Testing the No-Hair & Area Theorems with LIGO

Saul Teukolsky
Cornell University & Caltech

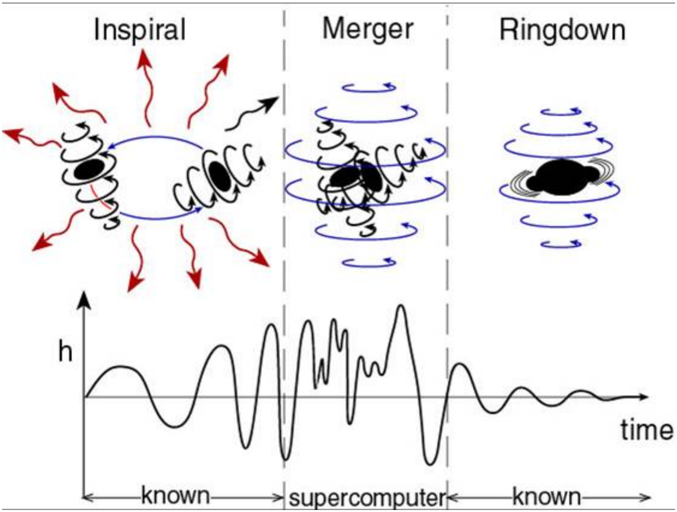
ICTP, July 12, 2022

Collaborators

- Matt Giesler (Caltech)
- Max Isi (MIT)
- Mark Scheel (Caltech)
- Will Farr (CCA/Stony Brook)

arXiv:1903.08284, 1905.00869, 2012.04486

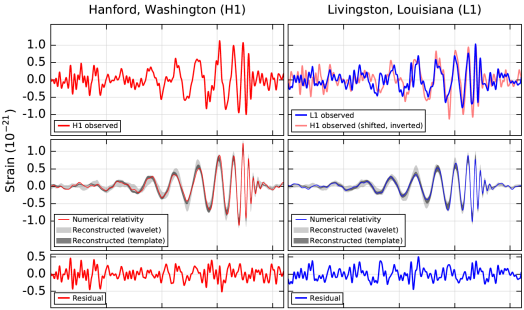
Waves from Binary Black Holes



(Figure: Kip Thorne)

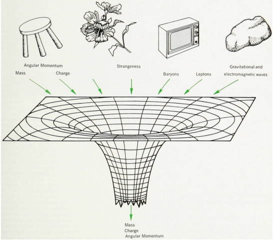
GW150914: GR Is Pretty Good!

- No PN inspiral - all NR (or models)
- Residuals $\sim$ noise. GR violations $< 4\%$
- Consistency:
  - Inspiral $\to M_1, M_2, S_1, S_2$
  - NR $\to M_f, S_f$
  - Ringdown: 1 QNM $\to M_f, S_f$



Quasi-normal modes: No Hair?

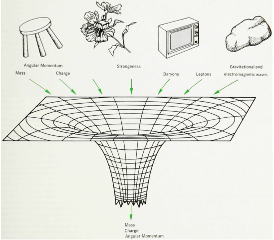
- Stationary BH described only by M and J (Kerr)
- “A black hole has no hair”
- Not necessarily true in alternative theories



(Ruffini & Wheeler 1971)

Quasi-normal modes: No Hair?

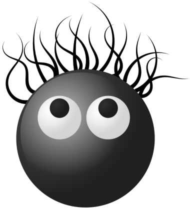
- Stationary BH described only by M and J (Kerr)
- "A black hole has no hair"
- Not necessarily true in alternative theories



(Ruffini & Wheeler 1971)

- Measure 2 least-damped QNMs
- Check M, J from ω and τ
- Low SNR: next-gen LIGO, LISA

Dreyer et al (2004)



Kerr Perturbations

$$\Psi_4 = \int d\omega\, e^{-i\omega t} \sum_{lm} e^{im\phi} S_{lm}(\theta, a\omega) R_{lm}(r, \omega)$$

$$\frac{d^2 R}{dr_*^2} + \left[\omega^2 - V(r)\right] R = 0 \qquad (a = 0)$$

$$\Psi_4 \sim \frac{d^2 h}{dt^2}$$

Late times: $$h \sim \sum C_{lmn} e^{-i\omega_{lmn}(t-r_*)} S_{lm}(\theta, a\omega_{lmn})$$

Modes: $$h_{lm} = \sum_{n=0}^{\infty} C_{lmn} e^{-i\omega_{lmn} t} \qquad (Y_{lm} \text{ vs. } S_{lm})$$

Overtones

Modes: $$h_{lm} = \sum_{n=0}^{\infty} C_{lmn} e^{-i\omega_{lmn} t}$$

$$\omega = \omega_{\mathrm{r}} + i\omega_{\mathrm{i}} = \omega_{\mathrm{r}} - i/\tau$$
$$h \sim \cos(\omega_{\mathrm{r}} t) e^{-(t/\tau)}$$

- $n =$ overtone index
- No-hair: $\omega_{lmn} = \omega_{lmn}(M_f, a_f)$
- n sorts by decreasing damping times
- Increasing $n \quad \rightarrow \quad$ lower frequency
- overtones often ignored (“subdominant”)

Ringdown Waveform Modeling

- Buoannano, Cook, Pretorius (2007): equal mass BBH
  - (2,2,0) + 3 overtones good even before peak of Ψ_4
    - $t(\Psi_{4,\mathrm{peak}}) \sim t(h_{\mathrm{peak}}) + 10M$

Ringdown Waveform Modeling

- Buoannano, Cook, Pretorius (2007): equal mass BBH
  - (2,2,0) + 3 overtones good even before peak of Ψ_4
    - $t(\Psi_{4,\text{peak}}) \sim t(h_{\text{peak}}) + 10M$
- EOB ringdown modeled with QNMs including overtones

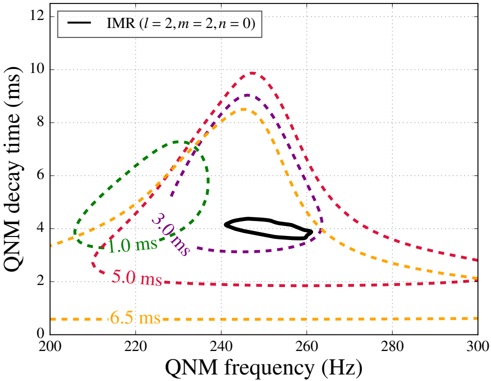
Ringdown Waveform Modeling

- Buoannano, Cook, Pretorius (2007): equal mass BBH
  - (2,2,0) + 3 overtones good even before peak of Ψ_4
    - $t(\Psi_{4,\mathrm{peak}}) \sim t(h_{\mathrm{peak}}) + 10M$
- EOB ringdown modeled with QNMs including overtones
- Matching to inspiral-merger: sometimes pseudo-QNMs

Ringdown Waveform Modeling

- Buoannano, Cook, Pretorius (2007): equal mass BBH
  - (2,2,0) + 3 overtones good even before peak of Ψ_4
    - $t(\Psi_{4,\mathrm{peak}}) \sim t(h_{\mathrm{peak}}) + 10M$
- EOB ringdown modeled with QNMs including overtones
- Matching to inspiral-merger: sometimes pseudo-QNMs
- Community: QNMs good for modeling, but h still non-linear at t_{peak}

Observing the Ringdown

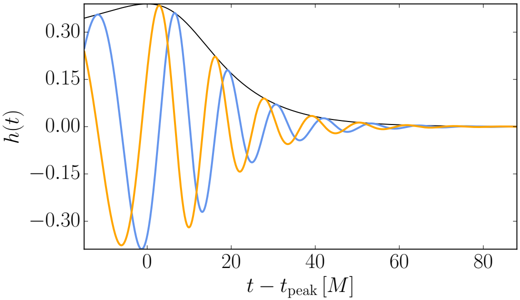


LVC 1602.03841

- IMR: NR $\rightarrow$ (M_f, χ_f) $\rightarrow$ ω_{220}
- Single damped sinusoid model
- Sensitive to start time
- Discrepancy: non-linearities?
- When does ringdown start?

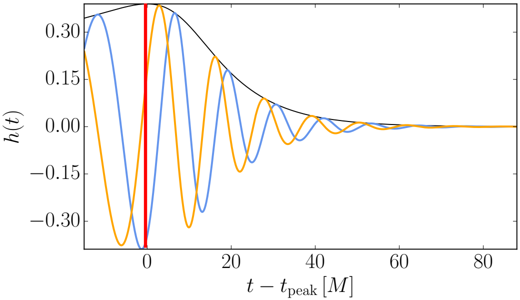
Ringdown Start Time

At what point do QNMs provide the correct description?



Ringdown Start Time

At what point do QNMs provide the correct description?

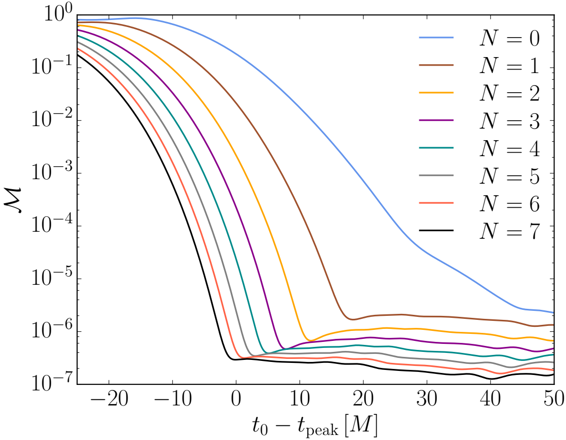


At t_{peak} (or even before) by including overtones!

(Giesler, Isi, Scheel, Teukolsky 2020)

$$h_{22} = \sum_{n=0}^{N} C_{22n} e^{-i\omega_{22n}(t-t_0)}$$

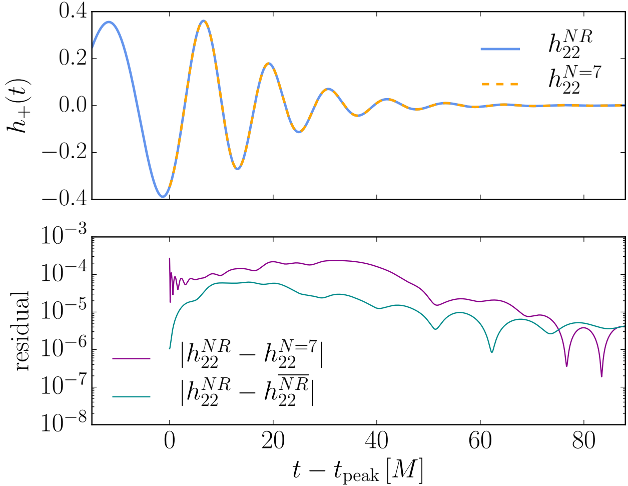
Least-squares $\rightarrow \; C_{22n}, \quad (M_f^{\mathrm{NR}}, a_f^{\mathrm{NR}}) \; \rightarrow \; \omega_{22n}$



$$\mathcal{M} = 1 - \frac{\langle h_{22}^{\mathrm{NR}}, h_{22}^{N} \rangle}{\sqrt{\langle h_{22}^{\mathrm{NR}}, h_{22}^{\mathrm{NR}} \rangle \langle h_{22}^{N}, h_{22}^{N} \rangle}} \qquad \langle x(t), y(t) \rangle = \int_{t_0}^{T} x(t)\overline{y(t)}\,dt$$

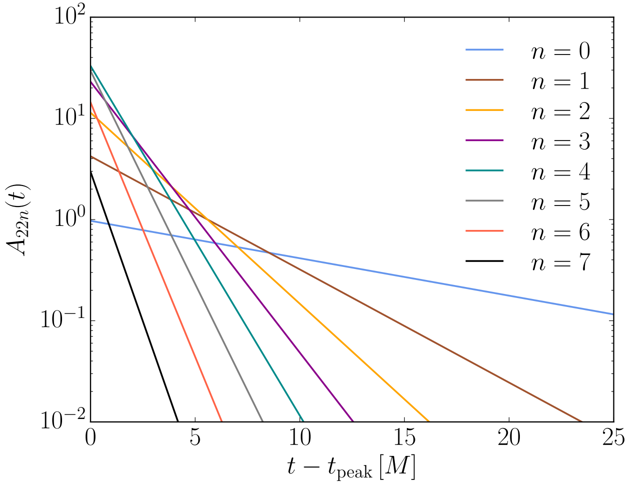
Non-Linearities are Small!

Overtones $\rightarrow$ linear description



h_{22}^{NR} = SXS:BBH:0305

Overtone Decomposition



- Fundamental not dominant until $\sim 10M$ (GW150914: ~ 3 ms)
- Early part dominated by overtones, not non-linearities!

Are We Just Overfitting?

- Damped sinusoids are overcomplete

Are We Just Overfitting?

- Damped sinusoids are overcomplete
- Completeness = red herring!

Are We Just Overfitting?

- Damped sinusoids are overcomplete
- Completeness = red herring!
  - QNMs $\rightarrow$ an asymptotic expansion (“Any” wave eqn)

Are We Just Overfitting?

- Damped sinusoids are overcomplete
- Completeness = red herring!
  - QNMs $\rightarrow$ an asymptotic expansion ("Any" wave eqn)
- Fitting $\neq$ expansion in c.o.s.

Are We Just Overfitting?

- Damped sinusoids are overcomplete
- Completeness = red herring!
  - QNMs $\rightarrow$ an asymptotic expansion (“Any” wave eqn)
- Fitting $\neq$ expansion in c.o.s.
- Instability of QNMs? (Pseudospectra . . .)

Are We Just Overfitting?

- Damped sinusoids are overcomplete
- Completeness = red herring!
  - QNMs $\rightarrow$ an asymptotic expansion ("Any" wave eqn)
- Fitting $\neq$ expansion in c.o.s.
- Instability of QNMs? (Pseudospectra ...)
  - arXiv:2111.05415 ("Elephant and Flea")

Are We Just Overfitting?

- Damped sinusoids are overcomplete
- Completeness = red herring!
  - QNMs $\rightarrow$ an asymptotic expansion (“Any” wave eqn)
- Fitting $\neq$ expansion in c.o.s.
- Instability of QNMs? (Pseudospectra . . .)
  - arXiv:2111.05415 (“Elephant and Flea”)

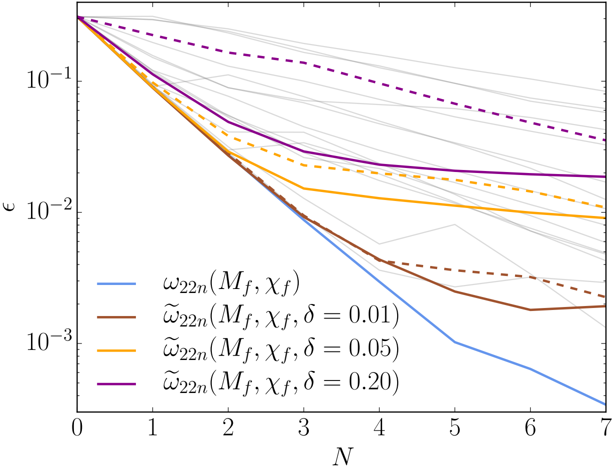
- Math is an experimental science!

Are We Just Overfitting?

- Damped sinusoids are overcomplete
- Completeness = red herring!
  - QNMs $\rightarrow$ an asymptotic expansion ("Any" wave eqn)
- Fitting $\neq$ expansion in c.o.s.
- Instability of QNMs? (Pseudospectra . . .)
  - arXiv:2111.05415 ("Elephant and Flea")
  - Observationally irrelevant (arXiv:2205.08547)
- Math is an experimental science!

Consider *small* deviations from true (ω, τ):

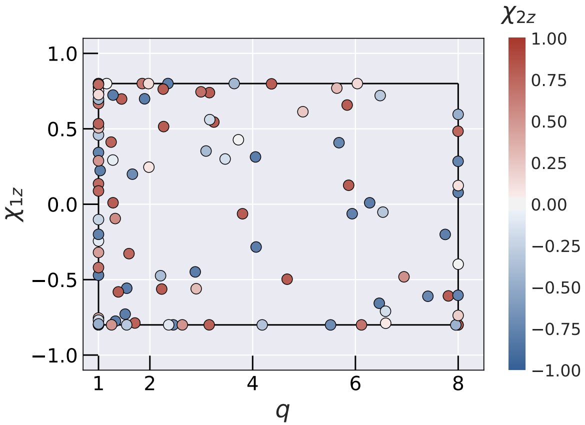
$$\widetilde{\omega}_{22n}(M_f, \chi_f) = \widetilde{\omega}_{22n}(M_f, \chi_f)(1+\delta), \quad n > 0$$



$$\epsilon^2 = (\delta M_f / M_f)^2 + (\delta \chi_f / \chi_f)^2$$

Robustness

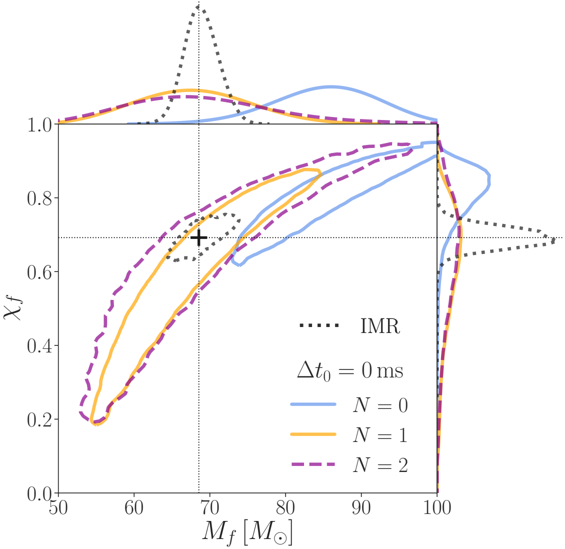
Tested on 80 waveforms:



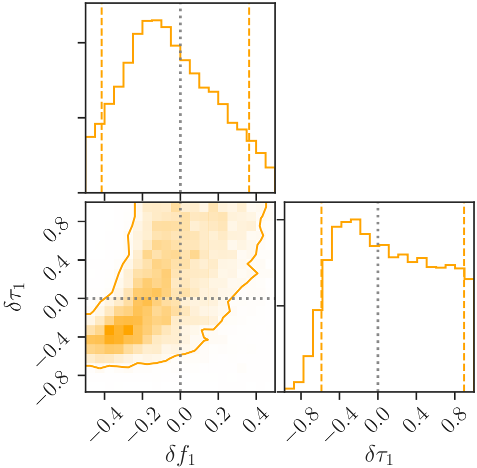
Recovered (M, χ): median $\epsilon \sim 10^{-3}$

Real Data: GW150914

Mass and spin with QNMs at $t = t_{\text{peak}}$:

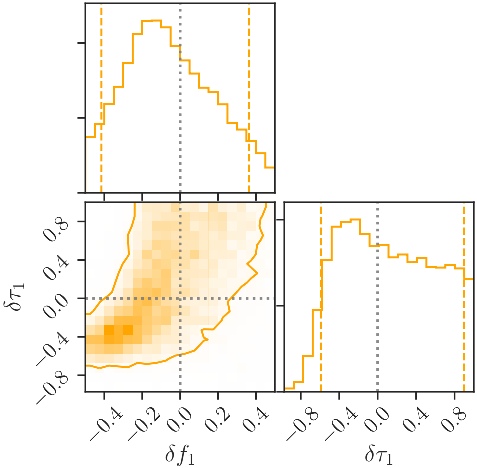


Testing the No-Hair Theorem with GW150914



- $f_{221}(M_f, \chi_f)(1 + \delta f_1)$
  $\tau_{221}(M_f, \chi_f)(1 + \delta\tau_1)$
- $\delta f_1 = 0$ to 20%
  ($\delta\tau_1$ to 100%)
- Bayes factor for no-hair vs. floating $(f, \tau) = 1.75$
- SNR ≈ 24
  ≈ 14 in ringdown
  ≈ 8 in LSC analysis

Testing the No-Hair Theorem with GW150914



- $f_{221}(M_f, \chi_f)(1 + \delta f_1)$
  $\tau_{221}(M_f, \chi_f)(1 + \delta\tau_1)$
- $\delta f_1 = 0$ to 20%
  ($\delta\tau_1$ to 100%)
- Bayes factor for no-hair vs. floating $(f, \tau) = 1.75$
- SNR ≈ 24
  ≈ 14 in ringdown
  ≈ 8 in LSC analysis

Cotesta et al arXiv:2201.00822 is wrong (Isi & Farr, Finch & Moore arXiv:2205.07809)

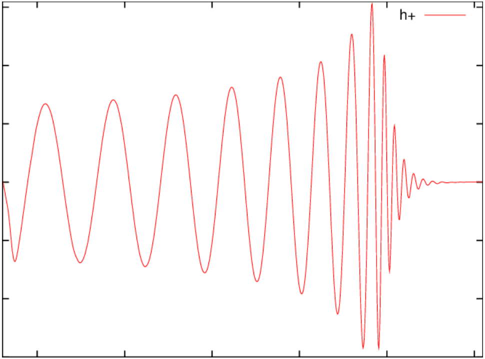
The Area Theorem

- Total horizon area of BHs cannot decrease

$$A = 8\pi M^2(1 + \sqrt{1-\chi^2}), \qquad \chi = J/M^2$$

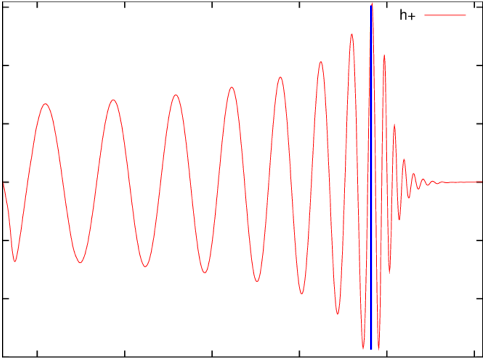
- Get M's and χ's for initial and final BHs:
  Split h at t_{peak}
  Analyze inspiral and ringdown *separately*
- Premerger: Use NRSur7dq4 templates (PN as $t \to -\infty$)
  Postmerger: Fit overtone model

Trickiness



- P.E.: Match computed in freq space
- Gibbs $\rightarrow$ taper in t, then use f $\rightarrow$ mixing or loss of SN
- Compute in t-domain!
  - Covariance = Toeplitz matrix, $\mathcal{O}(N^2)$ inversion

Trickiness



- P.E.: Match computed in freq space
- Gibbs $\rightarrow$ taper in t, then use f $\rightarrow$ mixing or loss of SN
- Compute in t-domain!
    - Covariance = Toeplitz matrix, $\mathcal{O}(N^2)$ inversion

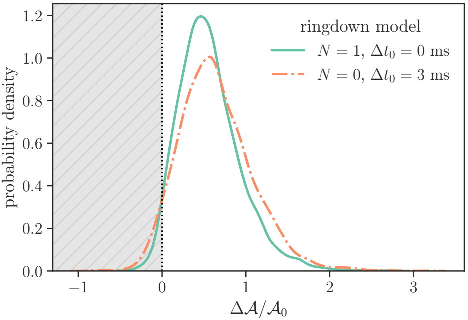
Results for GW150914

- Exclude $\Delta A < 0$ with

  97% prob $(N = 1)$

  95% prob $(N = 0)$

- Area th OK to $\gtrsim 2\sigma$



Summary

- Ringdown begins at peak strain (maybe earlier)
- Overtones dominate early ringdown
- Non-linearities in the ringdown surprisingly small

Summary

- Ringdown begins at peak strain (maybe earlier)
- Overtones dominate early ringdown
- Non-linearities in the ringdown are seemingly surprisingly small

Summary

- Ringdown begins at peak strain (maybe earlier)
- Overtones dominate early ringdown
- Non-linearities in the ringdown are seemingly surprisingly small
- Overtones enable a first test of the no-hair theorem

Summary

- Ringdown begins at peak strain (maybe earlier)
- Overtones dominate early ringdown
- Non-linearities in the ringdown are seemingly surprisingly small
- Overtones enable a first test of the no-hair theorem
- Similarly, first test of area theorem

Fitting a Sequence of Basis Functions

- Consider $y = 1 + x + x^2$ on $[-1, 1]$

Fitting a Sequence of Basis Functions

- Consider $y = 1 + x + x^2$ on $[-1, 1]$
- Basis $\{1, x, x^2\}$

Fitting a Sequence of Basis Functions

- Consider $y = 1 + x + x^2$ on $[-1, 1]$
- Basis $\{1, x, x^2\}$
- Fit 1
  Coefficient?

Fitting a Sequence of Basis Functions

- Consider $y = 1 + x + x^2$ on $[-1, 1]$
- Basis $\{1, x, x^2\}$
- Fit 1
  Coefficient?
- $4/3$

Fitting a Sequence of Basis Functions

- Consider $y = 1 + x + x^2$ on $[-1, 1]$
- Basis $\{1, x, x^2\}$
- Fit 1
  Coefficient?
- $4/3$
- Fit $\{1, x\}$, stays $4/3$

Fitting a Sequence of Basis Functions

- Consider $y = 1 + x + x^2$ on $[-1, 1]$
- Basis $\{1, x, x^2\}$
- Fit 1
  Coefficient?
- $4/3$
- Fit $\{1, x\}$, stays $4/3$
- Fit $\{1, x, x^2\}$. Changes to 1