ICTP Summer School on Cosmology 2022

2. "What causes the acceleration"

Filippo Vernizzi, IPhT CEA/Saclay

Trieste 5-12 July 2022

• What causes the acceleration?

Assuming GR : the dynamics of $a(t)$ determined by matter content

$$G_{\mu\nu} \equiv \underbrace{R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R}_{\text{geometry}} = 8\pi G \underbrace{T_{\mu\nu}}_{\text{matter}}$$

For a FLRW universe $T^{\mu}{}_{\nu} = (\rho + p)\, u^{\mu} u_{\nu} + p\, \delta^{\mu}{}_{\nu}$

energy density (ρ) pressure (p) 4-velocity (u^μ)

$u^{\mu} = (1, 0, 0, 0)$

Friedmann eqs: $H^2 \equiv \left(\frac{\dot a}{a}\right)^2 = \frac{8\pi G}{3}\rho - \frac{k}{a^2}$ $H \equiv \frac{\dot a}{a}$ Hubble rate

(Ex 1.2) $\frac{\ddot a}{a} = -\frac{4\pi G}{3}(\rho + 3p)$ $\dot{} \equiv \frac{d}{dt}$ ← proper time

Acceleration : $\ddot a > 0$; In FLRW : $\boxed{\ddot a > 0 \iff p/\rho < -\frac{1}{3}}$

Conservation : $\dot{\rho} + 3H(\rho + p) = 0$

For different non-interacting species $\rho = \Sigma_i \rho_i$; $p = \Sigma_i p_i$

$$\dot{\rho}_i + 3H(\rho_i + p_i) = 0$$

Integrating this eqn. for $w_i \equiv \frac{p_i}{\rho_i}$ (equation of state) constant :

$$\frac{d\rho_i}{da} = -3\frac{\rho_i}{a}(1+w_i) \Rightarrow \rho_i = \rho_{i,0}\left(\frac{a_0}{a}\right)^{3(1+w_i)}$$

Examples:

- $w=0$: non-relativistic matter (dust) $\rho \propto 1/a^3$
- $w=\frac{1}{3}$: relativistic matter (radiation) $\rho \propto 1/a^4$
- $w=-1$: cosmological constant ρ = const. $\Leftarrow$ acceleration!

Smaller w dominate at late times : the fate of the Universe is dictated by the nature of DE. DE low energy phenomenon.

- Remember $H^2 + \frac{K}{a^2} = \frac{8\pi G}{3} \Sigma_i \rho_i = \frac{8\pi G}{3} \Sigma \rho_{i,0} (1+z)^{3(1+w_i)}$

Divide by H^2 : $1 = \Sigma_i \Omega_i$; $\Omega_K \equiv -\frac{K}{a^2 H^2}$

Divide by H_0^2 : $E(z) = \left(\Omega_{i,0} (1+z)^{3(1+w_i)} \right)^{1/2}$ where (Ex. 1.3)

$\Omega_{i,0} \equiv \frac{8\pi G}{3 H_0^2} \rho_{i,0}$; $w_i = \frac{p_i}{\rho_i}$ (& $w_K = -\frac{1}{3}$)

- $\frac{\ddot{a}}{a} = -\frac{4\pi G}{3} (\rho_m + \rho_{DE} + 3 p_{ED}) \Rightarrow q = \frac{1}{2}\left[\Omega_m + (1 + 3 w_{DE}) \Omega_{DE}\right]$

For a C.C. $w_\Lambda = -1 \Rightarrow q_0 = \frac{1}{2} \Omega_{m,0} - \Omega_{\Lambda,0}$

- For $\Omega_{m,0} = 0.3$, $\Omega_{\Lambda,0} = 0.7 \Rightarrow q_0 = 0.15 - 0.7 = -0.55$

Note also that matter & C.C. are comparable at z s.t.

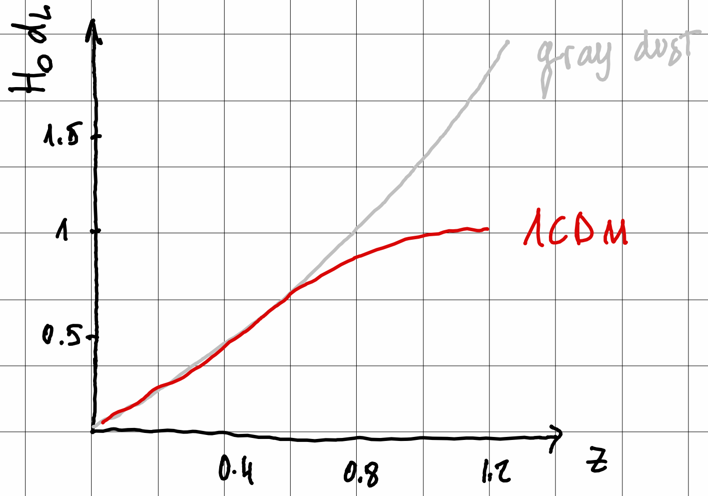
$\Omega_{m,0} (1+z)^3 = \Omega_{\Lambda,0} \Rightarrow z \simeq 1.3$. Dark energy dominates only recently.

• transition deceleration - acceleration

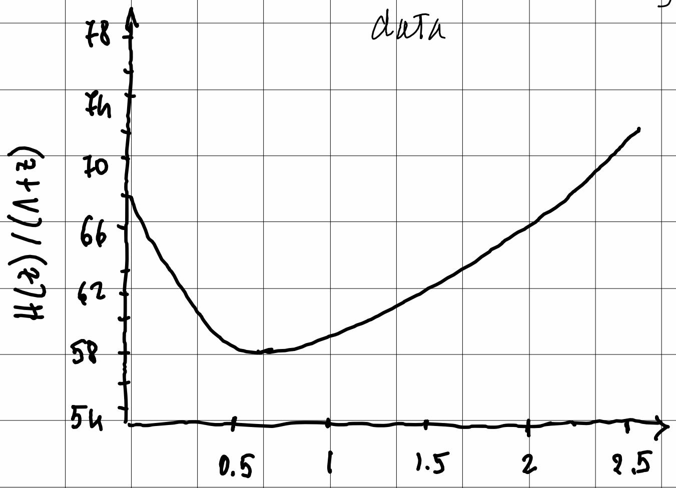
$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}\left(\rho_m - 2\rho_\Lambda\right) \propto \Omega_{m,0}(1+z)^3 - 2\Omega_{\Lambda,0}$$

the universe starts accelerating at $z = \left(2\frac{\Omega_{\Lambda,0}}{\Omega_{m,0}}\right)^{\frac{1}{3}} - 1 \simeq 0.7$ very recently!

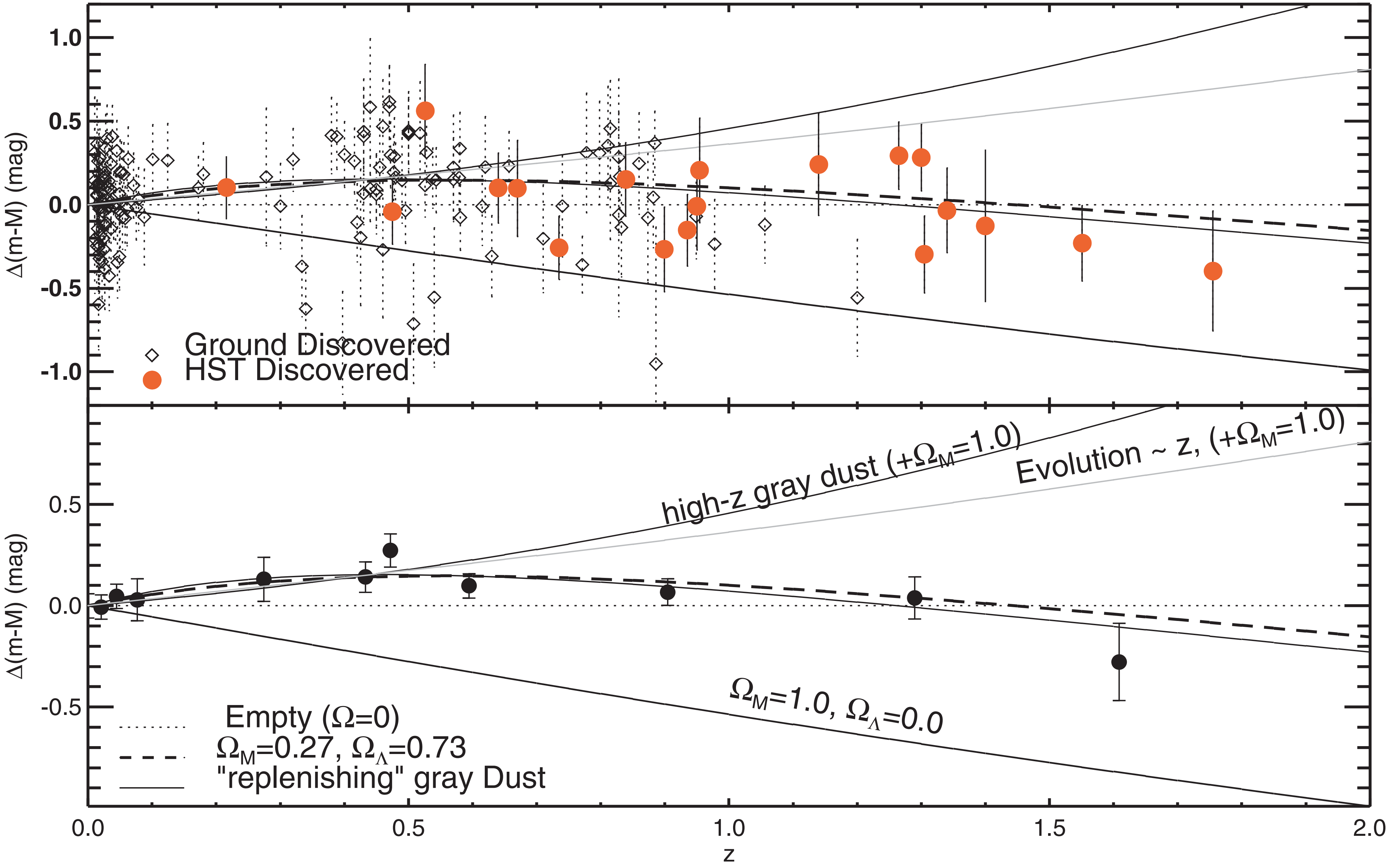
Supernovae at $z \gtrsim 1$ can measure transition from deceleration



From LSS (quasars, Ly-α) data

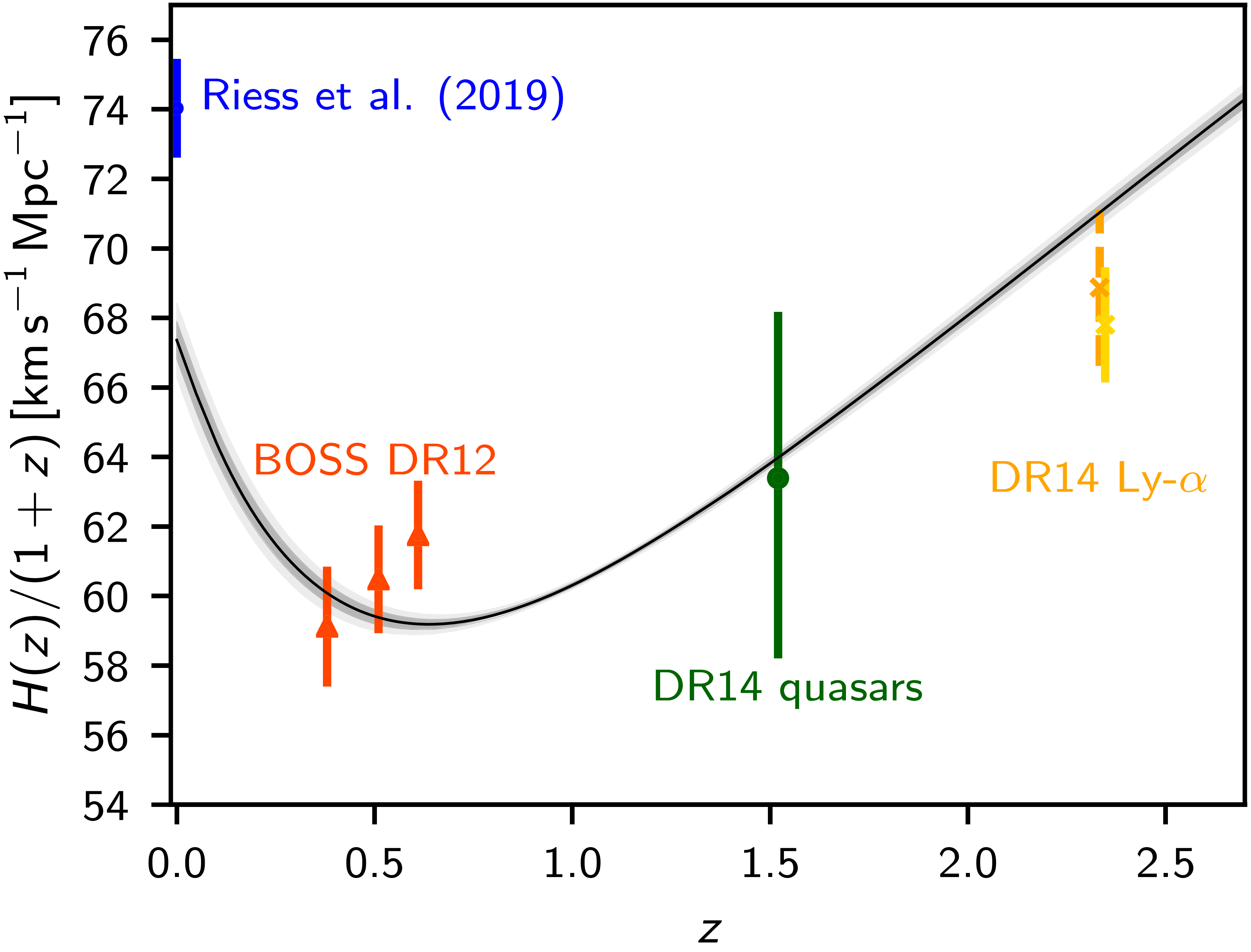


Deceleration-acceleration transition



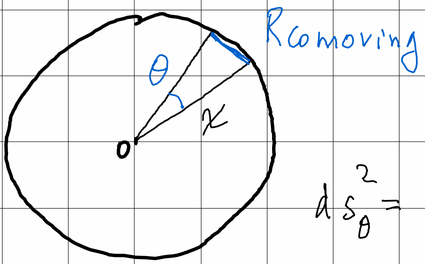
Riess et al. 2004

Deceleration-acceleration transition



• Angular diameter distance

Take a standard ruler: an object whose physical length is known



$$d_A = \frac{R_{phys}}{\theta} = \frac{a_0}{1+z} f_K(\chi) = \frac{d_L}{(1+z)^2}$$

$$ds_\theta^2 = a^2(t) f_K^2(\chi) d\theta^2 \Rightarrow R_{phys} = a(t) \underbrace{f_K(\chi)}_{R_{comoving}} \theta$$

• CMB position of peaks is an example of standard ruler

Oscillations in the photon/baryon plasma by pressure/gravity opposing each other. Oscillations stop at decoupling. Position of peaks determined by **sound horizon**: distance travelled by photons since the beginning.

$$r_s \equiv \int_0^{\tau_{dec}} c_s \, d\tau = \int_0^{t_{dec}} c_s \, a(t) dt \simeq \int_{z_{dec}}^{\infty} \frac{c_s \, dz}{H(z)} \simeq \frac{1}{\sqrt{3}} H_{dec}^{-1} \; ; \quad d_A = \frac{r_s}{\theta}$$

$\ell \sim \frac{\pi}{\theta} \Rightarrow$ For $\ell = 220$; $\theta^{rad} = \frac{2\pi}{220}$ $\left(\theta^{deg} = \frac{360^\circ}{2\pi} \theta^{rad} \right)$

$\Rightarrow d_A(z_{dec}) = \frac{r_s}{\theta_{CMB}}$ ← depends on Ω_m & $\Omega_b h^2$

← measured very well

$$r_s = \frac{1}{H_0 \Omega_m^{1/2}} \int_0^{a_{dec}} \frac{c_s}{\sqrt{a + a_{eq}}}$$

$$\chi(z_{dec}) = \int_0^{z_{dec}} \frac{dz}{a_0 H(z)} \underset{\Omega_\Lambda = 0}{=} \frac{1}{a_0 H_0} \int_0^{z_{dec}} \frac{dz}{\sqrt{\Omega_{m,0}(1+z)^3 + \Omega_{K,0}(1+z)^2}} \underset{z_{dec} \to \infty}{=} 2 \, \mathrm{arcsinh} \sqrt{\frac{\Omega_K}{\Omega_m}}$$

(EX. 2.1)

CMB angular diameter distance sensitive to curvature!

- BAO peaks, same effect imprinted in galaxy distribution

$\Big\} \Delta z = \frac{r_s}{H(z)}$

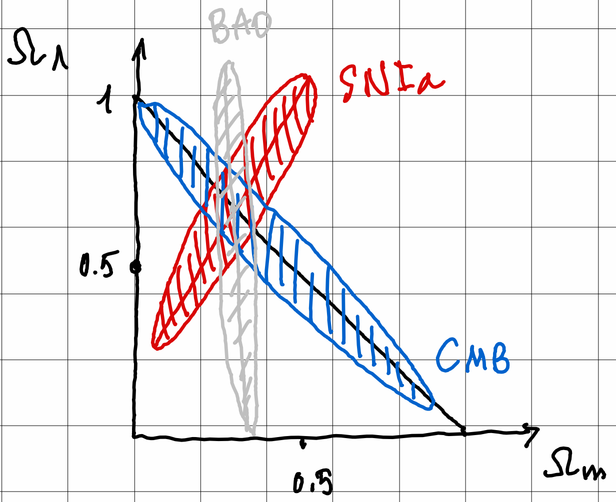
$\theta = \frac{r_s}{(1+z) d_A(z)}$ R_{phys}

In practice LSS surveys are not large enough to separate radial & angular direction.

$$D_V(z) = \left[\underbrace{(1+z)^2 d_A^2(z)}_{\text{comoving } d_A} \frac{z}{H(z)} \right]^{1/3}$$

Spherical average, geometric mean

• ΛCDM model



- Supernovae $\rightarrow$ $\cancel{\Omega_M - 2\Omega_\Lambda}$

  $2\Omega_M - \Omega_\Lambda$

  due to a partial cancellation between z^2 & z^3 terms

- CMB $\rightarrow$ Ω_K
- BAO $\rightarrow$ Ω_m

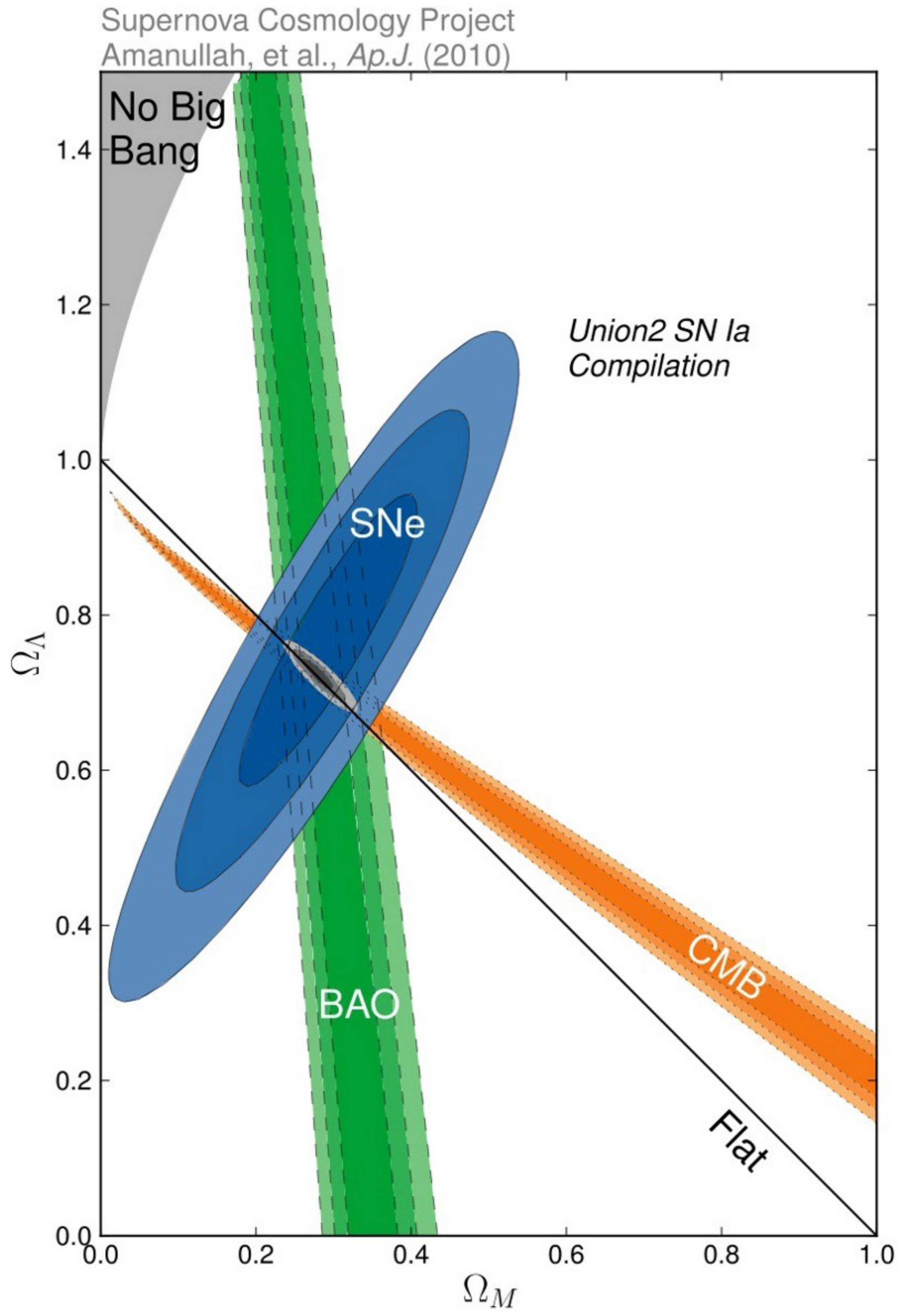
- Age of the universe : age of globular clusters ~13 Gyr

  Age of the universe for Ω_Λ : $t = \frac{2}{3H_0} \simeq$ 8-10 Gyr

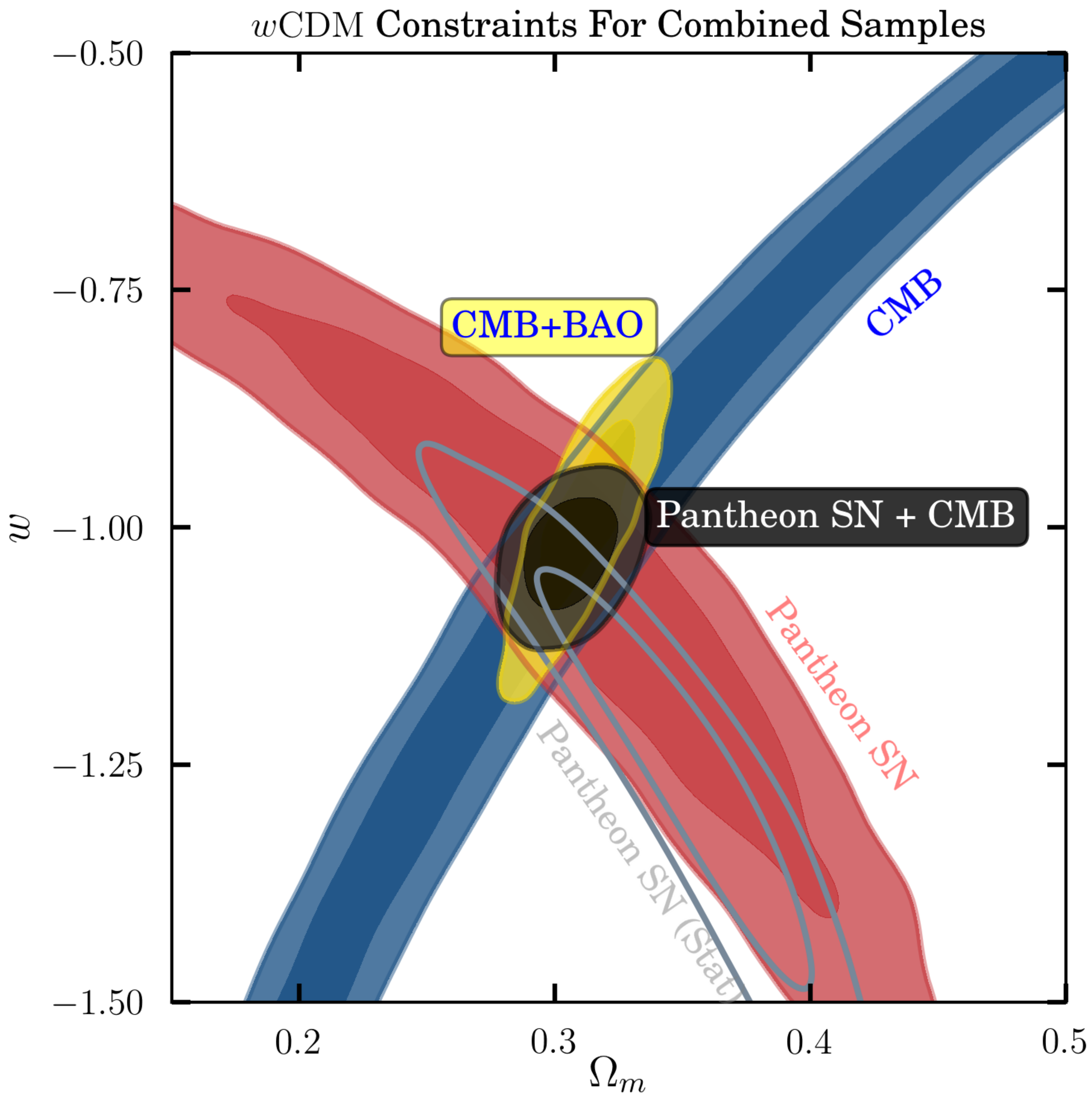
- Growth of structures : Galaxy surveys, weak lensing, etc.

- Clusters number counts : $\frac{dN}{dz\,d\Omega}(z) = \frac{dV}{dz\,d\Omega}(z) \int_{M_{min}}^{\infty} \frac{dn}{dM}$; $\frac{dV}{dz\,d\Omega} = \frac{\chi^2(z)}{H(z)}$

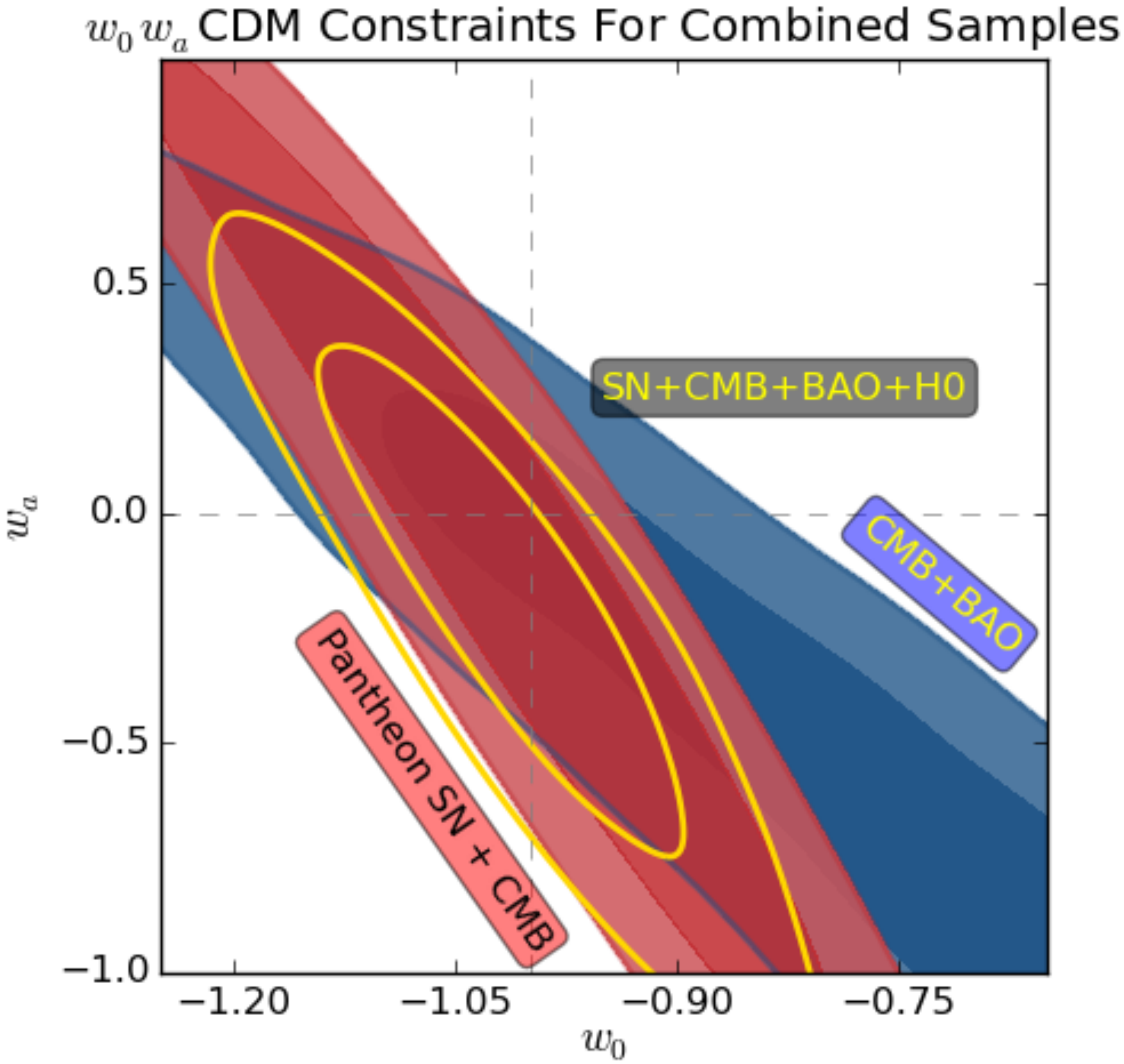
Concordance model



Equation of state parameter



Equation of state parameter



Conclusions Lecture 2

- The cosmic acceleration requires a component with **equation of state $w< -1/3$.** Observations consistent with **$w=-1$**. We also measure the deceleration/acceleration transition, implying that dust or evolution models are not enough to explain observations.

- It looks as if the acceleration is due to a cosmological constant. Aren't we happy?

EXERCISES

2.1 Show that the comoving distance to $z=\infty$ for $\Omega_\Lambda = 0$ is $\chi(\infty) = 2\,\mathrm{arcsinh}\sqrt{\frac{\Omega_K}{\Omega_m}}$. Use the replacement $1+z = y^{-2}$ to solve the integral.