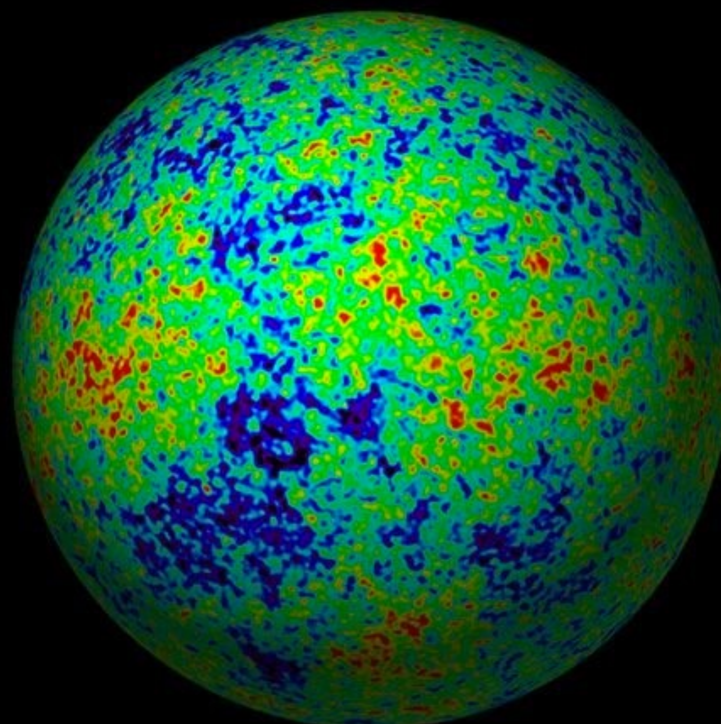
The Cosmic Microwave Background
Lecture 1: Basics



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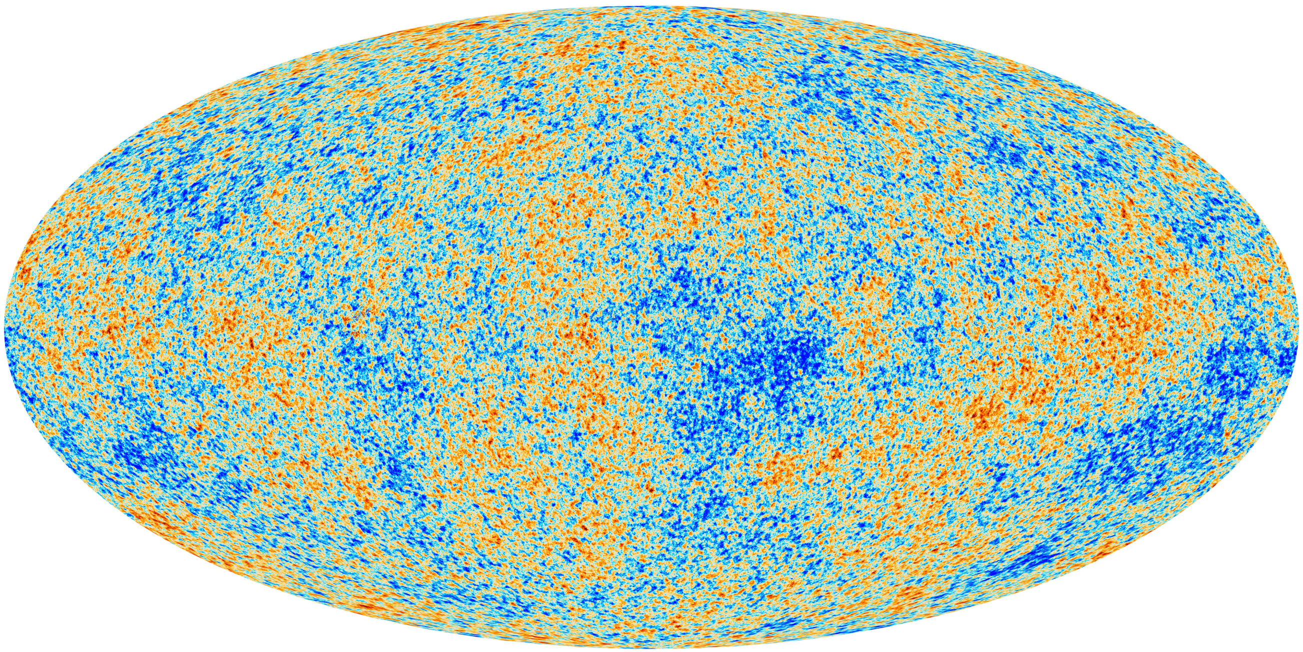
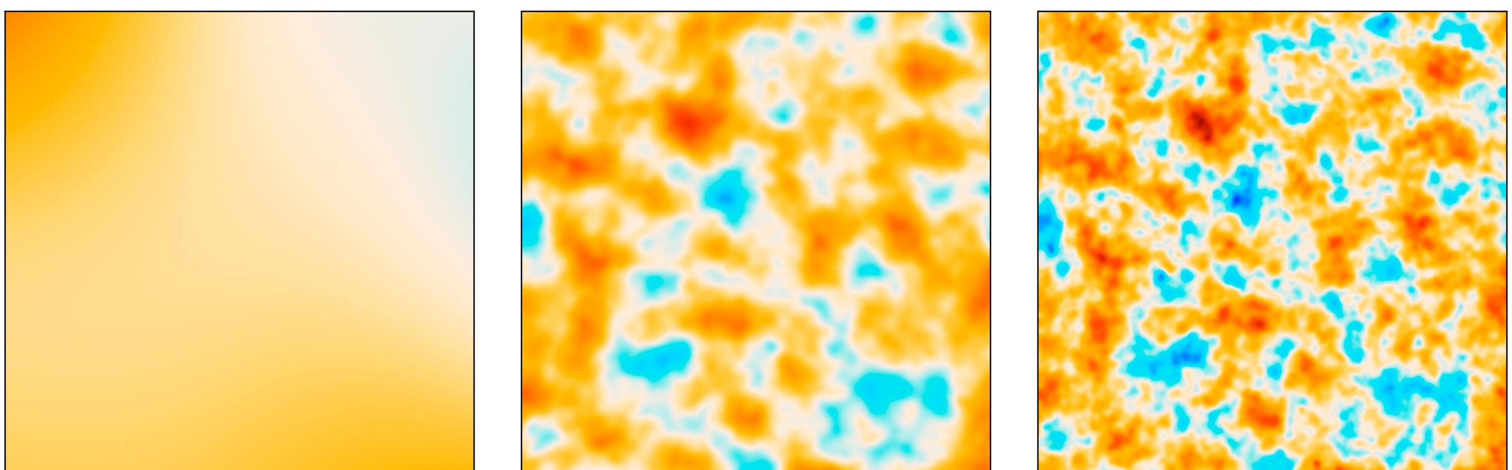
Brief Motivation



- Why watch lectures on CMB? Powerful probe of:
  - Cosmic history and evolution (“baby picture”)
  - Cosmic composition
  - Early universe and inflation
  - New physics at high energies/weak interactions (+tensions?)
  - Basic cosmology: historical proof of hot big bang

Course Overview

- Lecture 1: CMB basics: properties and propagation
- Lecture 2: From initial conditions to CMB power
- Lecture 3: CMB power spectra and cosmological parameters
- Lecture 4: CMB temperature and polarization as a test of inflation
- Lecture 5: CMB gravitational lensing

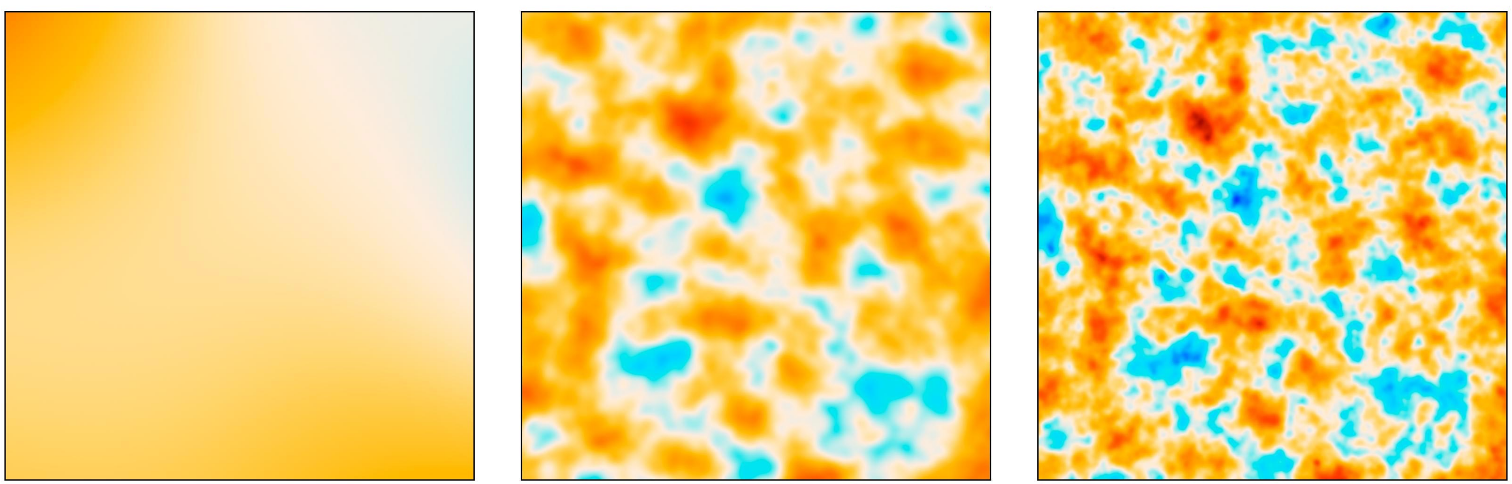


Other References

- Wayne Hu's notes (arXiv:astro-ph/9706147) and website (http://background.uchicago.edu/~whu/)
- Daniel Baumann's new book, "Cosmology"
- Anthony Challinor's lecture notes (arXiv:0903.5158) and Eiichiro Komatsu's lecture notes [many image credits!]
- Lewis and Challinor CMB lensing review (arXiv:astro-ph/0601594)
- Ruth Durrer's book, "The Cosmic Microwave Background"
- Scott Dodelson's book "Modern Cosmology"

Outline: Today's Lecture 1

- Introduction: CMB from the hot Big Bang
- Basic CMB statistics
- CMB photon propagation

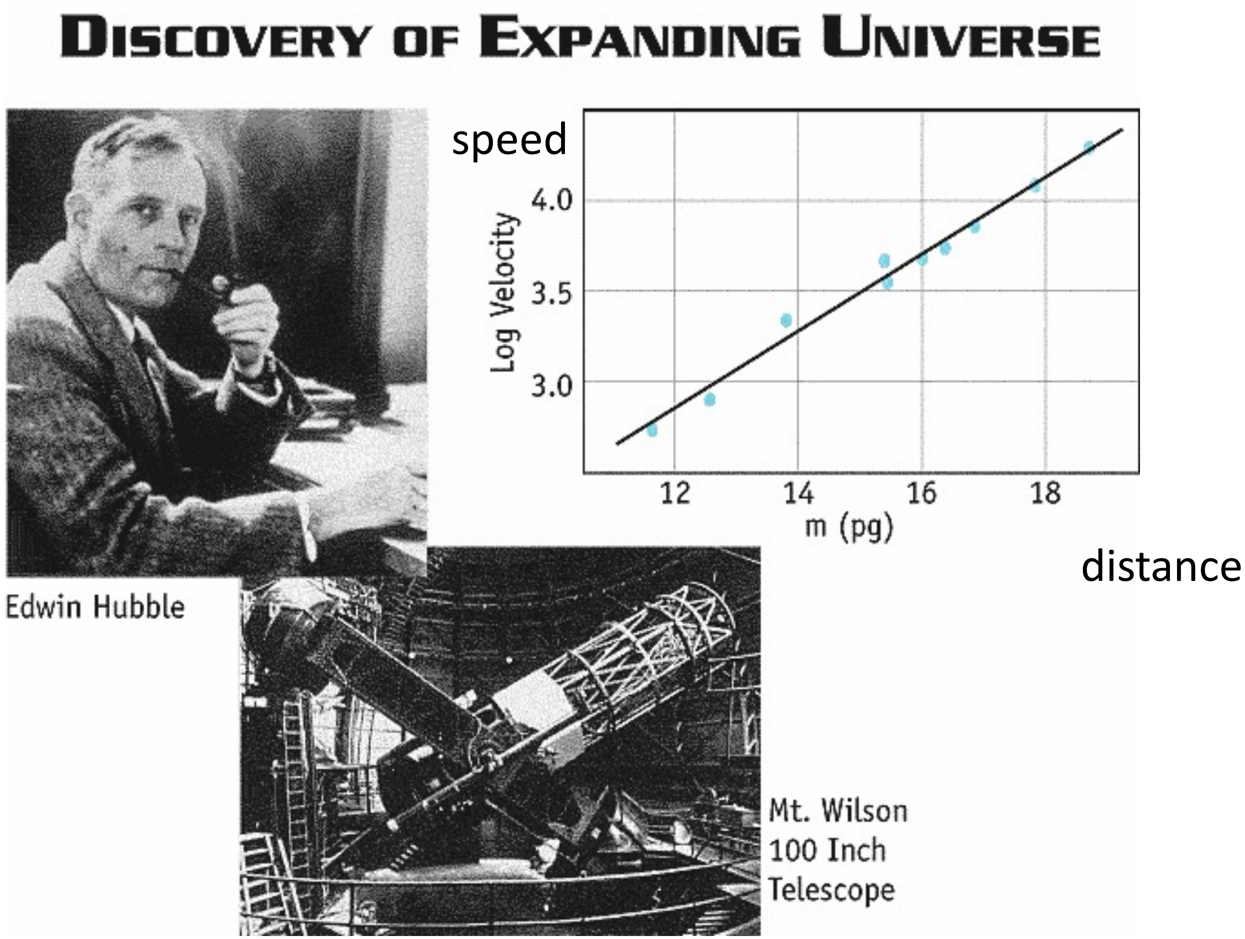


We Have Observed an Enormous Number of Galaxies

Image: Hubble Deep Field

All Galaxies Moving Away from Us!

- The further away galaxies are, the faster they seem to be moving away from us (!?)



Explanation: All distances are expanding with time

- Space itself is expanding. The more space between two objects, the more it expands, so the faster they (seem to) move away. Quantitatively, distances given by:

$$ds^2 = dt^2 - a^2(t)\left[dx^2 + dy^2 + dz^2\right]$$

conformal time

$$= a^2(\eta)\left(d\eta^2 - \left[dx^2 + dy^2 + dz^2\right]\right)$$

- Distances depend on scale factor a. Evolution described by Friedmann equation:

Reminder: Cosmological expansion

$$\left(\frac{\dot{a}}{a}\right)^2 \equiv H^2(t) = H_0^2\left[\Omega_\Lambda + \Omega_m a^{-3} + \Omega_r a^{-4}\right]$$

Hubble rate

Hubble constant today

Dark energy density~0.7

Matter (CDM and baryon) density ~0.3

Radiation density

- It follows that first a~$t^{1/2}$ in radiation domination, then in matter domination a(t)~$t^{2/3}$, finally dark energy domination (and inflation) a(t)~e^{Ht}. a increasing with time.

Reminder: Cosmological expansion

$$\left(\frac{\dot{a}}{a}\right)^2 \equiv H^2(t) = H_0^2 \left[\Omega_\Lambda + \Omega_m a^{-3} + \Omega_r a^{-4}\right]$$

Radiation density

Hubble rate

Hubble constant today

Dark energy density

Matter (CDM and baryon) density

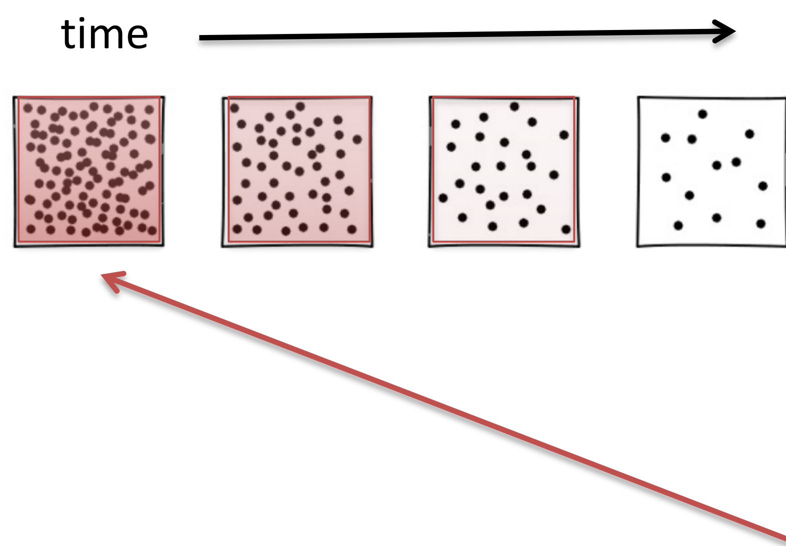
- It follows that first a~t$^{1/2}$~η in radiation domination, then in matter domination a(t)~t$^{2/3}$~η^2, finally dark energy domination (and inflation) a(t)~e^{Ht} [a=1 today]

- We will often use conformal time $\eta = \int_0^t \frac{dt}{a(t)}$ and $\mathcal{H} \equiv \frac{\frac{da}{d\eta}}{a}$ and generally assume flat LCDM universe and c=1 units

The big bang and the CMB

- Can show that temperature of the universe scales as

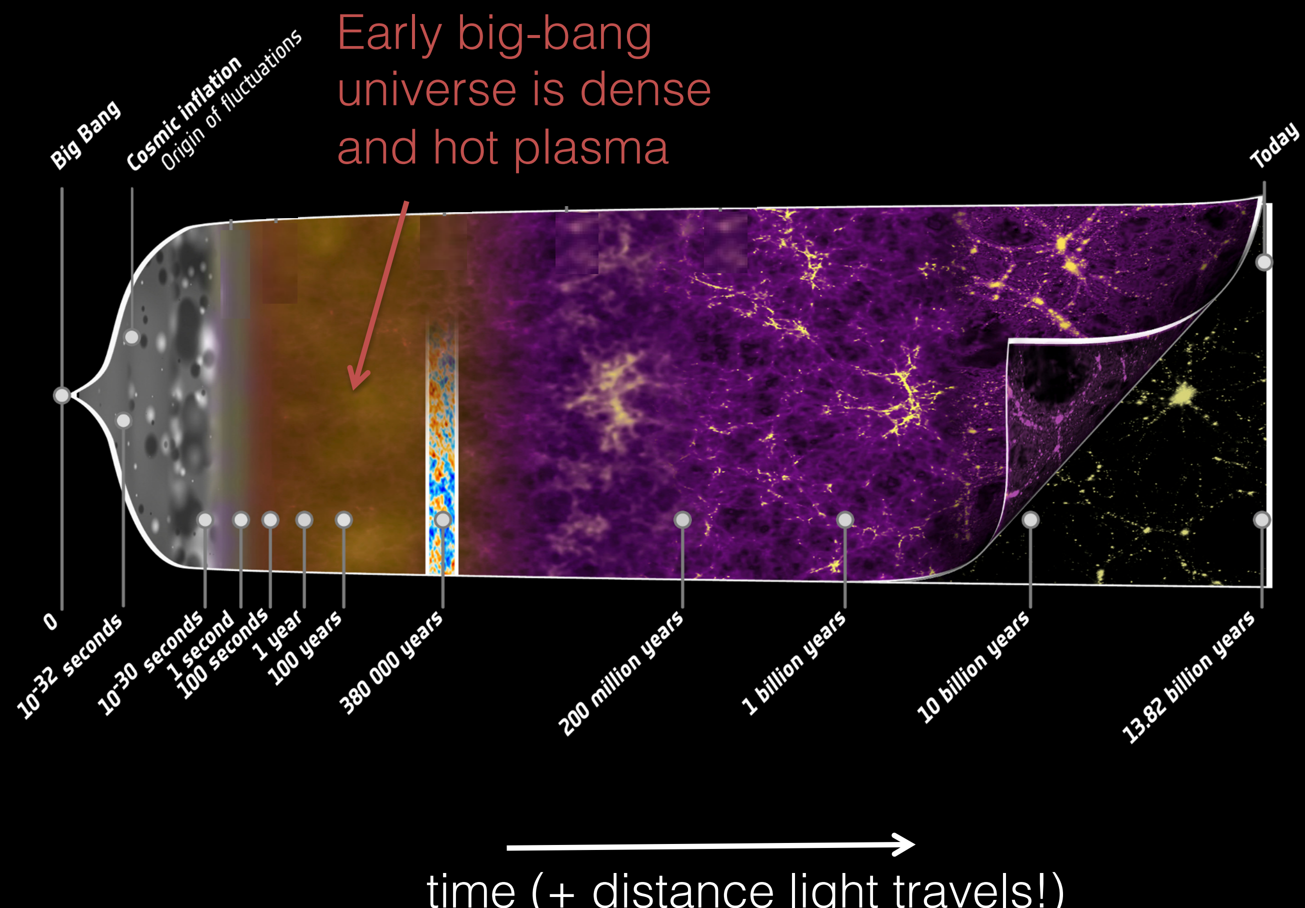
$$T \propto E_{\text{photon}} \propto 1/a \propto (1+z)$$



- Expansion implies universe used to be denser and hotter, has a beginning in the big bang when a approaches 0

Emission of the CMB after “Recombination”

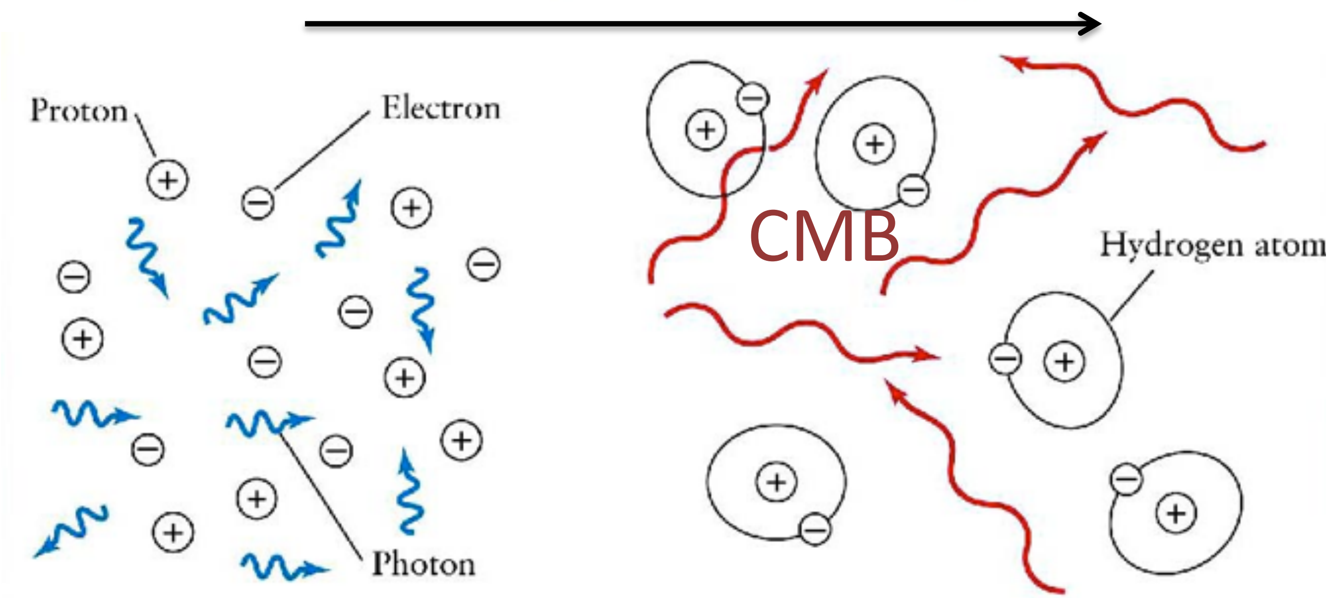
History of the universe



Emission of the CMB after Recombination

- Very early universe: so hot that consists of electron / proton / photon plasma. Photons often Thomson scatter.
- But: cools with expansion…H atoms form after ~400000 years

$$e^- + p^+ \quad \leftrightarrow \quad \mathrm{H} + \gamma$$



Hot early universe: **opaque**

Cool late universe: **transparent**

Emission of the CMB after Recombination

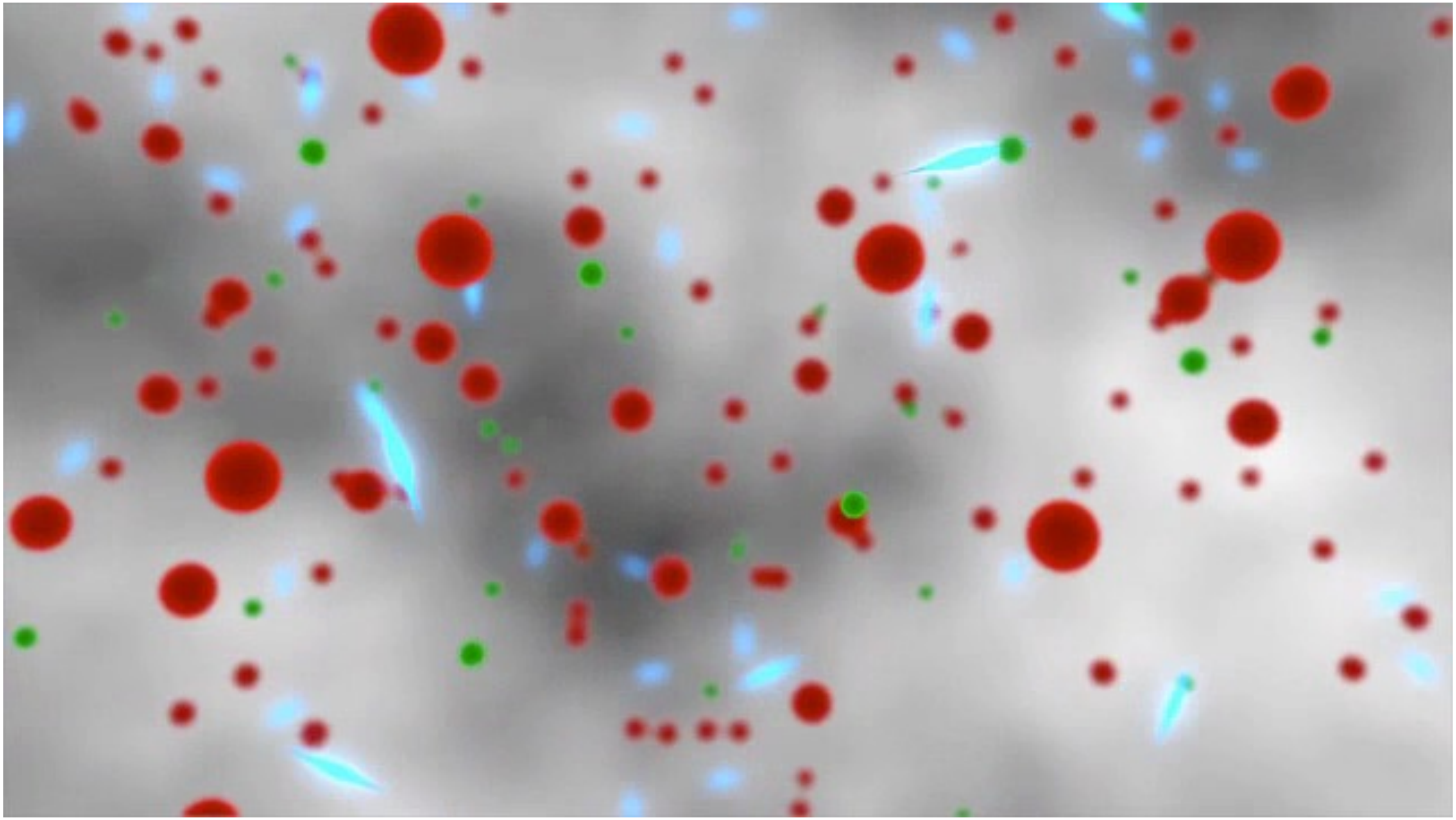
- Recombination: Hydrogen forms quickly at z~1300, T~0.3eV

$$e^- + p^+ \quad \leftrightarrow \quad \mathrm{H} + \gamma$$

- Before recombination, Thomson scattering efficient and mean free path $1/(n_e \sigma_T)$ short relative to expansion time / scale 1/H

- Little scattering after recombination. Photons free stream to us after z~1100

Emission of the CMB at Recombination



Recombination and Decoupling: Quantitative Details

$$e^- + p^+ \quad \leftrightarrow \quad \mathrm{H} + \gamma$$

- Recombination, i.e. formation of neutral hydrogen: use equilibrium expressions for n_e, $n_b \sim n_p + n_H$ and rearrange to get Saha equation for $X=n_e/n_b$. Gives $T\sim0.3$eV!

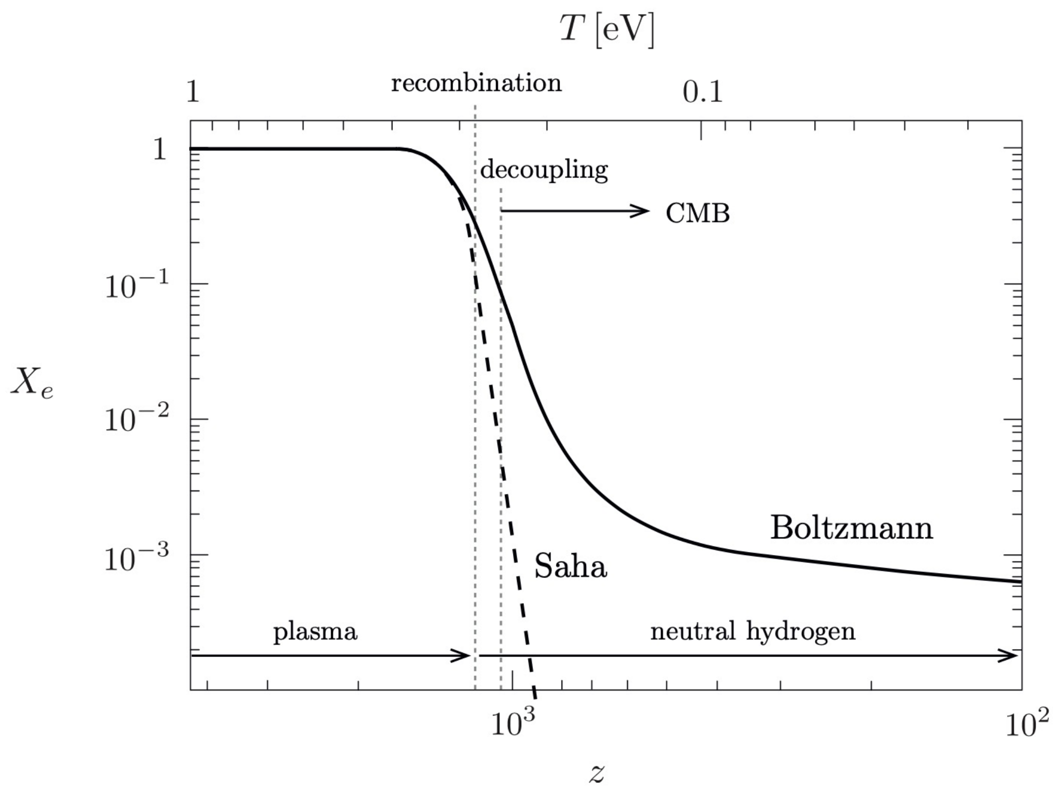
$$n_i^{\mathrm{eq}} = g_i \left(\frac{m_i T}{2\pi}\right)^{3/2} \exp\left(\frac{\mu_i - m_i}{T}\right)$$

Binding energy 13.6eV
$= m_e + m_p - m_H$

$$\left(\frac{1-X_e}{X_e^2}\right)_{eq} = \frac{2\zeta(3)}{\pi^2}\eta\left(\frac{2\pi T}{m_e}\right)^{3/2} e^{B_H/T}$$

Baryon-photon ratio $\sim 10^{-9}$

Recombination and Decoupling: Quantitative Details



- Set $\Gamma \sim n_e \sigma_T \sim H$ to see when photons decouple. Decoupling is right after recombination as n_e falls quickly, z~1100. Recombination / decoupling set by local T.

Exercise and Quiz Question

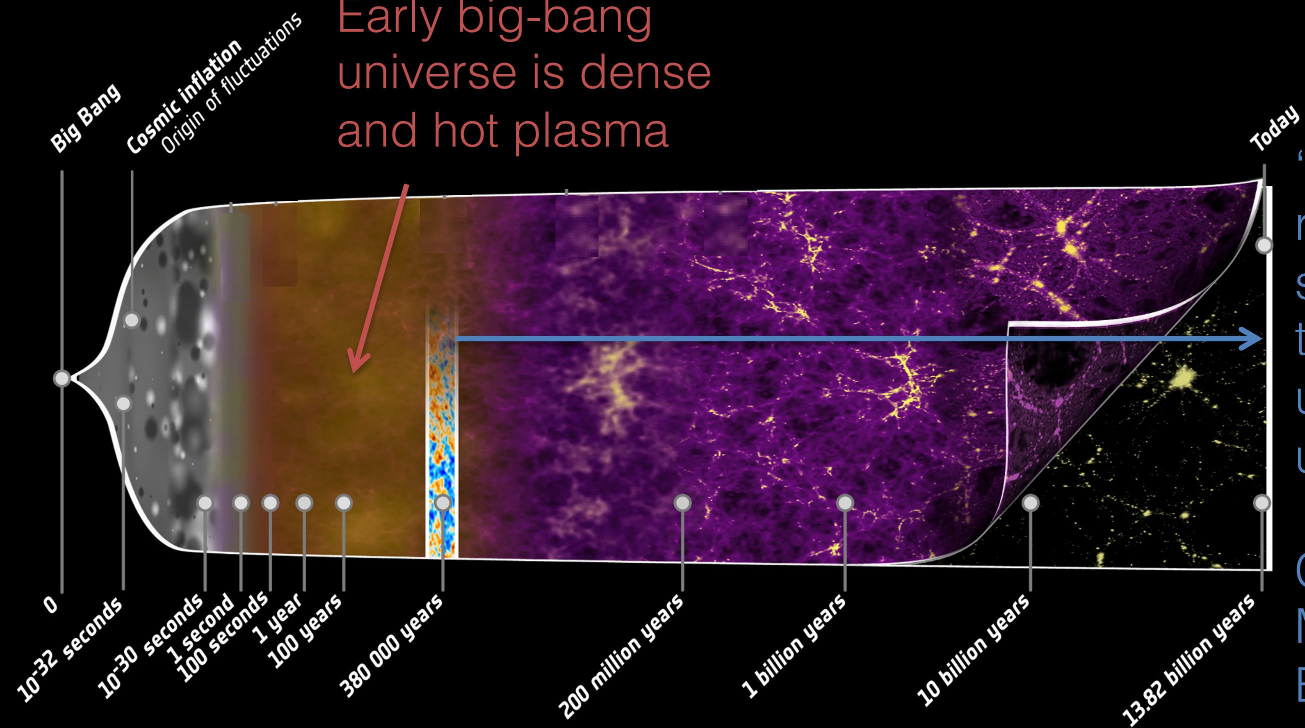
- Exercise: derive Saha Equation!

- Question: CMB temperature anisotropies probe fluctuations in the radiation density. How is this possible if photons are emitted at a ~fixed local CMB temperature (independent of radiation density)?

Emission of the CMB after Recombination

History of the universe

Early big-bang universe is dense and hot plasma



“afterglow” radiation should stream through the universe… to us:

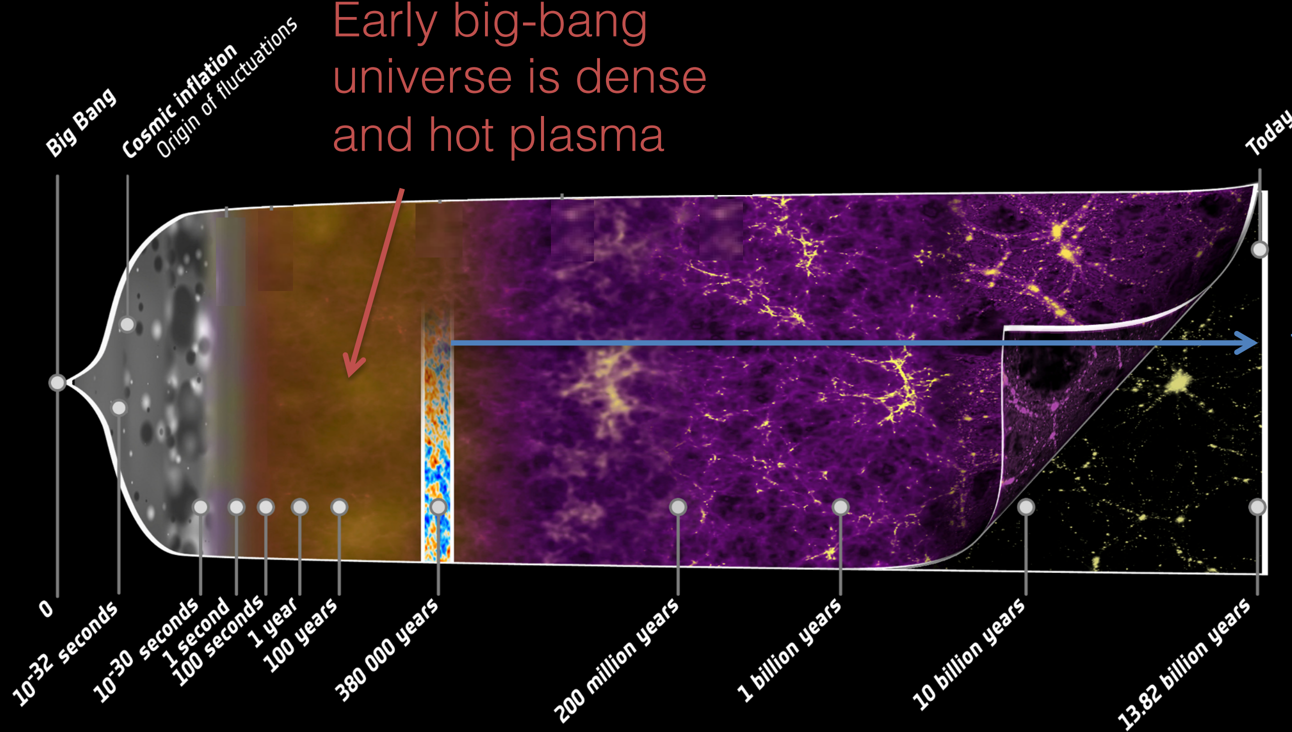
Cosmic Microwave Background

time (+ distance light travels!)

Emission of the CMB after Recombination

Big bang picture makes a clear prediction:
<u>we should see radiation from the primordial plasma in the sky</u>

Early big-bang universe is dense and hot plasma



"afterglow" radiation should stream through the universe… to us:

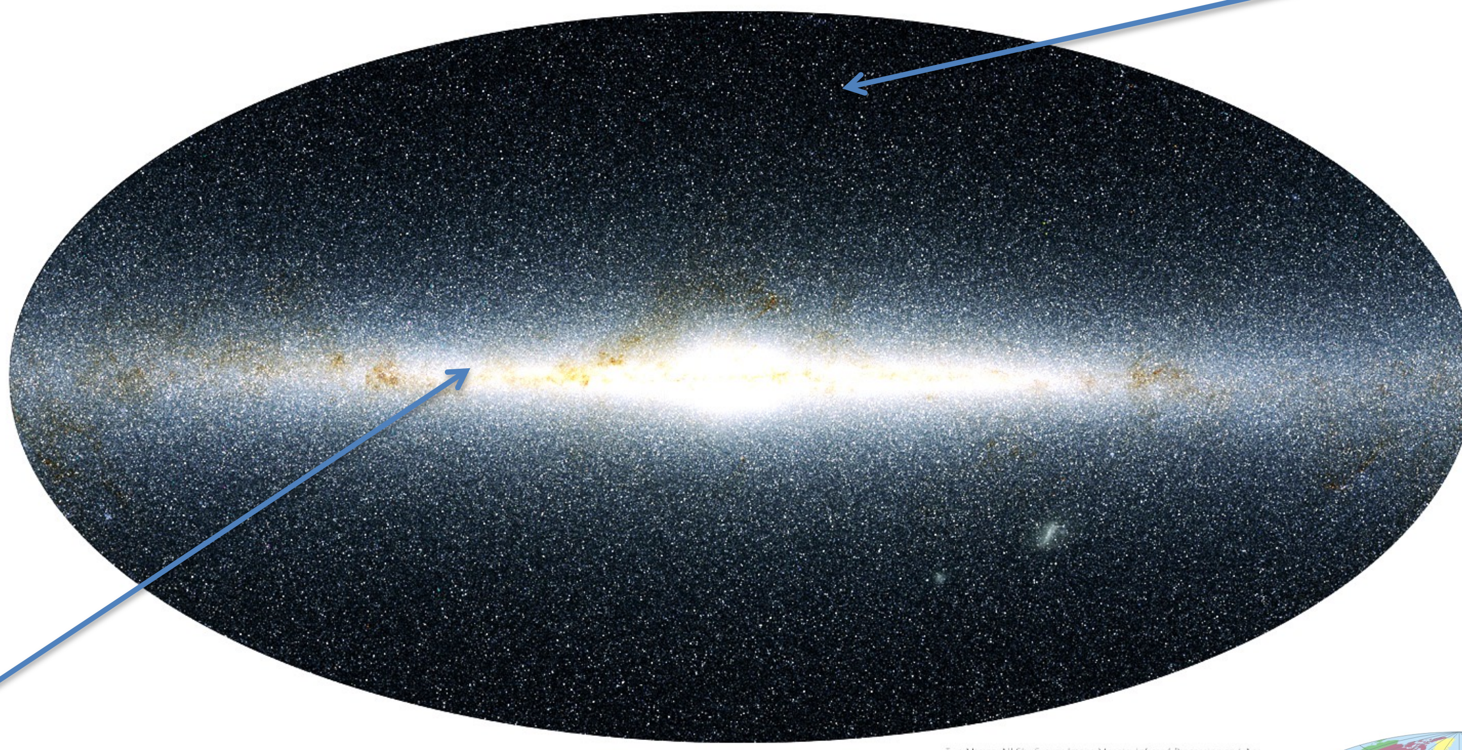
Cosmic Microwave Background

time (+ distance light travels!)

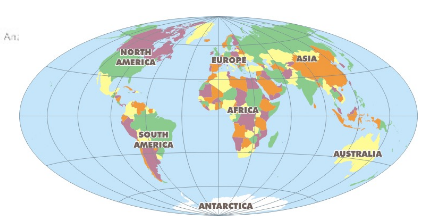
The CMB in the Sky?

- Can’t see in optical due to expansion / redshifting

No fireball ☹

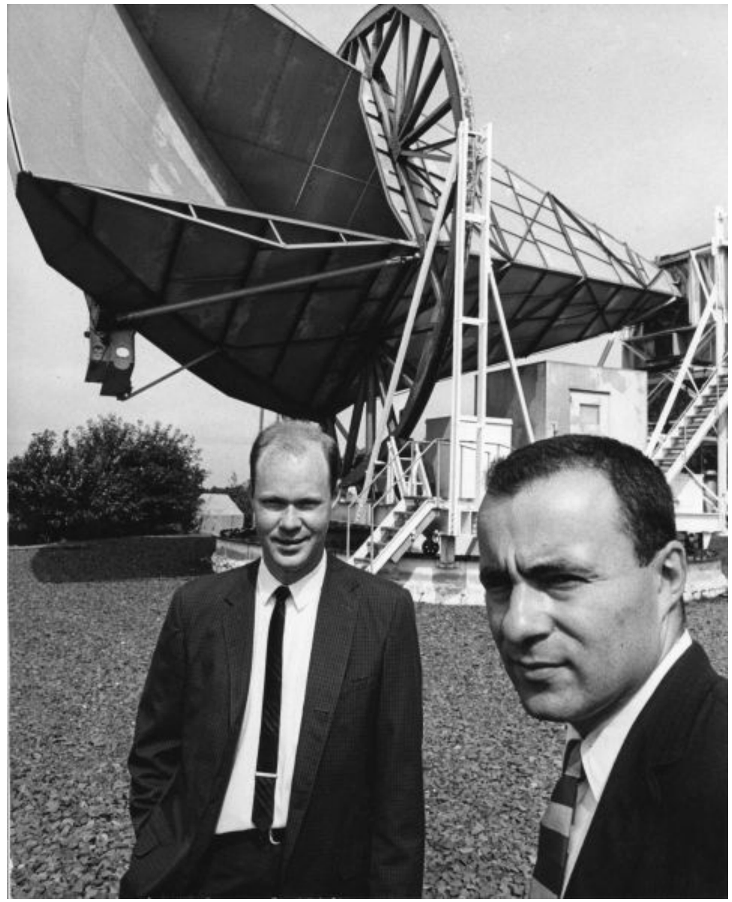


just see Milky Way stars ☹



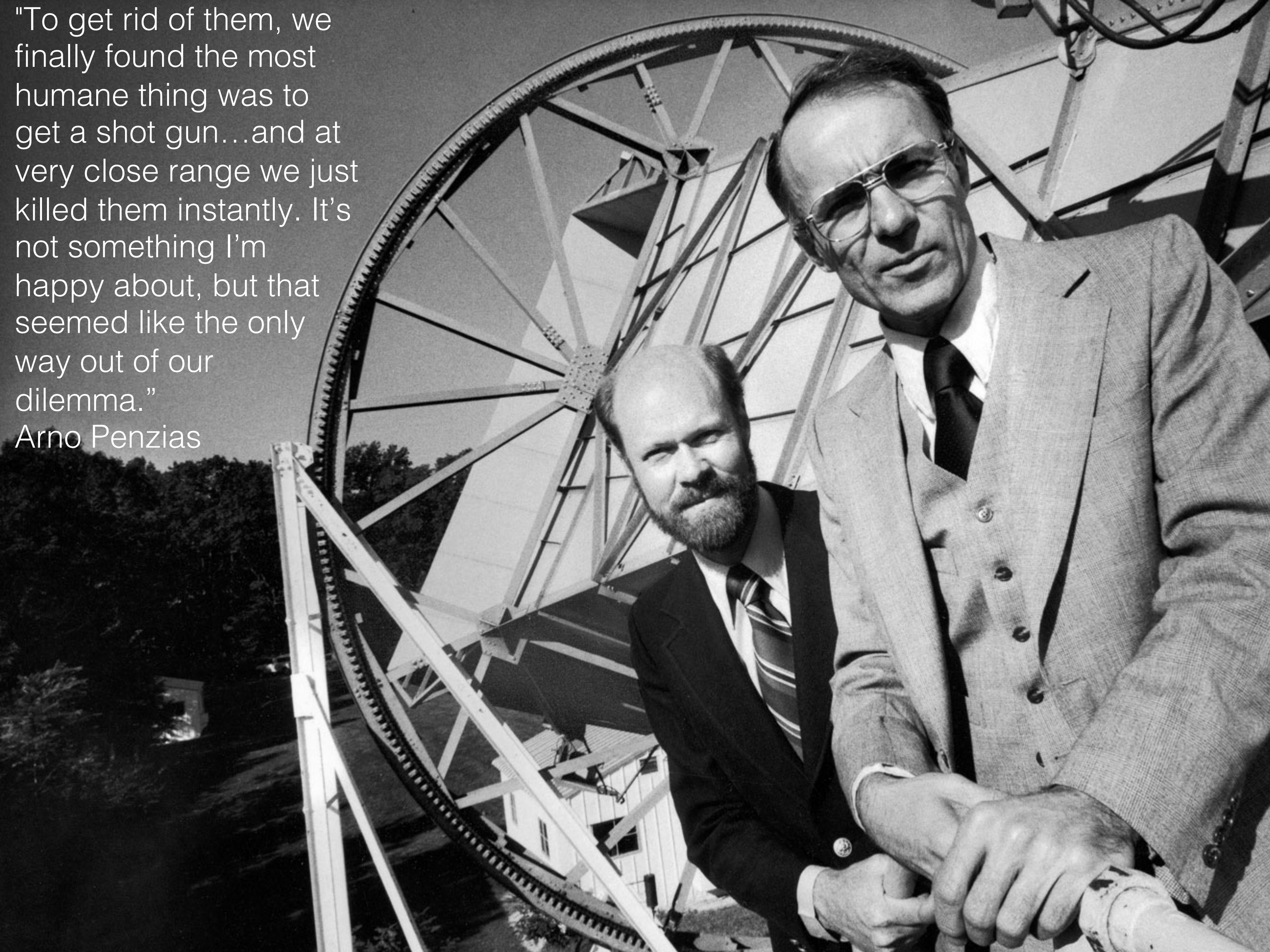
The CMB in the Microwave Sky

- Build microwave telescope to see this radiation!



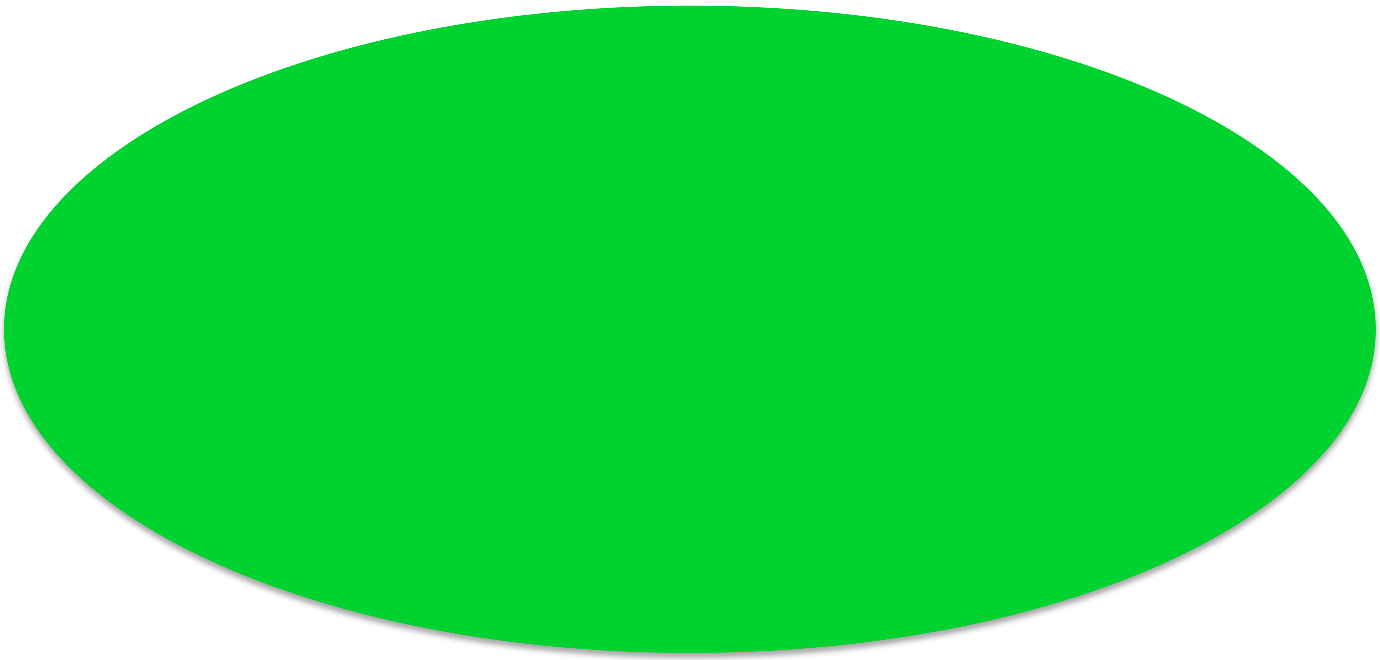
Penzias/Wilson
1968

"A Measurement of Excess Antenna Temperature at 4080 Mc/s"


"To get rid of them, we finally found the most humane thing was to get a shot gun…and at very close range we just killed them instantly. It's not something I'm happy about, but that seemed like the only way out of our dilemma."
Arno Penzias

Cosmic Microwave Background Light!

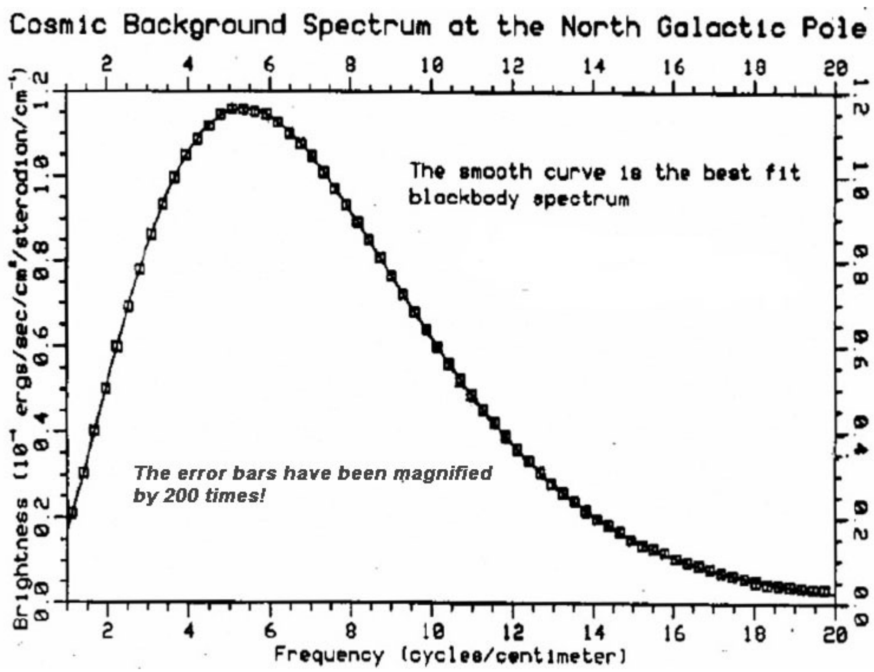
- CMB: blackbody radiation



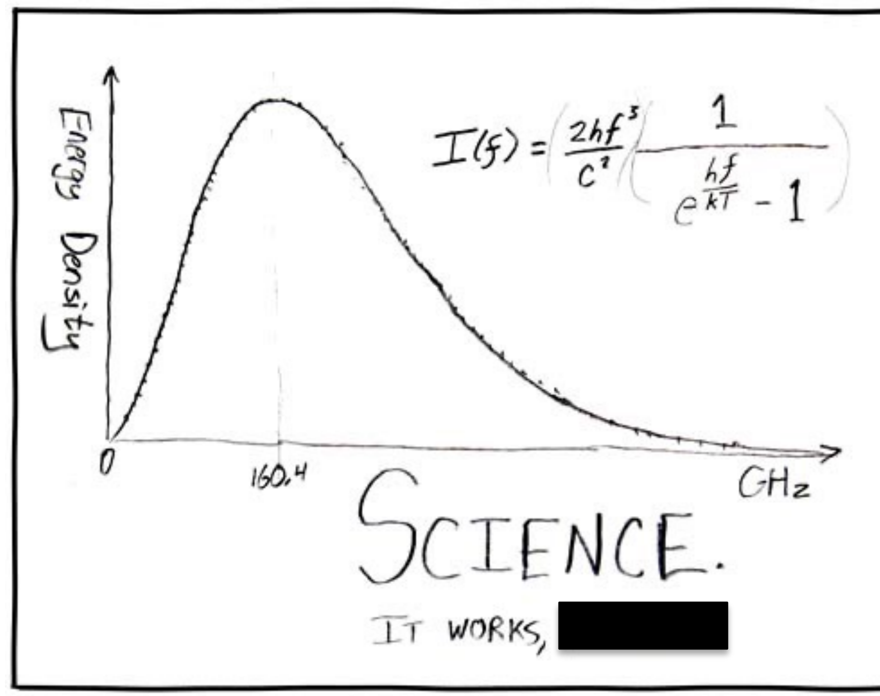
Cosmic Microwave Background Light!

- Compare CMB spectrum measurement with predicted blackbody spectrum: great confirmation of big bang

CMB Brightness



Frequency

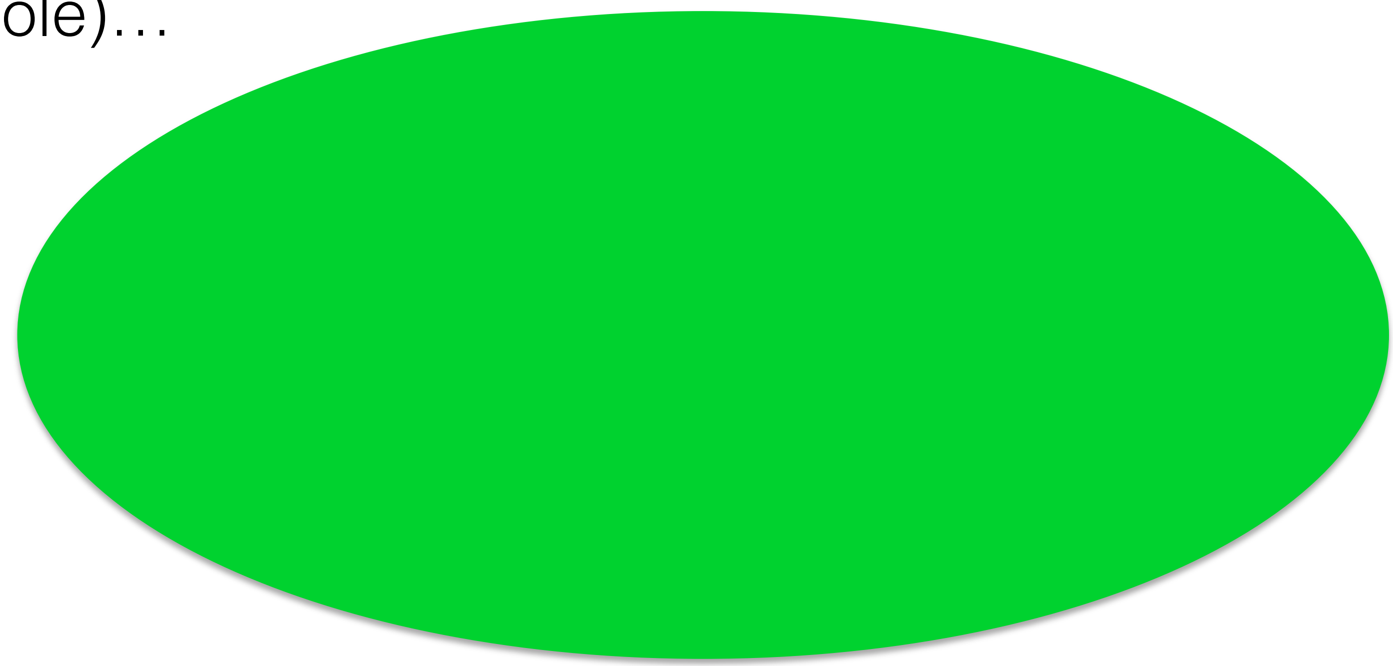


xkcd

- Near-perfect blackbody with T=2.7255

Is the CMB Temperature Uniform?

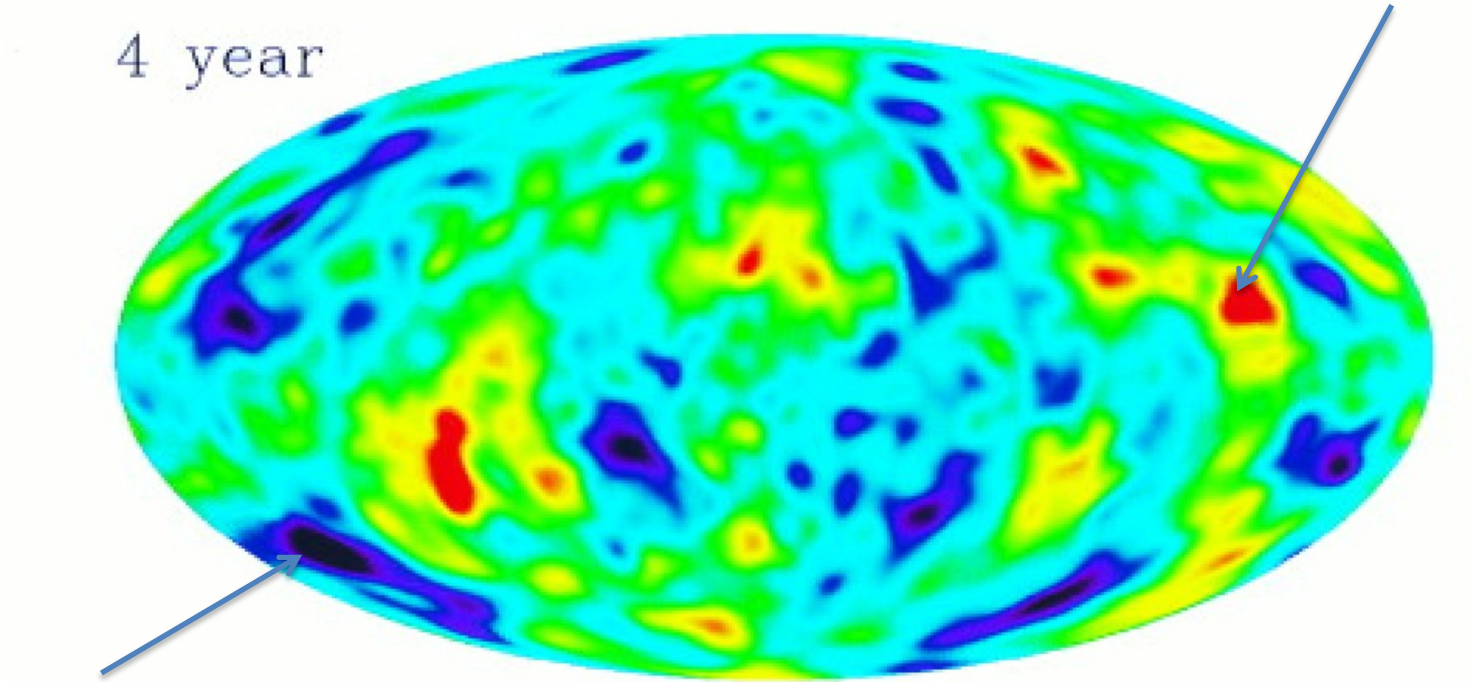
- Radiation that looks like the blackbody radiation of a 2.7 degree K object
- Increase contrast by 1000: still uniform (modulo dipole)…



Is the CMB Temperature Uniform?

- No! Turn up “contrast” by 100000
- See fluctuations in temperature / brightness across the sky, $\Theta \equiv \Delta T(\hat{\mathbf{n}})/\bar{T}$

brighter

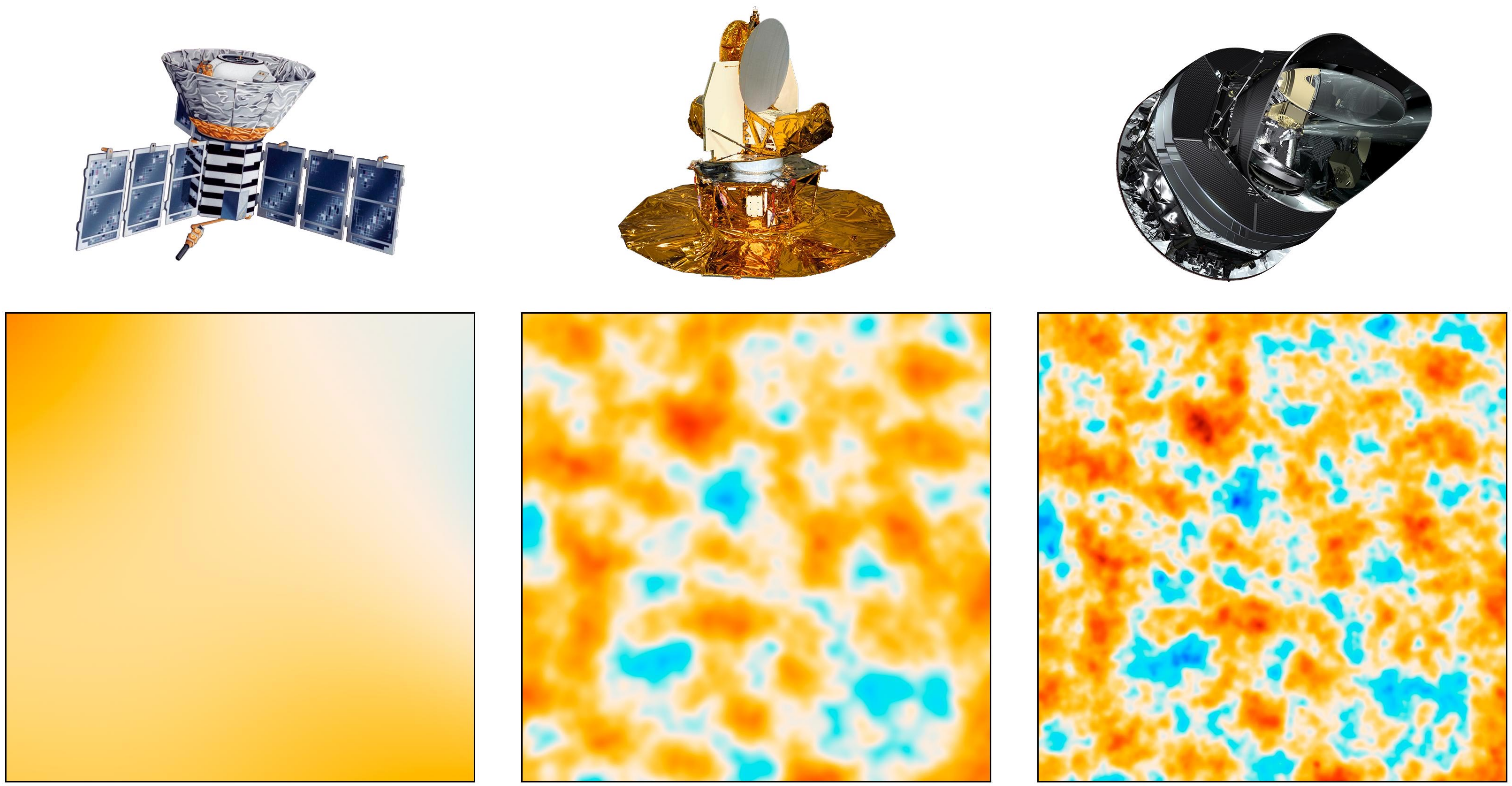


less bright

COBE Satellite Data
(Nobel Prize for Mather/Smoot)

CMB Fluctuations – Learning More

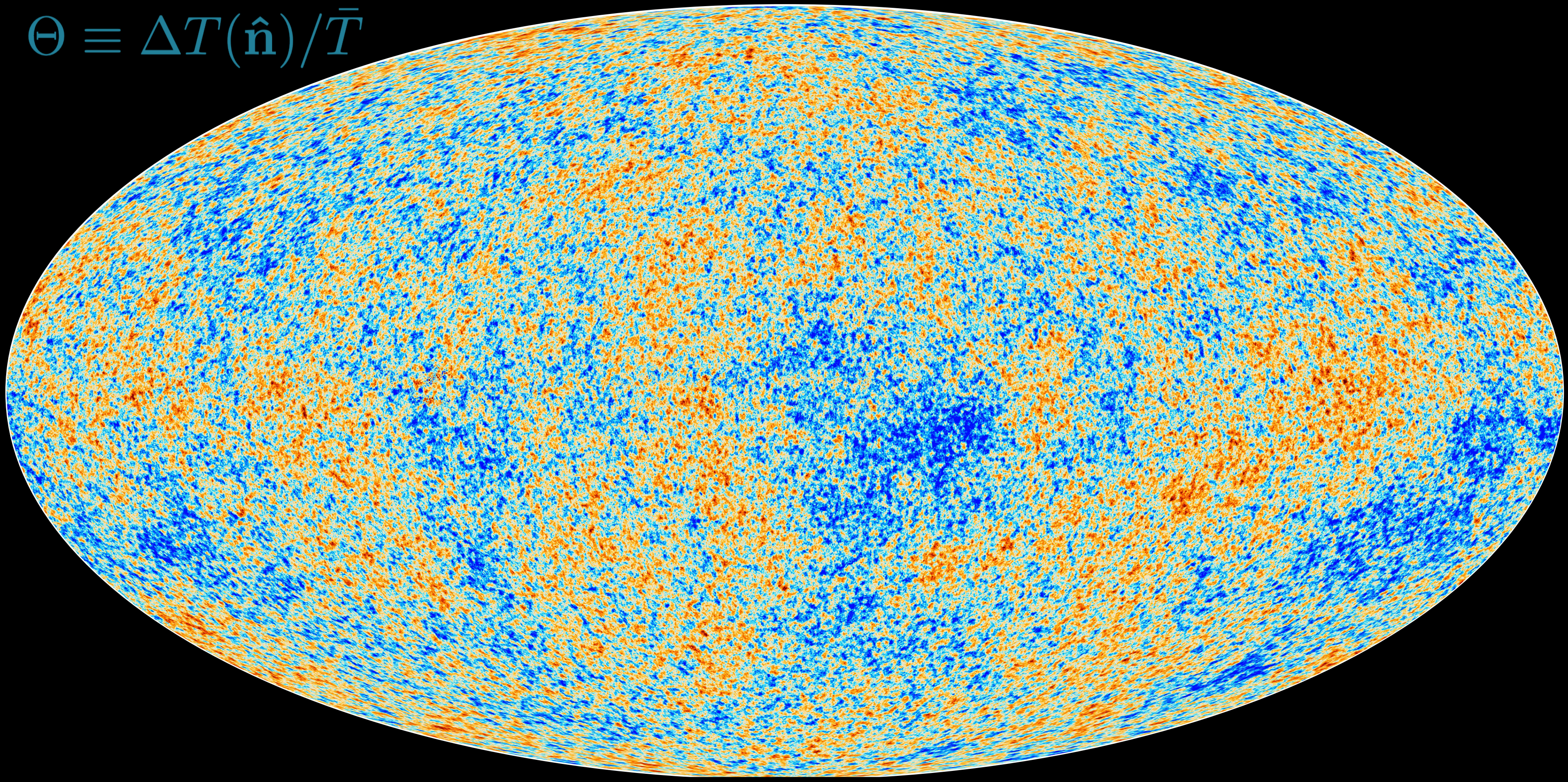
- Enhance: send up better satellites or use better ground-based telescopes...



COBE WMAP Planck

CMB Fluctuations –
Best Picture From Planck Satellite

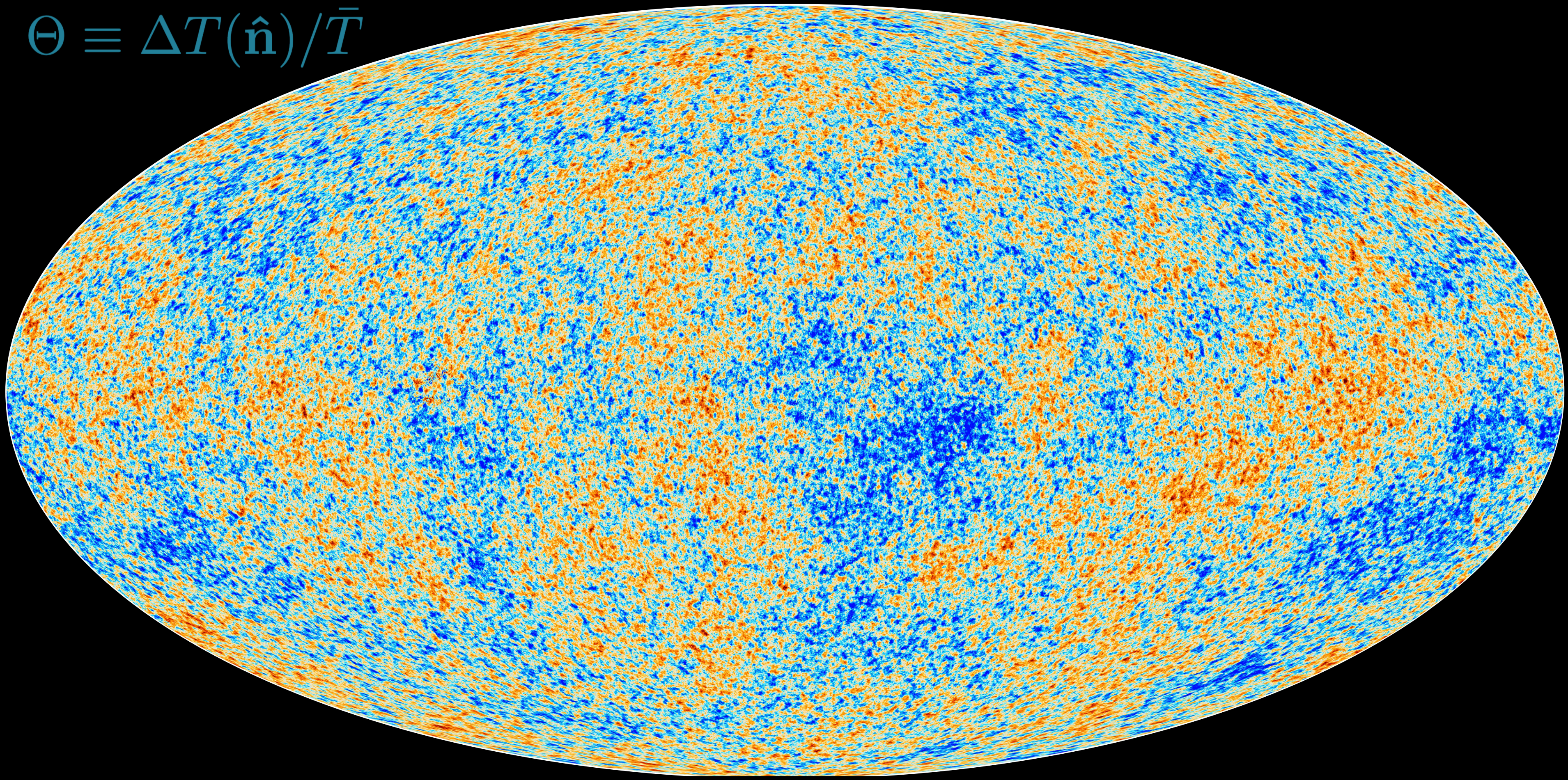
$$\Theta \equiv \Delta T(\hat{\mathbf{n}})/\bar{T}$$



CMB Brightness Fluctuations

CMB Fluctuations – Best Picture From Planck Satellite

$\Theta \equiv \Delta T(\hat{\mathbf{n}})/\bar{T}$



What exactly is it that we are looking at?

Last Scattering: The CMB “Surface”

$$dI = -In_e\sigma_T a d\eta$$

- $e^{-\tau}$ is probability of no scattering to time η
- where optical depth to conformal time η is:

$$\tau(\eta) = \int_{\eta}^{\eta 0} a n_e \sigma_T d\eta'$$

Last Scattering: The CMB "Surface"

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- where optical depth to conformal time η is:

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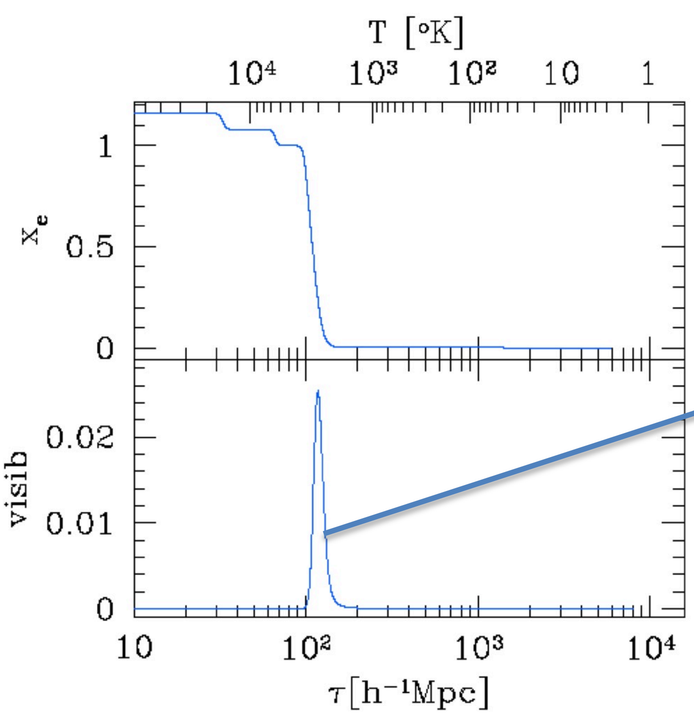
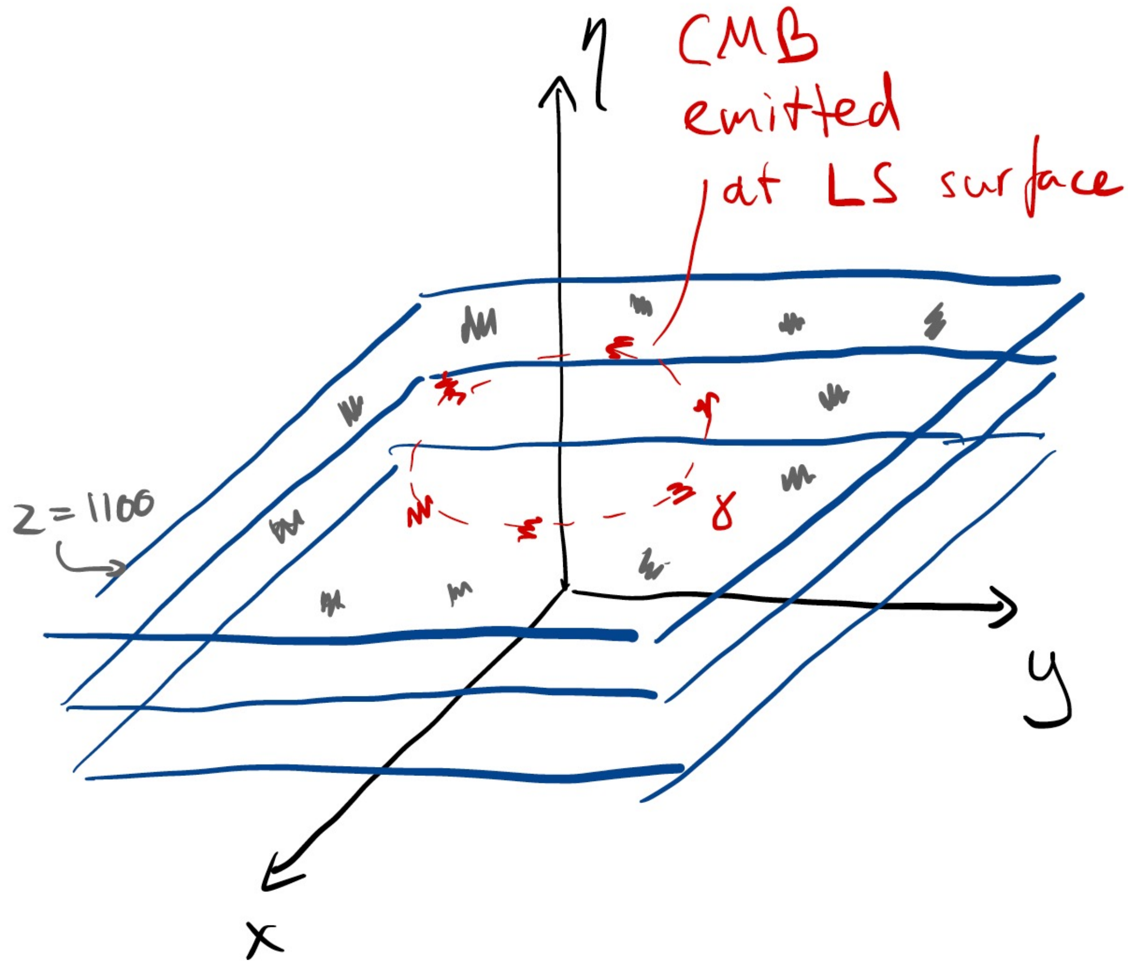
- Visibility: probability of last scattering at η $= -\dot{\tau} e^{-\tau}$



sharp peak at z~1100
last scattering ~surface

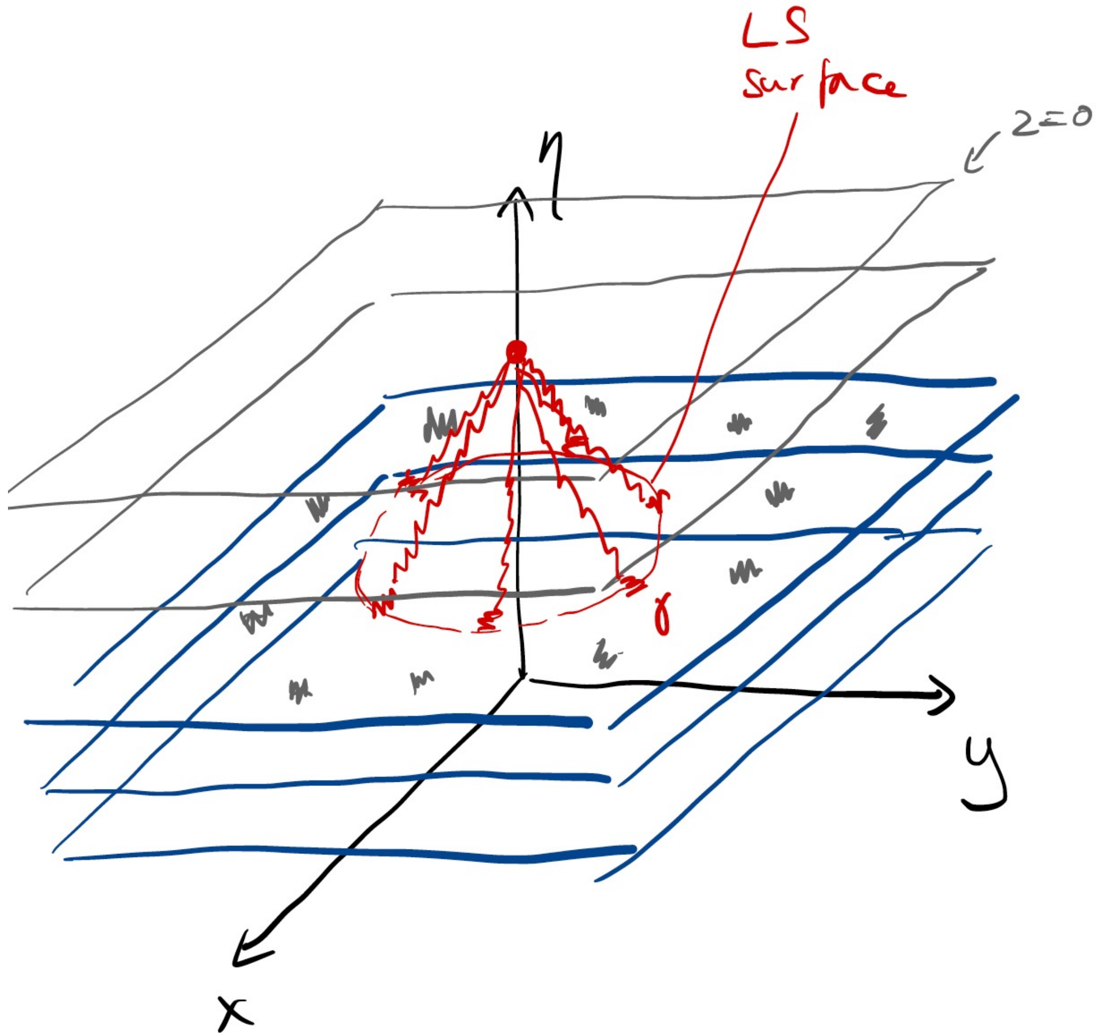
Figure: A. Challinor

Physical Picture: CMB Last-Scattering Sphere



We are seeing a “surface” in both time and space – 14 billion years in the past
(although see later: also some effects from intermediate distances / times)

Physical Picture: CMB Last-Scattering Sphere



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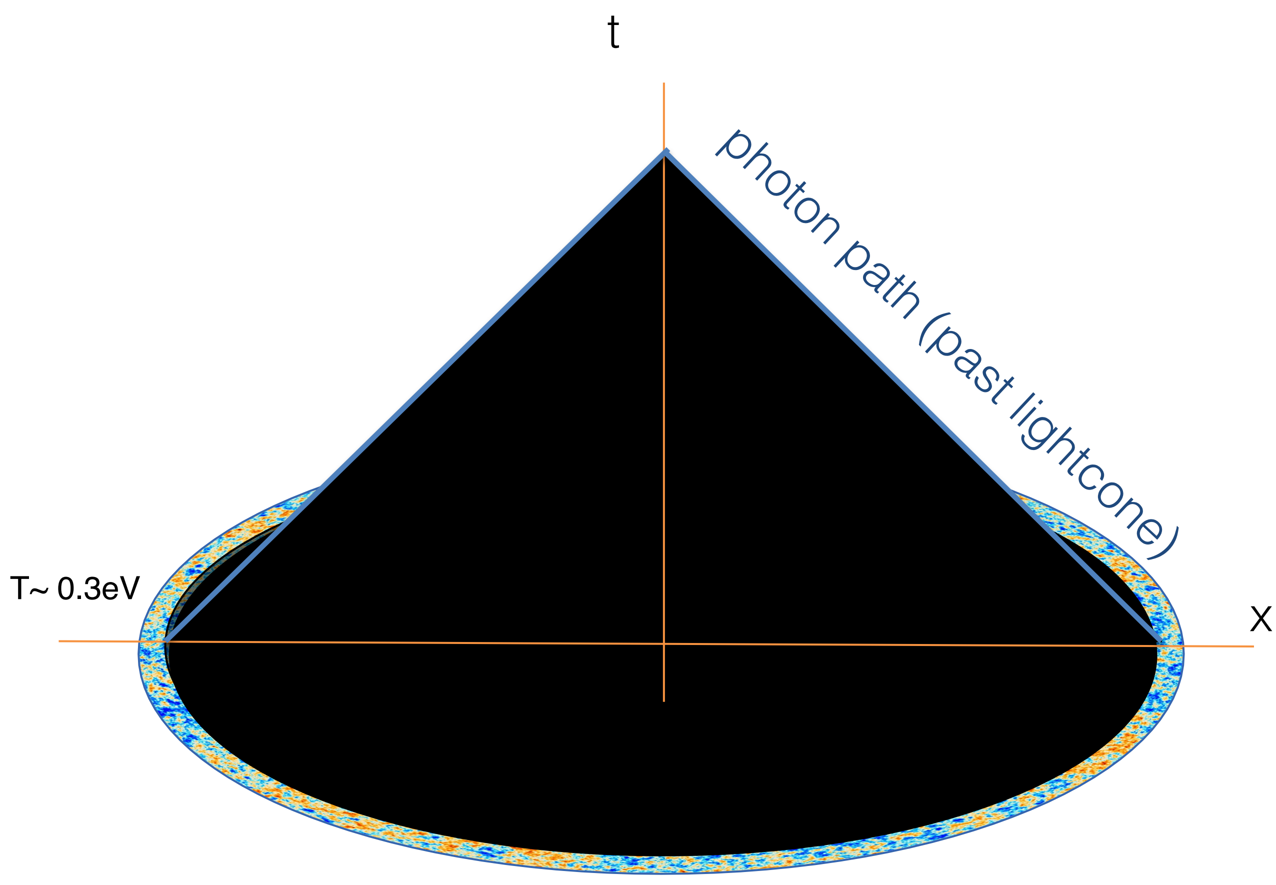
Physical Picture: CMB Last-Scattering Sphere

The observable universe:
(lightcone seen from top)

Primordial CMB emitted ~400000 years after big bang - t_e

We will assume instantaneous recombination!



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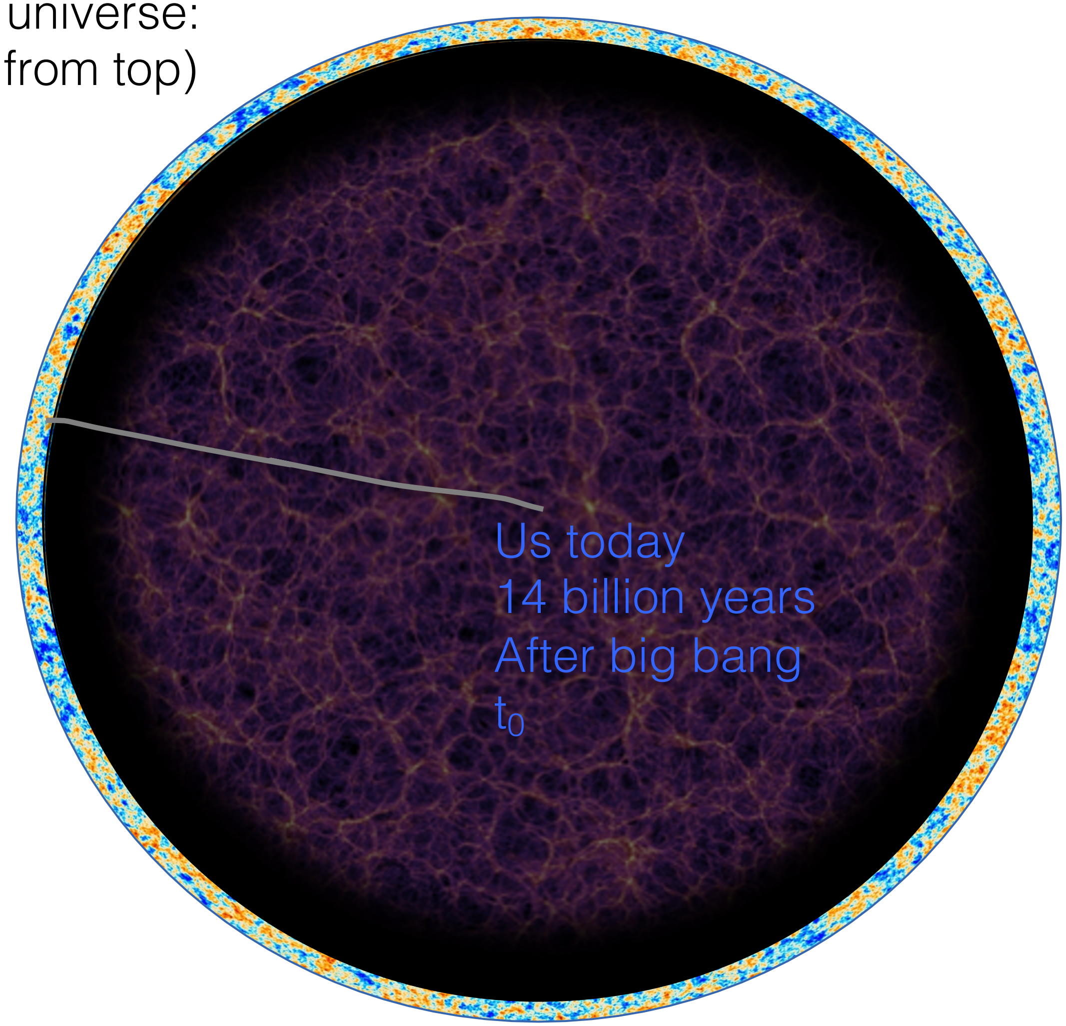

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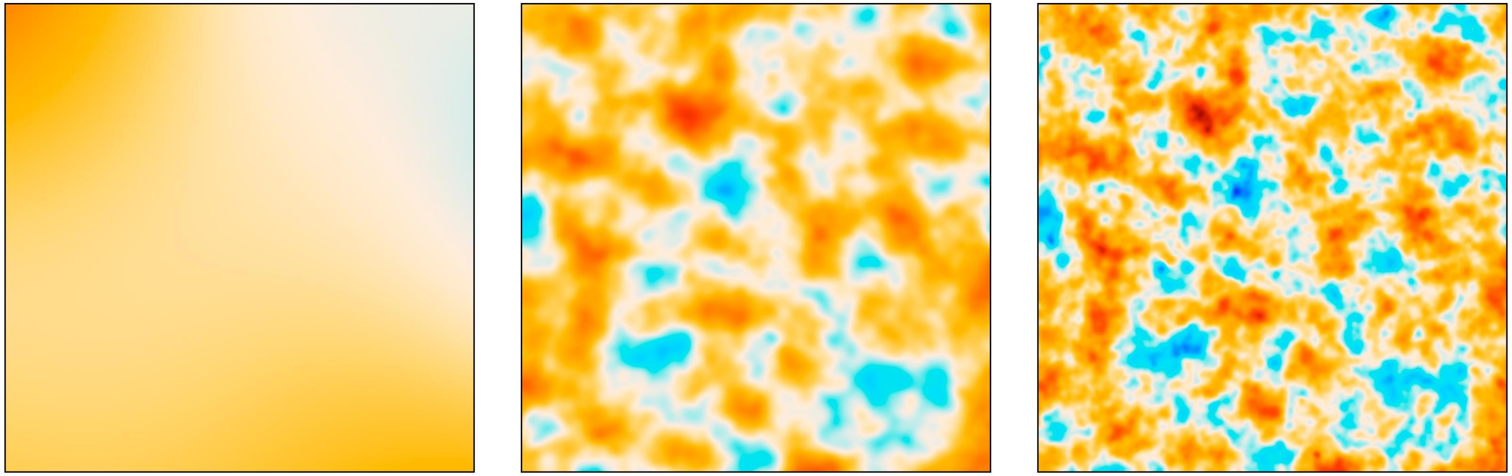
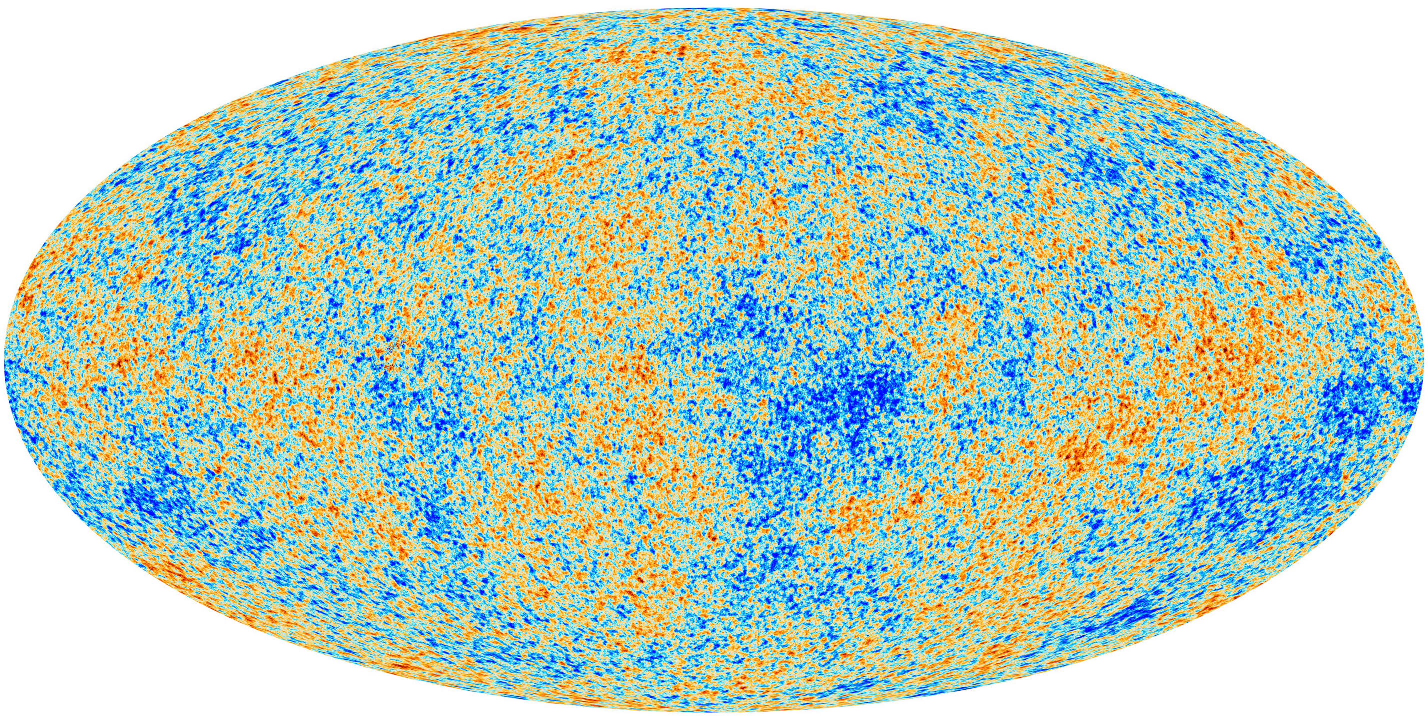
- Basic CMB statistics
- CMB photon propagation



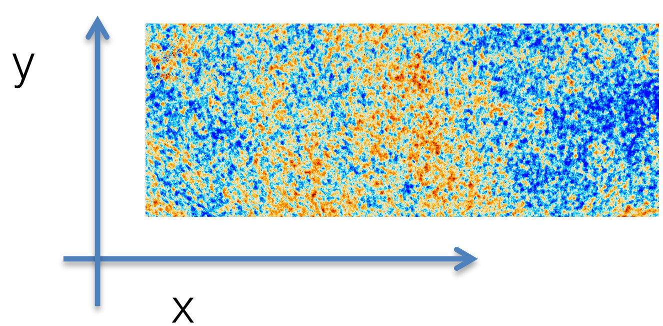
Understanding the CMB quantitatively / statistically

- Our theories cannot predict the actual realization of the fields
- We can only predict its statistical properties defined by correlation functions



Understanding the CMB statistically

- Our theories cannot predict the actual realization of the fields
- We can only predict its statistical properties defined by correlation functions.
- Let's first consider a small 2D cutout of the CMB and assume it is flat (flat sky approximation).



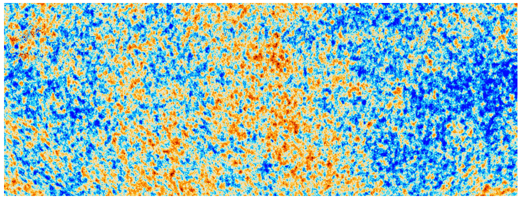
Understanding the CMB statistically

- Can define two-point correlation function, which fully describes Gaussian random field:

$$\xi(\mathbf{x},\mathbf{x}') \equiv \langle\Theta(\mathbf{x})\Theta(\mathbf{x}')\rangle = \int \mathcal{D}\Theta\ \Theta(\mathbf{x})\Theta(\mathbf{x}')\mathrm{Pr}[\Theta]$$

- Require statistical isotropy / translation invariance

$$\xi(\mathbf{x},\mathbf{x}') = \xi(\mathbf{x}+\mathbf{a},\mathbf{x}'+\mathbf{a})$$



- Now consider Fourier Transform $\Theta(\mathbf{x}) = \int \frac{d^2l}{(2\pi)^2}\Theta(\mathbf{l})e^{i\mathbf{l}\cdot\mathbf{x}}$

Understanding the CMB statistically

- What does this translation invariance imply for the Fourier coefficients? Require

$$\xi(\mathbf{x}, \mathbf{x}') = \xi(\mathbf{x} + \mathbf{a}, \mathbf{x}' + \mathbf{a})$$

for

$$\xi(\mathbf{x}, \mathbf{x}') \equiv \langle \Theta(\mathbf{x}) \Theta(\mathbf{x}') \rangle = \int \frac{d\mathbf{l}}{(2\pi)^2} \int \frac{d\mathbf{l}'}{(2\pi)^2} e^{i\mathbf{l}\cdot\mathbf{x} + i\mathbf{l}'\cdot\mathbf{x}'} \langle \Theta(\mathbf{l}) \Theta(\mathbf{l}') \rangle$$

$+a$

- This implies (along with rotational invariance for | |):

$$\langle \Theta(\mathbf{l}) \Theta(\mathbf{l}') \rangle = (2\pi)^2 C_{|\mathbf{l}|} \delta^{(D)}(\mathbf{l} + \mathbf{l}')$$

with C the power spectrum of the CMB

Large-scales: spherical harmonics

- On large scales we need a better basis to describe the full sphere. Use spherical harmonics instead of Fourier modes. But ideas the same. Expand:

$$\Theta(\hat{\mathbf{n}}) = \sum_{l=0}^{\infty} \sum_{m=-l}^{l} a_{lm} Y_{lm}(\hat{\mathbf{n}})$$

- Spherical harmonics are orthonormal over sphere

$$\int d\hat{\mathbf{n}} Y_{lm}(\hat{\mathbf{n}}) Y^*_{l'm'}(\hat{\mathbf{n}}) = \delta_{ll'} \delta_{mm'}$$

- Therefore the spherical multipole coefficients are

$$a_{lm} = \int d\hat{\mathbf{n}} Y^*_{lm}(\hat{\mathbf{n}}) \Theta(\hat{\mathbf{n}})$$

Large-scales: spherical harmonics

$$a_{lm} = \int d\hat{\mathbf{n}} Y^*_{lm}(\hat{\mathbf{n}})\Theta(\hat{\mathbf{n}})$$

- As in flat sky, translation invariance means the fields obey:

$$\langle a_{lm} a^*_{lm} \rangle = C_l \delta_{ll'} \delta_{mm'}$$

- Which defines the power spectrum. We can measure it as follows:

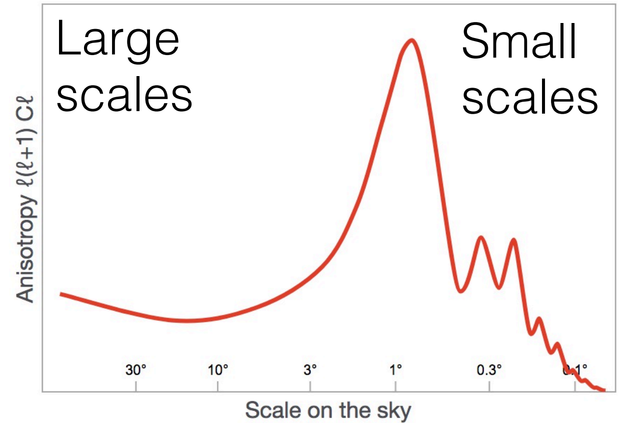
$$\hat{C}_l = \Sigma_m |a_{lm}|^2/(2l+1)$$

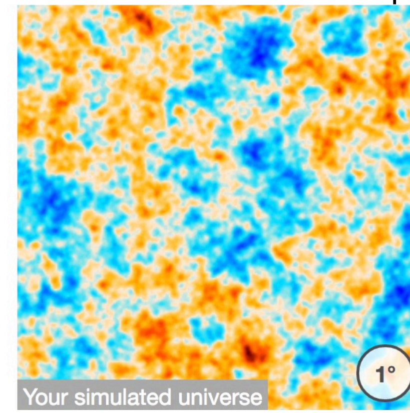
though with an error as we only have finite m modes.

Power Spectrum vs. Map Intuition

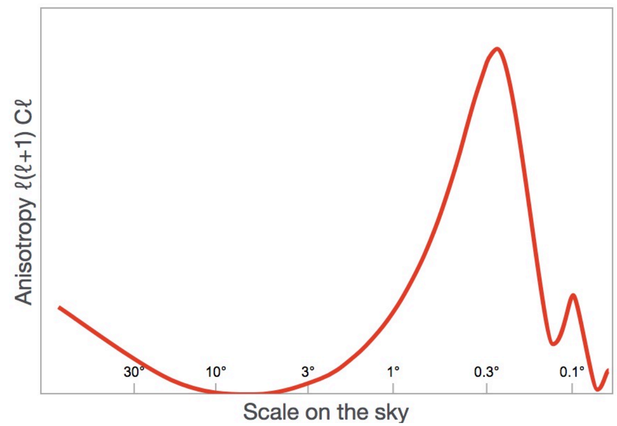
CMB map

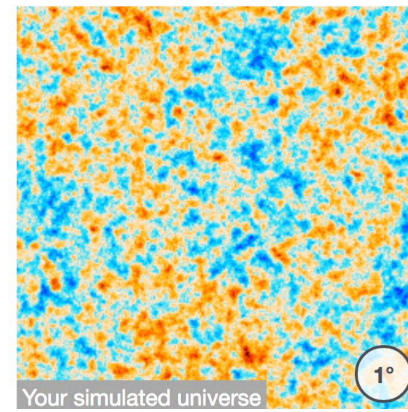
- Standard CMB



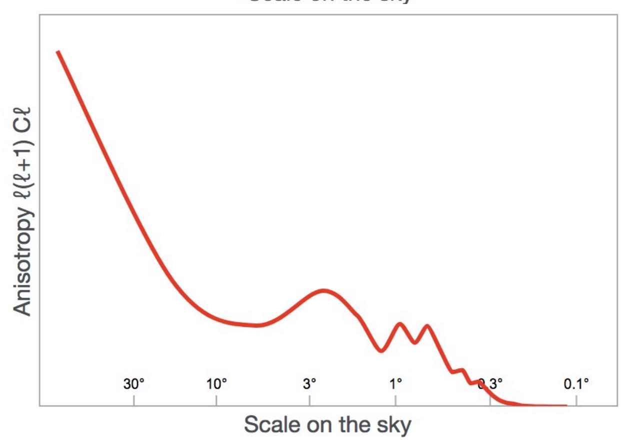


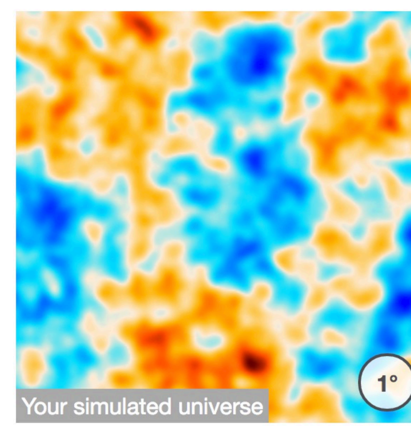
- Power on smaller scales



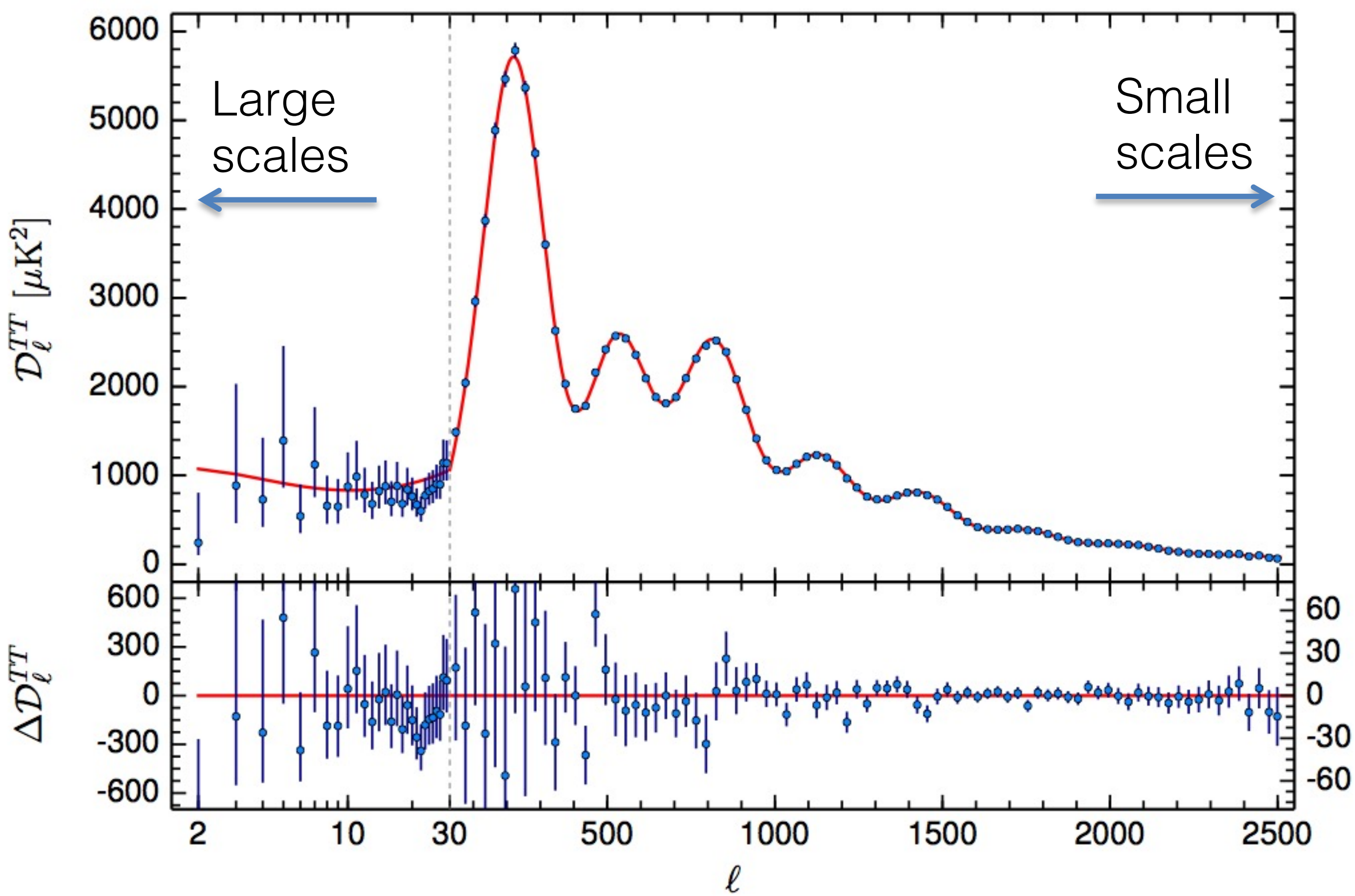


- Power on larger scales





Measurement: The Planck CMB Power Spectrum



$$\mathcal{D}_l \equiv l(l+1)C_l/2\pi$$

Goal for this and next two lectures: understand this CMB power spectrum

- Necessary steps: need to understand

I.: photon propagation from when the CMB was emitted

II.: initial conditions

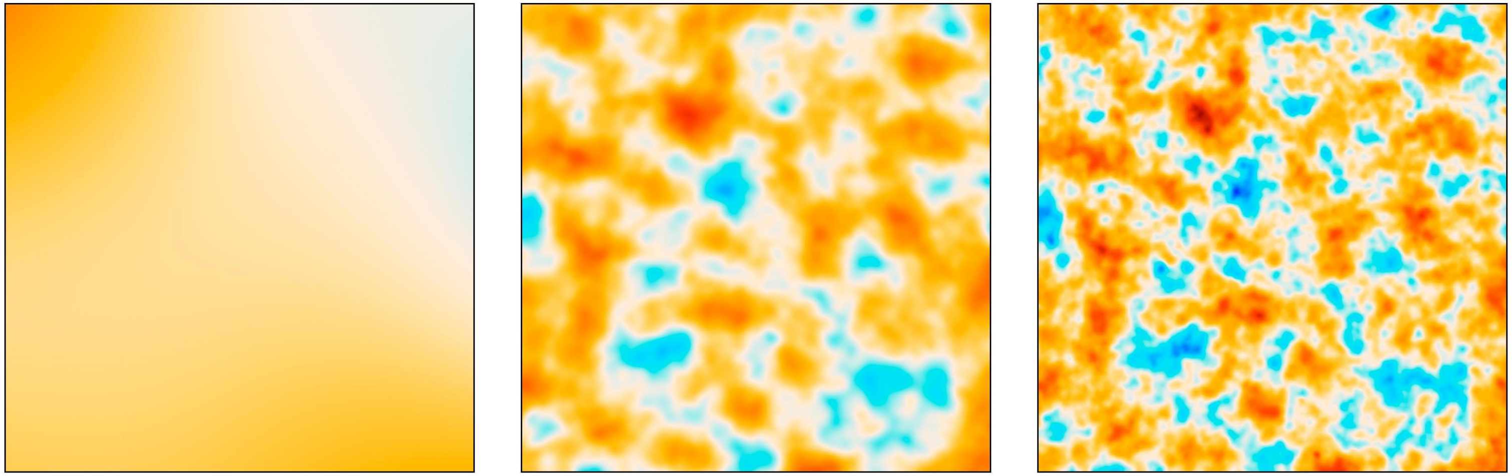
III.: evolution in plasma (acoustic processing) from initial conditions to perturbations at emission

$$\underset{\text{II.}}{\mathcal{R}(\mathbf{k},0)} \underset{\text{III.}}{\longrightarrow} (\delta_r, \Phi, \mathbf{v})_{\eta *} \underset{\text{I.}}{\longrightarrow} \delta T(\hat{\mathbf{n}})$$

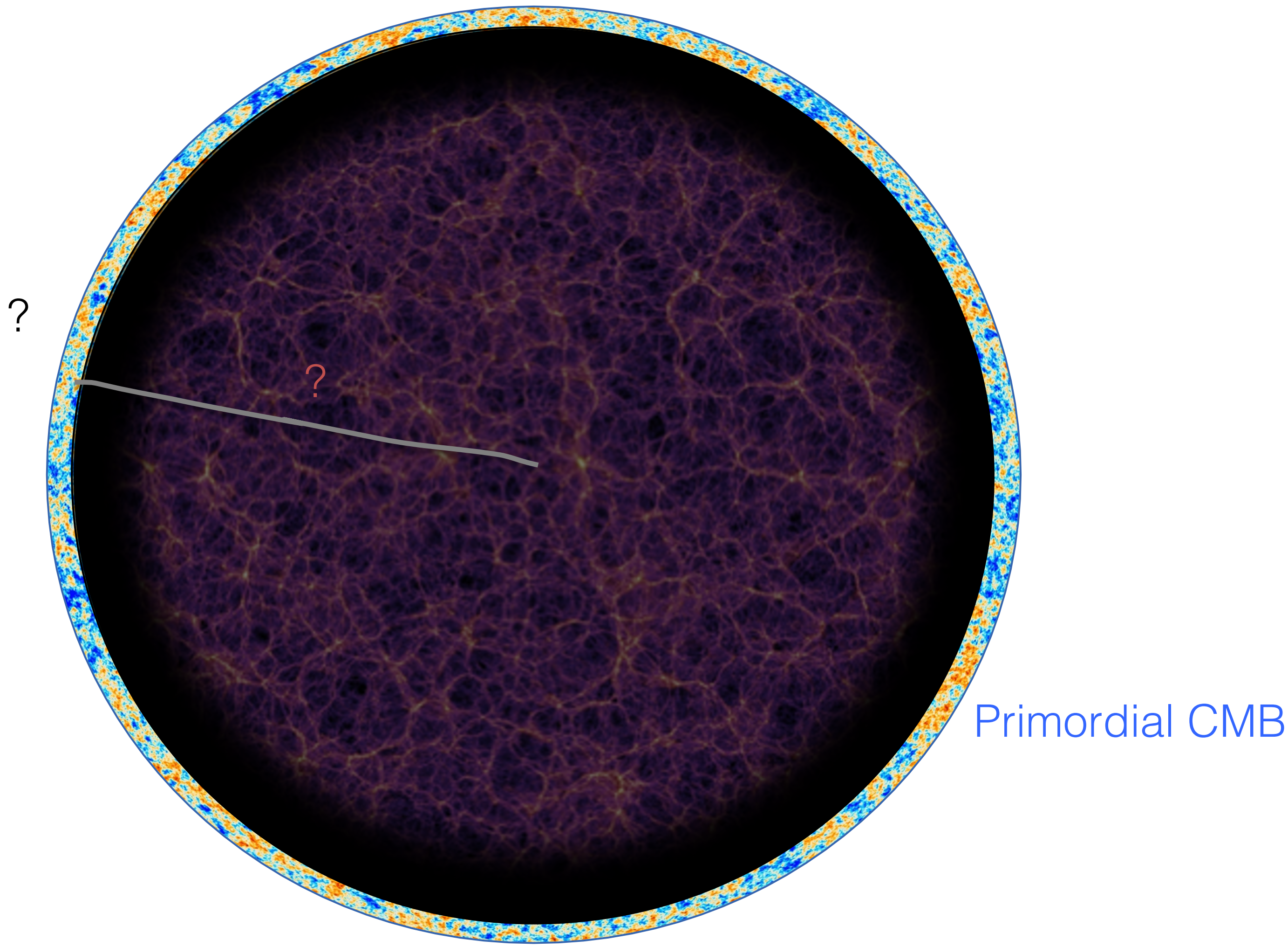
Start with I; will discuss II/III next lectures.

Outline: Today's Lecture 1

- Introduction: CMB from the hot Big Bang
- Basic CMB statistics
- CMB photon propagation and anisotropy basics



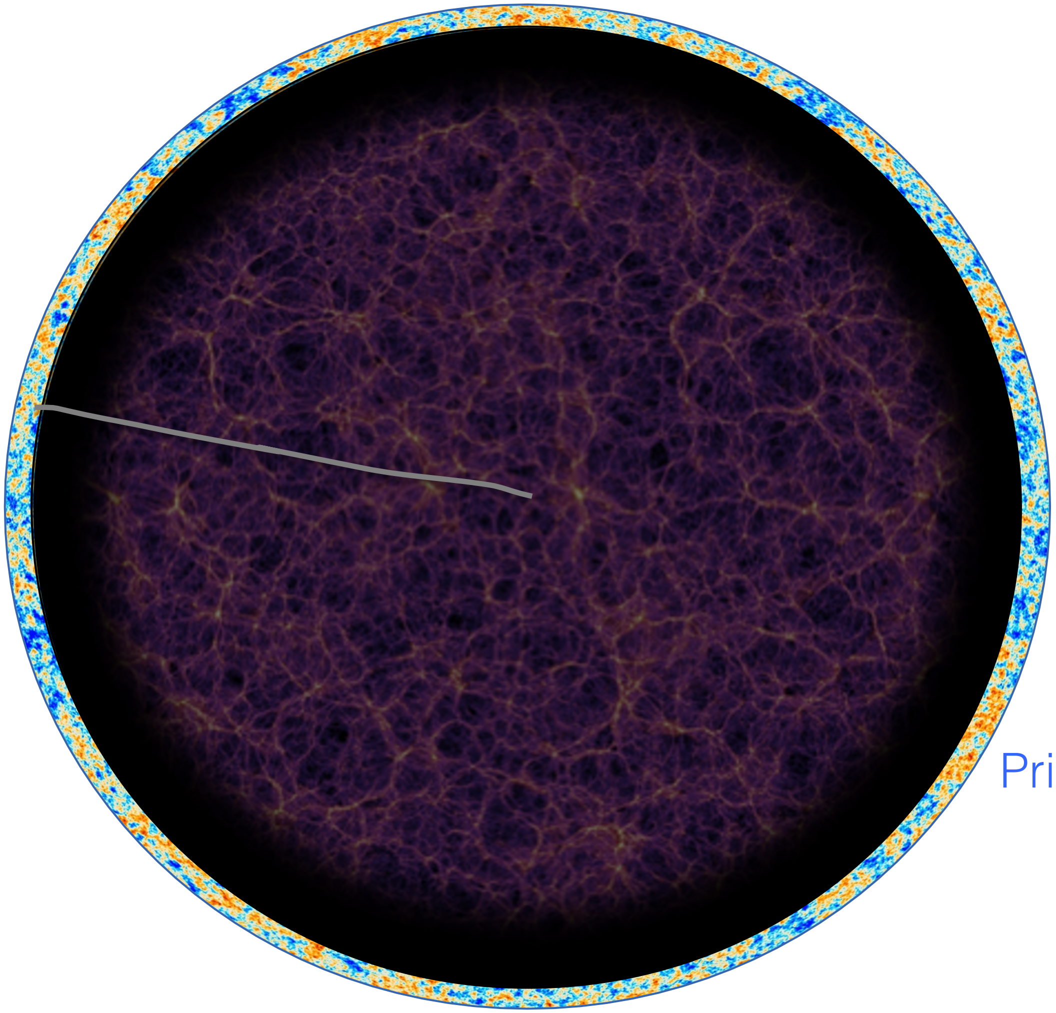
What determines the structure we see in the CMB?



- We will now explain how the CMB temperature fluctuations we see relate to cosmological quantities

Physical Picture: CMB Last-Scattering Sphere

CMB is emitted from last scattering surface whenever local temperature is T_*



- Calculate propagation to relate CMB to conditions on last scattering surface and conditions on the way.

Propagation of CMB photons

- Need to calculate propagation of light in clumpy universe. Differences in density cause changes to spacetime distance

$$ds^2 = a^2(\eta)\left[(1+2\Phi)d\eta^2 - (1-2\Phi)\delta_{ij}dx^i dx^j\right]$$

$$= g_{\mu\nu}dx^\mu dx^\nu$$

metric

Standard Newtonian potential
(affects spacetime distances!)
Neglect anisotropic stress.

- What is trajectory $x^\mu = (t, x^i)$ described in terms of a parameter λ ?

Propagation of CMB photons

- Particle path is determined such that proper spacetime distance is minimized. This gives the geodesic equation

$$\frac{d^2x^\mu}{d\lambda^2} + \Gamma^\mu_{\nu\rho}\frac{dx^\nu}{d\lambda}\frac{dx^\rho}{d\lambda} = 0$$

Christoffel symbol (function of metric)
correction to make coord. invariant

- Can write in terms of momenta $p^\mu = dx^\mu/d\lambda$ (ideal for photons):

$$\frac{dp^\mu}{d\lambda} + \Gamma^\mu_{\nu\rho}p^\nu p^\rho = 0$$

What determines CMB temperature fluctuations?

- Can show: $p^0 = E(1-\Phi)/a$
- (Since $E = p^\nu u_\nu$, and $u^\nu = \frac{dx^0}{ds}\delta^{0\nu} = (1-\Phi)\delta^{0\nu}/a$)
- Evaluate geodesic equation for p^0 component

$$\frac{dp^\mu}{d\lambda} + \Gamma^\mu_{\nu\rho} p^\nu p^\rho = 0$$

n.b. photons on null geodesics $p^\mu p_\mu = 0$

What determines CMB temperature fluctuations?

- This give

$$\frac{d\ln(aE)}{d\eta} = -\frac{d\Phi}{d\eta} + 2\Phi'$$ partial deriv.

- Integrate and Taylor expand the exponential:

$$E_0/(a(\eta_e)E(\eta_e)) = \exp\left[\Phi_e - \Phi_0 + \int_e^0 d\eta 2\phi'\right]$$

$$E_0 = a(\eta_e)E(\eta_e)\left[1 + \Phi_e - \Phi_0 + \int_e^0 d\eta 2\Phi'\right]$$

0=today, e=perturbed LS, *= average LS

- E and T evolve in the same way so (also adding Doppler shift term by hand):

$$\frac{T_{obs}}{T_*} = a(\eta_e)\left[1 + \Phi_e - \Phi_0 - \hat{\mathbf{n}}\cdot\mathbf{v} + \int_e^0 d\eta 2\Phi'\right]$$

What determines CMB temperature fluctuations?

- What is the scale factor when the CMB is emitted?

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- Can show this means $a(\eta_e) = a(\eta_*)(1 + \delta_r/4)$

$$\delta_r \equiv \frac{\delta\rho_r}{\bar{\rho}_r}$$

perturbation to radiation energy density / mean radiation energy density

What determines CMB temperature fluctuations?

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- Same T means $(\bar{\rho_r} + \delta\rho_r)[\eta_* + \delta\eta] = \bar{\rho}_r[\eta_*] = bT_*^4$ and hence

$$\delta\eta = -\frac{\delta\rho_r}{\bar{\rho}'_r} = -\frac{\delta\rho_r}{d\rho_r/da \times da/d\eta} = \frac{\delta_r}{4\frac{a'}{a}}$$

Since $\rho_r \propto a^4$

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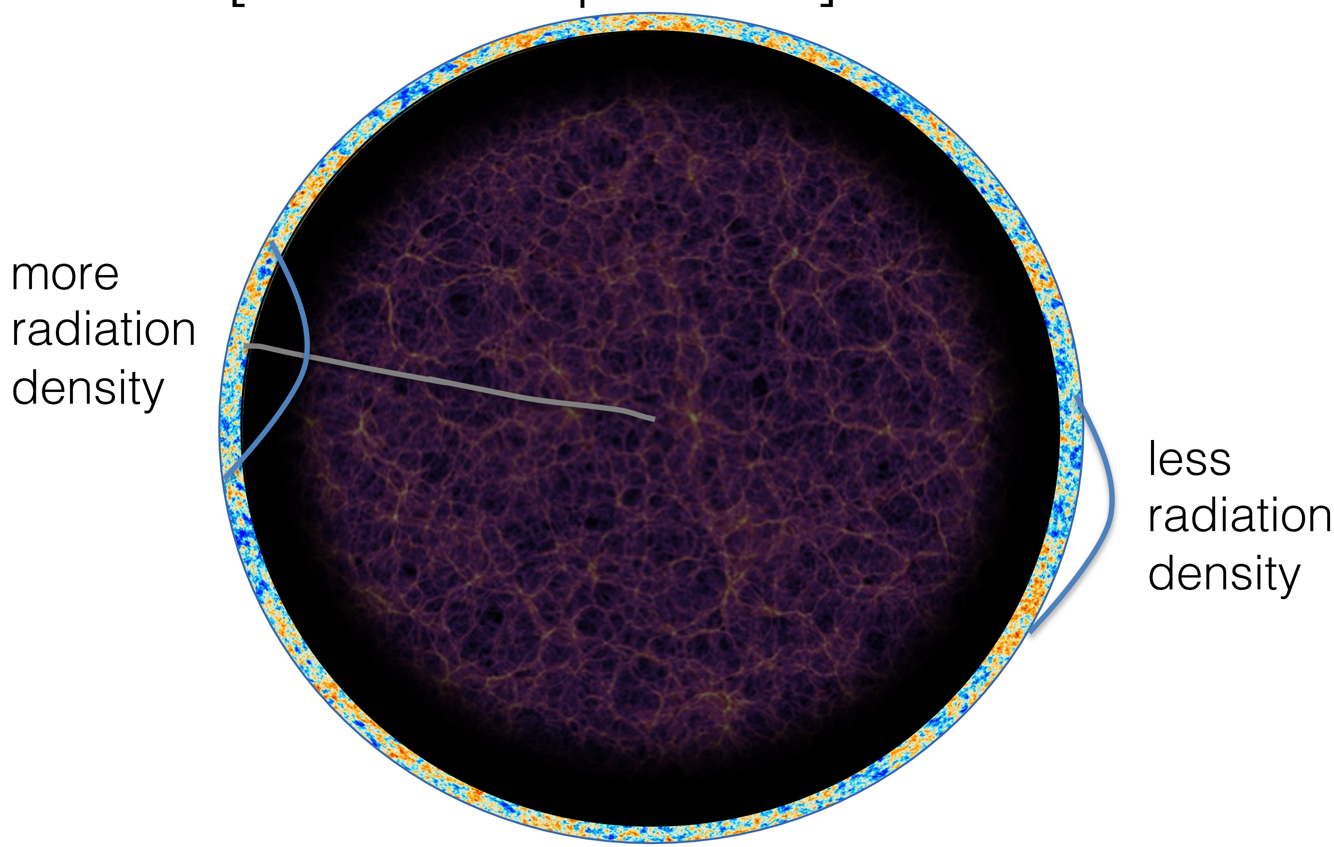
to get result add $a(\eta_e) = a(\eta_* + \delta\eta) = a(\eta_*)(1 + \frac{a'}{a}\delta\eta)$

Relating CMB to perturbations

- In other words, where we have a higher density, the CMB is emitted later with a larger scale factor, giving less redshifting (a=1/(1+z)) and a positive temperature fluctuation. [Answer to question!]



Relating CMB to perturbations

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- Putting this all together, Taylor expanding to first order, using $\Theta \equiv \Delta T(\hat{\mathbf{n}})/\bar{T}$ (and absorbing monopole local potential)

$$\Theta(\hat{\mathbf{n}}) = \frac{\delta_r}{4} + \Phi_e - \hat{\mathbf{n}} \cdot \mathbf{v} + \int_e^0 d\eta 2\Phi'$$

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Note all these terms are 3D quantities evaluated on last scattering surface: at position, time = $((\eta_0 - \eta_*)\hat{\mathbf{n}}, \eta_*) \equiv (\chi_* \hat{\mathbf{n}}, \eta_*)$

Relating CMB to perturbations

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Density perturbation
causes temperature increase

$$\delta_r \equiv \frac{\delta\rho_r}{\bar{\rho}_r}$$

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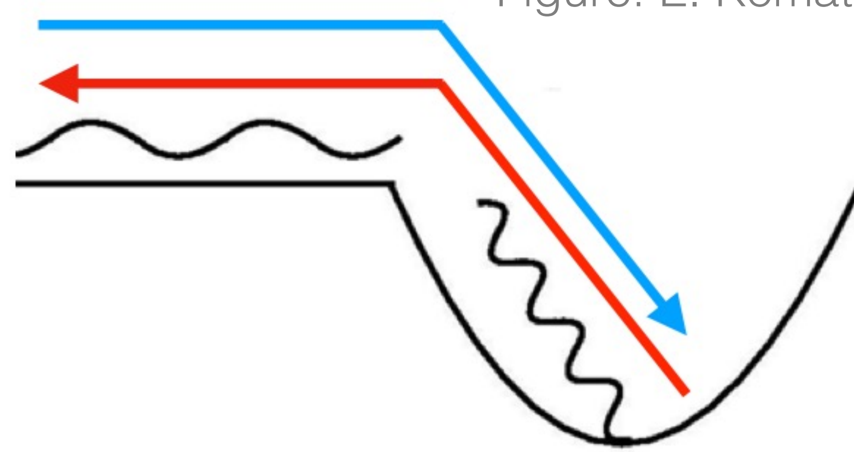
Potential minimum / maximum
Causes photon redshifting / blueshifting
Temperature –ve / positive

Figure: E. Komatsu



$$\Theta \sim S \equiv \frac{\delta_r}{4} + \Phi_e$$

N.B. Largest terms S

Relating CMB to perturbations

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Last scattering surface
moving towards observer causes
blueshifting

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ISW effect: potentials decaying causes blueshifting and positive T

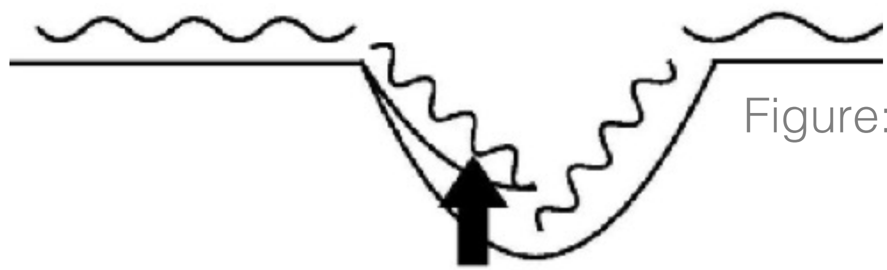

Figure: E. Komatsu

If potential decays, more energy gained falling in than lost climbing out

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- Next lecture: connect with initial conditions, derive CMB power!