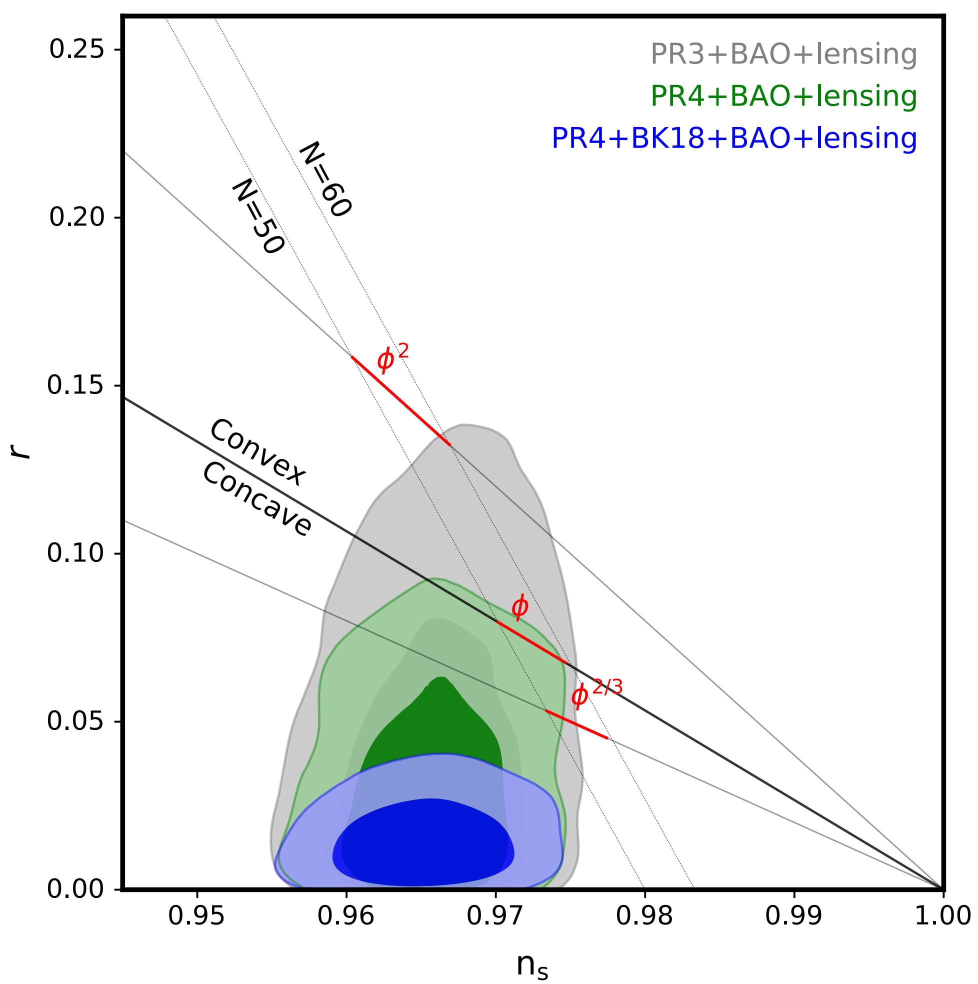
Slides lecture 1

Tristram et al '21

$$P_\zeta \propto \lambda^{1-n_s} \qquad n_s - 1 = 6\epsilon - 2\eta$$

$$r \equiv \frac{P_{\rm GW}}{P_\zeta} \qquad r = 16\epsilon$$



$$\begin{cases} P_\zeta = \frac{H^2}{8\pi^2 M_p^2 \epsilon} = 2.2 \cdot 10^{-9} \\ P_{\rm GW} = \frac{2H^2}{\pi^2 M_p^2} \equiv r\, P_\zeta \end{cases}$$

We measured the combination H^2/ϵ. Measuring GW ($\equiv$ knowing r)

$\rightarrow$ scale of inflation

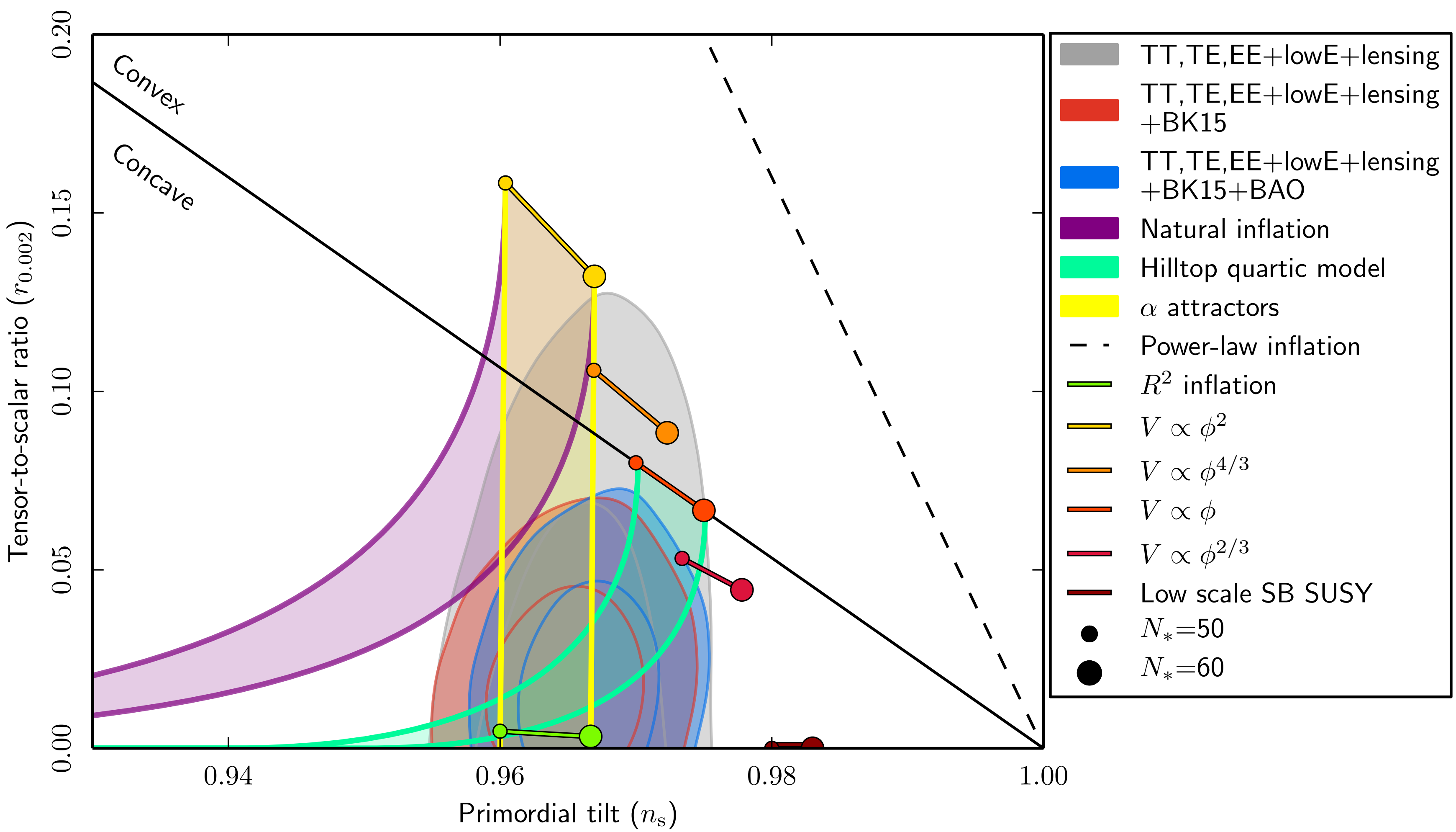
$$H \simeq 4.5 \cdot 10^{13}\,\text{GeV} \sqrt{\frac{r}{0.032}} \qquad \rho^{1/4} \simeq 1.4 \cdot 10^{16}\,\text{GeV} \left(\frac{r}{0.032}\right)^{1/4}$$

Slides lecture 2

$$N \simeq 55.6 + 2\ln\frac{V_k^{1/4}}{10^{16}\,\mathrm{GeV}} + \ln\frac{10^{16}\,\mathrm{GeV}}{\rho_{\rm end}^{1/4}} + \frac{1-3w}{12\,(1+w)}\ln\frac{\rho_{\rm reh}}{\rho_{\rm end}}$$

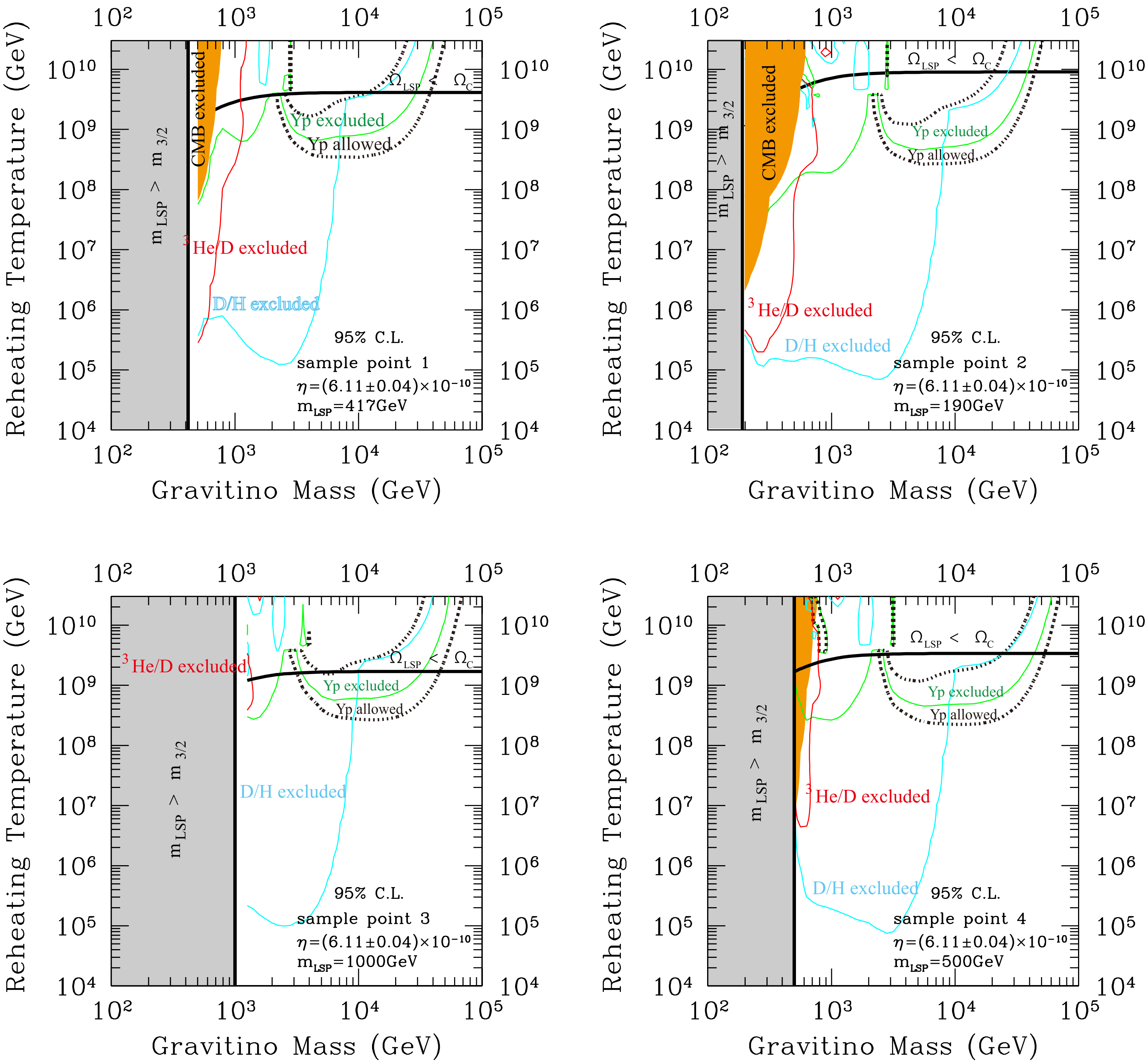
(i) instantaneous reheating after inflation $\longrightarrow$ $\Delta N = 0$

(ii) slowest possible decay $T_{\rm reh} \sim$ MeV $\longrightarrow$ $\Delta N \sim -15$

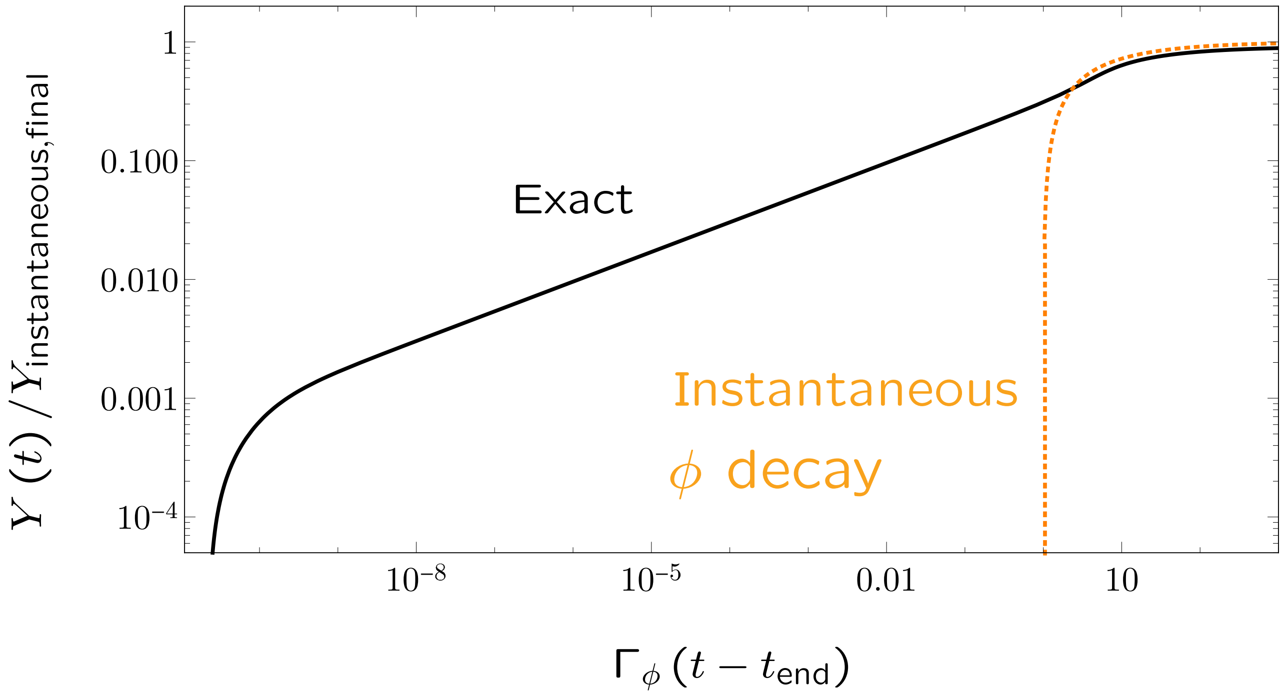


Slides lecture 3

Limits in Minimal Supersymmetric Standard Model for ≠ benchmark scenarios (≠ superparticle masses)



Kawasaki, Kohri, Moroi, Takaesu '17

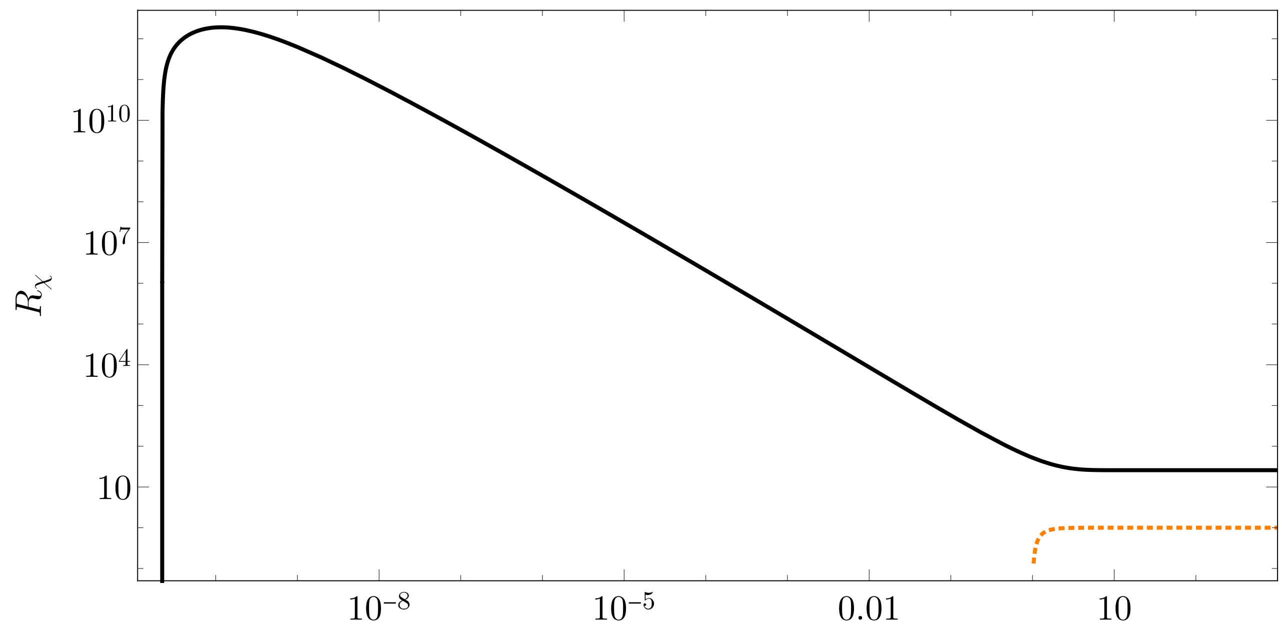


Example: gravitinos,

$$\sigma \simeq \frac{1}{M_p^2} \quad \rightarrow \quad n = 0$$

Massive spin 2

$$\sigma \propto \frac{1}{\Lambda^4\, M^4}\, T^6$$



See for instance 1709.01549 and 1803.01866

Nonperturbative reheating

- In computing Γ_ϕ we treat ϕ as collection of independent quanta
- Coherent oscillations $\phi(t) \rightarrow$ faster decay, at sufficiently large couplings

Shtanov, Taschen, Brandenberger '94

Kofman, Linde, Starobinsky '94; '97

Example: massive inflaton ϕ, oscillating about minimum of potential,

with quartic coupling with another scalar field χ

$$V = \frac{1}{2} m^2 \phi^2 + \frac{g^2}{2} \phi^2 \chi^2$$

- Next slide: Result of lattice simulation (Latticeeasy: Felder, Tkachev).

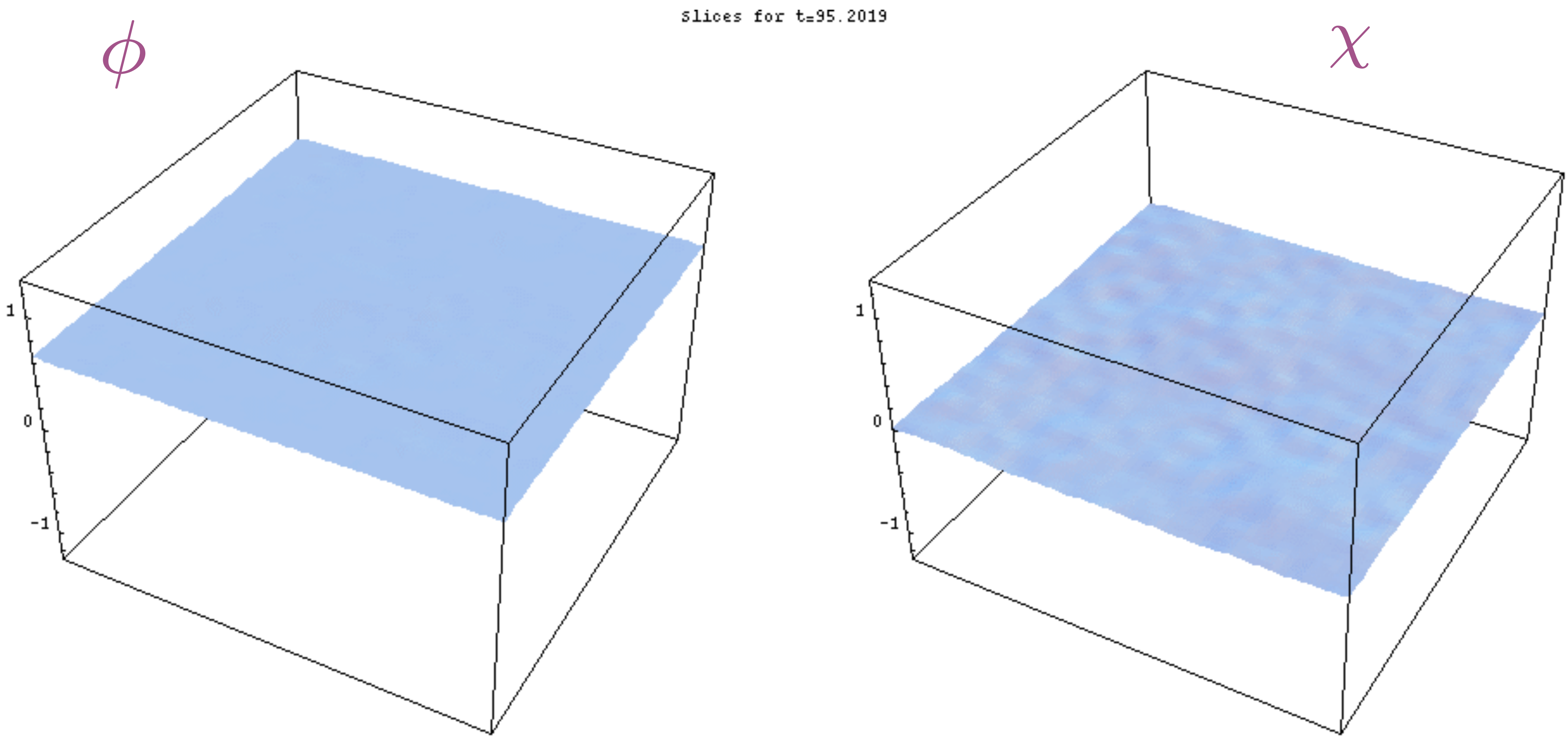
Three phases:

(1) Coherent inflaton oscillations

(2) χ excitations

(3) ϕ excitations

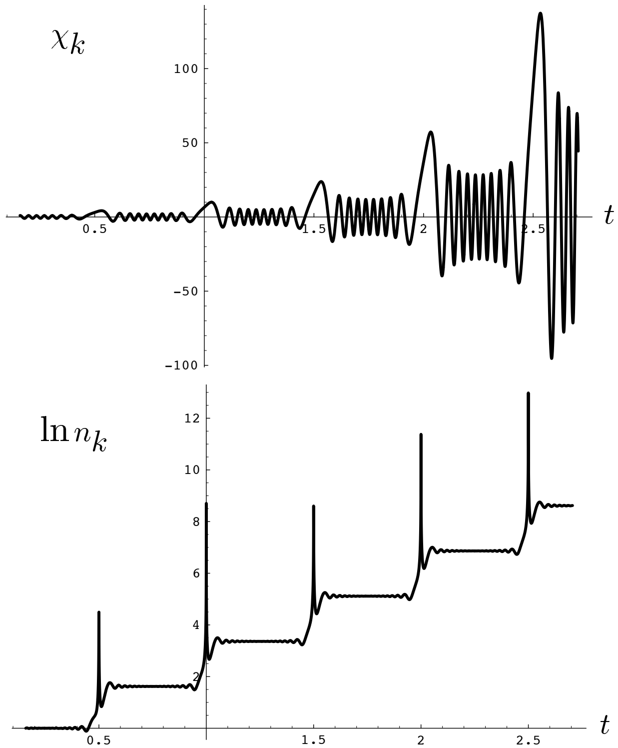
$$V = \frac{1}{2}m^2\phi^2 + \frac{g^2}{2}\phi^2\chi^2$$



G. Felder, I. Tkachev

$$V = \frac{1}{2}m^2\phi^2 + \frac{g^2}{2}\phi^2\chi^2$$

While lattice simulations required for the full dynamics, the early stages, and initial excitations of χ, can be obtained analytically



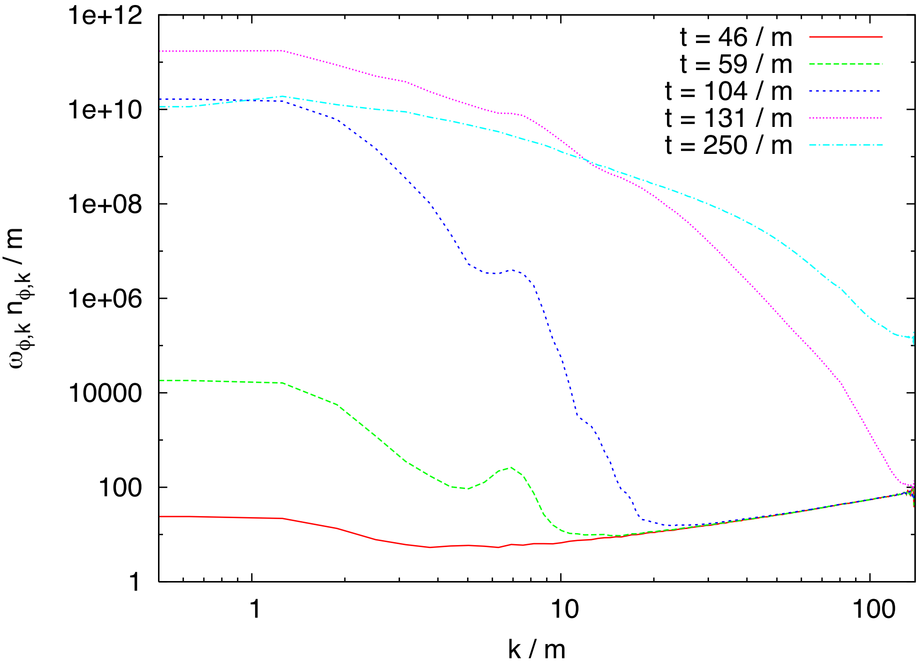
In this example, $m^2_{\chi,\text{effective}} = g^2\,\phi^2\,(t)$

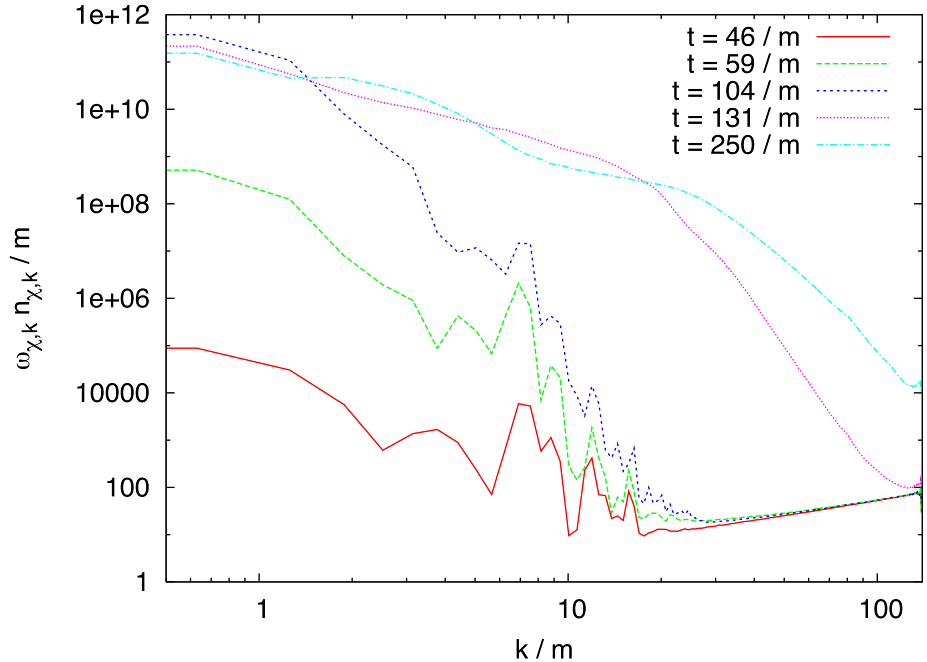
with $\phi\,(t) \simeq \Phi_0 \sin\,(mt)$

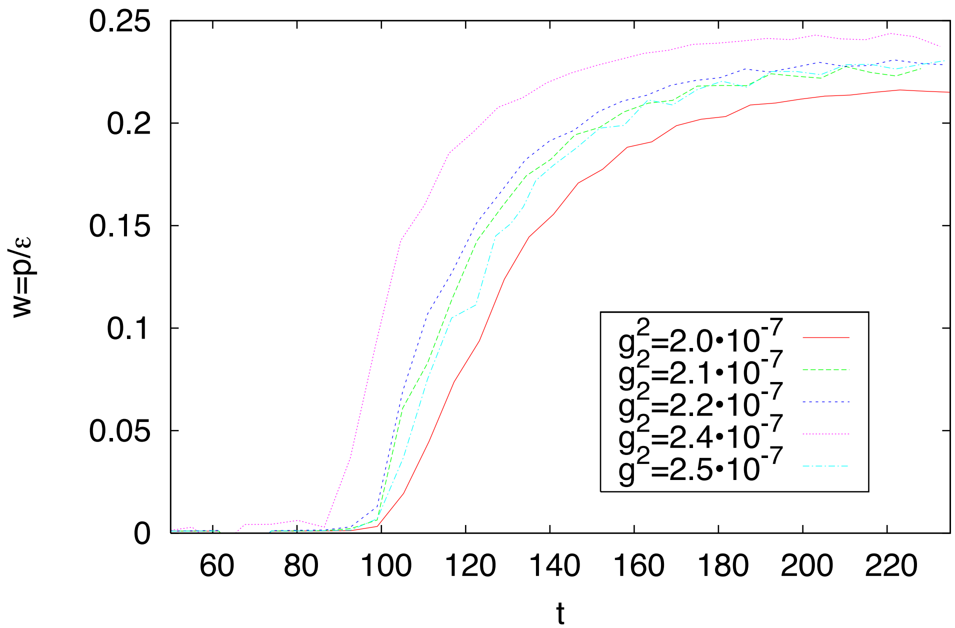
$$\Rightarrow \quad \omega = \sqrt{k^2 + g^2\,\Phi_0^2\,\sin^2\,(mt)}$$

- $\dot{\omega}/\omega^2$ maximum whenever $\phi = 0$
- Oscillator with periodically changing frequency $\rightarrow$ Resonance

Spectra from lattice simulations show initial growth, then saturation, then slow propagation towards UV. Still very far from thermal equilibrium







Intermediate equation of state between matter and radiation

Thermalization timescale well beyond the reach of the lattice

Slides lecture 4

Particle production / field amplification during inflation

$\phi(t) \;\rightarrow\; X(t, \vec{x}) \;\rightarrow\; \delta\phi(t, \vec{x}), \delta g(t, \vec{x})$

$\delta\phi/\phi \sim 10^{-5}$. Even dragging a small amount of energy out of ϕ could drastically change $\delta\phi$

Two possibilities considered in the literature:

- Isolated event(s) of particle production
- Continuous particle production

Consequences:

Modified density perturbations, $\left\langle \delta\phi^2 \right\rangle$, $\left\langle \delta\phi^3 \right\rangle$

Modified gravitational waves (GW), $\left\langle \delta g^2 \right\rangle$, $\left\langle \delta g^3 \right\rangle$

Primordial black holes (PBH)

Modified evolution of $\phi(t)$ (extra friction)

Isolated episodes of particle production

$$V = V(\phi) + \frac{g^2}{2}(\phi - \phi_*)^2 \chi^2$$

Chung, Kolb, Riotto, Tkachev '99

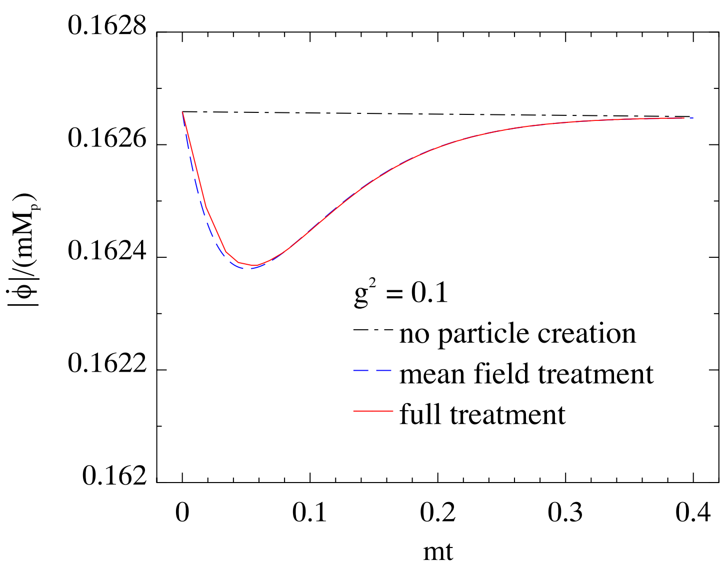
- $\phi(t) = \phi_*$ during inflation; at this point χ suddenly produced, with

$$n_\chi = \exp\left(-\frac{\pi}{g\,|\dot{\phi}_*|}\frac{k^2}{a_*^2}\right)$$

⟵ $* \equiv$ evaluated when $\phi = \phi_*$

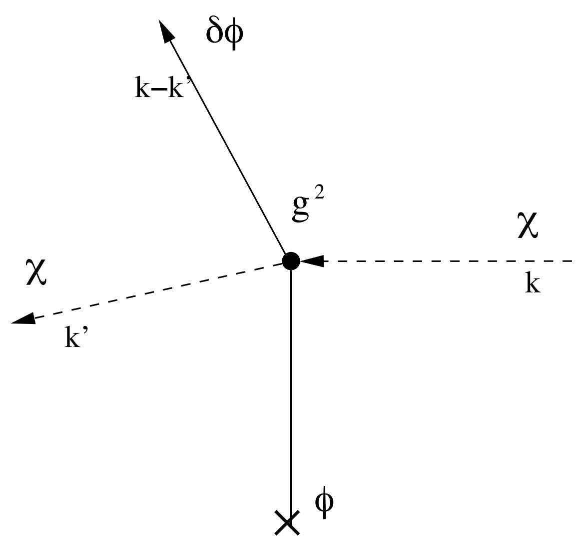
(solution of eqs. for α, β. See MP, Sorbo '00 for all details)

- $P_\zeta \propto \frac{1}{\epsilon} \propto \frac{1}{\dot{\phi}^2}$ Production of χ slows down inflaton



Bump on P_ζ at scales that leave the horizon at this time

- Main effect: scattering $\delta\chi + \phi \to \delta\phi + \ldots$



Barnaby, Huang, Kofman, Pogosyan '09

Pearce, MP, Sorbo '17

Perturbation theory for cosmological correlators: in-in formalism

Weinberg '05

$$\langle Q(t)\rangle = \sum_{N=0}^{\infty} i^N \int_{-\infty}^{t} dt_N \int_{-\infty}^{t_N} dt_{N-1} \ldots \int_{-\infty}^{t_2} dt_1$$

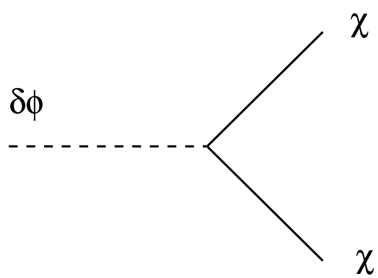
$$\times \left\langle \left[H_{\text{int}}(t_1), \left[H_{\text{int}}(t_2), \ldots \left[H_{\text{int}}(t_N), Q^{(0)}(t)\right] \ldots\right]\right]\right\rangle$$

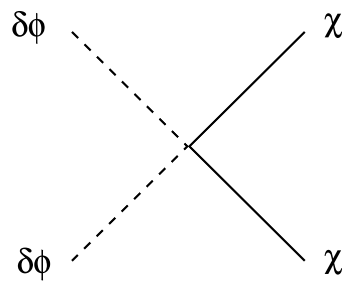
- Analogous to $S-$matrix elements computation, but here operators evaluated at same t (rather than $|\text{in}\rangle \to |\text{out}\rangle$)

$$V = V(\phi) + \frac{g^2}{2}(\phi-\phi_*)^2\chi^2 \longrightarrow H_{\text{int}} = a^4 g^2 \int d^3x \left\{[\phi-\phi_*]\,\delta\phi\,\chi^2 + \frac{1}{2}\delta\phi^2\chi^2\right\}$$

- Computations similar to standard QFT. In momentum space

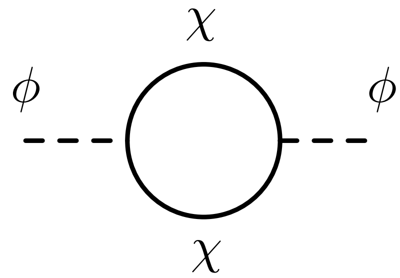
$$H_{\text{int},1} \propto \int d^3p_1 d^3p_2\, \delta\chi_{\vec{p}_1}\, \delta\chi_{\vec{p}_2}\, \delta\phi_{-\vec{p}_1-\vec{p}_2} \qquad H_{\text{int},2} \propto \int d^3p_1 d^3p_2 d^3p_3\, \delta\chi_{\vec{p}_1}\, \delta\chi_{\vec{p}_2}\, \delta\phi_{\vec{p}_3}\, \delta\phi_{-\vec{p}_1-\vec{p}_2-\vec{p}_3}$$

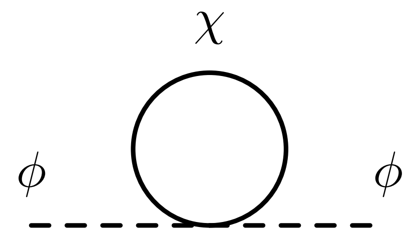




momentum conservation at the vertex

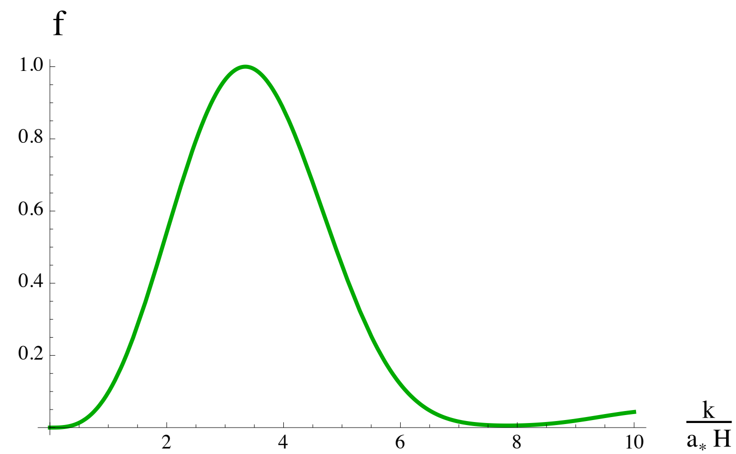
One loop diagrams



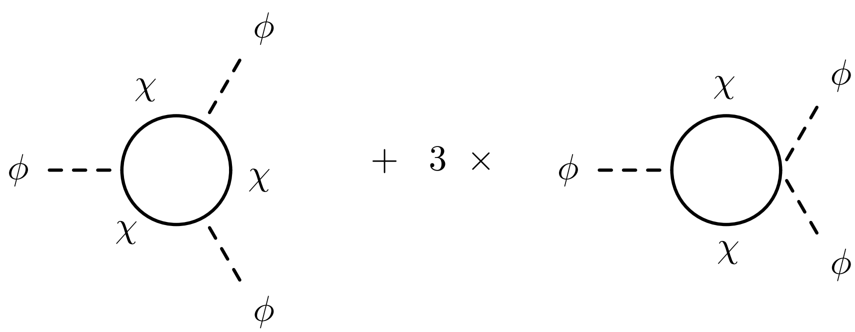


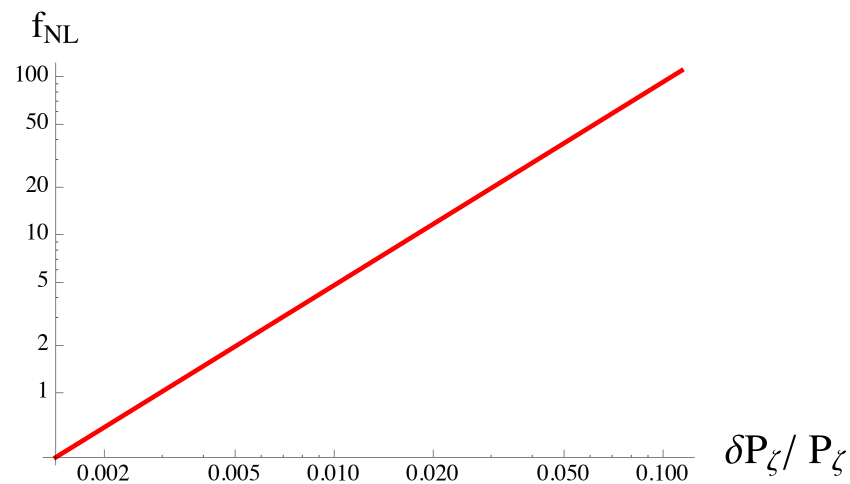
- First diagram dominates, leading to

$$\frac{\delta P_\zeta}{P_\zeta^{(0)}} \simeq 300\, g^{7/2}\, f\left(\frac{k}{a_* H}\right)$$



- Bispectrum $\left\langle \delta\phi^3 \right\rangle$ from





Analogous peak for $k_1 = k_2 = k_3 \simeq 4\, a_*\, H$

Planck 2018 : $f_{NL,\text{equilateral}} = -26 \pm 47$ (weaker limit for localized signal)

Trapped inflation

Green, Horn, Senatore, Silverstein '09

$$V = V(\phi) + \frac{g^2}{2} \sum_i (\phi - \phi_{*i})^2 \chi_i^2$$

- Inflaton coupled to many species χ_i, produced at $\neq$ values ϕ_{*i}

$$\ddot{\phi} + 3H\dot{\phi} + \frac{\partial V(\phi)}{\partial \phi} + g^2 \sum_i (\phi - \phi_{*i}) \left\langle \chi_i^2 \right\rangle = 0$$

$g(\phi - \phi_{*i}) =$ effective mass for χ

mass^2 $\times$ amplitude^2 = energy density

mass $\times$ amplitude^2 = # density

Integrating $\int d^3k \, n_k$ given above

$$\ddot{\phi} + 3H\dot{\phi} + \frac{\partial V(\phi)}{\partial \phi} + \sum_i \theta(t - t_{*i}) \left[\frac{a(t_{*i})}{a(t)}\right]^3 \frac{g^{5/2} \dot{\phi}^{3/2}}{(2\pi)^3} = 0$$

time when ϕ_{*i} is produced

Friction term. Particle production slows ϕ (energy conservation)

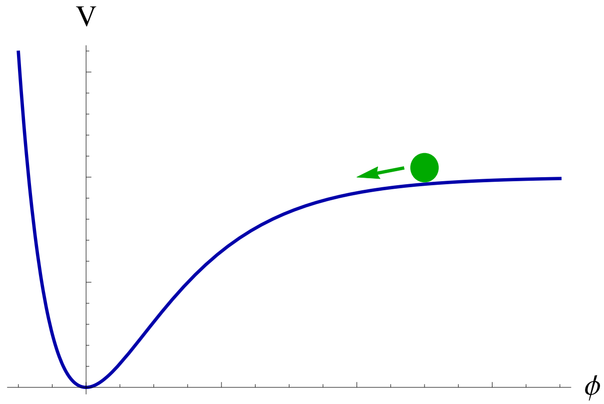
$$\ddot{\phi} + 3H\dot{\phi} + \frac{\partial V(\phi)}{\partial \phi} + \sum_i \theta(t - t_{*i}) \left[\frac{a(t_{*i})}{a(t)}\right]^3 \frac{g^{5/2}\,\dot{\phi}^{3/2}}{(2\pi)^3} = 0$$

- Assume dense prduction so that ϕ keeps being slowed down

- Particle production can allow slow roll inflation in $V(\phi)$ that is too steep if friction from expansion only. Realization of Warm inflation, Berera '95

- Perturbations studied in original paper through several approximations. Improved treatment in Pearce, MP, Sorbo '16

Continuous particle production / field amplification



Main theoretical difficulty is to keep the potential flat against radiative corrections

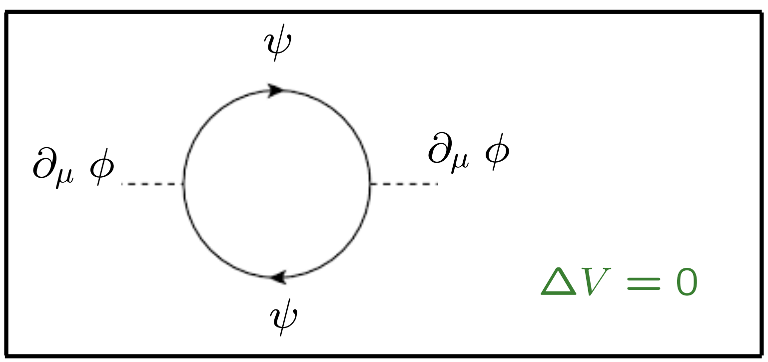
- Coupling to matter invariant under $\phi \to \phi +$ constant

Ex. axial symmetry: $\psi = \Psi e^{ia}$, $a \to a +$ const.. Called Axion / "Natural" Inflation

Freese, Frieman, Olinto '90, ...

Coupling to fermions : $\Delta\mathcal{L} = \frac{\partial_\mu \phi}{f} \bar{\psi}\gamma^5\gamma^\mu\psi$

to gauge fields : $\Delta\mathcal{L} = -\frac{\phi}{4f} F_{\mu\nu}\tilde{F}^{\mu\nu} \equiv \frac{\partial_\mu \phi}{2f}\epsilon_{\mu\nu\alpha\beta}A_\nu\partial_\alpha A_\beta$



Loops with these couplings do not modify the potential

Axion inflaton $\rightarrow$ gauge fields $\rightarrow$ $\delta\phi$, δg

$$\mathcal{L} = -\frac{1}{2}(\partial\phi)^2 - V(\phi) - \frac{1}{4}F^{\mu\nu}F_{\mu\nu} - \frac{\phi}{4f}\frac{\epsilon^{\alpha\beta\mu\nu}}{2}F_{\alpha\beta}F_{\mu\nu}$$

- Last term originally studied for magnetogenesis during inflation

Turner, Widrow '88

Garretson, Field, Carroll '92

Anber, Sorbo '06

...

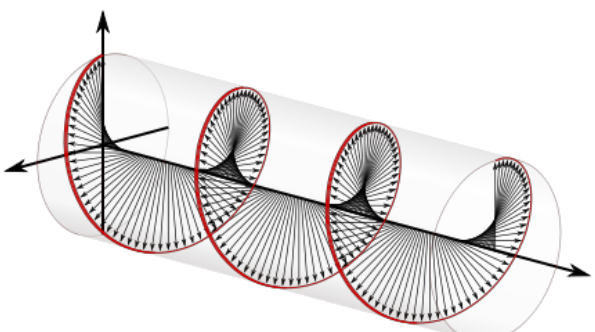
- $$-\frac{\phi}{4f}F_{\mu\nu}\tilde{F}^{\mu\nu} \equiv \frac{\partial_\mu\phi}{2f}\epsilon_{\mu\nu\alpha\beta}A_\nu\partial_\alpha A_\beta + \text{boundary term}$$

Production from $\phi(t) \Rightarrow \mu = 0$

One spatial momentum

- $\epsilon^{\mu\nu\alpha\beta}F_{\mu\nu}F_{\alpha\beta}$ changes sign under parity $\Rightarrow$ to have a scalar action, ϕ must be a pseudoscalar (also changes sign under $\vec{x} \rightarrow -\vec{x}$)

$\Rightarrow$ The motion $\phi(t)$ spontaneously breaks parity



$$\left(\frac{\partial^2}{\partial\tau^2} + k^2 \mp \frac{1}{f}\,a\,k\,\frac{d\phi}{dt}\right)A_\pm(\tau,k) = 0$$

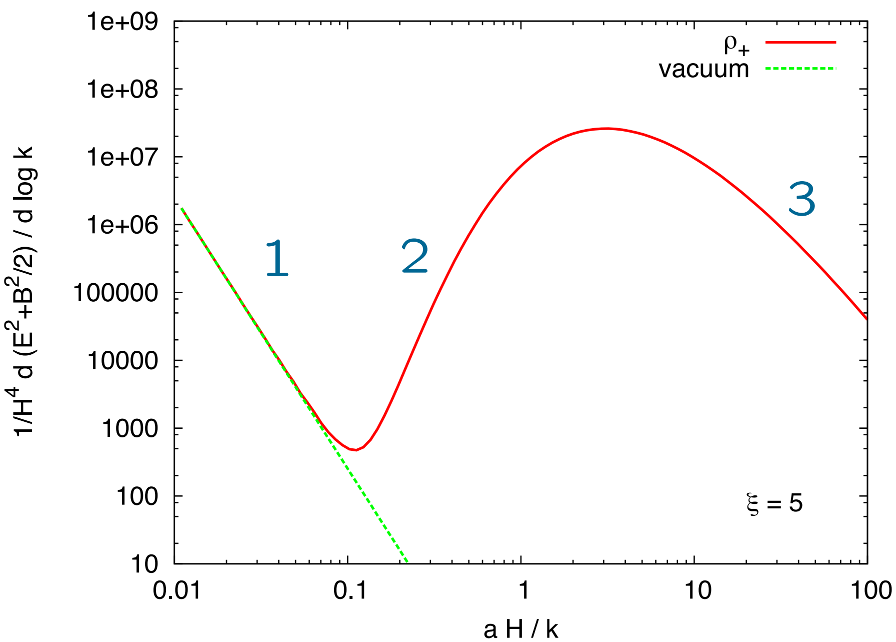
$+$ left handed

$-$ right handed

Convenient to define $\xi \equiv \frac{1}{2fH}\frac{d\phi}{dt} \quad \rightarrow \quad \left(\frac{\partial^2}{\partial\tau^2} + k^2 \mp 2\xi a\, H\, k\right) A_\pm(\tau,\, k) = 0$

★ As we shall see, phenomenology constrains $\xi = O(1)$. Last term dominates over second one for $aH \gtrsim k \;\Rightarrow\; \lambda \sim \frac{a}{k} \gtrsim H^{-1}$, right after horizon crossing

★ When last term dominant, A_+ tachyonic. One helicity highly amplified



1 k^2 dominates. Standard vacuum energy (must be renormalized away)

2 Tachyonic amplification of A_+ due to last term

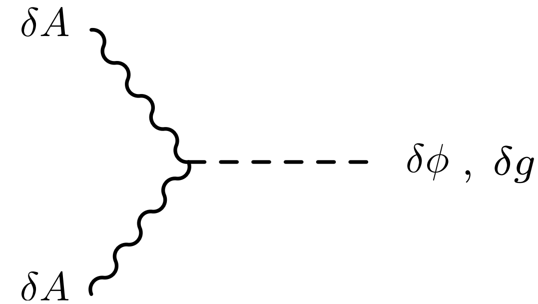
3 Dilution due to expansion

$A_{+,\text{max}} \propto \exp\left(\dot{\phi}\right)$

At any moment during inflation, modes of size comparable to the horizon at that time are produced. Each mode eventually dilutes away, and it is replaced by a mode that exits the horizon at the later time

- Each mode A_+, before being diluted, acts as a source of $\delta\phi$, δg

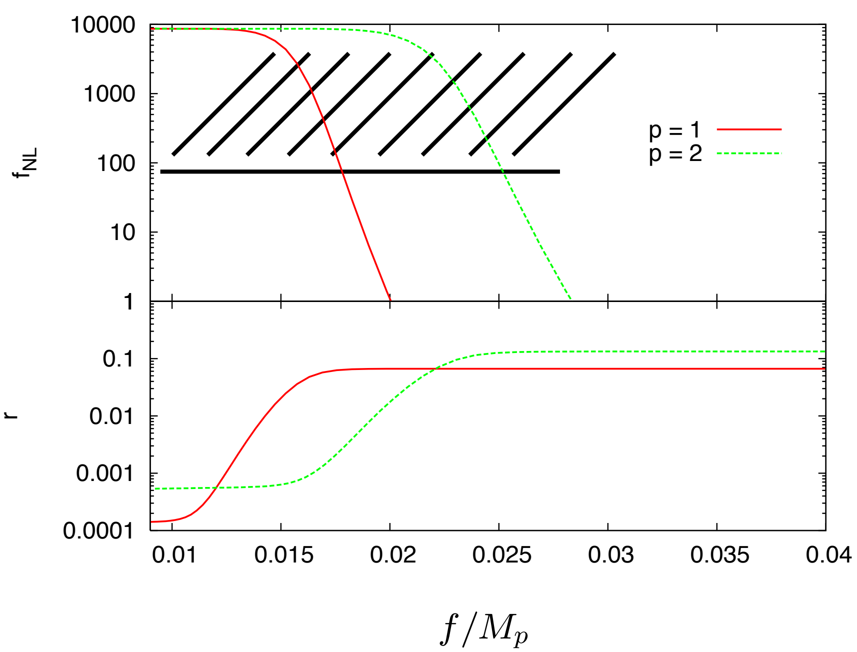
(see Barnaby, MP, Namba '11 for details)



Power spectrum : $\langle \delta\phi^2_{\rm vac} \rangle + \langle \delta\phi^2_{\rm sourced} \rangle$, Bispectrum : ~~$\langle \delta\phi^3_{\rm vac} \rangle$~~ $+ \langle \delta\phi^3_{\rm sourced} \rangle$

$$P_\zeta = \mathcal{P}_v \left[1 + 7.5 \cdot 10^{-5}\, \mathcal{P}_v \frac{e^{4\pi\xi}}{\xi^6} \right]$$

$$\mathcal{L} \supset -C\,\phi^p - \frac{\phi}{f} F\,\tilde{F}$$



Sourced perturbations highly non-Gaussian

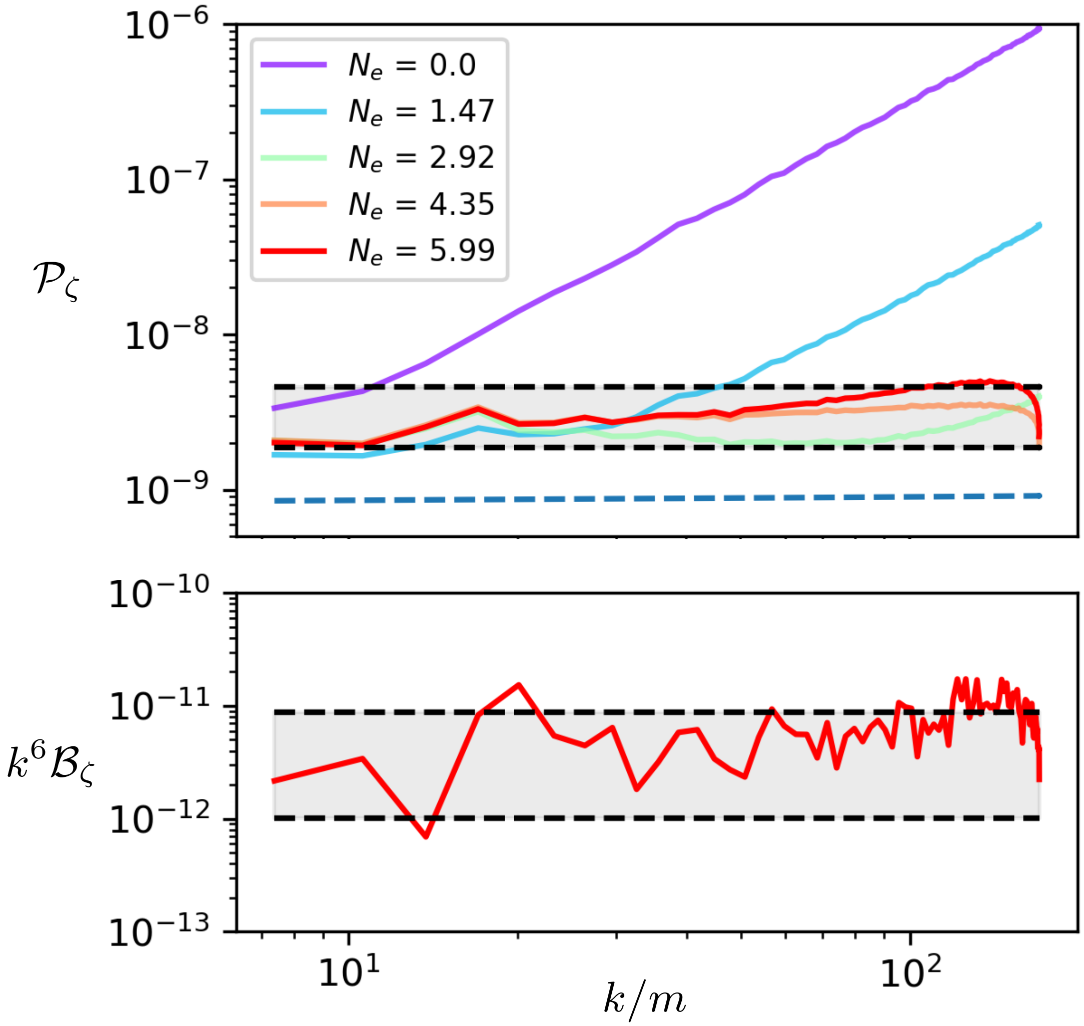
$f \gtrsim$ few $\times\ 10^{16}$ GeV

Gauge field also produces GW, $\delta A + \delta A \to h$

Chiral GW signal, $P_{\delta g,L} \gg P_{\delta g,R}$

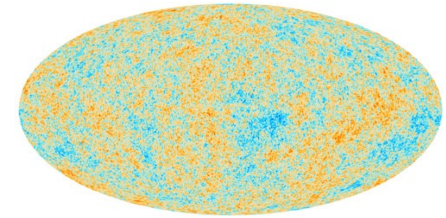
← More production

Fully numerical results in agreement with analytic computations



Caravano, Komatsu, Lozanov, Weller '22

GW as a probe of late inflation

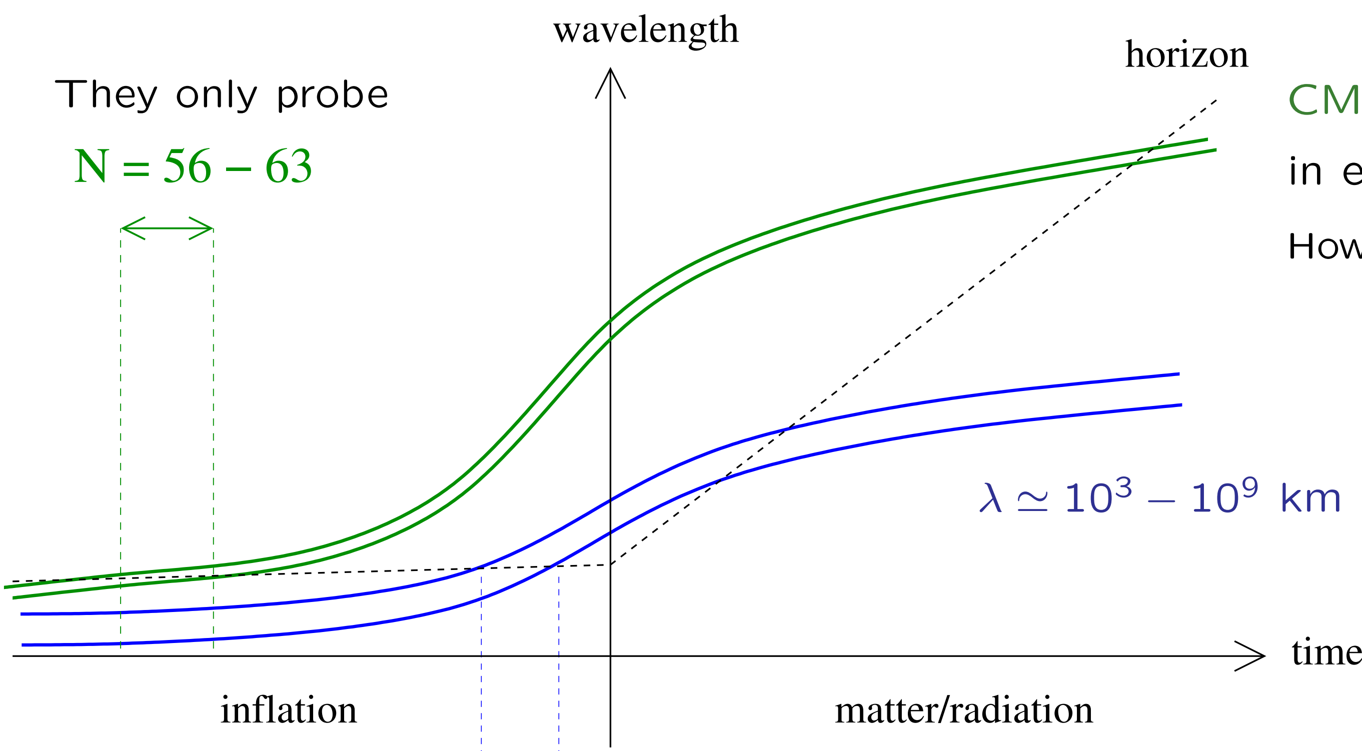


We give time in terms of e − foldings : $a \propto e^{Ht} = e^{-N}$

CMB modes produced at $N \simeq 60$ before the end of inflation, when $a \simeq e^{-60} a_{\rm end}$

They only probe

$N = 56 - 63$



$\lambda \simeq 10^3 - 10^9$ km

We can probe $N = 15 - 28$

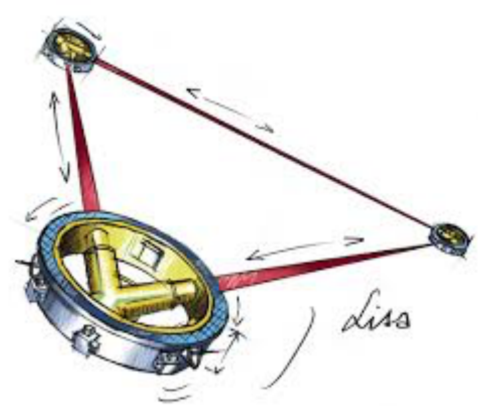
CMB and Large Scale Structure

in excellent agreement with inflation.

However, only probe $\lambda \simeq 10^2 - 10^5$ Mpc

Smaller scales / later times

essentially unprobed

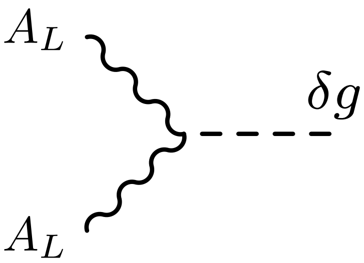


Development of GW interferometers

opens a new window on much

smaller scales

Sourced GW at interferometer scales in axion inflation



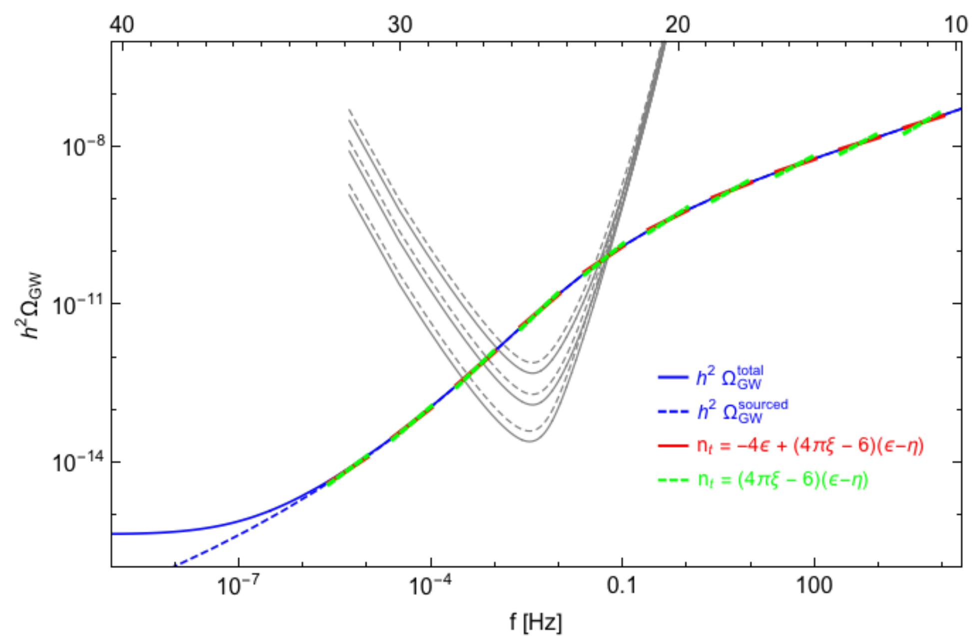
As discussed above, the amplified gauge fields also produce GW, though $A_+ A_+ \to h$ with amplitude $\propto \exp\left(\dot{\phi}\right)$

★ $\dot{\phi}$ grows during inflation (inflation ends because $\dot{\phi}$ too large) $\Rightarrow$ Blue GW and potentially visible at interferometers

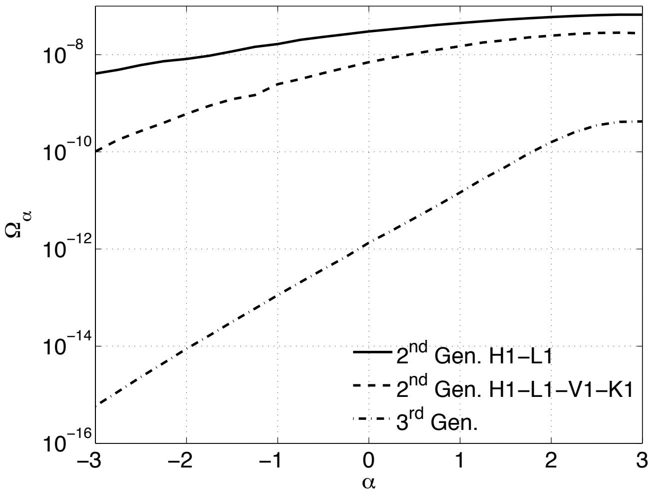
Cook, Sorbo '11; Barnaby, Pajer, MP'11;
Domcke, Pieroni, Binétruy '16; . . .

$$\Omega_{\rm GW} \equiv \frac{1}{\rho_c} \frac{d\rho_{\rm GW}}{d \ln f}$$

Bartolo et al '16; LISA cosmology WG



Measurement of GW chirality



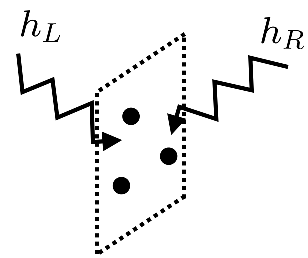
Crowder, Namba, Mandic, Mukohyama, MP '12

Assume $\Omega_{\mathrm{GW,L}} = \Omega_\alpha \left(\frac{f}{100\,\mathrm{Hz}}\right)^\alpha$ and $\Omega_{\mathrm{GW,R}} = 0$

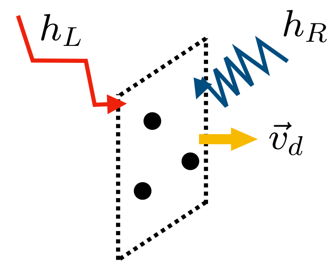
Amplitude needed to detect Ω_{GW}

and exclude $\Omega_{\mathrm{GW,R}} = \Omega_{\mathrm{GW,L}}$ at 2σ

Two GWs related by a mirror symmetry produce the same response in a planar detector. Cannot detect net circular polarization of an isotropic SGWB



Isotropy in any case broken by peculiar motion of the solar system. Assumption, $v_d \simeq 10^{-3}$ as CMB

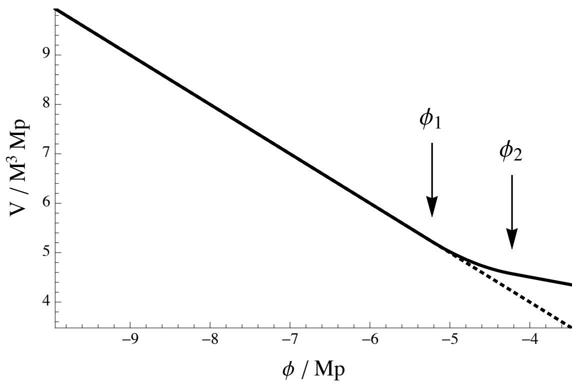


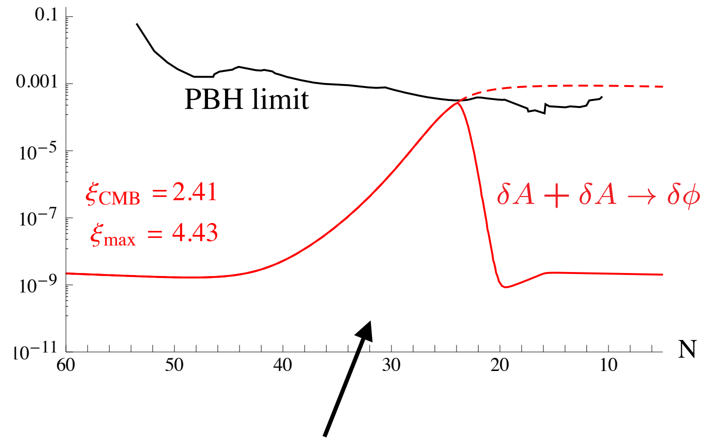
$$\mathrm{SNR}_{\mathrm{LISA}} \simeq \frac{v_d}{10^{-3}} \frac{\Omega_{\mathrm{GW,R}} - \Omega_{\mathrm{GW,L}}}{1.2 \cdot 10^{-11}} \sqrt{\frac{T}{3\,\mathrm{years}}}$$

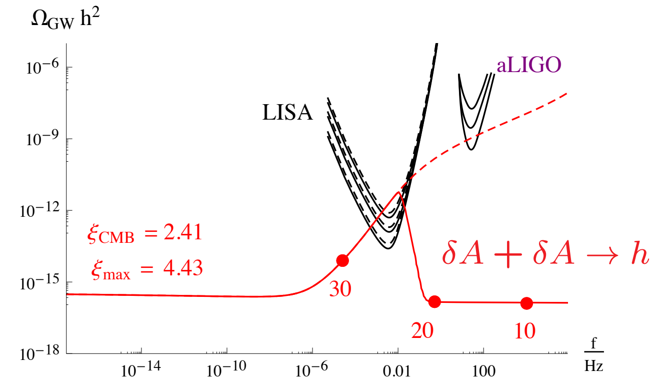
Domcke, García-Bellido, MP, Pieroni
Ricciardone, Sorbo, Tasinato '19

(one order of magnitude greater than estimate in Seto '06)

- Due to $\propto e^{\dot{\phi}}$, very sensitive to details in the potential





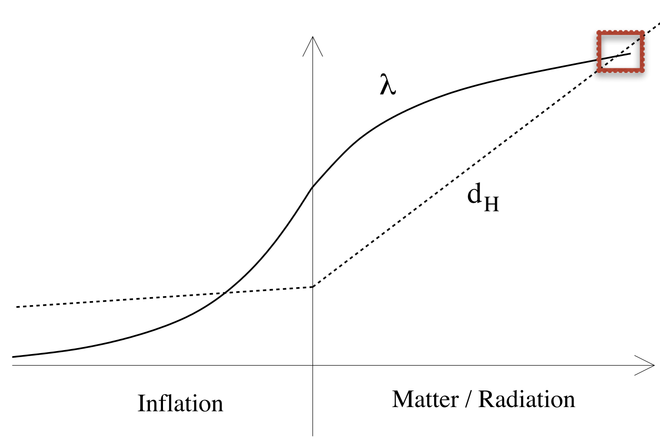


Linde, Mooij, Pajer '13

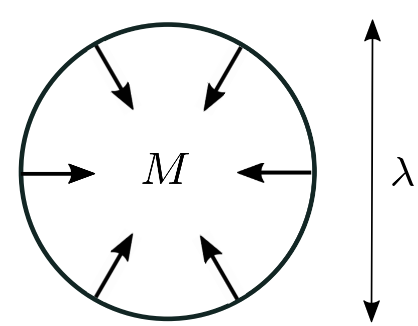
- Mechanism for a peaked distribution of Primordial Black Holes

Garcia-Bellido, MP, Unal '16

If sufficiently large, at horizon re-entry, the perturbation collapses to form a Primordial Black Hole (PBH)

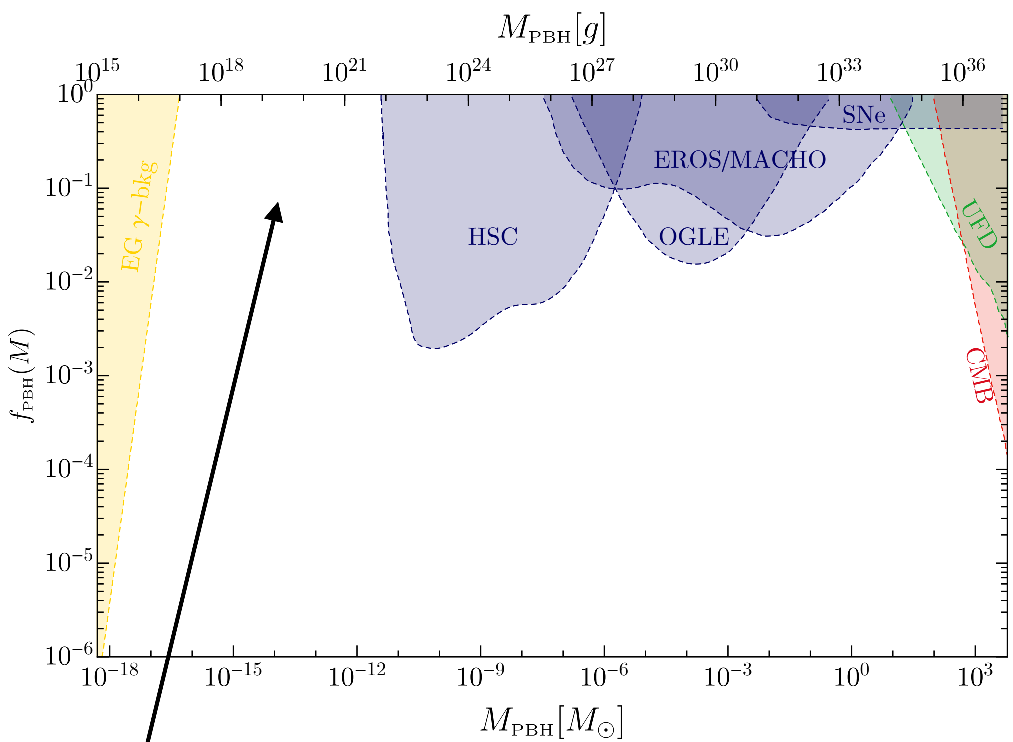


A significant fraction of the mass in the horizon collapses into the PBH. So, parametrically, $\lambda \leftrightarrow M_{\mathrm{PBH}}$



PBH dark matter

- PBH and PBH-DM long standing idea — Zel'dovich, Novikov '67; Hawking '71; Carr '75; Chapline '75
- Recent interest due to lack of detection of particle candidates, and LIGO / VIRGO events — Bird et al '16; Clesse, García-Bellido '16; Sasaki et al '16
- 2 windows, one at $\sim 10^{-12} M_\odot$, and (possibly) one at $\sim 10 - 100 M_\odot$



Credit: G. Franciolini, update of Carr, Kuhnel, Sandstad '16 and Inomata et al '17

Cut on HSC and on limits from femtolensing of γ-ray bursts. Schwarzschild radius$_{\rm PBH} < \lambda_\gamma$

Katz, Kopp, Sibiryakov, Xue '18

Limits from capture from NS and WD not shown due to uncertainty in DM astrophysical abundance, and on nuclear physics

Capela, Pshirkov, Tinyakov '13

Montero-Camacho, Fang, Vasquez, Silva, Hirata '19

PBH $\leftarrow$ enhanched $\delta\rho$ $\rightarrow$ GW

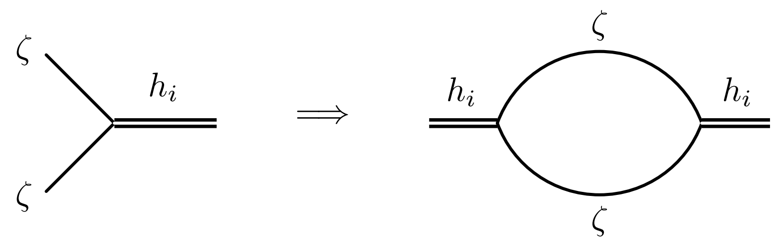
- Whenever $\delta\rho$ present GW produced

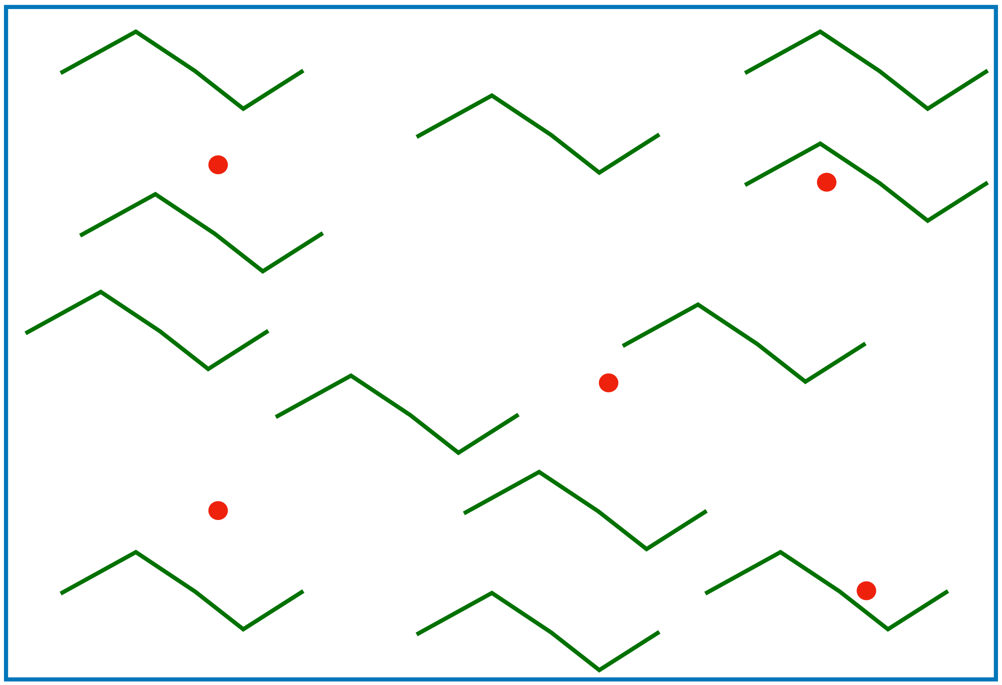
  1) during inflation, by the same source that produced $\delta\rho$

  2) by $\delta\rho$ at horizon re-entry after inflation

- Mechanism 2 is unavoidable and model-independent

Standard gravitational interaction:

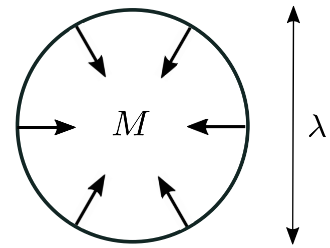


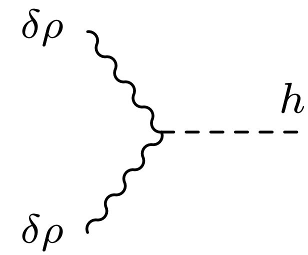


Warning: NOT the GW formed by the collapsing regions that form PBH (These regions have very small ρ at that time!) These are the GW produced everywhere by $\delta\rho$ that has been enhanced everywhere, so to produce the few collapsing regions

PBH mass $\rightarrow$ wavelength enhanced $\delta\rho$ modes $\rightarrow$ GW frequency

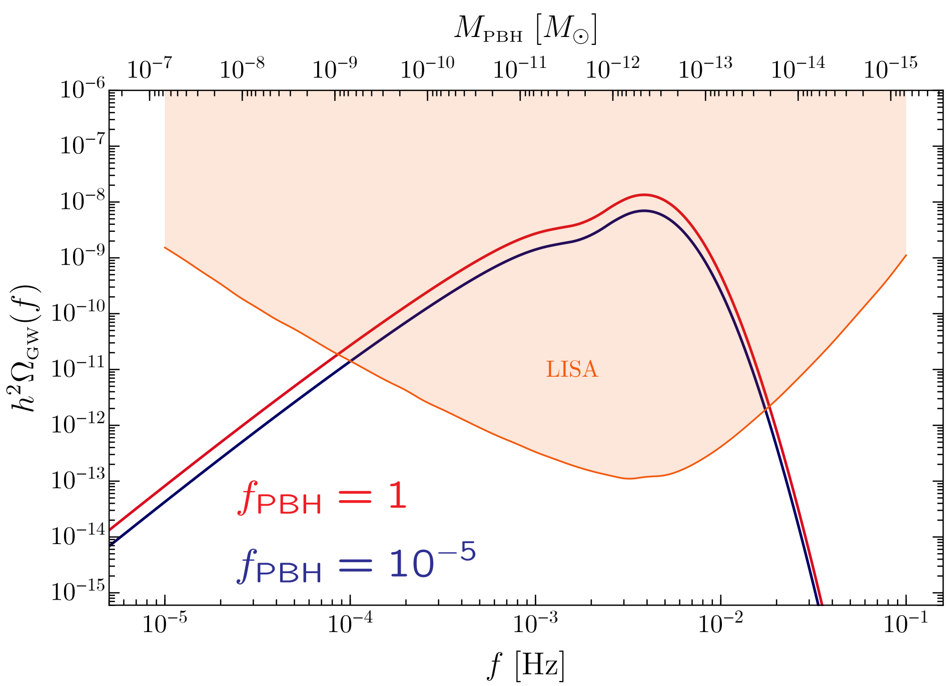
$$f_{\rm GW} \sim \frac{1}{\lambda} \sim 3\,{\rm mHz}\sqrt{\frac{10^{-12}\,M_\odot}{M}}$$





$M \sim 10\,M_\odot \quad \Rightarrow \quad f_{\rm GW} \sim {\rm nHz}$ PTA!

$M \sim 10^{-12}\,M_\odot \quad \Rightarrow \quad f_{\rm GW} \sim {\rm mHz}$ LISA!



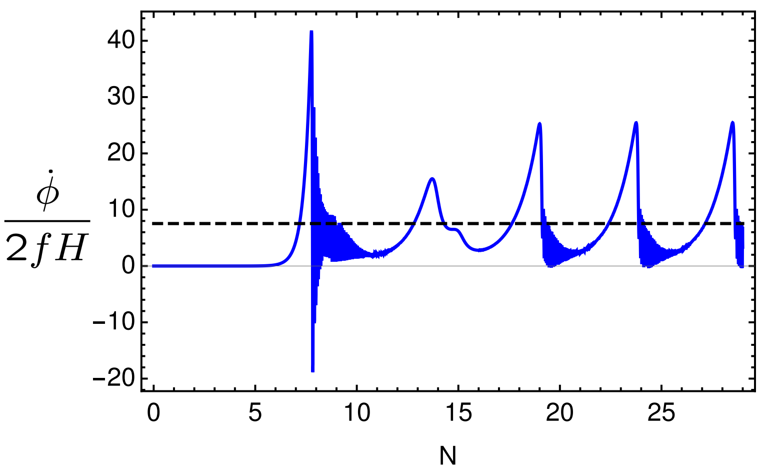
GW allow to probe existence of PBH of insignificant cosmological abundance

- Anber, Sorbo '09: QFT realization of warm inflation via $\frac{\phi}{f} F\tilde{F}$ coupling

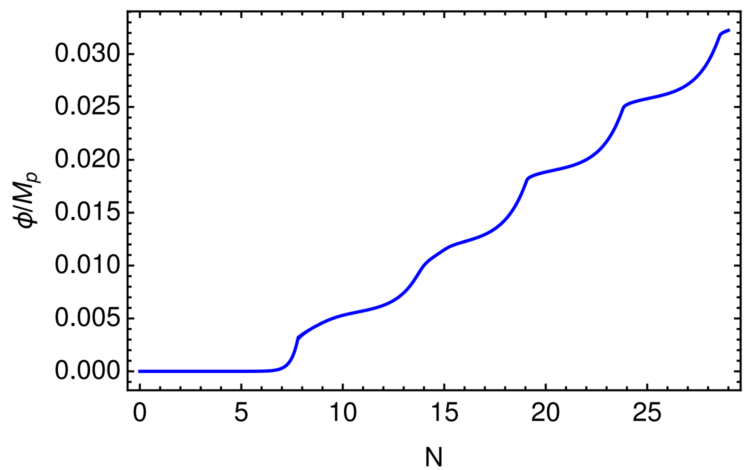
$$\ddot{\phi} + 3H\dot{\phi} + V' = -\frac{1}{4\pi^2 a^3 f}\int dk\, k^3 \frac{\partial |A|^2}{\partial \tau}$$

$$-\frac{2\times 10^{-4}}{f}\left(\frac{H}{\xi}\right)^4 e^{2\pi\xi} \quad , \quad \xi = \frac{\dot{\phi}}{2Hf}$$

$\longrightarrow$ steady state solution with $\simeq$ constant $\dot{\phi}$ and negligible $3H\dot{\phi}$



dashed line Anber-Sorbo solution



Burst of A production $\to$ inflaton slows down $\equiv$ negligible production,

and A redshifts away $\to$ inflaton speeds up $\equiv$ burst of production $\to$ . . .

Cheng, Lee, Ng '16; Notari, Tywoniuk '16; Dall'Agata, Gonzalez-Martın, Papageorgiou, MP '19; Domcke, Guidetti, Welling, Westphal '20; Caravano, Komatsu, Lozanov, Weller '22

Thank you for your attention

Enjoy the rest of the School!