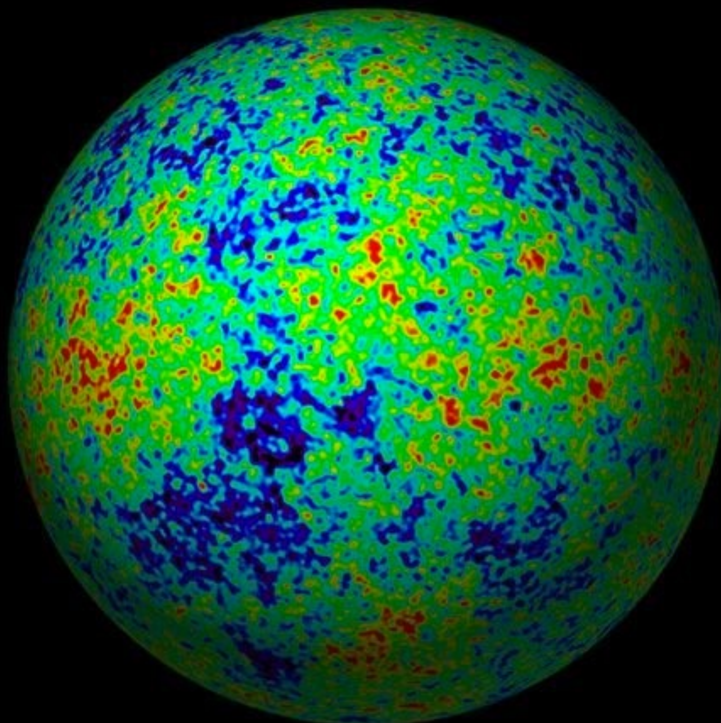
The Cosmic Microwave Background Lecture 2: From Initial Conditions to CMB Power



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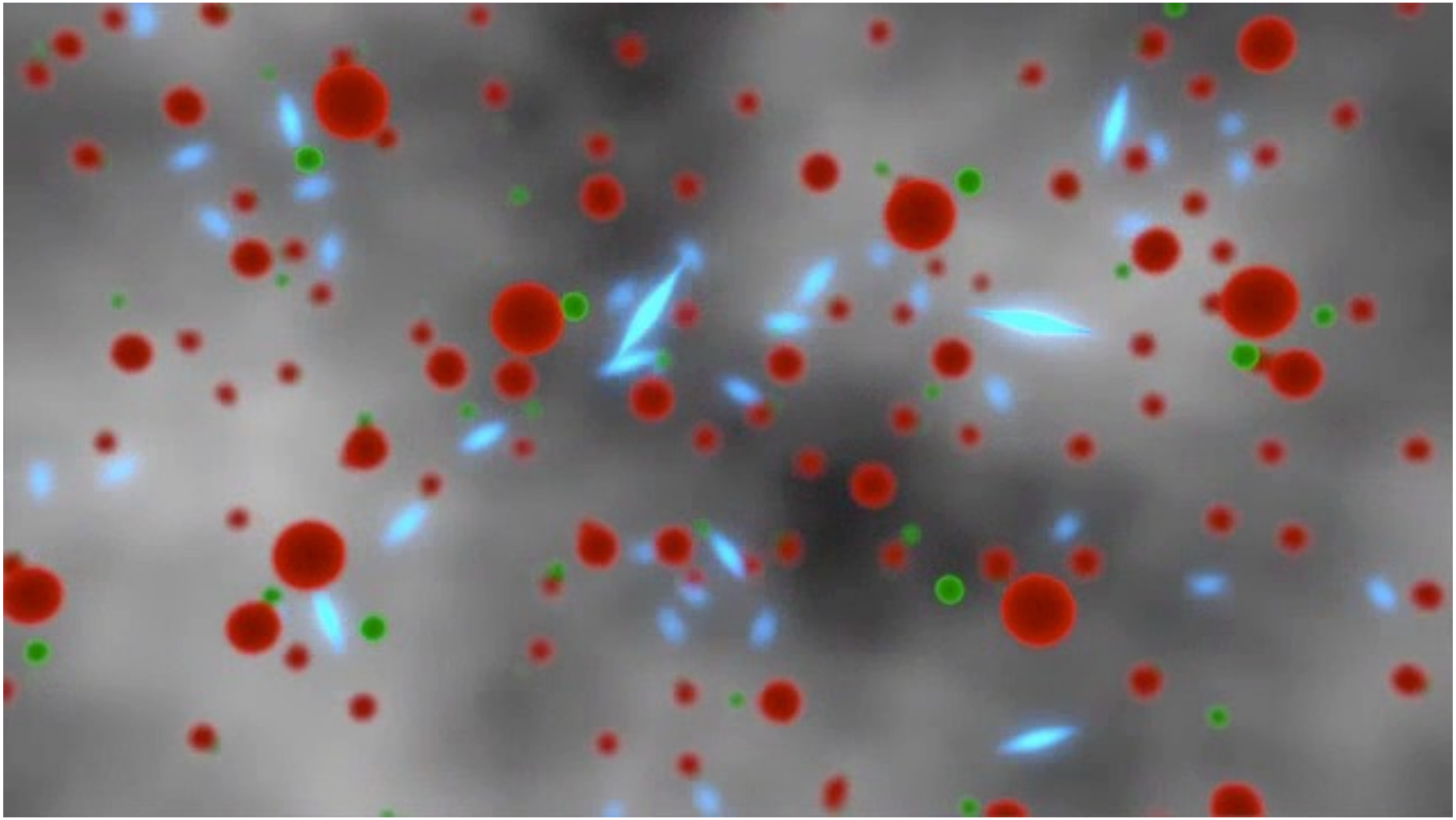
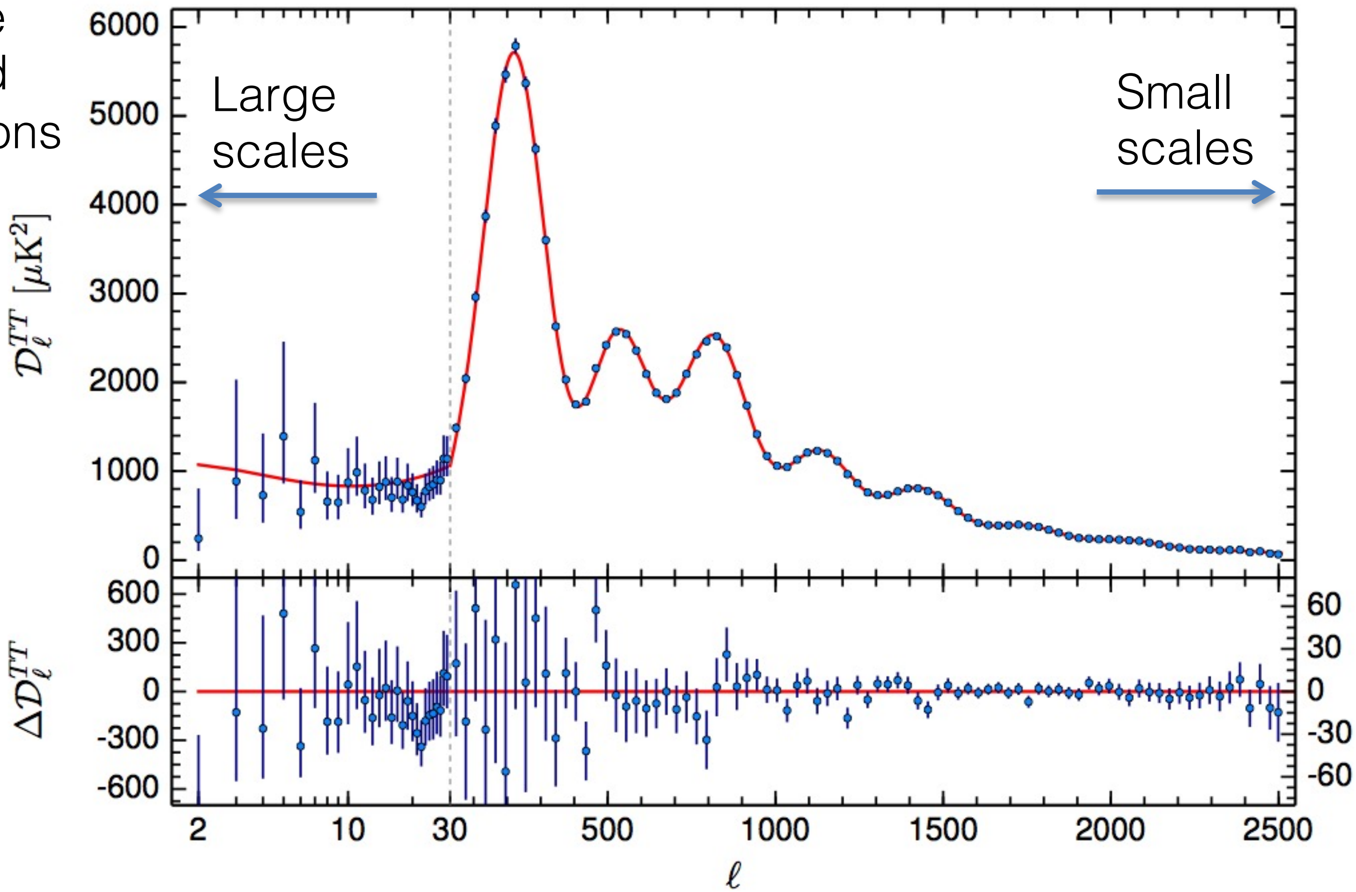


Measurement: The Planck CMB Power Spectrum

Magnitude of squared T fluctuations Θ



Inverse scale $\ell \sim \frac{2\pi}{\theta}$

$$\mathcal{D}_l \equiv l(l+1)C_l/2\pi$$

like $k \sim \frac{2\pi}{\lambda}$

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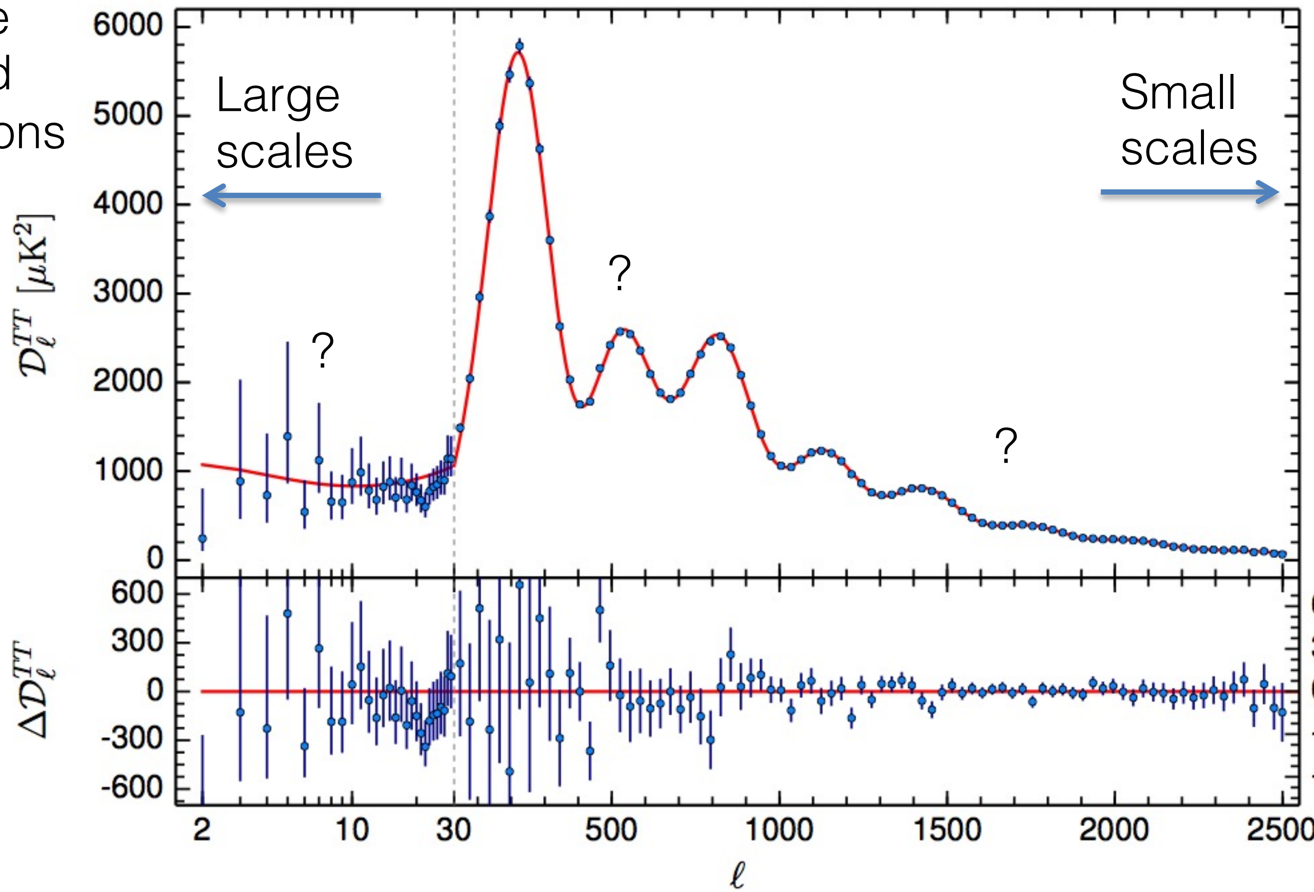


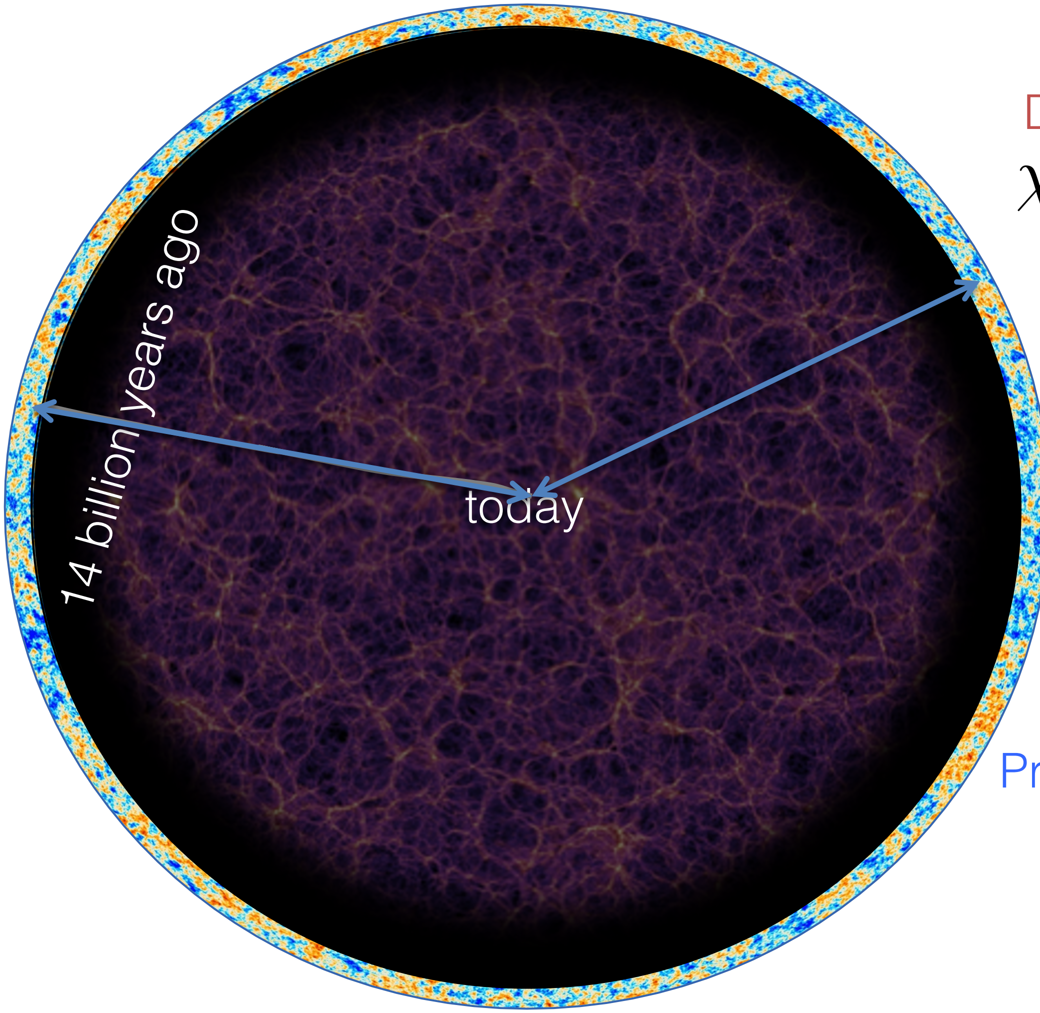
Inverse scale $\ell \sim \frac{2\pi}{\theta}$

$$\mathcal{D}_l \equiv l(l+1)C_l/2\pi$$

- Today: want to understand this spectrum! like $k \sim \frac{2\pi}{\lambda}$

Reminder: CMB Last-Scattering Sphere

CMB is emitted from last scattering surface whenever local temperature is T_*



Distance

$$\chi_* = \eta_0 - \eta_*$$

Primordial CMB

- Calculated propagation to relate CMB to conditions on last scattering surface and conditions on the way.

Reminder: CMB Temperature

$$\Theta(\hat{\mathbf{n}}) = \frac{\delta_r}{4} + \Phi_e - \hat{\mathbf{n}} \cdot \mathbf{v} + \int_e^0 d\eta 2\Phi'$$

Density perturbation
causes temperature increase

Last scattering surface
moving towards observer cause
blueshifting

Potential minimum / maximum
causes photon redshifting / blueshifting
Temperature –ve / positive

ISW effect: potentials decaying causes
blueshifting and positive T

Note first 3 terms are 3D quantities
evaluated on last scattering surface:
at position, time = $((\eta_0 - \eta_*)\hat{\mathbf{n}}, \eta_*) \equiv (\chi_* \hat{\mathbf{n}}, \eta_*)$

Reminder: CMB Temperature

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ISW effect: potentials decaying causes
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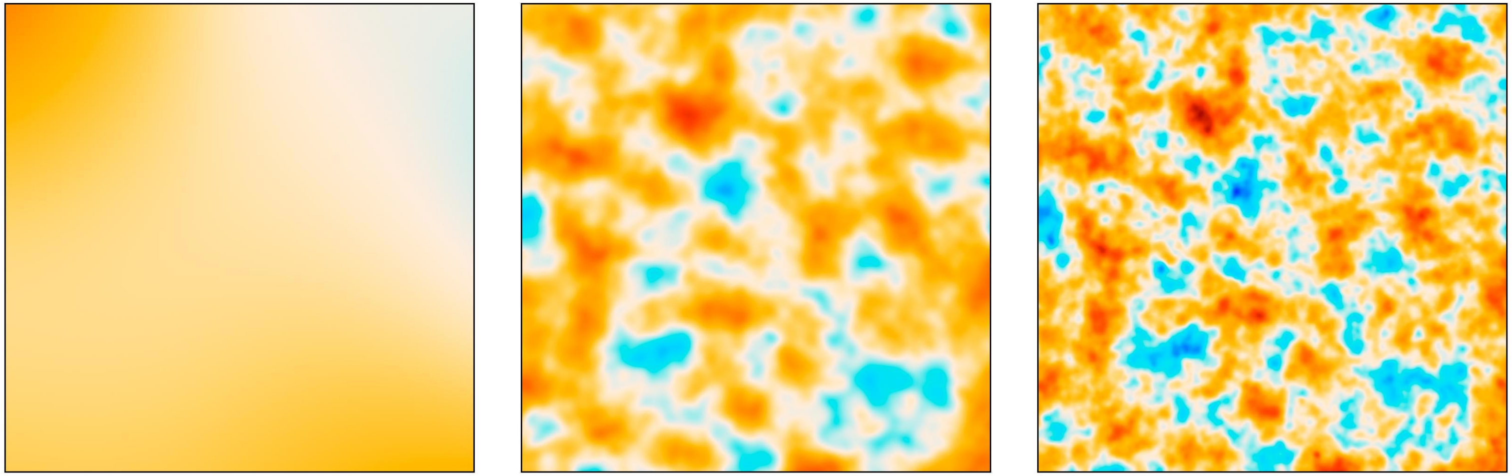
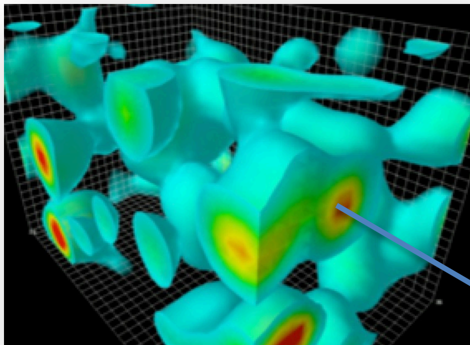
- Use this to connect power spectrum to initial conditions

Outline

- Connection of power spectrum to initial conditions
- Acoustic processing: basic equations
- Acoustic processing: detailed solution

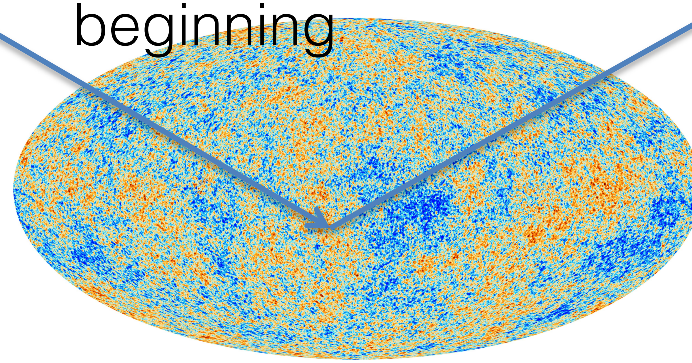


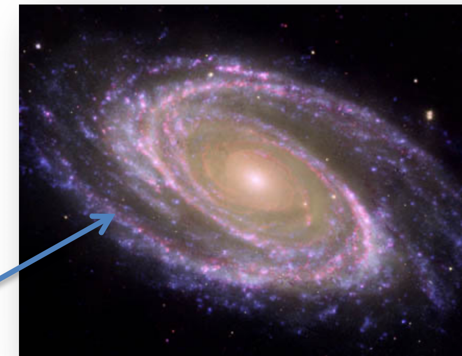
Initial Conditions: Inflationary Quantum Fluctuations Make Everything!



Inflation: accelerated expansion blows quantum fluctuations up, generates small differences in density

We can see these density differences in the CMB, a picture of (nearly) the beginning

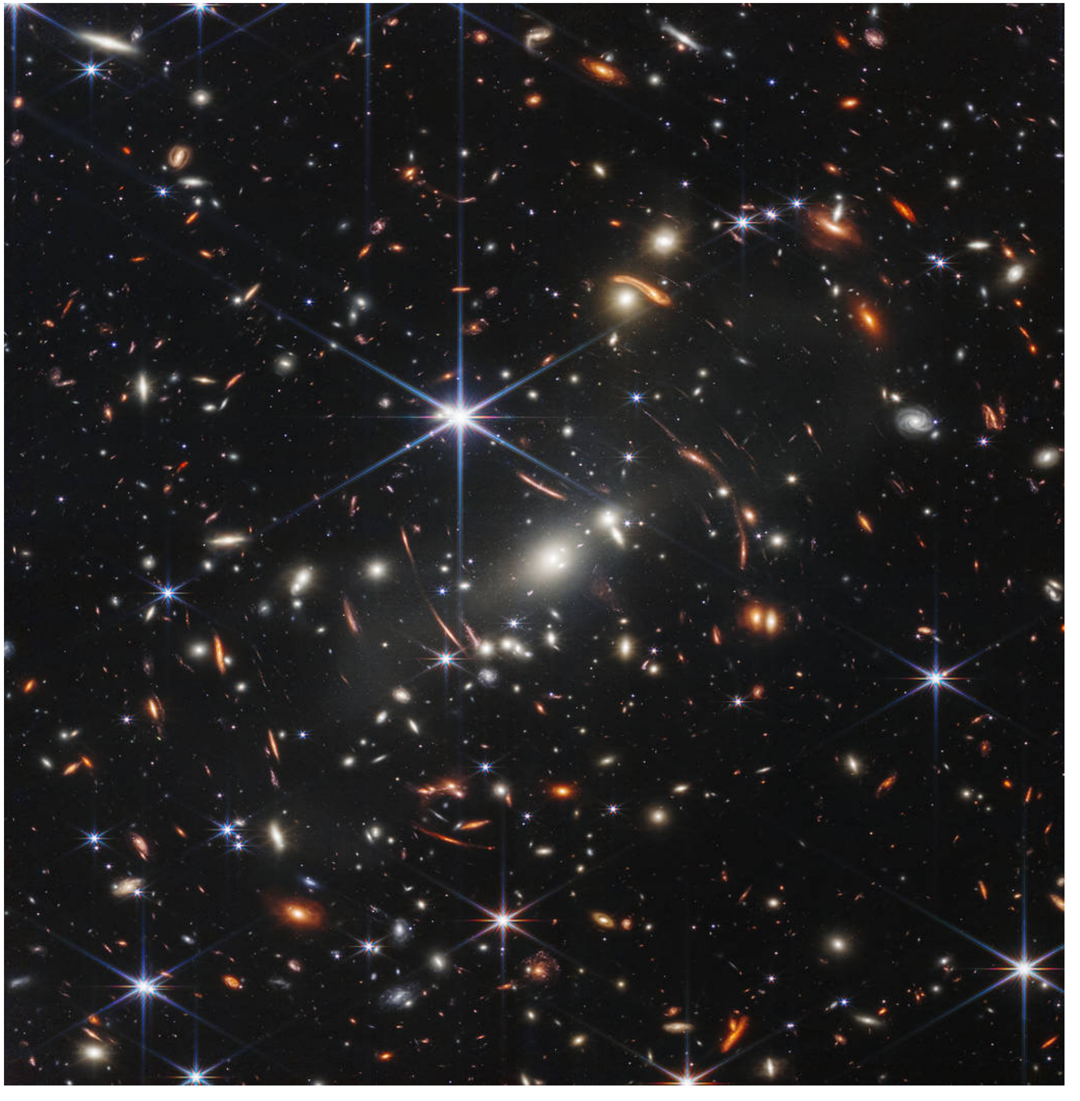




Over time, the smaller scales collapse into the stars and galaxies we see around us today!

Inflation: we (and everything) exist because of random quantum vacuum fluctuations in the early universe

Initial Conditions: Inflationary Quantum Fluctuations Make Everything! (including this)



Initial conditions: assumptions (for now)

- See later and other courses for more details on generation of inflationary perturbations

- Predicted quantity is the comoving curvature perturbation (constant and conserved outside horizon.) You can think of this as ~ the potential perturbation initial conditions at early times:

$$\mathcal{R} \sim \Phi$$

$$\mathcal{R} = -\frac{5+3w}{3+3w}\Phi$$

Initial conditions: assumptions (for now)

- Inflation tells us that the curvature perturbation is well described by a power spectrum

$$\langle \mathcal{R}(\mathbf{k})\mathcal{R}^*(\mathbf{k}')\rangle = (2\pi)^3 P_{\mathcal{R}}(k)\delta^{(D)}(\mathbf{k}-\mathbf{k}')$$

- Where the power spectrum is nearly scale invariant

$$\left[\frac{k^3}{2\pi^2}P_{\mathcal{R}}(k)\right] \approx \text{const.}$$

Linear Evolution and Transfer Functions

- Since perturbations are very small, linear evolution holds and all quantities evaluated at the CMB last scattering surface

$$\Theta(\hat{\mathbf{n}}) = \frac{\delta_r}{4} + \Phi_e - \hat{\mathbf{n}} \cdot \mathbf{v} + \int_e^0 d\eta 2\Phi'$$

can be related to initial conditions with simple linear transfer functions T(k).

- E.g. for radiation, Fourier transforming $\delta_r(\mathbf{x}) \rightarrow \delta_r(\mathbf{k})$ we can write

$$\delta_r(\mathbf{k}, \eta) \equiv T_r(k, \eta)\mathcal{R}(\mathbf{k}, 0)$$

Radiation density perturbation

Transfer function

Initial condition curvature perturbation

Linear Evolution and Transfer Functions

- The most relevant terms in $\Theta(\hat{\mathbf{n}}) = \frac{\delta_r}{4} + \Phi_e - \hat{\mathbf{n}} \cdot \mathbf{v} + \int_e^0 d\eta 2\Phi'$ are

$$\Theta \sim S \equiv \frac{\delta_r}{4} + \Phi_e$$

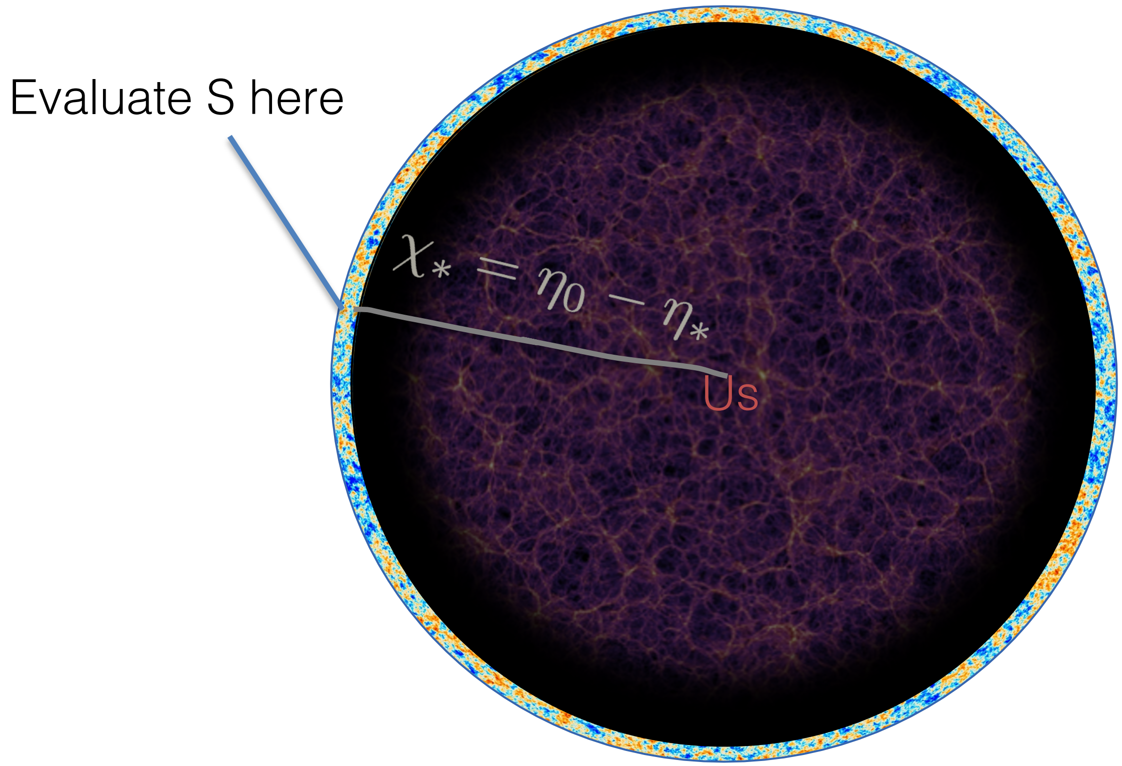
first two, where Sachs-Wolfe term S(**x**,η) is a 3D quantity.

- We can thus define a Fourier-space transfer function:

$$\Theta \sim S(\mathbf{k}, \eta_*) \equiv T_S(k, \eta_*)\mathcal{R}(\mathbf{k}, 0)$$

plasma processing — initial conditions

CMB power and projection



$$\chi_* = \text{distance to CMB}$$

N.B. This transform evaluated on one equal time slice

$$\Theta(\hat{\mathbf{n}}) = S(\mathbf{x} = \hat{\mathbf{n}}\chi_*, \eta_*) = \int \frac{d^3k}{(2\pi)^3} e^{+i\mathbf{k}\cdot\hat{\mathbf{n}}\chi_*} S(\mathbf{k}, \eta_*)$$

$$S(\mathbf{k}, \eta_*) \equiv T_S(k, \eta_*)\mathcal{R}(\mathbf{k}, 0) \quad e^{i\mathbf{k}\cdot\mathbf{x}} = 4\pi \sum_{lm} i^l j_l(kx) Y^*_{lm}(\hat{\mathbf{k}}) Y_{lm}(\hat{\mathbf{x}})$$

CMB power and projection

$$e^{i\mathbf{k}\cdot\mathbf{x}} = 4\pi \sum_{lm} i^l j_l(kx) Y^*_{lm}(\hat{\mathbf{k}}) Y_{lm}(\hat{\mathbf{x}})$$

$$\Theta(\hat{\mathbf{n}}) = S(\mathbf{x} = \hat{\mathbf{n}}\chi_*, \eta_*) = \int \frac{d^3k}{(2\pi)^3} e^{+i\mathbf{k}\cdot\hat{\mathbf{n}}\chi_*} S(\mathbf{k}, \eta_*)$$

$$= 4\pi \int \frac{d^3k}{(2\pi)^3} T_S(k, \eta_*) \mathcal{R}(\mathbf{k}, 0) \sum_{lm} i^l j_l(k\chi_*) Y^*_{lm}(\hat{\mathbf{k}}) Y_{lm}(\hat{\mathbf{n}})$$

- Read off spherical multipole coefficient

$$a_{lm} = 4\pi i^l \int \frac{d^3k}{(2\pi)^3} T_S(k, \eta_*) \mathcal{R}(\mathbf{k}, 0) j_l(k\chi_*) Y^*_{lm}(\hat{\mathbf{k}})$$

CMB power and projection

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- Read off spherical multipole coefficient

$$a_{lm} = 4\pi i^l \int \frac{d^3k}{(2\pi)^3} T_S(k, \eta_*) \mathcal{R}(\mathbf{k}, 0) j_l(k\chi_*) Y^*_{lm}(\hat{\mathbf{k}})$$

- Exercise: computer power spectrum

$$\langle a_{lm} a^*_{lm} \rangle = C_l \delta_{ll'} \delta_{mm'}$$

using $\langle \mathcal{R}(\mathbf{k}) \mathcal{R}^*(\mathbf{k}') \rangle = (2\pi)^3 P_{\mathcal{R}}(k) \delta^{(D)}(\mathbf{k} - \mathbf{k}')$

CMB power and projection

- Result:

$\chi_* =$ distance to CMB

$$C_l = 4\pi \int d\ln k \left[\frac{k^3}{2\pi^2} P_{\mathcal{R}}(k)\right] \times [T_S(k,\eta_*)]^2 \times [j_l(k\chi_*)]^2$$

primordial power — plasma processing — projection (Bessel)

- Which 3D wavenumber k projects to which 2D multipole l?

CMB power and projection

- $j_l(k\chi_*)$ peaks when $k\chi_* \sim l$

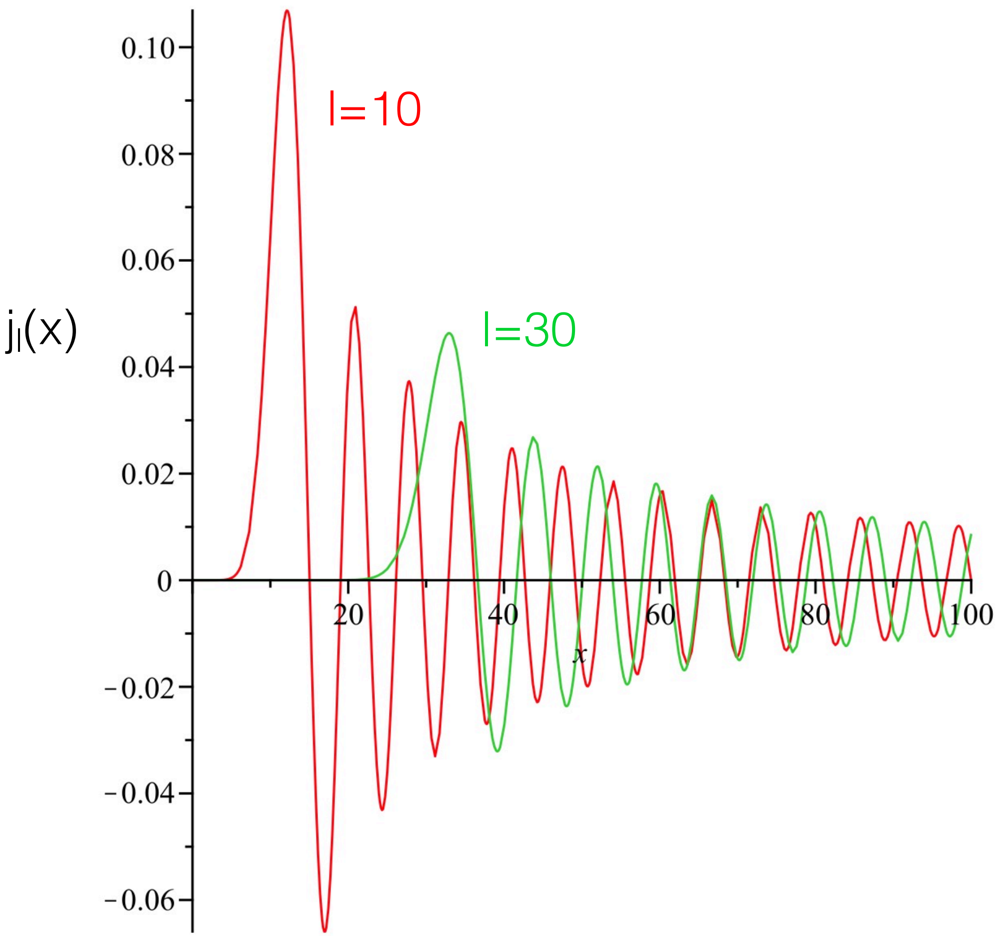


Image:
A. Challinor

CMB power and projection

$\chi_* =$ distance to CMB

- $j_l(k\chi_*)$ peaks when $k\chi_* \sim l$ (although some contributions from higher k, due to LOS-perpendicular wavefronts)

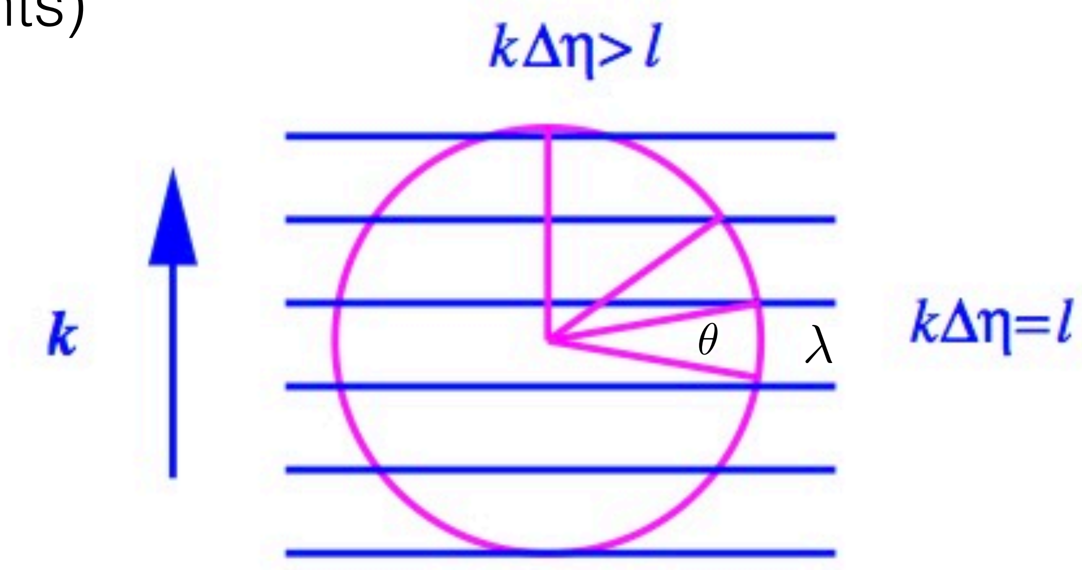


Image: A. Challinor

- CMB at multipole l mainly arises from fluctuations with wavenumber $k\chi_* \sim l$ or equivalently $\chi_*\theta = \lambda$

$k \sim (2\pi)/\lambda; \quad l \sim (2\pi)/\theta$

CMB power: approximate expression

- S slowly varying in k relative to the Bessel function, so

$$\frac{l(l+1)}{2\pi}C_l \approx T_S^2(\eta_*, k = l/\chi_*) \times \left[\frac{(l/\chi_*)^3}{2\pi^2} P_{\mathcal{R}}(k = l/\chi_*)\right]$$

i.e. here $k = l/\chi_*$

projection

$\chi_* =$ distance to CMB

transfer fn.: plasma processing (?)

Initial primordial power

CMB power: approximate expression

- S slowly varying in k relative to the Bessel function, so

$$\frac{l(l+1)}{2\pi}C_l \approx T_S^2(\eta_*, k = l/\chi_*) \times \left[\frac{(l/\chi_*)^3}{2\pi^2}P_{\mathcal{R}}(k = l/\chi_*)\right]$$

[ ]~constant

What is the plasma processing that determines CMB power via transfer function T_S?

Goal for this lecture: understand this CMB power spectrum

- Necessary steps: need to understand

  I.: photon propagation from when the CMB was emitted + projection to power spectrum ✓

  II.: initial conditions ✓

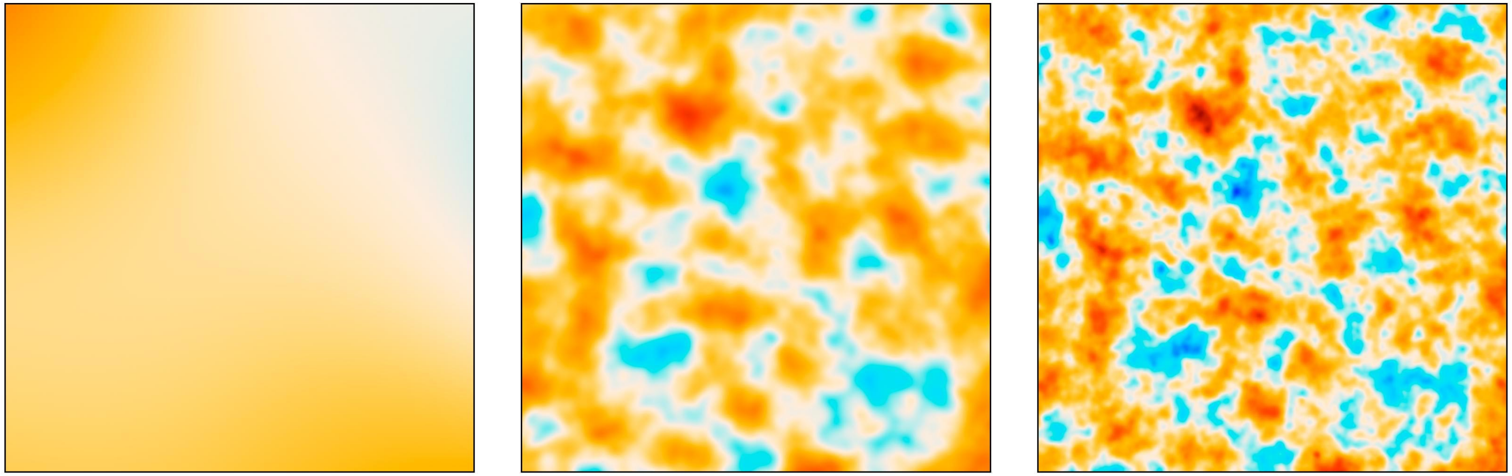
  III.: evolution in plasma (acoustic processing) from initial conditions to perturbations at emission

$$\mathcal{R}(\mathbf{k},0) \underset{\text{II.}}{} \xrightarrow[\text{III.}]{} (\delta_r, \Phi, \mathbf{v})_{\eta*} \xrightarrow[\text{I.}]{} \delta T(\hat{\mathbf{n}})$$

Will discuss III now.

Outline

- Connection of power spectrum to initial conditions
- Acoustic processing: basic equations
- Acoustic processing: detailed solution



Processing and Evolution

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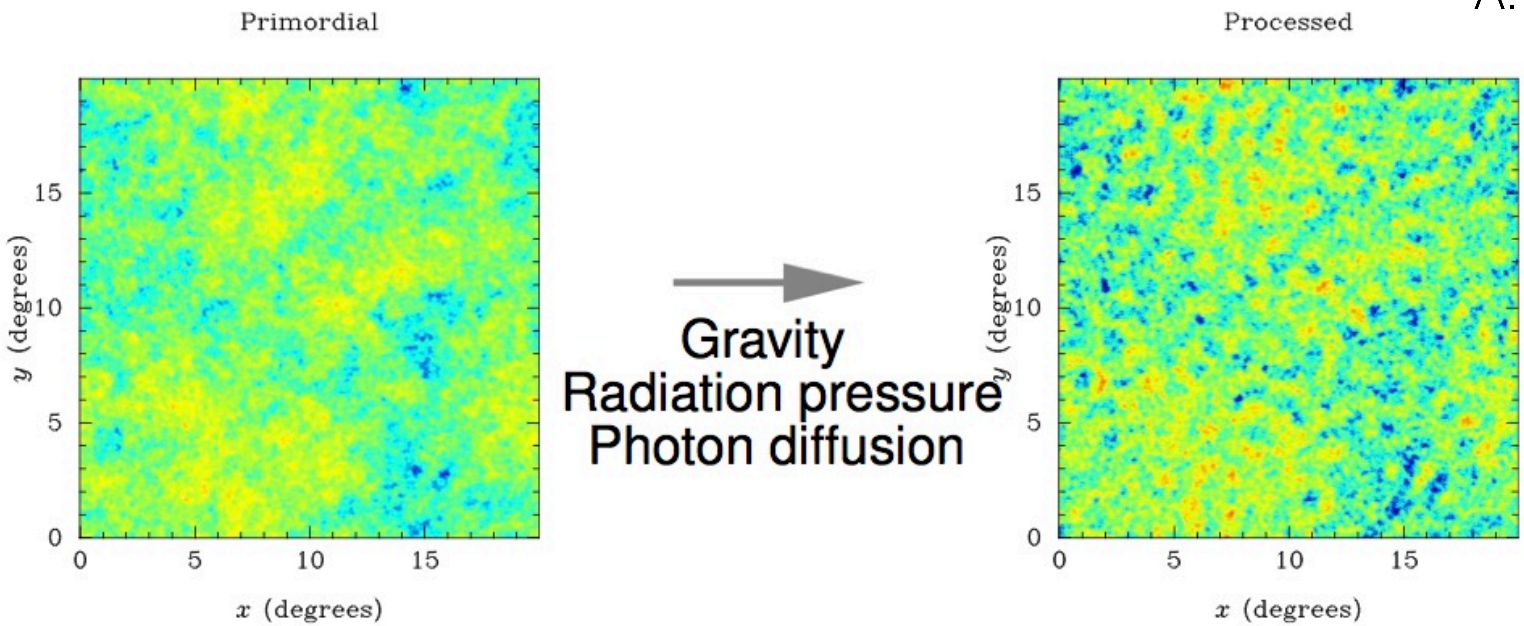
A. Challinor



- Want to derive transfer function T_S taking us from initial power spectrum to final CMB observables.

Processing and Evolution

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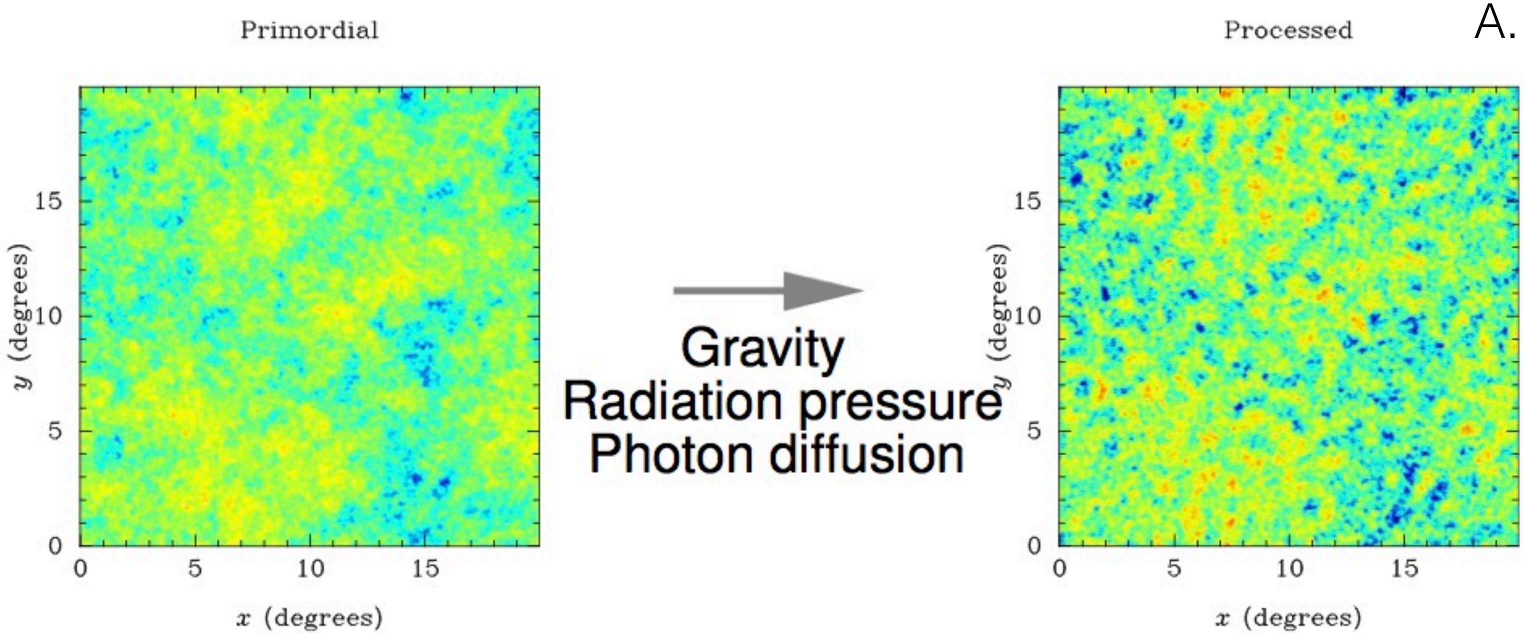
A. Challinor



- Want to derive transfer function T_S taking us from initial power spectrum to final CMB observables.
- Main relevant quantities are evolution of radiation density $\delta_r \equiv \frac{\rho_r - \bar{\rho}_r}{\bar{\rho}_r}$ and potential Φ

Deriving Evolution: Perturbation Theory

- Perturb stress-energy tensor of a fluid (ok for low l)

$$\bar{T}^{\mu}_{\ \nu} = (\bar{\rho} + \bar{P})\bar{U}^{\mu}\bar{U}_{\nu} + \bar{P}\delta^{\mu}_{\ \nu}$$

$$\bar{T}^{\mu}_{\ \nu} \to \bar{T}^{\mu}_{\ \nu} + \delta T^{\mu}_{\ \nu} = \bar{T}^{\mu}_{\ \nu} + (\delta\rho + \delta P)\bar{U}^{\mu}\bar{U}_{\nu} + (\bar{\rho} + \bar{P})[\bar{U}^{\mu}\delta\bar{U}_{\nu} + \delta\bar{U}^{\mu}\bar{U}_{\nu}] + \delta P\delta^{\mu}_{\ \nu} + \Pi^{\mu}_{\ \nu}$$

- And perturb the metric

$$ds^2 = a^2(\eta)\left[(1 + 2\Phi)d\eta^2 - (1 - 2\Phi)\delta_{ij}dx^i dx^j\right]$$

Perturbation Theory: Basic Equations

- Conserving Stress-Energy gives, for each component: The continuity equation (energy conservation)

$$\delta' + 3\mathcal{H}\left(\frac{\delta P}{\delta\rho} - \frac{\bar{P}}{\bar{\rho}}\right)\delta = -\left(1 + \frac{\bar{P}}{\bar{\rho}}\right)(\nabla\cdot\mathbf{v} - 3\Phi')$$

- As well as the Euler equation (momentum conservation):

$$\mathbf{v}' + 3\mathcal{H}\left(\frac{1}{3} - \frac{\bar{P}}{\bar{\rho}}\right)\mathbf{v} = -\frac{\nabla\delta P}{\bar{\rho} + \bar{P}} - \nabla\Phi$$

N.B. For radiation we set $P = \rho/3$

Perturbation Theory: Basic Equations

- Conserving Stress-Energy gives, for each component: The continuity equation (energy conservation)

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$$\mathbf{v}' + 3\mathcal{H}\left(\frac{1}{3} - \frac{\bar{P}}{\bar{\rho}}\right)\mathbf{v} = -\frac{\nabla\delta P}{\bar{\rho} + \bar{P}} - \nabla\Phi$$

Or:

$$\mathbf{q}' + 4\mathcal{H}\mathbf{q} + \nabla\delta P + (\bar{\rho} + \bar{P})\nabla\Phi = 0. \qquad \mathbf{q} = (\bar{\rho} + \bar{P})\mathbf{v} = (\bar{\rho}_r + \bar{P}_r)\mathbf{v}_r$$

N.B. For radiation we set $P = \rho/3$

Perturbation Theory: Basic Equations

- Conserving Stress-Energy gives, for radiation, the continuity equation (energy/mass conservation)

$$\delta_r' = -\frac{4}{3}(\nabla \cdot \mathbf{v}_r - 3\Phi')$$

- As well as the Euler equation (momentum conservation):

$$\mathbf{v}_r' = -\frac{1}{4}\nabla\delta_r - \nabla\Phi$$

Combine by taking time deriv. of continuity, eliminate v'

Perturbation Theory: Basic Equations

- Continuity + Euler:

$$\delta_r' = -\frac{4}{3}(\nabla \cdot \mathbf{v}_r - 3\Phi') \qquad \mathbf{v}_r' = -\frac{1}{4}\nabla\delta_r - \nabla\Phi$$

$$\delta_r'' = -\frac{4}{3}(\nabla \cdot (-\frac{1}{4}\nabla\delta_r - \nabla\Phi) - 3\Phi'')$$

$$\frac{\delta_r''}{4} - \frac{1}{3}\nabla^2\frac{\delta_r}{4} = \Phi'' - \frac{1}{3}\nabla^2\Phi$$

Acoustic Oscillations: Approximate Equation

- Fourier transformed and taking $\nabla \rightarrow -ik$ we obtain:

$$\left(\frac{\delta_r}{4}\right)'' + \frac{1}{3}k^2\left(\frac{\delta_r}{4}\right) = \Phi'' - \frac{1}{3}k^2\Phi$$

Radiation pressure: restoring force

Driving force from potentials

Note that we have neglected baryons in this treatment!

Acoustic Oscillations: Approximate Solution

- We will at first also only consider matter-dominated large scales where potential is constant. We obtain

$$\left(\frac{\delta_r}{4}+\Phi\right)'' + \frac{1}{3}k^2\left(\frac{\delta_r}{4}+\Phi\right) = 0$$

- Simple harmonic oscillator equation for each k! This system supports “acoustic” oscillations of the plasma. Frequency of oscillations is $k/3^{1/2}$ – higher k oscillates faster.

Acoustic Oscillations: Intuitive Picture

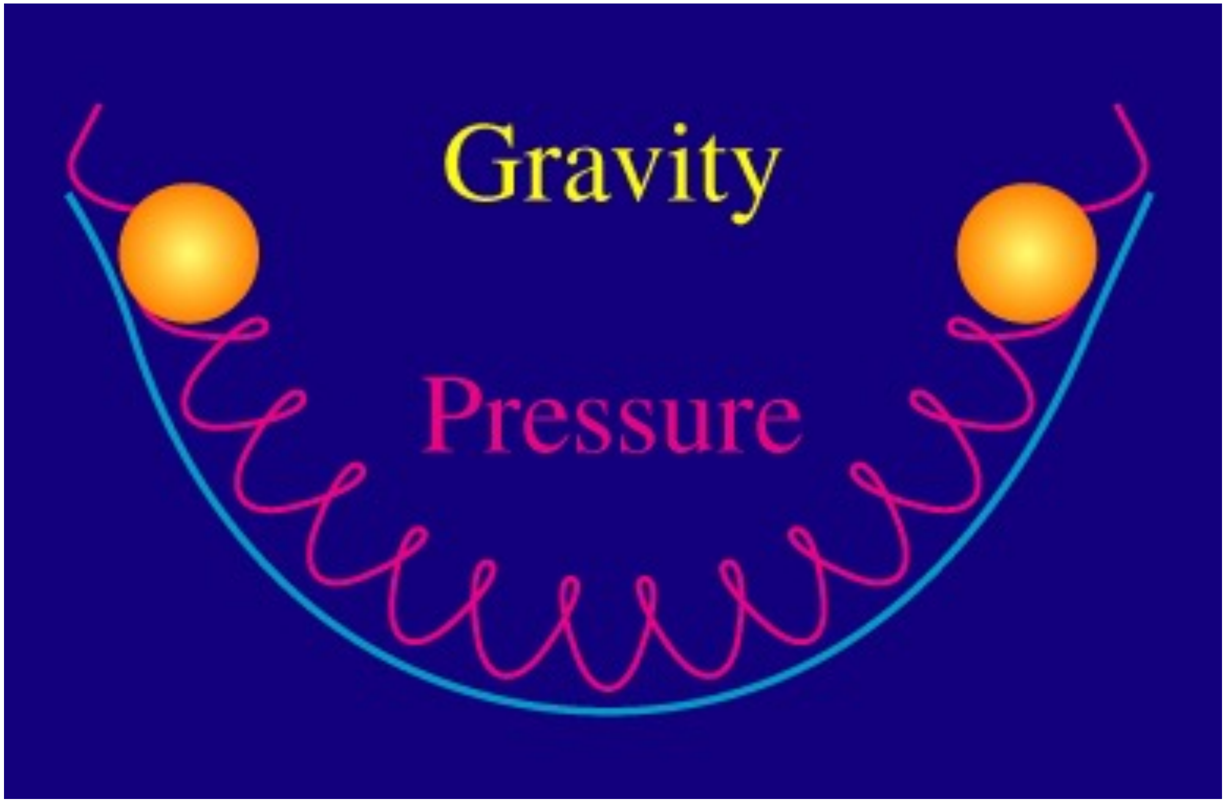
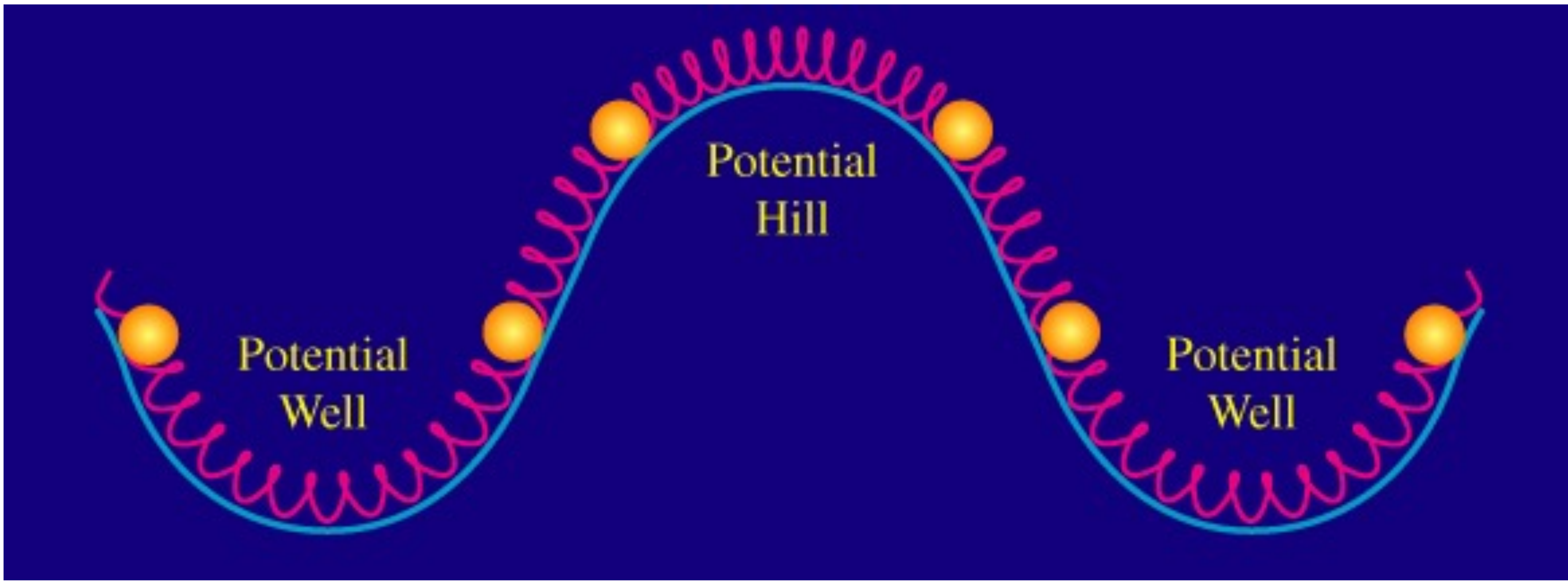


Image:
Wayne Hu

- Harmonic oscillator forced by constant potential

Acoustic Oscillations: Intuitive Picture



- Each k-mode oscillates as standing wave. 2 x k oscillates twice as fast

Image:
Wayne Hu

Acoustic Oscillations: Approximate Solution

- Recall $$\left(\frac{\delta_r}{4}+\Phi\right)'' + \frac{1}{3}k^2\left(\frac{\delta_r}{4}+\Phi\right) = 0$$

- Free solutions are $$\left(\frac{\delta_r}{4}+\Phi\right) \equiv S \propto$$

$$\cos(k\eta/\sqrt{3}), \sin(k\eta/\sqrt{3}) = \cos(kr_s(\eta)), \sin(kr_s(\eta))$$

- Where $r_s = \int_0^\eta d\eta' c_s(\eta')$ and $c_s(\eta') = 1/\sqrt{3}$

$$= \frac{\eta}{\sqrt{3}}$$

Acoustic Oscillations: Approximate Solution

- Free solutions are $\left(\frac{\delta_r}{4} + \Phi\right) \equiv S \propto$

$$\cos(k\eta/\sqrt{3}), \sin(k\eta/\sqrt{3}) = \cos(kr_s(\eta)), \sin(kr_s(\eta))$$

- Where $r_s = \int_0^\eta d\eta' c_s(\eta')$ and $c_s(\eta') = 1/\sqrt{3}$

$$= \frac{\eta}{\sqrt{3}}$$

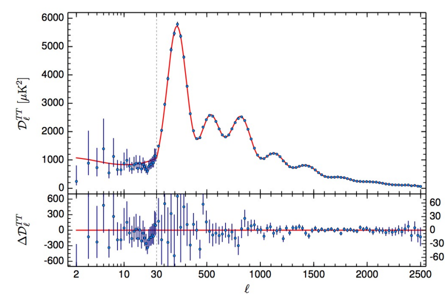
- Initial value $\propto \mathcal{R}$ with no time derivative. Therefore, assuming only cosine: T_S oscillates in k at recombination

$$T_S(k) \propto \cos(kr_s(\eta))$$

Acoustic Oscillations: Approximate Solution

- Found solution $T_S(k) \propto \cos(kr_s(\eta))$
- Since $$\frac{l(l+1)}{2\pi}C_l \approx T_S^2(\eta_*, k = l/\chi_*) \times \left[\frac{(l/\chi_*)^3}{2\pi^2}P_{\mathcal{R}}(k = l/\chi_*)\right]$$

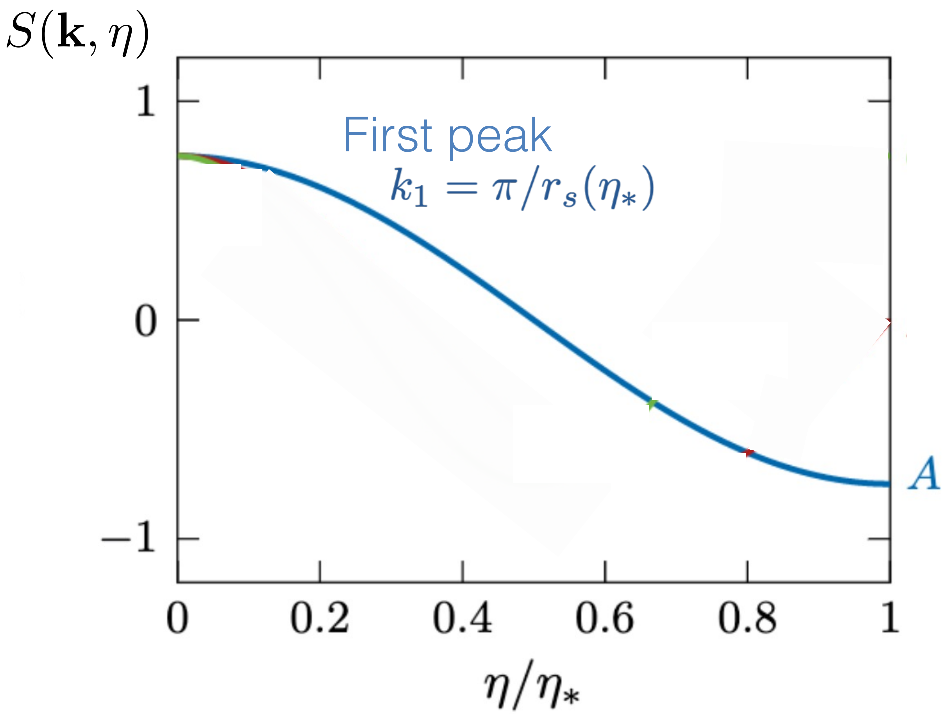
$$\rightarrow C_l \propto \cos^2\left(\frac{l}{\chi_*}r_s(\eta_*)\right)$$

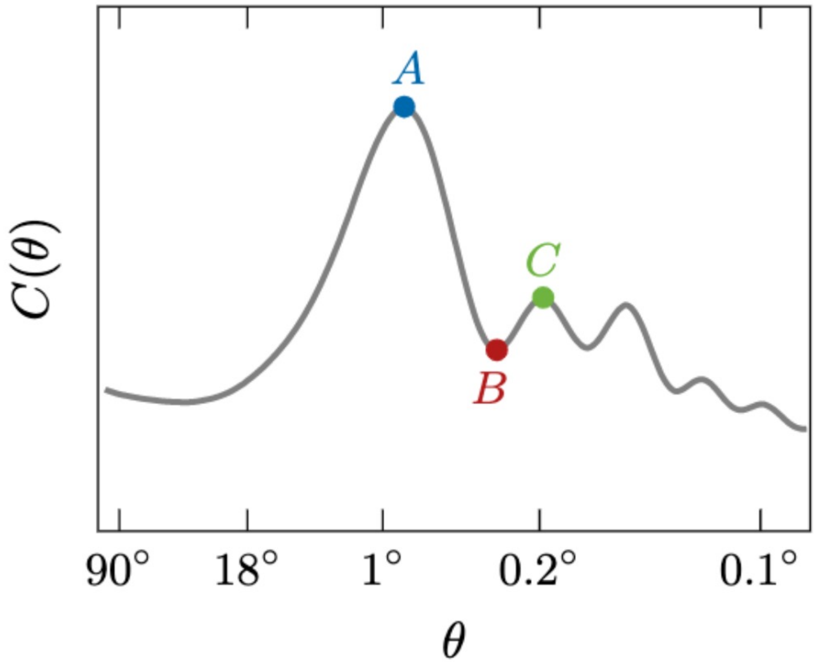


$\mathcal{D}_l \equiv l(l+1)C_l/2\pi$

- Series of peaks/troughs in multipole l! Seen in the CMB!

Acoustic Oscillations: Intuitive Picture

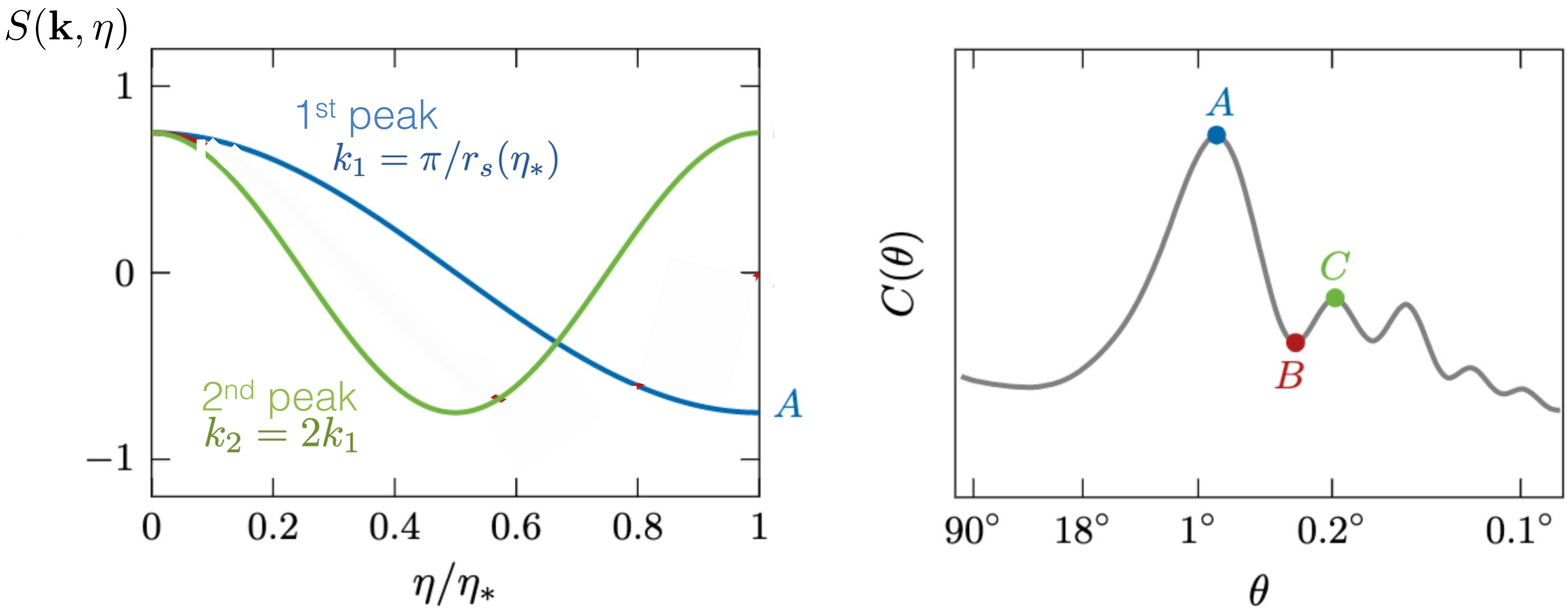




- Key: oscillation frequency depends on k [2 x k → 2 x freq.]
- At recombination, certain frequencies = certain k = n k_1 are at maximum of their oscillation. Series of peaks in k → l.

Image credit: D. Baumann

Acoustic Oscillations: Intuitive Picture



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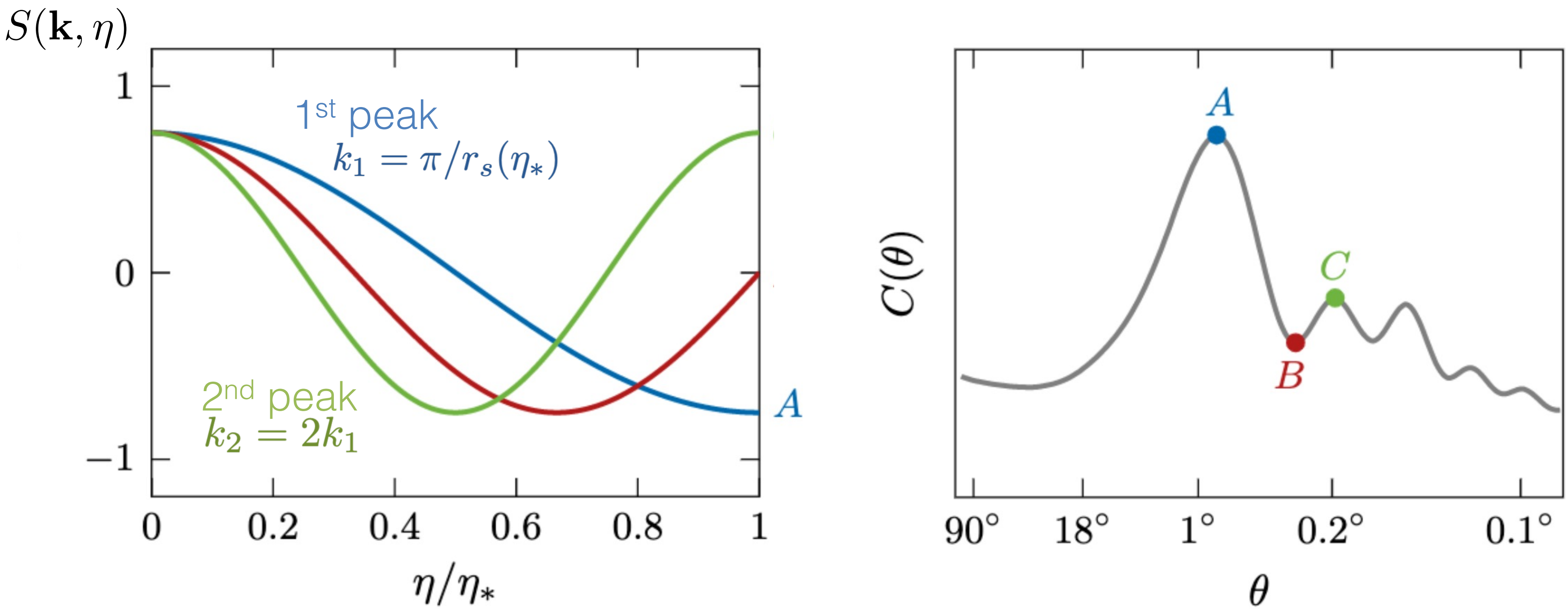
Acoustic Oscillations: Intuitive Picture



- Key: oscillation frequency depends on k [2 x k → 2 x freq.]
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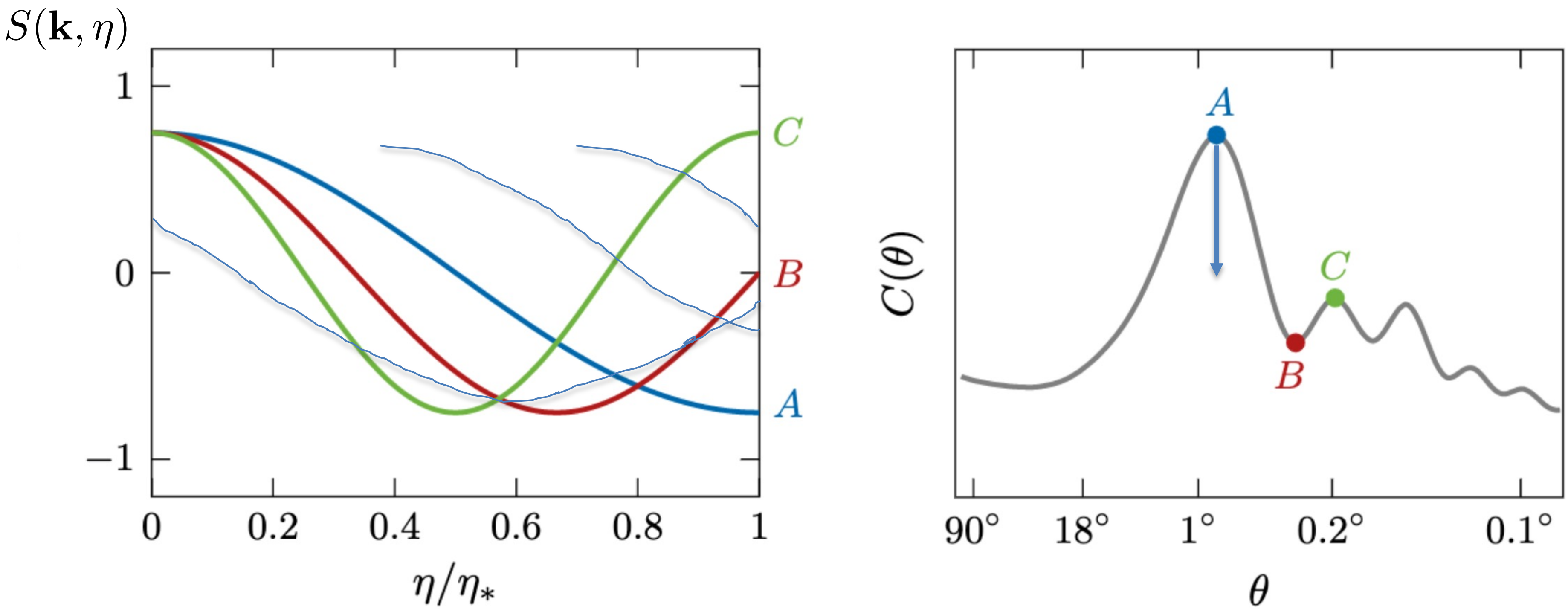
Aside: Coherent Phases Argument for Inflation



- Peak structure only arises because inflationary initial conditions generate pure cosine mode, with all fluctuations in phase.
- Other mechanisms for producing structure can't do this!

CMB Power Spectrum

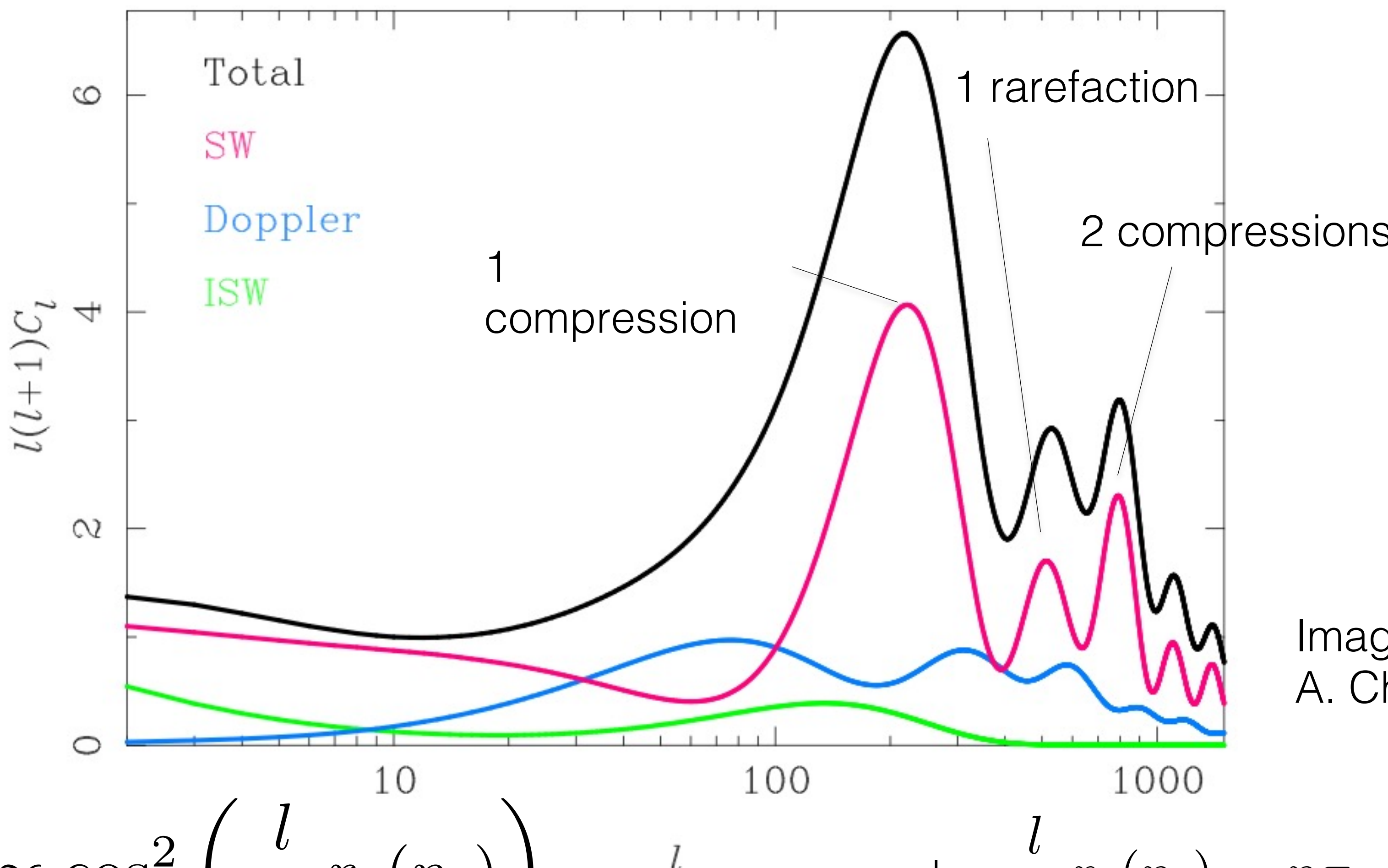


Image: A. Challinor

$$C_l \propto \cos^2\left(\frac{l}{\chi_*} r_s(\eta_*)\right)$$

peaks $\frac{l}{\chi_*} r_s(\eta_*) = n\pi$

- ~describes magenta curve. But why are odd peaks larger?

Outline

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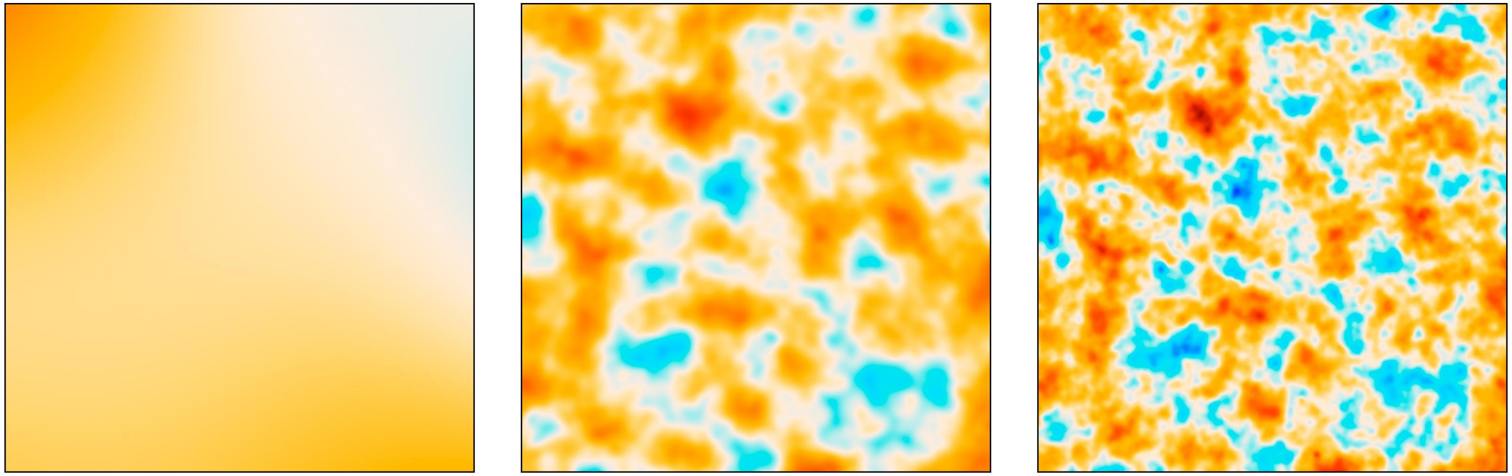


Acoustic Oscillations: Adding Baryons

- Baryons add to momentum density while contributing negligible pressure. Can add to Euler equation by rewriting it entirely in terms of momentum density q and adding the baryon contribution

$$\mathbf{q} = (\bar{\rho} + \bar{P})\mathbf{v} = (\bar{\rho}_r + \bar{P}_r)\mathbf{v}_r$$

$$\longrightarrow (\bar{\rho}_r + \bar{P}_r)\mathbf{v}_r + \bar{\rho}_b \mathbf{v}_b \approx \frac{4}{3}(1+R)\bar{\rho}_r \mathbf{v}_r$$

$$R \equiv \bar{\rho}_b / (\bar{\rho}_r + \bar{P}_r)$$

Acoustic Oscillations: Adding Baryons

$$\delta' + 3\mathcal{H}\left(\frac{\delta P}{\delta\rho} - \frac{\bar{P}}{\bar{\rho}}\right)\delta = -\left(1 + \frac{\bar{P}}{\bar{\rho}}\right)(\nabla\cdot\mathbf{v} - 3\Phi') \qquad \mathbf{q}' + 4\mathcal{H}\mathbf{q} + \nabla\delta P + (\bar{\rho} + \bar{P})\nabla\Phi = 0.$$

- Now the relevant evolution equations become

$$\left(\frac{\delta_r}{4}\right)'' + \frac{\mathcal{H}R}{1+R}\left(\frac{\delta_r}{4}\right)' + \frac{1}{3(1+R)}k^2\left(\frac{\delta_r}{4}\right) = \Phi'' + \frac{\mathcal{H}R}{1+R}\Phi' - \frac{1}{3}k^2\Phi$$

Damping due to velocity redshifting

Radiation pressure: restoring force

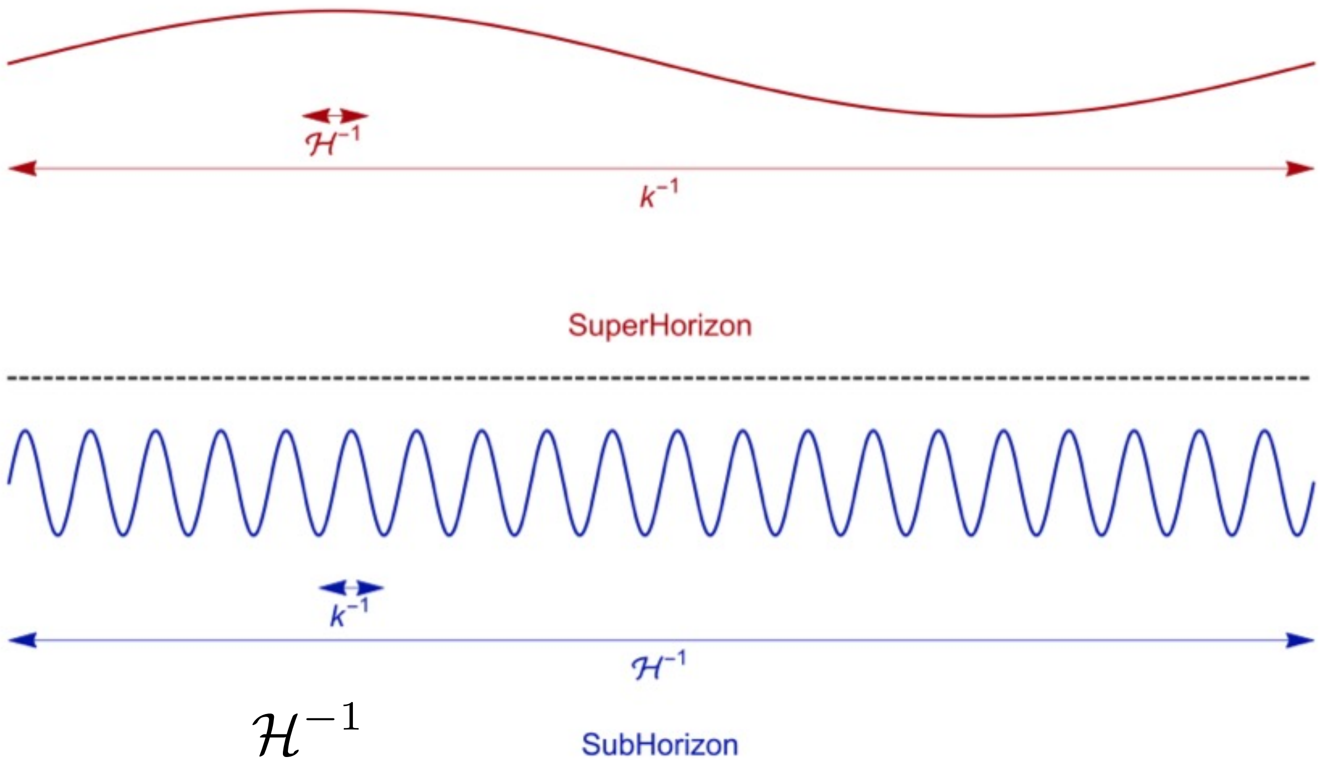
Driving force from potentials

- A damped, driven oscillator with sound speed

$$c_s \equiv \frac{1}{\sqrt{3(1+R)}}$$

For full solution, must understand potential evolution!

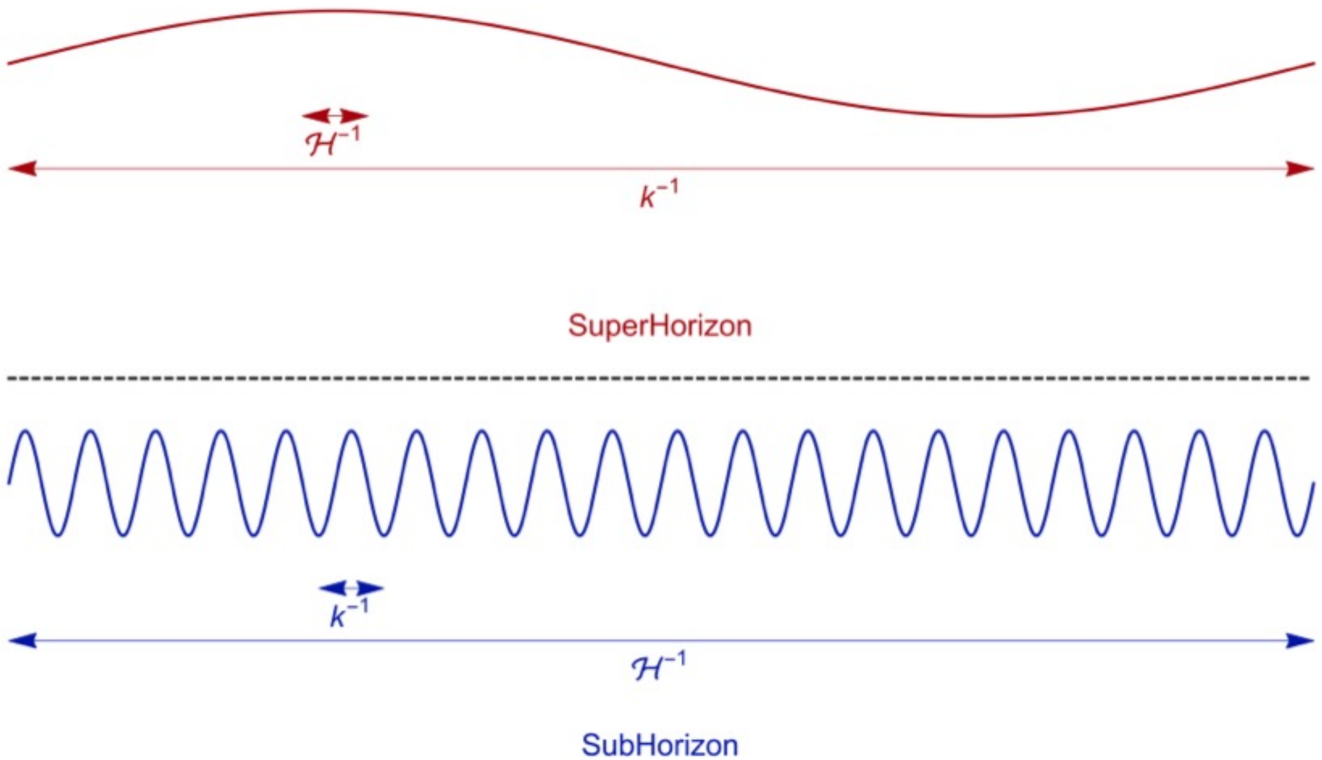
Reminder: Horizon Growth



- Horizon: distance over which causal physics acts and modes evolve. Horizon grows with time and larger modes enter.

Image credit: J. Fergusson

Reminder: Horizon Growth



- Small-scale modes enter the horizon earlier, during radiation domination $w = dP/d\rho = 1/3$
- Large-scale modes enter the horizon later, when the universe is already matter-dominated. $w \approx 0$

Critical value:

$k_{eq}^{-1} = \mathcal{H}^{-1}(\eta_{equality})$

Large and small scales: feel different potential

- From the Einstein equations for w=1/3; w = 0 obtain:

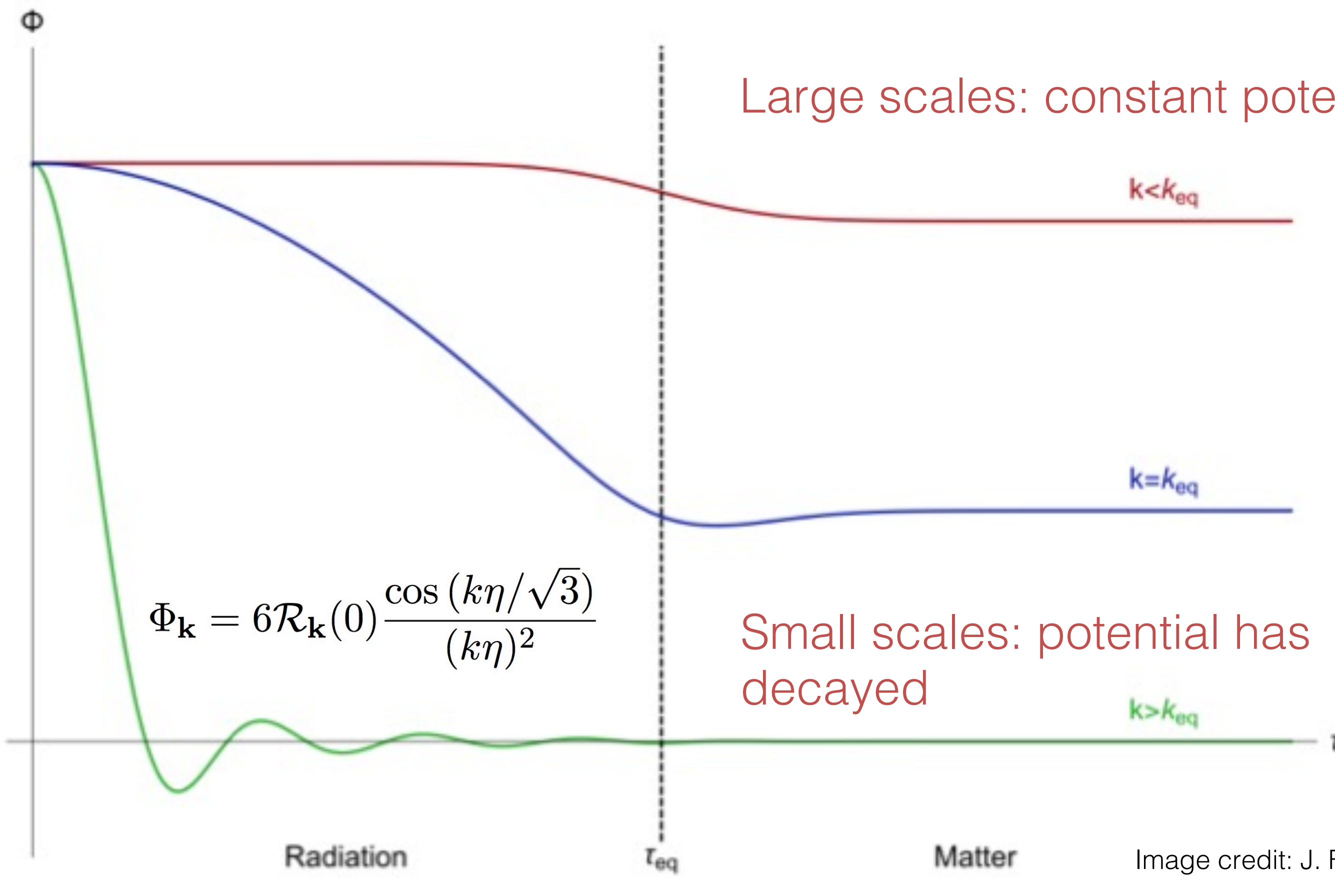


Image credit: J. Fergusson

Perturbation Evolution

- Now that we understand the potential, can evaluate radiation evolution more exactly:

$$\left(\frac{\delta_r}{4}\right)'' + \frac{\mathcal{H}R}{1+R}\left(\frac{\delta_r}{4}\right)' + \frac{1}{3(1+R)}k^2\left(\frac{\delta_r}{4}\right) = \Phi'' + \frac{\mathcal{H}R}{1+R}\Phi' - \frac{1}{3}k^2\Phi$$

Damping due to velocity redshifting

Radiation pressure: restoring force

Driving force from potentials

Large scales / matter dominated

- Solution: oscillations offset by constant potential:

$$\frac{\delta_r(\eta,\mathbf{k})}{4} = C(\mathbf{k})\cos(kr_s) + D(\mathbf{k})\sin(kr_s) - (1+R)\Phi$$

- For large scale modes $k \ll k_{eq}$ modes have entered in matter domination. Matching initial conditions, e.g.

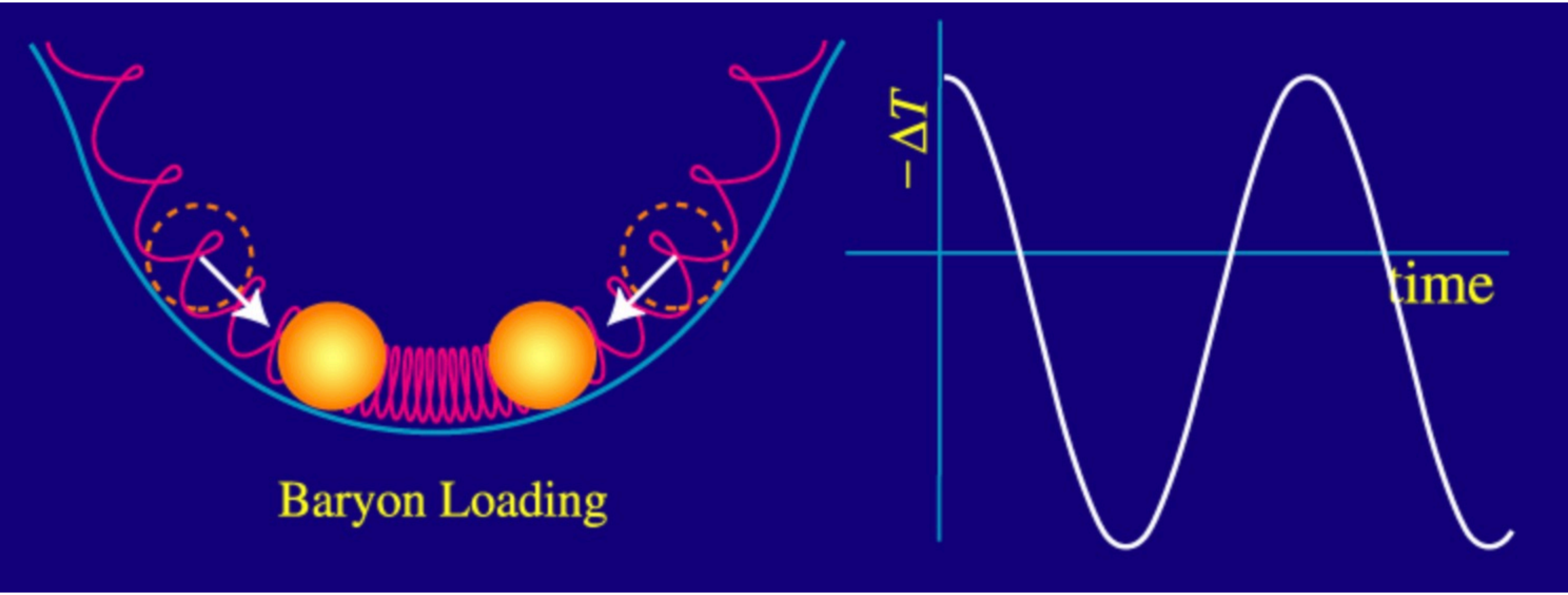
$\Phi(\eta,\mathbf{k}) = -3\mathcal{R}(\mathbf{k})/5$

$$\frac{\delta_r(\eta,\mathbf{k})}{4} + \Phi(\eta,\mathbf{k}) = -\frac{1}{5}\mathcal{R}(\mathbf{k})[(1+3R)\cos(kr_s) - 3R]$$

Effect of Baryons

$$-\frac{1}{5}\mathcal{R}(\mathbf{k})[(1+3R)\cos(kr_s)-3R]$$

Image: W. Hu



- Baryons enhance compression. This offsets oscillations, increasing size of odd peaks.

Small scales / radiation dominated

- During radiation domination it is easiest to go back to Einstein equations. Result is:

$$\delta_r(\mathbf{k}) \approx -\frac{2}{3}k^2\eta^2\Phi \approx -4\mathcal{R}(\mathbf{k})\cos(kr_s)$$

- Potentials have decayed far inside the horizon so are negligible; hence

$$\frac{\delta_r(\mathbf{k},\eta_*)}{4} + \Phi(\mathbf{k},\eta_*) \approx -\mathcal{R}(\mathbf{k})\cos(kr_s(\eta_*))$$

The CMB Temperature

- Summary: obtain equations

$$\frac{\delta_r(\mathbf{k},\eta_*)}{4} + \Phi(\mathbf{k},\eta_*) \approx \begin{cases} -\frac{1}{5}\mathcal{R}(\mathbf{k})[(1+3R)\cos(kr_s) - 3R] & k \ll k_{eq} \text{ Large scales} \\ -\mathcal{R}(\mathbf{k})\cos(kr_s(\eta_*)) & k \gg k_{eq} \text{ Small scales} \end{cases}$$

The CMB Temperature

- Summary: obtain equations

$$\frac{\delta_r(\mathbf{k},\eta_*)}{4} + \Phi(\mathbf{k},\eta_*) \approx \begin{cases} -\frac{1}{5}\mathcal{R}(\mathbf{k})[(1+3R)\cos(kr_s) - 3R] & k \ll k_{eq} \text{ Large scales} \\ -\mathcal{R}(\mathbf{k})\cos(kr_s(\eta_*)) & k \gg k_{eq} \text{ Small scales} \end{cases}$$

- We have now determined the transfer function T_s (above expressions without $\mathcal{R}$)

$$\frac{l(l+1)}{2\pi}C_l \approx T_S^2(\eta_*, k = l/\chi_*) \times \left[\frac{(l/\chi_*)^3}{2\pi^2} P_{\mathcal{R}}(k = l/\chi_*)\right]$$

The CMB Temperature

- Summary: obtain equations

$$\frac{\delta_r(\mathbf{k},\eta_*)}{4} + \Phi(\mathbf{k},\eta_*) \approx \begin{cases} -\frac{1}{5}\mathcal{R}(\mathbf{k})[(1+3R)\cos(kr_s) - 3R] & k \ll k_{eq} \\ -\mathcal{R}(\mathbf{k})\cos(kr_s(\eta_*)) & k \gg k_{eq} \end{cases}$$

- At fixed r_s – when we observe CMB – this gives $\cos^2$ oscillations, as before
- Can see: modes with $kr_s(\eta_*) = n\pi$ or $\frac{l}{\chi_*} r_s(\eta_*) = n\pi$ are at maximum at last scattering
- (Potential decay driving boots oscillations on small scales)

CMB Power Spectrum

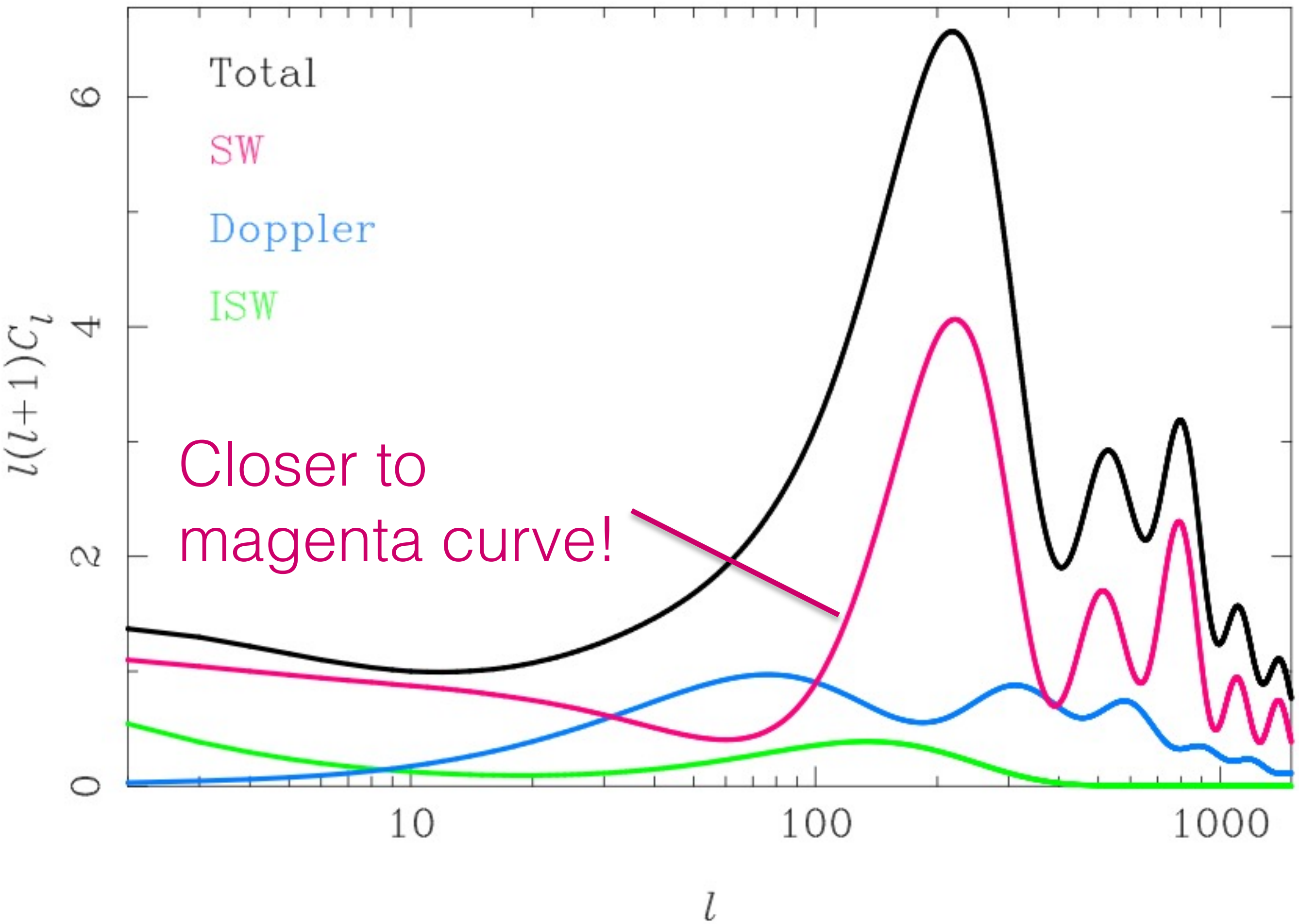


Image:
A. Challinor

- Already explained: baryons make odd peaks larger

Other terms in power spectrum

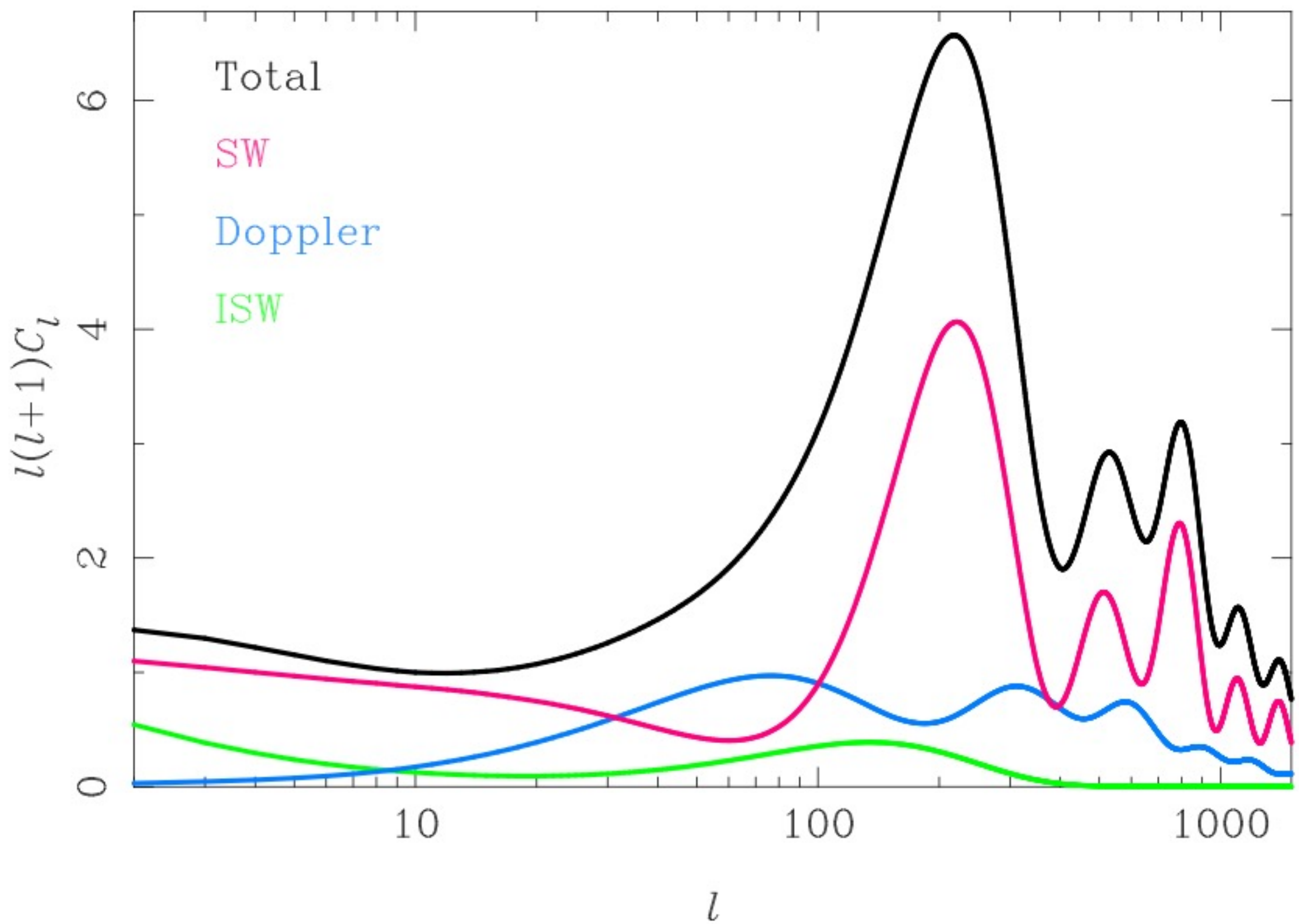
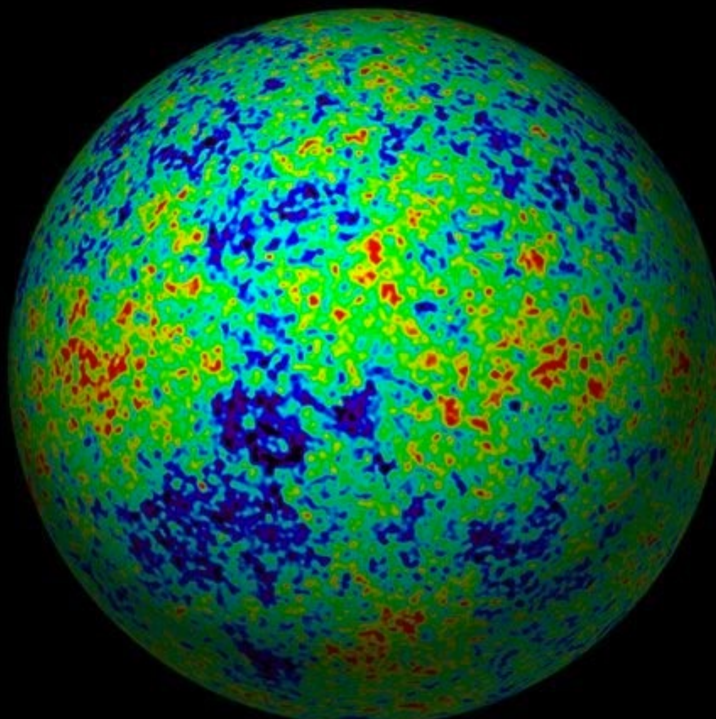


Image:
A. Challinor

- Have made progress on explaining SW term. Others subdominant but non-negligible. Refine next time!

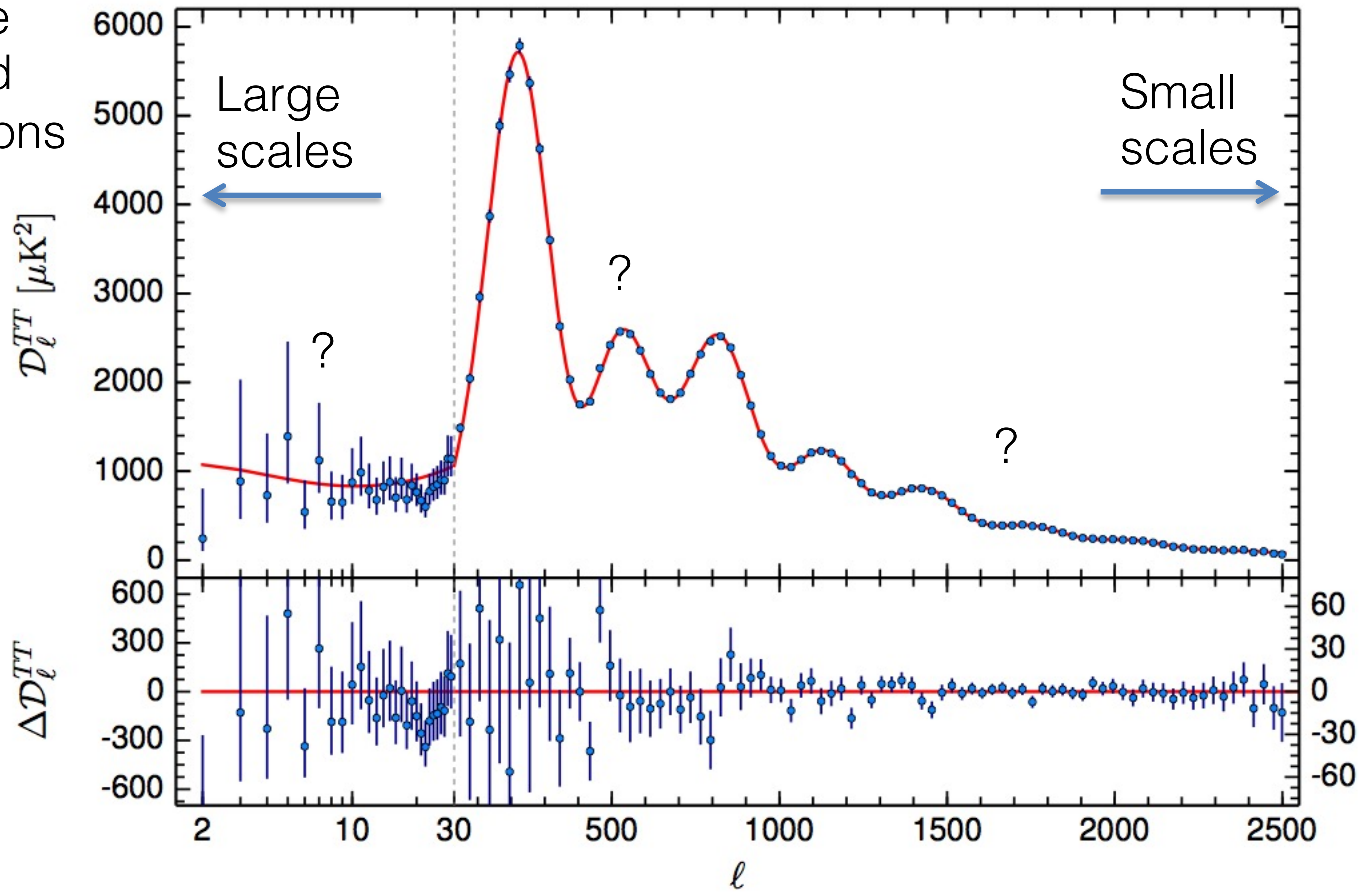
The Cosmic Microwave Background
Lecture 3: CMB Power Spectra and Parameters



Blake Sherwin
Department of Mathematics and Theoretical Physics / Kavli Institute for Cosmology
University of Cambridge

Measurement: The Planck CMB Power Spectrum

Magnitude of squared T fluctuations Θ



Inverse scale $\ell \sim \frac{2\pi}{\theta}$

$$\mathcal{D}_l \equiv l(l+1)C_l/2\pi$$

- Today: understand spectrum and parameter constraints!

Reminder: CMB Power and Initial Conditions

- CMB power spectrum depends on:

$$\frac{l(l+1)}{2\pi} C_l \approx T_S^2(\eta_*, k = l/\chi_*) \times \left[\frac{(l/\chi_*)^3}{2\pi^2} P_{\mathcal{R}}(k = l/\chi_*) \right]$$

i.e. here $k = l/\chi_*$

3. projection

$\chi_* =$ distance to CMB

2. transfer fn.: plasma processing (?)

1. initial primordial power

Reminder: Acoustic Oscillations

- Started discussion with matter-dominated large scales where potential is constant. Neglecting baryons:

$$\left(\frac{\delta_r}{4} + \Phi\right)'' + \frac{1}{3}k^2\left(\frac{\delta_r}{4} + \Phi\right) = 0$$

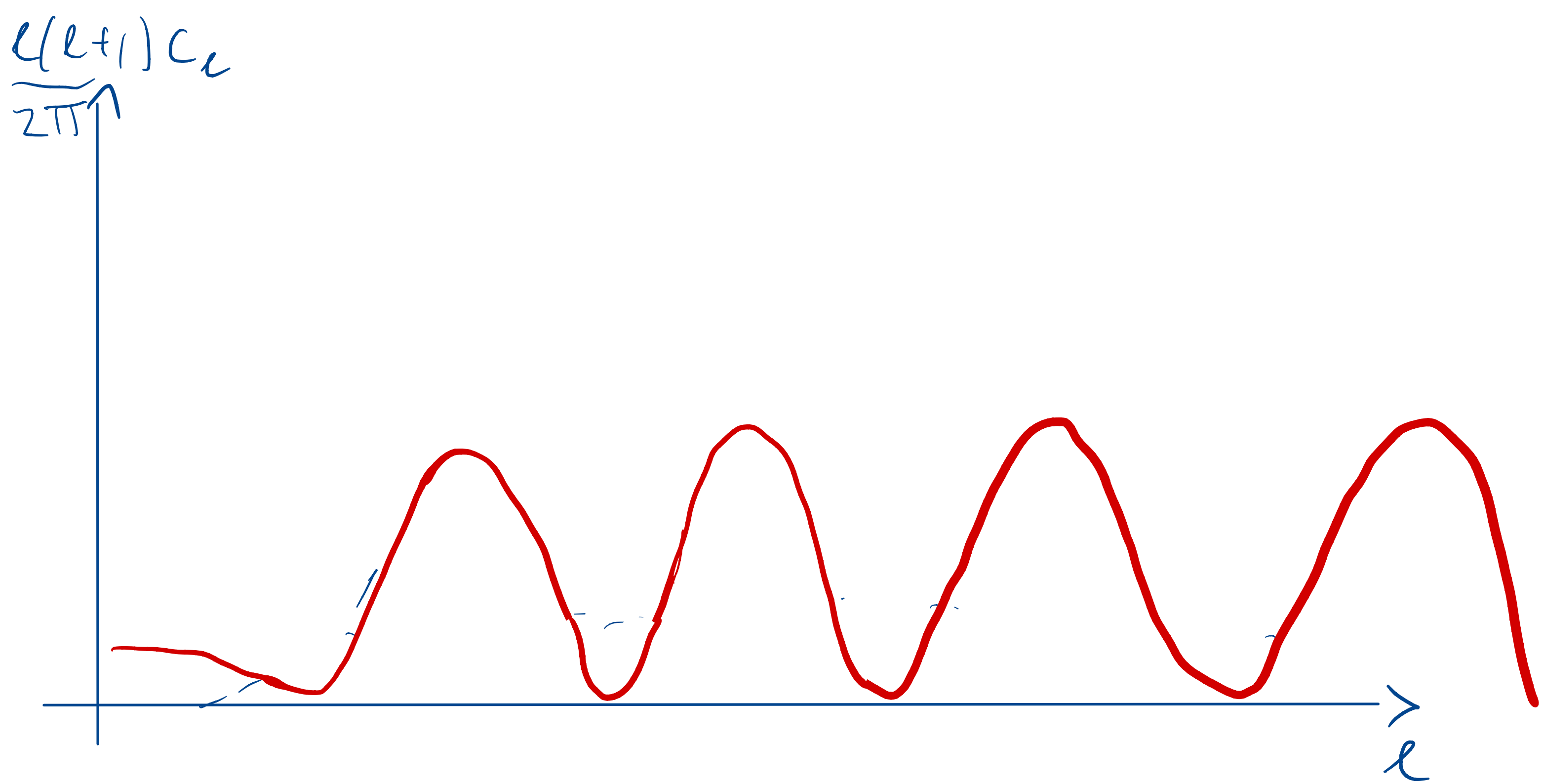
- Simple harmonic oscillator equation for each k, with frequency $k/3^{1/2}$.

$$T_S(k) \propto \cos(kr_s(\eta)) \qquad \rightarrow C_l \propto \cos^2\left(\frac{l}{\chi_*}r_s(\eta_*)\right)$$

Reminder: Acoustic Oscillations

- Approximate large-scale picture neglecting baryons

$$T_S(k) \propto \cos(kr_s(\eta)) \qquad \rightarrow C_l \propto \cos^2\left(\frac{l}{\chi_*} r_s(\eta_*)\right)$$



Reminder: Acoustic Oscillations

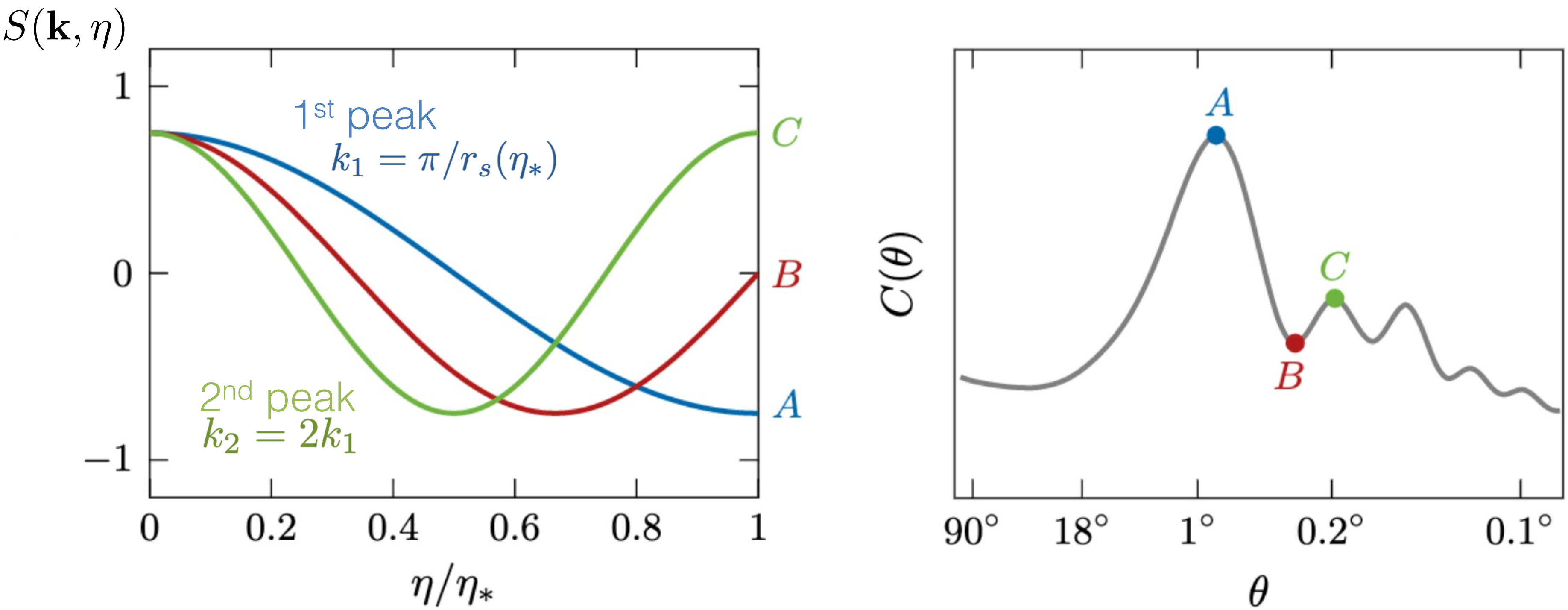


Image credit: D. Baumann

- Key: oscillation frequency depends on k [2 x k → 2 x freq.]
- At recombination, certain frequencies = certain k = n k_1 are at maximum of their oscillation. Series of peaks in k → l.

Reminder: Adding Baryons

- Adding baryons enhances momentum density, changes solution:

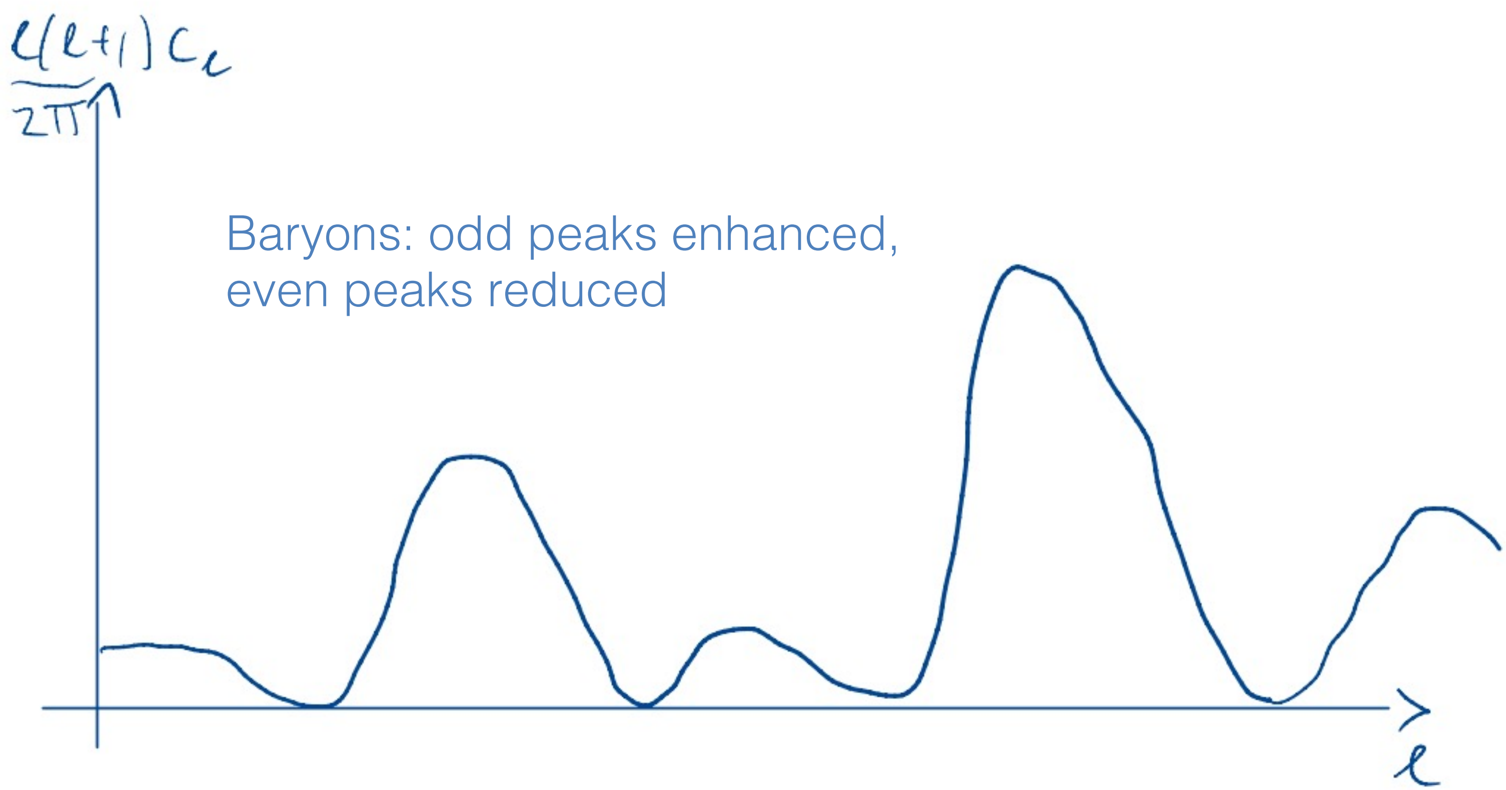
$$\frac{\delta_r(\mathbf{k},\eta_*)}{4} + \Phi(\mathbf{k},\eta_*) \approx \begin{cases} -\frac{1}{5}\mathcal{R}(\mathbf{k})[(1+3R)\cos(kr_s) - 3R] & k \ll k_{eq} \\ -\mathcal{R}(\mathbf{k})\cos(kr_s(\eta_*)) & k \gg k_{eq} \end{cases}$$

(Amplitude boosted by radiation driving)

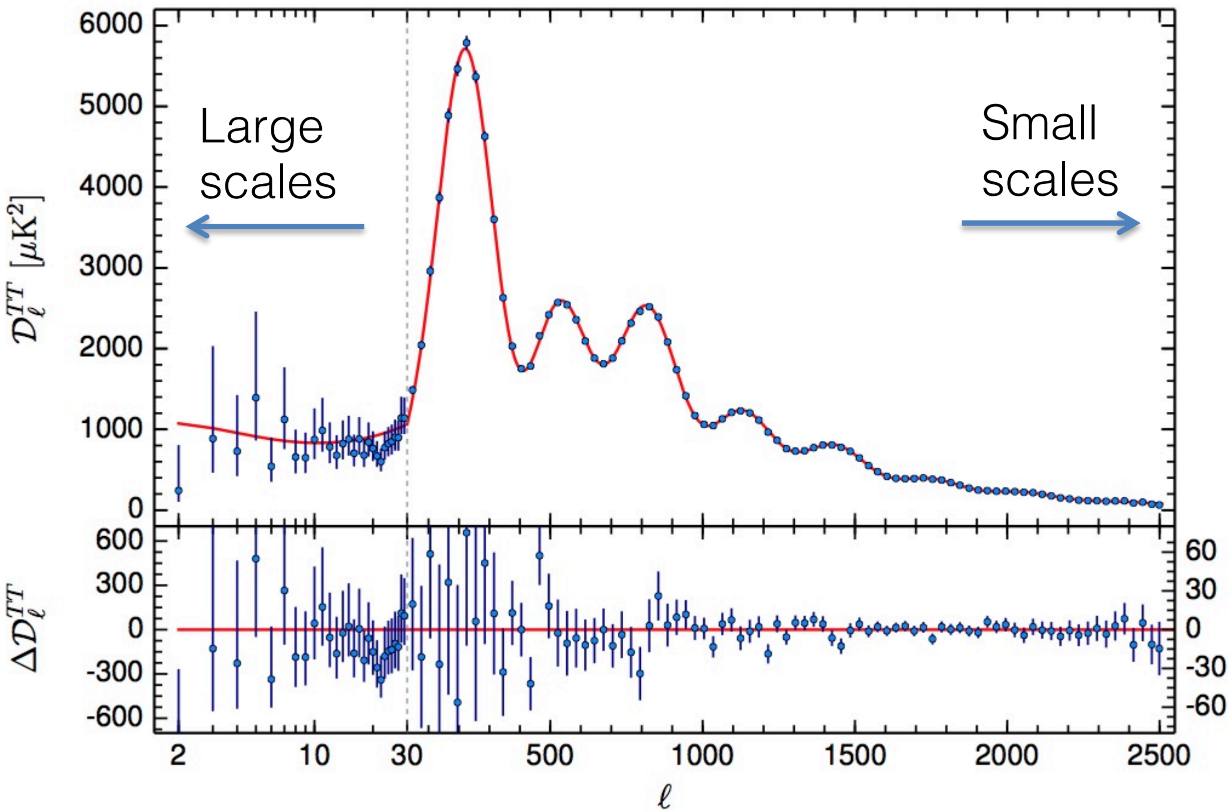
- At fixed r_s – when we observe CMB – this gives $\cos^2$ oscillations, as before, but even peaks enhanced

Reminder: Adding Baryons

$$\frac{\delta_r(\mathbf{k},\eta_*)}{4} + \Phi(\mathbf{k},\eta_*) \approx \begin{cases} -\frac{1}{5}\mathcal{R}(\mathbf{k})[(1+3R)\cos(kr_s) - 3R] & k \ll k_{eq} \\ -\mathcal{R}(\mathbf{k})\cos(kr_s(\eta_*)) & k \gg k_{eq} \end{cases}$$



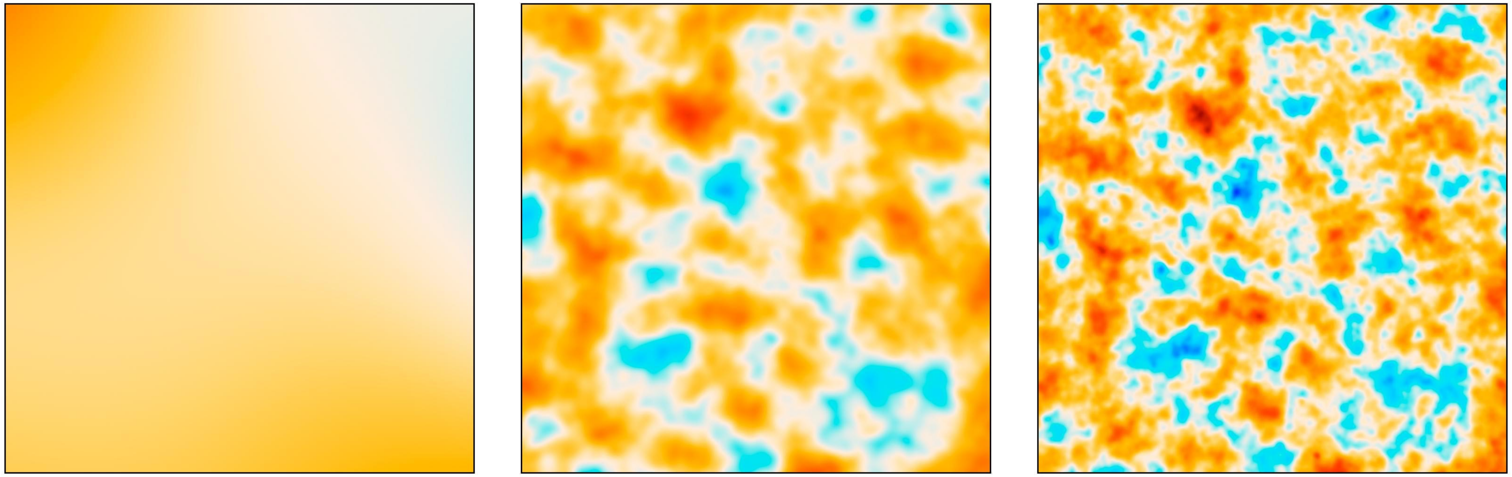
Not quite there… what are we missing?



- Have neglected: Doppler term, damping, reionization (+large scale limit) – discuss now!

Outline

- CMB power spectrum: details
- Determining cosmic composition from the CMB
- Tensions?



Complication: Doppler Terms

- We have so far only treated the Sachs-Wolfe terms, but have neglected the Doppler one

$$\Theta \sim -\hat{\mathbf{n}} \cdot \mathbf{v}$$

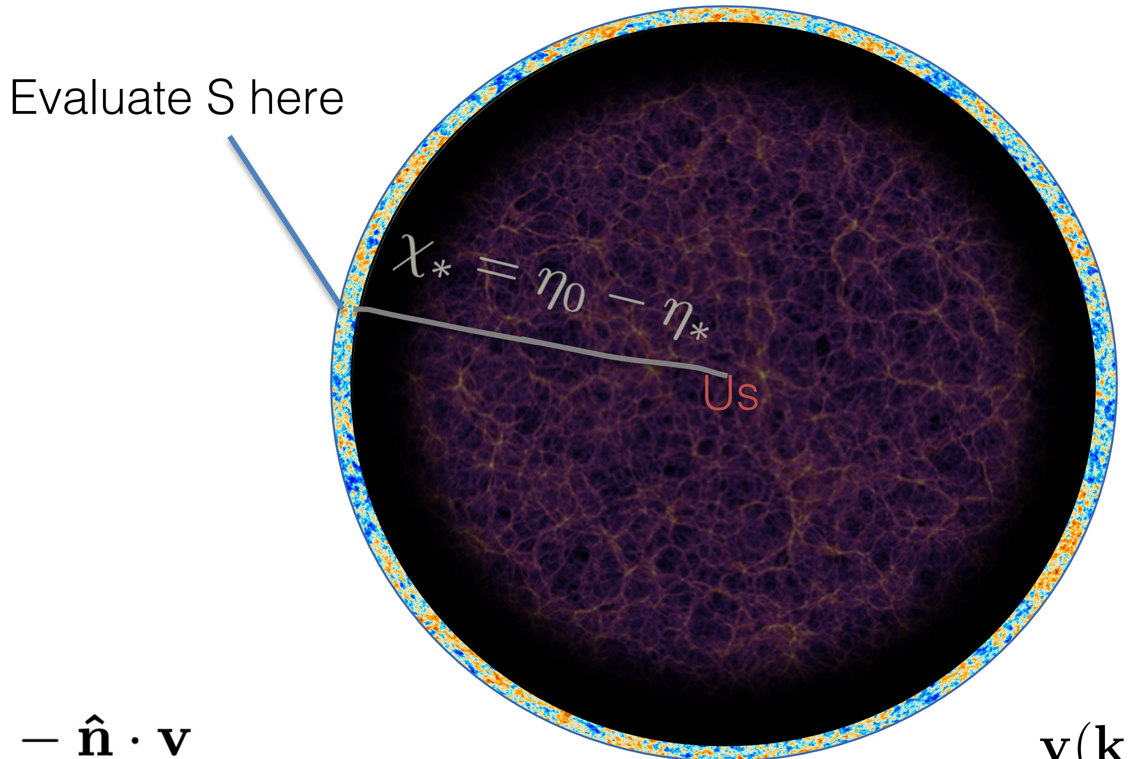
- Need to redo projection operation and transfer function calculation. Note: for scalar perturbations we know that

$$\mathbf{v}(\mathbf{k},\eta) = i\hat{\mathbf{k}}v(\mathbf{k})$$

Velocity potential

$$= i\hat{\mathbf{k}}T_v(k,\eta)\mathcal{R}(\mathbf{k})$$

Doppler term: project as before, with slight changes



$$\Theta \sim -\hat{\mathbf{n}} \cdot \mathbf{v}$$

$$\mathbf{v}(\mathbf{k}, \eta) = i\hat{\mathbf{k}} T_v(k, \eta) \mathcal{R}(\mathbf{k})$$

$$\Theta(\hat{\mathbf{n}}) = \int \frac{d^3k}{(2\pi)^3} e^{i\mathbf{k}\cdot\hat{\mathbf{n}}\chi_*} (-i\hat{\mathbf{k}} \cdot \hat{\mathbf{n}}) T_v(k, \eta_*) \mathcal{R}(\mathbf{k})$$

$$i\hat{\mathbf{k}} \cdot \hat{\mathbf{n}} e^{i\hat{\mathbf{k}}\cdot\hat{\mathbf{n}}k\chi_*} = \frac{d}{d(k\chi_*)} \left[e^{i\hat{\mathbf{k}}\cdot\hat{\mathbf{n}}k\chi_*} \right] = 4\pi \sum_{lm} i^l j_l'(k\chi_*) Y_{lm}^*(\hat{\mathbf{k}}) Y_{lm}(\hat{\mathbf{n}})$$

Result: CMB power from dipole term

- Take spherical harmonic transform, square, compare with power spectrum definition. Result:

$\chi_* =$ distance to CMB

$$C_l = 4\pi \int d\ln k \left[\frac{k^3}{2\pi^2} P_{\mathcal{R}}(k)\right] \times [T_v(k,\tau_*)]^2 \times [j_l'(k\chi_*)]^2$$

primordial power — plasma processing — projection (Bessel')

- What is velocity transfer function?

Velocity Transfer Function

- Continuity equation: $\delta_r' = -\frac{4}{3}(\nabla \cdot \mathbf{v} \; - 3\Phi')$
  Potential is constant (matter dom.) or small (rad.) so '= 0.

- Hence $v \; = 3\delta_r'/(4k)$ and

$$v(\mathbf{k}, \eta) \sim c_s \sin(kr_s)$$

- In other words, we have a sine transfer function, whereas Sachs-Wolfe has cosine.

Velocity Transfer Function

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$$v(\mathbf{k}, \eta) \sim c_s \sin(kr_s)$$

- In other words, we have a sine transfer function, whereas Sachs-Wolfe has cosine.

- Question: Why do we still have oscillations in the total power despite $\cos^2 + \sin^2 = 1$?

Velocity Transfer Function and Doppler Power

- Projection involves derivative of Bessel function! Less peaked w. contributions from much broader range of k.

$$C_l = 4\pi \int d\ln k \left[\frac{k^3}{2\pi^2} P_{\mathcal{R}}(k)\right] \times [T_v(k,\tau_*)]^2 \times [j_l'(k\chi_*)]^2$$

- Doppler power much less oscillatory:

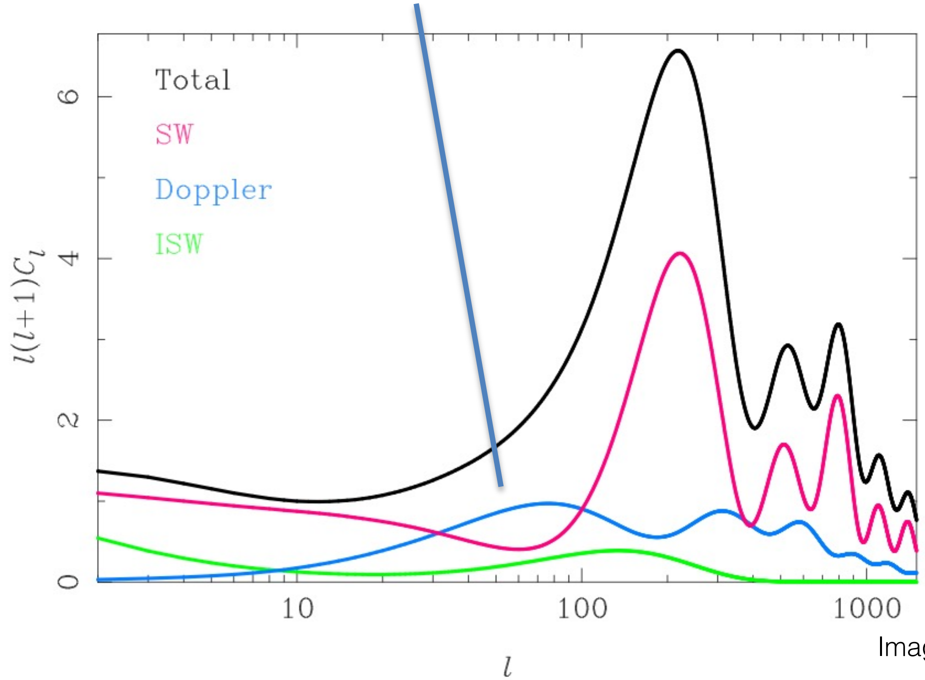
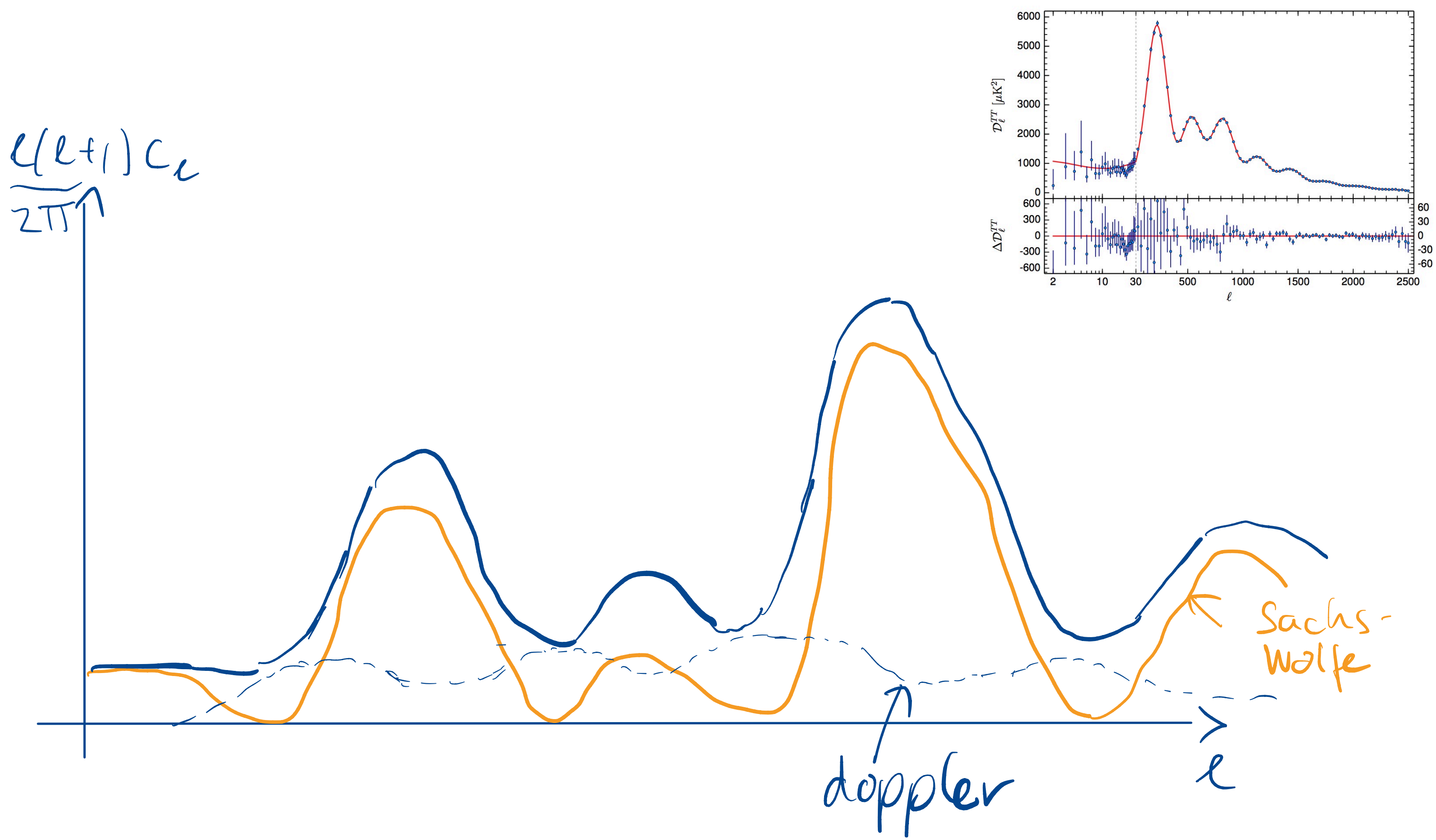


Image credit: A. Challinor

Power Spectrum Including Doppler



- Still not quite there: how can we explain the cutoff at high multipoles?

Complication: Photon Diffusion

- Photons diffuse, smoothing out inhomogeneities in photon density

- What is diffusion length?
  - The mean free path is $l_P = (a n_e \sigma_T)^{-1} = 1/|\dot{\tau}|$
  - Hence number of scatterings: $d\eta / l_p$
  - The distance random-walked is $L^2 \sim N l_p^2 \sim \frac{d\eta}{l_p} l_p^2 \sim d\eta\, l_p \sim d\eta |\dot{\tau}|^{-1}$
  - Hence diffusion length is $L^2 \sim k_D^{-2} = \int_0^{\eta_*} d\eta \; |\dot{\tau}|^{-1}$

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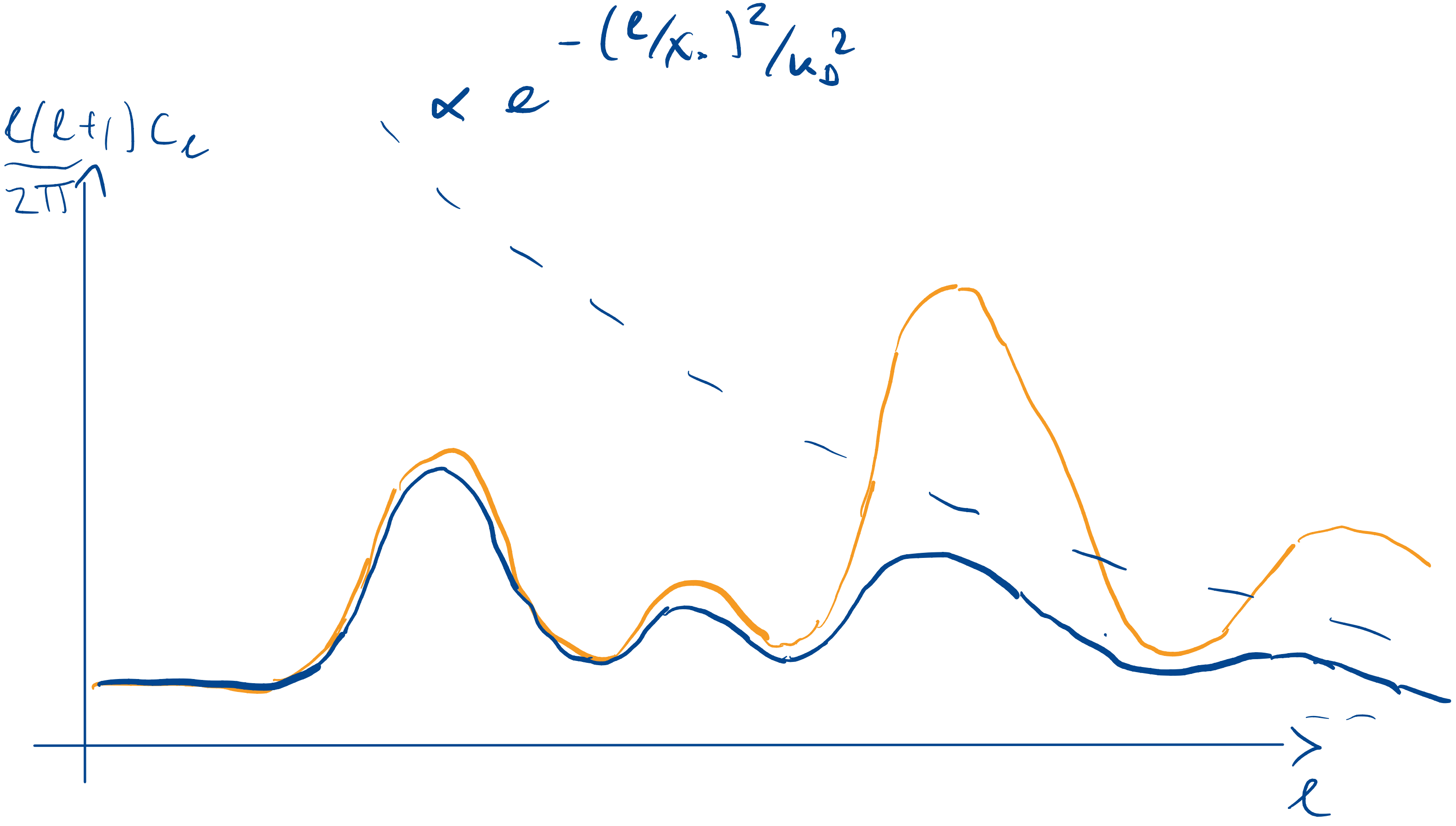
  - Hence diffusion length is $L^2 \sim \, k_D^{-2} = \int_0^{\eta_*} d\eta \; |\dot{\tau}|^{-1}$

- Recall diffusion equation $\frac{\partial \phi}{\partial t} = D \nabla^2 \phi \qquad \frac{\partial}{\partial t} \tilde{\phi}(\mathbf{k}, t) = -D |\mathbf{k}|^2 \, \tilde{\phi}(\mathbf{k}, t)$

$$\tilde{\phi}(\mathbf{k}, t) = \tilde{\phi}(\mathbf{k}, 0) e^{-Dk^2 t} = \tilde{\phi}(\mathbf{k}, 0) e^{-k^2 L^2}$$

where the diffusion length L is $L \sim (Dt)^{1/2}$

Complication: Diffusion



- Result: exponential suppression $\delta_r \propto e^{-k^2/k_D^2} \cos(kr_s)$

Complication: Diffusion

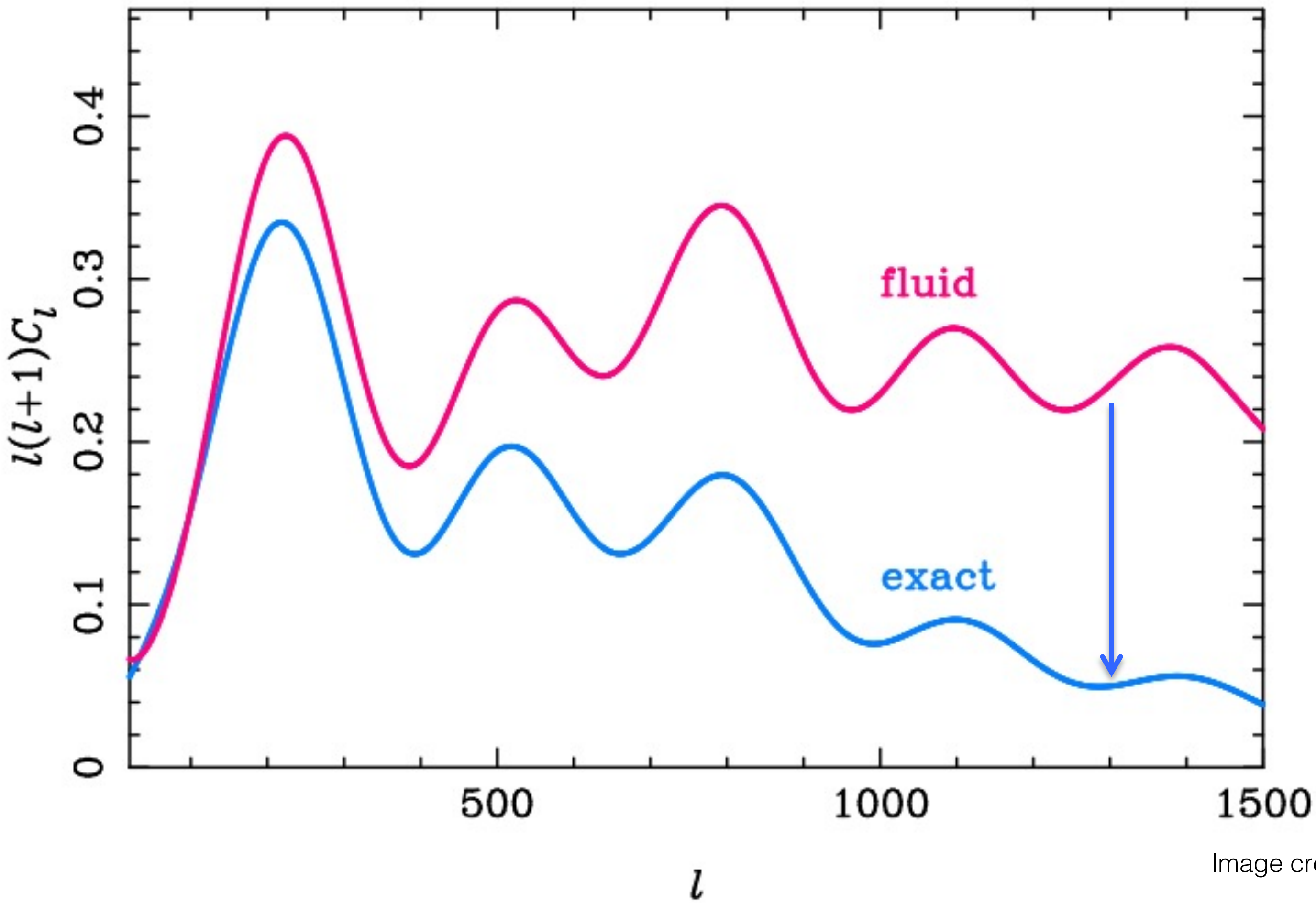


Image credit: A. Challinor

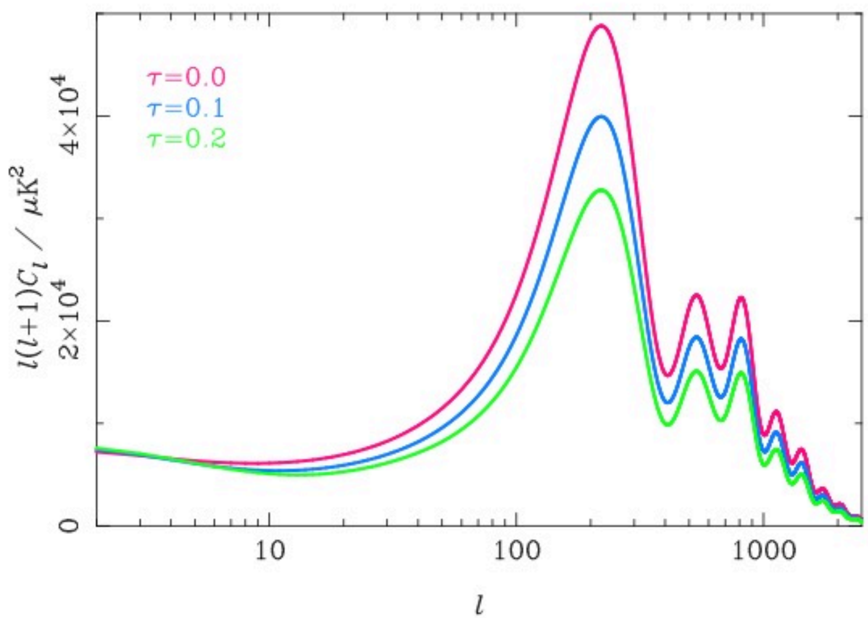
- Result: exponential suppression $\delta_r \propto e^{-k^2/k_D^2} \cos(kr_s)$

Complication: Reionization

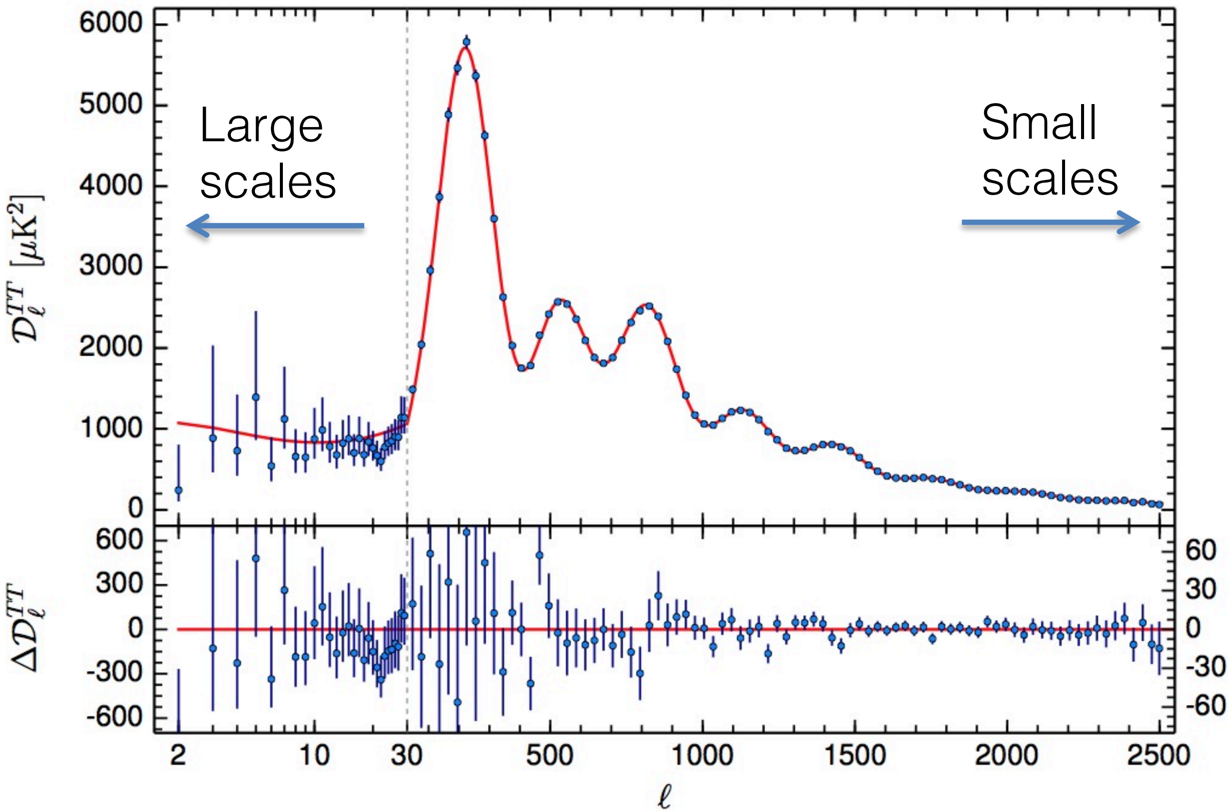
- Universe is reionized at z~6-10
  - CMB scatters off the ionized gas with optical depth

$$\tau = \int_{\eta_{re}}^{\eta_0} a n_e \sigma_T d\eta$$

  - Inside the horizon for small scales, anisotropies attenuated by $e^{-\tau}$, spectra by $e^{-2\tau}$



Measurement: The Planck CMB Power Spectrum



- Have only approximately calculated photon diffusion, finite visibility function,… full treatment with Boltzmann eq. explains this (CAMB, CLASS, CMBFAST…)

Aside: Sketch of Full Boltzmann Equation Treatment

- Don't treat photons as a fluid, analyze phase space distribution function f(time, position, energy, direction):

$$\frac{\partial f}{\partial \eta} + \mathbf{e} \cdot \boldsymbol{\nabla} f + \frac{\partial \bar{f}}{\partial \ln \epsilon} \frac{d \ln \epsilon}{d\eta} = \left. \frac{df}{d\eta} \right|_{\text{scatt.}}$$

- Rewriting in terms of the direction-dependent temperature:

$$f(\eta, \mathbf{x}, \epsilon, \mathbf{e}) = \bar{f}(\epsilon) \left[1 - \Theta(\eta, \mathbf{x}, \mathbf{e}) \frac{d \ln \bar{f}}{d \ln \epsilon} \right]$$

$$\frac{\partial \Theta}{\partial \eta} + \mathbf{e} \cdot \boldsymbol{\nabla} \Theta - \frac{d \ln \epsilon}{d\eta} = -a \bar{n}_e \sigma_{\mathrm{T}} \Theta + \frac{3 a \bar{n}_e \sigma_{\mathrm{T}}}{16\pi} \int d\hat{\mathbf{m}}\, \Theta(\hat{\mathbf{m}}) \left[1 + (\mathbf{e} \cdot \hat{\mathbf{m}})^2 \right] + a \bar{n}_e \sigma_{\mathrm{T}} \mathbf{e} \cdot \mathbf{v}_b$$

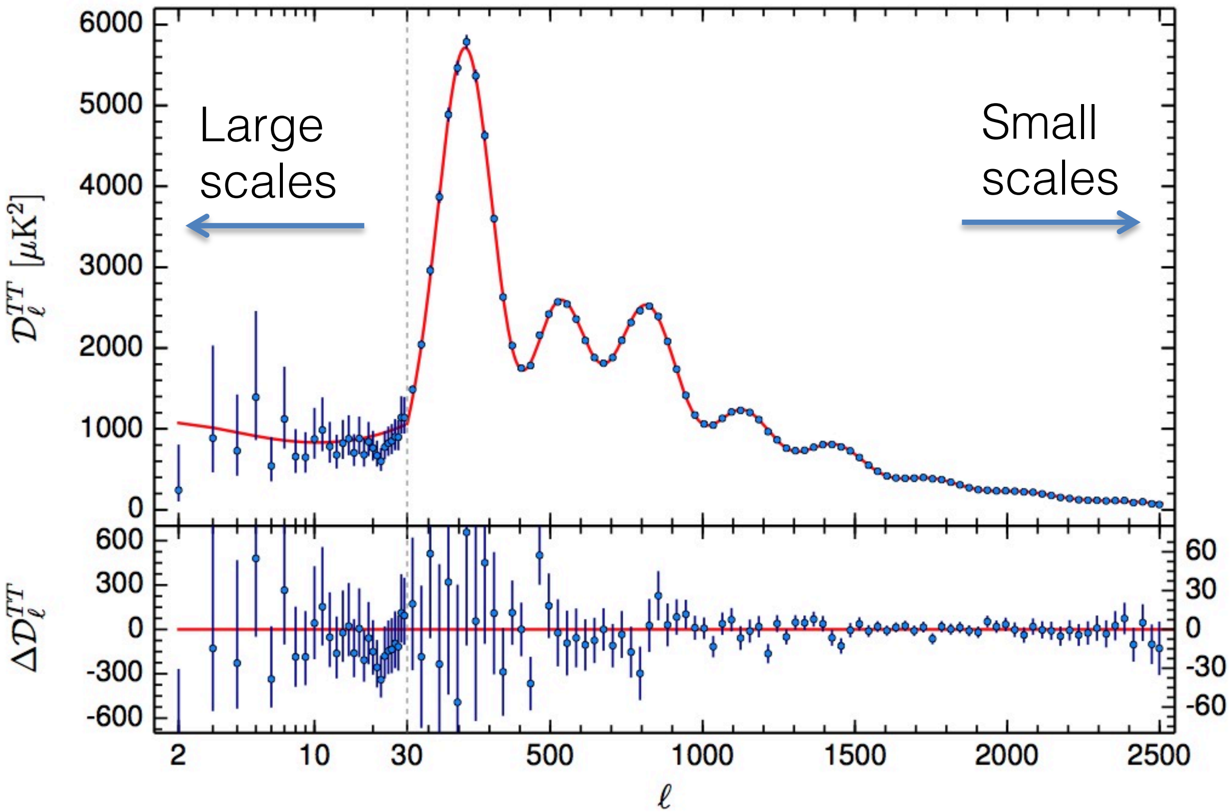
- For scalars evolution must be symmetric about k so expand in Legendre polynomials

$$\Theta(\eta, \mathbf{k}, \mathbf{e}) = \sum_{l \geq 0} (-i)^l \Theta_l(\eta, \mathbf{k}) P_l(\hat{\mathbf{k}} \cdot \mathbf{e})$$

$$\dot{\Theta}_l + k \left(\frac{l+1}{2l+3} \Theta_{l+1} - \frac{l}{2l-1} \Theta_{l-1} \right) = -\dot{\tau} \left[(\delta_{l0} - 1) \Theta_l - \delta_{l1} v_b + \frac{1}{10} \delta_{l2} \Theta_2 \right] + \delta_{l0} \dot{\phi} + \delta_{l1} k \psi$$

- Boltzmann ODE Hierarchy solved (cleverly) by CAMB/CLASS

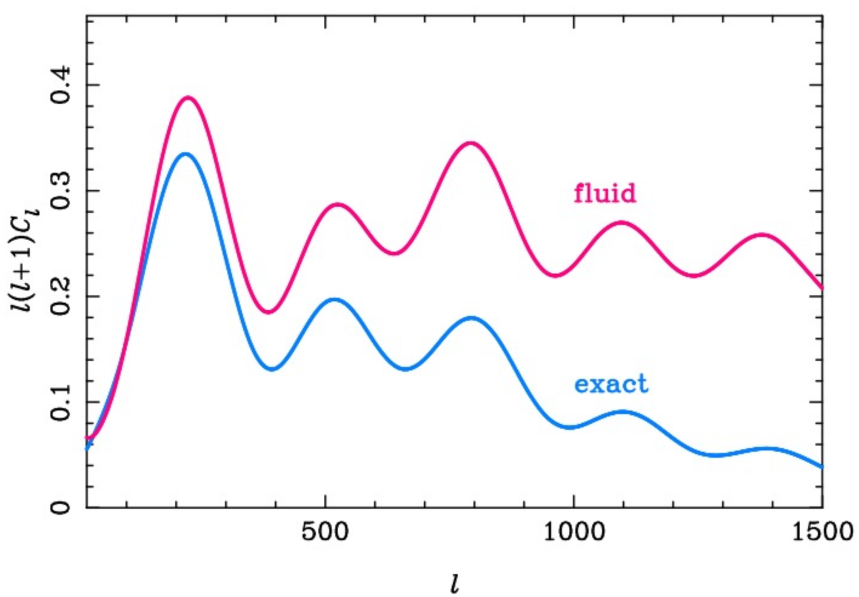
Measurement: The Planck CMB Power Spectrum



- Have only approximately calculated photon diffusion, finite visibility function,… full treatment with Boltzmann eq. explains this (CAMB, CLASS, CMBFAST…)
- But: have explained main features of spectrum!

The CMB as a Tool to Understand Cosmic Evolution

- We have explained the CMB power spectrum with
  - Acoustic oscillations
  - Peak heights affected by baryons
  - Small scales boosted by radiation driving
  - Very small scales reduced by diffusion damping
  - Amplitude reduced by scattering



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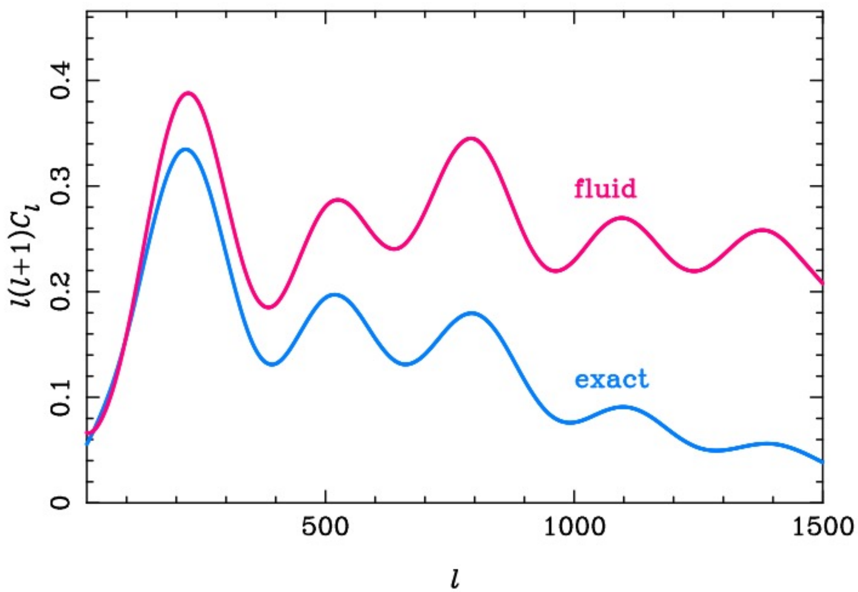
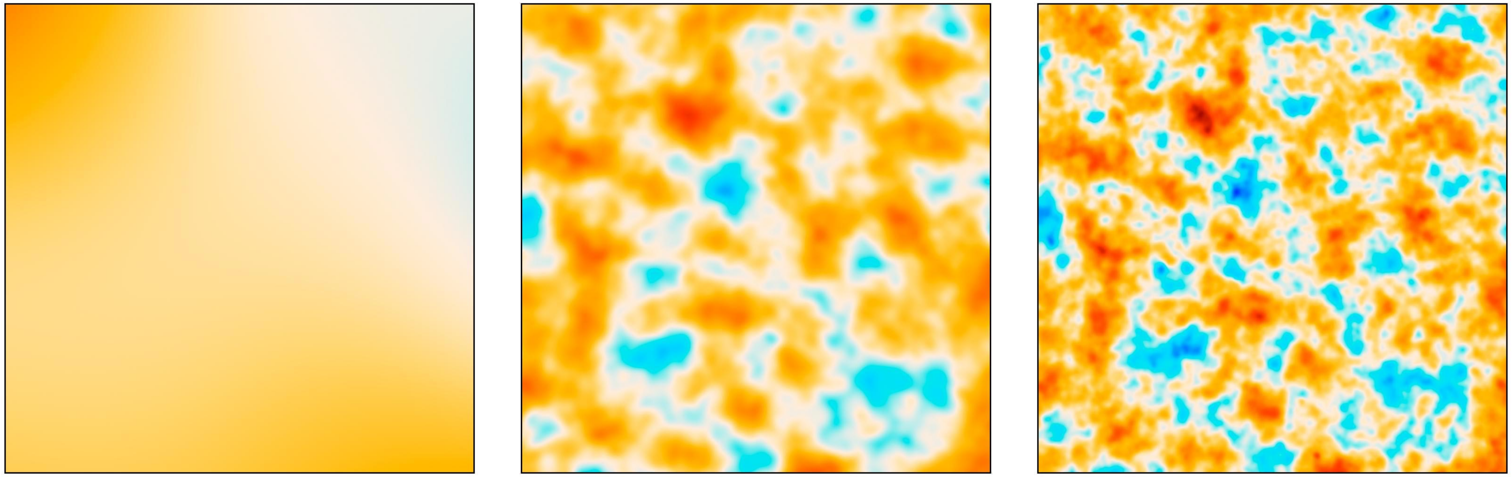


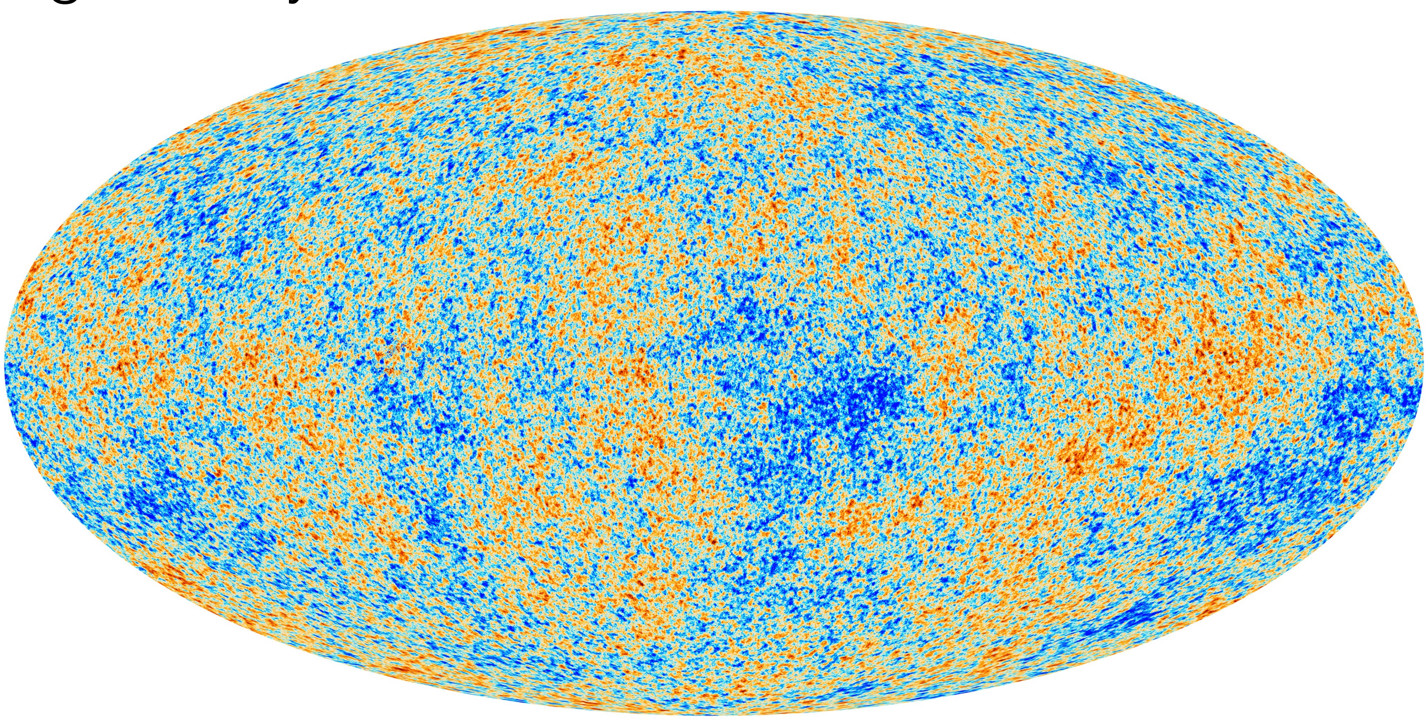
Outline

- CMB power spectrum: details
- Determining cosmic composition from the CMB
- Tensions?



Why is This Important – Understand Cosmic Properties / Composition

- Our measurements of the CMB are now so precise that we can **use this known physics to** learn cosmology
- Make PERCENT-level measurements of its composition, age, geometry…



The CMB as a Tool to Understand Cosmic Evolution

- We can use CMB to measure cosmic composition:
  - Baryon density
  - Dark matter density
  - Spectral index
  - Hubble constant
  - Number of relativistic species

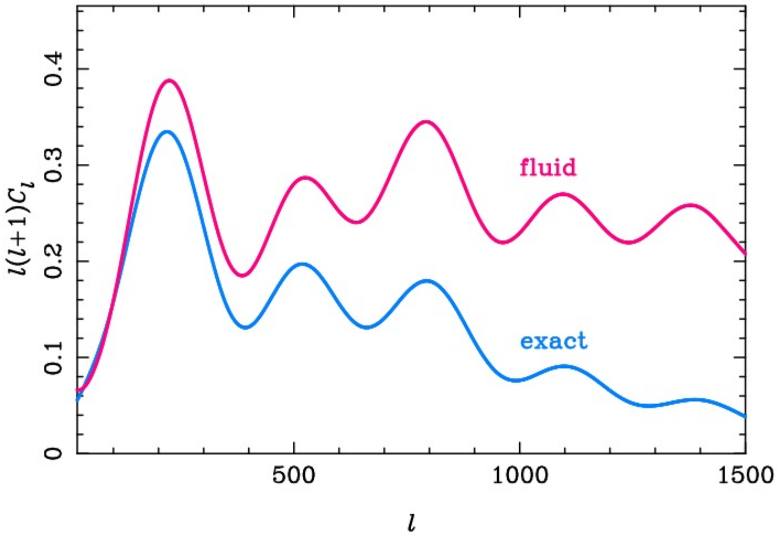
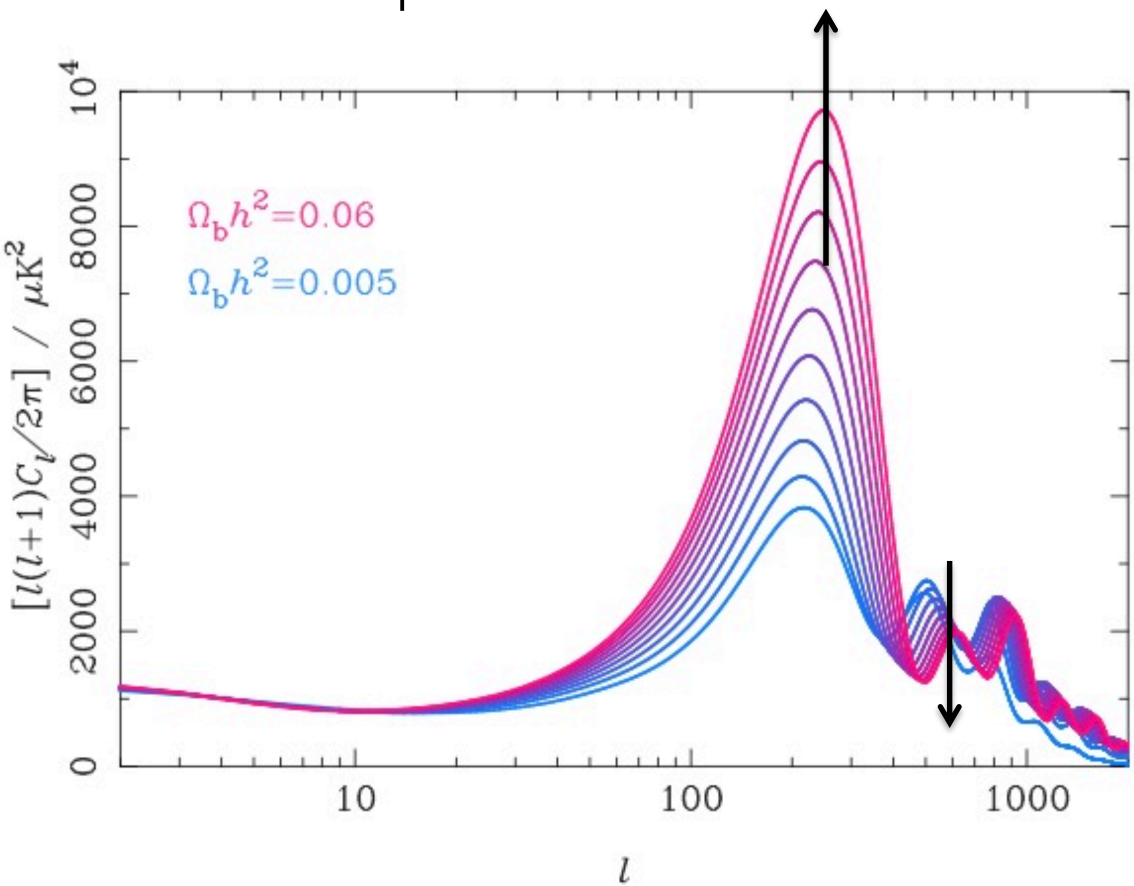


Image credit: A. Challinor

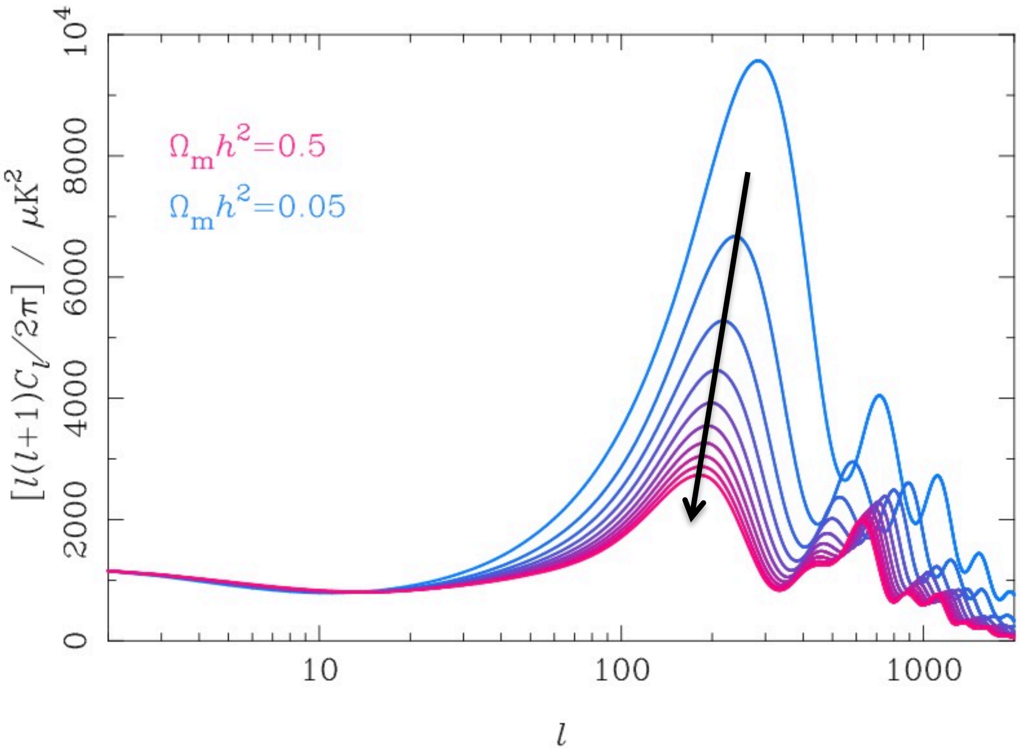
Constraining Baryon Density

- Increasing baryon density boosts compressional peaks relative to rarefaction peaks



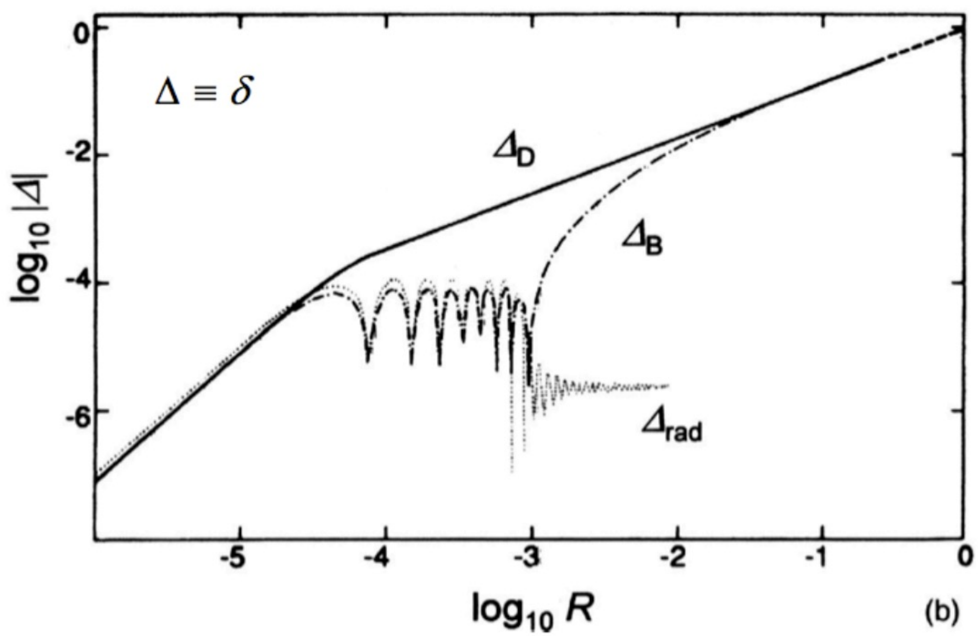
Constraining Matter Density

- Increasing matter density reduces radiation driving of low-order peaks. Shape of CMB implies dark matter!



Constraining Matter Density

- Increasing matter density reduces radiation driving of low-order peaks. Shape of CMB implies dark matter!
- Aside: baryon perturbations alone ~1e-5 (CMB) don't have time to grow ~a into O(1) density contrast!

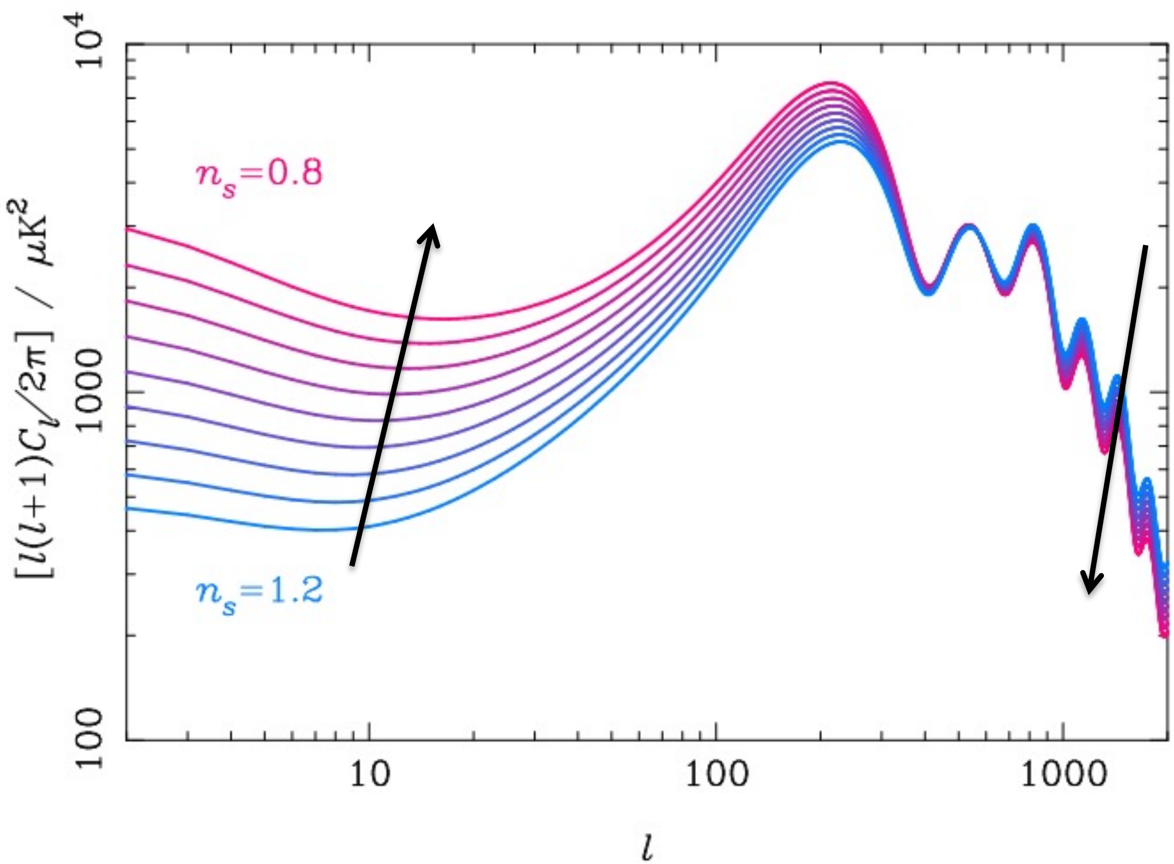


Bender, IMPRS Astrophysics Introductory Course

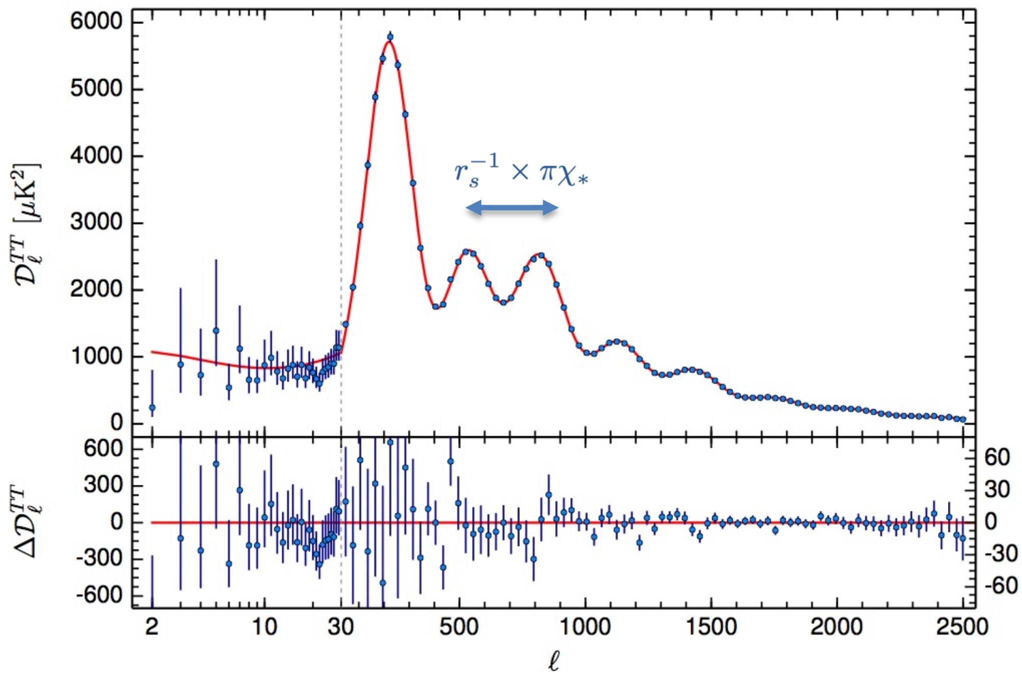
Constraining spectral index

- Changing scalar spectral index n_s just tilts overall spectrum! $k^3 P_{\mathcal{R}}(k) \propto k^{n_s - 1}$

$$\frac{l(l+1)}{2\pi} C_l \approx T_S^2(\eta_*, k = l/\chi_*) \times \left[\frac{(l/\chi_*)^3}{2\pi^2} P_{\mathcal{R}}(k = l/\chi_*) \right]$$



Measuring Hubble using the CMB



$\mathcal{D}_l \equiv l(l+1)C_l/2\pi$

- We saw: acoustic oscillations lead to a series of peaks with spacing $r_s^{-1} \times \pi\chi_*$
- Sound horizon known

$$r_s = \int_0^\eta d\eta' c_s(\eta')$$

sound speed

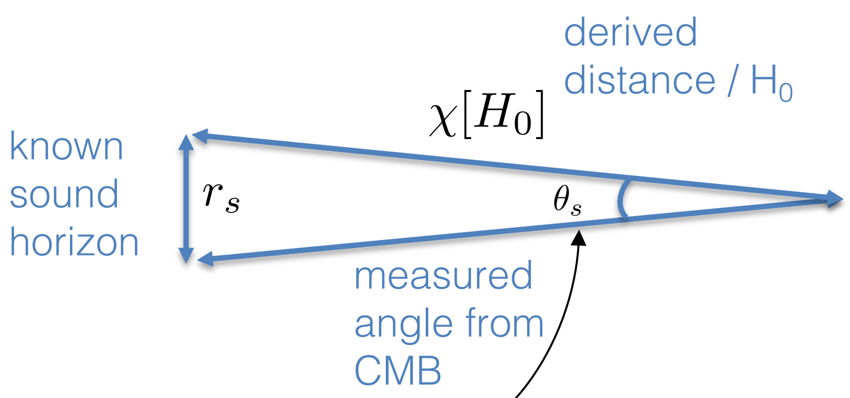
$$= \int_{z_r}^\infty \frac{c_s}{H(z)} dz$$

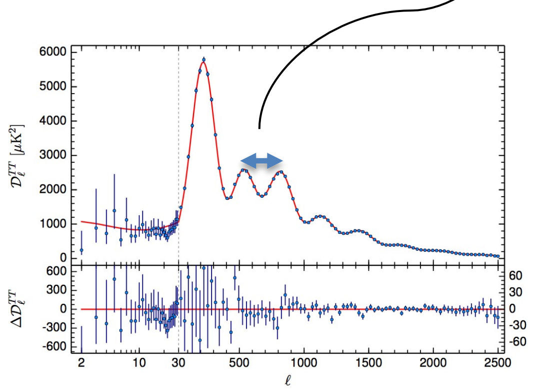
redshift

expansion rate

- Measured peak spacing, infer $\chi[H_0]$ and Hubble!

Measuring Hubble using the CMB





$\mathcal{D}_l \equiv l(l+1)C_l/2\pi$

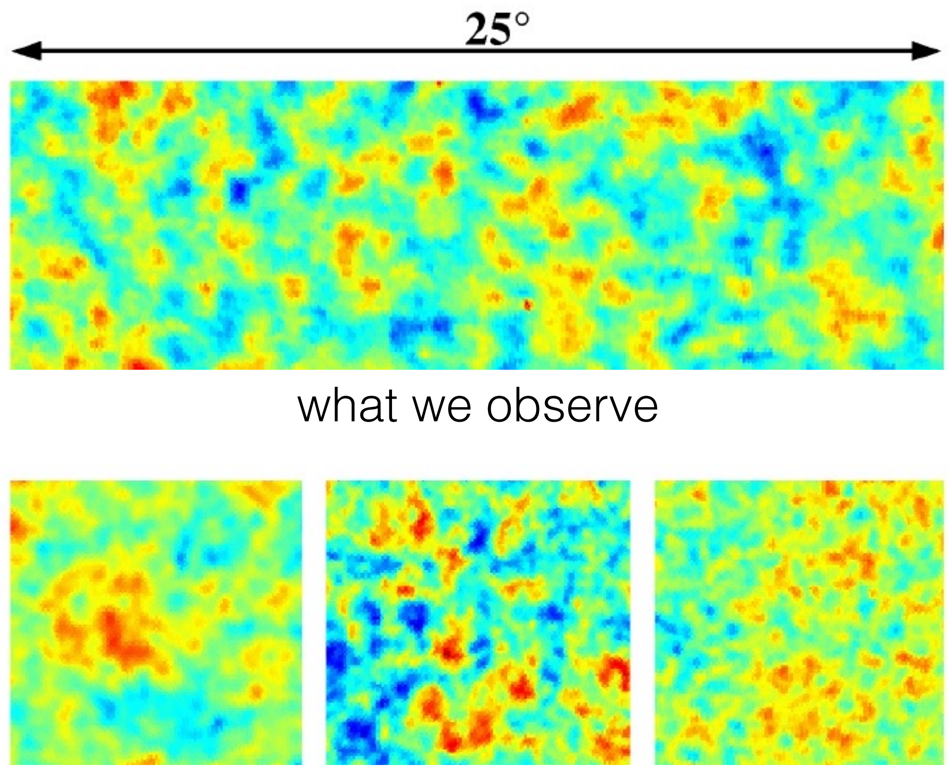
- Physical interpretation:
- Compute (calibr.) sound horizon r_s

$$r_s = \int_{z_r}^{\infty} \frac{c_s}{H(z)} dz$$

- Measure angle θ_s and infer distance $\chi[H_0] \sim r_s/\theta_s$
- Distance[H_0] => H_0 !

Intuition: constraining distance and H_0

- Determine size of the universe ~ 1/Hubble rate by looking at size of spots with known physical size



too close | correct distance | too far

Constraining Light Particle Number N_{eff}

- Cosmic Neutrino Background: in radiation dominated early era, large part of the energy density - 41% of total!

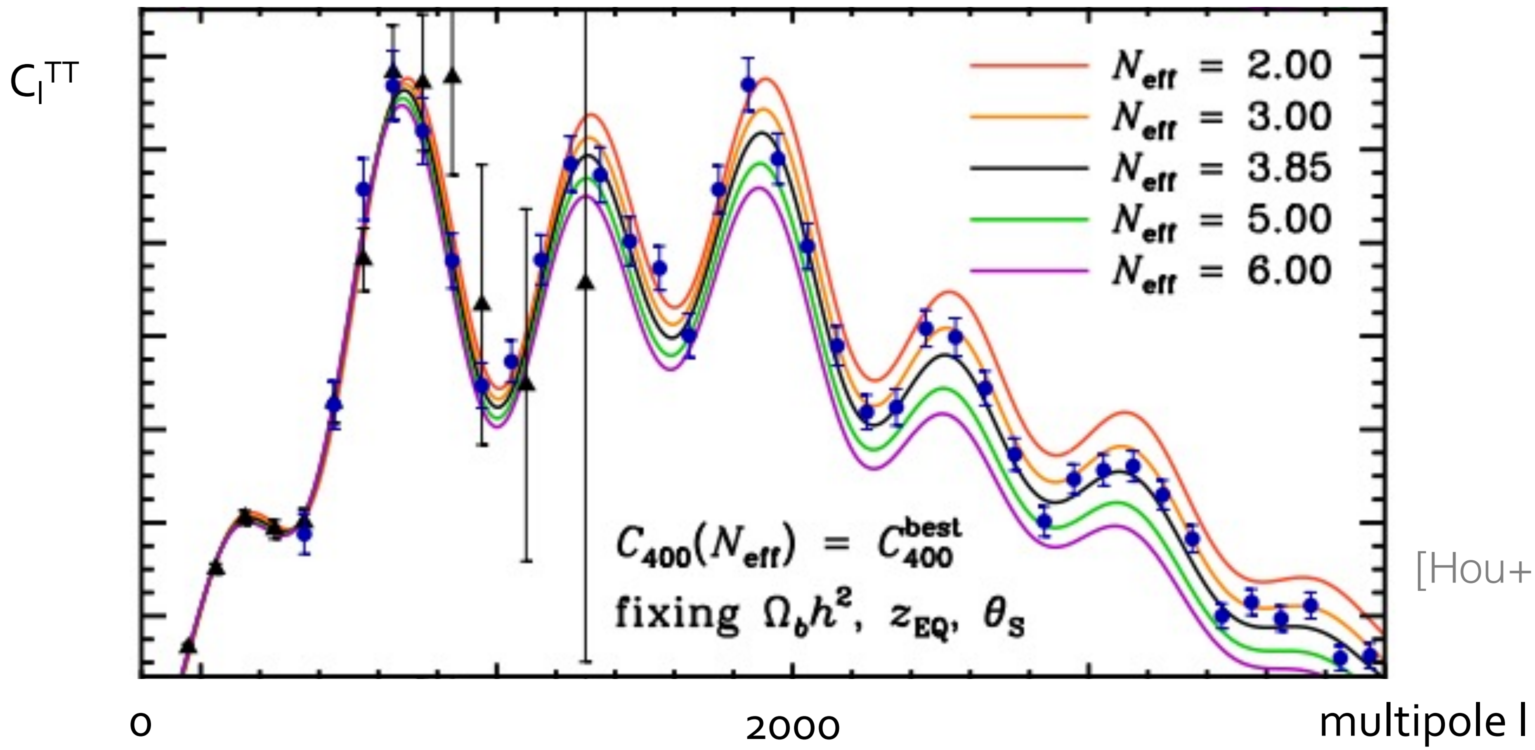
- Affects early expansion rate H (extra form of radiation):

$$3M_{\rm pl}^2 H^2 \simeq \rho_\gamma + \rho_\nu \longleftarrow \quad N_{\rm eff} \equiv \frac{8}{7}\left(\frac{11}{4}\right)^{4/3} \frac{\rho_\nu}{\rho_\gamma}$$

- Energy density parameterized via number of effective neutrino species N_{eff}. Measure via CMB!

Constraints on N_{eff} in the CMB Power Spectra

- Change in early expansion rate > change in diffusion time > small change in damping seen in CMB power spectrum.



[Hou++ 2013]

- (Also: small shift in phases.) Measure with Planck:

$$N_{\rm eff} = 3.04 \pm 0.18$$

Constraining Light Particle Number N_{eff}

- Not just sensitive to neutrinos!

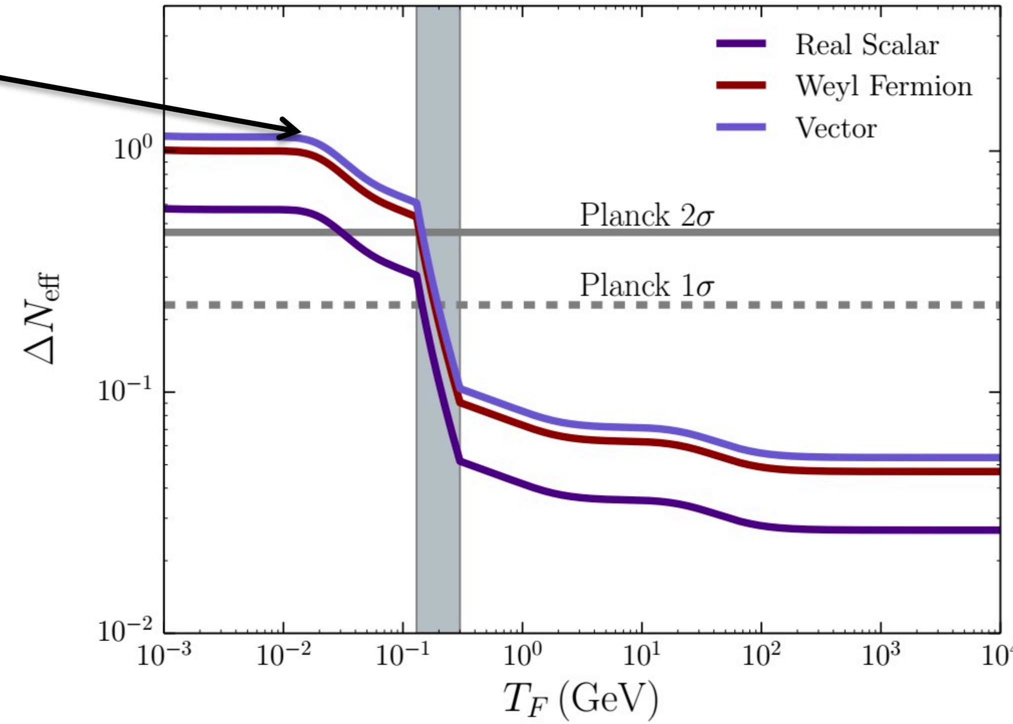
- Gravity sees everything: cosmology probes all that is neutrino like (radiation, free-streaming)

$$3M_{\rm pl}^2 H^2 \simeq \rho_\gamma + \rho_\nu$$

- Can hunt for any new light (relativistic, weakly coupled) particles!

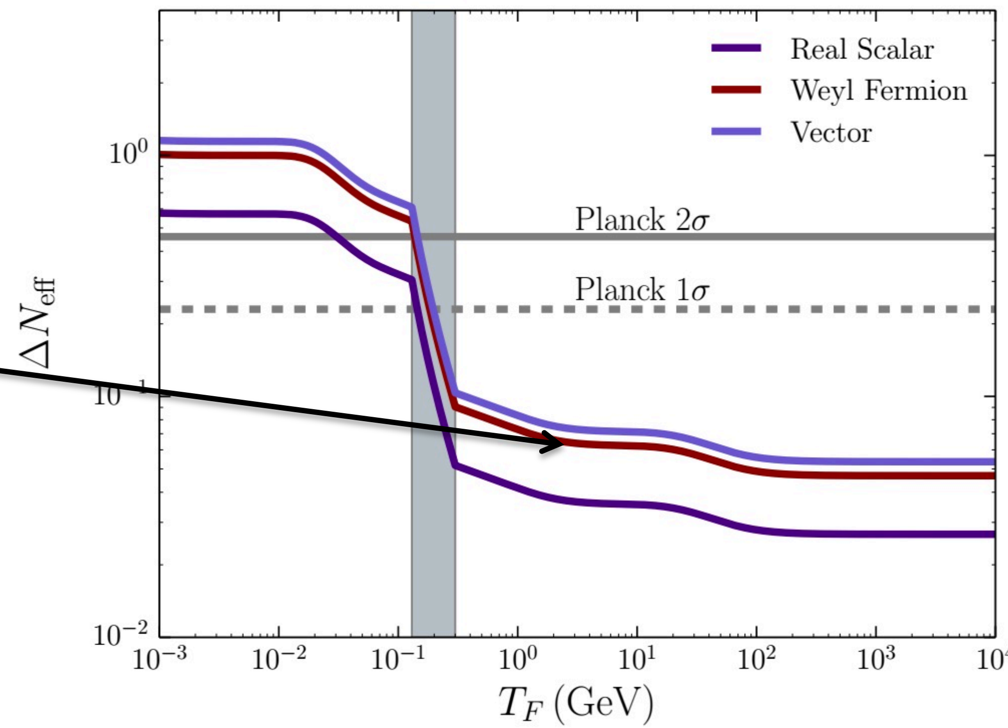
How Much N_{eff} Does A New Light Particle Contribute?

- Freeze out late / low energy: $\Delta N_{eff} \sim 1$



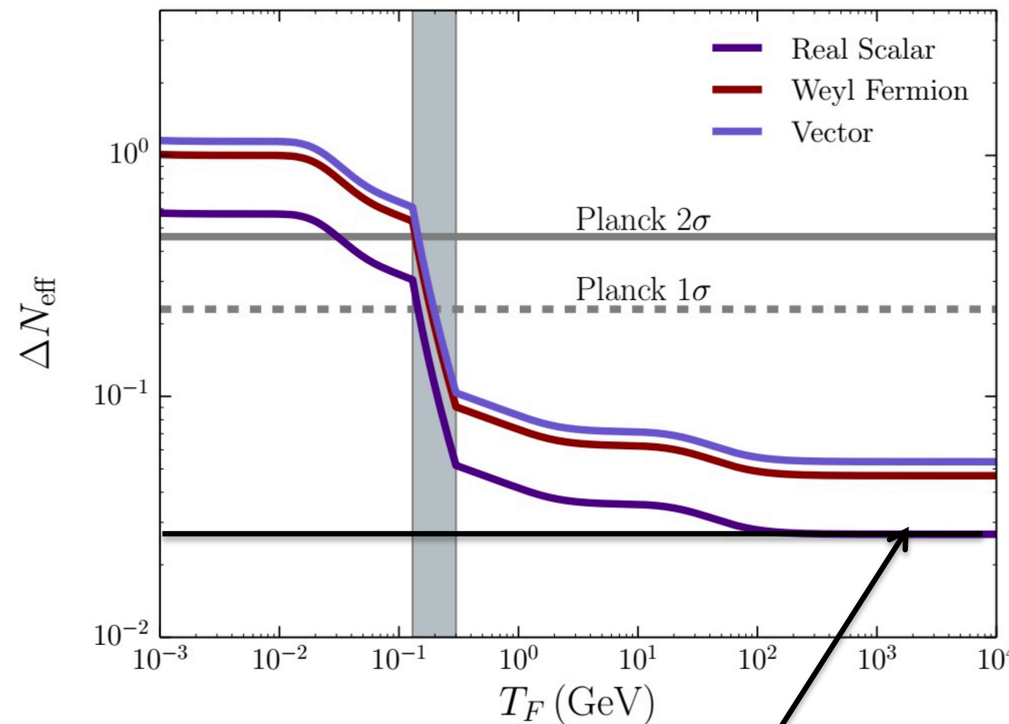
How Much N_{eff} Does A New Light Particle Contribute?

- Impact of extra species is diluted if it decouples at early times. Because then it misses out on energy from known phase transitions!



[Baumann++ 2015]

How Much N_{eff} Does A New Light Particle Contribute?



- If we can measure this: target sensitivity, can see any new light particle that was ever in equilibrium

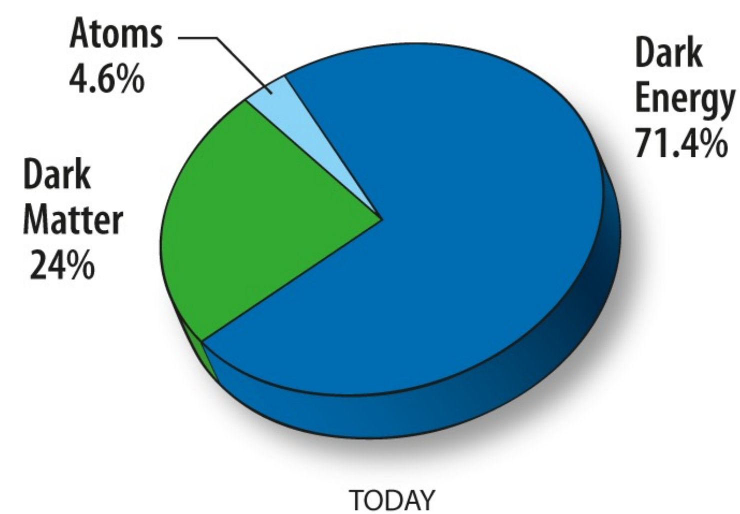
Real Scalar: $\Delta N_{\rm eff} = 0.027$

Weyl Fermion: $\Delta N_{\rm eff} = 0.047$

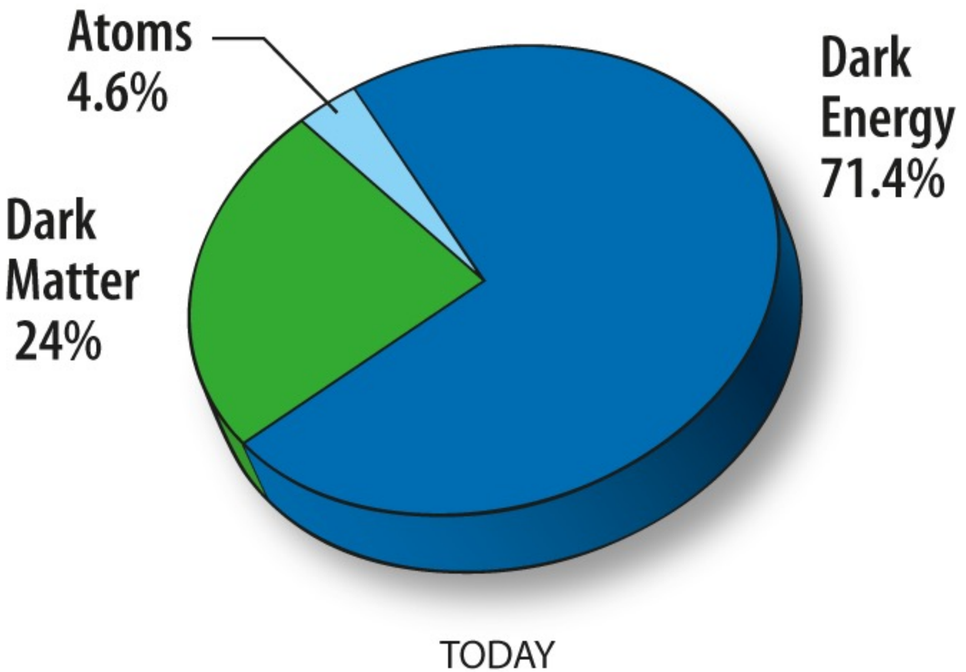
Vector boson: $\Delta N_{\rm eff} = 0.054$

We have learned a LOT from the CMB about cosmic composition

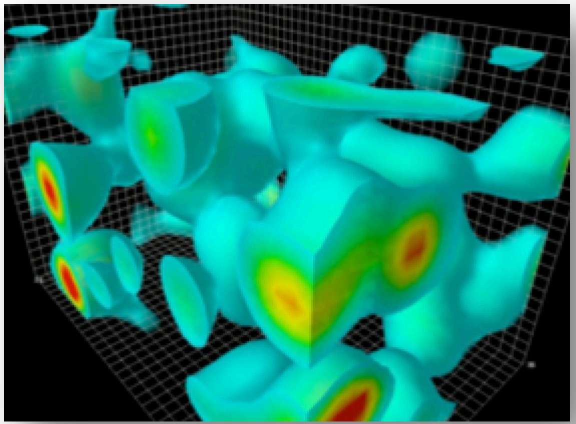
- Examples:
  - How old is the universe?: 13.798±0.037 billion years
  - What is it made out of: 5% normal matter, 24% dark matter(?), 71% dark energy(??). Described by only 6 numbers!
  - What is going to happen to it: expands faster and faster, leading to cold dilute universe



Standard Cosmological Model: Simple…



A_s, n_s



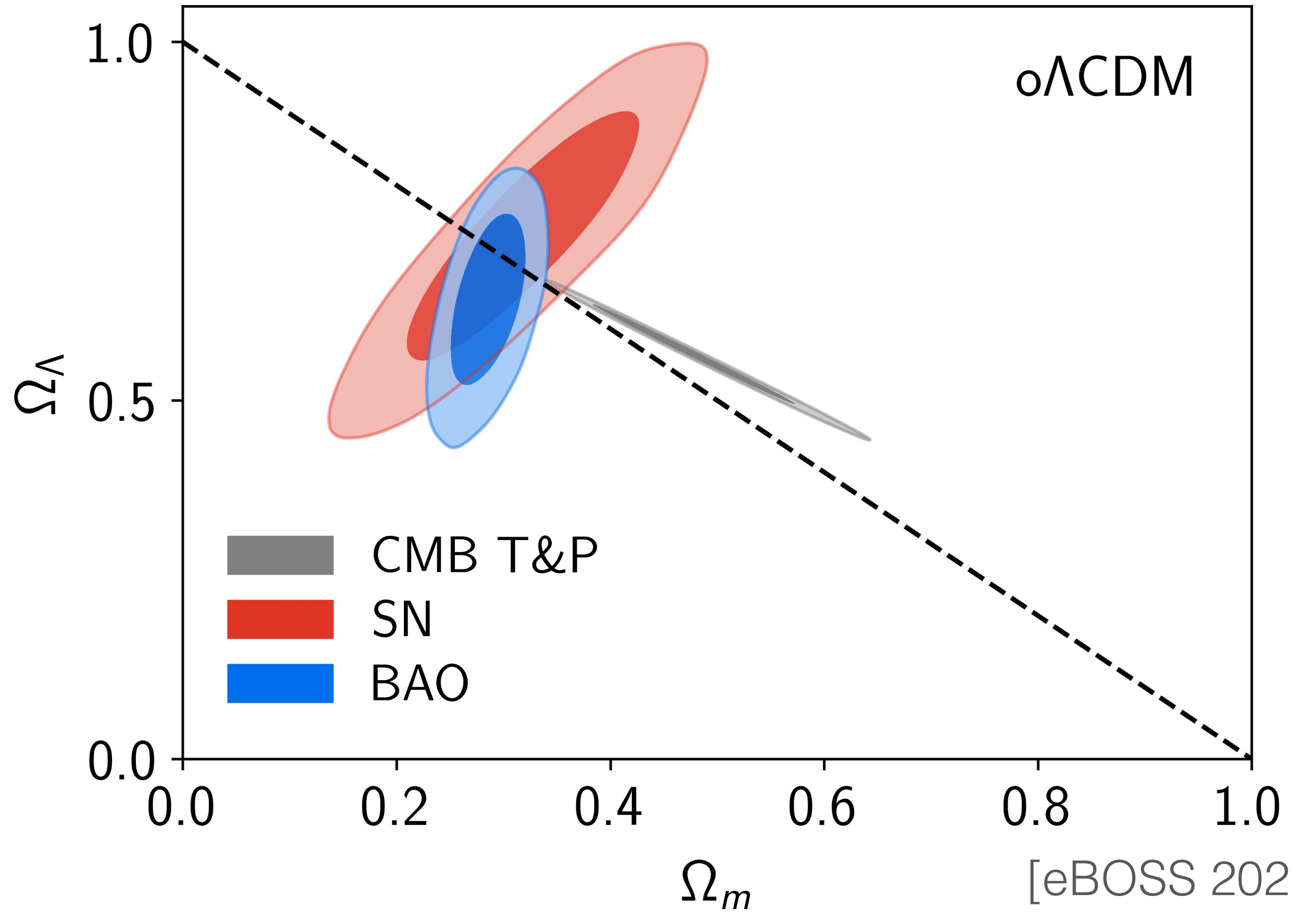
Described by only 5(+1) parameters

Standard Cosmological Model: robust and difficult to modify…

Can get independent, consistent measurements:

From CMB only OR SNae only OR BAO only

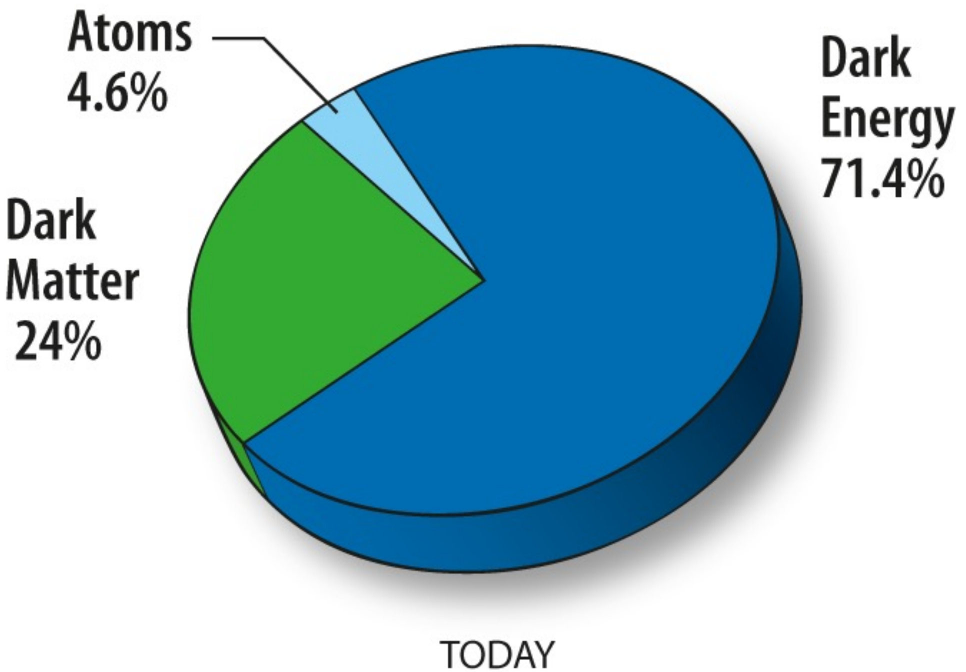
From evolution OR from distance



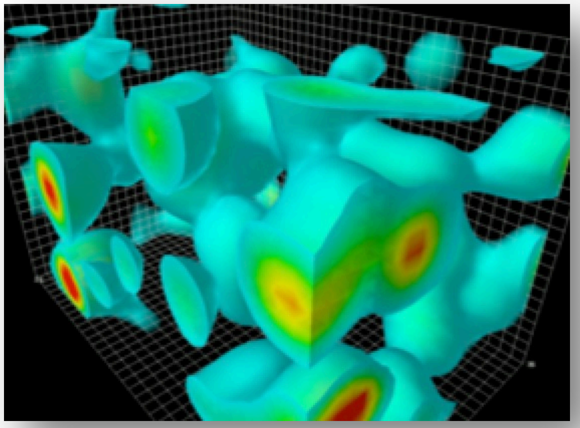
[eBOSS 2021]

Interlocking web of hundreds of -often very robust and well-understood- observations underpinning this model

Standard Cosmological Model: … but very strange



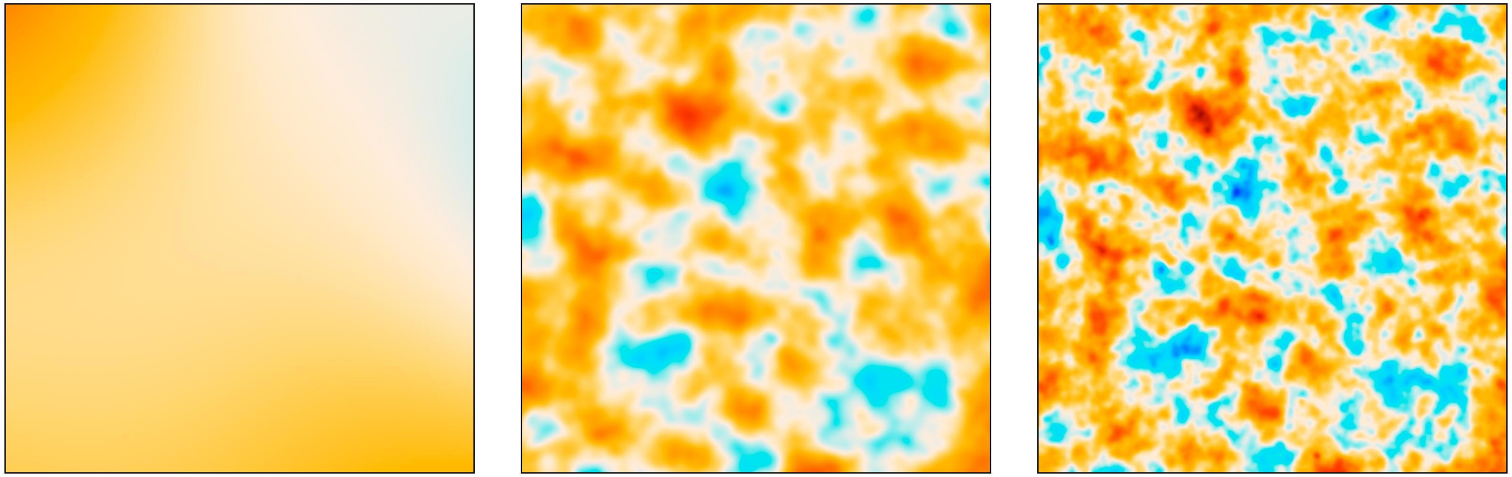
A_s, n_s



Described by only 5(+1) parameters,
but all of them poorly understood

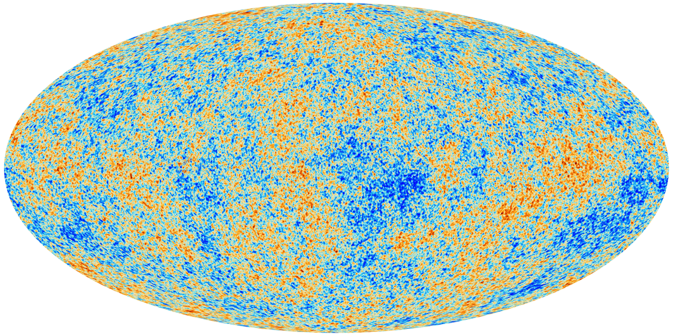
Outline

- CMB power spectrum: details
- Determining cosmic composition from the CMB
- Tensions?

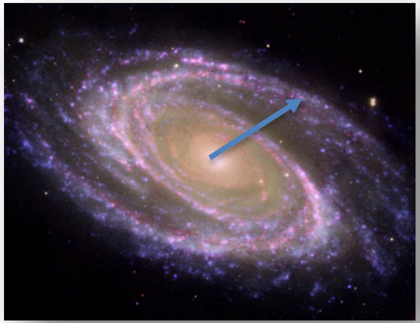


Recently: claims of tensions in late vs. early-time measurements

- Consistency test:



Fit LCDM model to CMB (early times, t=0.004 Gyr)



Predict expansion (H_0) or structure size (S8) at late times (t>10 Gyr) + compare with observations

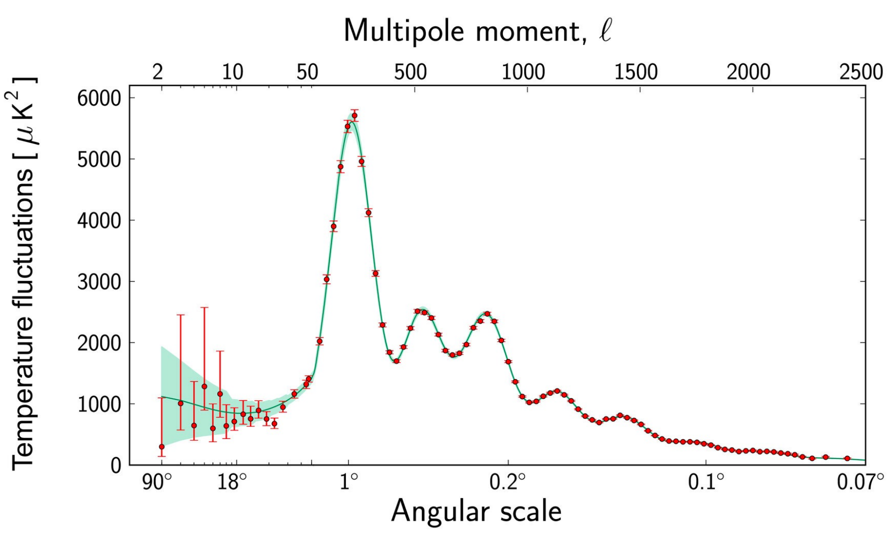
Claims of discrepancies in late vs. early measurements:

- Expansion rate of the universe: "Hubble/H_0 tension"
- Growth of structure "S8 tension"

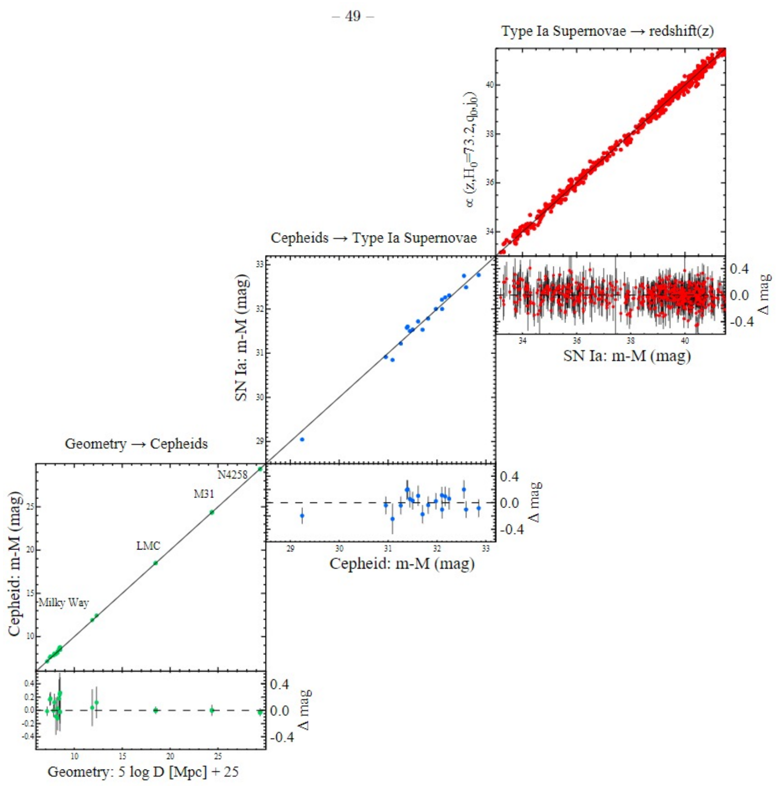
Ways to measure Hubble constant $H_0 = \frac{\dot{a}}{a}$ i.e., expansion rate of Universe

a: scale factor

CMB power spectrum / early / indirect

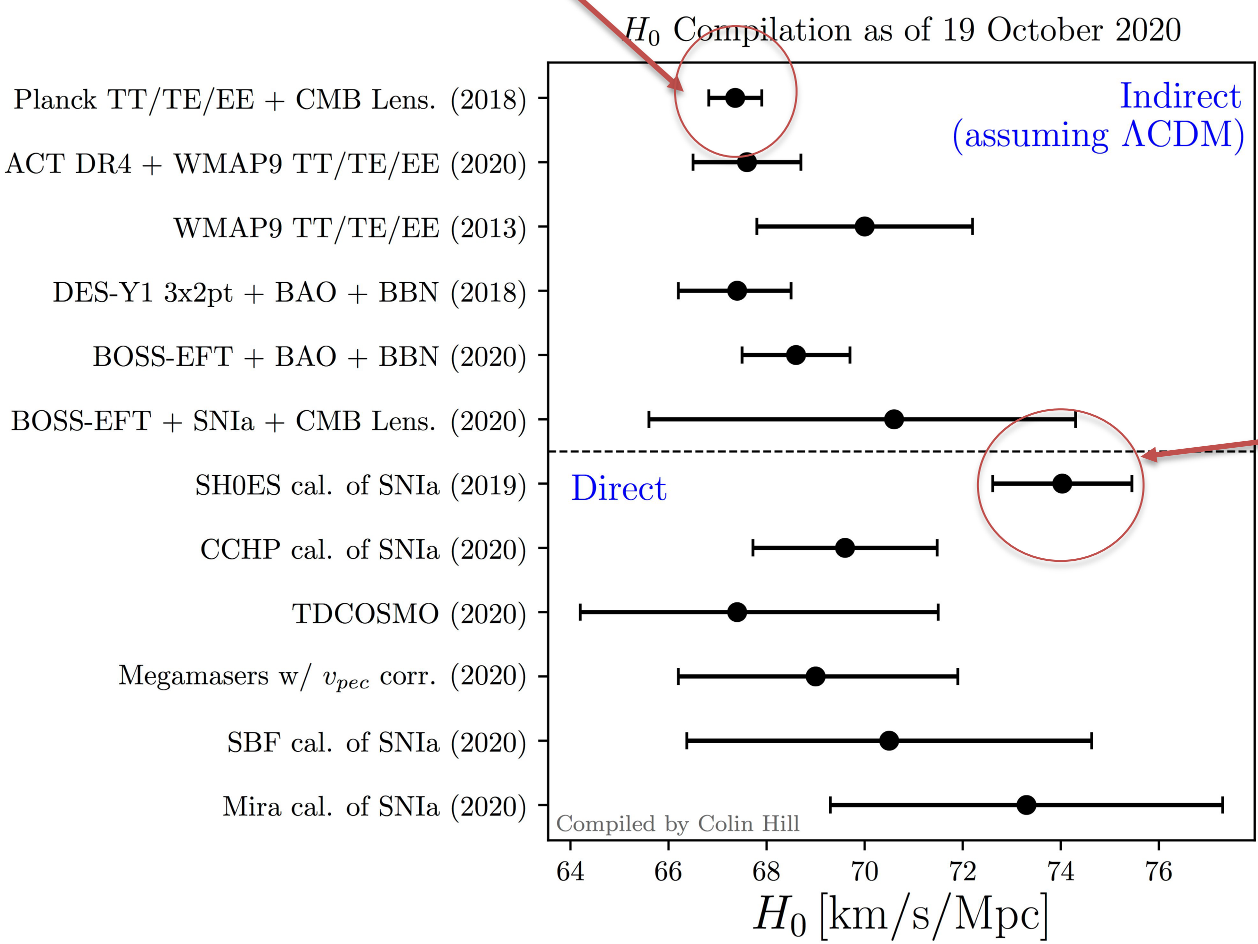


Cosmic distance ladder / late / direct



A big puzzle: the current Hubble constant tension

~68 km/s/Mpc ?



~74 km/s/Mpc ?

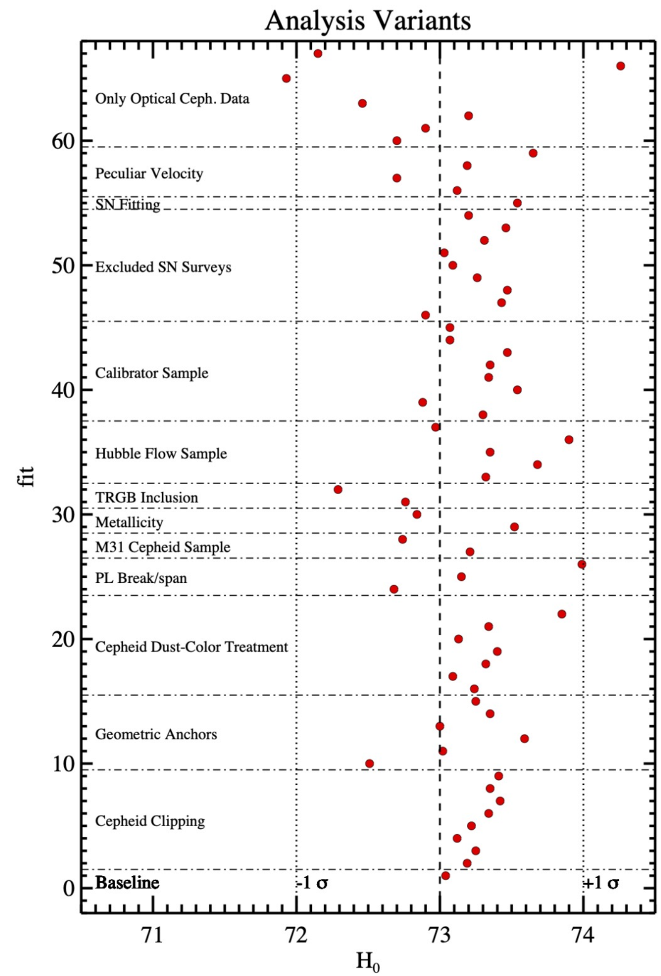
Latest:
73 km/s/Mpc

persistent tension between distance ladder and early-time/indirect measurements

Recently: new distance ladder measurements

[Riess++ 2021]

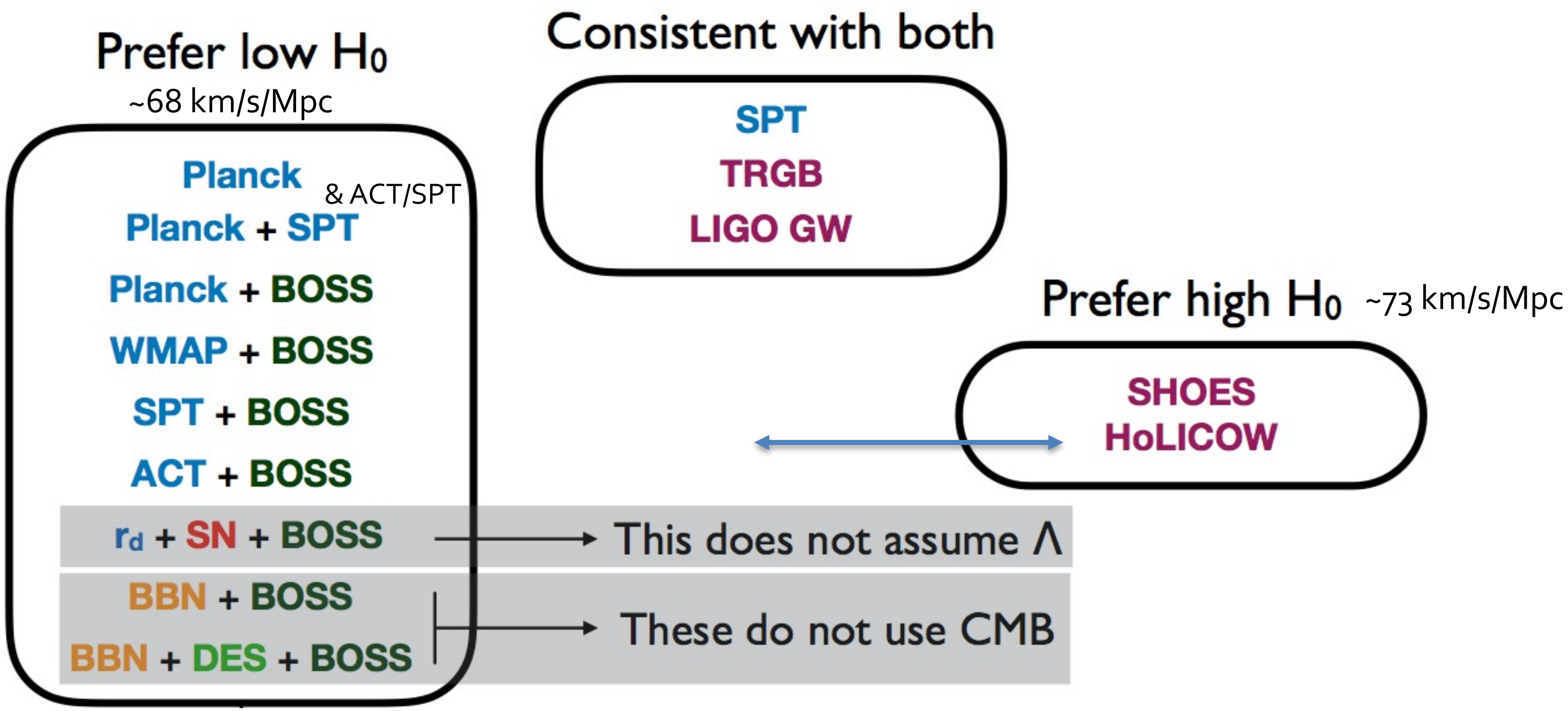
$$H_0 = 73.04 \pm 1.04 \ \mathrm{km\ s^{-1}\ Mpc^{-1}}$$



More than doubles supernova calibrator sample; lots of new checks, result: ~same
Formal claim of 5 sigma tension with Planck.

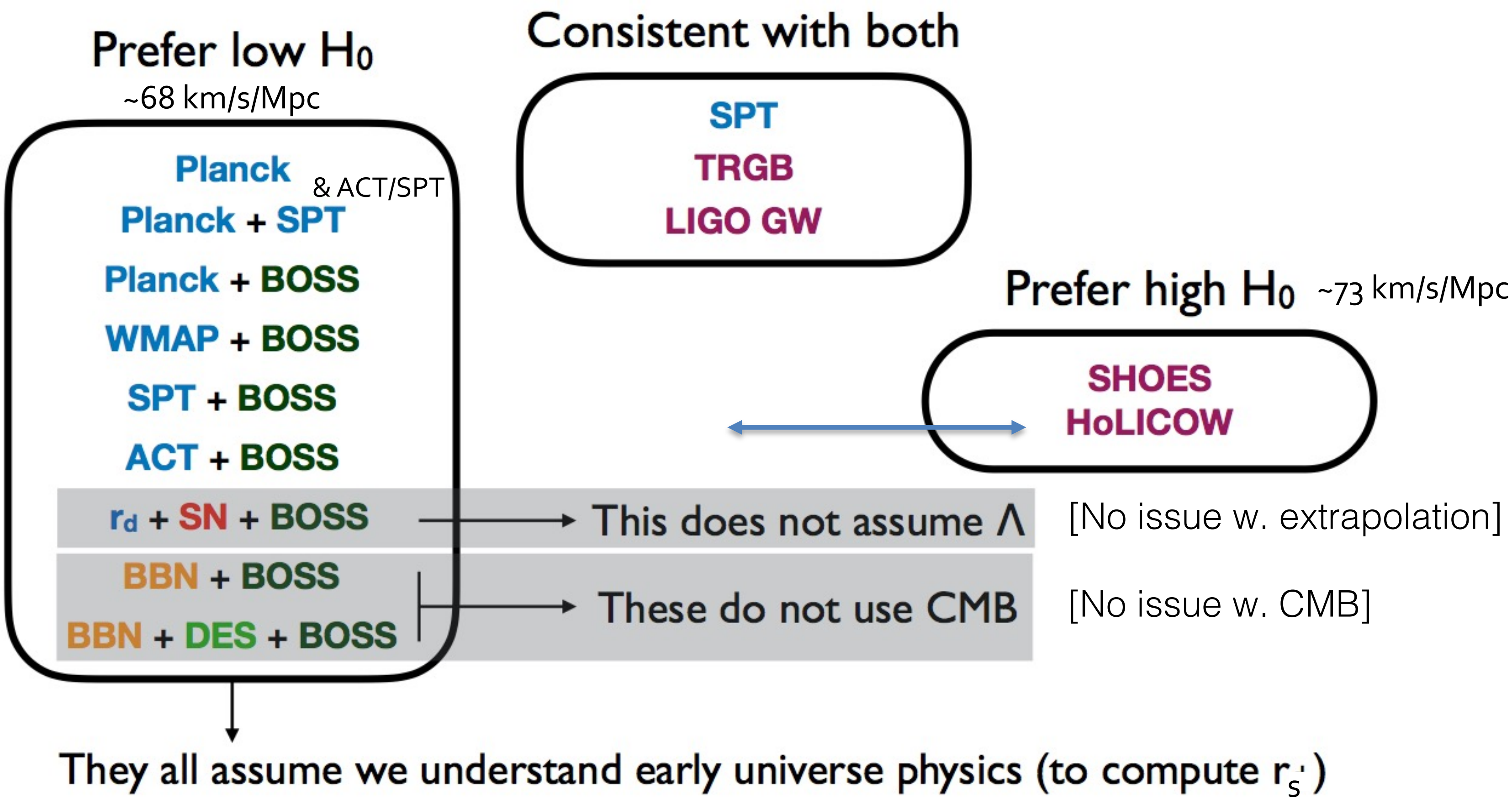
Hubble tension: several cross-checks

Figure credit: A. Font-Ribera

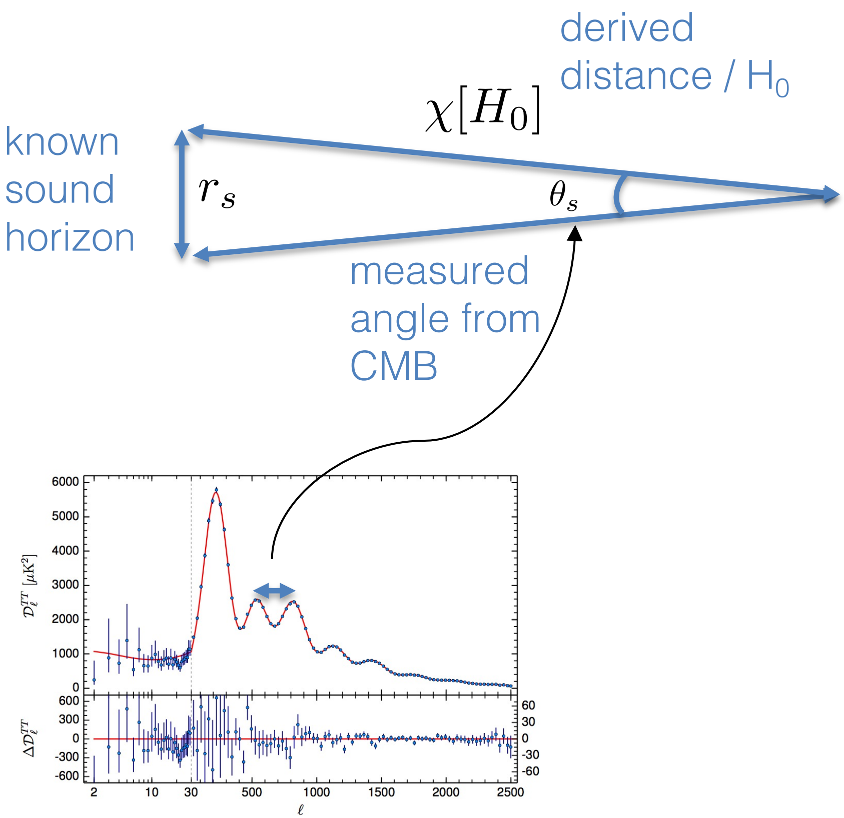


Hubble tension: several cross-checks

Figure credit: A. Font-Ribera



Measuring Hubble using the CMB



$\mathcal{D}_l \equiv l(l+1)C_l/2\pi$

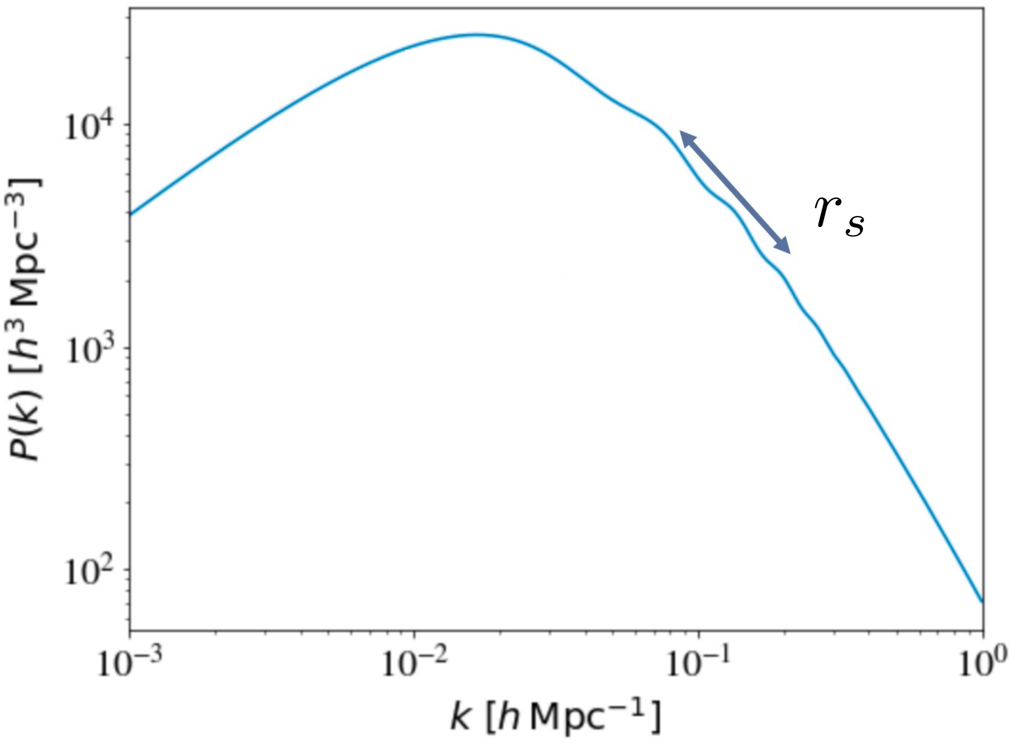
- Compute (calibr.) sound horizon r_s

$$r_s = \int_{z_r}^{\infty} \frac{c_s}{H(z)} dz$$

- Measure angle θ_s and infer distance $\chi[H_0] \sim r_s/\theta_s$
- Distance[H_0]
  => H_0 !

Similar: Measuring Hubble using Matter Power

- Matter power spectrum similar to CMB: Currently mainly get H_0 from "BAO oscillations", imprint of sound horizon on matter.



Idea for resolving tension: is new physics changing r_s?

Possible explanation for tension:
New physics has changed the sound horizon

- Arguably simplest new physics solution
- Can shrink sound horizon with e.g. early energy density

The final category is the set of solutions that introduces new components to increase $H(z)$ in the decade of scale factor evolution prior to recombination. We see these as the most likely category of solutions. They are also

$$r_s = \int_{z_r}^{\infty} \frac{c}{H(z)} dz$$

[Knox + Millea 2019]

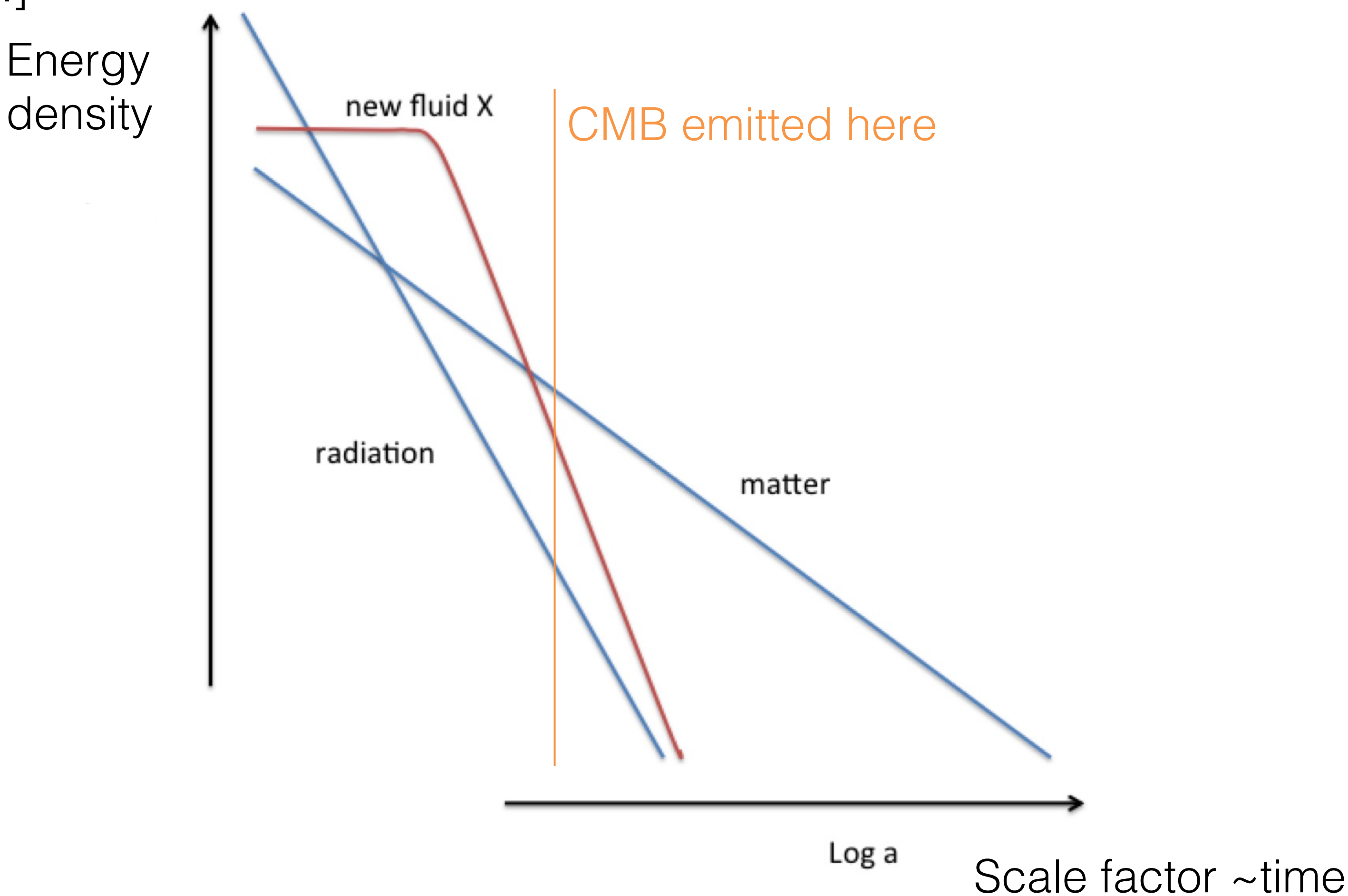
- But note: huge change!

z

Example: Early Dark Energy

[Poulin et al. 2018, ...]

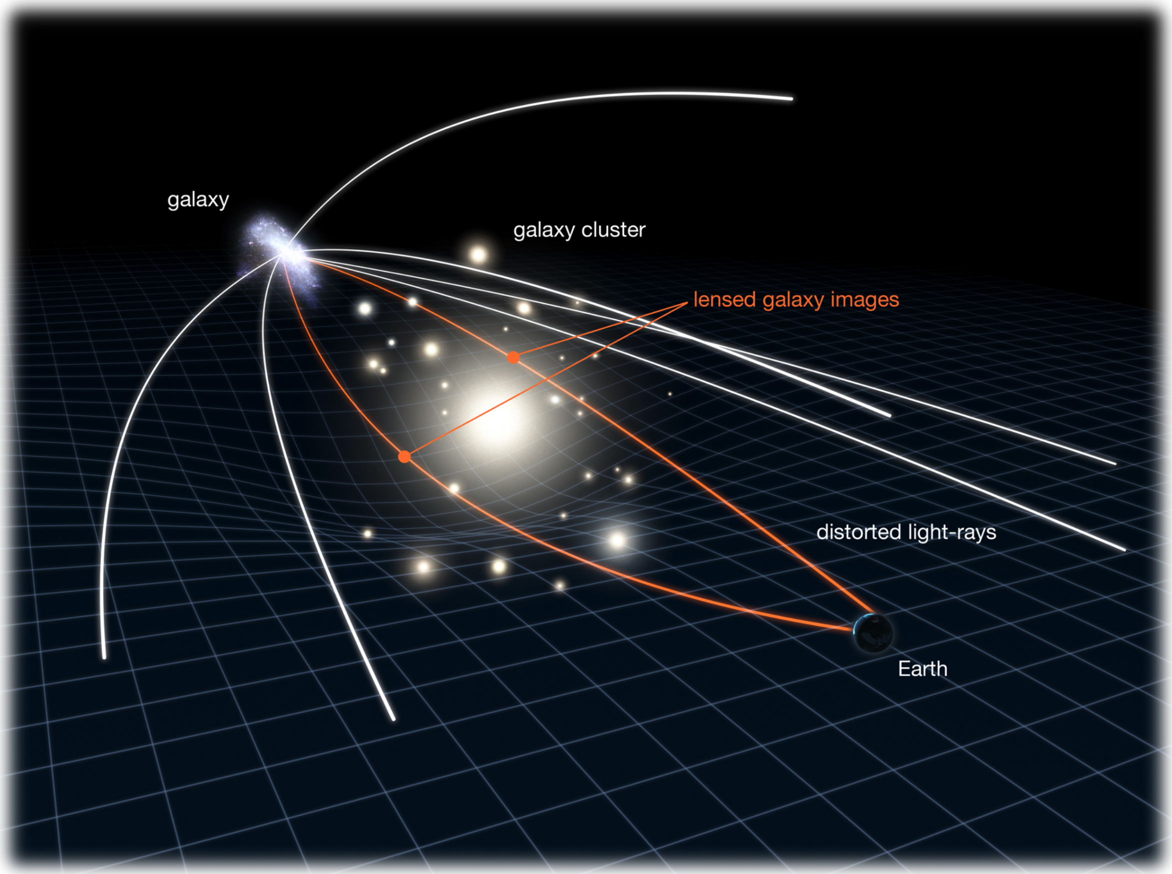
Model actually favored by ACT at ~3 sigma! But: more investigation of systematics needed [Hill++ 21]



Many models (e.g. di Valentino++ 21). Perhaps none entirely compelling? And many don't agree well with large-scale structure.

S8 Tension: Gravitational Lensing

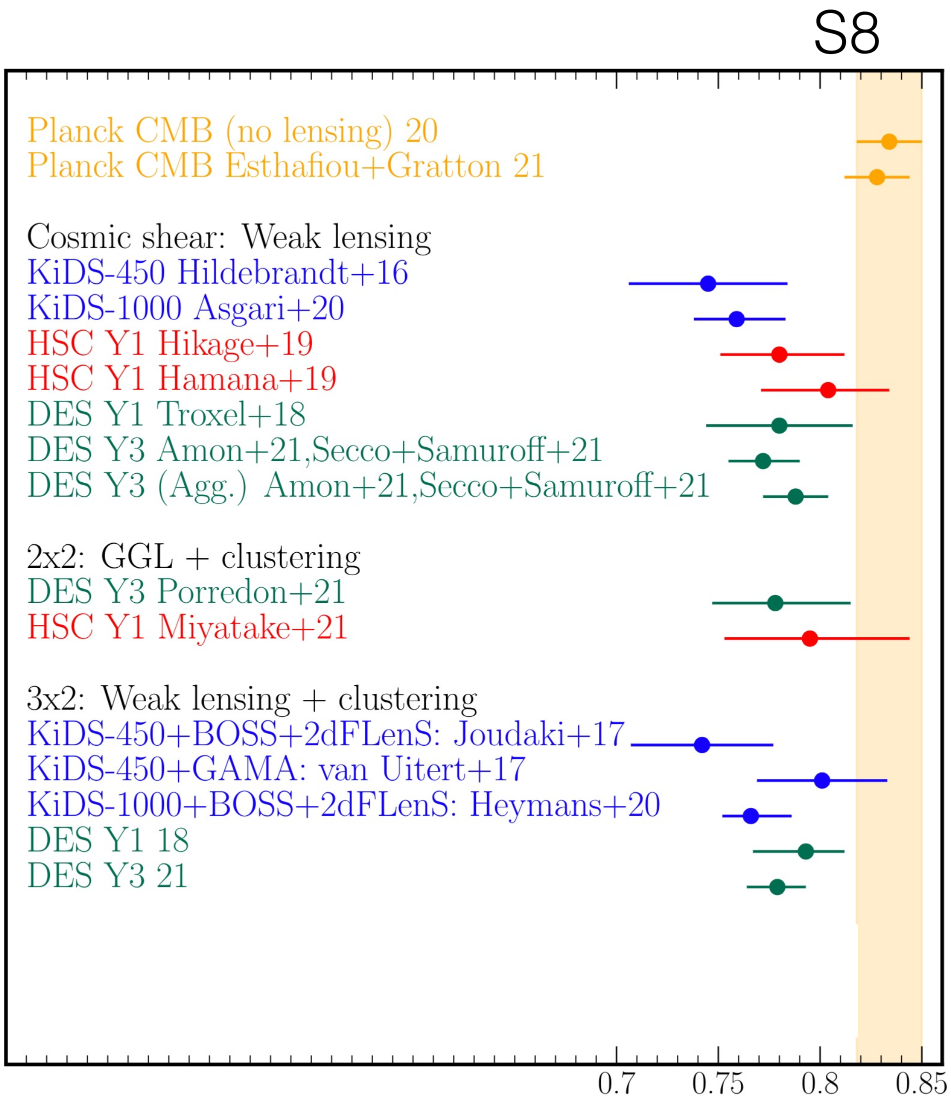
- Distribution of dark matter deflects light from galaxies. Can measure strength of lensing by warping of galaxy images



S8 tension from galaxy lensing

- Strength of lensing signal depends on how clumpy the mass distribution is (and how much matter there is). Parametrized by S8.

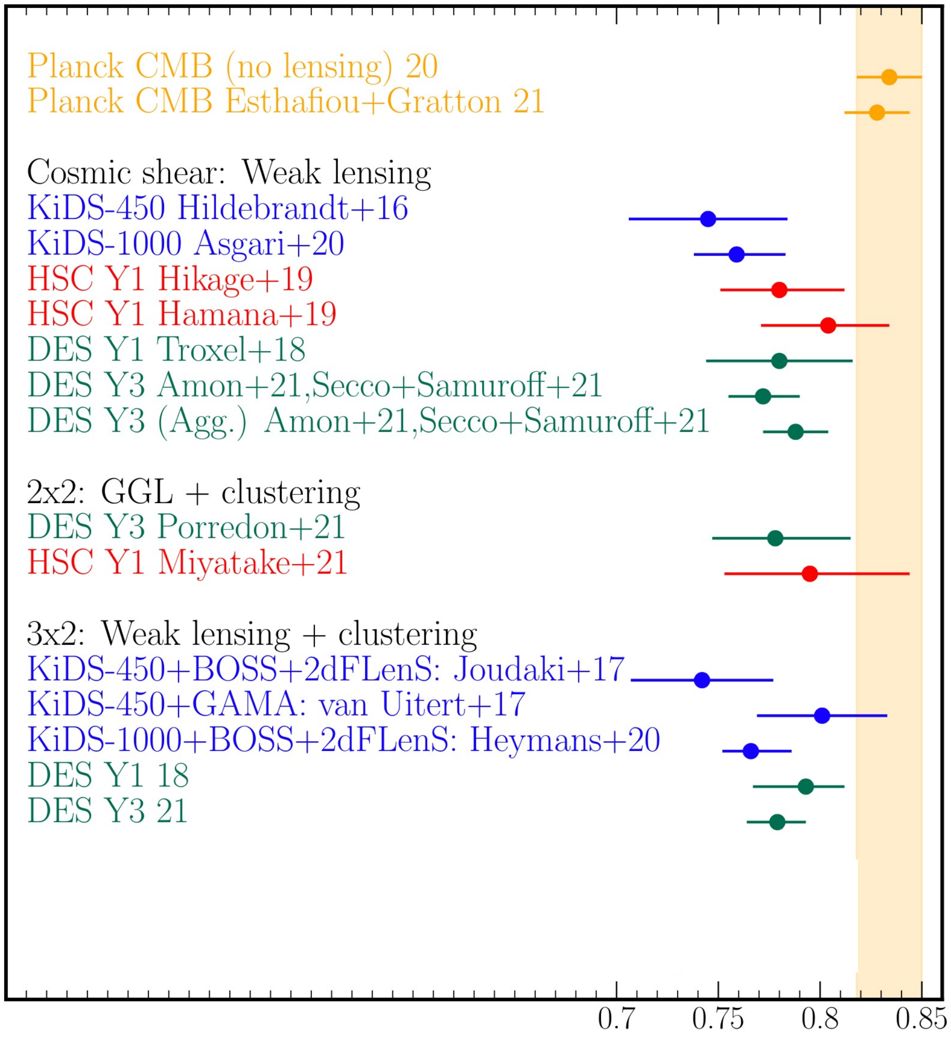
- Problem: S8 measured by lensing at low redshifts is ~2-3 sigma low compared to expectation from Planck.



In progress: test S8 tension with CMB lensing

- Galaxy lensing, while powerful, is challenging…
- Want to test further (e.g. with CMB lensing). Do we also find a low S8? [See last lecture]

S8



We have learned a LOT from the CMB about cosmic composition

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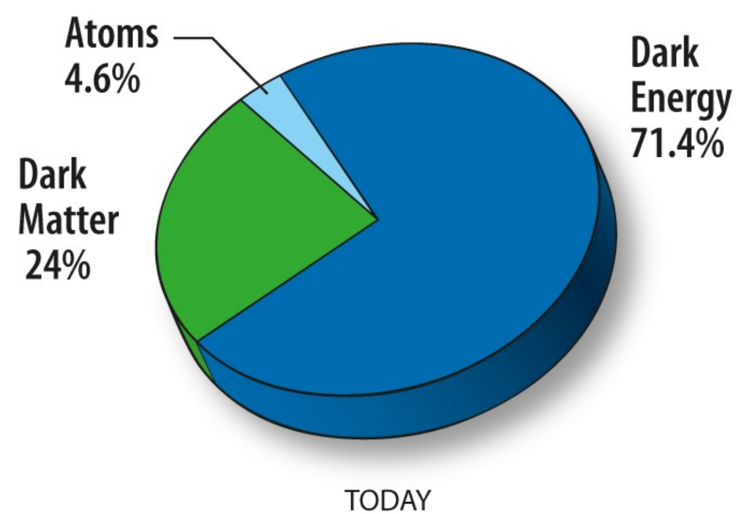

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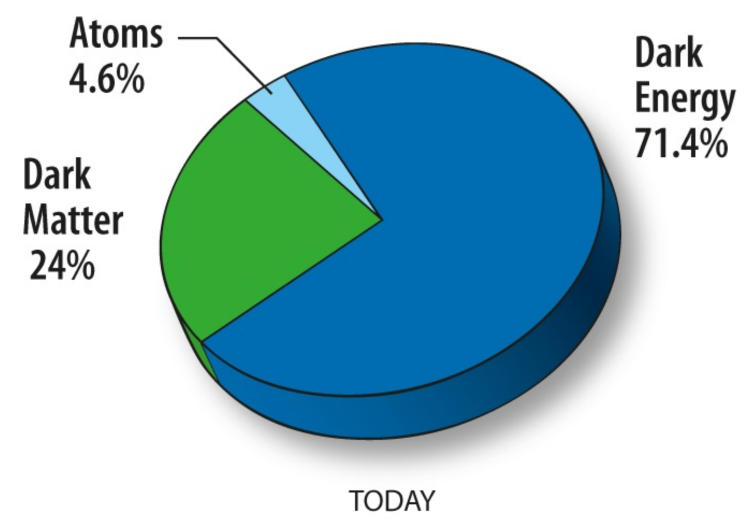

  - But lots more to come – Neff, polarization, lensing, SZ…

Next: active research areas – probing inflation with polarization; lensing