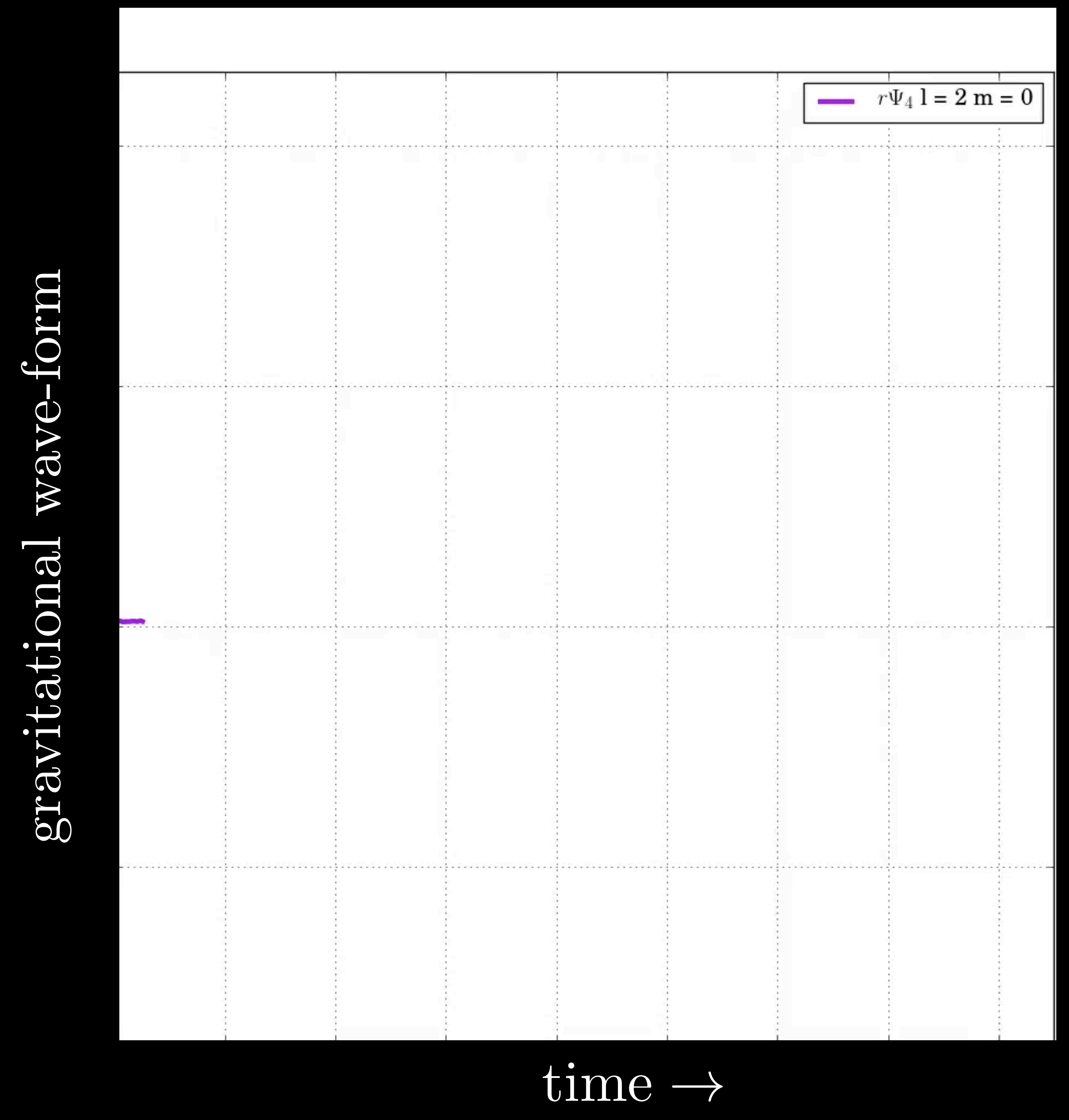
$$+\frac{1}{r^2c^2}\left\{\frac{\boldsymbol{S}_1}{M_1}\cdot\frac{\boldsymbol{S}_2}{M_2}-3\left(\frac{\boldsymbol{S}_1}{M_1}\cdot\hat{\boldsymbol{r}}\right)\left(\frac{\boldsymbol{S}_2}{M_2}\cdot\hat{\boldsymbol{r}}\right)+\sum_{a=1}^{2}\frac{C_{ES^2}^{(a)}}{2M_1M_2}\left[S_a^2-3(\boldsymbol{S}_a\cdot\hat{\boldsymbol{r}})^2\right]\right\}$$

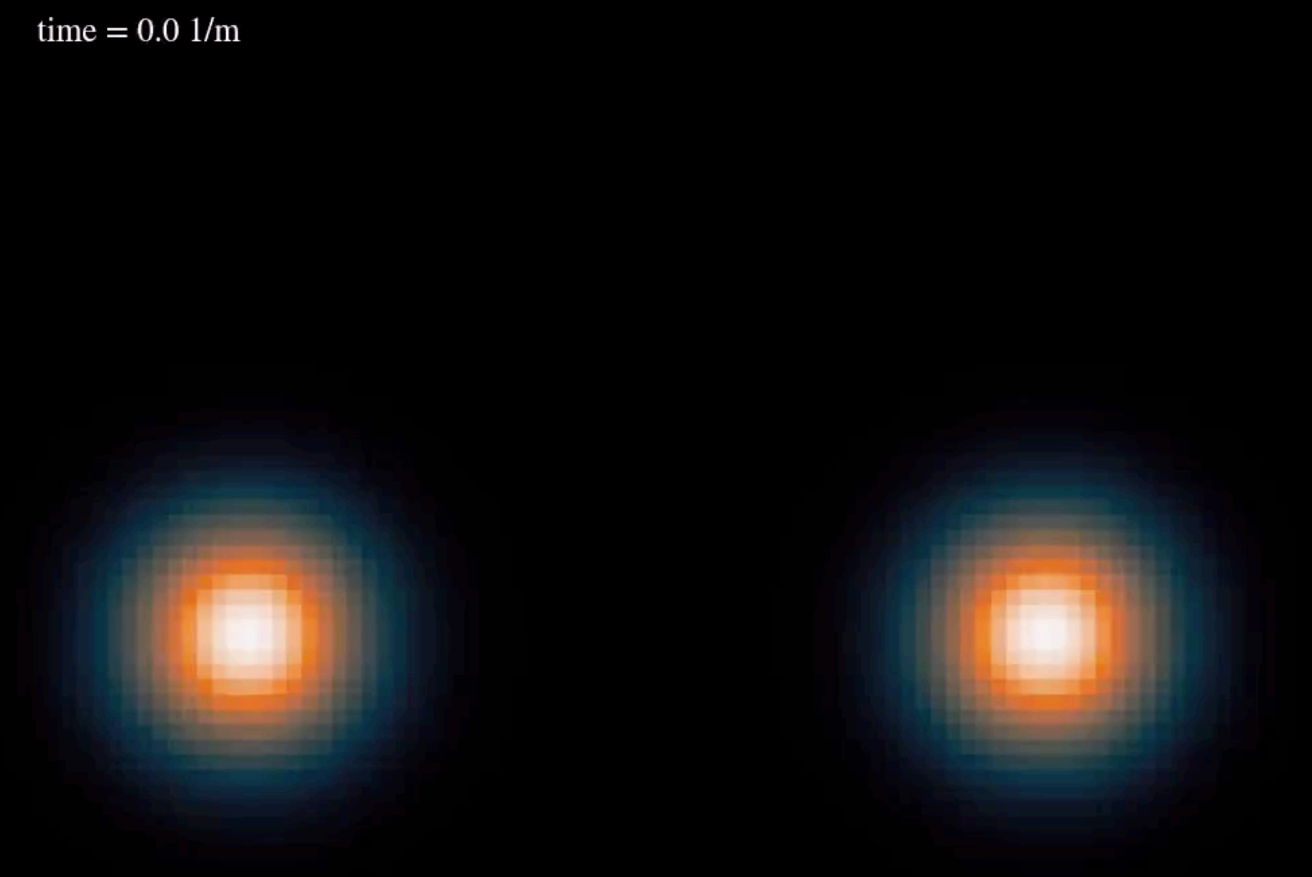
$$+\dots\Bigg]$$

gravitational waves ?

different for vector solitons?

with T. Helfer





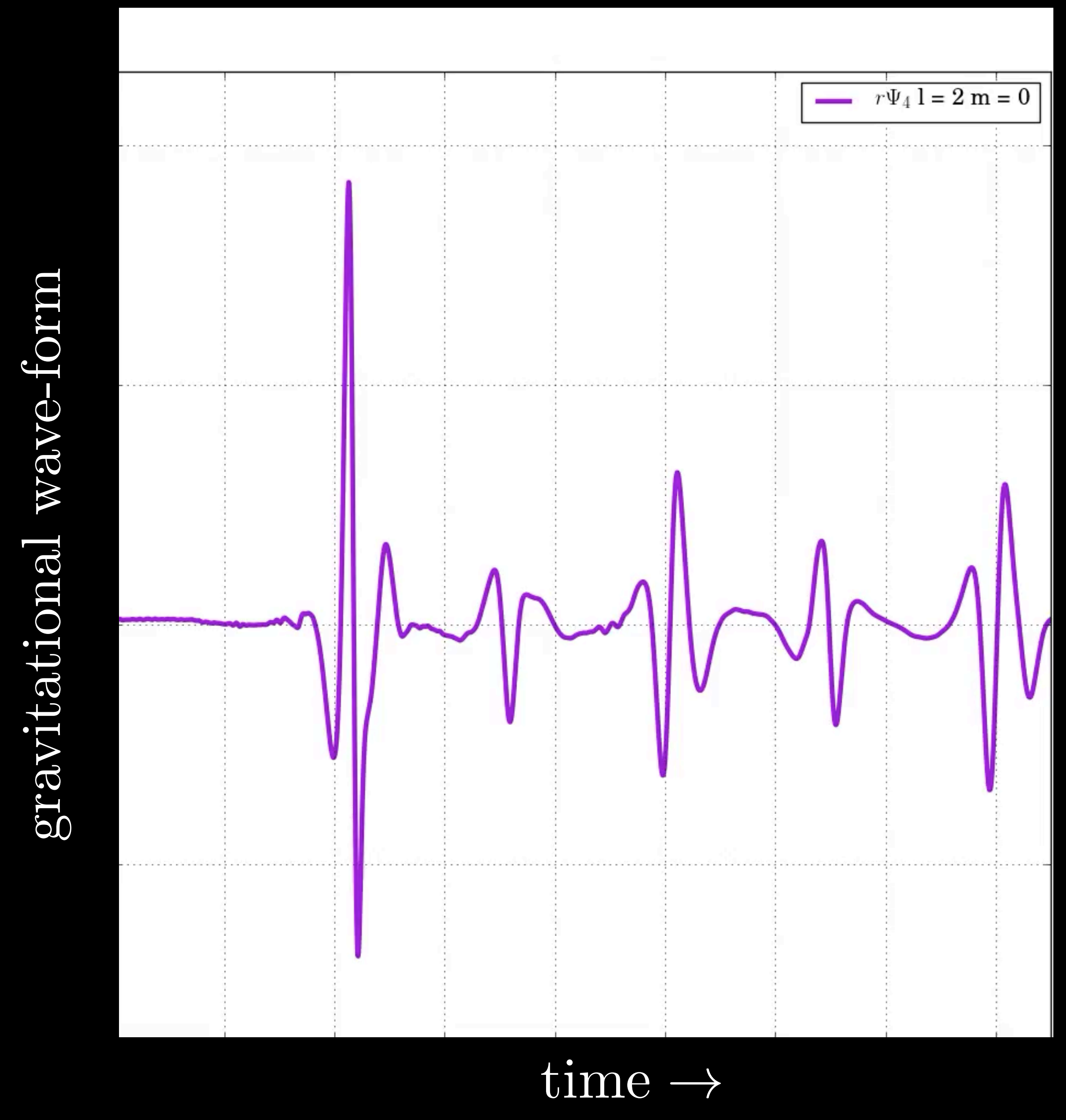
Helfer, Lim, Garcia, MA (2016)
above for compact scalar solitons using full numerical GR

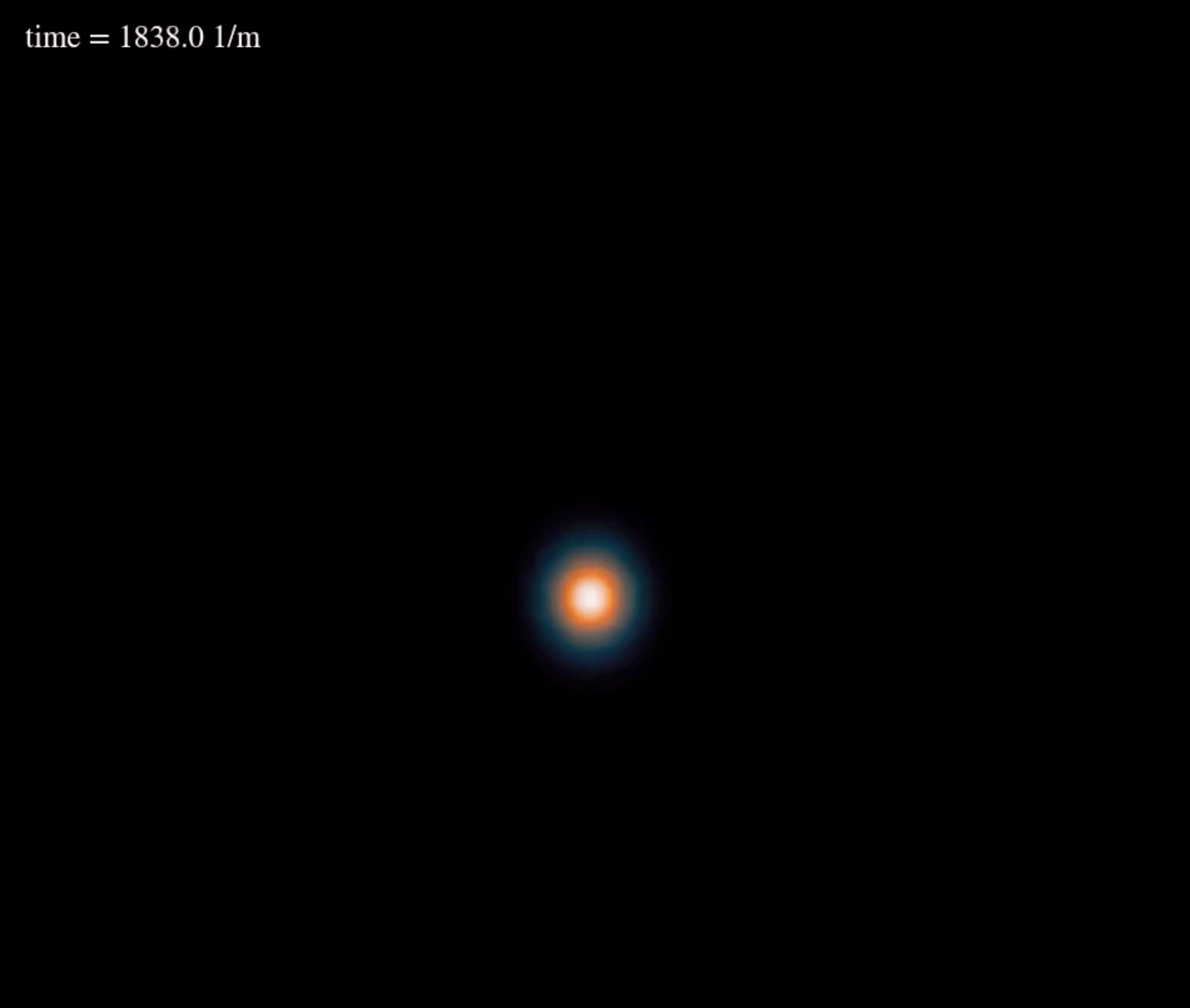
also see: Muia, Cicoli, Clough, Pedro, Quevedo (2019)

gravitational waves ?

different for vector solitons?

with T. Helfer





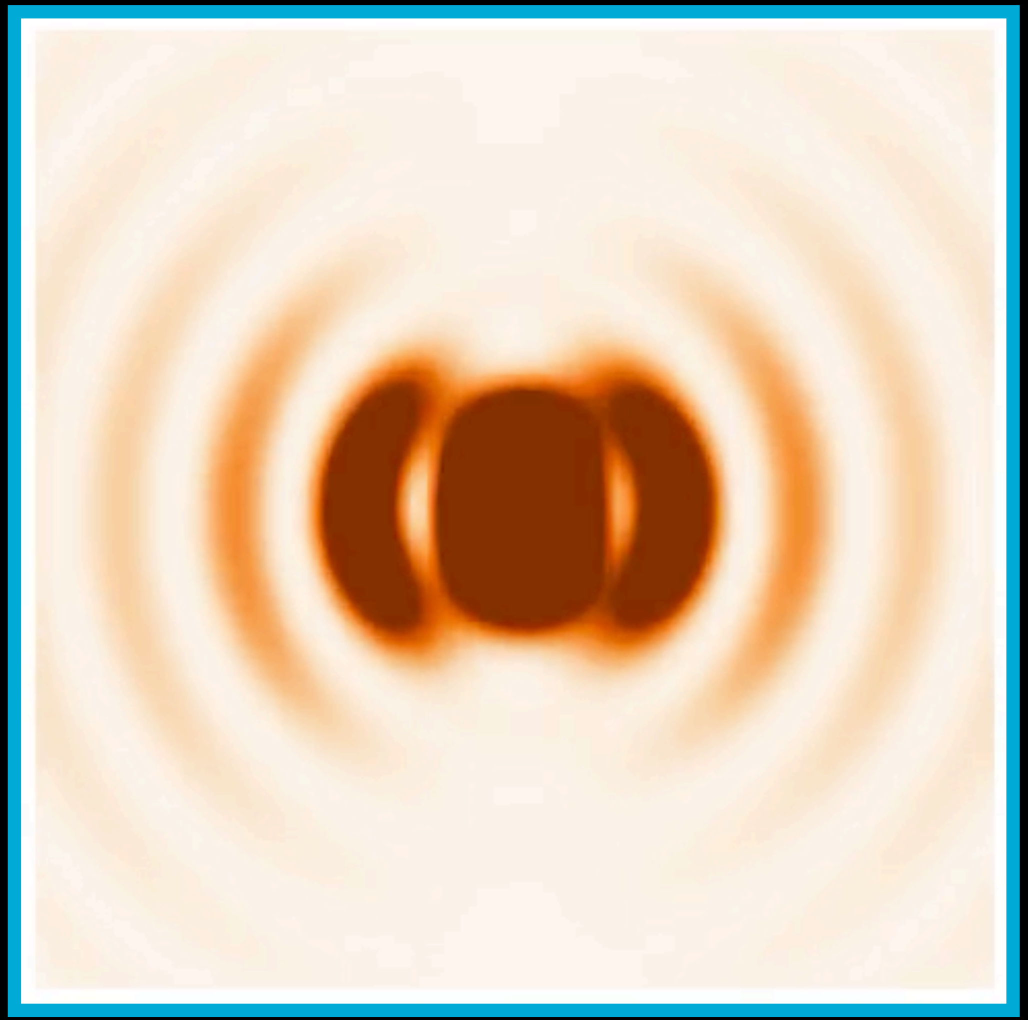
above for compact scalar solitons using full numerical GR

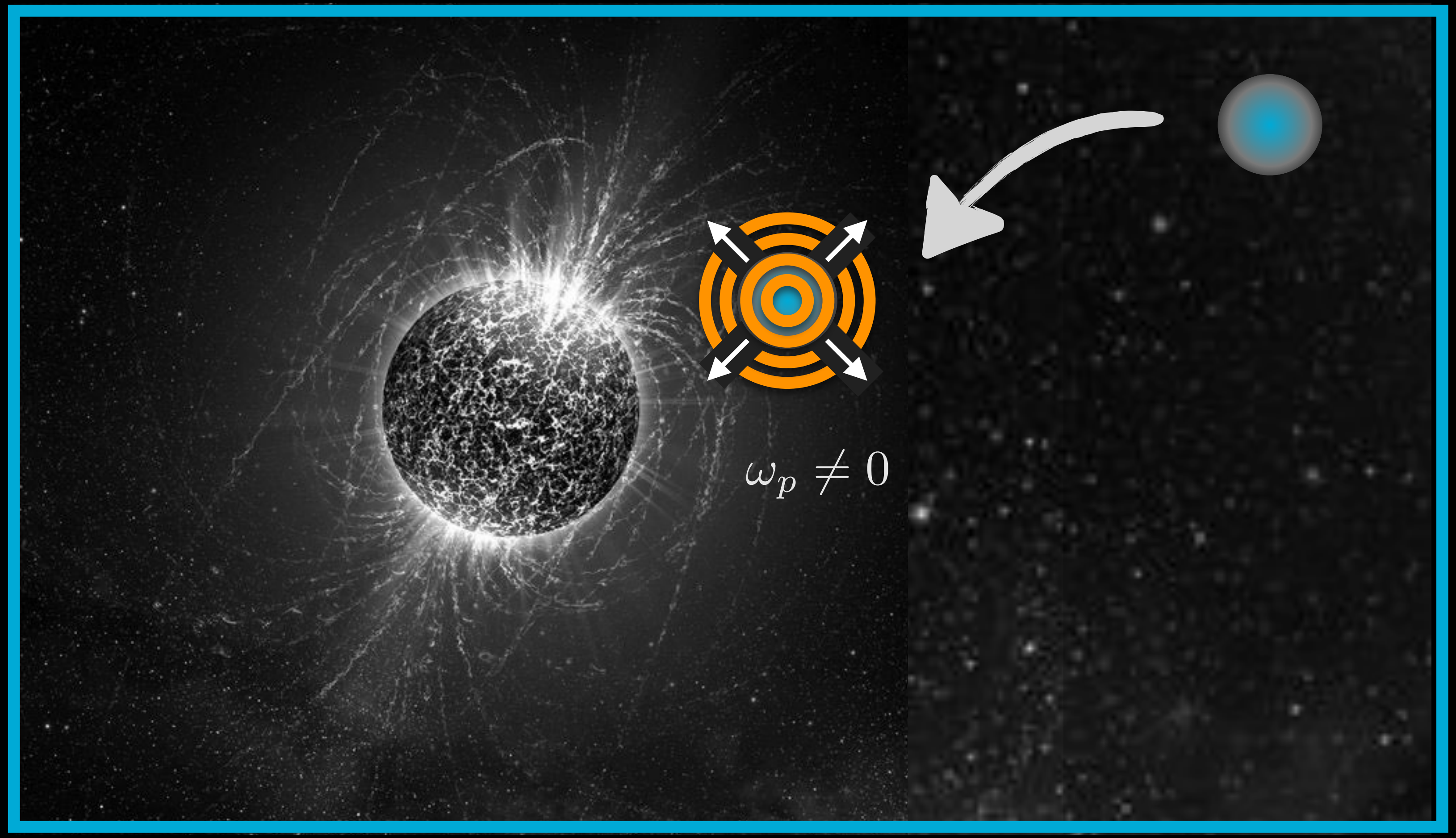
Helfer, Lim, Garcia, MA (2016)

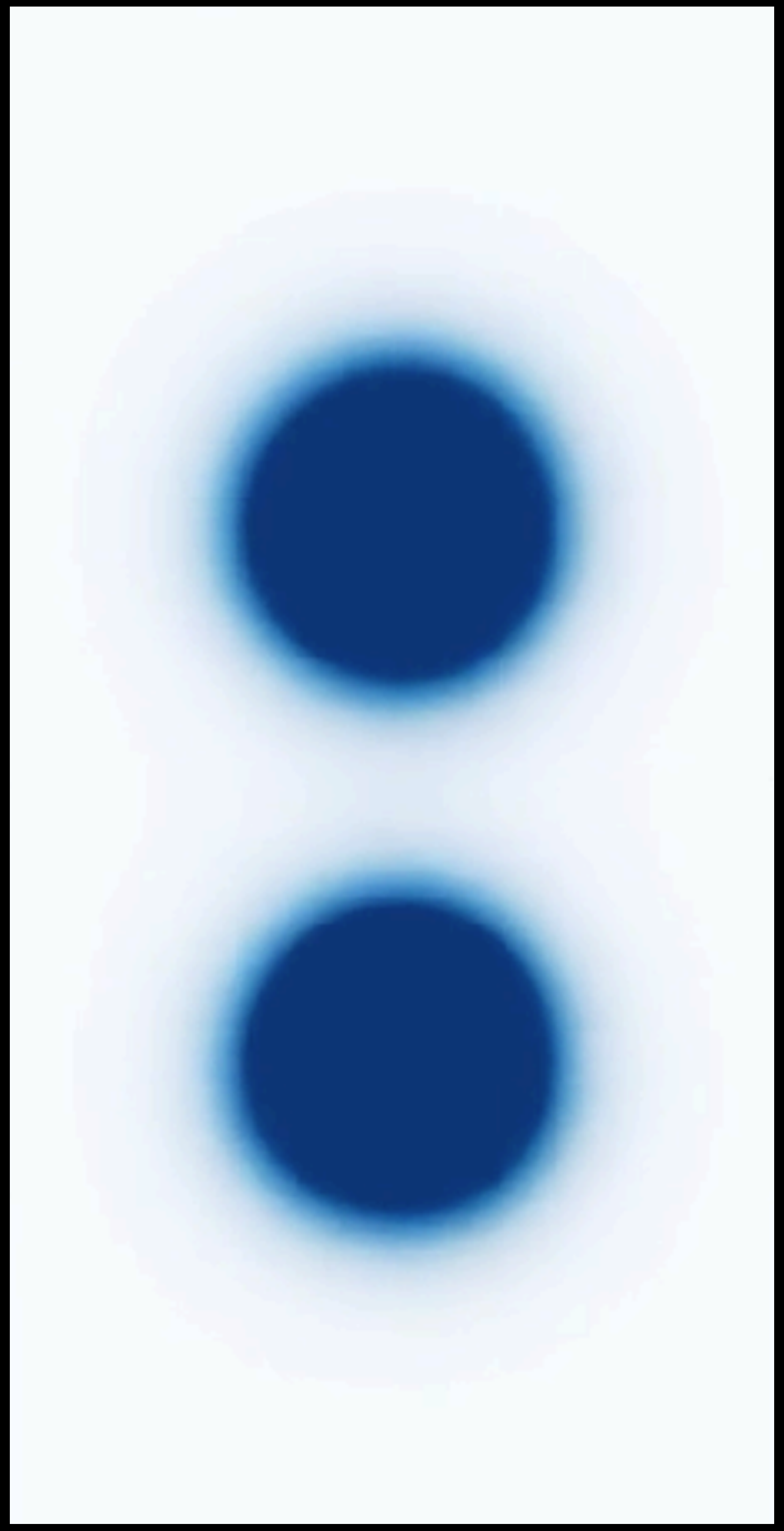
also see: Muia, Cicoli, Clough, Pedro, Quevedo (2019)

electromagnetic coupling and radiation (axion + photons)

MA, Long, Mou & Saffin (2021)







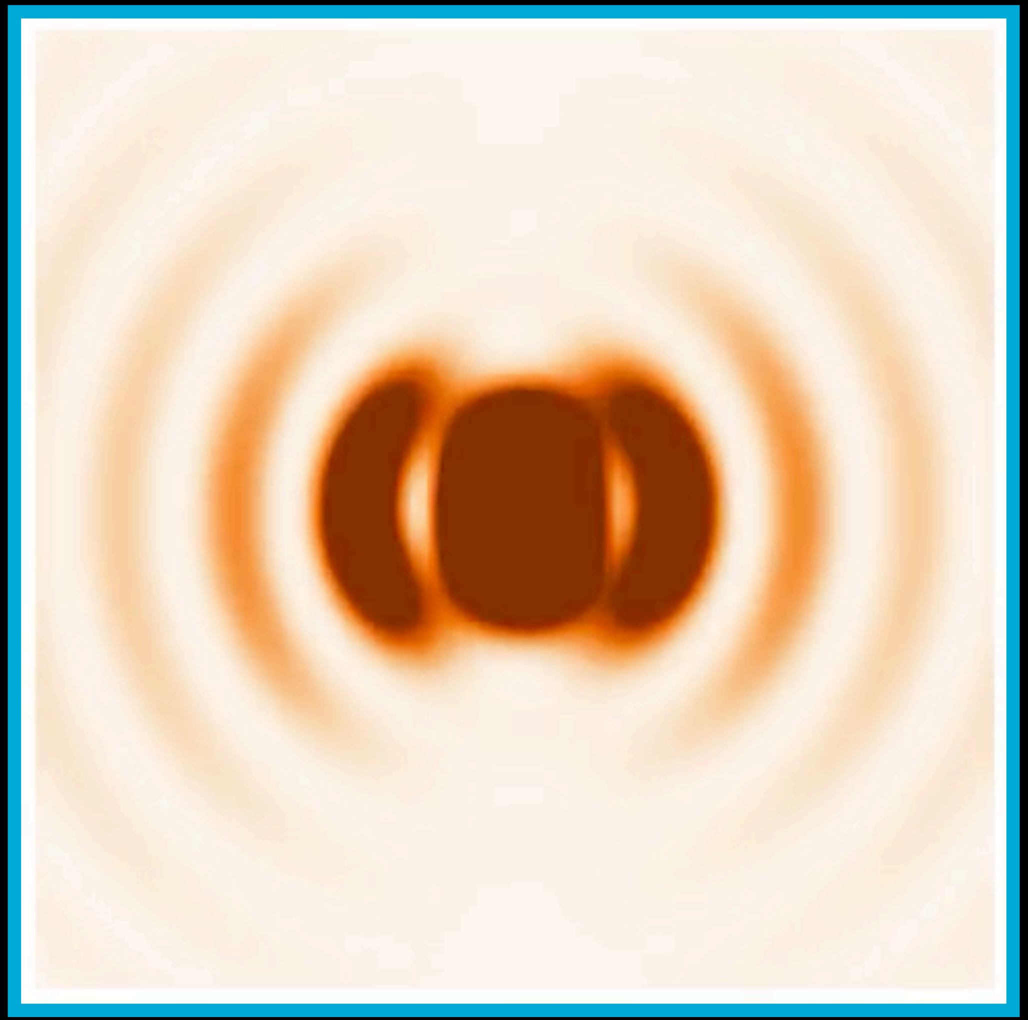
MA & Mou (2020)

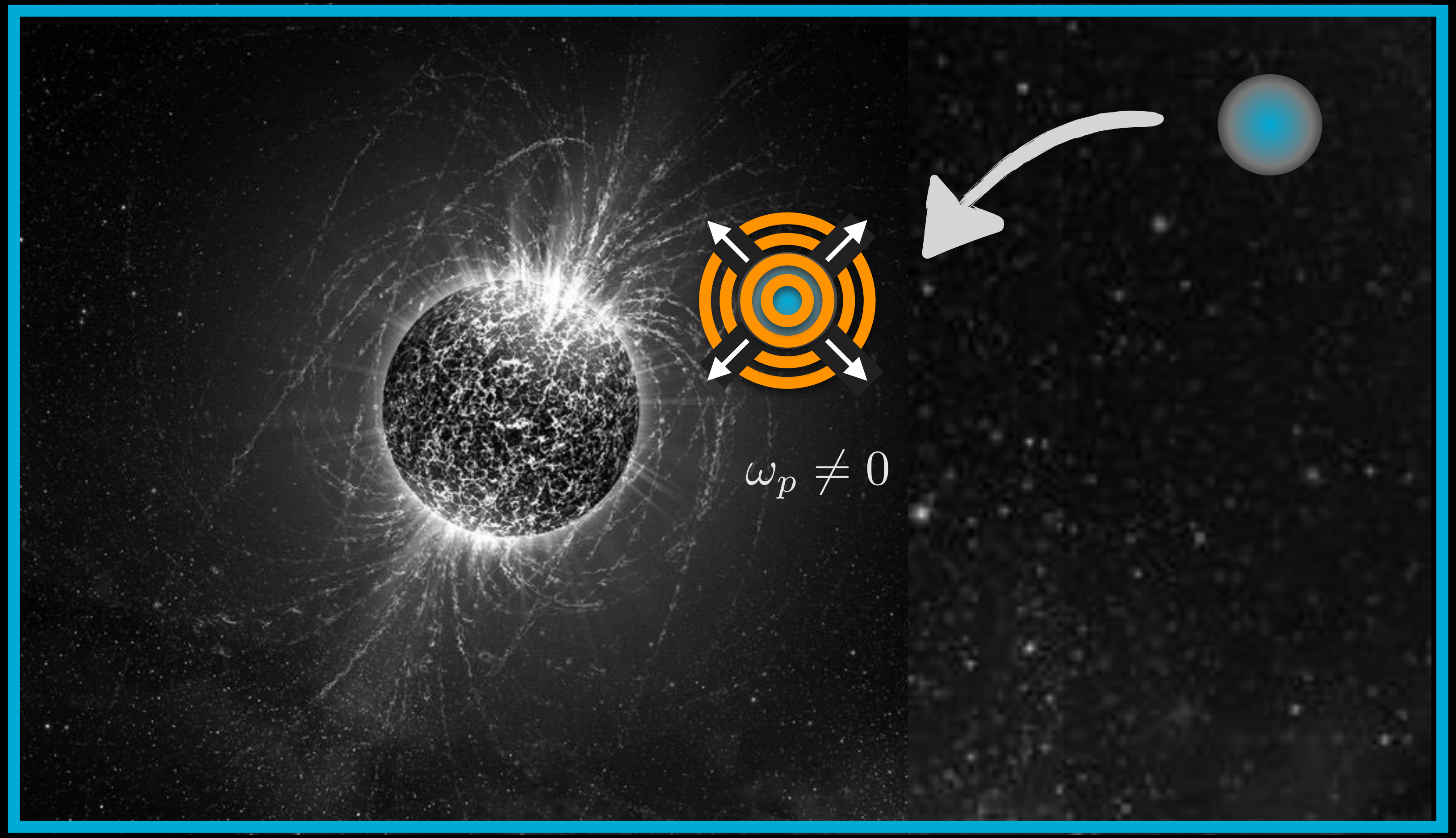
$$\mathcal{L}_{int} \sim g_{\phi\gamma}\phi F_{\mu\nu}\tilde{F}^{\mu\nu}$$

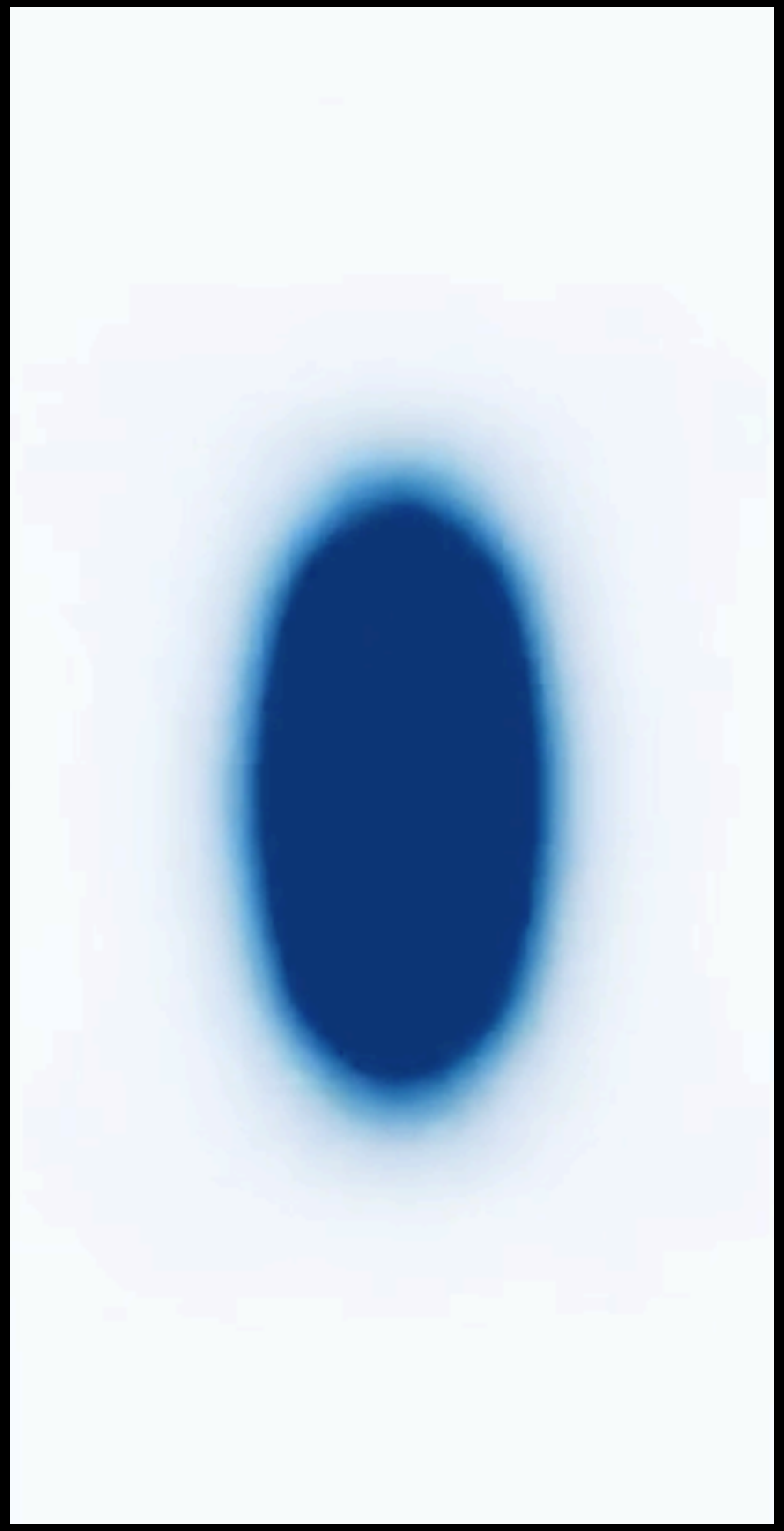
Motivated by earlier work by Hertzberg & Schippacaise (2018)

electromagnetic coupling and radiation (axion + photons)

MA, Long, Mou & Saffin (2021)

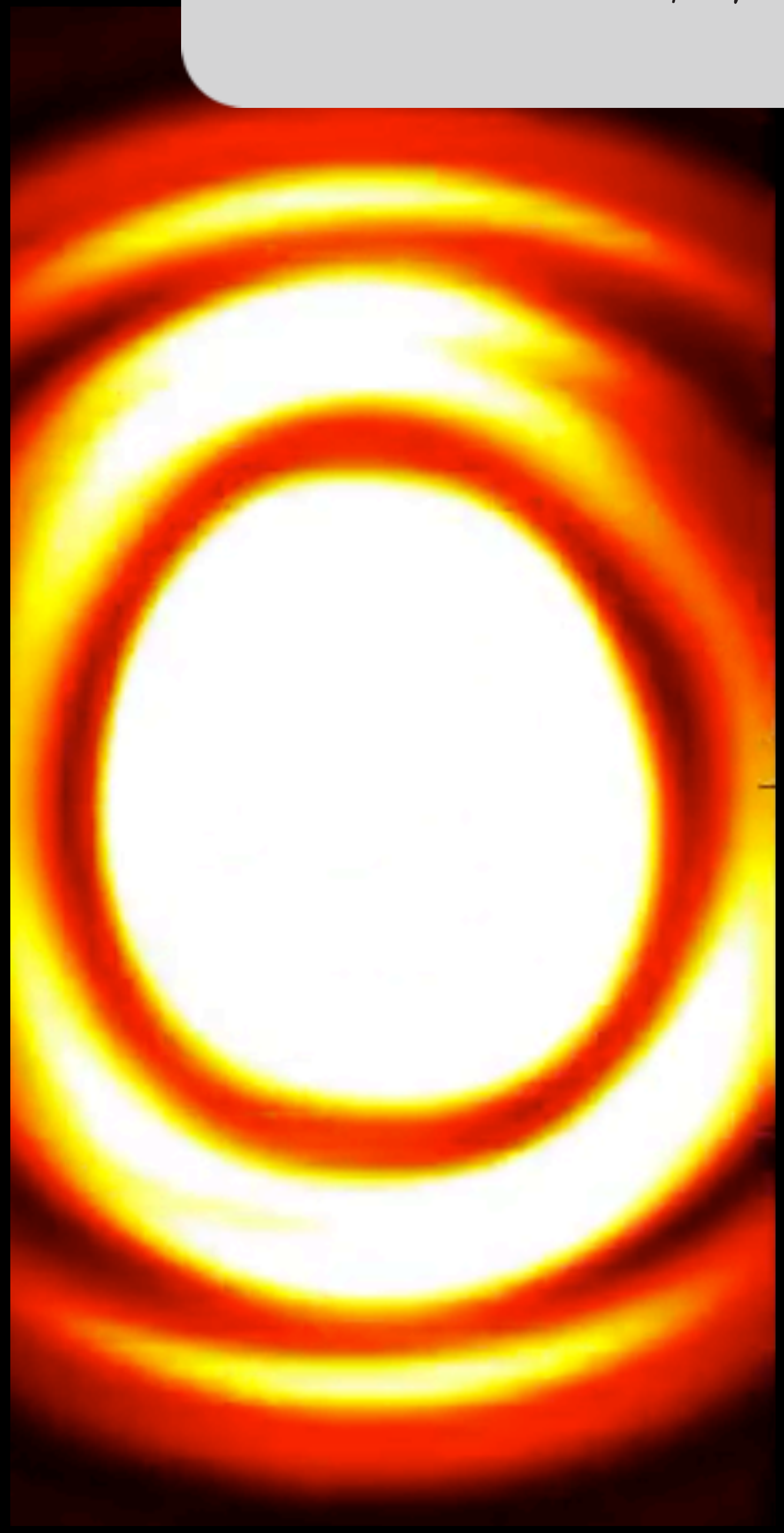






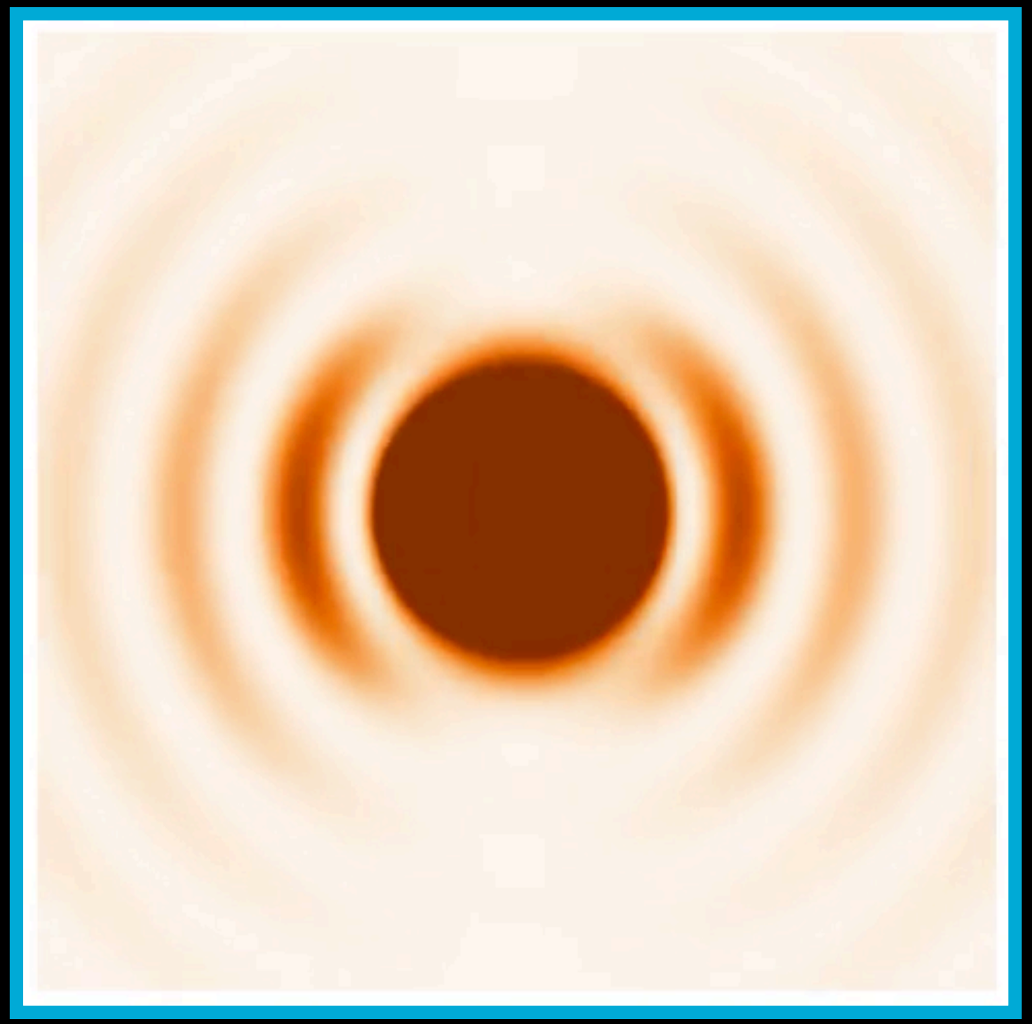
MA & Mou (2020)

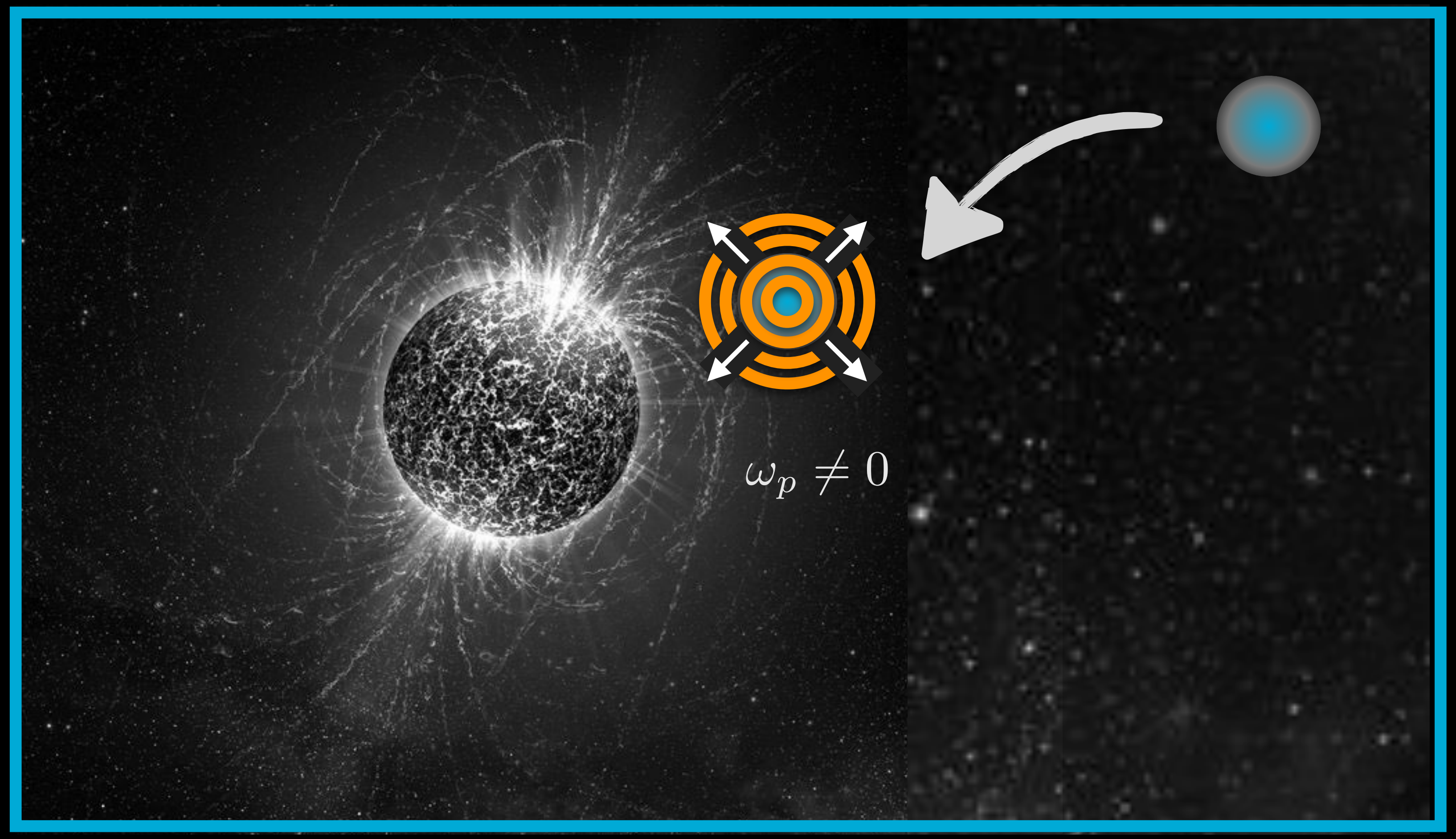
$$\mathcal{L}_{int} \sim g_{\phi\gamma}\phi F_{\mu\nu}\tilde{F}^{\mu\nu}$$



Motivated by earlier work by Hertzberg & Schippacaise (2018)

spin-s + photons: spin of soliton & polarization of photons

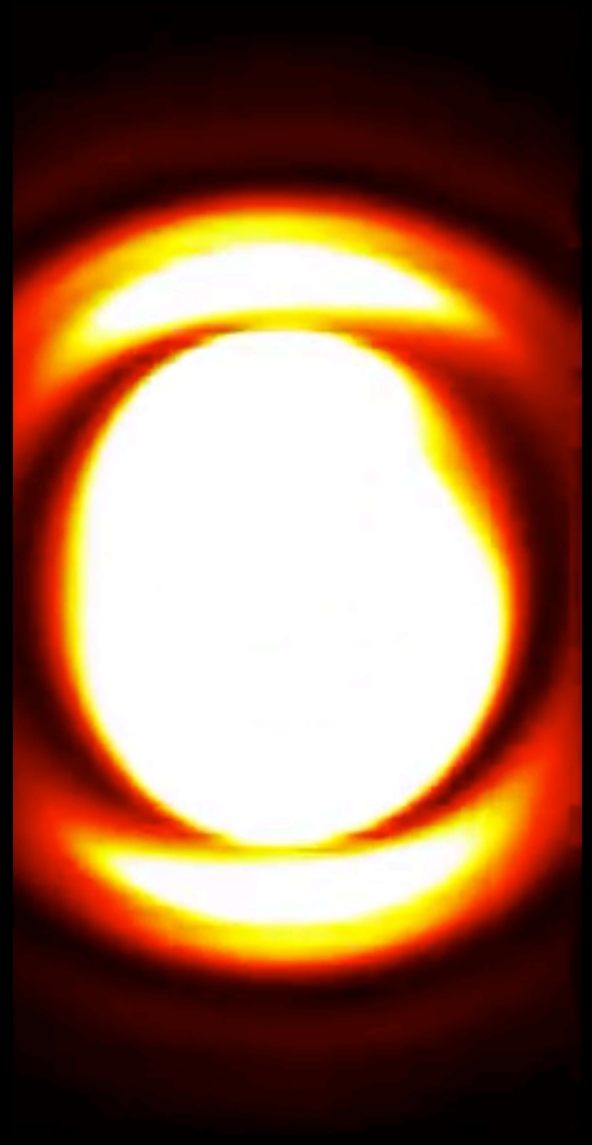




$$\mathcal{L}_{int} \sim \begin{cases} g^2_{W\gamma} W_\mu W^\mu \, F_{\alpha\beta}\tilde{F}^{\alpha\beta} & \text{spin}-1 \\ g^2_{H\gamma}(H_{\mu\nu}H^{\mu\nu} - H^2)\, F_{\alpha\beta}\tilde{F}^{\alpha\beta} & \text{spin}-2 \end{cases}$$

$$\sim g^2_{\mathcal{F}\gamma}\, \mathrm{Tr}[\boldsymbol{\mathcal{F}}\boldsymbol{\mathcal{F}}]\, F_{\alpha\beta}\tilde{F}^{\alpha\beta} \qquad \text{NR limit}$$

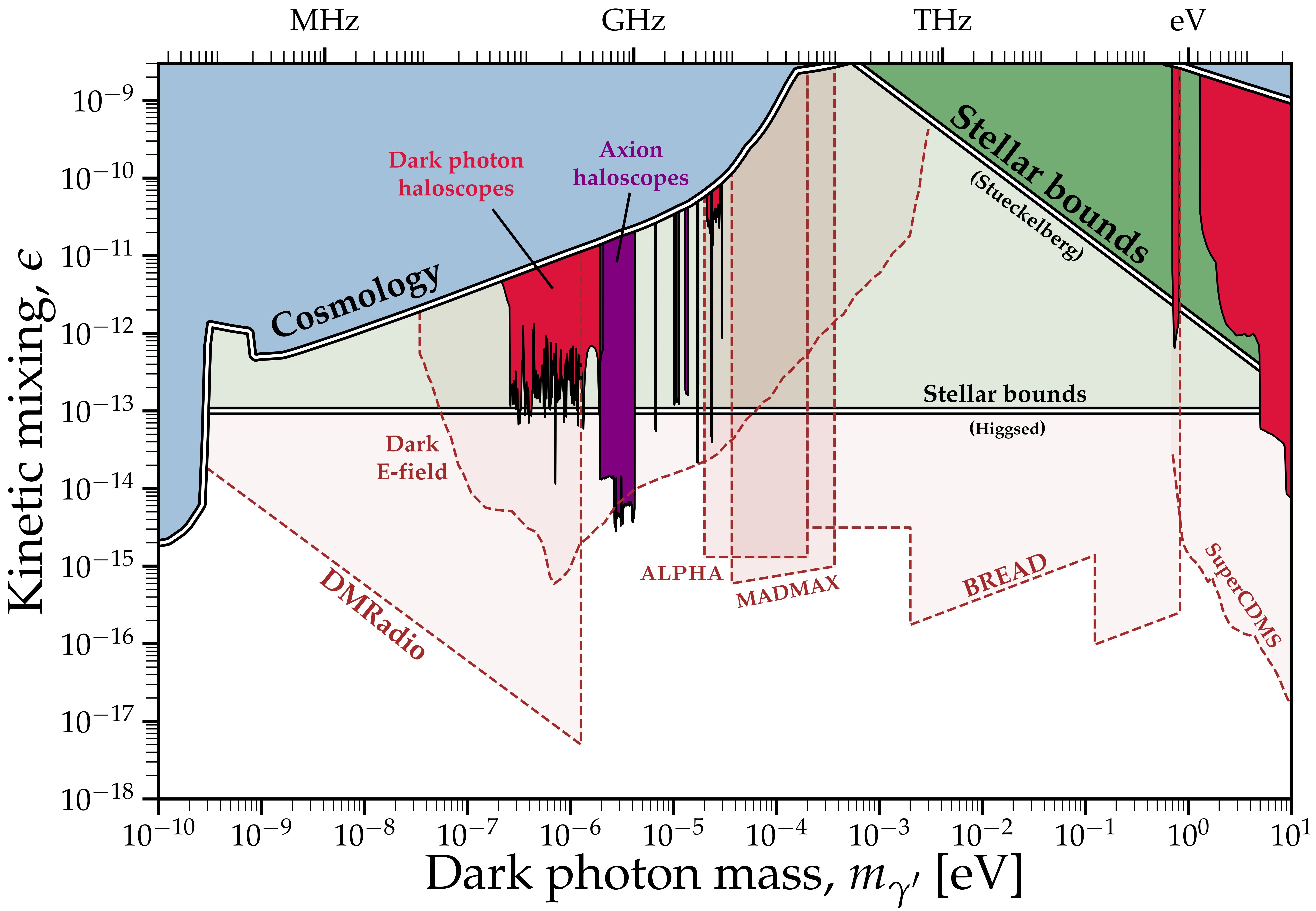
$$\epsilon F_{\mu\nu}\mathcal{G}^{\mu\nu}$$



kinetic mixing $\epsilon F_{\mu\nu} \mathcal{G}^{\mu\nu}$

https://arxiv.org/abs/2203.14915

assumed to be all of dark matter
(apart from stellar cooling)



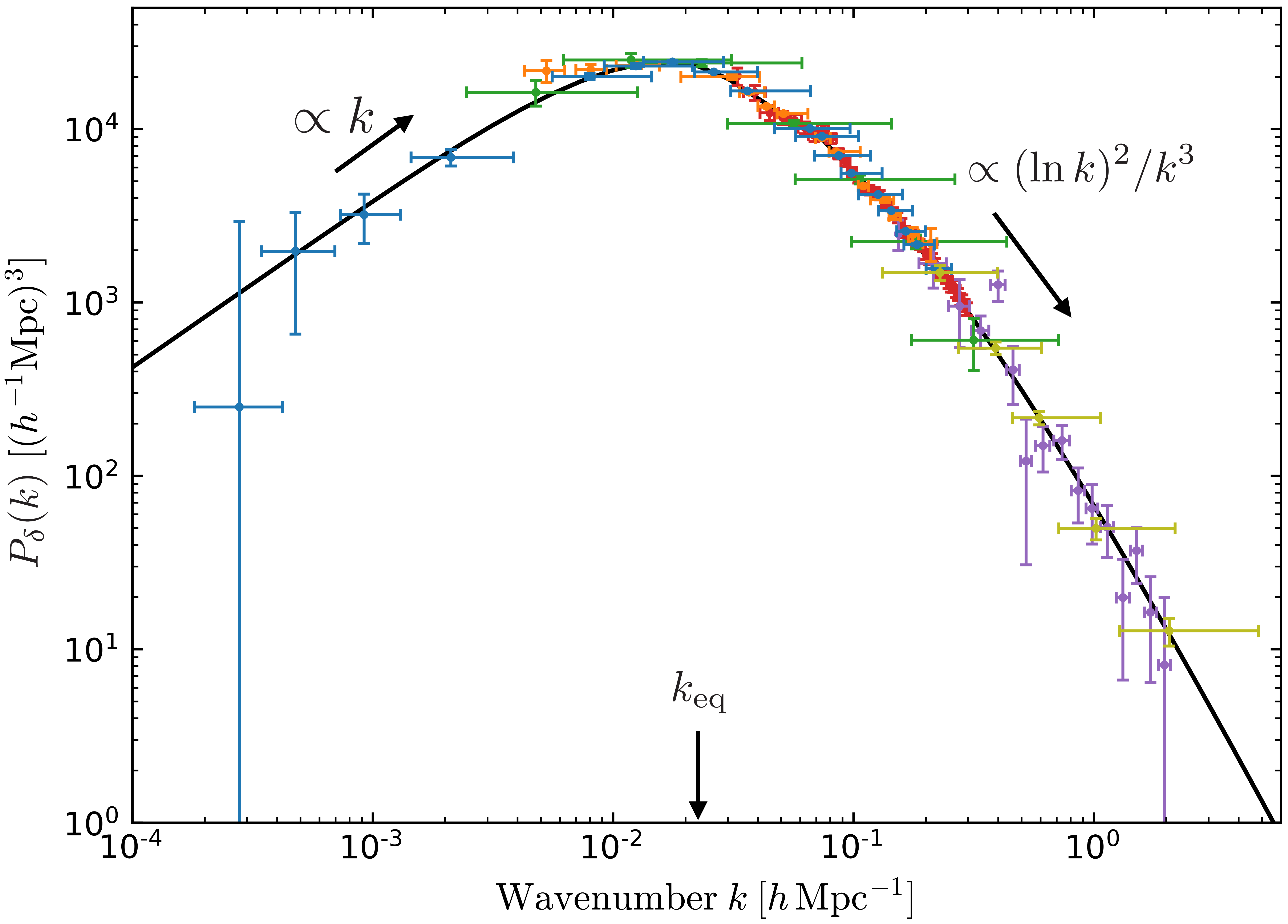
formation mechanism:

initial power spectrum — nonlinear structure

formation mechanism:

initial power spectrum — ‘usual’ CDM, SDM and then VDM

Review: linear matter power spectrum in CDM



$\sqrt{k^3 P_\delta(k)} \sim$ typical amplitude of matter density fluctuations on length scale k^{-1}

Review: linear perturbation theory

$$\delta(a, \boldsymbol{x}) \equiv \frac{\rho_{\rm dm}(a, \boldsymbol{x}) - \bar{\rho}_{\rm dm}(a)}{\bar{\rho}_{\rm dm}(a)}$$

$$\left[a^2 \partial_a^2 + \frac{3}{2} a \partial_a - \frac{3}{2}\right] \delta = 0$$

subhorizon, matter domination eq. only

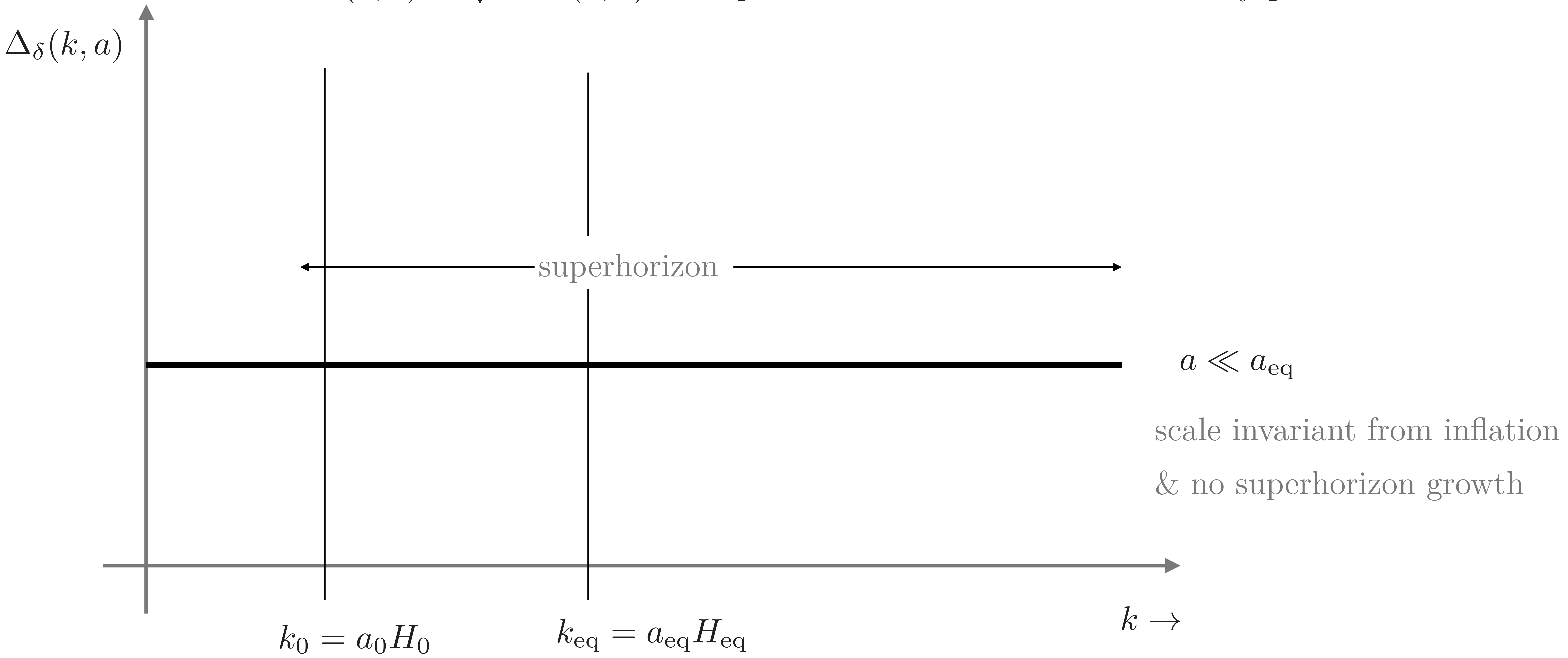
$$\implies \delta \propto a$$

$$\delta \propto a^0$$

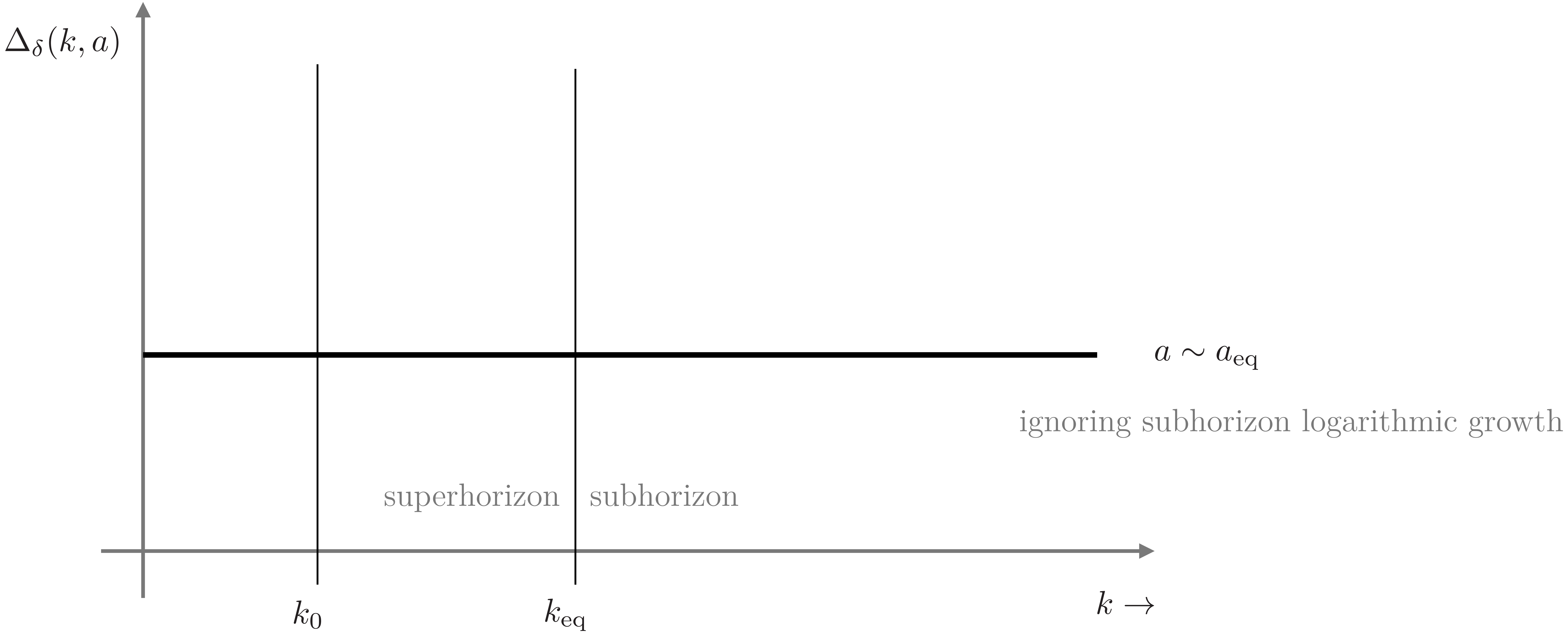
superhorizon (all eras, but gauge dep.)

initial conditions: scale invariant power spectrum

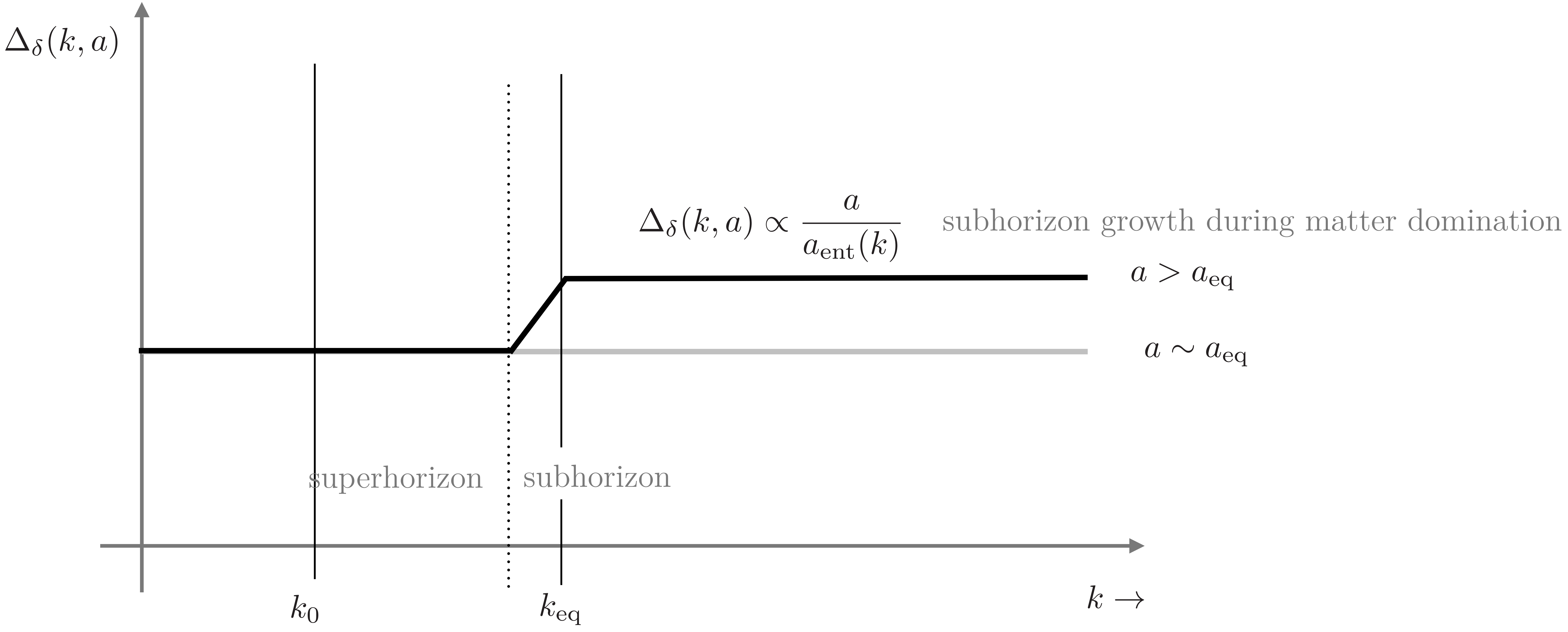
$\Delta_\delta(k,a) \sim \sqrt{k^3 P_\delta(k,a)} \sim$ amplitude of fractional matter density perturbation on scale k^{-1}



“no” growth until matter radiation equality

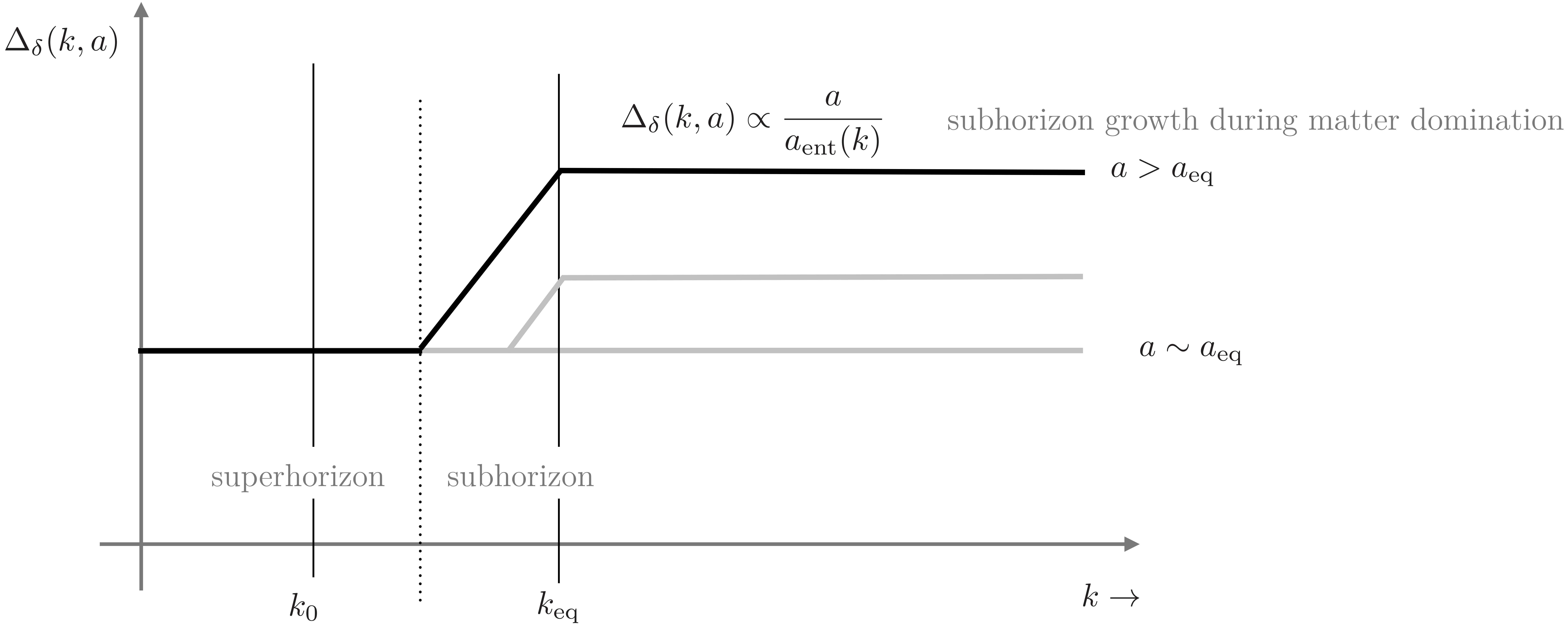


subhorizon growth during matter domination



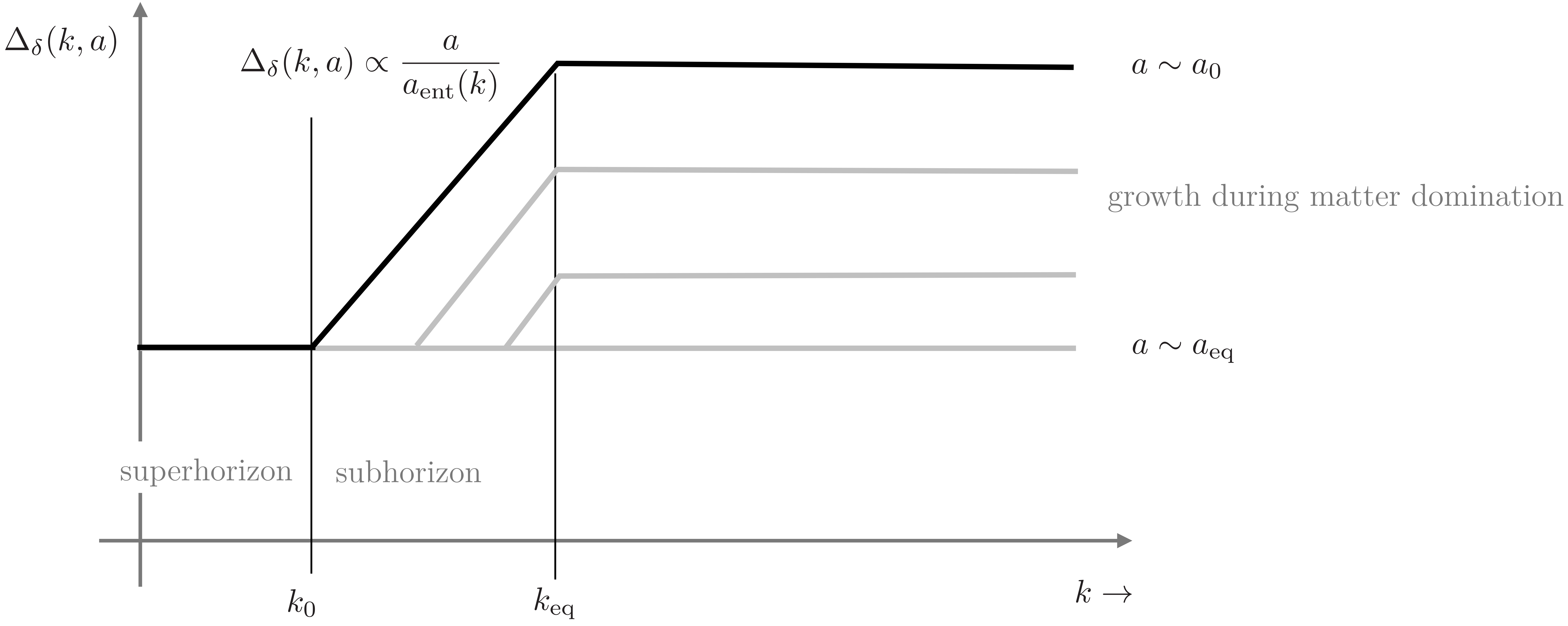
$$a_{\mathrm{ent}}(k)H(a_{\mathrm{ent}}(k)) = k$$

subhorizon growth during matter domination



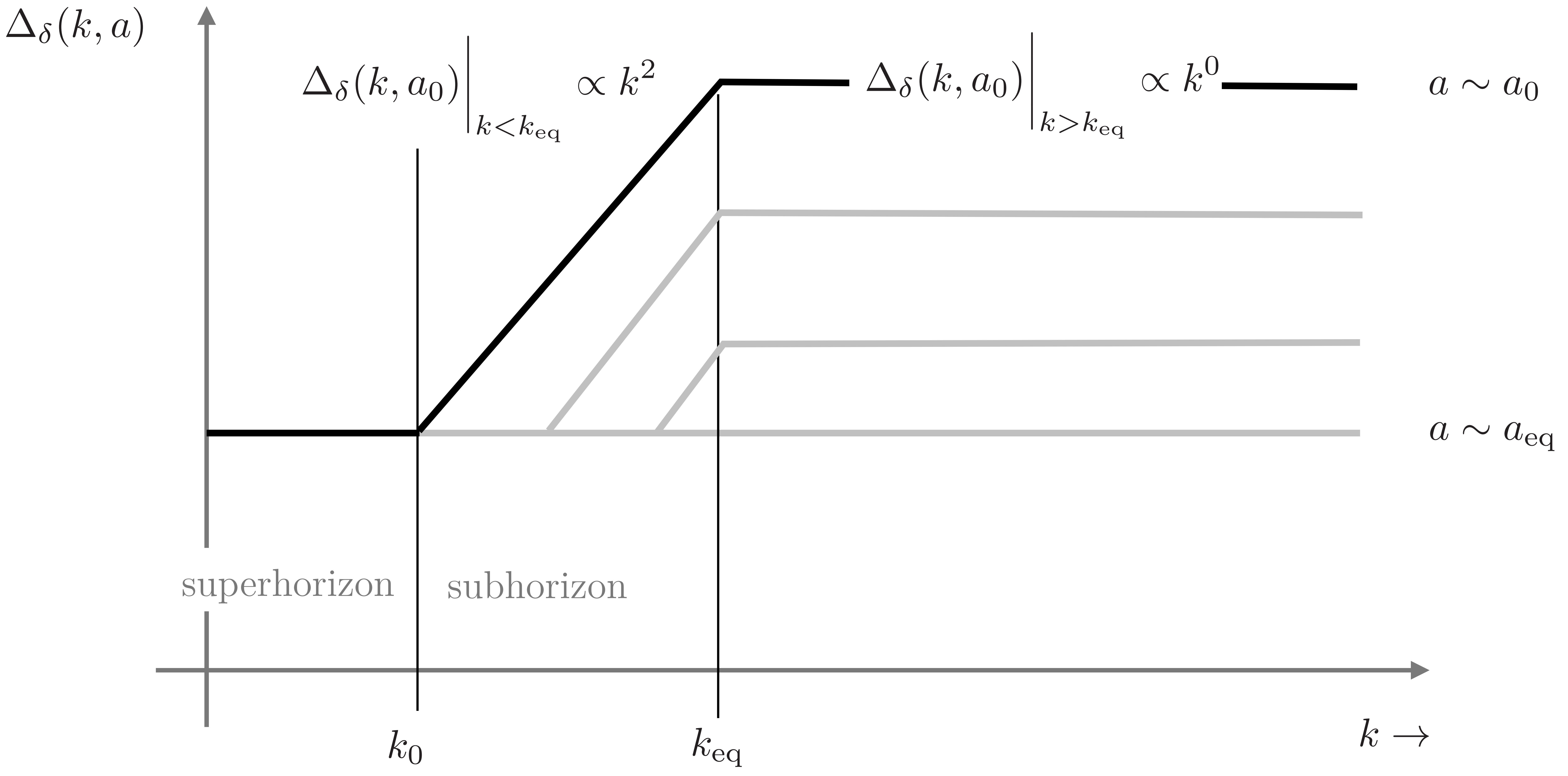
$$a_{\mathrm{ent}}(k)H(a_{\mathrm{ent}}(k)) = k$$

subhorizon growth during matter domination



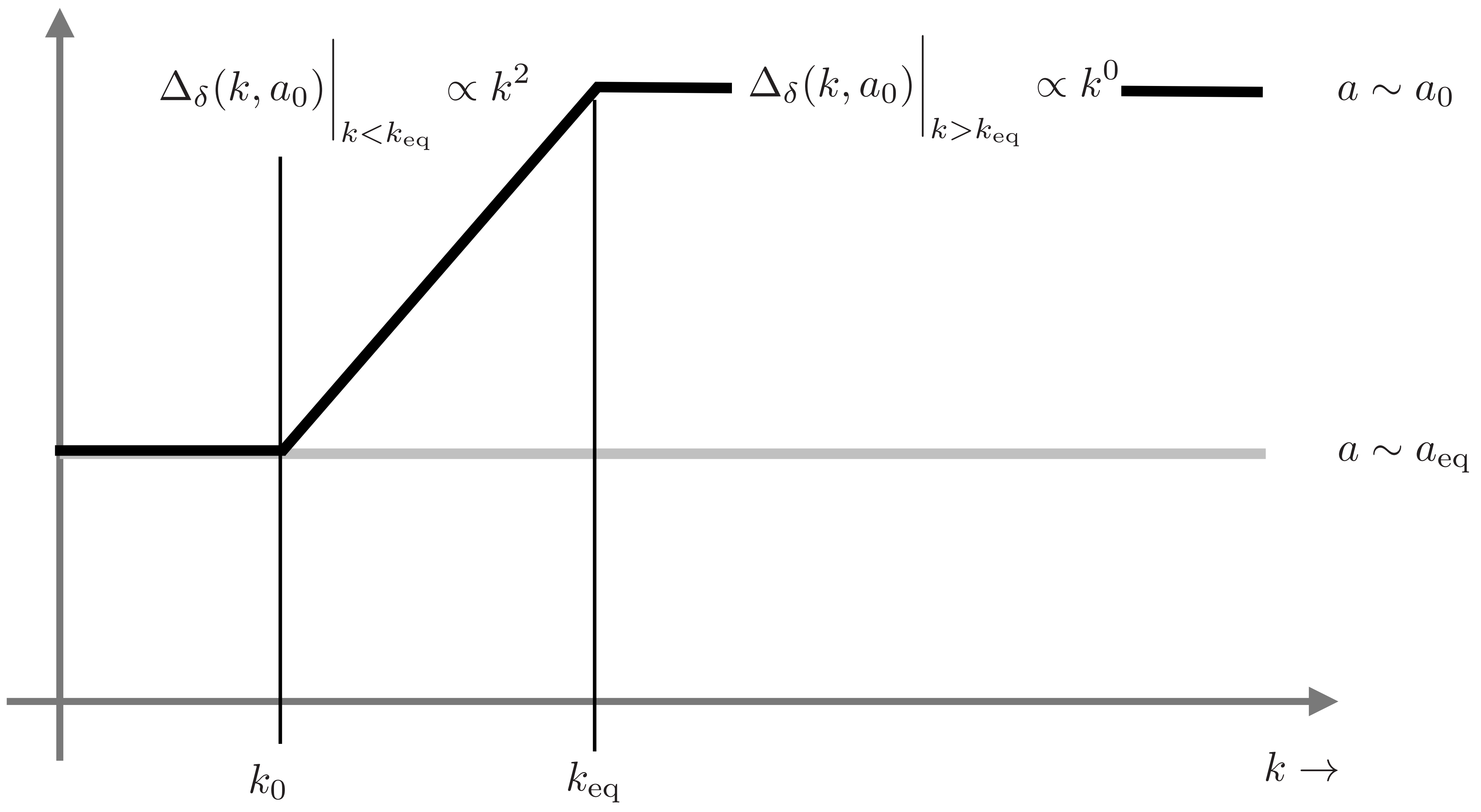
reason for slope — horizon entry

$$a_{\rm ent}(k)H(a_{\rm ent}(k)) = k \quad \Longrightarrow a_{\rm eq}(k) \propto k^{-2} \Longrightarrow \Delta_\delta(k,a)\Big|_{k<k_{\rm eq}} \propto k^2 \quad \text{and} \quad \Delta_\delta(k,a)\Big|_{k>k_{\rm eq}} \propto k^0$$

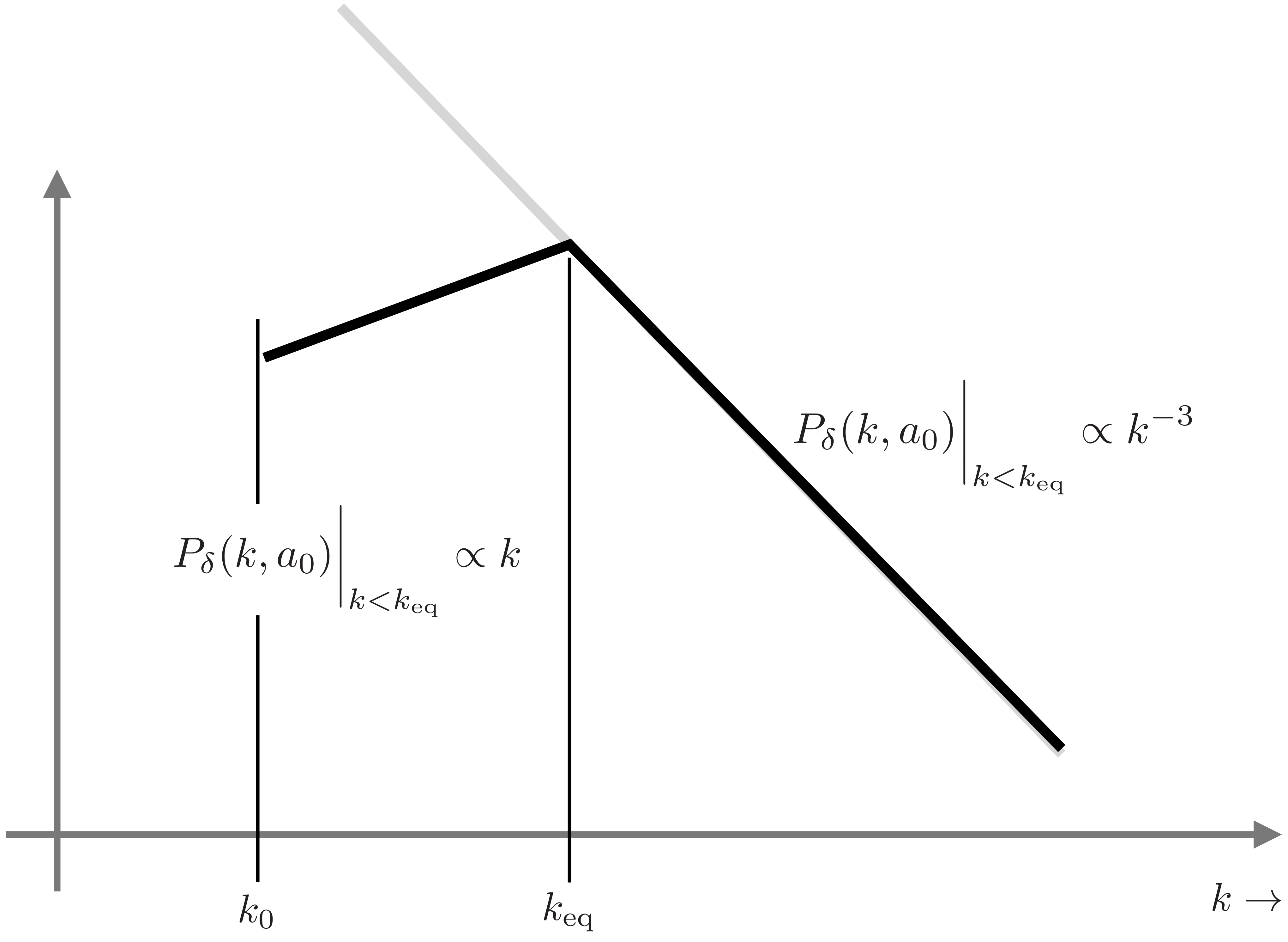


dimensionless to dimensional linear matter power spectrum

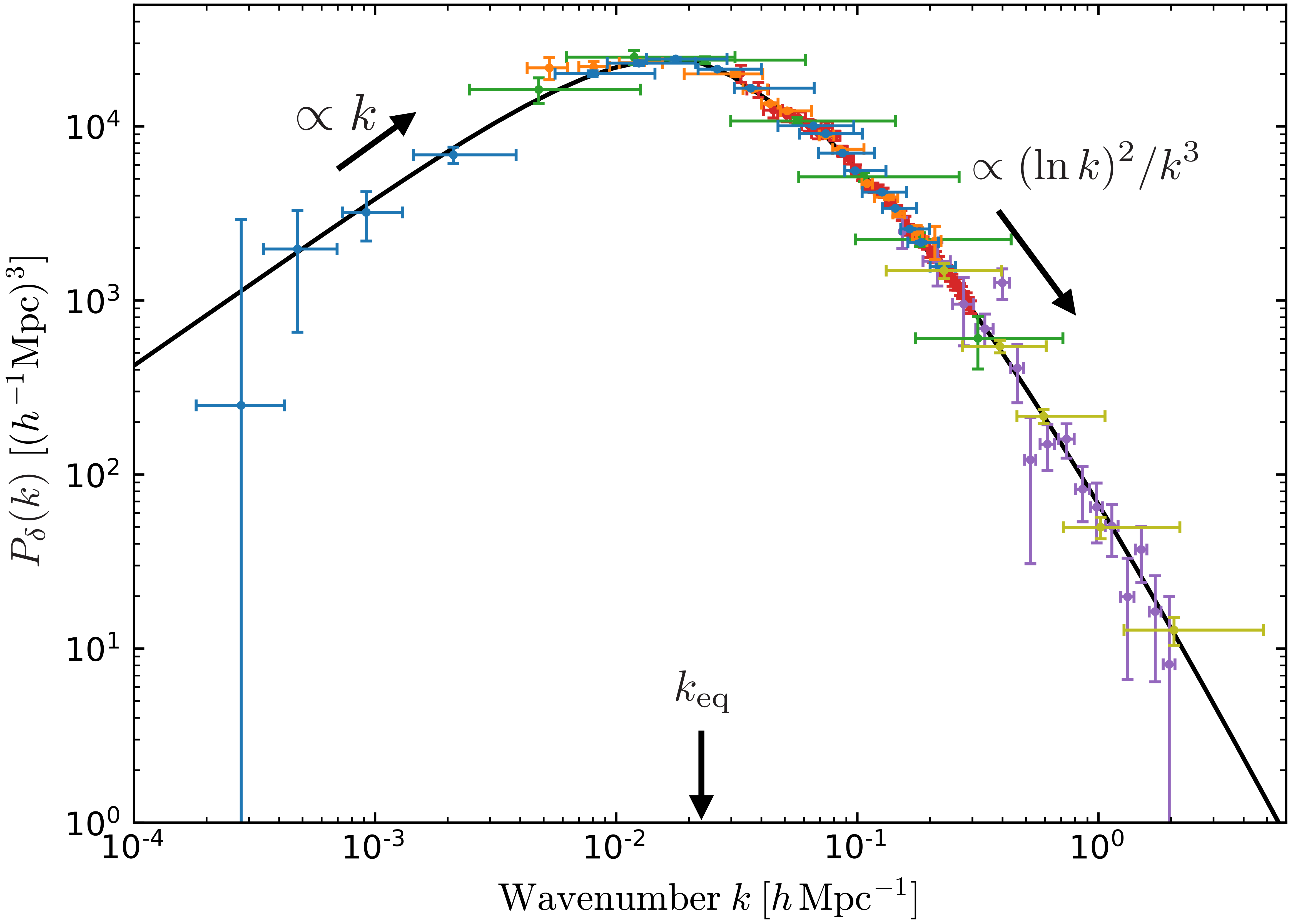
$$\Delta_\delta(k,a) \equiv \sqrt{\frac{k^3}{2\pi^2} P_\delta(k,a)} \Longrightarrow P_\delta(k,a) = \frac{2\pi^2}{k^3}\Delta_\delta^2(k,a)$$

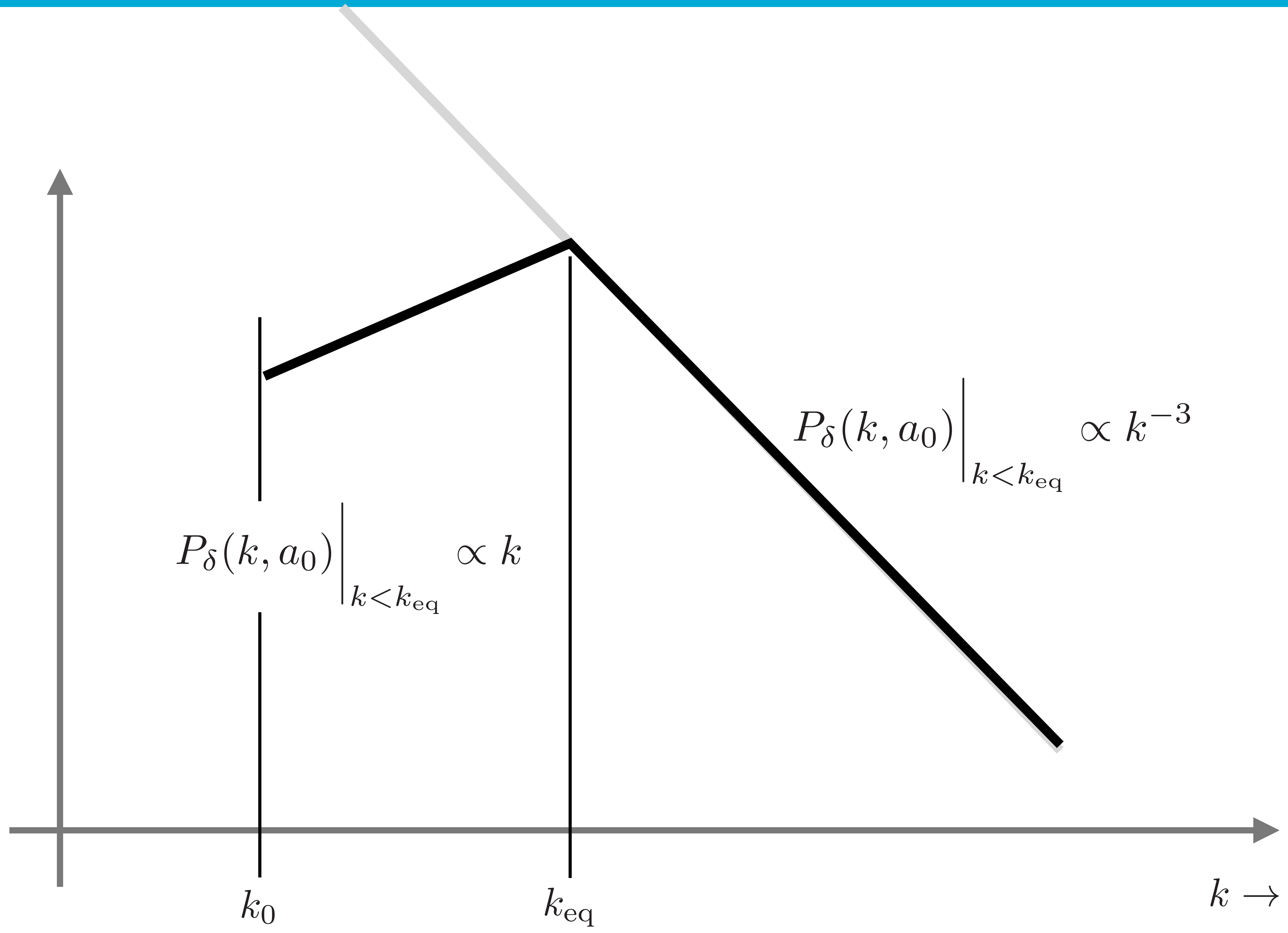


$\Longrightarrow$



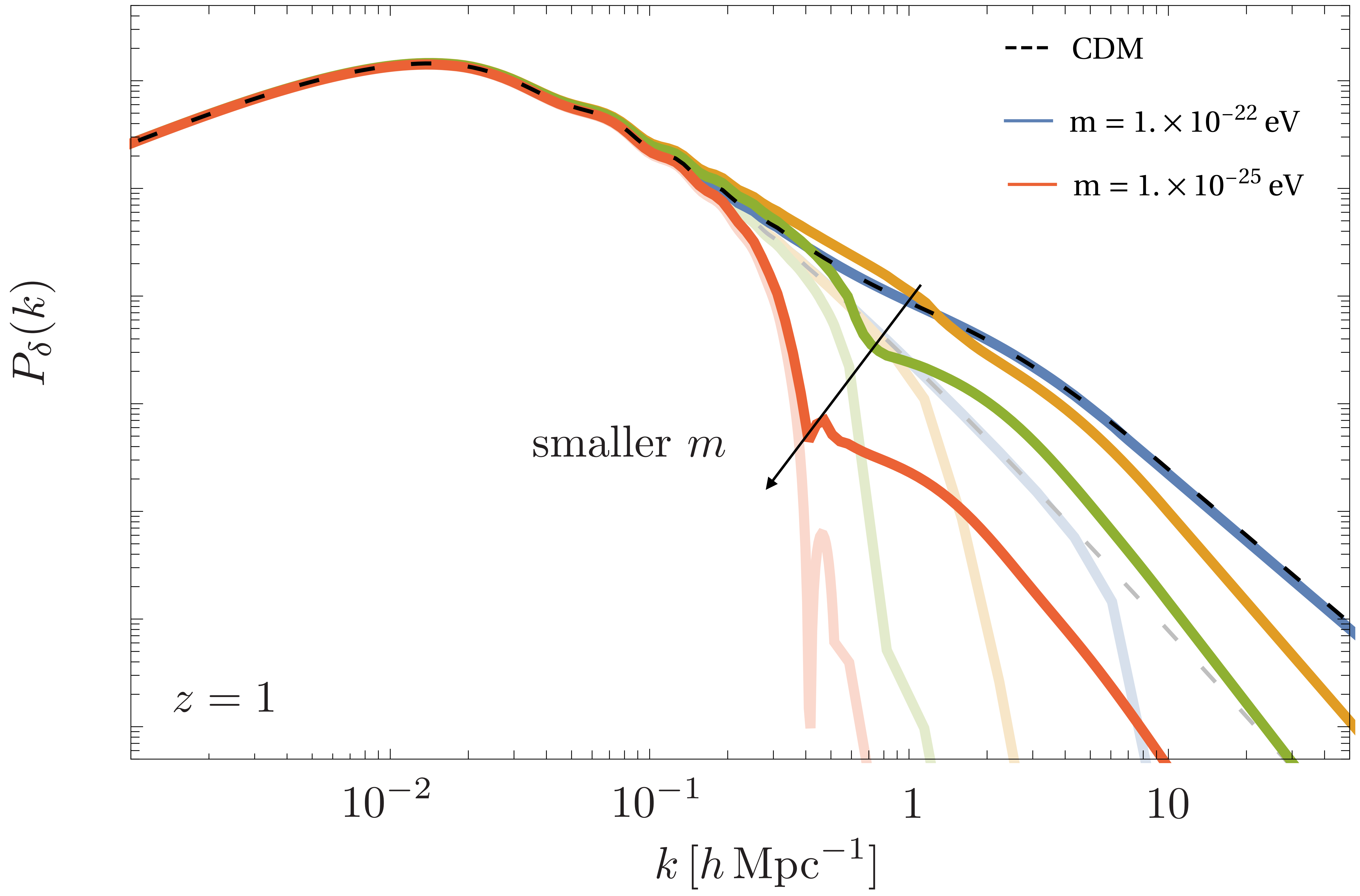
success! linear matter power spectrum in CDM





if we had included the logarithmic growth during radiation domination, we would have recovered the log correction as well

SDM density power spectrum



suppression of structure above

$$k_J^{\rm eq} \sim a_{\rm eq}\sqrt{mH_{\rm eq}}$$

a wave-dynamics “Jeans” scale

*bold opaque lines include some nonlinear evolution

fig. adapted from Dentler et. al 2021
https://arxiv.org/abs/2111.01199

scalar DM: linear perturbation theory matter domination

due to gradient pressure

$$\left[a^2\partial_a^2 + \frac{3}{2}a\partial_a - \frac{3}{2}\left(1 - \frac{k^4}{k_J^4}\right)\right]\delta = 0$$

subhorizon, matter domination eq. only

$$\delta \propto \begin{cases} a & k \ll k_J \\ a^0 & k \gg k_J \end{cases}$$

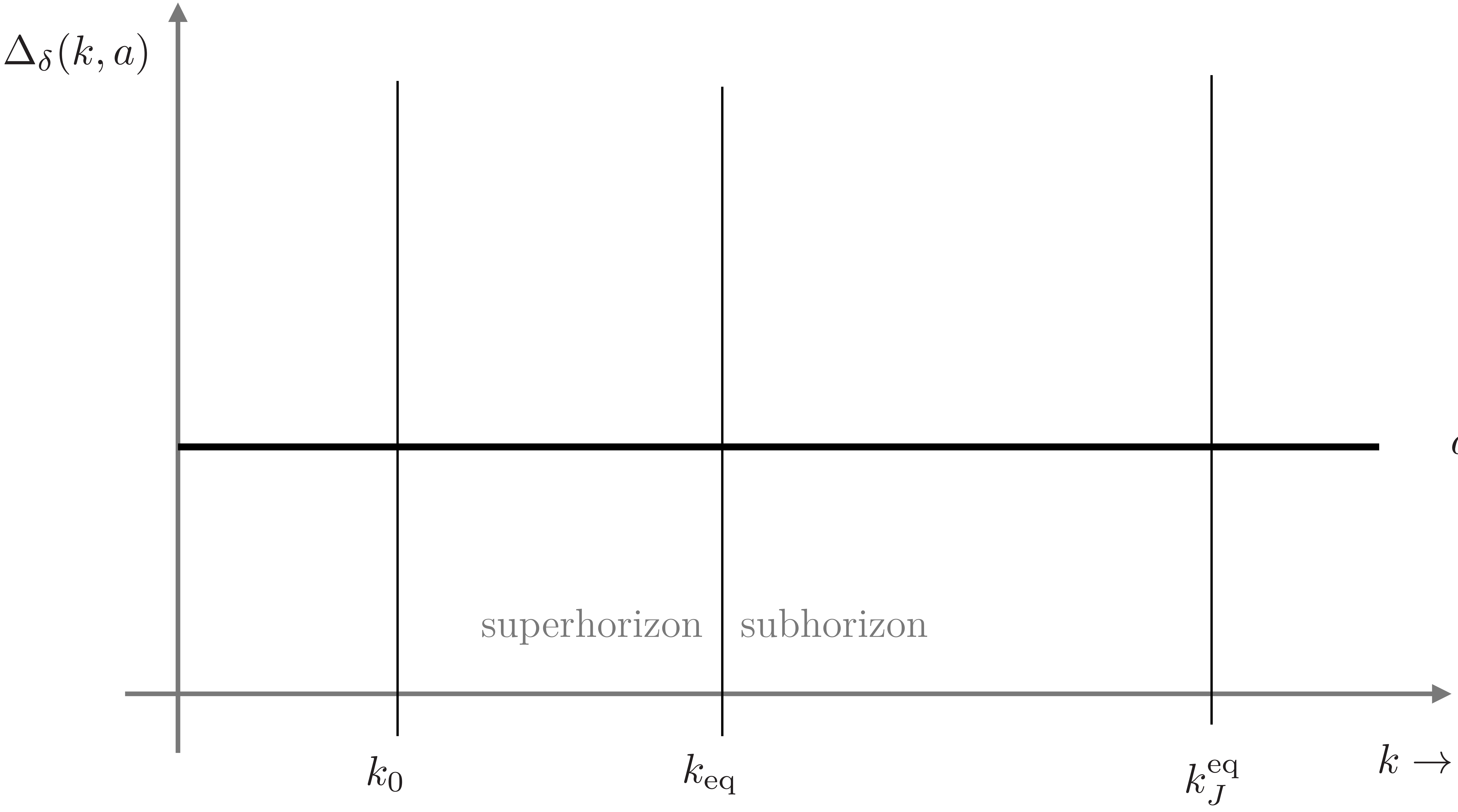
usual growth

oscillatory, due to gradient pressure

$$\delta \propto a^0$$

superhorizon (all eras)

SDM power spectrum (gravitational production)

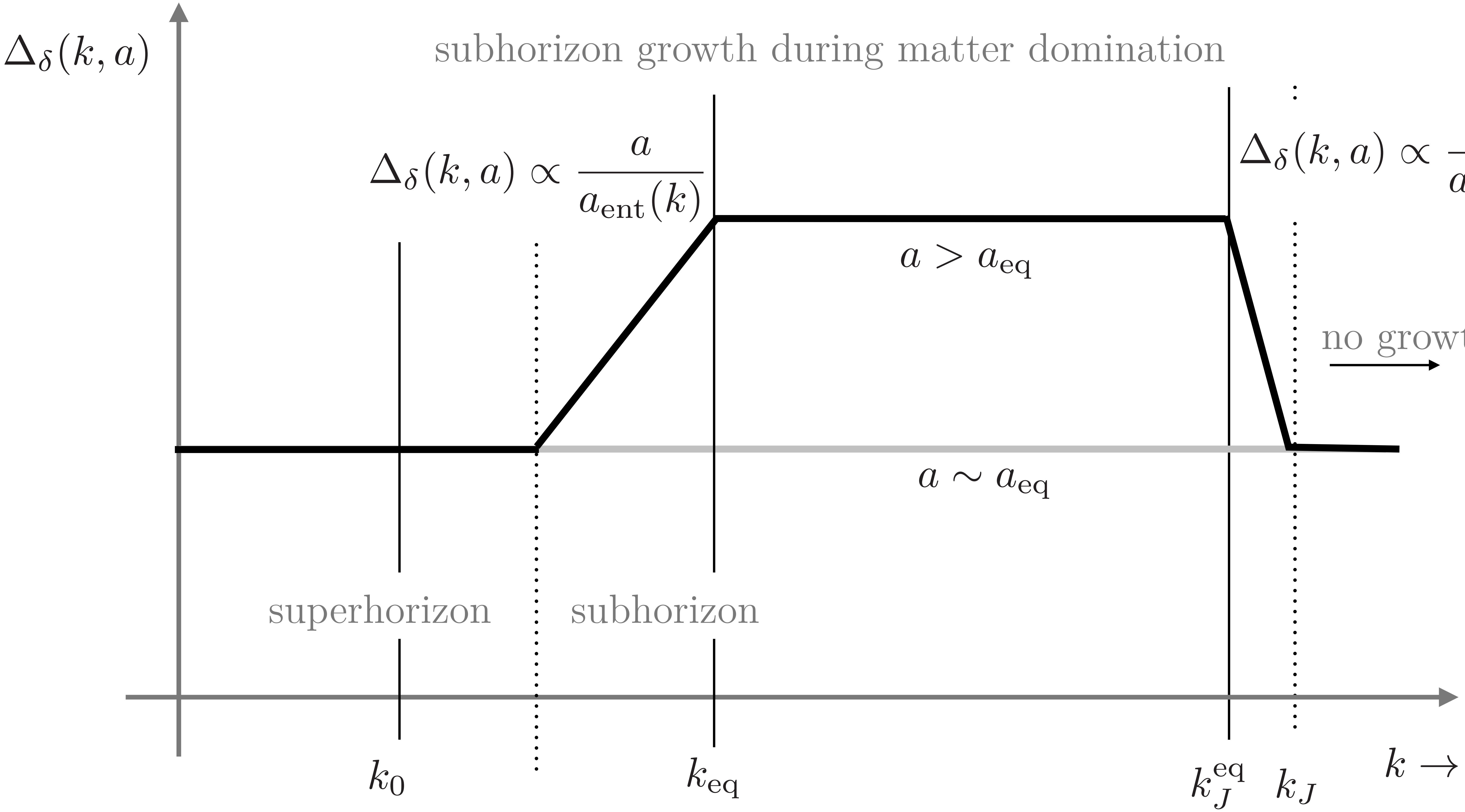


$$k_J \equiv a\sqrt{mH} \sim \lambda_{\rm dB}$$

$$= k_J^{\rm eq}(a/a_{\rm eq})^{1/4}$$

no growth for length scales smaller than deBroglie

$k_J = k_J^{\rm eq}(a/a_{\rm eq})^{1/4}$

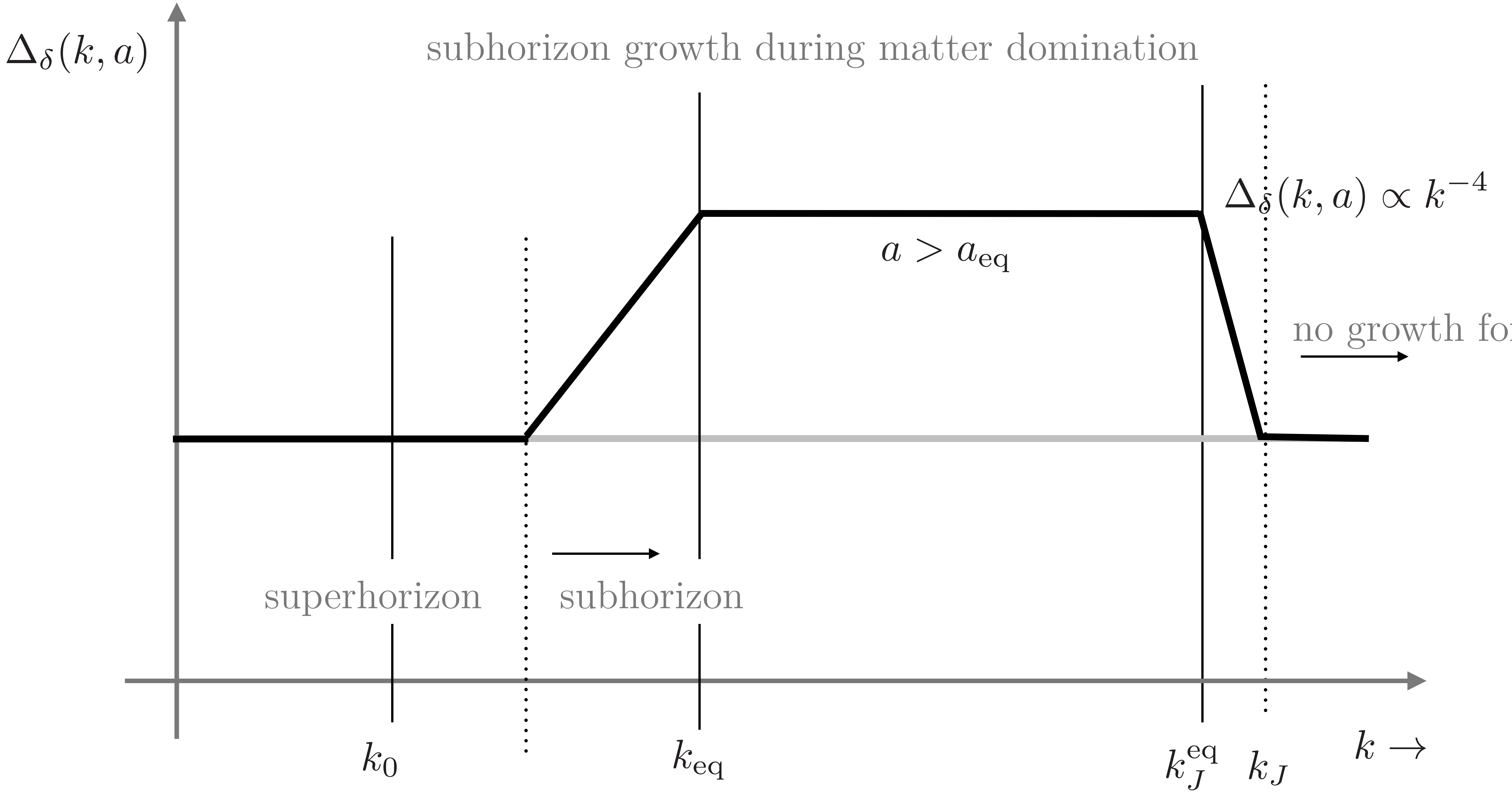


$a_{\rm ent}(k)H(a_{\rm ent}(k)) = k \implies a_{\rm eq}(k) \propto k^{-2}$

$k = k_{\rm eq}^J(a_J(k)/a_{\rm eq})^{1/4} \implies a_J(k) \propto k^4$

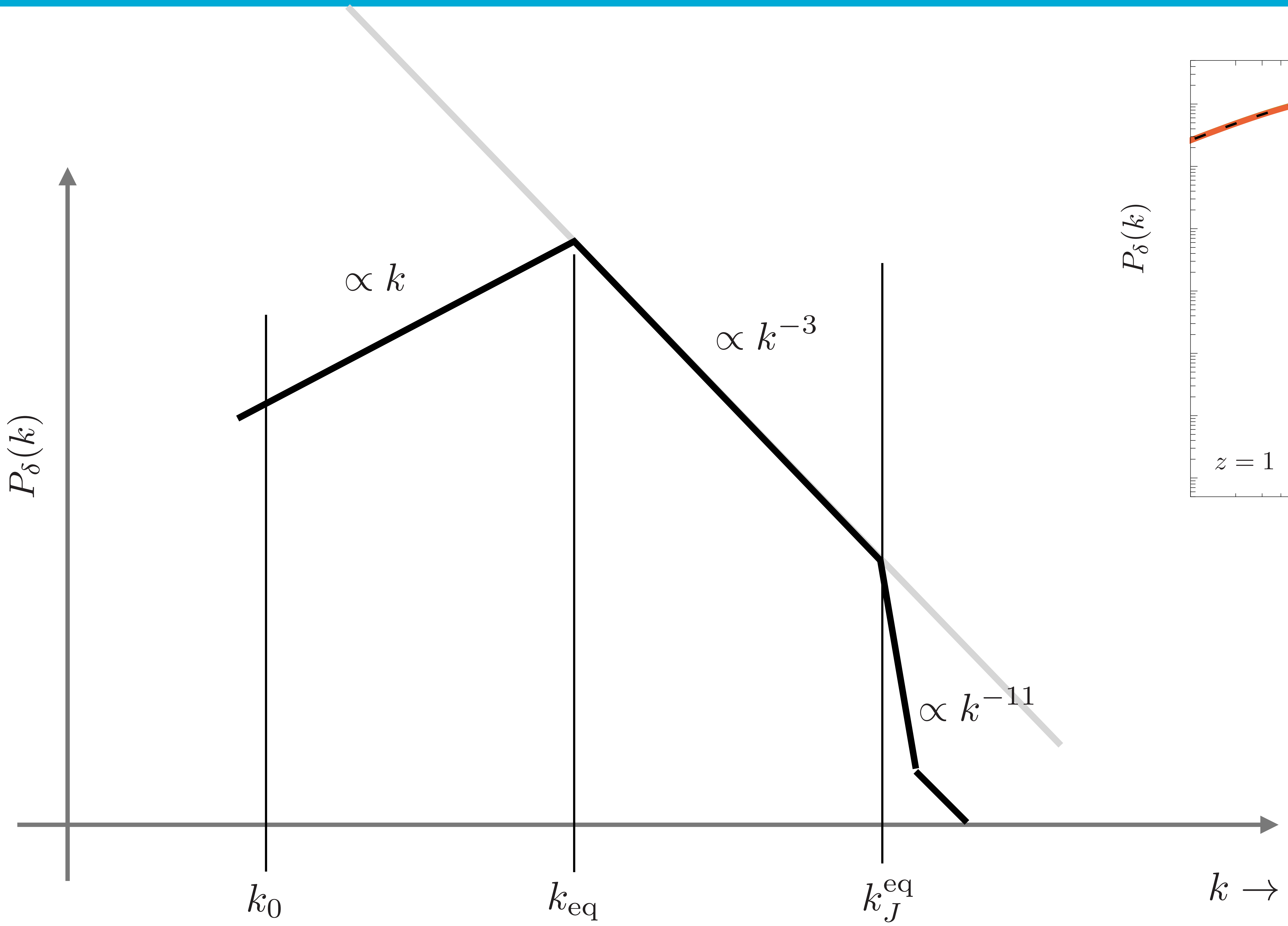
no growth for length scales smaller than deBroglie

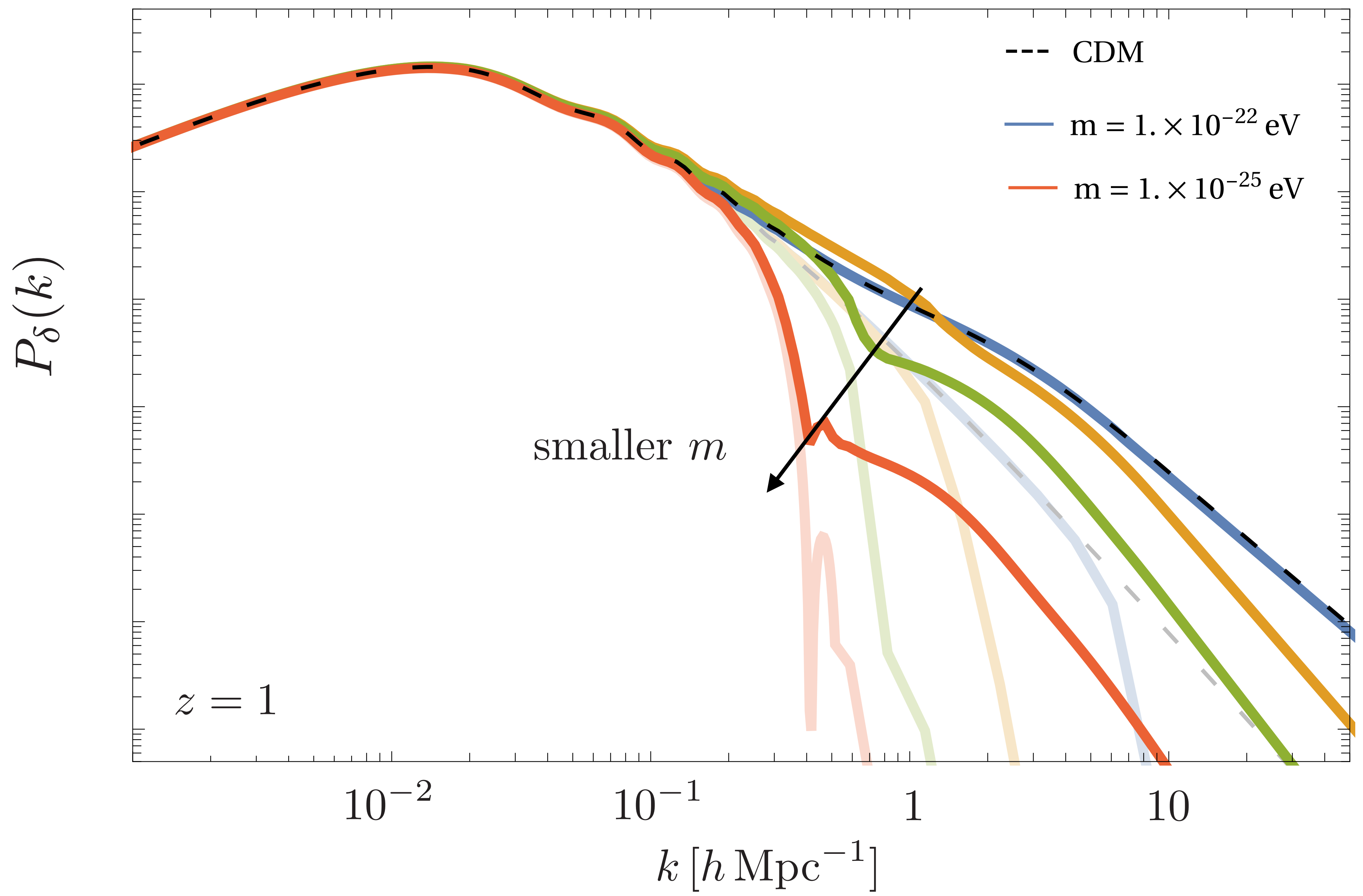
$$k_J = k_J^{\rm eq}(a/a_{\rm eq})^{1/4}$$



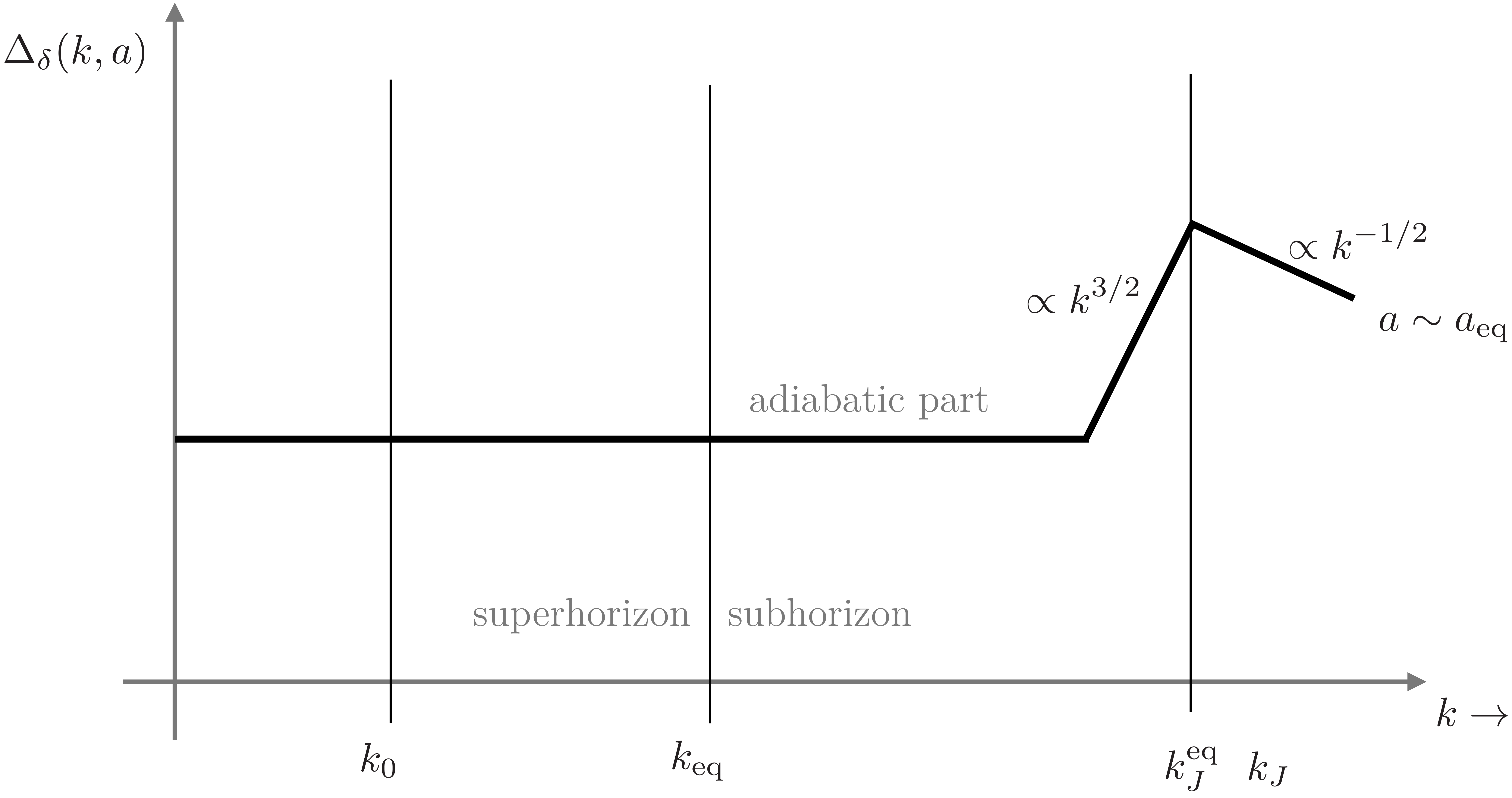
$$k = k_{\rm eq}^J(a_J(k)/a_{\rm eq})^{1/4} \implies a_J(k) \propto k^4$$

suppression of small scale structure (linear)

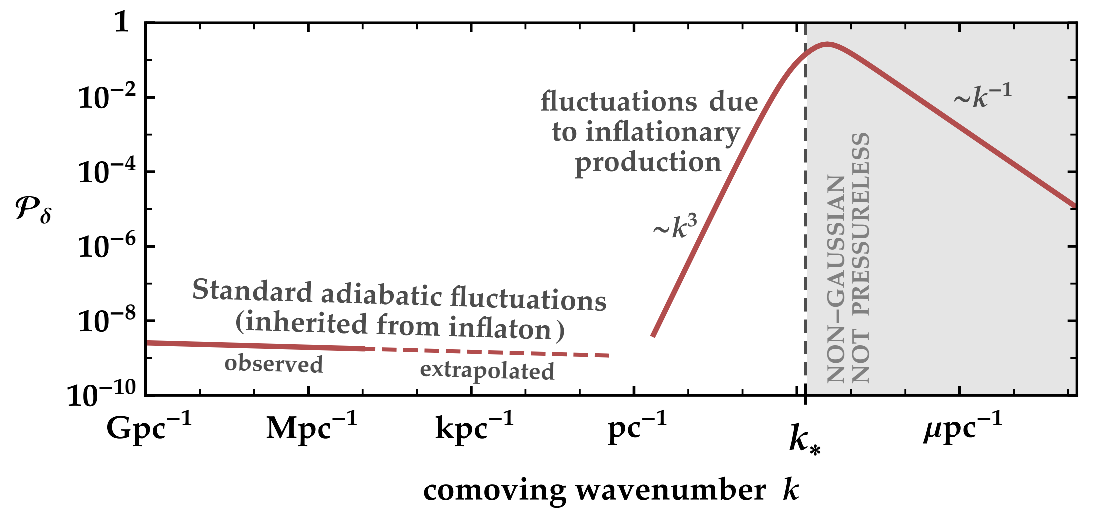




VDM power spectrum (gravitional production)



Graham, Mardon, Rajendran (2016)



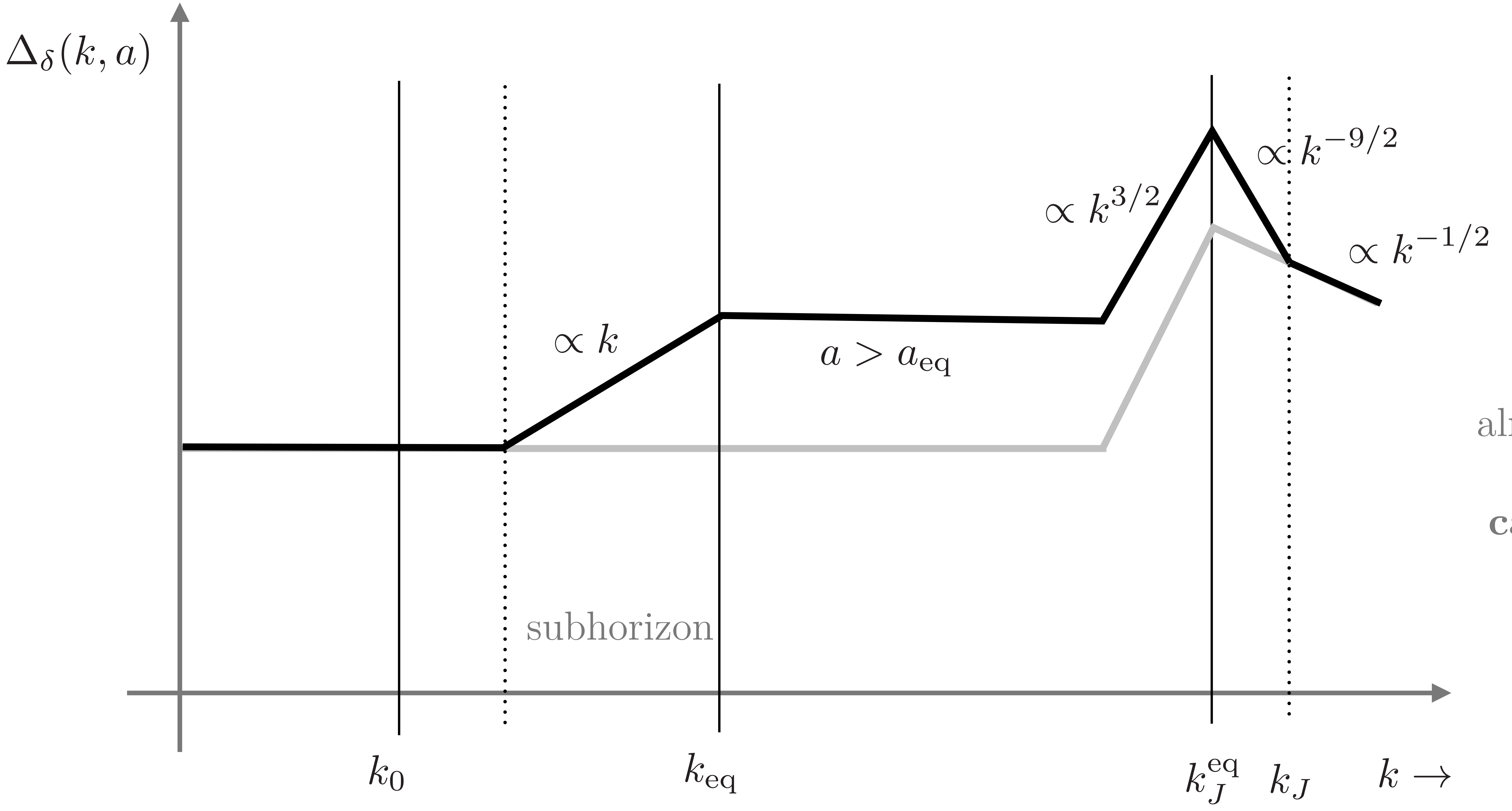
powers related to initial conditions & and relativistic-nonrelativistic transition

early derivation of action for longitudinal mode: Himmetoglu, Contaldi & Peloso (2008)

Also see: Ahmed, Grzadkowski, Socha (2020), Kolb & Long (2020)

VDM power spectrum (gravitational production)

$$k_J \equiv a\sqrt{(mc/\hbar)(H/c)} \sim \lambda_{\rm dB}$$
$$= k_J^{\rm eq}(a/a_{\rm eq})^{1/4}$$



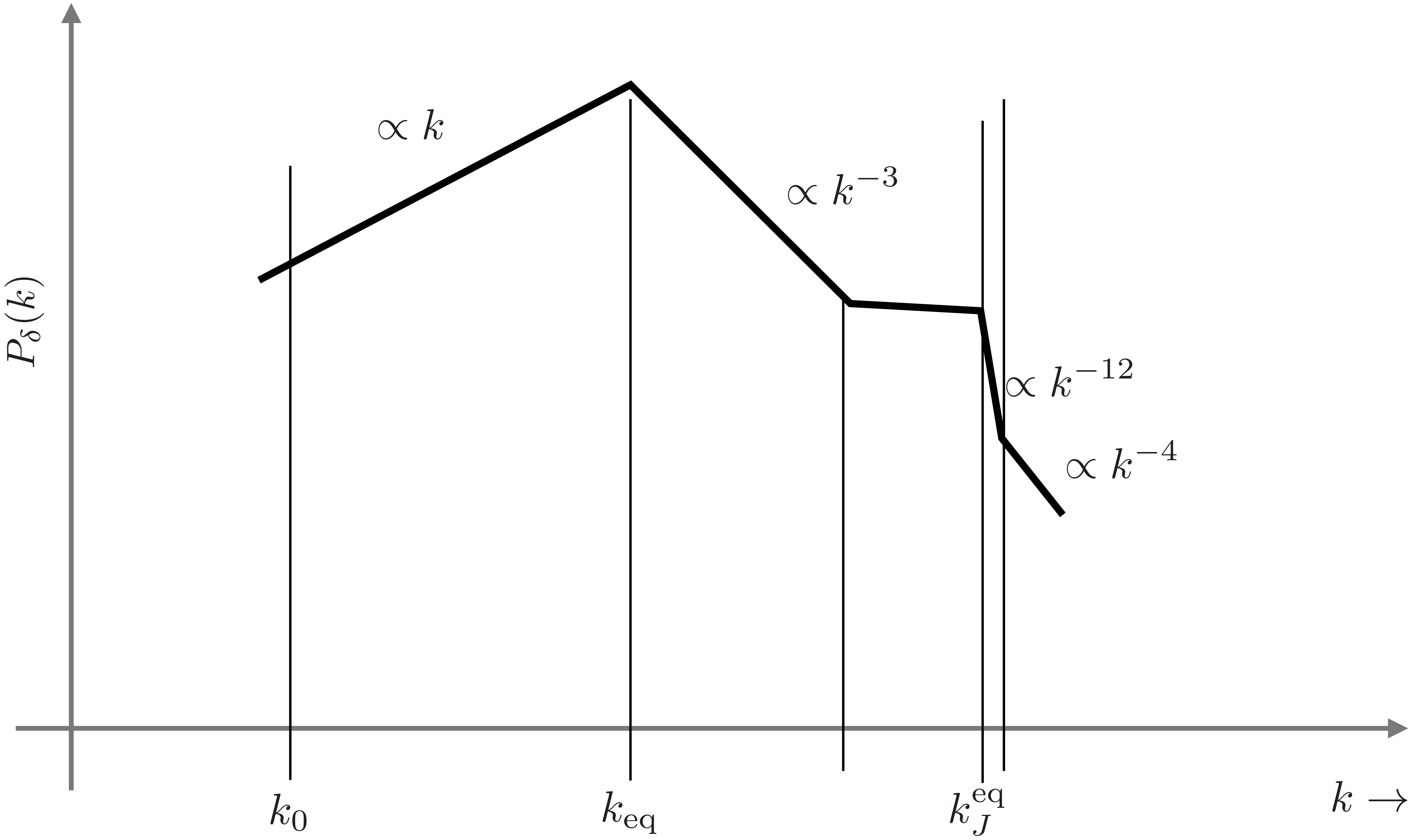
already peaked spectrum !

caution — already nonlinear

high k peak/powers related to vector nature and relativistic-nonrelativistic transition

peak at small scales !

$k_J \equiv a\sqrt{mH} \sim \lambda_{\rm dB}$



extra power on “small” length scales!

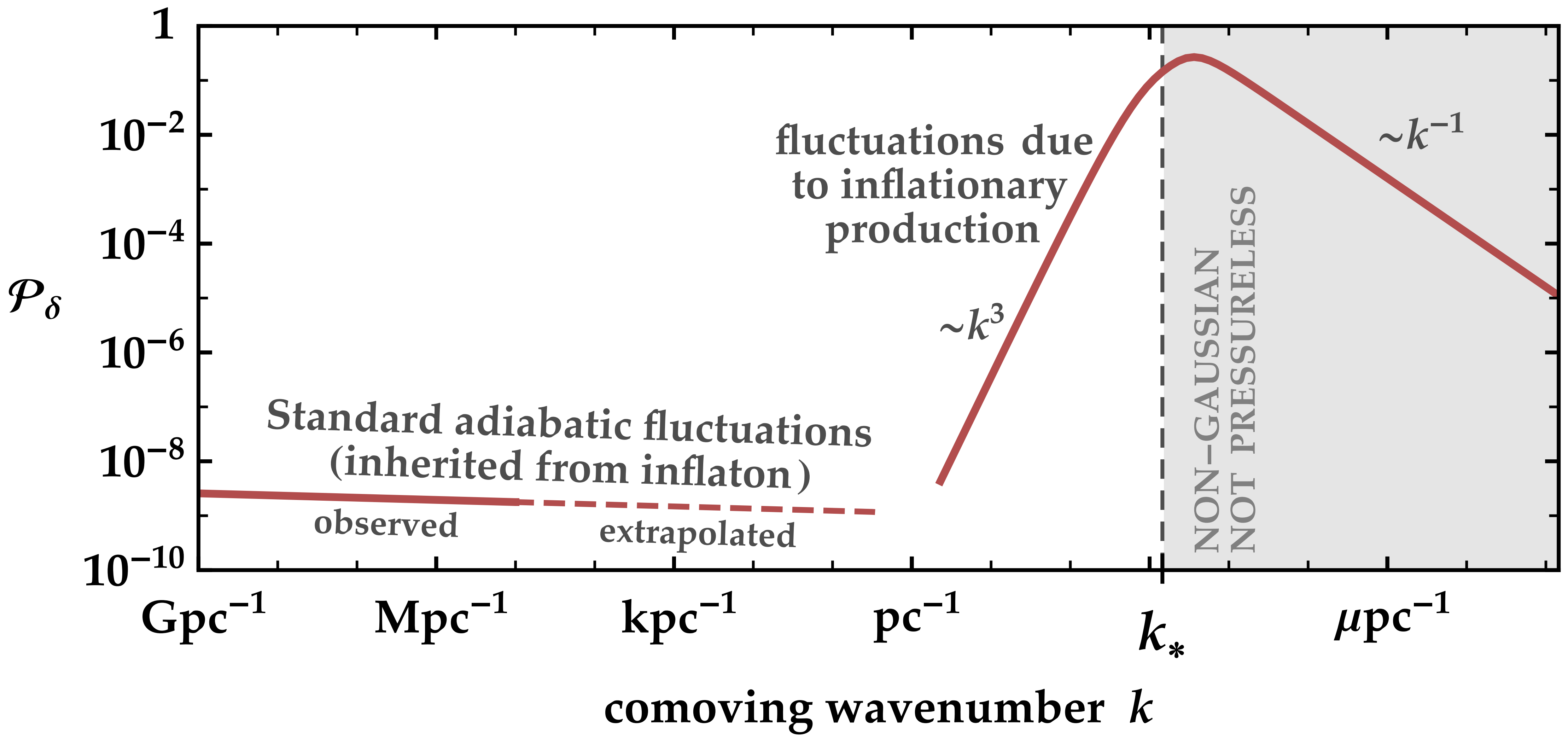
For simplified “analytic" treatment of implications, see Blinov et. al (2021)

gravitational particle production to nonlinear structures

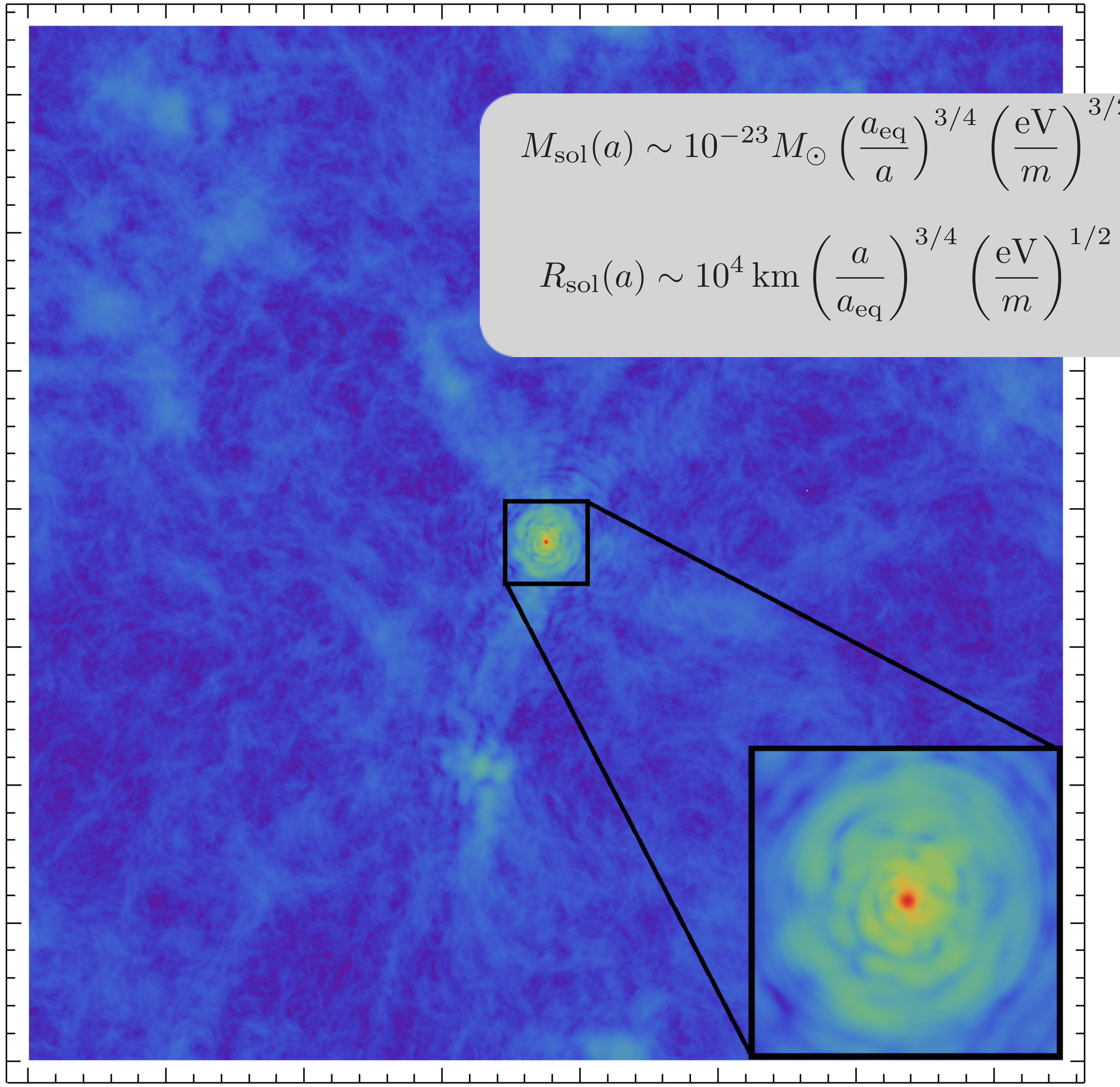
cannot easily do ultralight dark photons

$$\Omega_{\rm vdm} \sim 0.3 \left(\frac{m}{10^{-5}\,{\rm eV}}\right)^{1/2} \left(\frac{H_{\rm inf}}{10^{14}\,{\rm GeV}}\right)^4$$

Graham, Mardon, Rajendran (2016)



Gorghetto, Hardy, March-Russell, Song & West (2022)



$$M_{\rm sol}(a) \sim 10^{-23} M_\odot \left(\frac{a_{\rm eq}}{a}\right)^{3/4} \left(\frac{\rm eV}{m}\right)^{3/2}$$

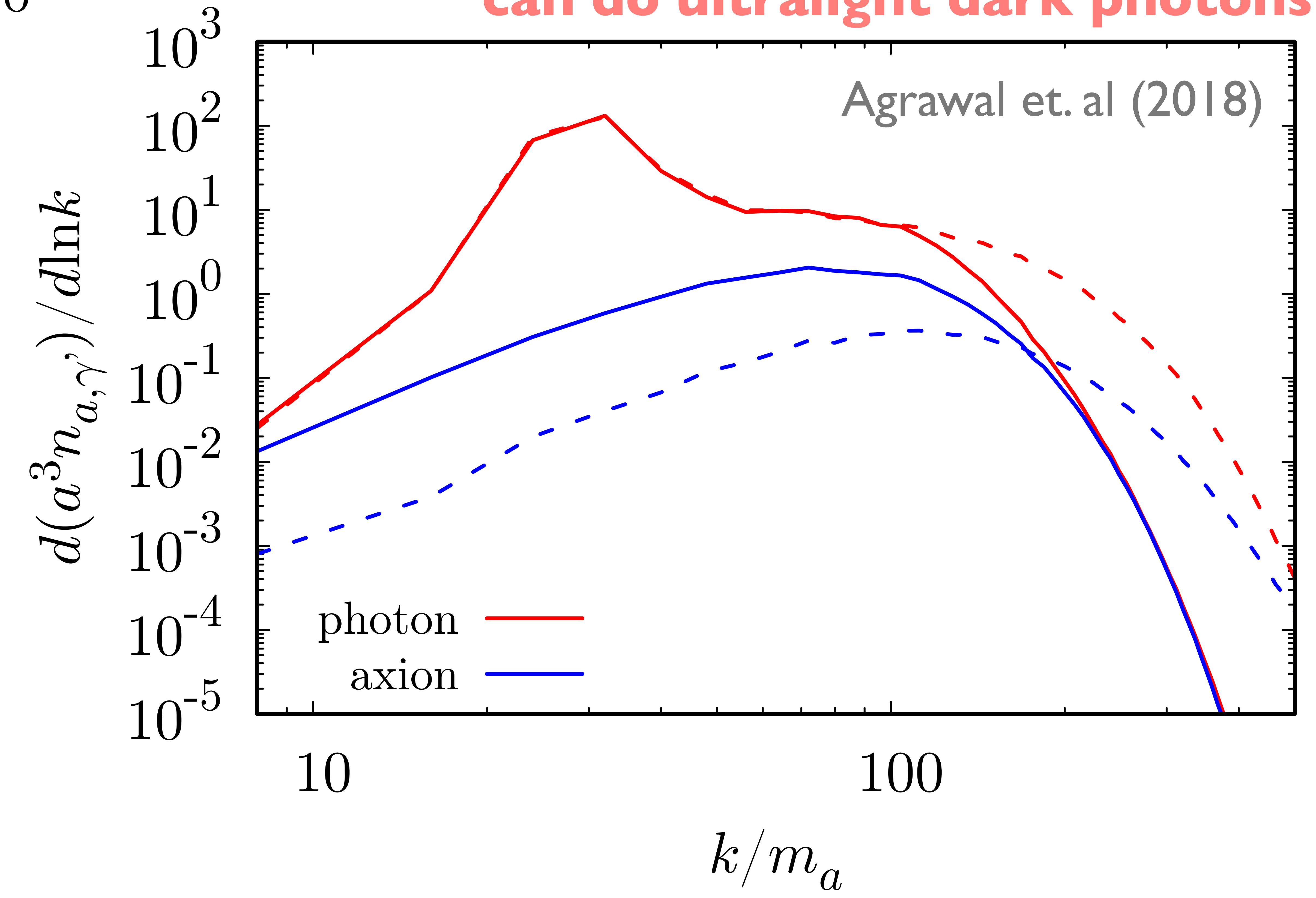
$$R_{\rm sol}(a) \sim 10^4\,{\rm km} \left(\frac{a}{a_{\rm eq}}\right)^{3/4} \left(\frac{\rm eV}{m}\right)^{1/2}$$

non-gravitational production of dark photons to nonlinear structures

$$\ddot{\mathbf{A}}_{\mathbf{k},\pm} + H\dot{\mathbf{A}}_{\mathbf{k},\pm} + \left(m_{\gamma'}^2 + \frac{k^2}{a^2} \mp \frac{k}{a}\frac{\beta\dot{\phi}}{f_a}\right)\mathbf{A}_{\mathbf{k},\pm} = 0$$

In Progress

can do ultralight dark photons



Co, Pierce, Zhang, and Zhao (2018)
Dror, Harigaya, and Narayan (2018)
Bastero-Gil, Santiago, Ubaldi, Vega-Morales (2018)

Also see: Long & Wang for production from strings and Co et. al for production from axion rotations

generalization to arbitrary spin

spin-s field as dark matter

Einstein +

Klein-Gordon $(s = 0)$

Proca $(s = 1)$

Fierz-Pauli $(s = 2)$

non-relativistic limit = multicomponent Schrödinger-Poisson

spin-s fields as light dark matter

Einstein + Klein-Gordon ($s = 0$)

Einstein + Proca ($s = 1$)

Einstein + Fierz-Pauli ($s = 2$)

non-relativistic limit →

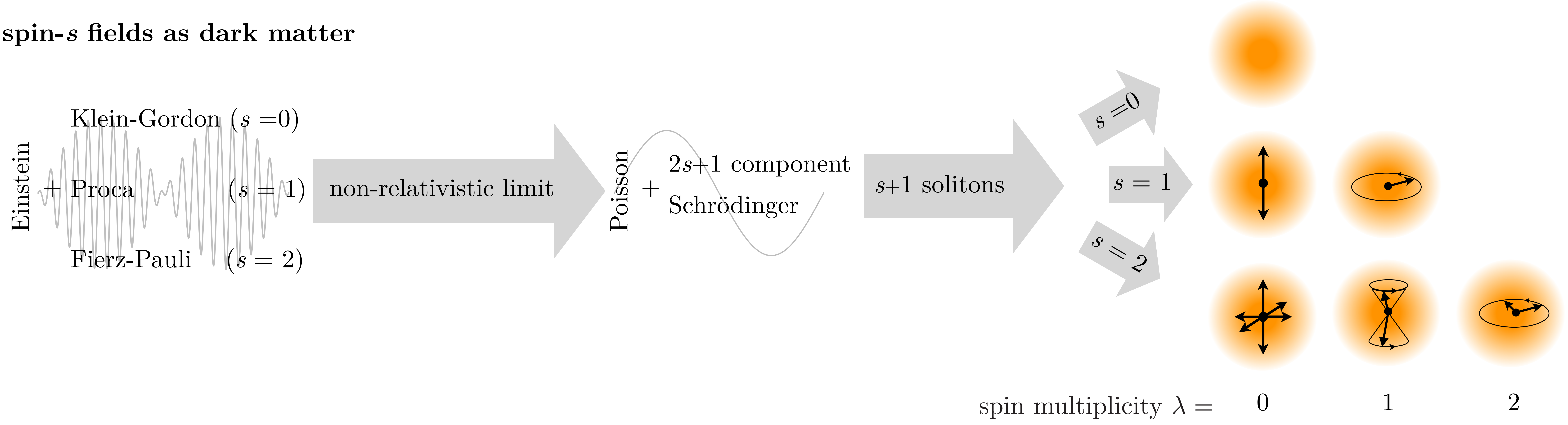
Poisson + $2s$+1 component Schrödinger

-phenomenology/numerical simulations

- interference ~1/(2s+1)

extremally polarized solitons

spin-s fields as dark matter



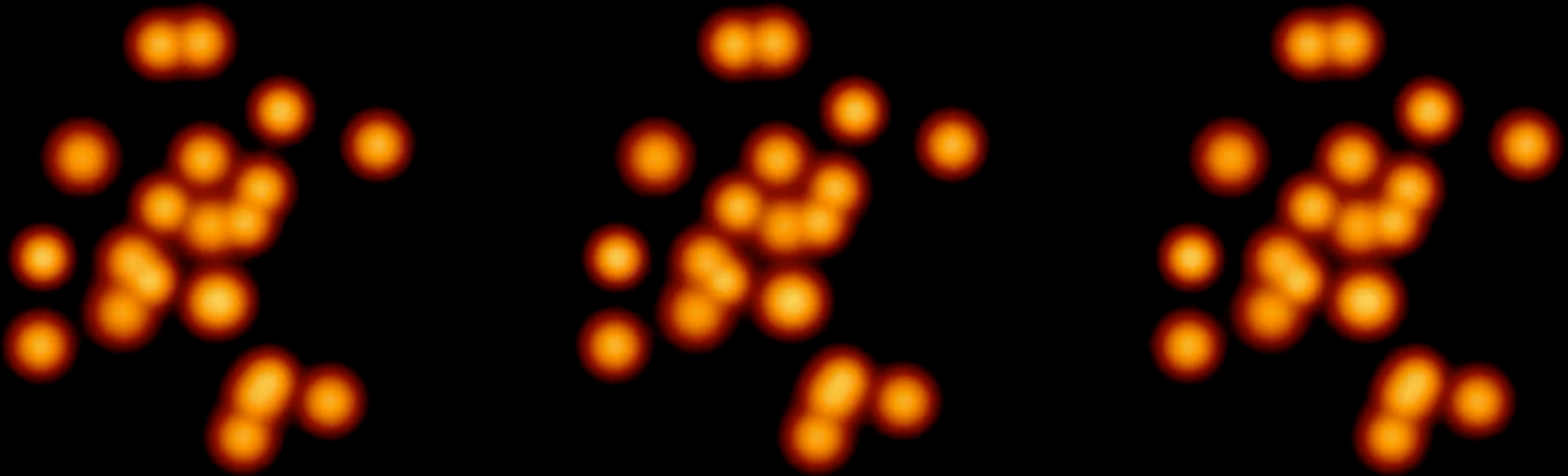
macroscopic spin $\boldsymbol{S}_{\rm tot}/\hbar = \lambda N \hat{z}$

$N =$ # of particles in soliton

Jain & MA (2021)

future possibilities …

- formation and survival mechanisms (cosmological simulations remain to be done)
- BH superradiance with higher spin fields (already done)
- dynamical friction
- vortices
- direct/indirect detection (interaction with baryons? photons? kinetic mixing)
- condensation time scales
- lifetimes of higher spin solitons
- initial power spectrum of fluctuations in the fields



extra slides

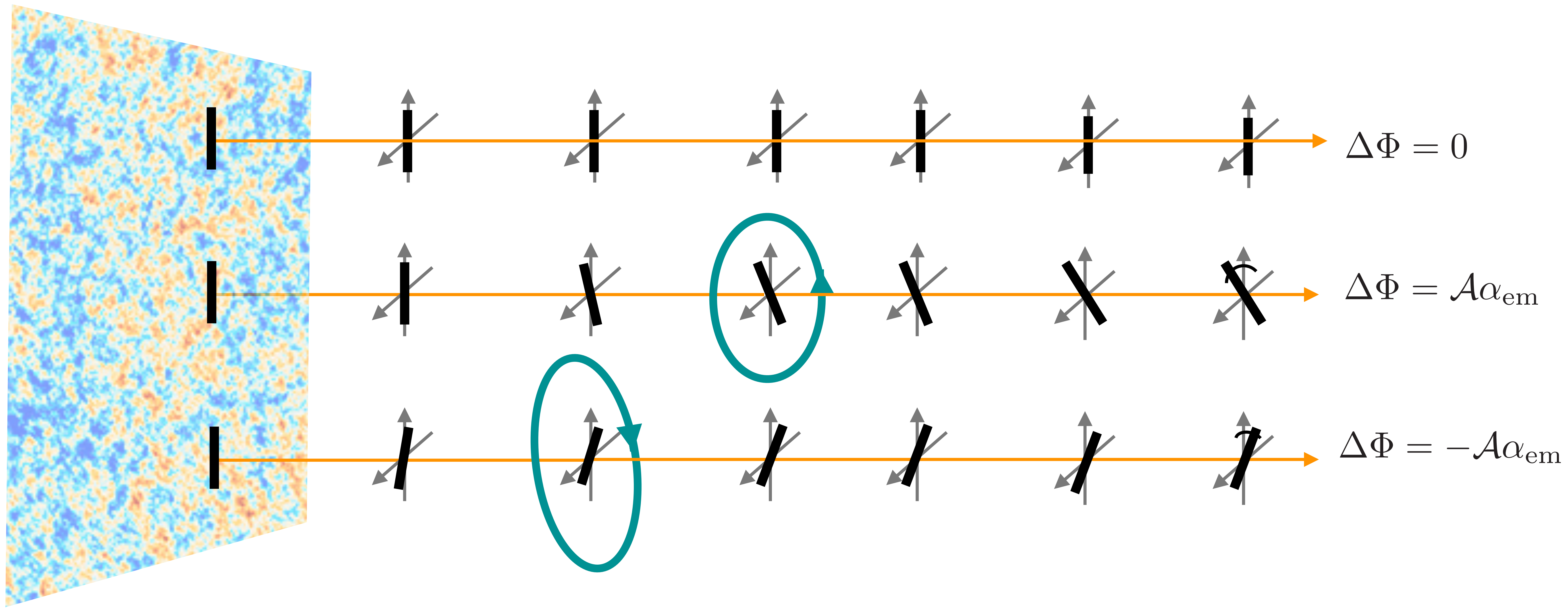
CMB birefringence from ultralight-axion string networks

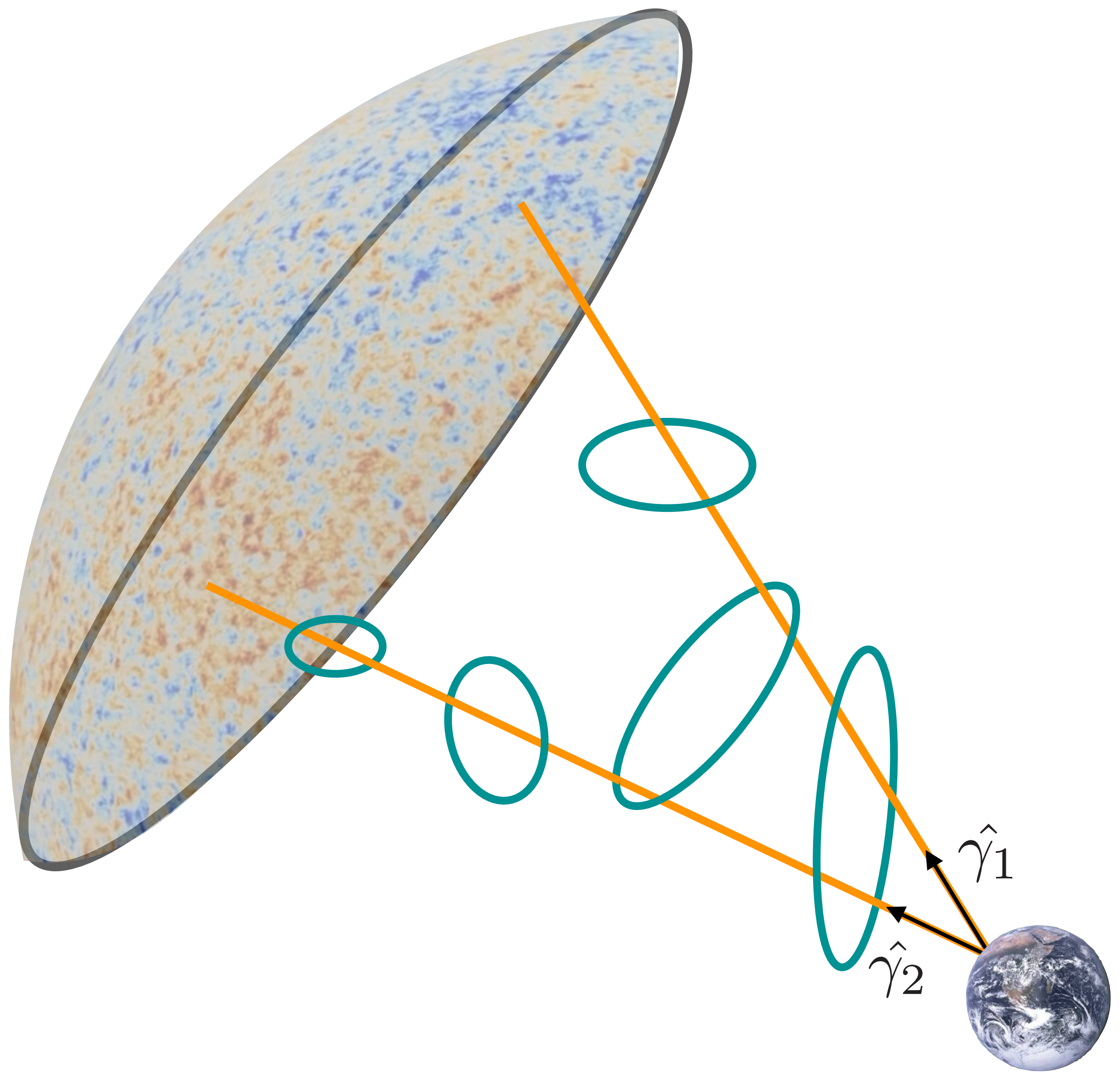
[arXiv:2103.10962]

with Andrew Long and **Mudit Jain**

$$\mathscr{L}_{\rm int} = -\frac{1}{4}\, g_{a\gamma\gamma}\, a\, F_{\mu\nu}\tilde{F}^{\mu\nu}$$

$$\Delta\Phi = \frac{g_{a\gamma\gamma}}{2}\int_C \mathrm{d}X^\mu\, \partial_\mu a(x) \quad = \pm g_{a\gamma\gamma}\pi f_a = \pm \mathcal{A}\alpha_{\rm em}$$

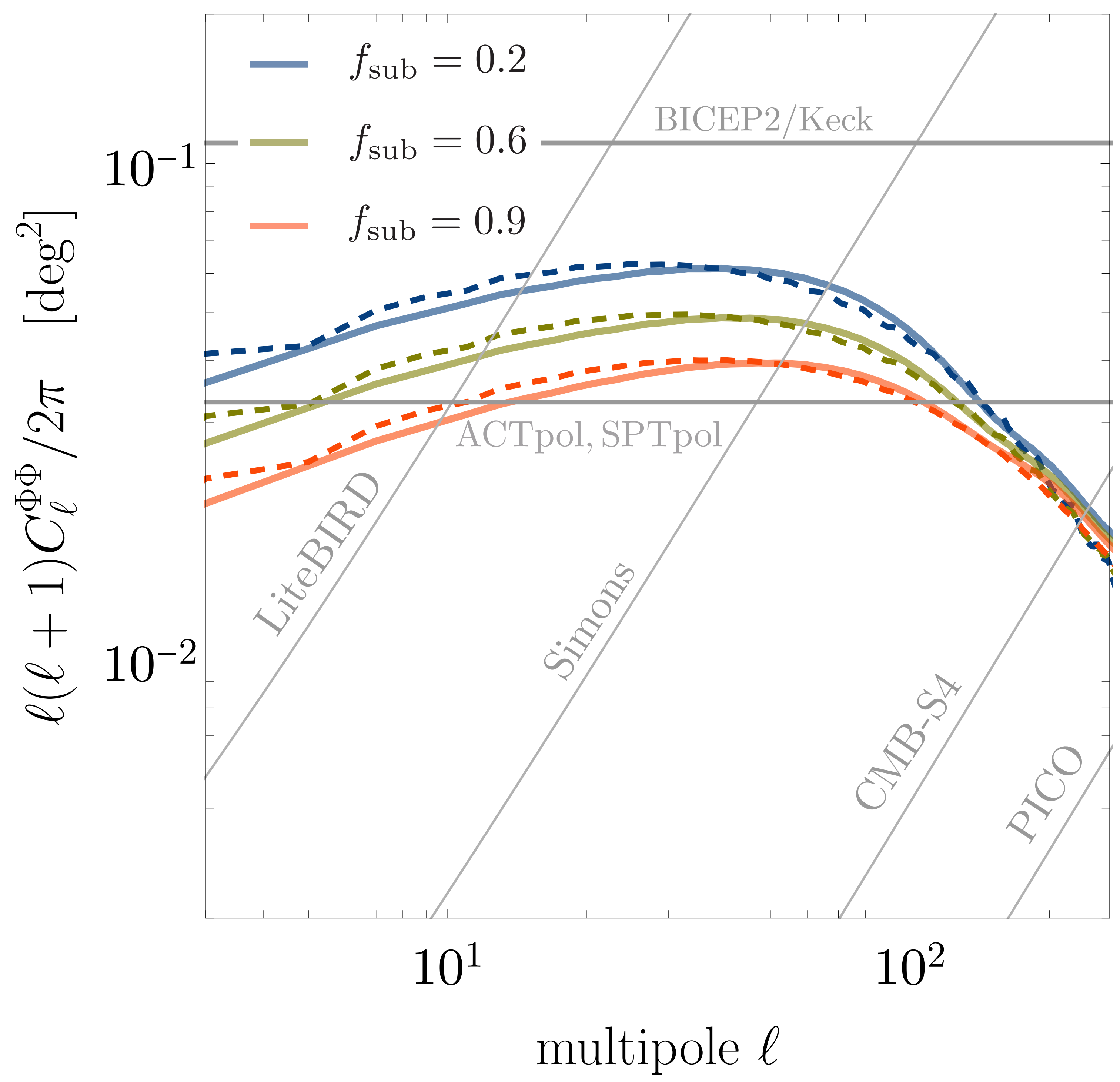




[arXiv:2103.10962] with Andrew Long and **Mudit Jain**

CMB birefringence from ultralight-axion string networks

models with sub/super horizon loops



models with decaying networks