ICTP Summer School on Cosmology 2022

1. "Measuring the accelerated expansion"

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- FLRW metric

The Universe is roughly homogeneous & isotropic on large scales.
FLRW metric is a good approximation

$$ds^2 = -\underset{\substack{\uparrow \\ c=1}}{c^2} dt^2 + \underset{\substack{\uparrow \\ \text{scale factor}}}{a^2(t)} \Big[\underbrace{d\chi^2 + f_K^2(\chi)\left(d\theta^2 + \sin^2\theta \, d\varphi^2\right)}_{\text{spatial metric}} \Big]$$

$$f_K(\chi) = \begin{cases} \sin\chi & K = 1 \quad \text{closed} \\ \chi & K = 0 \quad \text{flat} \\ \sinh\chi & K = -1 \quad \text{open} \end{cases}$$

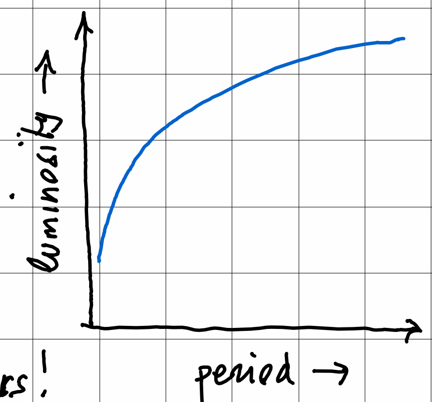
• Measuring the acceleration

Want to measure $a(t)$. We observe distances & redshifts (c.f. Hubble).
In cosmology there are two main ways of measuring distances.

- standard candles : objects of known luminosity
- " rulers : " " " size

* Example of standard candle : Cepheids

Pulsating stars with stable periodic luminosity.
Relation between period & luminosity discovered in 1908 by Henrietta Swan Leavitt.
Crucial for all cosmological observations.
Allowed to go from $d \sim 10^2$ l.y. to 10^{10} light years!



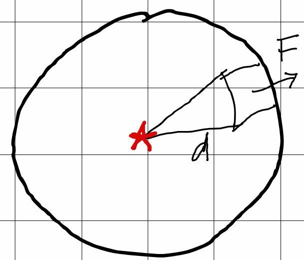
In cosmology we need more luminous objects & to define distances in an expanding spacetime : Supernovae & luminosity distance!

- Luminosity distance

Luminosity : radiated EM energy per unit time (power) $\frac{dE}{dt}$

Flux :

$$F = \frac{L}{A} = \frac{L}{4\pi d^2}$$



Amount of energy reaching a surface. Telescopes measure this.

In an expanding universe $A = \iint d\hat{S}_2 = a^2 f_K^2(\chi) \iint d\Omega_2 =$

induced metric on the sphere

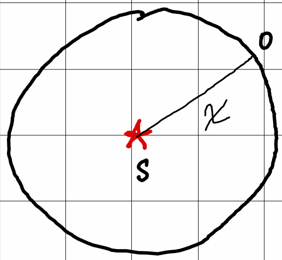
$= 4\pi a^2 f_K^2(\chi)$

We use the same definition as in flat spacetime

$$F_{obs} = \frac{L_{obs}}{A(t_{obs}, \chi)} \; ; \; L_{obs} = \frac{dE_{obs}}{dt_{obs}} = \frac{dE_{obs}}{dE_{em}} \frac{dt_{em}}{dt_{obs}} \frac{dE_{em}}{dt_{em}} = \frac{1}{(1+z)^2} L_{em}$$

$$F = \frac{L}{4\pi d_L^2} \quad \text{where} \quad d_L = (1+z)\, a_0\, f_K(\chi)$$

Luminosity distance



We need to find χ (comoving distance)

Photons travel along $ds^2 = 0 \Rightarrow dt = \pm a(t)\, d\chi$

$$\chi = \int_{t_s}^{t_0} \frac{dt}{a(t)} = \int_{a_s}^{a_0} \frac{da}{a} \frac{dt}{da} = \int_{a_s}^{a_0} \frac{da}{a^2 H(t)} = \frac{1}{a_0} \int_0^{z_s} \frac{dz}{H(z)}$$

$$a(z) = \frac{a_0}{1+z} \qquad da = -\frac{dz}{(1+z)^2}\, a_0$$

Def: $E(z) \equiv \dfrac{H(z)}{H_0}$ with $H_0 \equiv H(z=0)$

Def: $\Omega_{K,0} \equiv -\dfrac{K}{a_0^2 H_0^2} \Rightarrow a_0 = \dfrac{H_0^{-1}}{\sqrt{|\Omega_{K,0}|}}$

$$\boxed{d_L(z) = (1+z) \frac{H_0^{-1}}{\sqrt{|\Omega_{K,0}|}} f_K\left(\sqrt{|\Omega_{K,0}|} \int_0^z \frac{dz'}{E(z')} \right)}$$

- For small z: $E(z) = \frac{H(z)}{H_0} \simeq \frac{H_0}{H_0} = 1$: $\quad d_L(z) \simeq H_0^{-1} z$ Hubble law

- Supernovae are found for $z \lesssim 1$. We can expand for small z. Moreover, curvature today is small and enters only at third order in z. Up to second order in z: (Ex. 1.1)

$$d_L(z) = (1+z)\int_0^z \frac{dz'}{H(z')} = d_L(0) + d_L'(z)\Big|_{z=0} z + \frac{1}{2} d_L''(z)\Big|_{z=0} z^2 + \dots$$

$$d_L'(z)\Big|_{z=0} = H_0^{-1} \; ; \quad d_L''(z)\Big|_{z=0} = H_0^{-1}\left(1 - \frac{\ddot{a}}{Ha}\right)_{z=0}$$

- Def: traditionally $\quad q \equiv - \frac{\ddot{a}/a}{H^2} = - \frac{\ddot{a}\,a}{\dot{a}^2}$ $\quad$ dimensionless quantity

Acceleration for $q_0 < 0$!

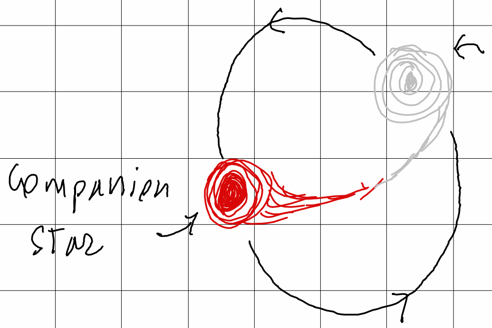
the sign is historical: people were expecting to measure $\ddot{a} < 0$!

$$\Rightarrow \boxed{d_L(z) = H_0^{-1} z \left[1 + \frac{1}{2}(1-q_0) z + \dots\right]}$$

Supernovae type Ia

- **Supernovae type Ia**

Supernovae type Ia can be used as standard candles: very bright! Ideal to measure acceleration



accreating white dwarf, sustained by electron pressure ($m < 1.44\, M_\odot$)

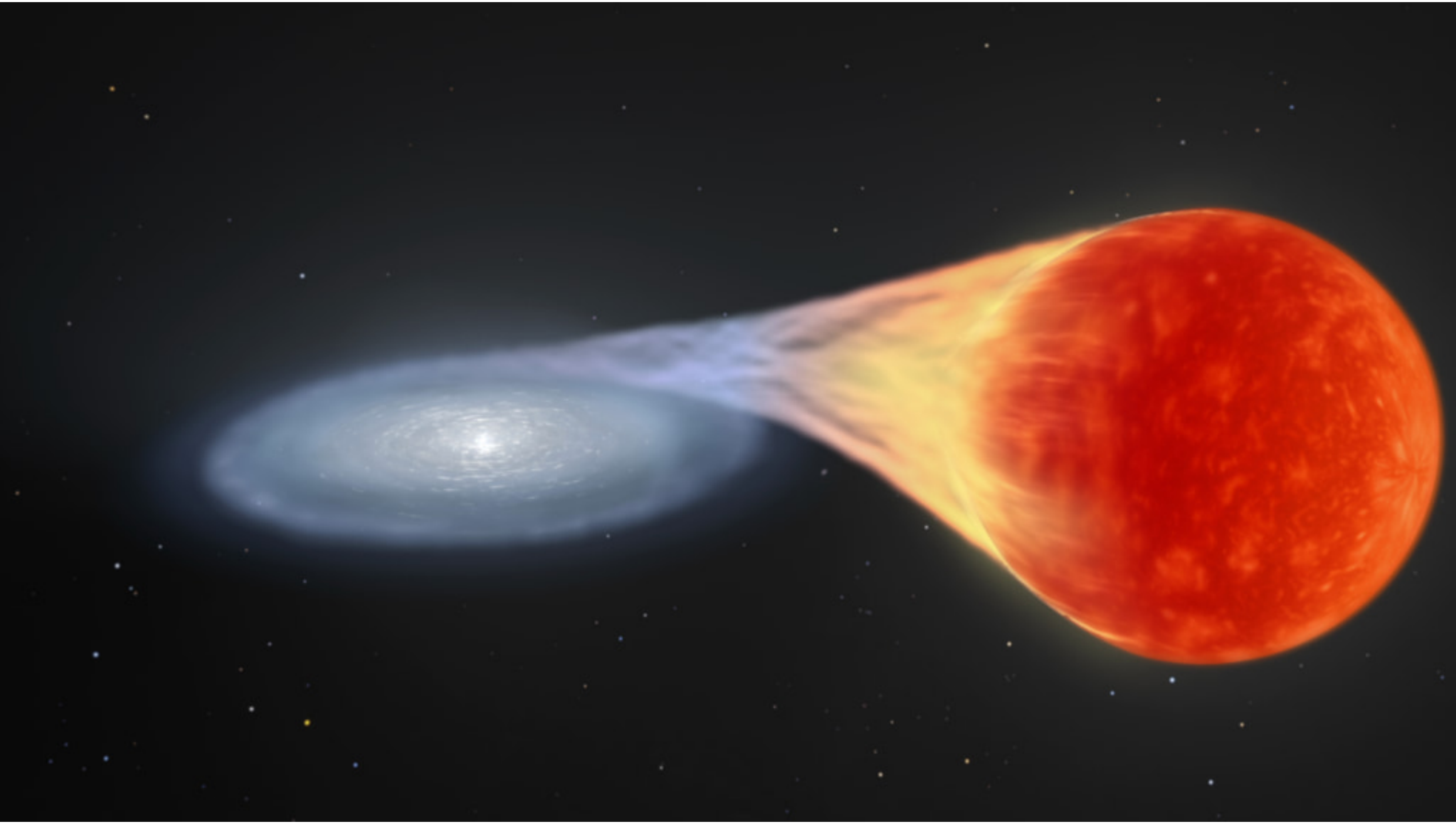
Mass of white dwarf increases by accreation. Internal temperature raises, carbon starts fusing:

C + C → heavier elements

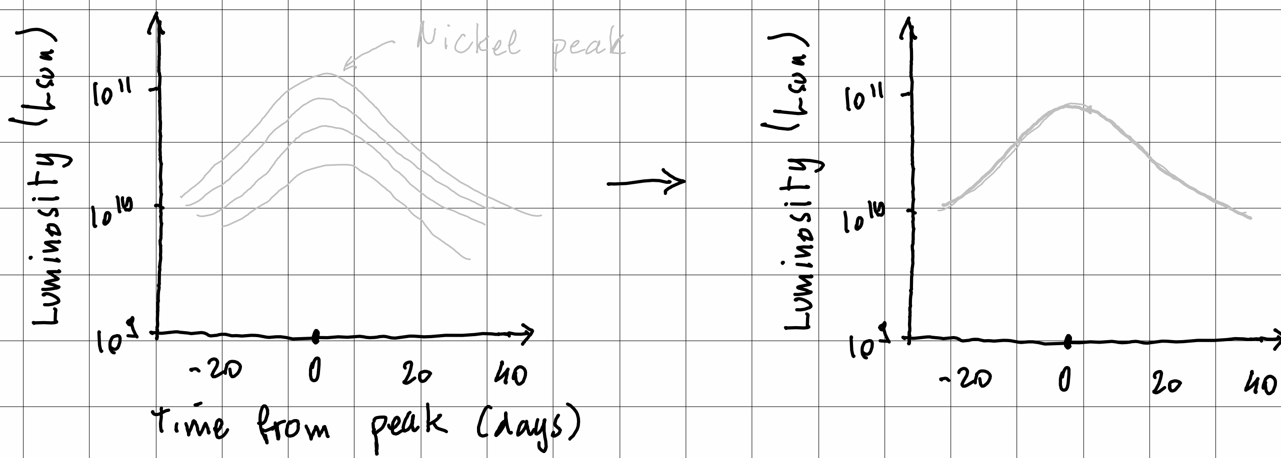
Runaway reaction: violent process, carbon detonation!

This happens at the critical (Chandrasekhar) mass $1.44\, M_\odot$, when gravity overcomes electron degeneracy pressure.

Supernovae type Ia



Characteristic luminosity vs time light-curve :



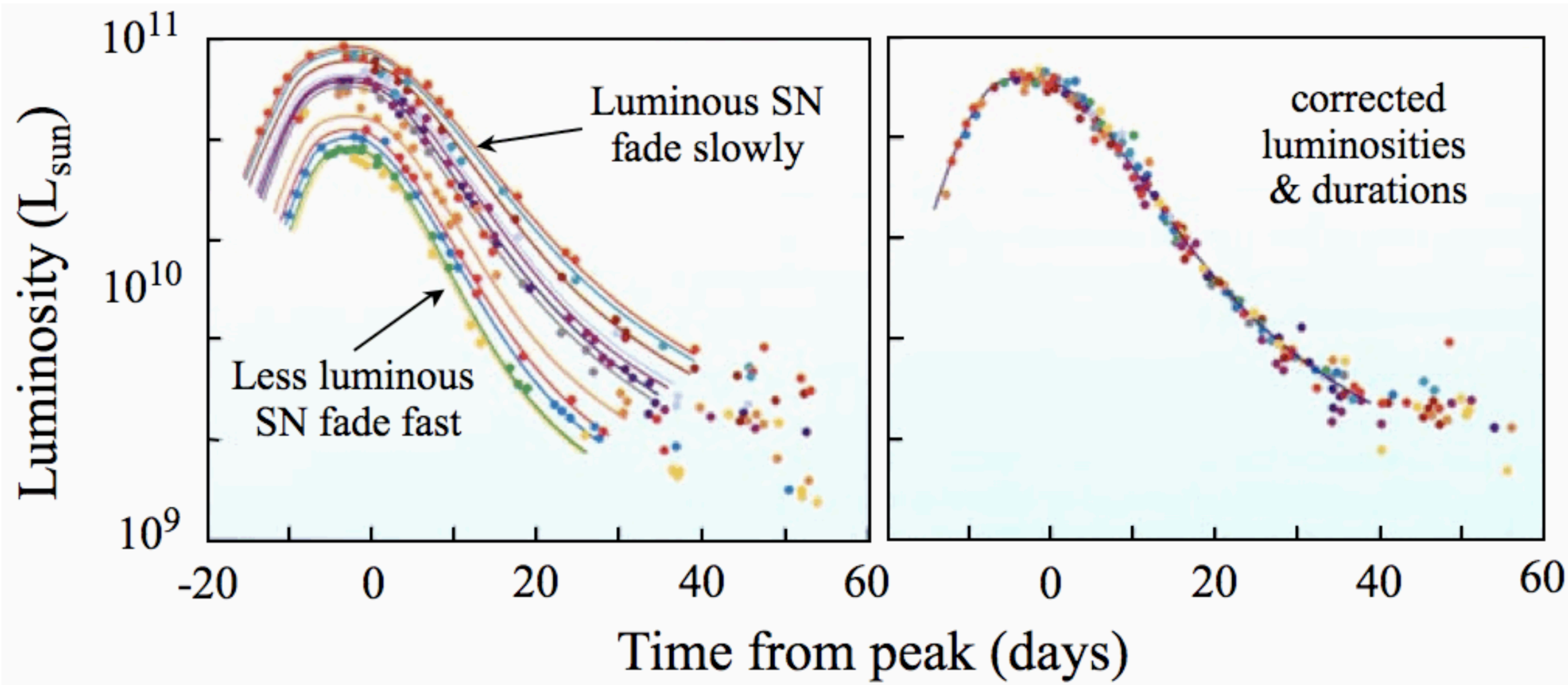
Luminosity peak determined by amount of Nickel production : more Ni = = higher peak and slower decay. Simple models (e.g. Phillips relation) can be used to standardise supernovae : **standard candles!**

Distant supernovae **calibrated** with near supernovae, that are calibrated with Cepheids, that are calibrated by parallax : **distance ladder**.

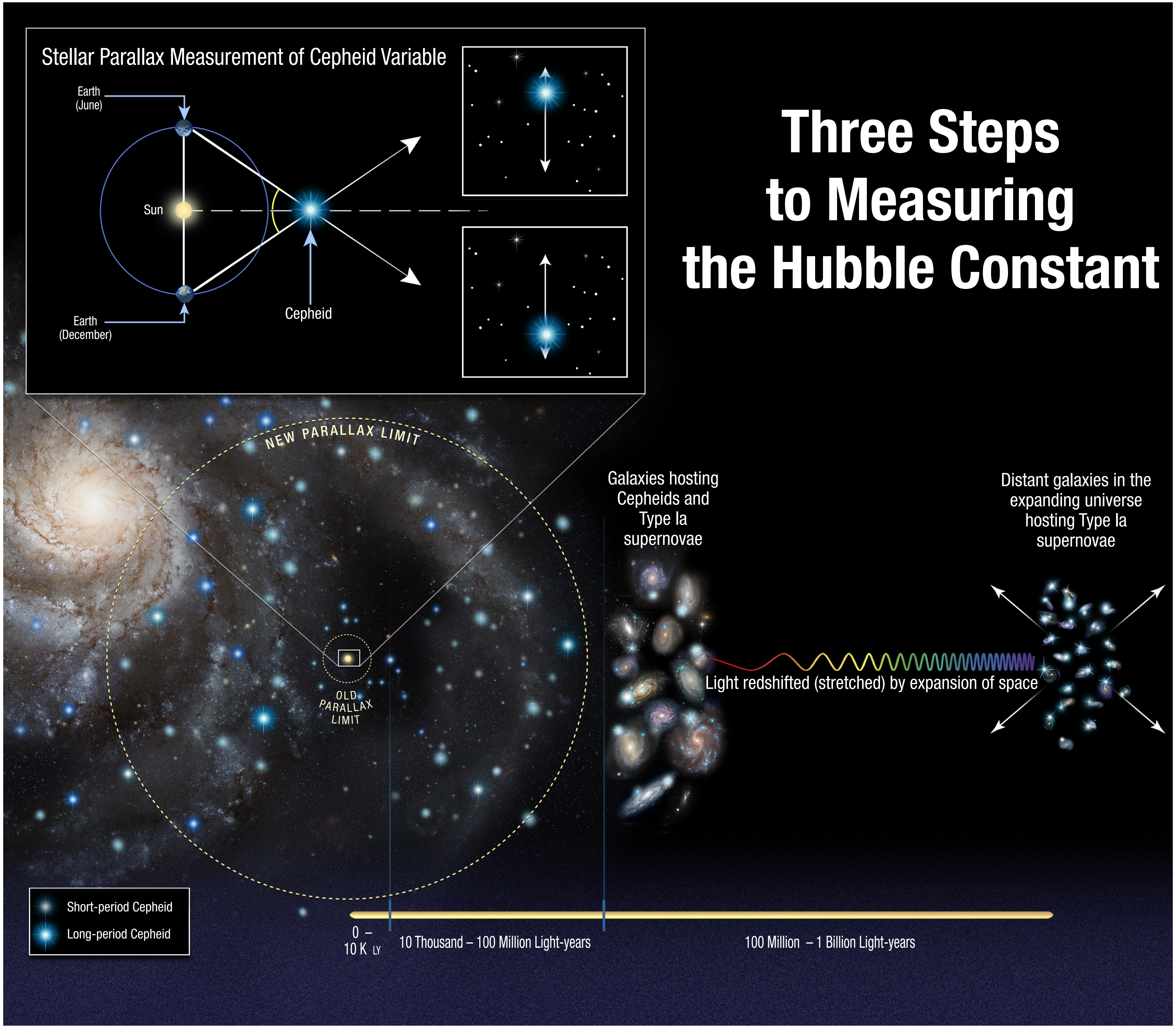
SN calibration limited factor : intrinsic scatter (few Cepheids), possibly redshift dependence (we use the same model at all z's).

Supernovae type Ia

Characteristic luminosity vs time light-curve:

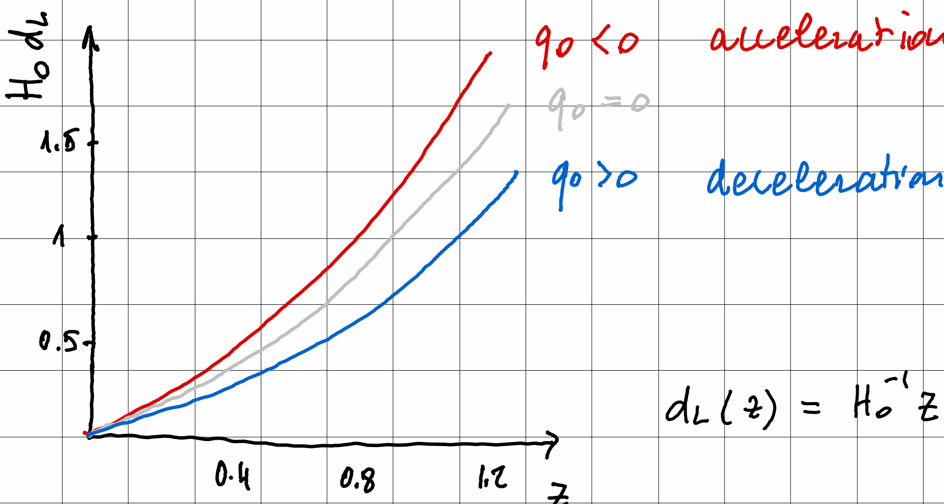


The distance ladder



- The discovery of $q_0 < 0$

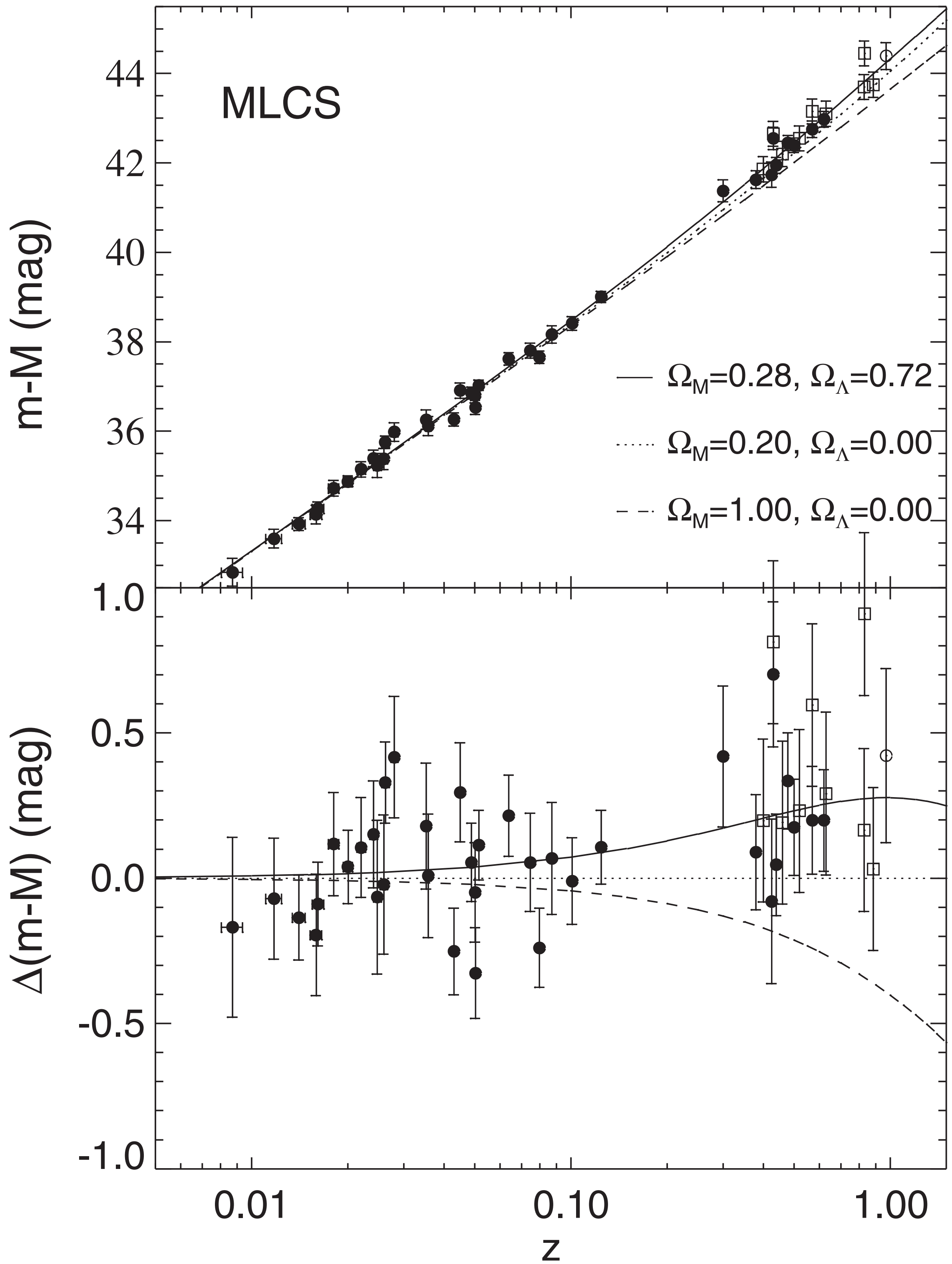
Nobel price in 2011 for 1998 discovery:
Saul Perlmutter (Supernovae Cosmology Project) and
Adam Riess and Brian P. Schmidt (High-z Supernovae Search team).
[book on the history of the discovery: "The 4% Universe" by R. Panek]



← supernovae appear dimmer (at fixed z) and slower (at fixed distance)

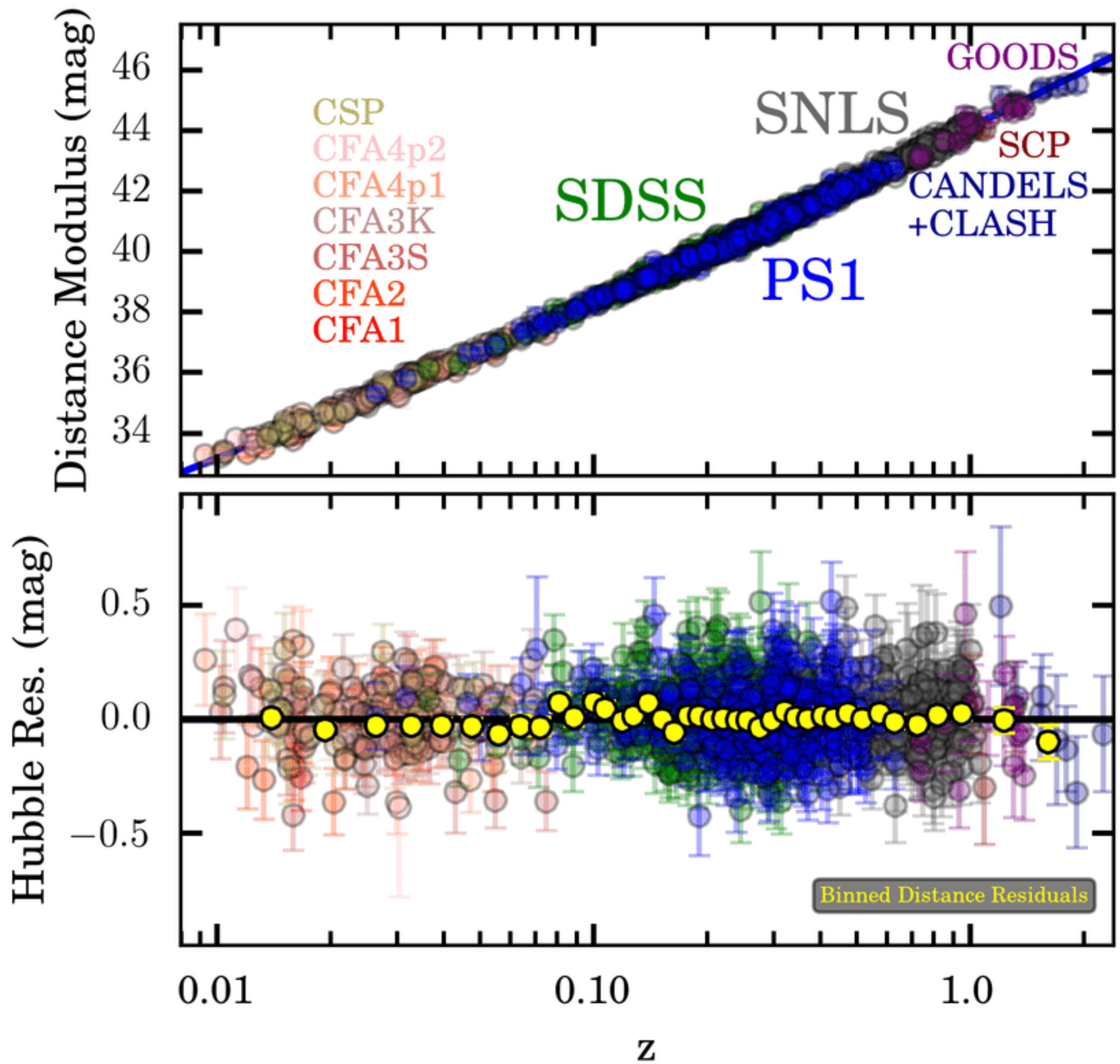
$$d_L(z) = H_0^{-1} z \left[1 + \frac{1}{2} (1 - q_0) z + \dots \right]$$

Luminosity distance/redshift relation



A. V. Filippenko and A. G. Riess 2000

Luminosity distance/redshift relation



The Hubble diagram for the Pantheon sample. Scolnic et al. ‘21

• Standard sirens

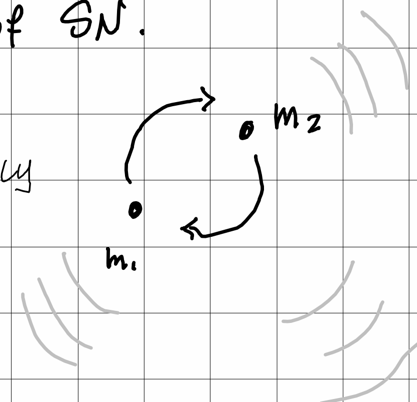
Use GW instead of light. BH binaries instead of SN.

$$h_{+,\times} \sim \frac{4}{d_L}(G\mathcal{M}_c)^{5/3}(\pi f)^{2/3}$$

GW frequency

GW strain

Chirp mass $\mathcal{M}_c \equiv \dfrac{(m_1 m_2)^{3/5}}{(m_1+m_2)^{1/5}}(1+z)$



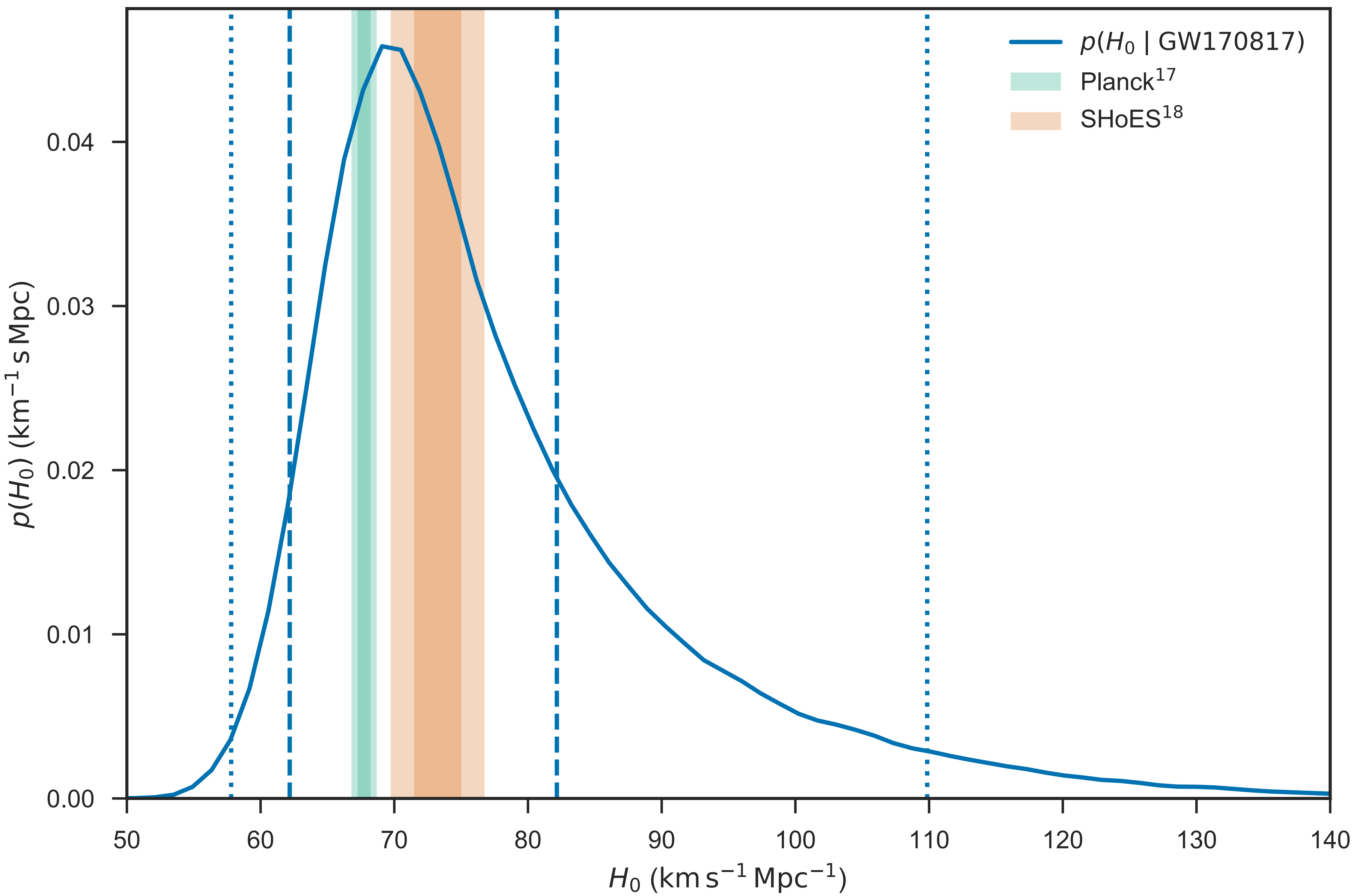
the chirp mass can be inferred from $\dot{f}$:

$$\frac{df}{dt} \sim \frac{96}{5}\pi^{8/3}(G\mathcal{M}_c)^{5/3} f^{11/3} \qquad \text{(from quadrupole formula)}$$

Advantage: the system is self-calibrated (and can go at high z if $\mathcal{M}_c$ is large). However, impossible to deduce z without an associated optical counterpart (e.g. GW170817): **mass-redshift degeneracy**.

Same signal from $m_{1,2}$ at $z=0$ & $\dfrac{m_{1,2}}{1+z}$ at z.

H_0 from GW170817



$$H_0 = 69^{+17}_{-8}\ \mathrm{km/Mpc/s}$$

Errors:

1) Hubble flow velocity: need to account for peculiar velocities, caused by local grav field, attraction to nearby galaxies

$$v_H = 3017 \pm 166\,\mathrm{km\,s^{-1}}.$$

2) Distance error large,

$$d = 43.8^{+2.9}_{-6.9}\,\mathrm{Mpc}$$

3) statistical measurement error from noise in detectors + instrumentation calibration uncertainties:

Several ways of determining the redshift

- Direct EM counterpart with z measured (GW 170817)
- Collection of galaxies localized in the GW localization volume

$\Big\}$ Schutz '86

- Knowledge of the source-frame mass distribution (peaks, mass gaps, etc.)
- Tidal deformability breaks mass-redshift degeneracy (for NS)

Potentially interesting in the future

Conclusions Lecture 1

- To measure the expansion rate we need to measure distances and redshifts: **luminosity distance** and **angular diameter distance**. We have derived the dependence of the luminosity distance on the deceleration parameter, which can be measured by **standard candles**.

- **Type Ia supernovae** are very bright and their brightness depends on the time-luminosity relation: They are ideal standard candles to go at high redshifts. They have been used to measure the expansion at high redshifts and **discover the cosmic acceleration**. Concordance with other observations. Standard sirens potentially useful on the future.

EXERCISES

1.1 verify $d_L(z) = H_0^{-1} z \left[1 + \frac{1}{2}(1-q_0) z + \dots \right]$

SOLUTION:

$$d_L(z) = (1+z)\int_0^z \frac{dz'}{H(z')} = d_L(0) + d_L'(z)\Big|_{z=0} z + \frac{1}{2} d_L''(z)\Big|_{z=0} z^2 + \dots$$

and plug:

$$d_L'(z) = \int_0^z \frac{dz'}{H(z')} + \frac{1+z}{H(z)} \Rightarrow d_L'(z)\Big|_{z=0} = H_0^{-1}$$

$$d_L''(z) = \frac{1}{H} + \frac{1}{H} + \frac{d}{dz}\frac{1}{H} = \frac{2}{H} - \frac{1}{H(1+z)}\frac{d}{dt}\frac{a}{\dot{a}} = \frac{1}{H}\left[2 - \frac{1}{1+z}\left(1 - \underbrace{\frac{\ddot{a}}{aH}}\right)\right]$$

$$\frac{dz}{dt} = \frac{d}{dt}\left(\frac{a_0}{a} - 1\right) = -\frac{\dot{a} a_0}{a^2} = -H(1+z)$$

$$= \frac{1}{H}\left(2 - \frac{1+q}{1+z}\right)$$

$\equiv q$

1.2 If you have never done it, play with Friedmann eqs. by showing that the first is equivalent to the second upon use of the conservation eqn.

1.3 Generalize the expression of $E(z)$ to the case of $w_{DE}(z)$

1.4 Show that the comoving distance to $z=\infty$ for $\Omega_\Lambda=0$ is

$\chi(\infty) = 2\,\mathrm{arcsinh}\sqrt{\frac{\Omega_K}{\Omega_m}}$. Use the replacement $1+z=y^{-2}$ to solve the integral.